Pseudospectrum and (In)stability of Black Hole Total Transmission Modes

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What Are TTMs?

$$\left(\partial_t^2 - \partial_x^2 + V\right)\Psi = 0\,, \tag{1}$$

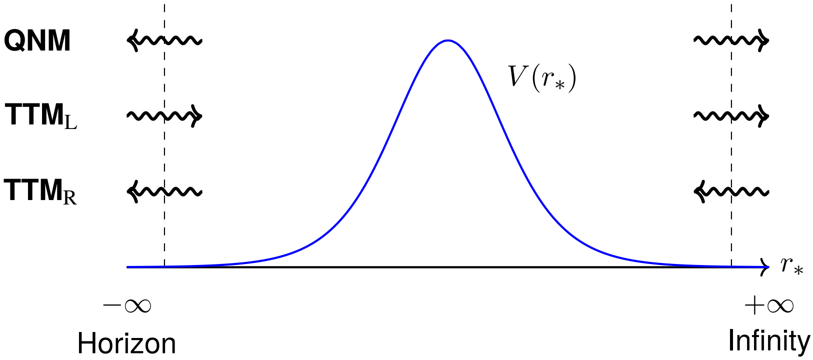
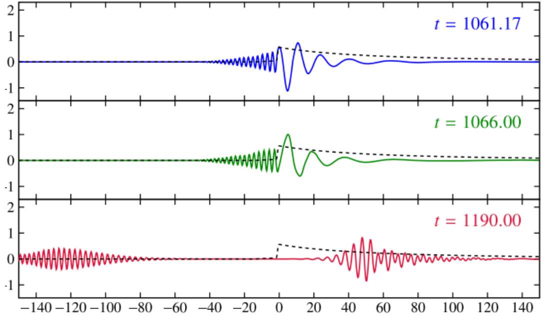


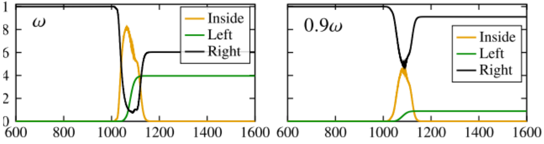
Fig. 1: The definition of TTM.

Why TTM?

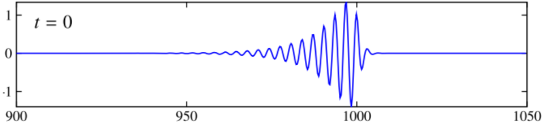
- TTMs' role was unclear: they are complex-frequency modes.
- Recently, Cardoso et. al.[1]: tailored scattering can excite them.
- An active, complementary to passive way to probe the black-hole spacetime.



wave evolution



energy distribution



initial data

[1] F. Tuncer, V. Cardoso, R. Panosso Macedo, and T. F. M. Spieksma, Phys. Rev. D 113, 104028 (2026); arXiv:2509.19451.

Background: Tangherlini Black Hole

- Generalization of Schwarzschild black holes.
- Static, shperically symmetric.

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{d-2}\,, \qquad f(r) = 1 - \left(\frac{r_\mathrm{h}}{r}\right)^{d-3}\,.$$

$$t_\pm = \frac{1}{r_h}\left(t \pm r_*\right)\,, \qquad \sigma = \frac{r_h}{r}\,,$$

$$L_1\psi = \pm i\omega_\mp r_h L_2\psi\,,$$

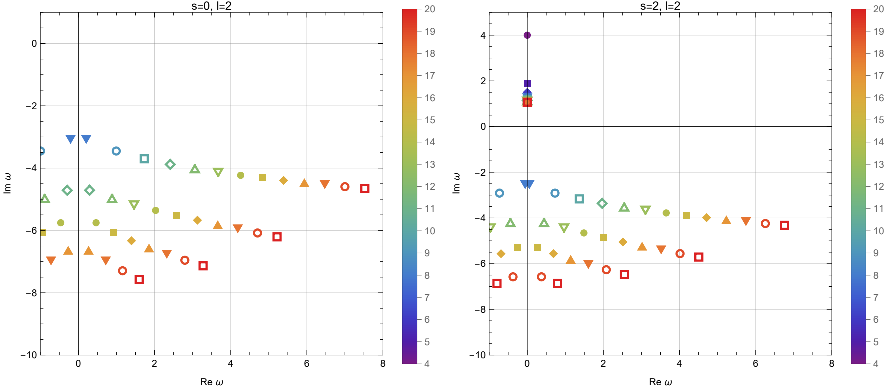
with

$$L_1 = \frac{\mathrm{d}}{\mathrm{d}\sigma}\left(p\frac{\mathrm{d}}{\mathrm{d}\sigma}\right) - \frac{r_h^2 V}{p}\,, \qquad L_2 = 2\frac{\mathrm{d}}{\mathrm{d}\sigma}\,, \qquad p(\sigma) = \sigma^2 f\big(r(\sigma)\big)\,,$$

with $+$ for $\mathrm{TTM_R}$ and $-$ for $\mathrm{TTM_L}$; we focus on $\mathrm{TTM_L}$,
and $s = 0$: massless scalar or gravitational tensor perturbations,
$s = 2$: gravitational vector perturbations("Regge-Wheeler"-like sector).

