Inside Black Holes, Singularity to Complementarity

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Through two exact solution families to the Einstein equation and the one-to-one correspondence between their free parameters, we show that the ensemble of collapsars with only close-to-implementing horizon in the Schwarzschild time definition and the over-cross-oscillatory solid-balls in the Lemaitre time definition constitute two complementary description for the microscopic state of black holes formed through gravitational collapse. We quantise the solutions in the Schwarzschild time definition and show that the area law formula of Bekenstein-Hawking entropy follows naturally from the wave-functional's degeneracy of the collapsing material. In two companion works, supports from the gravitational wave of binary merger process and predictions for the fast radio burst of single body perturbations of this complementarity will be reported independently.

Introduction What's the form of matter's existence inside black holes (BHs) is a fundamental question to modern theoretical physics and astronomy. The mainstream answer to this question is based on the serial of singularity theorems proved by Penrose, Hawking and Geroch et al [1–3]. According to their theorems, any matter system controlled by general relativity (GR) and strong energy condition, when trapped surface forms, then finite affine length light-like (FALL) geodesics exist, which implies the incompleteness or singularity of spacetime because of contradiction with causality. Yet, with concrete example in 2023 [4], Roy Kerr point out that the terminal of FALLs is not necessarily the non-derivable point of spacetime, so the statement that gravitational collapse must cause singularity or unpredictability is a belief more than logic. Nevertheless, the prevailing view remains that by GR the only form of matter's existence inside BHs is singularity, i.e., a point or circular-line with infinite mass-energy density and divergent spacetime curvature. At the same time, by arguing that such singularities happen always behind the event horizon so are undetectable to any outside probes [5, 6], this belief places itself in a position unfalsifiable forever.

However, this belief contradicts the basic fact of BH thermodynamics very directly, especially the fact that BHs are entropic so microscopically diversifying. Facing with this contradiction, both string theory [7–10] and loop quantum gravity [11–14] choose giving up the singularity belief but embracing the thermodynamics differently; while some classic [15] and semi-classic magicians [16] try dancing with the two simultaneously under musics inaudible to all gravitational probes. The common feature of these attempts is, their inner structure picture of BHs are indistinguishable from that of singularity belief by the current and near future observations. In contrast to this status, we will report here a purely general relativistic way [17] of reconciling the contradiction between the BH thermodynamics and singularity theorem. Our key point is the complementary feature of gravitational collapse physics, according to which the metrics describing the microscopic states of BHs will be written out explicitly and their quantisation will yield degeneracies consistent with the area law formula of Bekenstein-Hawking (Bek-Hwk) entropy. The observational support and predictions of this reconciling way is rather remarkable and will be discussed in two other works [29, 30] independently.

Our logic goes as follows. (i) in the Schwarzschild time definition, the gravitation collapse of a massive star causes only a continuously contracting collapsar with close-to-implmenting but never successfully implemented event horizon; gravitational time dilation will assign the collapsar, spherically symmetric case e.g., an approximately $m(r) = r/2G_N$-type radial mass profile; an ensemble of collapsars with the same symmetry and total mass is needed to account for the initial state's uncertainty. (ii) in the Lemaitre time definition, the singularity happens inevitably but is not the terminal of any physical evolution; just like the horizon is a null hyper-surface in the Penrose-Carter diagram, the singularity is only an equal-time hyper-surface with divergent spacetime curvature; the unpredictability of physics across that surface implies that oscillations over-cross the central point of the collapsar are ergodic. (iii) the Ensemble of collapsars in the Schwarzschild time definition (EiS) and the Ergodically evolving over-cross-oscillatory solid-ball in the Lemaitre time definition (EiL) are two equally right and complete descriptions for the microscopic state of BHs formed through gravitation collapse, i.e. EiS=EiL.

We will call this relationship of equivalence and completeness as Complementarity inside BHs (CiB). Comparing with the BH complementarity principle of refs. [18–20], CiB is not a principle customised for resolving the information missing puzzle, although it has such potentials [17]; it is a general feature of gravitational physics derivable from the general covariance of GR. In our case, this only means that EiS can be obtained from the modes of EiL through the time coordinate's redefinition. On our developments of this recognition, we recommend readers to [21–26] and [27, 28]. In the remaining part of this work, we elaborate details of this relationship, from exact solutions to the Einstein equation in two time coordinates to the proof of the area law formula of Bek-Hwk entropy through quantisation, and to the potential observational evidence.

