Inside Black Holes
Singularity to Complementarity!

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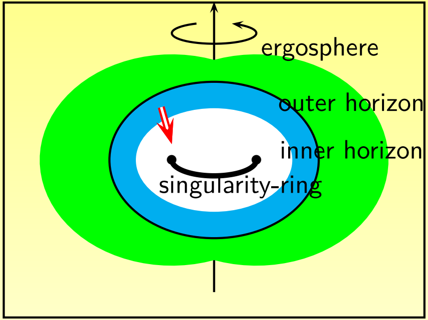
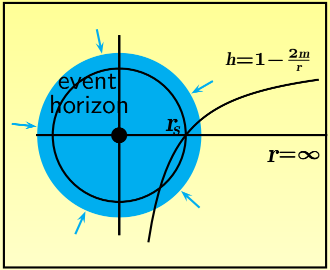
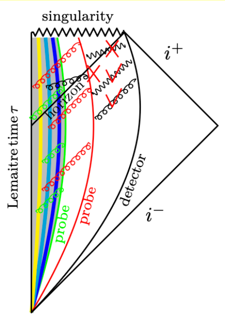
existence of singularity is a physical reality rather than physical relation







Kerr: inferring from physics between the two horizons -> singularity is a belief rather than logic because unpredictability across the inner horizon

me[EPJC2024]: the existence of horizon in the spherical symmetric collapse is coordinate dependent, but physical. “coordinate dependence = unphysical” is a wrong recognition basing on the mixing of two key concepts: physical reality and physical law.

physical reality: the output of single type physical measurement, they are coordinate dependent, esp. time definition dependent

physical law: links between different physical realities, such as physical laws, they are coordinate independent

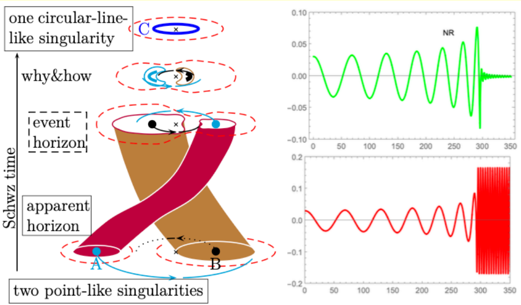
A Stupid Question: Why the merger product of two Schwzs one Kerr?

If you believe that black holes in numerical relativity (NR) and astronomical observations (AO) are singular, then you need to face a basic question : why the merger product of two such Schwz-BHs is one Kerr? [2505.14749v2]

The system composed of two point-like singularities cannot become a circular-line-like one simply through inspiral and radiation. But the damping of gravitational waves from their binary merger process tells such thing happens indeed.

∼ BHs in both NR and AO are regular internally and carry no horizon at all. Technically, this is because they are not defined through event horizon and singularity. They are defined ...

Physically, this is because the formation of horizon and singularity does not happen in the coverage of time definition all outside probes were selected and monitored



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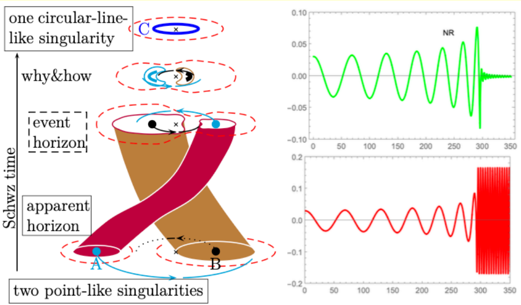

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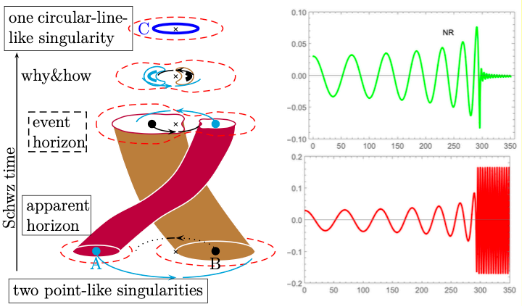
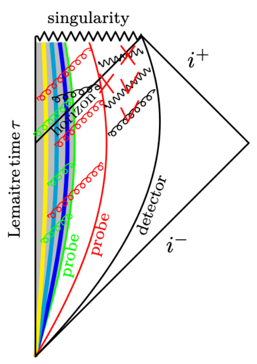
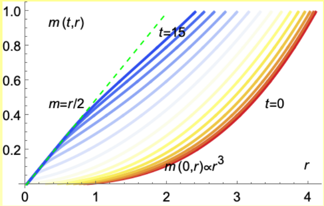
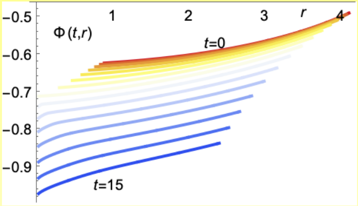


What Penrose et al's singularity theorem ignores?

