On the Fate of Inflaton after Inflation:
Fragmentation, Oscillon Formation & Decay

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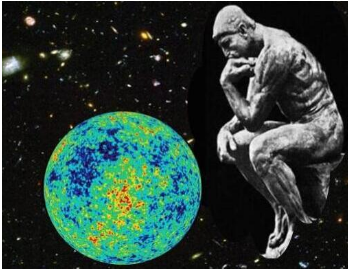
Initial Conditions for Cosmological Structure

LSS $\longleftarrow$ **CMB** $\longleftarrow$ **Initial** $\boldsymbol{\zeta}(\vec{x})$ (gravitational instability)

Properties of (initial) primordial fluctuations:



[The Thinker by Auguster Rodin (Edited by Siddharth Bhatt)]

1. **Adiabatic** & **Super-Hubble** $\zeta(\vec{x})$
2. Almost **scale-invariant**

$$\boxed{\mathcal{P}_\zeta = A_S\left(\frac{k}{k_*}\right)^{n_S-1}} \quad k_* = 0.05\,\mathrm{Mpc}^{-1}$$

$$A_S \ll 1\,, \;\; |n_s - 1| \ll 1$$

3. Nearly **Gaussian** (Variance: $\sigma \simeq 10^{-4}$)

$$\boxed{\mathbf{P}[\zeta] = \mathcal{B}\exp\left[\frac{-\zeta^2}{2\sigma^2}\left(1 + \cancelto{?}{f_{\mathrm{NL}}}\,\zeta + \ldots\right)\right]}$$

$\rightarrow$ **LSS, CMB** $\Rightarrow$ **Large-scale primordial fluctuations**

$\rightarrow$ **Origin** $\Rightarrow$ **Quantum fluctuations during Inflation** ($T \gg 1\,\mathrm{MeV}$)

Inflationary Framework: Single Scalar Field

$$\boxed{\textbf{System} = \textbf{Gravity GR } (g_{\mu\nu}) + \textbf{Scalar Field } (\phi)}$$

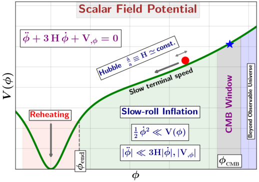
$$S[g_{\mu\nu},\phi] = \int d^4x\,\sqrt{-g}\left(\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\phi\,\partial^\nu\phi - V(\phi) + \ldots\right)$$

1. **Background Dynamics:**

Quasi-Exponential expansion $\ddot{a} > 0$

$$a_{\rm end} \simeq a_i\, e^{\Delta N}, \qquad \Delta N \gtrsim 60$$

$\Rightarrow$ **Isotropic, uniform & flat Universe**



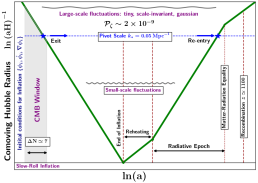
[Typical Inflaton Potential]

2. **Small (linear) Perturbations:**

Quantum fluctuations in qdS spacetime

$\Rightarrow$ **Super-Hubble Correlations**

Origin of CMB, LSS in the Universe !



[Comoving Causal Diagram]

Inflationary Predictions for Linear Perturbations

$$\boxed{\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\left(e^{-2\Psi(t,\vec{x})}\,\delta_{ij} + h_{ij}(t,\vec{x})\right)\mathrm{d}x^i\mathrm{d}x^j\right]}$$

Two types of inflationary fluctuations ($m \ll H$):

1) **Comoving Scalar Perturbations:** $-\zeta(t,\vec{x}) = \Psi + \frac{1}{\sqrt{2\epsilon_1}}\frac{\delta\phi}{m_p}$

2) **Transverse & traceless Tensor Perturbations:** $h_{ij}(t,\vec{x})$

Slow-roll regime: (**'Slow terminal motion' of ϕ**) $\epsilon_1,\ |\epsilon_2| \ll 1$

Scalar power spectrum

$$\boxed{\mathcal{P}_\zeta(k) = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2\frac{1}{\epsilon_1} = A_S\left(\frac{k}{k_*}\right)^{n_S-1}}$$

Tensor power spectrum

$$\boxed{\mathcal{P}_\mathcal{T}(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 = A_\mathcal{T}\left(\frac{k}{k_*}\right)^{n_\mathcal{T}}}$$

$$\boxed{|n_S - 1|,\, |n_\mathcal{T}|,\, r \ll 1}$$

Scalar spectral index

$$\boxed{n_S - 1 = -2\,\epsilon_1 - \epsilon_2}$$

Tensor spectral index

$$\boxed{n_\mathcal{T} = -2\,\epsilon_1}$$

Tensor-to-scalar Ratio

$$\boxed{r = A_\mathcal{T}/A_S = 16\,\epsilon_1}$$

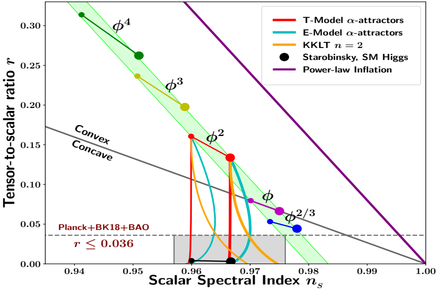
$\Rightarrow$ **Tiny & nearly Scale – invariant Fluctuations**

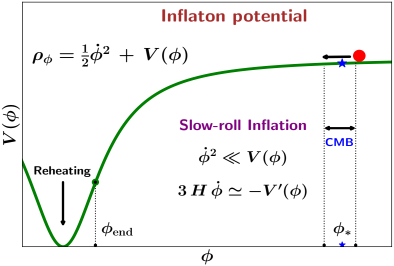
Observational Constraints(Planck ACT SPT+BICEP/Keck)

$$A_s = 2.1 \times 10^{-9}\ ; \quad n_s \simeq 0.969\ ; \quad r \leq 0.036$$

$$\boxed{\Rightarrow \quad H^{\rm inf} \leq 10^{13}\,{\rm GeV} \quad \Rightarrow \quad E^{\rm inf} \leq 10^{16}\,{\rm GeV}}\ ; \ \epsilon_1 \lesssim -\epsilon_2$$

(Hierarchy)





<u>Latest CMB Data $\Rightarrow$ On CMB scales :</u>

Single-field slow-roll paradigm of Inflation &

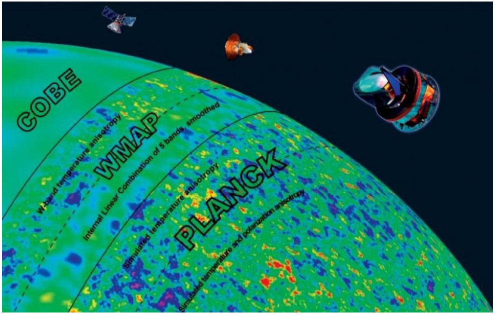
Concave, *e.g.* asymptotically-flat potentials!

**Planck(2018); **BICEP/Keck(2021); **SSM & Sahni(2022), **Bhatt, SSM *et.al.*(2022)

Graceful-Exit from Inflation → hot Big Bang

Cosmic Microwave Background (CMB) Radiation : TT , TE & EE

+ **BBN** + **LSS** + **SN 1a**



→ **Origin of the fluctuations in the plasma** (✓)

Inflationary quantum fluctuations

→ **Origin of the constituents of the plasma (hot Big Bang) ?**

Inflaton Decay & Reheating

What happened to other fields during inflation?

