Do cosmic string segments emit gravitational waves?

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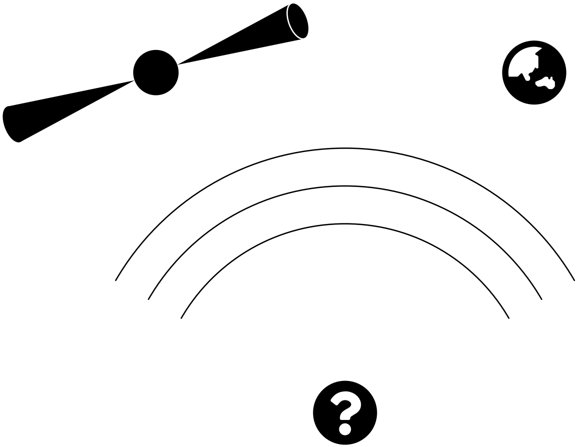
Stochastic gravitational wave background

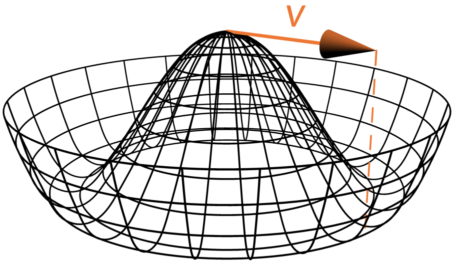
- Recently evidenced by PTAs (NANOGrav, EPTA, InPTA, PPTA, CPTA)