It assumes the formation of trapped surface. But the time it forms is gauge dependent







$$ds^2_{schwz} = -e^{\Phi} dt^2 + h^{-1} dr^2 + r^2 d\Omega^2,\ h = 1 - \frac{2m(t,r)}{r} \qquad (1)$$

Φ contains gauge ambiguity related with t's definition, it needs elimination by e.g., requiring that the collapsing speed of r-shell at time t equal to the freely falling velocity of test-particles r-away from a Schwarzschild-BH of mass $m(t,r)$,

$$v(t,r)^{volume\ element}_{collapsing\ body} = v(r)^{freely\ falling\ partciles}_{outside\ a\ Schwz\ m(t,r)} \Rightarrow\ m(t,r) \xrightarrow{t\to\infty} \frac{r}{2} \qquad (2)$$

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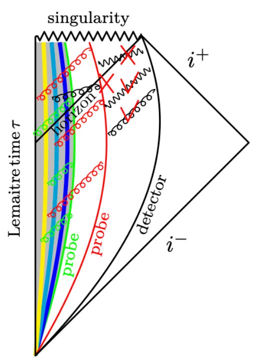

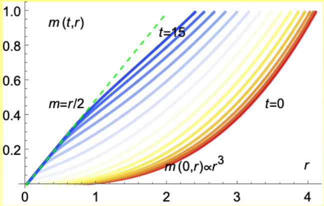

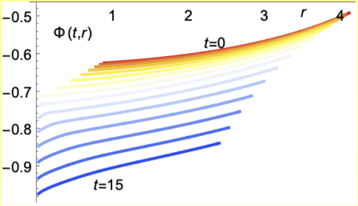

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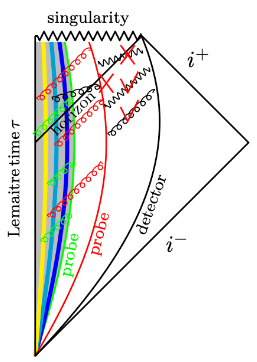

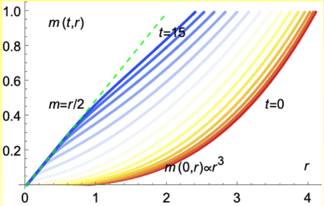

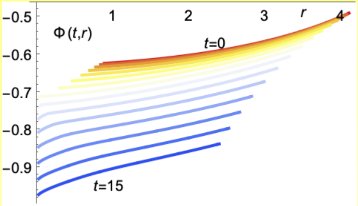

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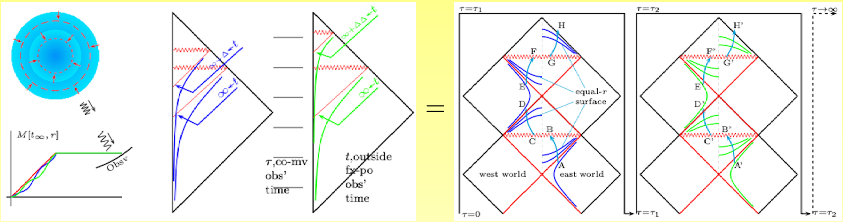
lasting-for-ever contraction v.s. periodically-forming-resolving singularity



◇ the collapsar's lasting-for-ever contraction in the Schwarzschild time definition and periodically-forming-resolving horizon and singularity in the Lemaítre time definition are two contradicting descriptions to each other

♣ an ensemble of collapsars with equal total-mass but different radial profile is needed to account for the initial status' uncertainty. While over-cross-oscillations passing through the singularity is non-predictability thus ergodic

♠ Letting EnS=ErL or Ensemble in the Schwz.t.def = Ergodicity in the Lemaítre.t.def resolves this contradiction naturally, we call this relation as CGC or Complementarity of Gravitational Collapse