Exact solution families In the case of the gravitational force dominates over all other interactions, neglecting the material pressure of a collapsar is reasonable for the purpose of its microscopic state counting. So to avoid splitting attention to non-gravitational physics, we will focus on the collapse of spherical dust balls in this work only, leaving studies of the astronomically interesting stellar's collapse to ref.[30]. By the Schwarzschild time definition, it can be proven [17, 21] that the full space metric of such collapsar family can be written as

$$ds^2_{\text{full}}{=}-hFdt^2{+}h^{-1}dr^2{+}r^2d\Omega^2, h{=}1{-}\frac{2GM[t,r]}{r}, \quad (1)$$
$$F=1+h^{-2}\dot{M}^2/M'^2, M[t,r]^{\text{outside.matt.}}_{\text{occup.region}}\equiv 2GM_{\text{tot}},$$
$$M[t,r]\xrightarrow[r<2GM_{\text{tot}}]{t\to\infty}\frac{r}{2G}-\text{small fluct.}$$

The function form F here is determined by the normalisation of the collapsing volume element's four velocity $u^\mu=\{1,\frac{\dot M}{M'}\}\cdot\frac{dt}{d\tau}$. Substituting (1) into Einstein equation $R_{\mu\nu}{-}\frac{1}{2}g_{\mu\nu}=\varepsilon^{S}u_\mu u_\nu$ and using short-hand notation $m=GM[t,r]\equiv m^{S}$, we can derive the local energy density as $\varepsilon^{S}{=}\frac{2m'}{r^2}\cdot\frac{m'^2(1-2m/r)^2}{\dot m^2+m'^2(1-2m/r)^2}$ and

$$\frac{\dot m\dot m'}{m'^2}-[\frac{2m}{r(r{-}2m)}+\frac{m''}{m'}]\frac{\dot m^2}{m'^2}-\frac{(r{-}2m)m'}{r^2}{=}0, \quad (2)$$
$$\frac{\ddot m}{m'}-[\frac{3m}{r(r{-}2m)}+\frac{m''}{m'}]\frac{\dot m^2}{m'^2}-\frac{(r{-}2m)(m{+}2rm')}{r^3}{=}0. \quad (3)$$

Given the initial mass distribution $m(0,r)$ and its contracting speed at the boundary of the matter occupation region $\dot m(0,2GM_{\text{tot}})$, eq(2) allows us to determine the collapsing rate throughout the whole body $\dot m(0,r)$. With the help of which, eq(3) can be integrated routinely to yield the full form of $m(t,r)$, see ref.[17] for concrete examples. The key point here is, the microscopic state of the collapsar is totally determined by $m(0,r)$ and $\dot m(0,2GM_{\text{tot}})$.

Referring the upper part of FIG.1, due to the gravitational time dilation, the event horizon of the collapsar is only an close-to-implmenting but never successfully implemented physical boundary. To all probes defined in the Schwarzschild time definition, i.e. probes whose feature is measured in $-\infty{<}t{<}\infty$, the collapsar is only a continuously contracting object with approximately $M[\infty,r]\approx\frac{r}{2G}$-type radial radial mass function. An ensemble of collapsars with the same symmetry but different radial mass profile is needed to account for the initial distribution's uncertainty due to thermo fluctuations. So even though the collapsar under consideration develops no successfully implemented horizon, it is entropic. The next section will prove that it is just this abandoned information that forms the basis of Bek-Hwk entropy. But before that proof, we need show that the Schwarzschild time is not a wrongly chosen time coordinate. In the Lemaitre time definition, a totally peer-to-peer physical picture exists.

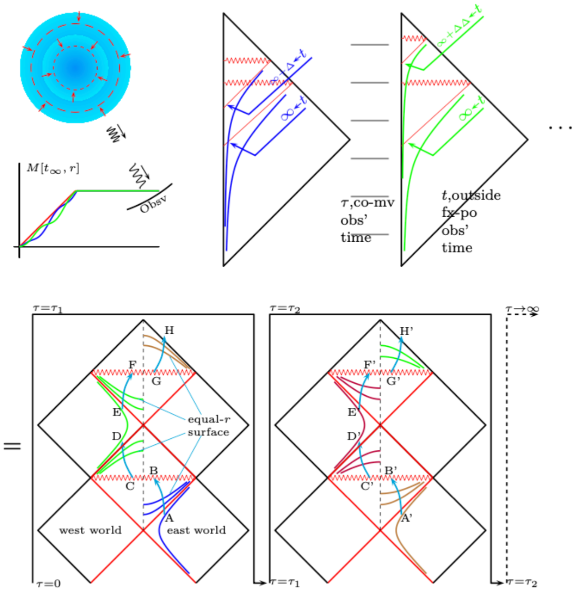


FIG. 1: In the Schwarzschild time definition, the horizon of a collapsar is only close-to-implmenting but never successfully implemented; to any macroscopic collapsar, an ensemble of microscopic collapsars with equal mass and symmetry but different radial mass profile is needed to account for the initial state's uncertainty. In the Lemaitre time definition, the singularity is inevitable but it is not the terminal of any physical evolution. The unpredictability of physics across the singularity only implies that collapsar material's oscillation across the central point is ergodic. This two pictures are equivalent due to general coordinate invariance.