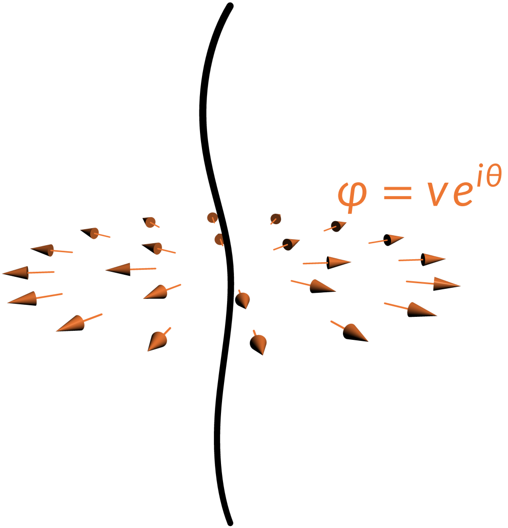


Cosmic strings

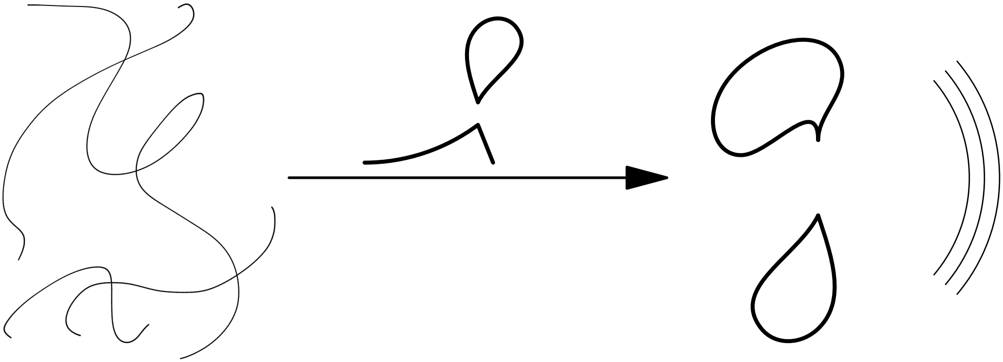
Cosmic string from phase transition

$U(1)$: $\varphi \to e^{i\alpha}\varphi$

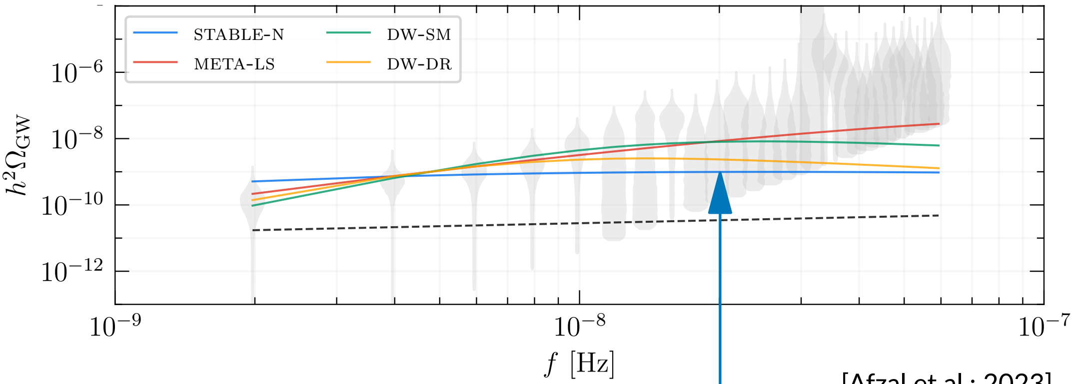




Cosmological evolution and GW



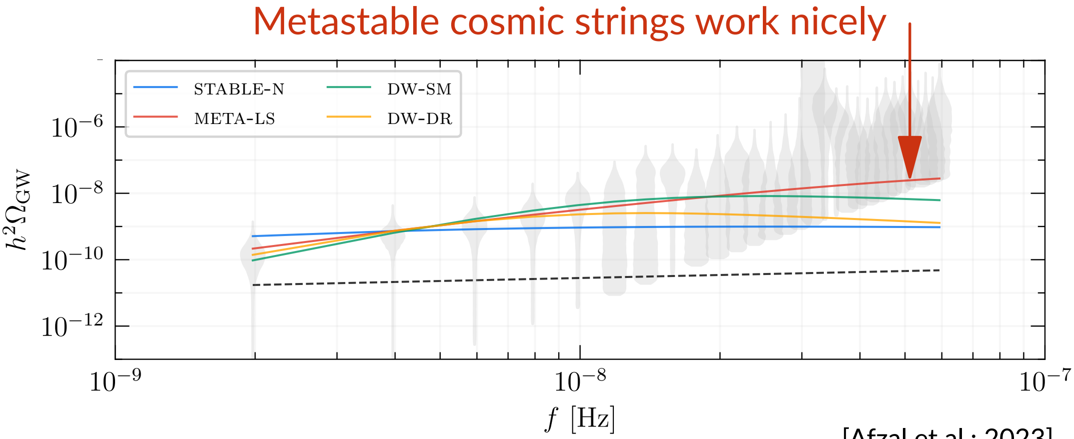
Cosmic strings for PTA?



[Afzal et al.; 2023]

Blue tilt not reproduced

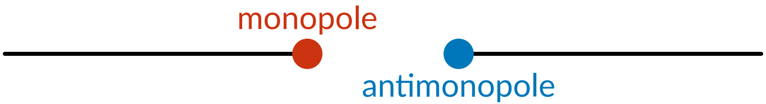
Metastable cosmic strings for PTA



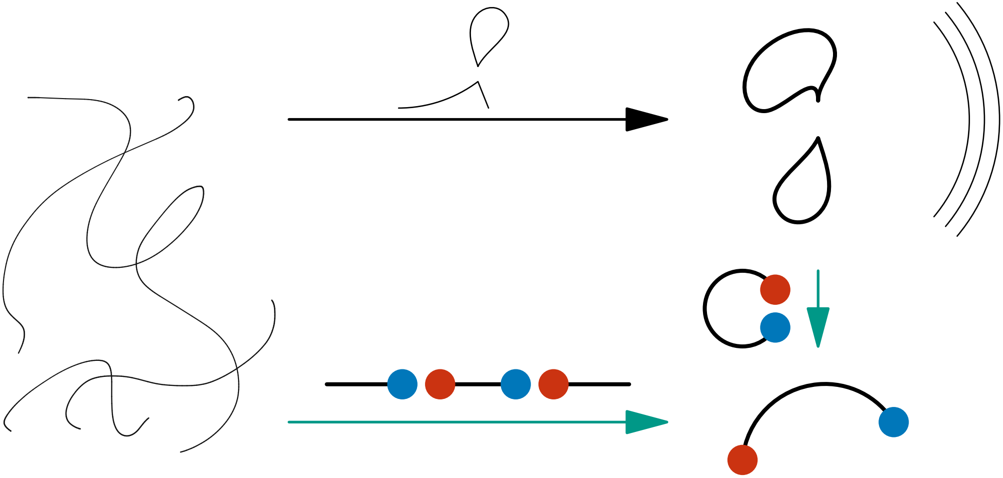
[Afzal et al.; 2023]

Metastable cosmic strings

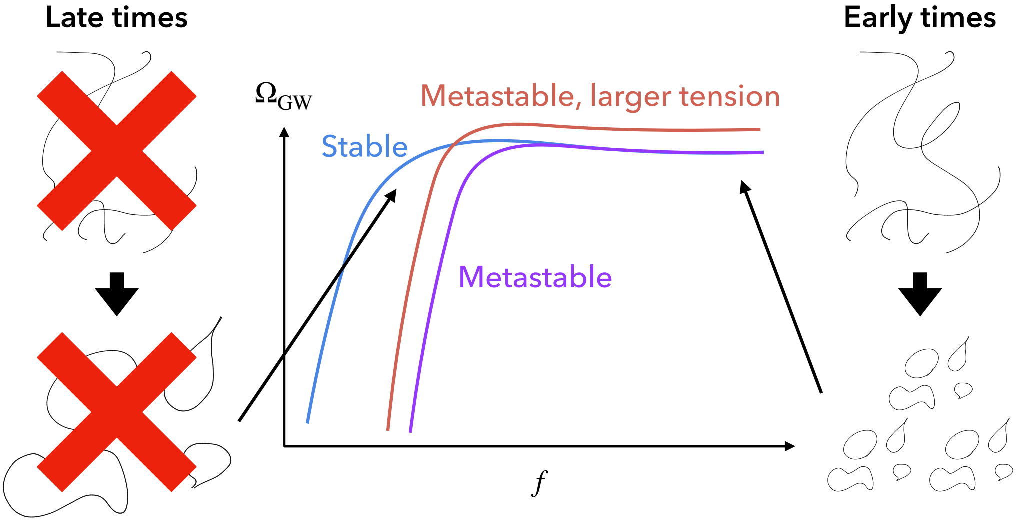
- Decay via monopole-antimonopole pair production
  - Decay rate: new parameter → more general spectrum
- Arise from e.g. some grand unification models