- Observations favour '**single-field slow-roll**' inflation.
- '**Cold inflationary paradigm**:' $\Rightarrow\ \rho_\chi\,,\ \rho_\psi \ll \rho_\varphi$

  & **Negligible coupling to external fields** $\boldsymbol{g^2, h \ll 1}$

$$\boldsymbol{S[\varphi,\chi,\psi]} = -\int \mathrm{d}^4x\sqrt{-g}\Big[\ \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi + V(\varphi) + \cancelto{0}{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi} + \cancelto{0}{\frac{1}{2}m_{0\chi}^2\chi^2}$$

$$+\ \cancelto{0}{\bar{\psi}\,(i\gamma^\mu\partial_\mu + m_{0\psi})\,\psi}$$

$$+\ \frac{1}{2}\cancelto{0}{\boldsymbol{g^2\,\varphi^2\,\chi^2}} + \cancelto{0}{\boldsymbol{h\,\psi\bar{\psi}\varphi}} + \ldots\Big]$$

$\Rightarrow$ **particle production during inflation can be neglected.**

- **Effects of the small coupling ?**
  1. **Primordial Non-Gaussianity**: inflaton interactions.
  2. Decay of the inflaton field: **Reheating the universe**.

General Reheating Dynamics

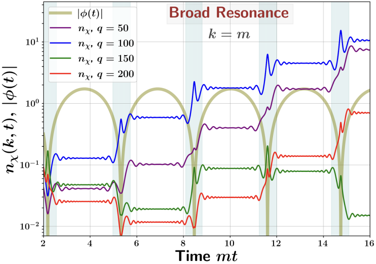
1) **Non-perturbative inflaton decay:** (Early stages)

$\mathcal{L}_{\rm int} \supset -\frac{1}{2} g^2 \varphi^2 \chi^2$ for $g^2 \geq 4 \left(\frac{m}{\phi_0}\right)^2$

$\Rightarrow$ **Broad-band resonance** for

$\Rightarrow$ **Explosive growth** $\boxed{n_\chi(k) \propto e^{\mu_k t}}$

$m \simeq 10^{-5}\, m_p\,,\ \phi_0 \simeq 0.2\, m_p \Rightarrow \boxed{g^2 \geq 10^{-8}}$



2) **Perturbative inflaton decay:**

$$\varphi\varphi \longrightarrow \chi\chi \ \text{ for } \ g^2 < 10^{-8}\,; \quad \varphi \longrightarrow \bar{\psi}\psi \ \text{ for } \ h \lesssim 10^{-2}$$

3) **Coherent Oscillations:** $(m \gg H)$

For $h,\, g \sim 0$, the **inflaton condensate oscillates for a long time**

$$\boxed{\phi(t) = \phi_0(t)\, \cos(mt)\,; \quad \langle w_\phi \rangle \simeq 0 \ \Rightarrow \ \rho_\phi \propto a^{-3}}$$

$\Rightarrow$ universe remains **condensate-dominated** × (**Not Correct!**)

We have ignored **two important effects**:

→ **Gravitational clustering:** **(Very late times)**

Metric fluctuations are important on time scales $t \sim \mathcal{O}\left(\frac{1}{H}\right) \gg m^{-1}$

$$\boxed{\mathrm{d}s^2 = -e^{-2\boldsymbol{\Psi}(t,\vec{x})}\mathrm{d}t^2 \; + \; a^2(t) \, \left[\left(e^{2\boldsymbol{\Psi}(t,\vec{x})} \, \delta_{ij} + h_{ij}(t,\vec{x}) \right) \, \mathrm{d}x^i \mathrm{d}x^j \, \right]}$$

→ **Asymptotically flat potentials:**

(CMB observations)

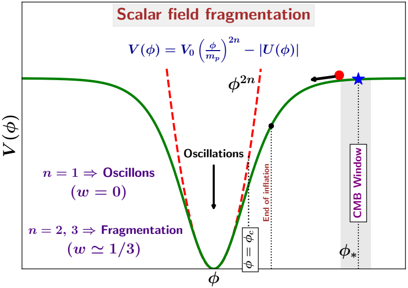
$$\boldsymbol{V(\phi) = V_0 \left(\frac{\phi}{m_p}\right)^2 - |V_{\mathrm{nl}}(\phi)|}$$

have '**attractive self-interaction**'

⇒ **Scalar field fragmentation**

⇒ **Cosmological Solitons :**

'**Oscillons**'



**Many important Work 2010-2020

(e.g. Richard Easther's Work)

Existence of quasi-Solitons: Oscillons

→ **Self-supported, localised, long-lived non-linear 'solitary' configurations**.

→ **Solitons are ubiquitous in nature**!

(**1834 J. S. Russell: solitary wave in a canal in Edinburgh**)

(Appearing in fluids, smoke rings, condensed matter physics, optics, HEP, topological defects and Cosmology.)

→ **Oscillons are oscillating non-topological pseudo-solitons**!

→ **Analytical results** based on **small-amplitude oscillations**

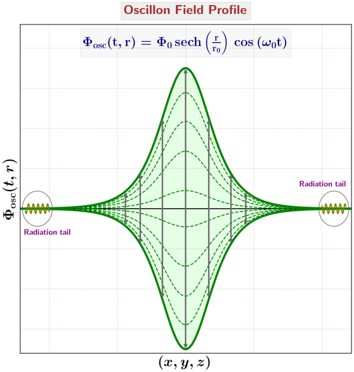
$$\boldsymbol{V}(\varphi) \approx \frac{1}{2}\, m^2 \boldsymbol{\varphi^2} - \frac{\lambda}{4}\, \boldsymbol{\varphi^4} + \frac{\mu}{6}\, \boldsymbol{\varphi^6}$$

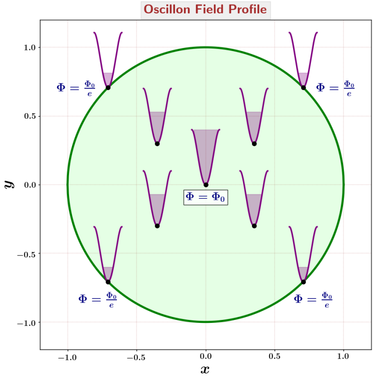
→ Supports **Oscillon-like solution** of the form

$$\boldsymbol{\varphi_{\rm osc}(t, r) \approx \Phi_{\rm osc}(r) \cos(\omega_0\, t) + ...}\,; \qquad \boxed{\boldsymbol{\Phi_{\rm osc}(r) \approx \Phi_0\, \mathrm{sech}\left(\frac{r}{r_0}\right)}}$$

****Rajaraman(1987), **Gleiser** *et. al*; ****Amin** *et. al* ; ****Mahbub, SSM (2023)**

Oscillon Solution $\boxed{\phi_{\rm osc}(t,r) \approx \Phi_{\rm osc}(r)\cos(\omega_0 t)}$





Adiabatic Invariant (approx. Conserved Charge)

$$\mathcal{I}(\omega_0) = \int \mathrm{d}^3\vec{x} \oint \pi_\phi(t,\vec{x})\,\mathrm{d}\varphi_{\rm osc}(t,\vec{x})$$

Number of Inflatons $\quad \frac{\mathcal{I}}{2\pi} \equiv \boxed{\mathcal{N}(w_0) = \frac{\omega_0}{2}\int \mathrm{d}^3\vec{x}\;\Phi^2_{\rm osc}(r)}$

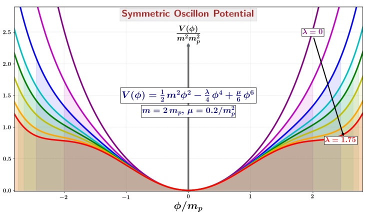
Existance of Oscillons: Known & Unknown

→ **Oscillons exist as analytic (stationary) solutions**

(of real scalar field theories with attractive potentials)