Lemaitre time is the proper time of probes co-moving with the collapsing material. By this time definition, the spacetime metric [17, 22] of dust collapsars with inhomogeneous radial mass profile can be written as

$$ds^2_{\text{in}}{=}-d\tau^2{+}\frac{\big[1{-}\big(\frac{2m}{\varrho^3}\big)^{\frac{1}{2}}\frac{m'\varrho}{2m}\tau\big]^2d\varrho^2}{a[\tau,\varrho]}{+}a[\tau,\varrho]^2\varrho^2d\Omega_2^2, \quad (4)$$
$$ds^2_{\text{out}}{=}-d\tau^2{+}\frac{r_s^{2/3}d\varrho^2}{[\frac{3}{2}(\varrho{-}\tau)^{\frac{2}{3}}]}{+}[\frac{3}{2}(\varrho{-}\tau)^{\frac{2}{3}}]^2r_s^{\frac{2}{3}}d\Omega_2^2, \quad (5)$$
$$a[\tau\in|_0^{\frac{p^\varrho}{4}},\varrho]{=}a_0\big(1{-}\frac{4\tau}{p^\varrho}\big)^{\frac{2}{3}},\ a[\tau|^{\frac{p^\varrho}{2}}_{\frac{p^\varrho}{4}},\varrho]{=}{-}a[\frac{p^\varrho}{2}{-}\tau,\varrho], \quad (6)$$
$$a[\tau|^{p^\varrho}_{p^\varrho/2},\varrho]{=}{-}a[p^\varrho{-}\tau,\varrho],\ a[\tau|^{p^\varrho+}_{p^\varrho},\varrho]=a[\tau{-}p^\varrho,\varrho],$$

where $m=2GM[\varrho]\equiv m^{L}$ is the collapsar mass profile on the co-moving grid $\{\varrho,\theta,\phi\}$ determined by initial conditions, while $a[\tau,\varrho]$ is an oscillatory function of τ with ϱ dependent period $p^\varrho\equiv\frac{8}{3}\big(\frac{\varrho^3}{2GM[\varrho]}\big)^{\frac{1}{2}}$. The function form (6) follows from the Einstein equation in this time definition $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\{\frac{m'/\varrho^2}{a^{\frac{3}{2}}-\frac{t\varrho m'}{2\sqrt{m\varrho^3}}+\frac{3t^2m'}{4\varrho^2}}(\equiv\varepsilon^{L}),0,0,0\}$. It's easy to see that if one set $a_0 = 2^{-\frac{2}{3}}$ and $m^L[\varrho] = m^S[0, r]$, then ε^L will reduce to ε^S with $\dot{m}^S = 0$ directly. At the same time, outside the matter occupation region, by the following coordinate transformation

$$dt+\sqrt{\frac{r_s}{r}}\big(1-\frac{r_s}{r}\big)^{-1}dr\equiv d\tau, dt+\sqrt{\frac{r}{r_s}}\big(1-\frac{r_s}{r}\big)^{-1}dr\equiv d\varrho, \quad (7)$$

the metric (5) will reduce to the outside part of metric (1) directly. It can be checked that all initial mass profile and collapsing rate allowable by (4) have correspondences in (1), and vice versa. This means that, if the Lemaitre time geometry can completely describe the inner structure of a collapsar, then the Schwarzschild time geometry will do equally well.

In the Lemaitre time definition, the horizon forms in finite τ-duration and the singularity happens afterwards inevitably. But the point here is, the happening of singularity is not the terminal of any physical evolution, see [31, 32] for cosmic counterpart. In the Penrose-Carter diagram, such event corresponds just an equal-τ surface, see the lower part of FIG.1 for intuitions. After hitting on the central point, the collapsing materials will over-cross each other and oscillate afterwards. The unpredictability of physics across the singularity only means that such an oscillation is ergodic. That is, the oscillation will experience all possible modes characterised by $m^L[\varrho] \otimes a_0$. For example, before the singularity-crossing $\tau\in(-\frac{p^\varrho}{4}, \frac{p^\varrho}{4})$, the metric (4) has $m^L_1[\varrho] \otimes a_{0,1}$, but after the singularity-crossing $\tau\in(\frac{p^\varrho}{4}, \frac{3p^\varrho}{4})$, it has $m^L_2[\varrho] \otimes a_{0,2}$. What was written in (6) is an oscillatory mode which crosses the singularity smoothly. In the Schwarzschild time definition, this mode space will be characterised by $m^S[0, r] \otimes \dot{m}^S[t = 0, 2GM_{\rm tot}]$ At classic level, the size of this mode space is infinite and uncountable. But quantisations will change things abruptly.