Metastable cosmic strings



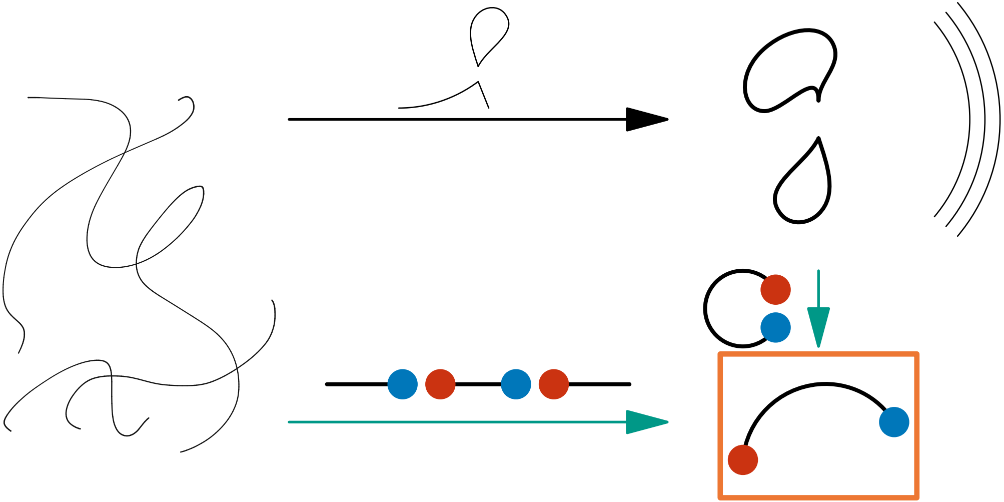
Metastable Cosmic Strings



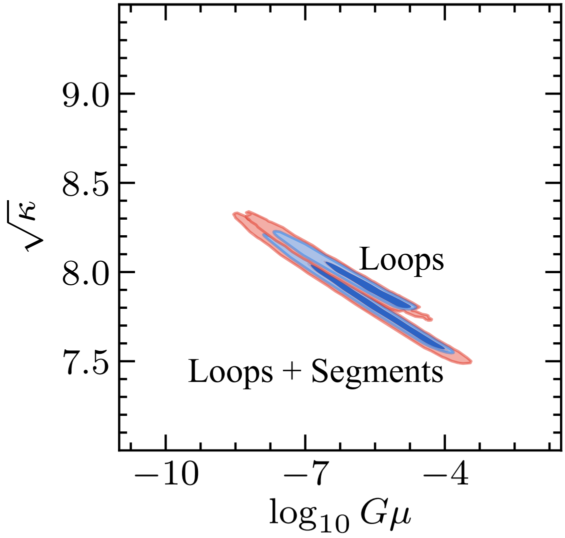
Do segments emit gravitational waves?

arXiv:2507.12386

Cosmic string segments

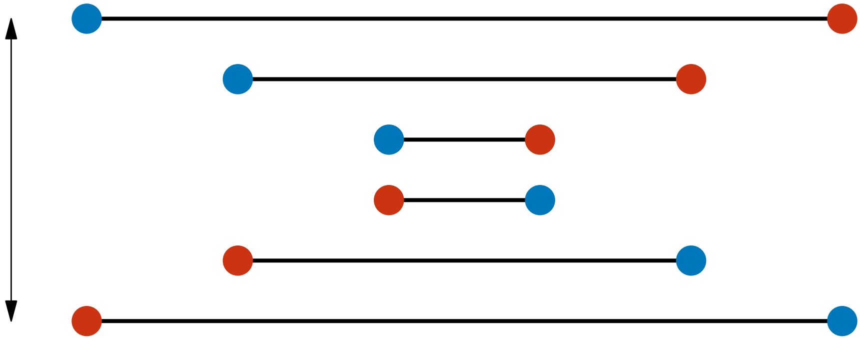


With vs. without GW from segments



[Afzal et al.; 2023]

GW from a perfectly straight segment [Martin, Vilenkin; 1997]



- $P_n = 4G\mu^2/n$ for nth harmonic mode
  - $P_{\mathrm{tot}} := \sum P_n \sim 4G\mu^2 \ln n_{\max}$

Conventional treatment

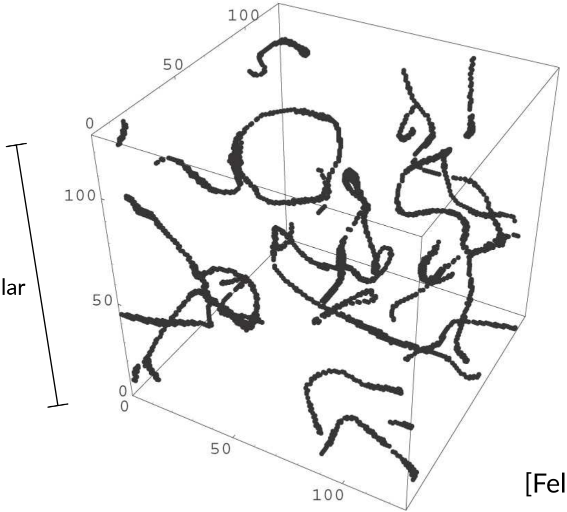
- Physical cutoff: $n \lesssim \mu L/M$
  - μ: string tension, L: maximum length, M: monopole mass
  - For PTA, $\sqrt{\mu} \sim M \sim 10^{16}$ GeV, $L \sim H^{-1}(T \sim \text{MeV}) \to P_{\text{tot}} \sim 700G\mu^2$
  - Monopoles are highly accelerated: $\gamma_m = E/M \sim 10^{40}$ ... really?
- Community assumption: P_{tot} is similar to that of a loop ($\sim 50G\mu^2$)
  - Corresponds to $n_{\text{max}} \sim 10^5$
  - Not too physical

What is the realistic picture?