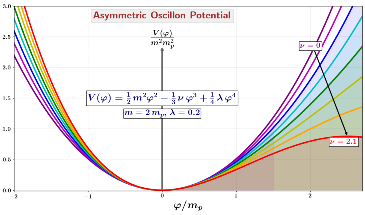
(a) For symmetric potentials:

$$V(\phi) \approx \frac{1}{2}\, m^2 \boldsymbol{\phi}^2 - \frac{\lambda}{4}\, \boldsymbol{\phi}^4 + \frac{g}{6}\, \boldsymbol{\phi}^6$$



(b) For asymmetric potentials?

$$V(\phi) \approx \frac{1}{2}\, m^2 \boldsymbol{\phi}^2 - \frac{\mu}{3}\, \boldsymbol{\phi}^3 + \frac{1}{4}\, \lambda\, \boldsymbol{\phi}^4$$



→ **Can they form dynamically ?**

(starting from **natural conditions at the end of inflation**)

*Copeland *et. al*(1995); *Amin *et. al*(2011); *Mahbub, **SSM**(2023); *Kim, McDonald

Self-resonance & Inflaton Fragmentation

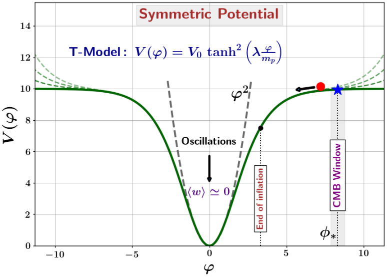
Particle Production in an Oscillating $\phi(t)$ Background

Inflaton fluctuations $$\boxed{\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi)\right]\delta\varphi_k = 0}$$

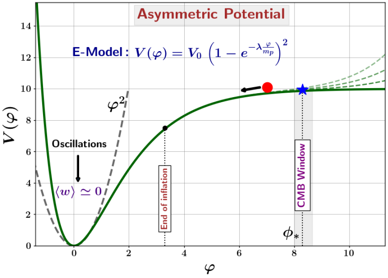
Parametric Oscillator $\Rightarrow$ Exponential growth $\delta\varphi_k(t) \propto e^{\mu_k m t}$

Band structure **similar to Mathieu resonance** for quadratic case

$$V(\phi) = \frac{1}{2}m^2\phi^2 - |U(\phi)|$$



Symmetric



Asymmetric

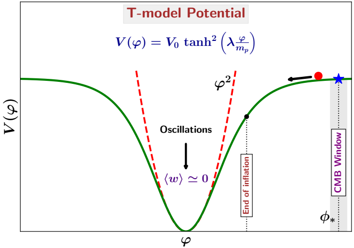
Self-resonance in α-attractor Pottentials

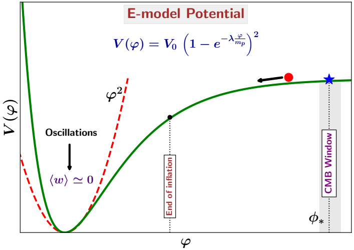
$$\boxed{V_{\rm T}(\phi) = V_0\, \tanh^2\left(\lambda_{\rm T}\,\frac{\phi}{m_p}\right)}$$

$$\boxed{V_{\rm E}(\phi) = V_0\left(1 - e^{-\lambda_{\rm E}\,\frac{\phi}{m_p}}\right)^2}$$

$$V_{\rm T}(\phi) \simeq \frac{1}{2}\,m^2\,\phi^2 - \left(\frac{\lambda_{\rm T}^2}{3}\,\frac{m^2}{m_p^2}\right)\phi^4 + \mathcal{O}\left(\phi^6\right)$$

$$V_{\rm E}(\phi) \simeq \frac{1}{2}\,m^2\,\phi^2 - \left(\frac{\lambda_{\rm E}}{2}\,\frac{m^2}{m_p}\right)\phi^3 + \mathcal{O}\left(\phi^4\right)$$





Parametric Oscillator

$$\boxed{\delta\ddot{\varphi}_k + 3\,H\,\delta\dot{\varphi}_k + \Omega_k^2\,\delta\varphi_k = 0}$$

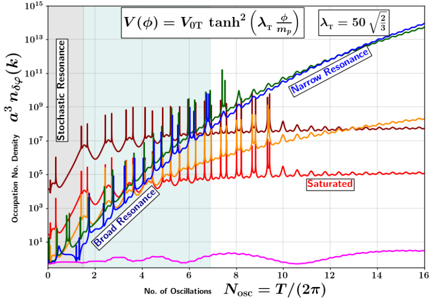
$$\Omega_k^2 = \frac{k^2}{a^2} + m^2\left[1 \pm (\#)\,\phi_e\,\cos(mt) \pm (\#)\,\phi_e^2\,\cos^2(mt) + \ldots\right]$$

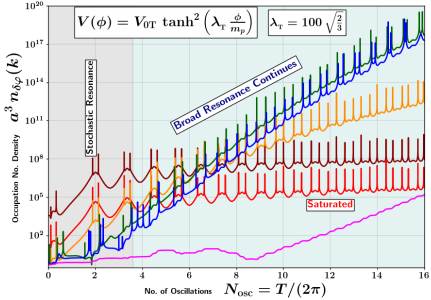
Resonant growth of Inflaton $\delta\varphi_k \propto e^{\mu t}$

Linear broad-band resonance for $\boldsymbol{\lambda \gtrsim 10}$

Maximally growing mode $\Rightarrow$ $\boxed{k_{\rm osc} \simeq 0.75\, m}$

$\Rightarrow$ Expected **Size of Oscillon** $\boxed{R_{\rm osc} \approx \dfrac{2\,\pi}{k_{\rm osc}} \simeq 8\, m^{-1}}$





Will the linear growth continue ? Backreaction !

Non-linear dynamics: $\mathcal{C}$osmo$\mathcal{L}$attice Simulations

Fully non-linear field equations

$$\ddot{\tilde{\varphi}} + 3\tilde{H}\dot{\tilde{\varphi}} - \frac{\widetilde{\nabla}^2}{a^2}\,\tilde{\varphi} + \widetilde{V}_{,\tilde{\varphi}} = 0$$

$$\tilde{H} \equiv \frac{\dot{a}}{a} = \frac{1}{3m_p^2}\left\langle \widetilde{K}_{\tilde{\varphi}} + \widetilde{G}_{\tilde{\varphi}} + \widetilde{V}(\tilde{\varphi}) \right\rangle$$

Where $\tilde{t} = m\,t$; $\tilde{x} = m\,x$; $\tilde{\varphi}, \tilde{\chi} = \frac{1}{\beta}\,\frac{\varphi,\chi}{m_p}$; $\tilde{F} = \frac{F}{\beta^2 m^2 m_p^2}$

$$\widetilde{K}_{\tilde{\varphi}} = \frac{1}{2}\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{t}}\right)^2 ; \quad \widetilde{G}_{\tilde{\varphi}} = \frac{1}{2a^2(\tilde{t})}\left[\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{x}}\right)^2 + \left(\frac{\partial\tilde{\varphi}}{\partial\tilde{y}}\right)^2 + \left(\frac{\partial\tilde{\varphi}}{\partial\tilde{z}}\right)^2\right]$$

$$\tilde{\rho}_{\tilde{\varphi}} = \widetilde{K}_{\tilde{\varphi}} + \widetilde{G}_{\tilde{\varphi}} + \widetilde{V}(\tilde{\varphi}) ; \quad \tilde{p}_{\tilde{\varphi}} = \widetilde{K}_{\tilde{\varphi}} - \frac{1}{3}\widetilde{G}_{\tilde{\varphi}} - \widetilde{V}(\tilde{\varphi})$$

Lattice specifications:

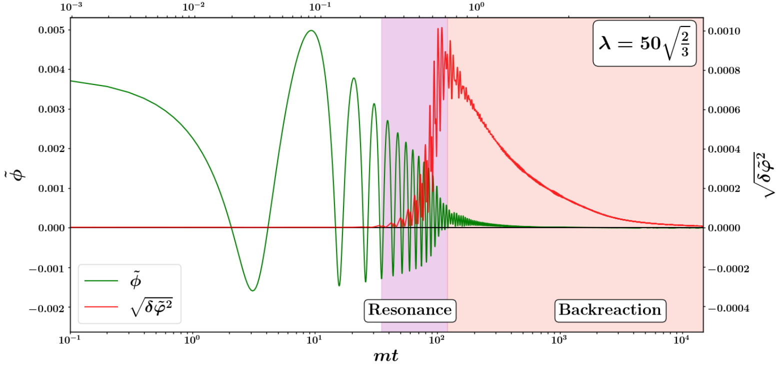
$$N = 128^3 \;\&\; \mathbf{256^3}\,; \quad \boxed{0.05\,m^{-1} \leq k \lesssim 15\,m^{-1}}$$

****Figueroa *et. al* (2020, 2021); **Mahbub, SSM (2023)**

Self-resonance and Inflaton Fragmentation

Strong self-resonance $\Rightarrow$ Inflaton fragmentation

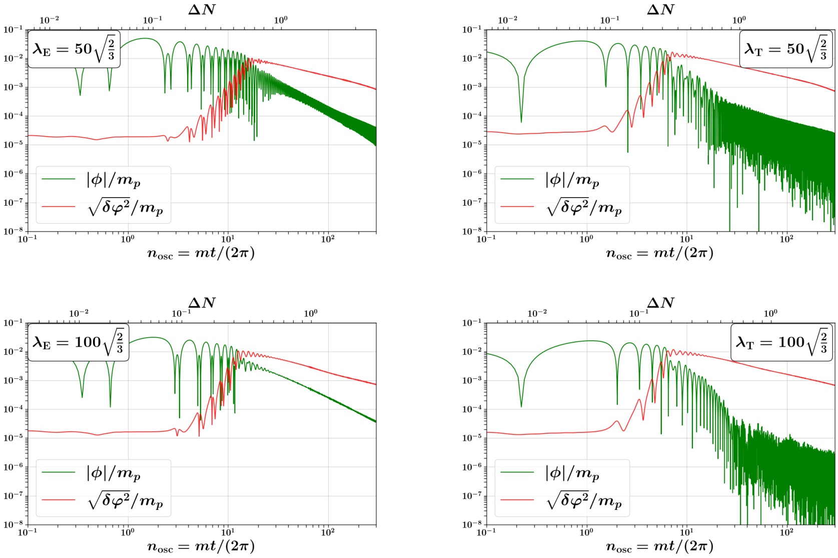
(Asymmetric E-Model potential)



Backreaction is significant within $\sim 100/(2\pi)$ oscillations

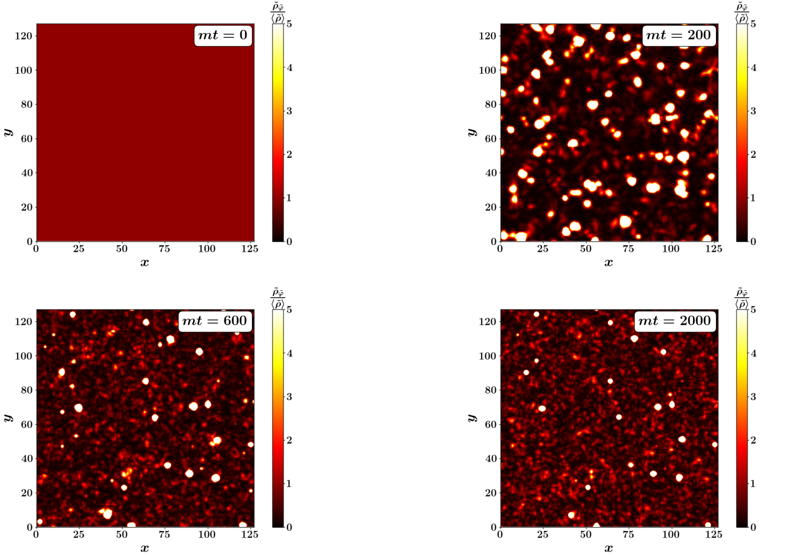
**Mahbub, SSM (2023); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

Inflaton Fragmentation within a Single e-fold



Coherent oscillations of ϕ break down within ONE e-fold!

Oscillon formation in real time (Asymmetric)



Shrinking Comoving Size $\Longrightarrow$ Fixed Physical Size Oscillons

Islands of Parameter Space in $\lambda_{\rm T,E}$

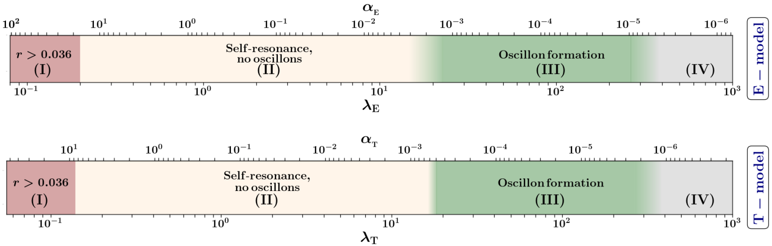
$\rightarrow$ **Relevant parameters:** $\{\boldsymbol{\lambda_{\rm T,E}},\, \boldsymbol{\phi_{\rm ini}},\, \boldsymbol{V_0}\}$

$\rightarrow$ Natural initial conditions $\Rightarrow$ $\boldsymbol{\phi_{\rm ini} = \phi_{\rm end}}$ (V_0 via CMB Normalisation)

$\rightarrow$ Inflaton Mass $\boxed{\boldsymbol{m = \sqrt{2\,V_0/\lambda^2_{\rm E,T}} \simeq 10^{-5}\, m_p}}$

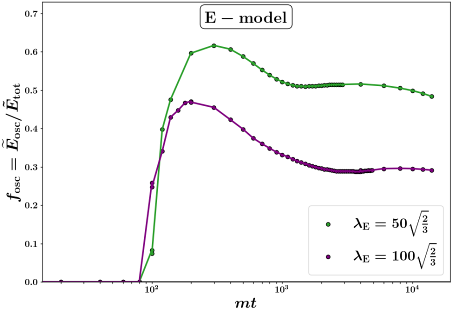
$\rightarrow$ **Oscillon Formation** for $\boxed{\boldsymbol{10 \leq \lambda_{\rm E,T} \lesssim 10^3}}$

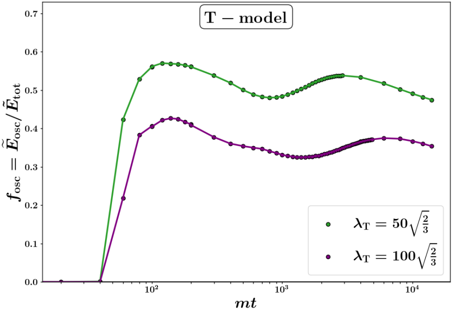
$\rightarrow$ For **E, T Models** $\boxed{\boldsymbol{\lambda_{\rm Int} \approx \left(\frac{m}{m_p}\right)^2 \lambda^{1,2}_{\rm E,T}}}$ $\boldsymbol{\ll 1}$ (**Weak Self-coupling**)



Fractional Energy Density of Oscillons

Energy/Mass fraction $$\boxed{f_{\rm osc} \equiv \frac{E_{\rm osc}}{E_{\rm tot}} = \frac{\int_{\delta\rho_\varphi \gtrsim 4\bar{\rho}_\varphi} \mathrm{d}^3\boldsymbol{x}\,\rho_\varphi(\boldsymbol{x},t)}{\int \mathrm{d}^3\boldsymbol{x}\,\rho_\varphi(\boldsymbol{x},t)}}$$





$\gtrsim$ 40% **of the total density** $\Rightarrow$ **Significant!**

**Mahbub, SSM (2023)

Inferences (so far)

1. **Oscillons do form** after inflation in absence of external coupling starting from **generic initial conditions**.
2. Oscillons form **for both Symmetric and Asymmetric plateau potentials**.