The Origin of Bek-Hwk Entropy Essentially, the entropy of all physical objects arises from the information given up by the investigator about their inner structure and motion state. For the BHs formed through gravitational collapse, we will show in this section that the amount of information about their initial state in the Schwarzschild time definition or the oscillatory mode in the Lemaitre time definition has the exponentiated area law feature. So their Bek-Hwk entropy arises from the investigator's giving up of these information in believing that they have successfully implemented event horizon. Since the classic metric families (1) and (4) form a pair of equally right and complete descriptions for the microscopic state of BHs formed through gravitational collapse. We choose to quantise the microscopic state of collapsing materials characterised by the $m^S[0, r] \otimes \dot{m}^S[t = 0, 2GM_{\rm tot}]$ parameters in the metric family (1) and calculate the resultant wave-functional's degeneracy.

For this purpose, we consider the collapsar as a combination of many concentric shells m_i and use $\{m_i\}$ to denote an arbitrary partition of the total mass $M_{\rm tot}$ [17, 25, 26]. The effective geometry controlling each shell m_i's motion can be written as

$$ds^2 = -h_i dt^2 + h_i^{-1} dr^2 + r^2 d\Omega^2, h_i = 1 - \frac{2GM_i}{r}, \quad (8)$$

where $M_i = \sum_{i'=1}^{i} m_{i'}$ is the mass of shell i and its inside partners together. This shell's motion is determined by the standard geodesic equation and four velocity normalisation of a representative volume element on it, which can be written into a hamiltonian constraint

$$m_i \dot{x}^2 - \frac{GM_i m_i}{x} - m_i(\gamma_i^2 - 1) = 0, \quad (9)$$

where $\gamma_i = h_i \dot{t}$ is an integration constant of the geodesic equation. $\gamma_i = 0$ and 1 corresponds to cases the shell is released from $x^i_{\rm ini} = 2GM_i$ and $x^i_{\rm ini} = \infty$ respectively. This equation can be quantised canonically

$$[-\frac{\hbar^2}{2m_i}\partial_x^2 - \frac{GM_i m_i}{x} - m_i(\gamma_i^2 - 1)]\psi_i(x) = 0, \quad (10)$$

where $\psi_i(x)$ denotes the probability amplitude the shell be measured of size x. All other shells can be done similarly. Directly multiplying their wave-functions together, we will get the wave functional of the whole collapsar as follows

$$\Psi[M(r)] = \psi_0 \otimes \psi_1 \otimes \psi_2 \cdots, \sum_i m_i = M_{\rm tot}. \quad (11)$$

Except the normalisation of ψ_i, equation (10) is almost the standard eigenstate Schrodinger equation with coulomb potentials. Its solution [23–26] can be written down immediately

$$\psi_i = N_i e^{-x} x L^1_{n_i-1}(2x), x \equiv m_i r(1\text{-}\gamma_i^2)^{\frac{1}{2}}/\hbar \quad (12)$$

$$n_i = \frac{GM_i m_i}{\hbar(1\text{-}\gamma_i^2)^{\frac{1}{2}}} = 1, 2, 3\cdots \quad (13)$$

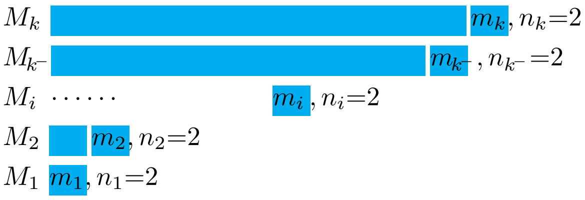
where $L^1_{n_i-1}(2x)$ is the associated Lagurre polynomial and N_i is the normalisation of single shell's wave-function; n_i is the corresponding radial excitation level.

The wave-functional (11) of the whole collapsar is determined by the scheme of shell **p**artition $\{m_i\}$ and radial **e**xcitation $\{n_i\}$. Any $\mathbf{p} \otimes \mathbf{e}$ scheme satisfying

$$\begin{aligned}&(i)\textstyle\sum_i m_i{=}M_{\rm tot}; (ii)\text{the position } r_{\rm max}^{|\psi_i^2|} \text{ of} |\psi_i^2|'s \text{ global}\\ &\text{maximal value lies outside } r_h^i{=}2GM_i \text{ for all } i \text{ (to}\\ &\text{avoid shells fall into horizon in finite } t \text{ time)}\\ &(iii) r_{\rm max}^{|\psi_i^2|}\leqslant r_{\rm max}^{|\psi_{i+1}^2|} \text{for all } i \text{ (to avoid repeating)}\end{aligned} \quad (14)$$

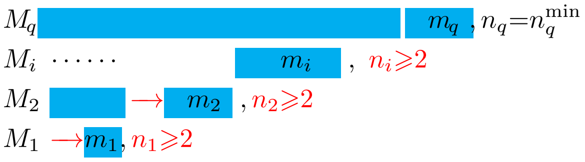
will lead to a possible microscopic state of the collapsar. These states can be constructed from the ground $\mathbf{p} \otimes \mathbf{e}$

scheme represented the following way,



(15)

through the shell's recombination and the radial excitation's reassigning,



(16)