Impact of fluctuations

Wiggles at formation

$40m_{\mathrm{scalar}}^{-1}$



[Felder, Kofman, Linde; 2012]

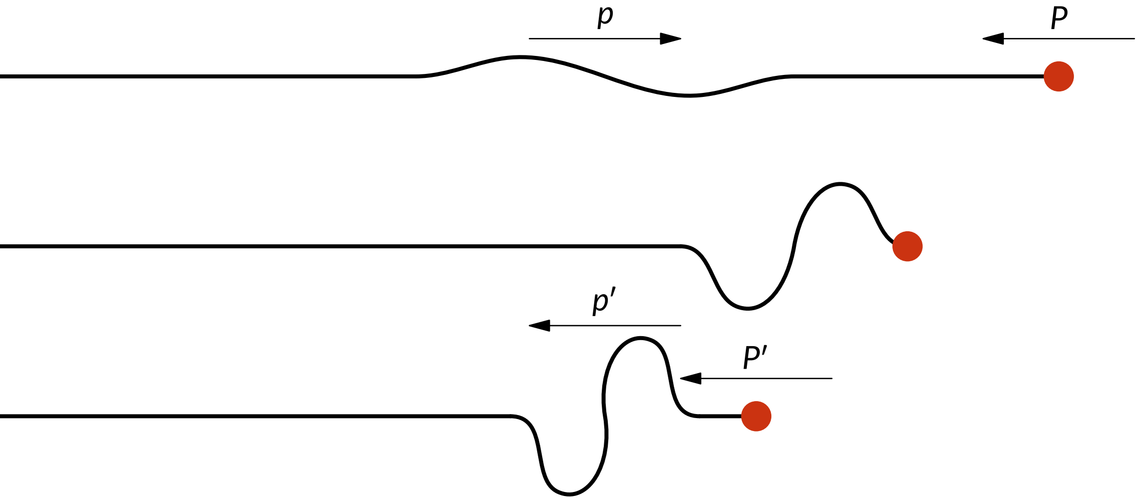
Thermal fluctuations on the strings

- Strings are wiggly
  - Brownian at formation
  - Smoothed out by drag force while in contact with the thermal bath
- $\rightarrow$ cannot be smoother than thermal fluctuation on string

$$f(p)\mathrm{d}p \simeq \frac{1}{\exp[|p|/T_s] - 1}\mathrm{d}p$$

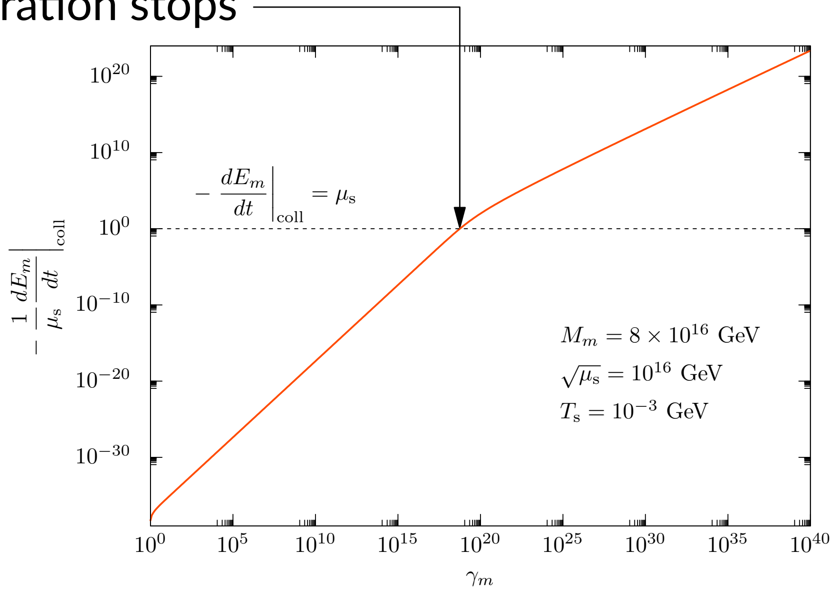
  - T_s: string temperature (cosmic temperature up to g_*)

Fluctuations take energy away



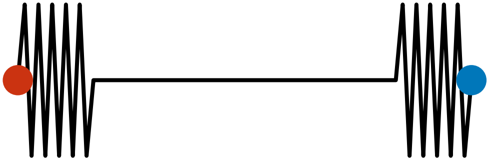
→ friction force to monopole

Acceleration stops

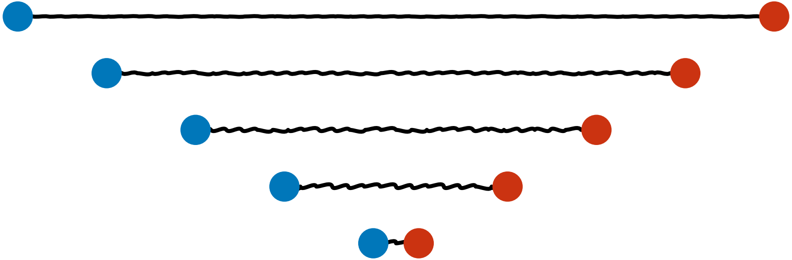


Energy loss from high-energy processes

- Short-wavelength ($\sim$ string thickness) structures
- $\to$ Nonlinear dynamics triggered
  - Massive particle emission
  - Microscopic loop formation
  - Cusp evaporation
  - ...



Segments do not oscillate



- Most energy is lost nongravitationally
- → GW suppressed

Summary

- GW from metastable cosmic strings can be a probe for new physics
- Different parameters preferred for loop only vs. loop+segment
- Small-scale fluctuations on segments prevent oscillation
  - Most of energy is released nongravitationally
  - Litte GW expected
- Always “loop only” scenario

Thank you!

Backup

How long and straight segments need be?

- Reality: various, curved / shorter segments
  - segments from loops
  - segments from long strings
  - 2nd, 3rd... generation segments
- For PTA, required straight length $\sim \dfrac{\gamma_m M_m}{\mu} \sim 10^{-20} H^{-1} (T \sim \mathrm{MeV})$
  - $\rightarrow$ applicable to realistic segments!

