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The Missing Mass Problem in Galaxy Clusters

1st BiCoQ Conference: from gravity to particles

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But first... What is a galaxy cluster?

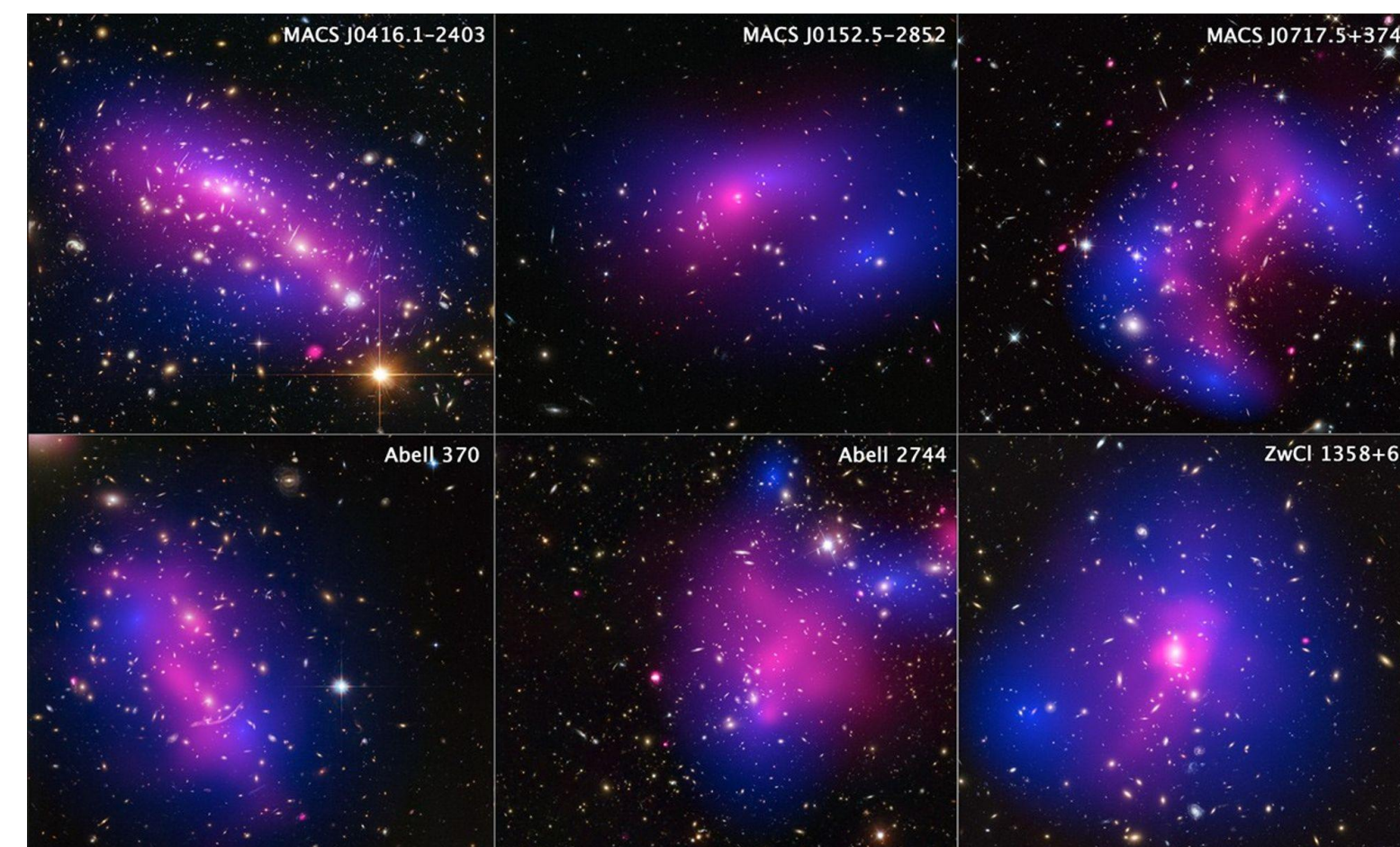
- **Largest gravitationally bound structures**, size of ~ 2 Mpc ($1 \text{ Mpc} = 3.26 \times 10^6 \text{ ly}$) \rightarrow exceptional probes for testing gravitational theories.
- **Thousands of galaxies** typically distributed around a *Brightest Cluster Galaxy* or **BCG**: only 2% - 5% of the total mass.
- **Hot ionized plasma** called *Intra Cluster Medium* or **ICM**: 10% - 20% of the total mass.