The $\mathbf{p}\otimes\mathbf{e}$ scheme (15) is called ground because in it the global maximal values of all shells' wave function happen on their minimal possible radial position allowed by the three conditions in (14) so that all $\gamma_i = 0$ and all n_i=2,

$$\frac{GM_i m_i}{\hbar(1-0)^{\frac{1}{2}}} = 2, \forall i \in \{1, 2, \cdots, k\} \Rightarrow k = \frac{GM_{\rm tot}^2}{2\sqrt{2}\hbar}. \tag{17}$$

This derivation holds as long as $M_{\rm tot}$ is mildly larger than $M_{\rm pl}$[17, 23]. The radial excitation number $n_q^{\min}$ of the excited $\mathbf{p}\otimes\mathbf{e}$ (16) is determined by the requirement that the global maximal value happens on the minimal position $r_{\max}^{|\psi_q^2|}$ allowed by (14)-(ii), i.e. $2GM_{\rm tot} < r_{\max}^{|\psi_q^2|}$. The number of ways recombining the shells in (15) is exactly 2^k. The correspondence between parameters characterising the quantum wave-function and classic inner-structure geometry reads

$$\{m_i\} \otimes \{\prod_i^{\otimes} n_i\} \leftrightarrow m^S(0, r) \otimes \acute{m}(0, 2GM_{\rm tot}). \tag{18}$$

For certain recombined-shell configuration [17], the radial excitation scheme $\prod_i^{\otimes} n_i$ allowed by (14) have a polynomial type non-uniqueness $X(A)$. So the degeneracy of the wave-functional (11) becomes

$$W = \exp\{c(\epsilon)[A/4G\hbar + \ln X(A)]\}, A = 4\pi(2GM_{\rm tot})^2, \tag{19}$$

where ϵ parameterises the precision of the shell partition and $c(\epsilon)$ is an ϵ-dependent constant and $M_{\rm tot}$ is only required to be mildly larger than $G^{-\frac{1}{2}} \equiv M_{\rm pl}$. The parameter ϵ enters here because we need a standard to define to what degree two partition schemes $\{m_i\}$ and $\{m_i'\}$ satisfying conditions (14) simultaneously and have equal $\{n_i\}$ are distinguishable from each other. This parameter affects the value of k linearly so by setting $\epsilon = \frac{\ln 2}{8\pi}$ we will get the exact Bek-Hwk entropy formula with logarithmic corrections.

Because the parameters $m^L[\varrho] \otimes a_0$ and $m^S[0, r] \otimes \acute{m}^S[0, 2GM_{\rm tot}]$ in the classic geometries are one-to-one correspondence, the above proof implies that the quantised over-cross-oscillatory solid-balls in the Lemaitre time defintion will lead to the same results as the quantised collapsars in the Schwarzschild time definition. So the two pictures are two equally right and complete descriptions for the microscopic state of BHs formed through gravitational collapse. The Bek-Hwk entropy of BHs arises from the investigator's giving up of the information embodied in the collapsing material's initial distribution or the over-cross-oscillatory material's oscillatory mode. This is possible, because in our interpretation, the carrier of the fundamental degrees of freedom is the collapsing material's collective motion mode instead of their composing particles. According to (13) the typical mass of these modes is $1/GM$, so their total number is $\mathcal{O}[GM^2]$. Since this is the typical mass of Hawking particles, it is very natural to link this fundamental degrees of freedom with the information missing puzzle [23, 24]. In fact, this is just the road we reach the quantitative CP in this work.

Potential observational signals Our interpretation for the origin of Bek-Hwk entropy implies that all BHs formed through gravitational collapse have extended internal-mass distribution and only close-to-implmenting but not successfully implemented horizon to all probes living in their Schwarzschild time definition. When two BHs inspiral and merge, both are probes and being probed objects of their partners, so will see no horizons on each other but will exhibit the change of internal mass distribution under the gravitation and radiation back-reactions. This signal may have been detected in the BH binary's merger events [33–35] but not extracted properly. In [29], we will refine the exact one body method proposed in [17] and calculate the gravitational wave (GW) form of such events theoretically and uncover that, their quasi-normal mode feature in late time stage is possible only when BHs have extended inner structure and not successfully implemented horizon. Numeric relativity [36–39] provides us such waveforms only because it replaces the change of internal mass distribution of practical BHs with the shape deformation of their apparent horizon. Other signals such as echos from BHs with incomplete absorbing horizon [40–46] and special features of the shadow image of string theory fuzzy balls [47, 48] are also possible according to our inner structure picture of BHs.