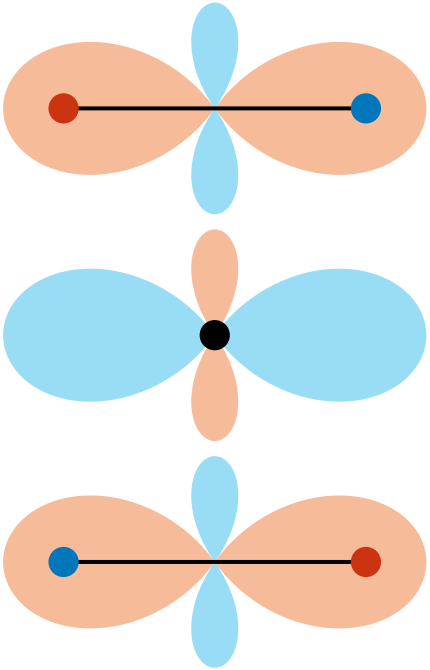
Nambu–Goto effective action

$$S = -\int \mathrm{d}^4x \left[\frac{1}{4e^2} F^2 + |D\varphi|^2 + V(|\varphi|) \right]$$

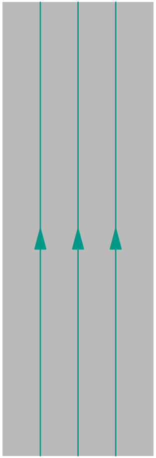
$$\Big\downarrow \ \text{integrate out heavy modes}$$

$$S = -\mu \int_{\text{worldsheet}} \mathrm{d}^2 x$$

$\mu \approx 2\pi v^2$: tension = energy per unit length



Magnetic flux



$$
\begin{aligned}
\int_{\text{across string}} B &= \int_{\text{across string}} \mathrm{d}A \\
&= \oint_{\text{around string}} A \\
&= \oint_{\text{around string}} \frac{i}{\varphi}(\overbrace{D\varphi}^{=0} - \mathrm{d}\varphi) \\
&= \int_0^{2\pi} \frac{i}{ve^{i\theta}}(-ive^{i\theta}\mathrm{d}\theta) \\
&= 2\pi
\end{aligned}
$$

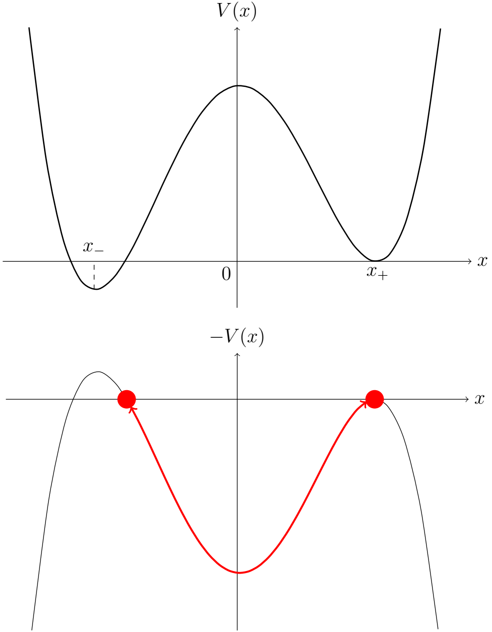
→ string cannot be cut

String breaking rate

Tunneling and bounce

see e.g. [Coleman, 1985]

- Procedure:
  - Go to imaginary time
    - $\approx$ invert the potential
  - Find the bounce solution
    - Action: S_B
  - Decay rate: $\Gamma \sim \exp[-S_B]$



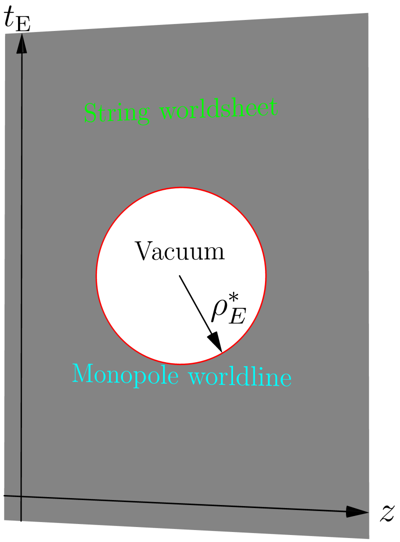
String breaking rate

Preskill-Vilenkin approximation [Preskill & Vilenkin, 1992]

- Neglect monopole size and string width
- $S_E = 2\pi\rho_E^* M_M - \pi\rho_E^{*2} T_{\rm str}$
  - $\rightarrow \rho_E^* = M_M / T_{\rm str}\ ,\ S_B = \pi M_M^2 / T_{\rm str} = \pi\kappa$
    - M_M: monopole mass, $T_{\rm str}$: string tension
- String width $T_{\rm str}^{-1/2} \ll \rho_E^*$ required

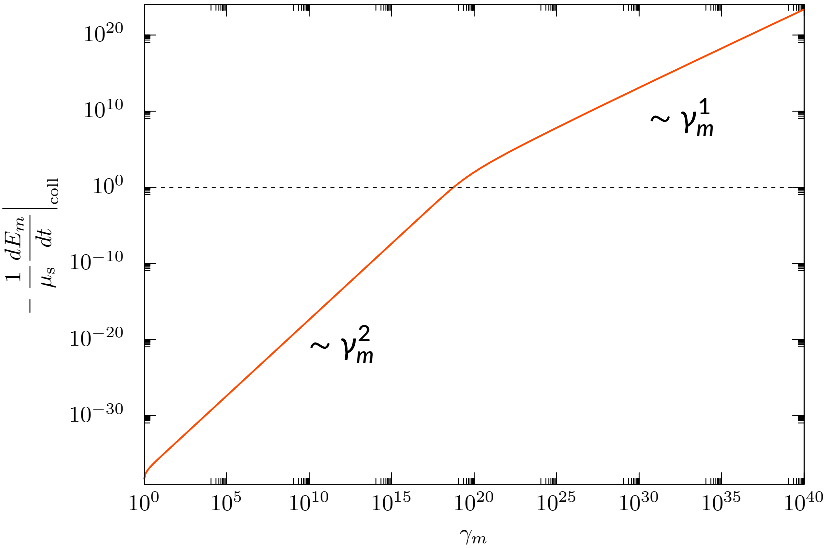
$\rightarrow \sqrt{\kappa} \gg 1 \cdots$ **Is this OK for PTA ($\sqrt{\kappa} \sim 8$)?**