Important Open Questions

1. We have **ignored external coupling;** $g \to 0$

   External couplings are important for Reheating !

   What happens if $g \neq 0$? Do oscillons form?

   Rest of this Talk!

2. What is the **lifetime of oscillons**? How do they **decay?**

****Hertzberg (2010); **Zhang, Amin, Copeland, Saffin, Lozanov (2020)**

Preheating via Self & External Resonance

$\rightarrow$ For inflaton decay, $|\phi(t)| \gg \delta\varphi(t,\vec{x}),\ \chi(t,\vec{x})$

$$\varphi(t,\vec{x}) = \boldsymbol{\phi}(t) + \boldsymbol{\delta\varphi}(t,\vec{x})$$

$$\chi(t,\vec{x}) = \cancelto{0}{\bar{\chi}(t)} + \boldsymbol{\delta\chi}(t,\vec{x}) \quad (\boldsymbol{\chi} \ \ \textbf{field is in vacuum state})$$

$\rightarrow$ At the end of inflation, $\rho_\phi \gg \rho_\chi,\ \rho_{\delta\varphi}$ (**Condensate dominated**)

$\rightarrow$ Resulting **equations of dynamics in the linear regime**

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$\boldsymbol{\ddot{\delta\varphi}_k + 3H\dot{\delta\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi)\right]\delta\varphi_k} = 0 \quad \textbf{Self} - \textbf{resonance}$$

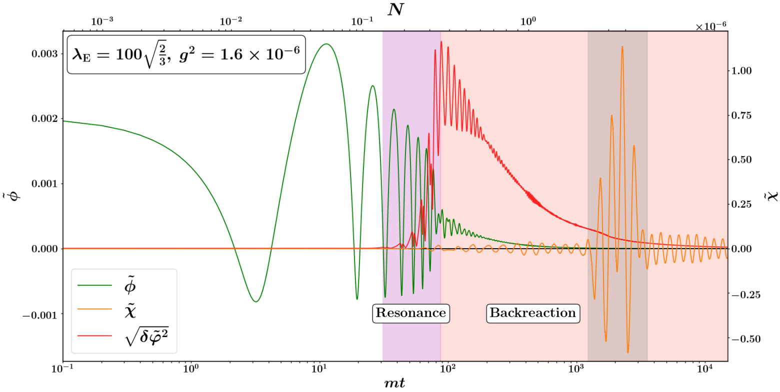
$$\boldsymbol{\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2\,\phi^2\right]\chi_k} = 0 \quad \textbf{External} - \textbf{resonance}$$

and the **Hubble parameter** $H^2 \simeq \frac{1}{3\,m_p^2}\left[\frac{1}{2}\,\dot{\phi}^2 + V(\phi)\right]$

****SSM Lecture Notes (2024)**

Self-resonance and Oscillon decay for $g \neq 0$

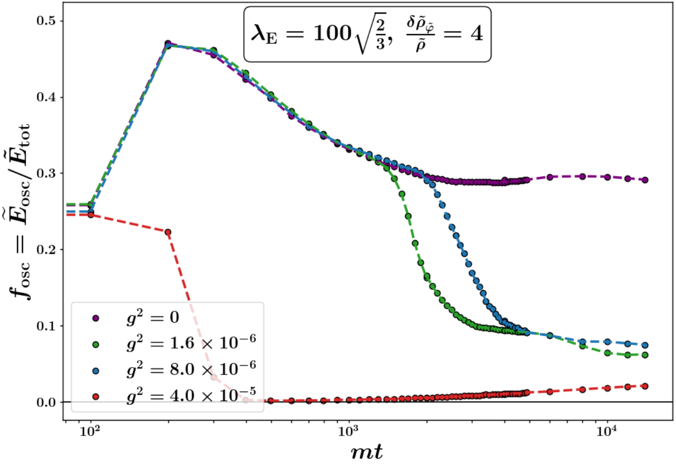
Self-resonance → Oscillons → χ production



Asymmetric E-model potential

**Mahbub, SSM (2023); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

Energy(Mass) Fraction of Oscillons



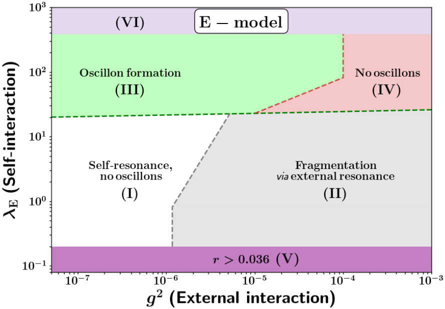
Reduction in $f_{\rm osc}$ due to χ-production

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

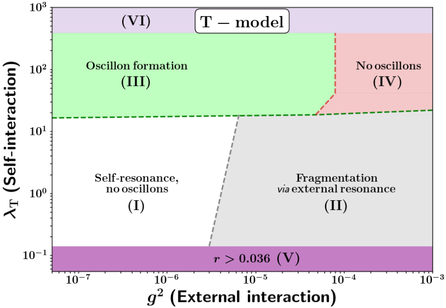
Islands of the Extended Parameter Space $\{\lambda, g^2\}$

External Coupling $\boxed{\mathcal{I}(\phi, \chi) = \frac{1}{2}\, g^2\, \phi^2\, \chi^2}$

$$V_{\rm E}(\phi) = V_{\rm 0E}\, \left(1 - e^{-\boldsymbol{\lambda}_{\rm E}\, \frac{\phi}{m_p}}\right)^2 \qquad V_{\rm T}(\phi) = V_{\rm 0T}\, \tanh^2\left(\boldsymbol{\lambda}_{\rm T}\, \frac{\phi}{m_p}\right)$$

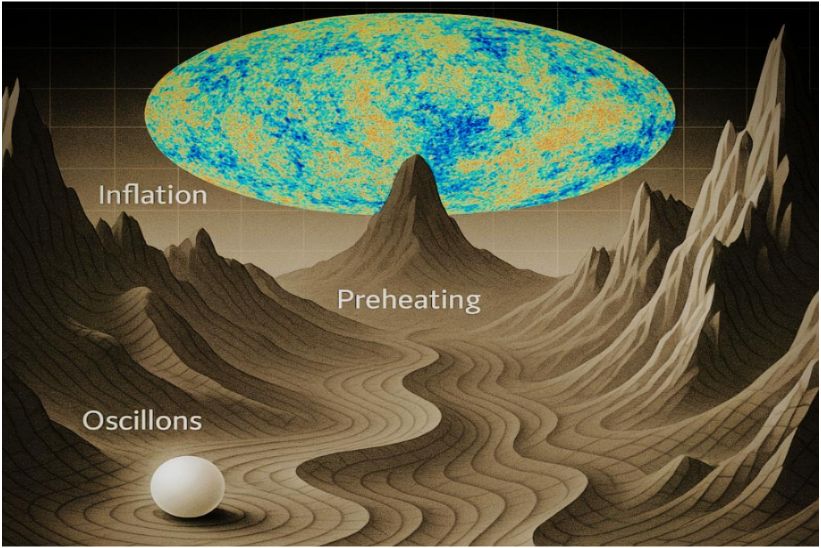


(**Asymmetric E-Model** $\boldsymbol{\lambda}_{\rm E}$ *vs* $\boldsymbol{g^2}$)



(**Symmetric T-Model** $\boldsymbol{\lambda}_{\rm T}$ *vs* $\boldsymbol{g^2}$)

Lumpy Road from Inflation to Hot Big Bang



Summary: Oscillons plausibly form after inflation

1. **Do oscillons form in presence of external coupling(s) ?**

   YES! (Preheating via Oscillon decay into χ)

2. **Lifetime of oscillons? How does an oscillon decay?**

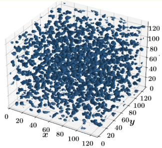
   Ongoing work (analytical)!