To astronomical BHs, possessing extended inner-mass distribution and only close-to-implmenting but never successfully implemented horizon will make them Neutron Star (NS) like collapsars more than no hair BHs. Their differences relative to NSs may only be, BHs have continuously contracting matter core and plasmonic crusts while NSs have static and balanced matter cores and crusts; BHs have magnetic dipoles parallel with their angular momentum while NSs can have these two quantities point to different directions. We will show in [30] that perturbations in the dynamically contracting plasmonic crust of this BHs caused by the inhomogeneous neutron synthesis process will oscillate as relativistic sound waves and stimulate fast radio bursts by changing the equivalent molecular current of the magnetic field. In the case of no perturbation, such BHs will not radiate as NSs because their magnetic field aligns with angular momentum. This is a new mechanism of fast radio bursts which is not allowed by other BH inner-structure pictures reviewed above. We will take it as an exclusive prediction of our inner-structure picture for BHs and discuss in depth in [30].

Conclusion and discussion Our analysis here shows that GR permits two complementary description for the microscopic state of BHs formed through gravitational collapse: (1) continuously contracting collapsars with close-to-implmenting but never successfully implemented horizon in the Schwarzschild time definition, and (2) over-cross oscillatory solid-balls ergodically but unpredictably crossing the singularity in the Lemaitre time definition. This complementarity emerges naturally from the general coordinate invariance of GR rather than apriori assumptions required of any unknown quantum gravitation theory. The area-law formula of Bekenstein-Hawking entropy arises from the degeneracy of the quantised wave functional of the collapsing matter's configuration. This resolves the contradiction between the statistic feature BH thermodynamics and the deterministic scenario of singularity theorems. It suggests that BHs maintain extended matter distribution of their progenitors rather than develop successfully implemented singularity or event horizon, of course in the Schwarzschild time definition, whose domain corresponds to all outside observations. This is a remarkable prediction testable through gravitational wave spectroscopy and multi-messenger astronomy.

In contrast with the thermodynamic analogue [49–51] and the Euclidean path integration proof [52–54] based on the saddle point approximation, our proof of the area-law formula here provides a truly microscopic interpretation for the origin of Bek-Hwk entropy. Our interpretation challenges the conventional ones but are fully compatible with GR. Unlike string theory's fuzz-balls or loop quantum gravity's quantised geometries, our complementarity requires not any beyond-GR mechanisms. It only adopts the inherent state-counting freedom within Einstein's theory itself. The predicted "horizonless" phenomenology differs crucially from traditional BH models in three aspects: (1) the ring-down feature of binary merger process contain effects from the BH internal structure's deformation; (2) electromagnetic counterparts may show characteristic timing signatures from their crust plasmonic oscillation; (3) quantum emission processes maintain unitary evolution without information loss. The former two aspects enable direct observational tests using existing gravitational wave detectors and radio transient surveys.

The last aspects concerns the nature of quantum gravity. By our complementarity, BHs are just super massive atoms bounded by gravitation, they will experience gravity induced spontaneous radiation just as usual atoms experience electromagnetic force induced spontaneous radiation [23, 24]. This radiation have exactly thermal spectrum due to the BHs' exponentially high degrees of microscopic state degeneration. It allows for explicitly hermitian hamiltonian description thus no information missing in any sense, see [17] for analysis related with the fire wall paradoxes and this complementarity. The analsys there implies that the contradiction between GR and quantum mechanics is not so remarkable as they are commonly imagined. This is obviously a positive news for their unification [55].

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[1] R. Penrose, "Gravitational Collapse and Space-Time Singularities", *Phys. Rev. Lett.* **14 (3)**, 57(1965)

[2] S. Hawking, "Breakdown of Predictability in Gravitational Collapse", *Phys. Rev.* **D14,** 2460 (1976)

[3] R. Geroch and G. Horowitz, "Global structure of spacetimes, in General Relativity: An Einstein Centenary Survey", **pp. 212–293**, 1979.

[4] R. P. Kerr, "Do Black Holes have Singularities?" `doi:10.48550/arXiv.2312.00841`

[5] R. Penrose, "Gravitational collapse: The role of general relativity", *Riv. Nuovo Cim.* **1** (1969) 252, *Gen. Rel. Grav.* **34** (2002) 1141, `DOI:10.1023/A:1016578408204`.

[6] R. Penrose, "Singularities and time-asymmetry", Penrose, R. Israel, W. (Ed.). (1979). Singularities and time-asymmetry. United Kingdom: University Press.

[7] A. Strominger, C. Vafa, "Microscopic Origin of the Bekenstein-Hawking Entropy", *Phys. Lett.* **B379** (1996)99, `arXiv:hep-th/9601029`.

[8] G. Horowitz, A. Strominger, "Counting States of Near-Extremal Black Holes", *Phys. Rev. Lett.* **77** (1996) 2368-2371, `doi.org/10.1103/PhysRevLett.77.2368`

[9] O. Lunin and S. D. Mathur, "Statistical Interpretation of the Bekenstein Entropy for Systems with a Stretched

Horizon", *Phys. Rev. Lett.* **88**(2002) 211303, DOI:10.1103/PhysRevLett.88.211303

[10] K. Skenderis and M. Taylor, "Fuzzball Solutions for BHs and D1-Brane–D5-Brane Microstates", *Phys. Rev. Lett.* **98** (2007) 071601, DOI:10.1103/PhysRevLett.98.071601.