Numerical Results

Sector	pure imag.	complex
$s=0$	no	$d\geq 8$
$s=2$	yes	$d\geq 8$



The spectral (in)stability

- TTMs: An **open, non-Hermitian** system, similar to QNM.
- How robust are TTMs under small perturbations?

Condition Number

For a generalized eigenvalue problem $Ax = \lambda Bx$:

$$\kappa(\lambda) = \frac{\|y\|\|x\|}{|\langle y, Bx\rangle|}.$$

ambiguities:

- κ can drop below 1, and the value of κ of a single mode tells us nothing.
- A global rescaling:

$$\begin{aligned} \mathbf{Ax} = \lambda\mathbf{Bx} \quad &\longrightarrow \quad (a\mathbf{A})\mathbf{x} = \lambda(a\mathbf{B})\mathbf{x}\,, \\ \kappa \quad &\longrightarrow \quad \kappa/a\,. \end{aligned}$$

  fixed by

$$L_2 = 2\frac{\mathrm{d}}{\mathrm{d}\sigma}\,.$$

- which norm to choose?
  energy norm and 2 norm.

Condition Number Under Energy Norm

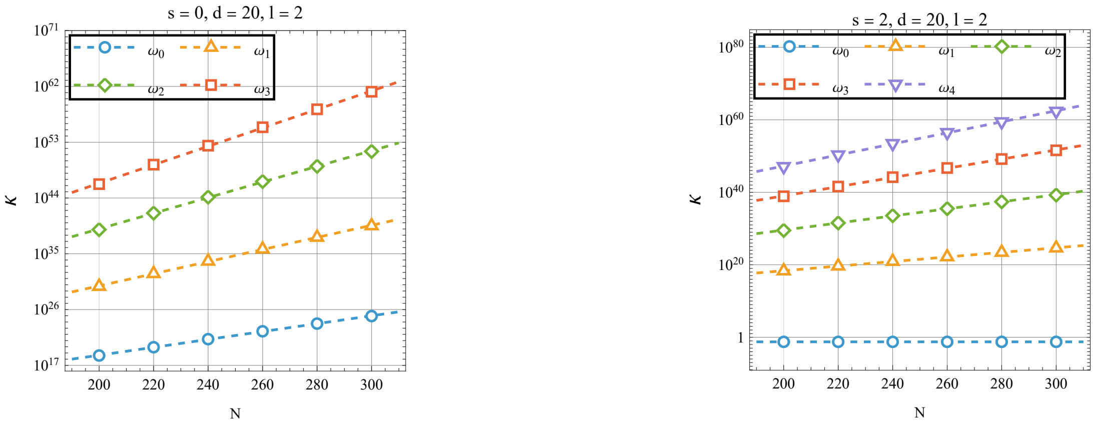


Fig. 3: Condition numbers

Condition Number of the Pure Imaginary Mode

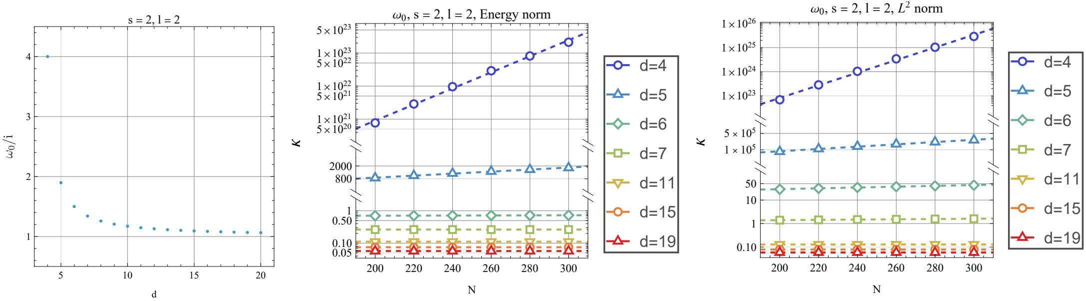


Fig. 4: Pure imaginary modes.