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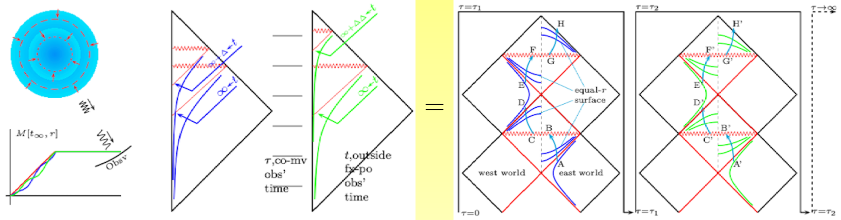

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the essence of CGC is general coordinate invariance

In the Schwz.t.def, the collapsars ensemble is described by the following metric families [2505.14750], where $M[t,r]$ is determined by Einstein equation and $M[t_0,r]$ completely

$$♣\quad ds^2_{schwz}=-e^{\Phi}dt^2+h^{-1}dr^2+r^2d\Omega^2,\, h=1-\frac{2GM}{r} \tag{3}$$

$\Phi:$ determinable by GF&$M(t,r)$ completely

$$M[t,r]\xrightarrow[r<2GM_{\rm tot}]{t\to\infty}\frac{r}{2G}-\text{devi}(initials)$$

In the Lemaítre.t.def, the system's over-cross oscillation is described by the following metric, where $a[\tau,\varrho]$ is a singular oscillatory function whose continuation across the central point is non-predictable

$$♠\quad ds^{2in}_{Lmt}=-d\tau^2+\frac{\left[1-\left(\frac{2GM}{\varrho^3}\right)^{\frac{1}{2}}\frac{M'\varrho}{2M}\tau\right]^2d\varrho^2}{a[\tau,\varrho]}+a[\tau,\varrho]^2\varrho^2d\Omega_2^2 \tag{4}$$

$$ds^{2out}_{Lmt}=-d\tau^2+\frac{r_s^{2/3}d\varrho^2}{[\frac{3}{2}(\varrho-\tau)^{\frac{2}{3}}]}+[\frac{3}{2}(\varrho-\tau)^{\frac{2}{3}}]^2r_s^{\frac{2}{3}}d\Omega_2^2$$

◇ Two solutions are connected to each other under gen.coor.trans. the one-to-one mapping between $\{M[t_0,r]\}$ and $\{M[\varrho]\}$ justifies the equality of EnS=ErL

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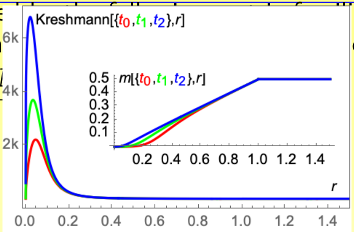

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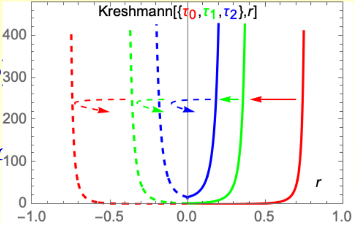

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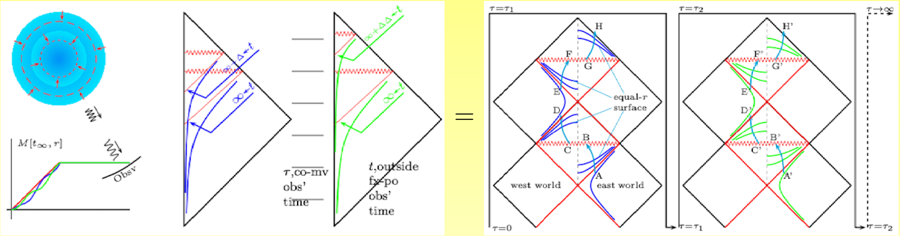
the Bek&Hwk entropy's existence support the idea of CGC



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♣ in $g^{Lmt}_{\mu\nu}[a(\tau,\varrho), M[\varrho]]$, statistic of BHs arises from the ergodic evolution caused by the unpredictability of the matter's motion passing through the singularity epoch

♡ BHs in the AO need be considered collapsars with only close-to-implementing horizon. Over-cross oscillatory ball is a valid picture only to the Lemaítre.t.users, both descriptions can be shown to have degeneracies consistent with ...