[11] C. Rovelli, "Black hole entropy from loop quantum gravity", *Phys. Rev. Lett.* **77** (1996) 3288, arXiv:gr-qc/9603063.

[12] A. Ashtekar, J. Baez, A. Corichi, K. Krasnov, "Quantum Geometry and Black Hole Entropy", *Phys. Rev. Lett.* **80** (1998) 904, arXiv:gr-qc/9710007.

[13] A. Ashtekar and M. Bojowald, "Quantum geometry and the Schwarzschildsingularity", *Classical and Quantum Gravity* **23** (2006) 391–411, arXiv:gr-qc/0509075.

[14] L. Modesto, "Loop quantum black hole", *Classical and Quantum Gravity* **23** (2006) 5587–5601, arXiv:gr-qc/0509078.

[15] S. Hawking, M. Perry, A. Strominger, "Soft Hair on Black Holes", *Phys.Rev.Lett.* **116** (2016) 231301, DOI:10.1103/PhysRevLett.116.231301, arXiv:hep-th/1601.00921

[16] V. Balasubramanian, A. Lawrence, J. Magan, M. Sasieta, "Microscopic origin of the entropy of astrophysical black holes", *Phys.Rev.Lett* **132** (2024) 141501, DOI:10.1103/PhysRevLett.132.141501, arXiv:hep-th/2212.08623.

[17] Ding-fang Zeng, "Microscopic State of BHs and an Exact One Body Method for Binary Dynamics in General Relativity", *EPJC* **84** (2024) 370, arXiv:2311.11764

[18] G. 't Hooft, "The BH Interpretation of String Theory", *Nucl. Phys.* **B335** (1990) 138, doi.org/10.1016/0550-3213(90)90174-C

[19] K. Schoutens, E. Verlinde, H. Verlinde "Quantum BH Evaporation", *Phys.Rev.* **D48** (1993) 2670, doi.org/10.1103/PhysRevD.48.2670

[20] L. Susskind, L. Thorlacius, and J. Uglum, "The stretched horizon and BH complementarity" *Phys. Rev.* **D48** (1993) 3743, doi.org/10.1103/PhysRevD.48.3743

[21] Ding-fang Zeng, "Resolving the Schwarzschild singularity in both classic and quantum gravities", *Nucl. Phys.* **B917** (2017) 178, arXiv: 1606.06178, doi.org/10.1016/j.nuclphysb.2017.02.005.

[22] Ding-fang Zeng, "Exact inner metric and microscopic state of AdS$_3$-Schwarzschild BHs", *Nucl. Phys.* **B954** (2020) 115001, arXiv: 1812.06777, doi.org/10.1016/j.nuclphysb.2020.115001.

[23] Ding-fang Zeng, "Spontaneous Radiation of BHs", *Nucl. Phys.* **B977** (2022) 115722, arXiv: 2112.12531, doi.org/10.1016/j.nuclphysb.2022.115722

[24] Ding-fang Zeng, "Gravity Induced Spontaneous Radiation", *Nucl. Phys.* **B990** (2023) 116171,arXiv: 2207.05158, doi.org/10.1016/j.nuclphysb.2023.116171

[25] Ding-fang Zeng, "Schwarzschild Fuzzball and Explicitly Unitary Hawking Radiations", *Nucl. Phys.* **B930** (2018) 533-544, arXiv: 1802.00675, doi.org/10.1016/j.nuclphysb.2018.03.012.

[26] Ding-fang Zeng, "Information missing puzzle, where is hawking's error?", *Nucl. Phys.* **B941** (2018) 665, arXiv: 1804.06726, doi.org/10.1016/j.nuclphysb.2019.02.023.

[27] E. Greenwood, D. Stojkovic, "Quantum gravitational collapse: non-singularity and non-locality" *JHEP* **0806** (2008) 042, 0802.4087

[28] A. Saini, D. Stojkovic, "Non-local (but also non-singular) physics at the last stages of gravitational collapse", *Phys. Rev.* **D89** (2014) 044003 1401.6182

[29] Ding-fang Zeng, "Inspiral to Merger, a Full Process Applicable One Body Method to the Two Body problem in General Relativity", to appear as the second cousin of this manuscript.

[30] Ding-fang Zeng, "Complementarity inside BHs, Fast Radial Burst as a Prediction", to appear as the third cousin of this manuscript.

[31] L. Boyle, K. Finn, and N. Turok, "CPT-Symmetric Universe", *Phys. Rev. Lett.* **121** (2018) 251301, DOI:10.1103/PhysRevLett.121.251301

[32] L. Boyle, K. Finn, N. Turok, "The Big Bang, CPT, and neutrino dark matter", *Annals of Physics* **438** (2022) 168767, DOI:10.1016/j.aop.2022.168767

[33] "GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs", GWTC-1: *Phys. Rev.* **X9** (2019) 031040, arXiv: 1811.12907, doi.org/10.1103/PhysRevX.9.031040.