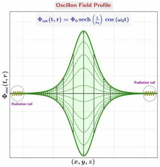
3. **More realistic interactions? Ongoing work (Lattice)!**

4. **Effects of metric curvature ?**

Phenomenological Implications

1. High Frequency GWs from Preheating
2. Late-time Clustering & GWs
3. Oscillons after Reheating
   (*e.g.* Phase Transitions)
4. Primordial Black Holes
5. Dark Matter Oscillatons





Extra Slides

Equations of Reheating Dynamics

$\boxed{\textbf{System : } \text{Inflaton } \varphi \longrightarrow \text{ massless offspring } \chi}\,; \quad \boldsymbol{m \gg m_{0\chi}}$

Described by the **action**

$$\boxed{S[\varphi,\chi] = -\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{1}{2}\partial_\mu\varphi\partial^\nu\varphi + V(\varphi) + \frac{1}{2}\partial_\mu\chi\partial^\nu\chi + \mathcal{I}(\varphi,\chi)\right]}$$

With interaction $\mathcal{I}(\phi,\,\chi) = \frac{1}{2}\,g^2\,\varphi^2\,\chi^2$

The corresponding **field equations** are

$$\ddot{\varphi} - \frac{\nabla^2}{a^2}\varphi + 3H\dot{\varphi} + V_{,\varphi} + \mathcal{I}_{,\varphi} = 0$$

$$\ddot{\chi} - \frac{\nabla^2}{a^2}\chi + 3H\dot{\chi} + \mathcal{I}_{,\chi} = 0$$

with **Hubble parameter**

$$\boxed{H^2 = \frac{1}{3\,m_p^2}\left[\frac{1}{2}\,\dot{\varphi}^2 + \frac{1}{2}\,\frac{\vec{\nabla}\varphi}{a}\frac{\vec{\nabla}\varphi}{a} + V(\varphi) + \frac{1}{2}\,\dot{\chi}^2 + \frac{1}{2}\,\frac{\vec{\nabla}\chi}{a}\frac{\vec{\nabla}\chi}{a} + \mathcal{I}(\varphi,\,\chi)\right]}$$

Universe Reheating after Inflation

Reheating may proceed in two distinct regimes –

1. **Perturbative Decay:**
   - Relevant primarily for **fermionic decay** $\varphi \longrightarrow \bar{\psi}\psi$
   - When bosonic couplings are extremely weak $g^2 \ll 10^{-8}$
   - **Slow** and **in-efficient**
   - Does not incorporate the presence of **coherently oscillating inflaton condensate** background
2. **Non-perturbative Decay:**
   - When bosonic couplings are high enough $g^2 \gtrsim 10^{-8}$
   - Particle production in presence of oscillating inflaton field $\Rightarrow$ **Parametric resonance - Collective phenomenon**
   - Fast, explosive, efficient, **highly non-thermal**
   - Not applicable to fermionic decay (Pauli exclusion)

Stages of Inflaton Decay

→ Reheating dynamics is complicated, non-perturbative physics

→ **Particle production in presence of oscillating inflaton condensate**

→ Dynamics can be broadly divided into three distinct phases :

1. **Preheating** (**linear parametric resonance**)

2. **Backreaction** (**quenching of resonant particle production**)

3. **Scattering & thermalization** (**perturbative decay, turbulence**)

Preheating in the linear regime

→ For inflaton decay, $|\phi(t)| \gg \delta\varphi(t,\vec{x}),\ \chi(t,\vec{x})$

$$\begin{aligned}
\varphi(t,\vec{x}) &= \phi(t) + \cancelto{0}{\delta\varphi(t,\vec{x})} \\
\chi(t,\vec{x}) &= \cancelto{0}{\bar{\chi}(t)} + \delta\chi(t,\vec{x}) \quad (\boldsymbol{\chi}\ \textbf{field is in vacuum state})
\end{aligned}$$

→ At the end of inflation, $\rho_\phi \gg \rho_\chi,\ \rho_{\delta\varphi}$ (**Condensate dominated**)

→ Resulting **equations of dynamics in the linear regime**

$$\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) &= 0 \\
\cancelto{0}{\delta\ddot{\varphi} + 3H\,\delta\dot{\varphi} + \left[-\frac{\nabla^2}{a^2} + V_{,\phi\phi}(\phi)\right]\delta\varphi} &= 0 \\
\ddot{\chi} + 3H\dot{\chi} + \left[-\frac{\nabla^2}{a^2} + g^2\,\phi^2\right]\chi &= 0
\end{aligned}$$

and the **Hubble parameter** $H^2 \simeq \frac{1}{3\,m_p^2}\left[\frac{1}{2}\,\dot{\phi}^2 + V(\phi)\right]$

SSM Lecture Notes (2024)

Preheating & linear Parametric Oscillator

Equation of dynamics of the Fourier modes of χ

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2\,\phi(t)^2\right]\chi_k = 0$$

Ignoring the expansion of space $H = 0$ (**adiabatic approx**)

And (remember!) for $V(\phi) \simeq \frac{1}{2}m^2\phi^2$, since

$$\boxed{\phi(t) \simeq \phi_0\,\cos(mt)}\,,$$

the mode functions χ_k satisfy the **Mathieu Equation**

$$\Rightarrow \boxed{\frac{\mathrm{d}^2\chi_k}{\mathrm{d}T^2} + \Big[A_k - 2q\,\cos(2T)\Big]\chi_k = 0}$$

With $T = mt - \frac{\pi}{2}$; and the **resonance parameters** $\{q,\, A_k\}$

$$q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 \;; \quad A_k = \left(\frac{k}{m}\right)^2 + 2q$$

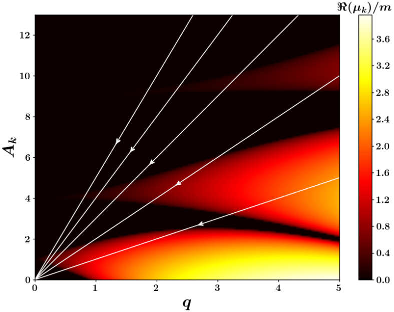
**SSM Lecture Notes (2024)

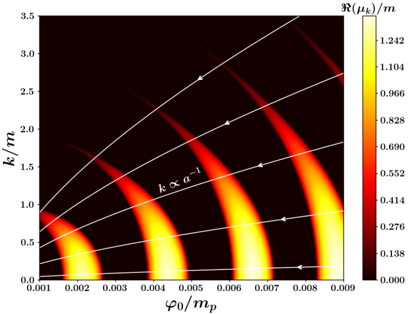
Linear parametric oscillator (**Mathieu Equation**)

$$\boxed{\frac{d^2 X_k}{dT^2} + \Omega_X^2(k,T)\, X_k = 0} \; ; \quad \boxed{\Omega_X^2 = A_k - 2q\,\cos(2T)}$$

Floquet Theorem $\boxed{X_k(T) = \mathcal{M}_k^{(+)}(T)\; e^{\mu_k T} + \mathcal{M}_k^{(-)}(T)\; e^{-\mu_k T}}$

$\Rightarrow$ **Exponentially growing** for $\mathbf{Re}(\mu_k) \neq 0$ (**in resonance bands**)