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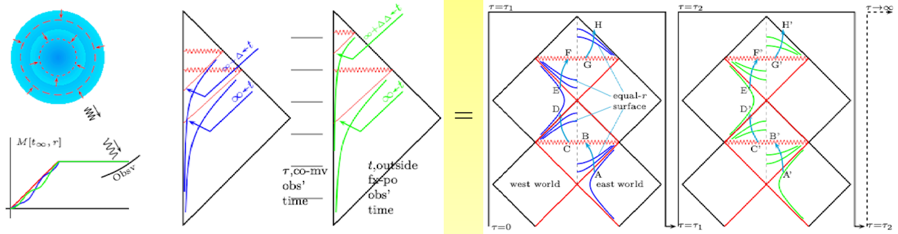

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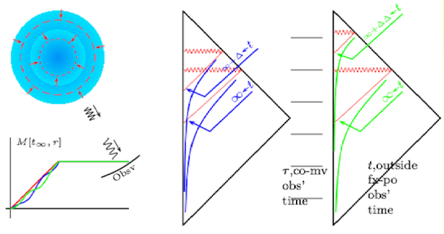

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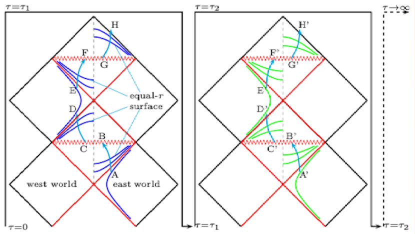

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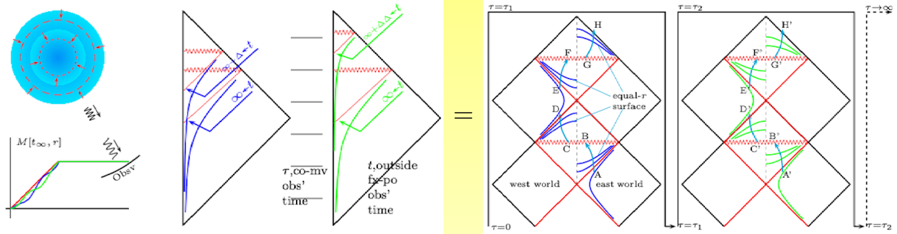

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quantization, wave functional and the microscopic state counting

$\{m(t_0, r),\ \dot{m}(t_0, r)\}$ or $\{a(\tau, \varrho),\ M[\varrho]\}$ are classic descriptions for the microscopic of BHs, infinitely many possibilities. Quantization change things abruptly [zeng2017-2021],

1. decompose the collapsar into many concentric shells $\{m_0, m_1, \cdots, m_\ell\}$, each shell freely falling in a Schwz-like geometry $ds^2 = -h_i dt^2 + h_i^{-1} dr^2 + \cdots$, $h_i = 1 - \frac{2GM_i}{r}$, so that

$$\ddot{x}^\sigma + \Gamma^\sigma_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0 \Rightarrow h_i \dot{t} = \gamma_i \ \&\ h_i \dot{t}^2 - h_i^{-1} \dot{r}^2 = -1 \xrightarrow[quan]{can} \tag{5}$$

$$[-\frac{\hbar^2 \partial_r^2}{2m_i} - \frac{GM_i m_i}{r} - m_i(\gamma_i^2 - 1)]\psi_i(r) = 0 \tag{6}$$

2. the wave functional of the whole collapsar can be written as

$$\Psi[m(r)] = \psi_{n_0}(r, m_0) \otimes \psi_{n_1}(r, m_1) \otimes \cdots \psi_{n_\ell}(r, m_\ell) \tag{7}$$

$$\ell_{\max} = \frac{GM_{\rm tot}^2}{2\sqrt{2}} \equiv k \tag{8}$$

3. how many shells? what's the mass of each shell?two constraints

$$A.\ \sum_{i=0}^{\ell} m_i = M_{\rm tot},\ \ B.\ \frac{GM_i m_i}{\hbar(2-2\gamma_i^2)^{1/2}} \equiv n_i = 1, 2, 3 \cdots \tag{9}$$

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quantization, wave functional and the microscopic state counting

$\{m(t_0, r), \dot{m}(t_0, r)\}$ or $\{a(\tau, \varrho), M[\varrho]\}$ are classic descriptions for the microscopic of BHs, infinitely many possibilities. Quantization change things abruptly [zeng2017-2021],

1. decompose the collapsar into many concentric shells $\{m_0, m_1, \cdots, m_\ell\}$, each shell freely falling in a Schwz-like geometry $ds^2 = -h_i dt^2 + h_i^{-1} dr^2 + \cdots$, $h_i = 1 - \frac{2GM_i}{r}$, so that

$$\ddot{x}^\sigma + \Gamma^\sigma_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0 \Rightarrow h_i \dot{t} = \gamma_i \;\&\; h_i \dot{t}^2 - h_i^{-1} \dot{r}^2 = -1 \xrightarrow[quan]{can} \tag{5}$$

$$[-\frac{\hbar^2 \partial_r^2}{2m_i} - \frac{GM_i m_i}{r} - m_i(\gamma_i^2 - 1)]\psi_i(r) = 0 \tag{6}$$