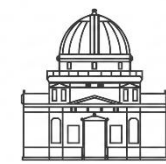
Credits: NASA, CXC, MIT, STScI

What is the “missing mass” problem?

- In **Newtonian gravity** **more mass is required** both in **galaxies** and **galaxy clusters** than it is observed → **dark matter (DM)** hypothesis.
- In a **Modified Newtonian Dynamics (MOND)** framework **no DM** is needed in galaxies **but more mass is still required** in galaxy clusters than it is observed.
- What would be the **nature** of this “missing mass” in a MOND framework? (cold gas clouds, sterile neutrinos ...)



Credits: NASA, ESA, STScI



Dataset & initial conditions

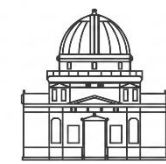
Sample of 5 galaxy clusters:

- Abell 1795
 - Abell 2029
 - Abell 2142
 - Abell 644
 - Abell 2319
- Signs of mergers detected

from the XCOP survey from the work of [Kelleher & Lelli \(2024\)](#) using X-ray data.

INITIAL HYPOTHESIS AND CONSTRAINTS

- Hydrostatic equilibrium.
 - Spherical symmetry.
 - Fitting procedure and analysis up to $r = 1$ Mpc.
- $M_N(r)$



The Modified Newtonian Dynamics

Generalised non-linear Poisson equation:

$$\nabla \cdot \left(\mu \left(\frac{|\nabla\Phi|}{a_0} \right) \nabla\Phi \right) = 4\pi G\rho \quad \longrightarrow \quad \begin{cases} \mu(x) = 1 & \text{if } x \gg 1 \text{ Newtonian limit} \\ \mu(x) = x & \text{if } x \ll 1 \text{ Deep MOND regime} \end{cases} \quad \text{e.g. } \mu(x) = \frac{x}{1+x}$$

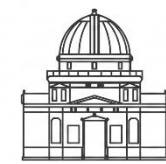
$$a_0 = 1.2 \times 10^{-10} \text{ m/s}^2 \quad \text{Milgrom (1983a, b, c)}$$

$$x = \frac{g_{\text{MOND}}}{a_0}$$

In spherical symmetry:

$$v \left(\frac{GM_{\text{tot}}}{r^2 a_0} \right) \frac{GM_{\text{tot}}}{r^2} = g_{\text{MOND}} \quad \longrightarrow \quad v_1(y) = \frac{1 + \sqrt{1 + 4/y}}{2} \quad v_2(y) = \frac{1}{1 - \exp(-\sqrt{y})}$$

$$y = \frac{g_N}{a_0}$$



Finding the total mass in MOND

Total mass in MOND with v_1 :

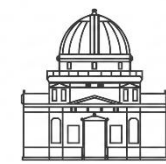
Putting $g_{\text{obs}} = g_{\text{MOND}} = \frac{GM_{\text{tot},N}}{r^2}$ and considering that $\mu\left(\frac{g_{\text{MOND}}}{a_0}\right) g_{\text{MOND}} = \frac{GM_{\text{tot},\text{MOND}}}{r^2}$ we can find

the **relation** between the **total MONDian mass** and the **total Newtonian mass**:

$$M_{\text{tot},\text{MOND}} = \frac{M_{\text{tot},N}}{(r^2 a_0 / GM_{\text{tot},N}) + 1} \longrightarrow M_{\text{mm}}(r) = M_{\text{tot},\text{MOND}}(r) - M_{\text{bar}}(r)$$

Total mass in MOND with v_2 : we have to perform a **numerical inversion**.

Goal of the analysis \longrightarrow test the “missing mass follows gas” hypothesis.