$\rightarrow$ **Alternative evaluation desired**



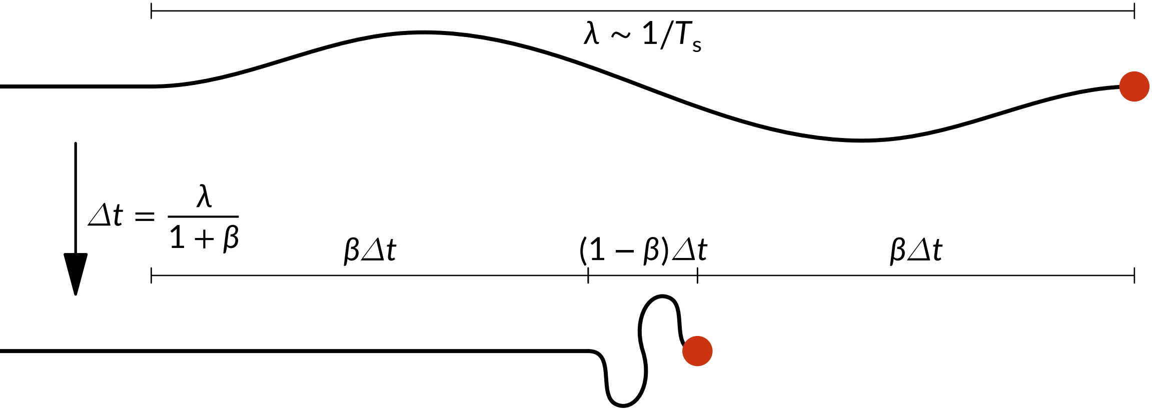
Local vs. Global

- Global string: from broken global U(1)
  - Couples to Nambu–Goldstone bosons → GW suppressed
    - Not discussed today
  - Example: post-inflationary axion models
- Local string: from broken gauged U(1)
  - Couples to gravity
  - Example: SO(10) $\to$ SM $\times$ U(1) $\to$ SM



10^{20}
10^{10}
10^{0}
10^{-10}
10^{-20}
10^{-30}
$-\frac{1}{\mu_s}\frac{dE_m}{dt}\Big|_{\rm coll}$
$\sim \gamma_m^1$
$\sim \gamma_m^2$
10^0 10^5 10^{10} 10^{15} 10^{20} 10^{25} 10^{30} 10^{35} 10^{40}
γ_m

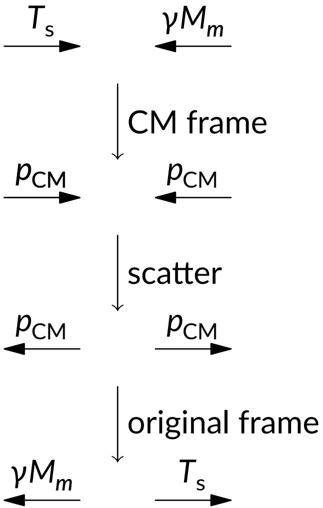
Small γ_m: heavy monopole limit



$$\text{friction} \sim [(1-\beta)\Delta t]^{-1}/\Delta t \sim \gamma^2 T_s^2$$

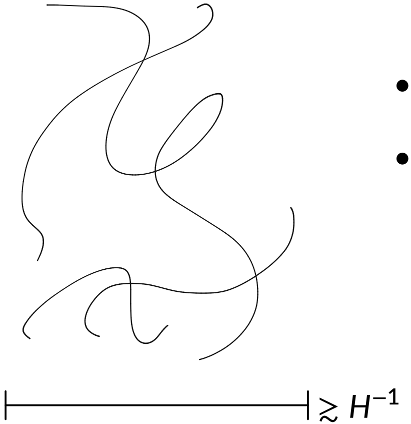
$$\gamma_{(\mathrm{cr})}^2 T_s^2 \sim \mu \to \gamma_{(\mathrm{cr})} \sim \sqrt{\mu}/T_s$$

Large γ_m: massless monopole limit



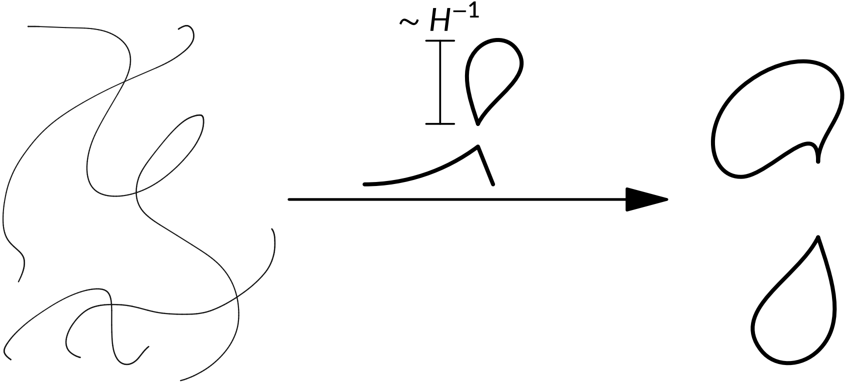
- Most of monopole's energy lost at one collision
- friction $\sim \gamma M_m \times T_s$