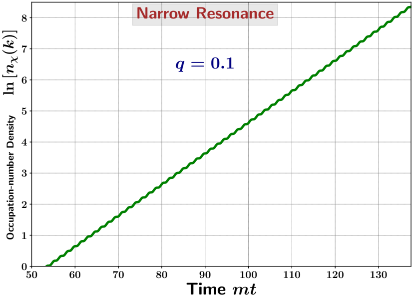


Kofman, Linde, Starobinsky (1990s) ****SSM Lecture Notes (2024) **Non-thermal**

Narrow and Broad Resonance of Mathieu Eqn

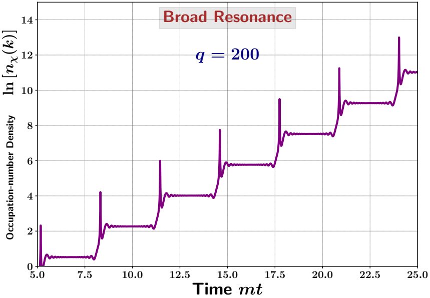
Narrow resonance

$$q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 \ll 1$$



Broad resonance

$$q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 \geq 1$$



Occupation number density

$$\boxed{n_\chi(k) \equiv \frac{\mathcal{E}_\chi(k)}{\Omega_\chi(k)} = \frac{1}{2\Omega_\chi}\left[\left|\frac{\mathrm{d}\chi_k}{\mathrm{dT}}\right|^2 + \Omega_\chi^2\,|\chi_k|^2\right] \propto e^{2\mu_k T}}$$

Backreaction & quenching of particle production

→ Equation of dynamics of the Fourier modes

$$\ddot{\chi}_k + 3\,H(t)\,\dot{\chi}_k + \left[\frac{k^2}{a^2(t)} + g^2\,\phi_0(t)^2\cos^2(mt)\right]\chi_k = 0$$

→ Resonance parameter $q = \frac{g^2\phi_0^2(t)}{4m^2}$ & physical momentum $k_p = \frac{k}{a(t)}$

The system moves from

Broad → **Narrow** resonance ;

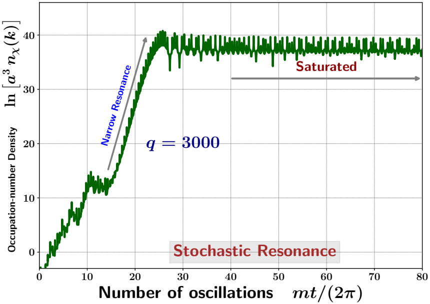
Particle production *via* resonance is quenched due to

→ **Redshifting of** $q(t)$, $k_p(t)$

→ **Backreaction of** $\chi(t,\vec{x})$ **on** $\phi(t)$

Perturbative processes take over

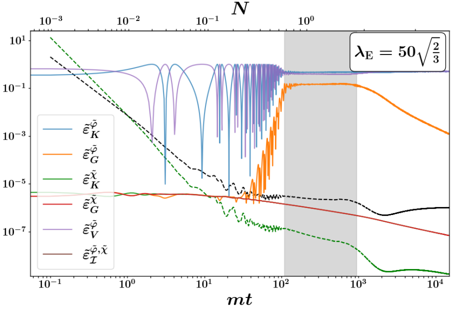
$$\varphi \longrightarrow \bar{\psi}\psi\,; \quad \varphi\varphi \longrightarrow \chi\chi$$



Evolution of energy density components

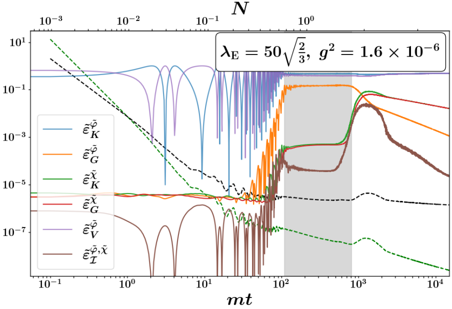
Absence of external interaction

Long-lived Oscillons



Presence of external interaction

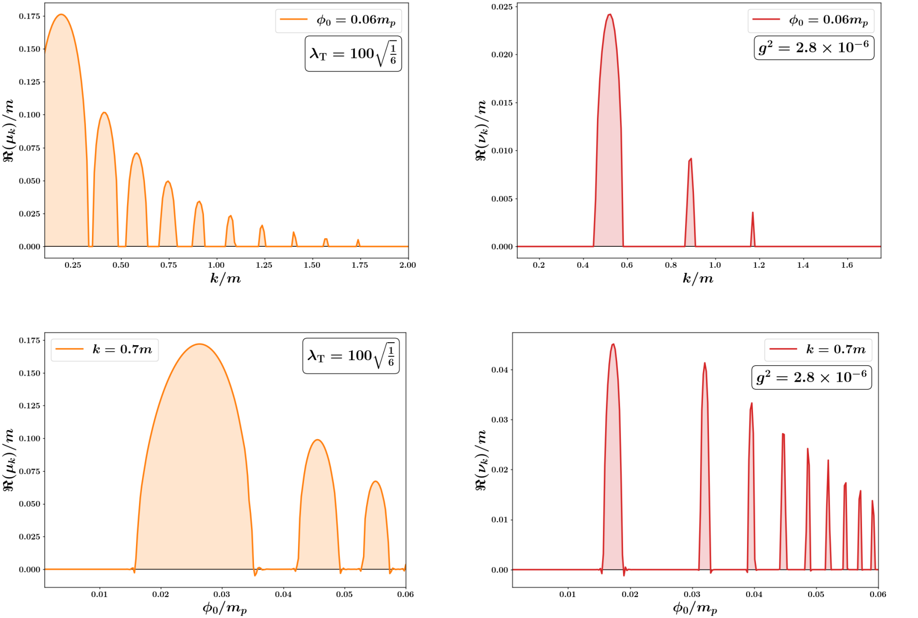
Oscillons decaying into χ



(**Production of χ-particles due to oscillon decay**!)

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

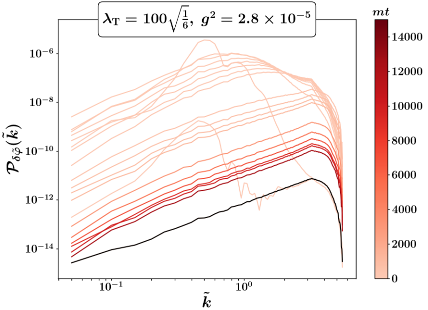
Structure of the resonance with $\{\lambda,\, g^2\}$

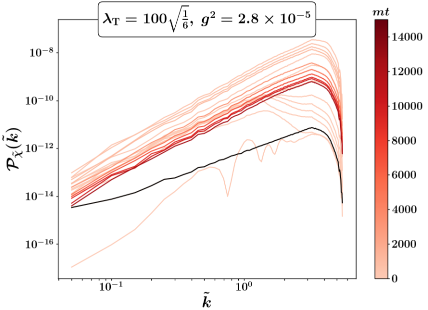


Power spectra of fluctuations: Resonance

Inflaton $\delta\varphi$-fluctuations

Offspring χ-fluctuations

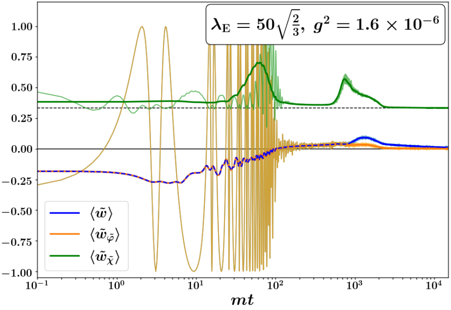




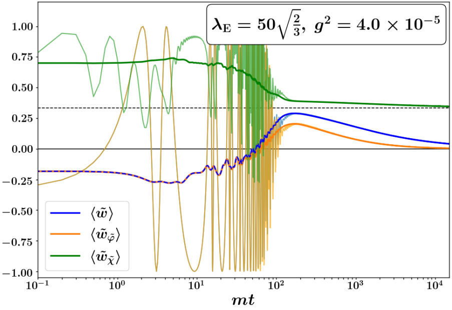
**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Evolution of Equation of State