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$$\Psi[m(r)] = \psi_{n_0}(r, m_0) \otimes \psi_{n_1}(r, m_1) \otimes \cdots \psi_{n_\ell}(r, m_\ell) \tag{7}$$

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shell-decomposition method, degeneracy counting

any shell partition $\{m_i\}$ and radial excitation $\{n_i\}$ scheme satisfying conditions

(i) $\sum_i m_i = M_{\rm tot}$; (ii) the position $r_{\max}^{|\psi_i^2|}$ of $|\psi_i^2|'s$ global maximal value lies outside $r_h^i = 2GM_i$ for all i; (iii) $r_{\max}^{|\psi_i^2|} \leqslant r_{\max}^{|\psi_{i+1}^2|}$ for all i to avoid repeated counting

contribute an allowable microscopic state, all constructible from the basic p⊗e scheme,

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\begin{array}{ll}
M_k & k \propto GM^2 \quad m_k, \; n_k = 2 \\
M_i & \cdots\cdots \quad m_i, \; n_i = 2 \\
M_1 & m_1, \; n_1 = 2 \\
M_0 & m_0, \; n_0 = 2
\end{array} \tag{10}
$$

through shell-recombination and radial-quantum re-excitation

$$
\begin{array}{ll}
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M_i & \cdots\cdots \quad m_i, \; n_i \geqslant 2 \\
M_1 & \longrightarrow m_2, \; n_1 \geqslant 2 \\
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\textbf{number possible ways}: \#\mathrm{p}\otimes\mathrm{e} = 2^{k-1} \cdot X(k) = \exp[c(\varepsilon) A_{\rm rea}/4G] + \log \mathrm{correc} \tag{12}
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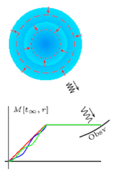
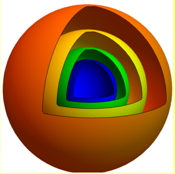
a typical example of shell-partition and radial-excitation, $M_{\rm tot}\!=\!3M_{\rm pl}$





$$M_3 \quad \boxed{}\ \boxed{m_3},\ n_3\!=\!2 \qquad (13)$$
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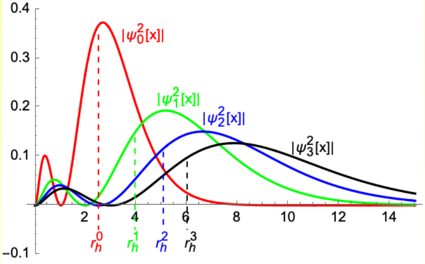

the most-finely shell-partition and radial-excitation assures all sub-shells have $n_i = 2$, so that the maximal value of the wave-function lies just outside the classic horizon corresponding with r_i.

wave functional of the whole system:
$\Psi[m(r)]\!=\!\psi_0(r,m_0)\otimes\psi_1(r,m_1)$
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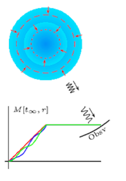

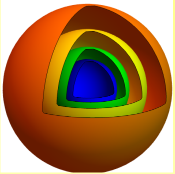

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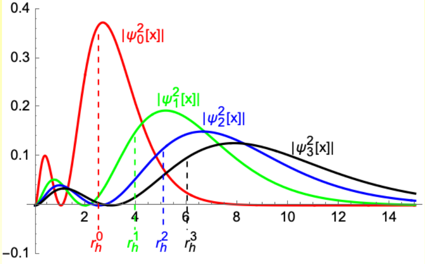

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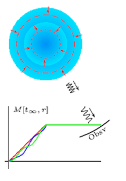

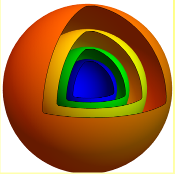

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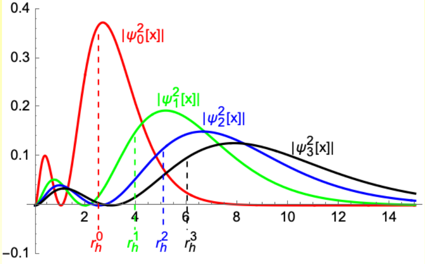