Cosmological evolution and GW: long strings



- Formed at $T \sim v$
- Scaling solution: attractor of the dynamics
  - $O(1)$ string / Hubble volume
  - Little initial condition dependence

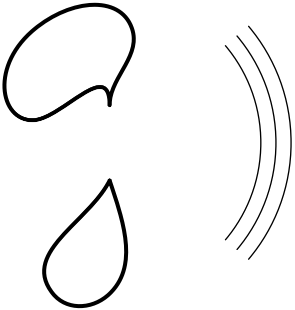
Cosmological evolution and GW: loop formation



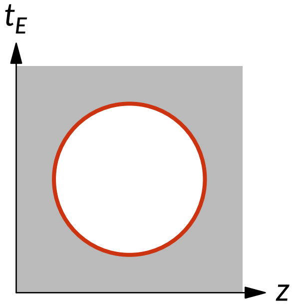
Formed continuously; number fixed by scaling

Cosmological evolution and GW: loops

- Vibrate and emit GW
  - $f \sim (\text{loop size})^{-1}$
  - power $\sim 50G\mu^2$
- Gradually shrink as they decay into GW

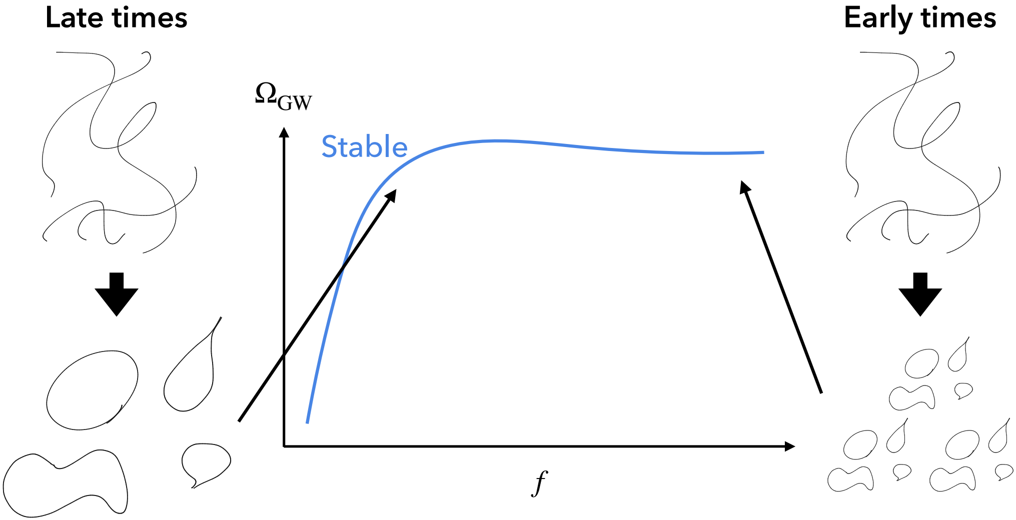


Decay rate

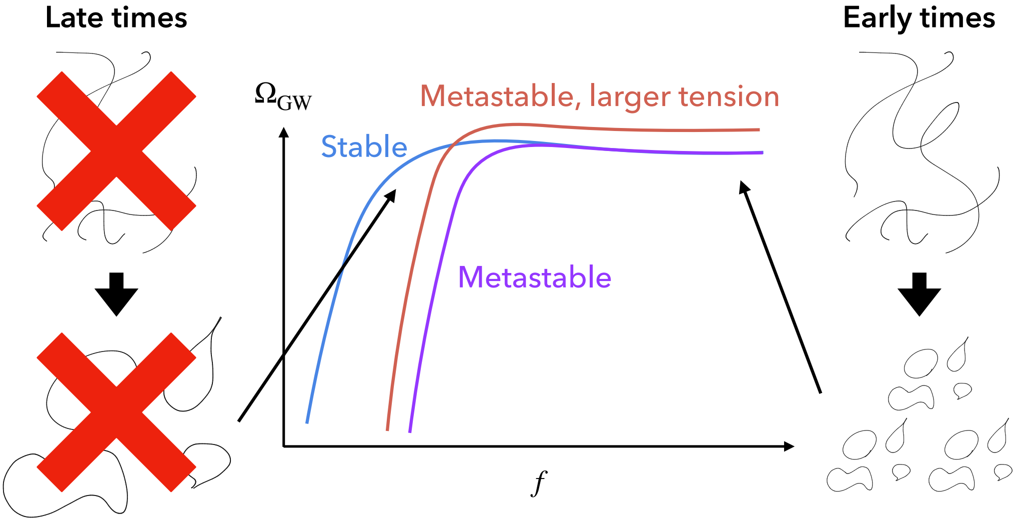


- $\Gamma = \dfrac{\mu}{2\pi}\exp[-\pi\kappa]$
  - PTA: $\sqrt{\kappa} \sim 8$
- Thin-defect approximation: $\kappa = M^2/\mu$
  - Not an excellent one [AC, Nakayama, Ibe, Shirai, Watanabe; 2024]