Pseudospectrum

$$\sigma_\epsilon(A,B) = \{\lambda \in \mathbb{C} : \exists\, \delta A,\ \|\delta A\| < \epsilon,\ \text{such that } \lambda \in \sigma(A+\delta A, B)\}\,.$$

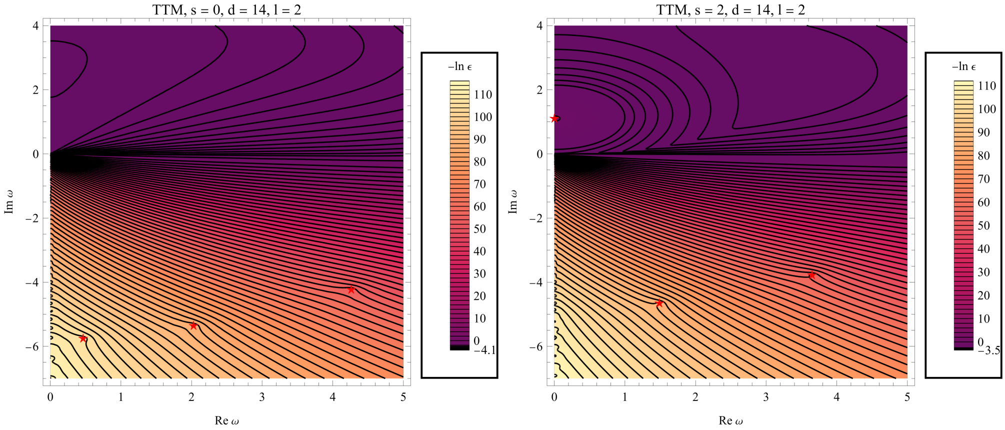


Fig. 5: Pseudospectrum

Pseudospectrum

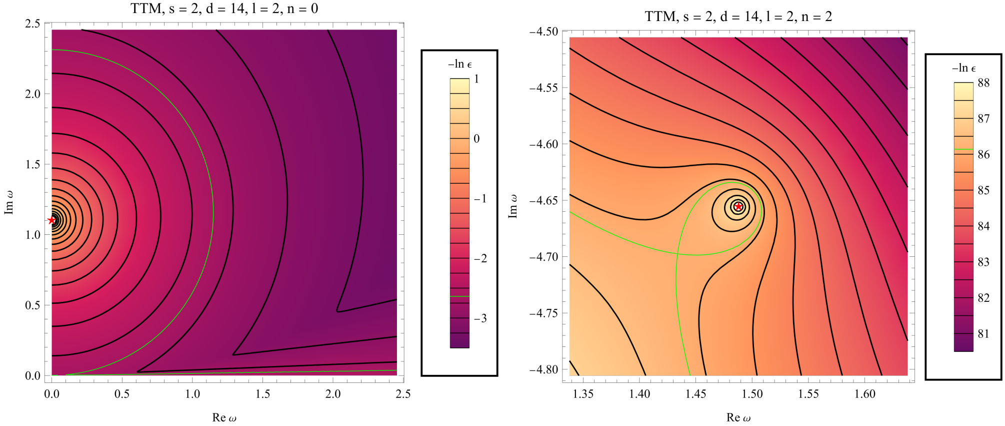


Fig. 6: Zoom in of pseudospectrum

Conclusions

- Purely imaginary TTMs appear only in the $s = 2$ sector.
- Generic complex TTMs appear from $d = 8$.
- A notable **exception** is the higher-dimensional, purely imaginary $s = 2$ mode, which appears stable.
- TTMs: generically **spectrally unstable**, much like QNMs.

Open Questions

- Rotating case: subtle norm choice.[2]
- Other possible coordinate systems.
- Why generic complex TTMs appear from $d = 8$.
- Why modes with positive imaginary part appears exactly on the imaginary axis?[3]

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[2] Cai, Cao, Chen, Guo, Wu, Zhou, PRD 111, 084011 (2025).

[3] Konoplya, Zhidenko, PRD 95, 104005 (2017); Ma, Mai, Yang, arXiv:2606.02066 (2026).

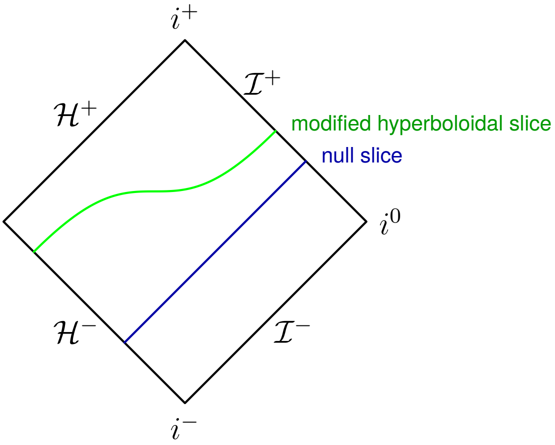

Fig. 7: Modified hyperboloidal slice (green) and null slice (blue).

Thank You

Questions and comments are welcome.