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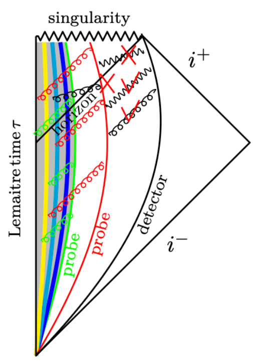
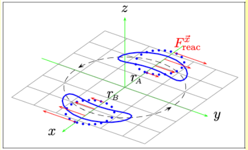
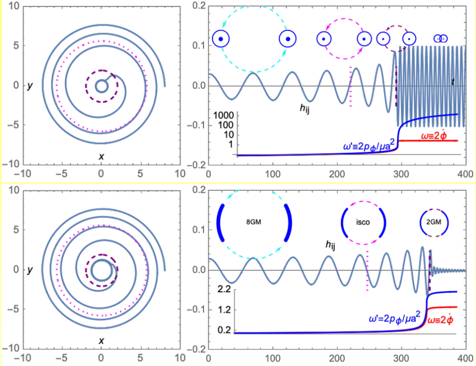
observational support, GW of binary merger process







♠ The late time damping GW of two initially spherical symmetric BHs' merger is a direct evidence for their no-horizon non-singular structure

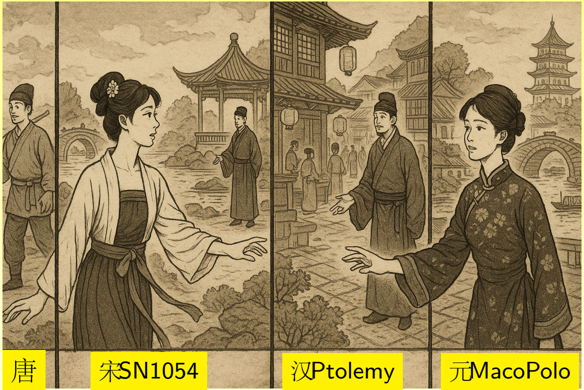
♡ NR yields us such GWs only because its shape-deformable apparent horizon and in-falling BC there gives an equivalent description for what happens to the collapsing materials there

◇ an eXact One-Body method to the binary problem of GR can show that the GW of exact Schwz's binary merger process does not damp [2505.14749]

conclusion

- ♠ If singularity exists in BHs of NR and AO, then the damping of GWforms from their binary merger process is difficult to understand on the bases of symmetry analysis
- ♡ geometrically definable $\neq$ physically realizable, the former is not necessarily physics measurable to us and computing worth-while when making physical predictions
- ♣ an ensemble of collapsars with equal mass but different radial profile in the Schwz time definition and an over-cross-oscillatory solid ball which ergodically experiences all possible modes are two equivalent description for the microscopic state of black holes formed through gravitational collapse
- ♡ all probes whose motion can be observed and reported lives in the coverage of Schwz time definition of their probing goal, so can only tell us inner structure of their probing goals before their horizon and singularity form
- ♠ both participants of the binary merger processes in NR and AO are just such probes, so what their behavior tells us is their probing goal's no-horizon and non-singular structure

Thanks for your attention!



I've waited for you long under the tree of time,
the dust pesters me and laughs at me for white head
. . .
in Tang's east Qiantang you're,
in Song's north Lin'an I am ,
you're gone coarse clothes blood-splashed,
I wailed you the waist yellow sashed ,
for compass errors of yamaking ,
i was reborn in Han's Quanting ,
as i reached Sui's city Hang， you're in pre-qin's Yuhang.

— "The Nymph" by DaoLang

At first sight, the song tells a love story. Deeply, a meditation on time, destiny and reincarnation. two youngs get known and fall in love each other, but they are separated. After each life, they are reincarnated with memory but each into different dynasties. the sequence of their reincarnation does not follow chronological order of true history ...

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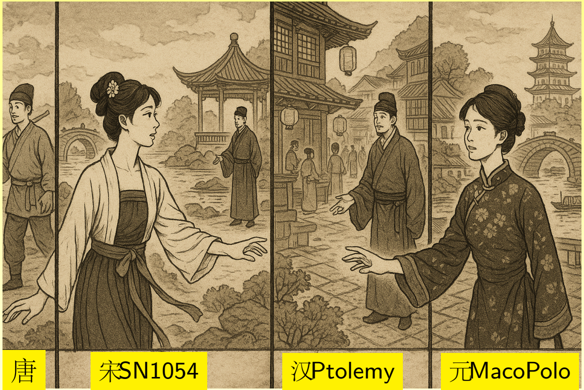

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