Metastable Cosmic Strings



Metastable Cosmic Strings

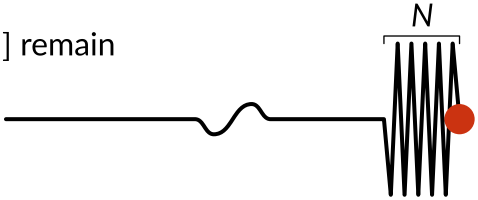


Ballpark figures for PTA

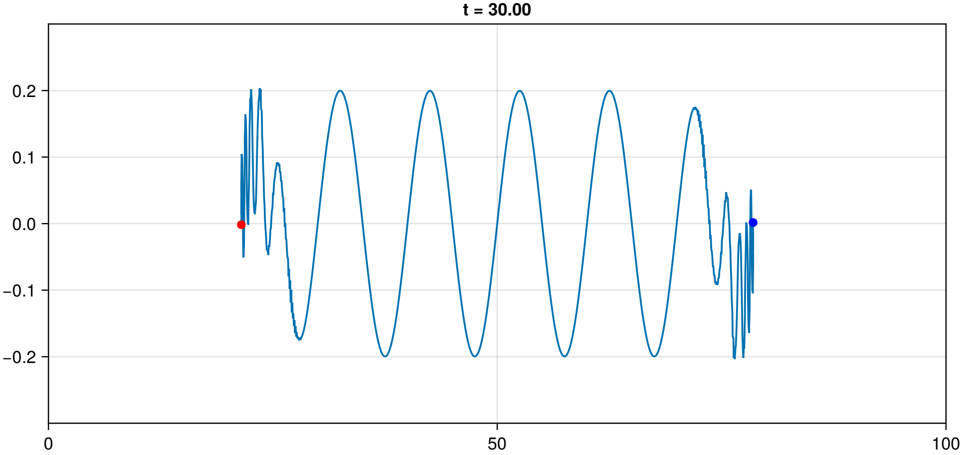
- $\sqrt{\mu} \sim 10^{15\text{–}17}$ GeV
- Decaying at $T \sim$ MeV

Nongravitational energy loss: rough sketch

- N quanta of scattered wave
- Scattering probability P
  - $(1-P)^N$: hit monopole; N increase by 1
  - $1-(1-P)^N$: hit scattered wave; N decrease by 1
    - Energy released $\sim \mu/T_s$
- $N \sim 1/\ln[(1-P)^{-1}]$ remain



Numerical simulation



Magnetic monopoles

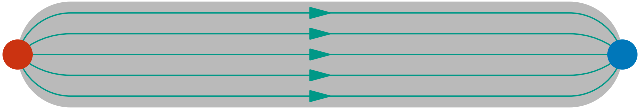
- Arise from e.g. SU(2) → U(1):

$$\underbrace{\phi^a}_{\text{SU(2) triplet}}(x) = V\frac{x^a}{|\vec{x}|}$$

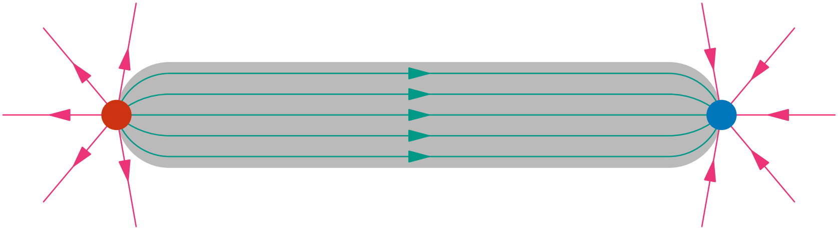
- Have magnetic charge
- Mass: $M \sim \dfrac{4\pi V}{g}$

Confined vs. Unconfined magnetic flux

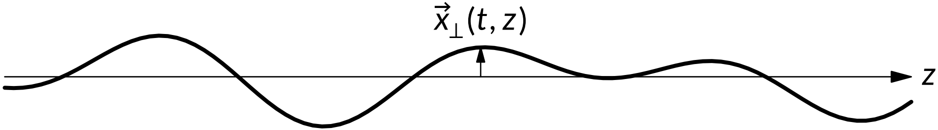
- Confined: possibly emit GW(?)
  - From e.g. $SU(2) \to U(1) \to 1$



- Unconfined: radiate $U(1)$ gauge boson $\to$ little GW
  - From e.g. $SU(2) \times U(1) \to U(1) \times U(1) \to U(1)$



Propagating waves on strings

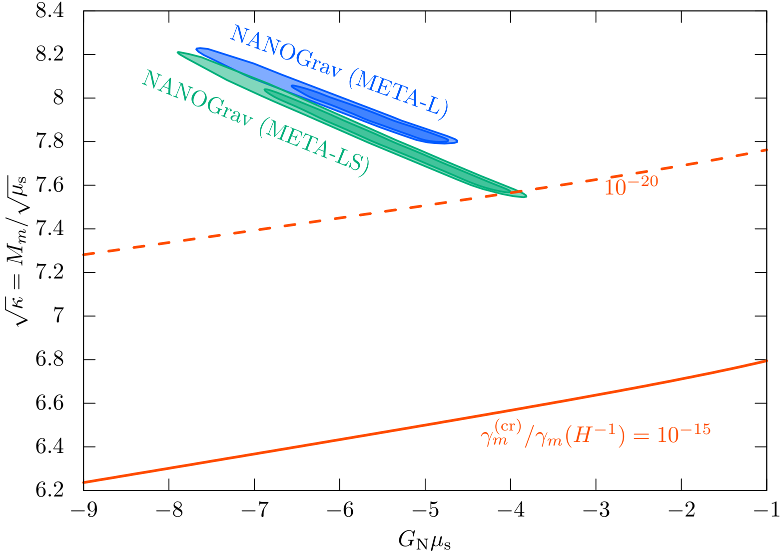


$$S = -\mu \int_{\text{worldsheet}} \mathrm{d}^2 S$$

String on z axis + small perturbation

$$S_{\text{zm}} = \frac{\mu}{2} \int \mathrm{d}t\,\mathrm{d}z \left[(\partial_t \vec{x}_\perp)^2 - (\partial_z \vec{x}_\perp)^2 \right]$$

No segment contribution for PTA



Failure of stable strings

- Only parameter: tension
- → cannot fit height & tilt