Low external interaction



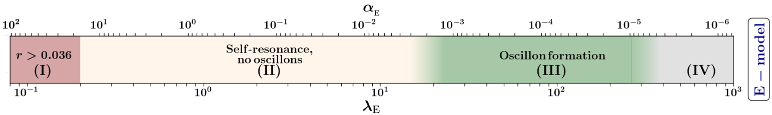
Large external interaction

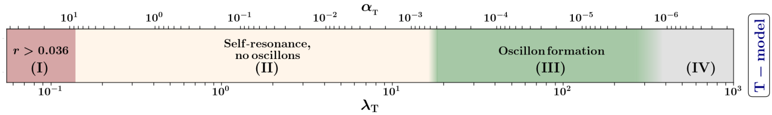


$\langle w_\varphi \rangle \rightarrow 0$ **asymptotically**

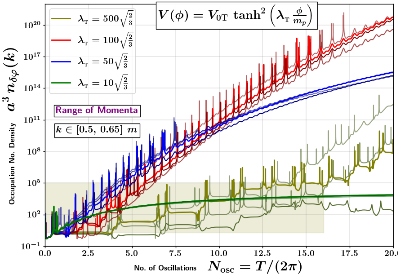
**Shafi, Copeland, Mahbub, SSM, Basak (2024)

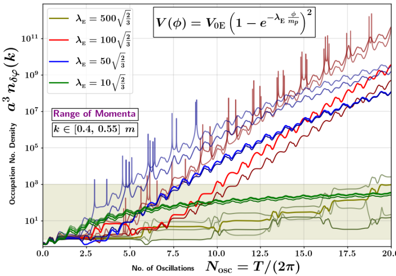
Islands of Parameter Space in $\lambda_{\mathrm{T,\,E}}$





Prominent Stochastic Resonance (for ultra-large $\lambda_{\mathrm{T,\,E}}$ values)





Are quantum corrections important ? (No)

→ **Relevant parameters:** $\{\boldsymbol{\lambda}_{\mathbf{T,E}},\, \boldsymbol{\phi}_{\mathbf{ini}},\, \boldsymbol{V_0}\}$

→ Natural initial conditions ⇒ $\boldsymbol{\phi}_{\mathbf{ini}} = \boldsymbol{\phi}_{\mathbf{end}}$ (V_0 via CMB Normalisation)

→ **Oscillon Formation** for $\boxed{\mathbf{10} \leq \boldsymbol{\lambda}_{\mathbf{T,E}} \lesssim \mathbf{10^3}}$

→ For **T-Model** $\boldsymbol{V_{\mathrm{T}}(\phi) = V_0 \tanh^2\left(\lambda_{\mathrm{T}} \frac{\phi}{m_p}\right)}$

Taylor expansion ⇒ $\boldsymbol{V_{\mathrm{T}}(\phi) = \frac{1}{2}\, m^2\, \phi^2 - \frac{\lambda_{\mathrm{Quart}}}{4}\, \phi^4} + \mathcal{O}\left(\phi^6\right)$

Where $\boldsymbol{m = \sqrt{\frac{2\, V_0}{\lambda_{\mathrm{T}}^2}} \simeq 10^{-5}\, m_p}$, $\boxed{\boldsymbol{\lambda_{\mathrm{Quart}} = \frac{\lambda_{\mathrm{T}}^2}{3}\, \frac{m^2}{m_p^2}}} \approx 10^{-10}\, \lambda_{\mathrm{T}}^2$

→ Therefore, $\boldsymbol{\lambda_{\mathrm{Quart}} \ll 1}$ for $\boldsymbol{\lambda_{\mathrm{T}} \lesssim 10^3}$ (**weak coupling!**)

→ In fact, **Tree-level perturbative Unitarity Bound**

$$\Rightarrow \boxed{\boldsymbol{\lambda_{\mathrm{T}}^{\mathrm{loop}} \lesssim 9.8 \times 10^4}}$$

Potential $\boxed{V(\phi) = \frac{1}{2}\, m^2 \phi^2 + V_{\rm nl}(\phi)}\, ; \quad V_{\rm nl}(\phi) < 0 \ \text{ for } \ \phi < \phi_c$

Radiation

Oscillon Solution $\varphi_{\rm osc}(t, r) = \Phi_{\rm osc}(r) \cos(\omega_0 t) + \cancel{\delta\varphi_R(t, r)}$

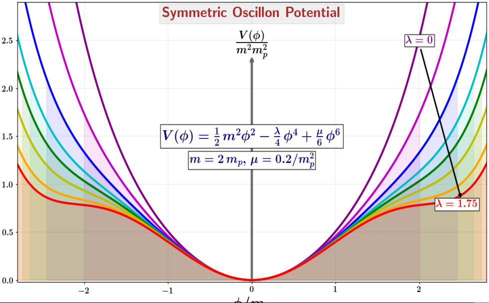
KG Field Eqn $\boxed{\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi + V_{{\rm nl},\varphi}(\varphi) = 0} \quad (H = 0)$

Define $$U_{\rm nl}(\Phi_{\rm osc}) = -\frac{\omega_0}{\pi} \int_{-\pi/\omega_0}^{\pi/\omega_0} {\rm d}\tilde{t}\, \cos(\omega_0 \tilde{t}) \times V_{\rm nl}\left(\Phi_{\rm osc}\, \cos(\omega_0 \tilde{t})\right)$$

Bound-state Condition:

$$\boxed{w_0 < m}$$

(violated for some $w \geq \# w_0$)



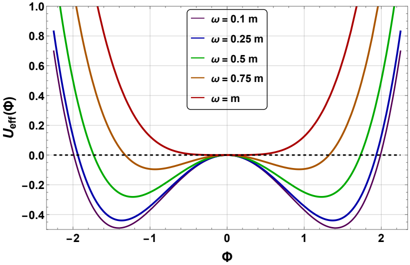
Example $$V_{\rm nl}(\phi) = -\frac{\lambda}{4}\, \phi^4 + \frac{\mu}{6}\, \phi^6$$

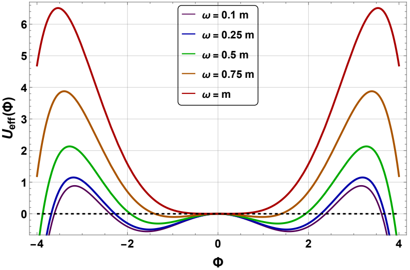
$$U_{\rm nl}(\Phi) = \left(\frac{3}{4}\right) \frac{\lambda}{4}\, \Phi^4 - \left(\frac{5}{8}\right) \frac{\mu}{6}\, \Phi^6$$

Oscillon Field $\boxed{\phi_{\rm osc}(t,r) \approx \Phi_{\rm osc}(r)\cos(\omega_0 t)}$

Spatial Profile
$$\frac{{\rm d}^2\Phi_{\rm osc}}{{\rm d}r^2} + \left(\frac{2}{r}\right)\frac{{\rm d}\Phi_{\rm osc}}{{\rm d}r} + \frac{\rm d}{{\rm d}\Phi}U_{\rm eff}(\Phi_{\rm osc}) = 0$$

$$U_{\rm eff}(\Phi) = -\frac{1}{2}\left(m^2-\omega^2\right)\Phi + U_{\rm nl}(\Phi)$$





$\Phi_{\rm osc}(r)$ can be determined by the **Shooting Method**

$$\Phi\Big|_{r=0} = \Phi_0\,, \quad \frac{{\rm d}\Phi}{{\rm d}r}\Big|_{r=0} = 0\,, \quad \& \quad \Phi\Big|_{r\to\infty} = \frac{{\rm d}\Phi}{{\rm d}r}\Big|_{r\to\infty} = 0$$