[34] "GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run", GWTC-2: *Phys. Rev.* **X11** (2021) 021053, doi.org/10.1103/PhysRevX.11.021053.

[35] "GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run", GWTC-3: doi.org/10.48550/arXiv.2111.03606.

[36] M. Ansorg, B. Brugmann, W. Tichy, "Single-domain spectral method for black hole puncture data", *Phys. Rev.* **D70** (2004) 064011, arXiv: gr-qc/0404056 doi.org/10.1103/PhysRevD.70.064011

[37] Z. B. Etienne, J. A. Faber, Y. T. Liu et al, "Filling the holes: Evolving excised binary black hole initial data with puncture techniques", *Phys. Rev.* **D76** (2007) 101503, doi.org/10.1103/PhysRevD.76.101503.

[38] V. Varma, M. A. Scheel,H. P. Pfeiffer, "Comparison of binary black hole initial data sets", *Phys. Rev.* **D98** (2018) 104011, arXiv: 1808.08228, doi.org/10.1103/PhysRevD.98.104011.

[39] D. Pook-Kolb, O. Birnholtz, B. Krishnan, and E. Schnetter, "Interior of a Binary Black Hole Merger", *Phys. Rev. Lett.* **123** (2019) 171102, arXiv:1903.05626. doi.org/10.1103/PhysRevLett.123.171102

[40] V. Cardoso, S. Hopper, C F. B. Macedo and et al, "Gravitational-wave signatures of exotic compact objects and of quantum corrections at the horizon scale", *Phys. Rev.* **D94** (2016) no.8, 084031, arXiv:1608.08637.

[41] V. Cardoso, V. F. Foit, M. Kleban, "Gravitational wave echoes from black hole area quantization", *JCAP* **08** (2019) 006, arXiv: 1902.10164, doi.org/10.1088/1475-7516/2019/08/006.

[42] E. Maggio, A. Testa, S. Bhagwat, P. Pani, "Analytical model for gravitational-wave echoes from spinning remnants", *Phys. Rev.* **D100** (2019) 064056 (2019), arXiv: 1907.03091, doi.org/10.1103/PhysRevD.100.064056

[43] G. D'Amico, N. Kaloper, "On Black Hole Echoes", *Phys. Rev.* **D102** (2020) 044001, arXiv: 1912.05584, doi.org/10.1103/PhysRevD.102.044001.

[44] I. Agullo, V. Cardoso, A. del Rio etal, "Potential gravitational-wave signatures of quantum gravity", *Phys. Rev. Lett.* **126** (2021) 041302, arXiv: 2007.13761, doi.org/10.1103/PhysRevLett.126.041302.

[45] R.f. Dong, D. Stojkovic, "Gravitational wave echoes

from black holes in massive gravity", *Phys. Rev.* **D103** (2021) 026058, arXiv: 2011.04032, `doi.org/10.1103/PhysRevD.103.024058`.

[46] S. Chakraborty, E. Maggio, A. Mazumdar, P. Pani, "Implications of the quantum nature of the black hole horizon on the gravitational-wave ring-down", *Phys. Rev.* **D106** (2022) 024041, arXiv: 2202.09111. `doi.org/10.1103/PhysRevD.106.024041`.

[47] F. Bacchini, D. R. Mayerson, B. Ripperda et al, "Fuzzball Shadows: Emergent Horizons from Microstructure", *Phys. Rev. Lett.* **127** (2021) 171601

[48] D. R. Mayerson, "Modave Lectures on Horizon-Size Microstructure, Fuzzballs and Observations", `arxiv:2202.11394`

[49] S. W. Hawking, "Gravitational Radiation from Colliding Black Holes", `Phys.Rev.Lett.26,1344`

[50] J. D. Bekenstein, "Black Holes and Entropy", `Phys.Rev.D7,2333`

[51] J. D. Bekenstein "Generalized second law of thermodynamics in black-hole physics", `Phys.Rev.D9,3292`

[52] A. Sen, "Logarithmic corrections to Schwarzschild and other non-extremal BH entropy in different dimensions", `DOI:10.1007/JHEP04(2013)156`

[53] G. W. Gibbons and S. W. Hawking, "Action integrals and partition functions in quantum gravity", *Phys. Rev.* **D15** (1977) 2752, `doi.org/10.1103/PhysRevD.15.2752`.

[54] J.D. Brown, J.W. York, "The Path Integral Formulation of Gravitational Thermodynamics", reports on the conference "The Black Hole 25 Years After", Santiago, Chile, January 1994 `doi.org/10.48550/arXiv.gr-qc/9405024`

[55] G. 't Hooft, "Conversations on Quantum Gravity", J. Armas (Ed.) (2021), pp275. Cambridge Press.