Missing mass profile and fitting procedure

Fitting model:

$$M_{mm}(r) = 4\pi \int_{r_0}^r dr' \eta r'^2 \rho_{gas}(r') e^{-r'/r_{cut}} \quad \text{Famaey, Pizzuti & Saltas (2025)}$$

MCMC sampling:

- Uniform flat priors $\rightarrow 10 < r_{cut} < 2000$ kpc and $0.5 < \eta < 100$
- Log-likelihood assumes Gaussian uncertainties

$$\ln \mathcal{L} = -0.5 \sum_i \left[\frac{(M_{mm,i} - M_{model,i})^2}{\sigma_i^2} + \ln(2\pi\sigma_i^2) \right]$$

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$\rho_{bar}(r')$ or $\rho_{star}(r')$

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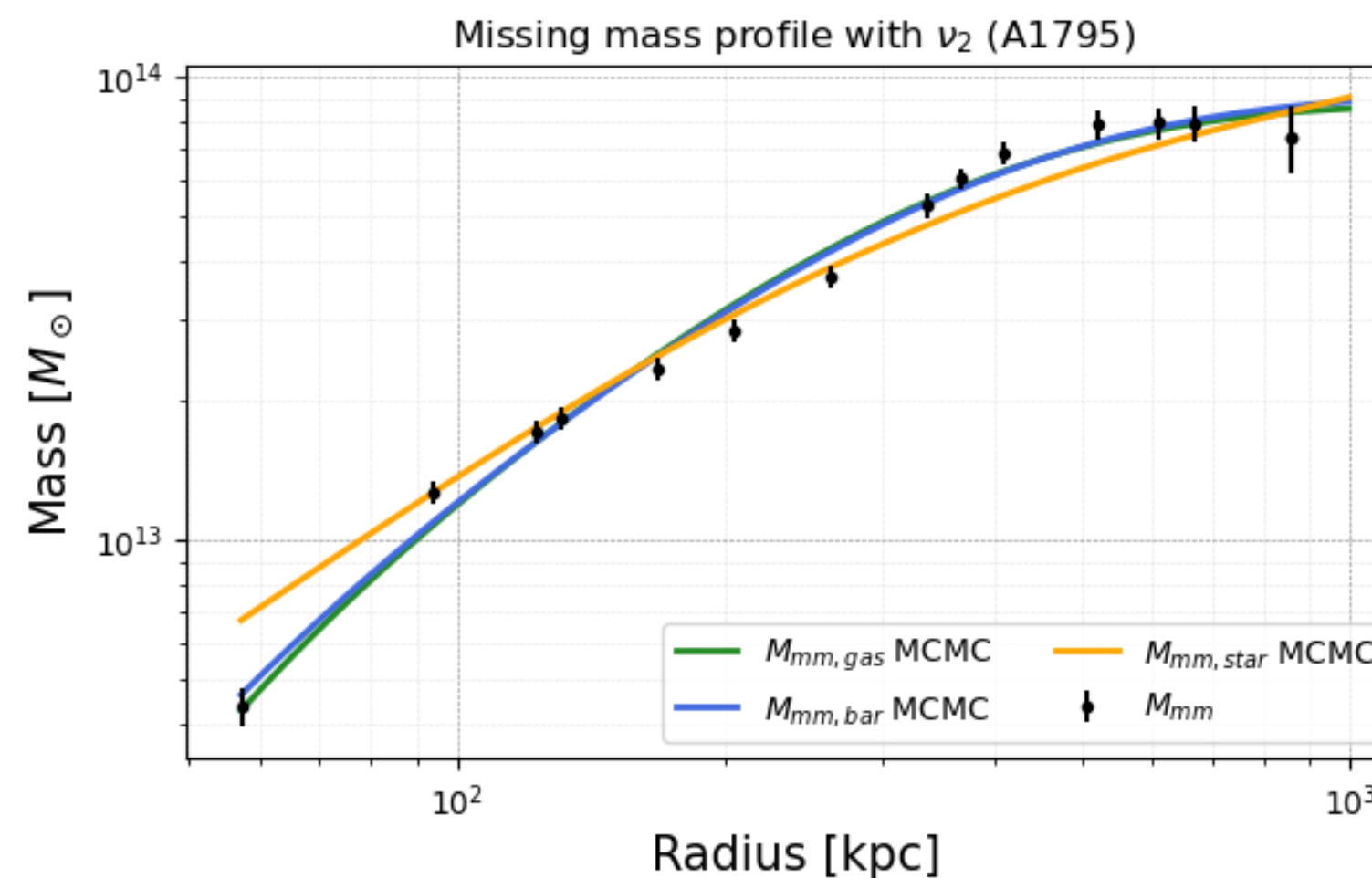
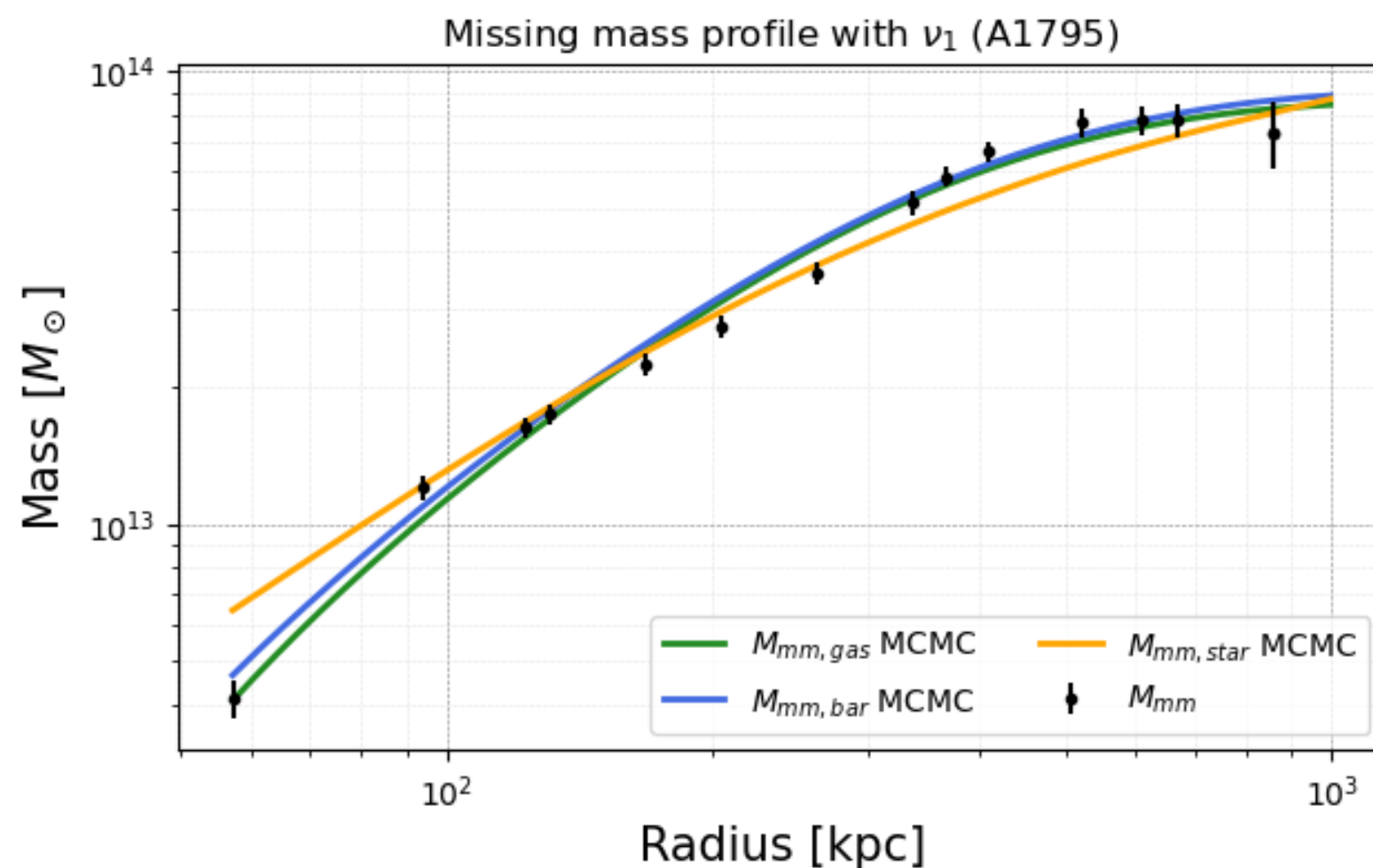
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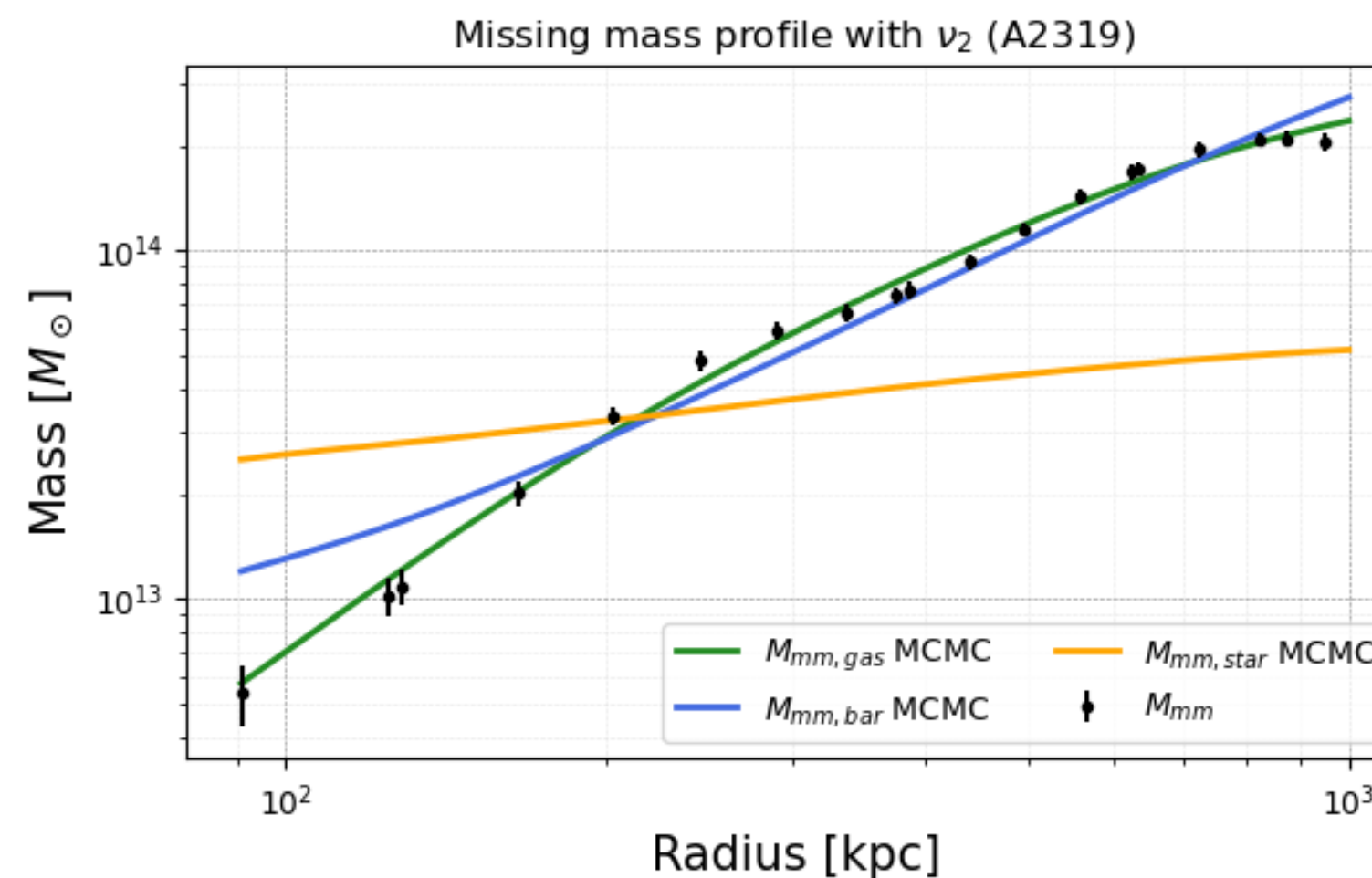
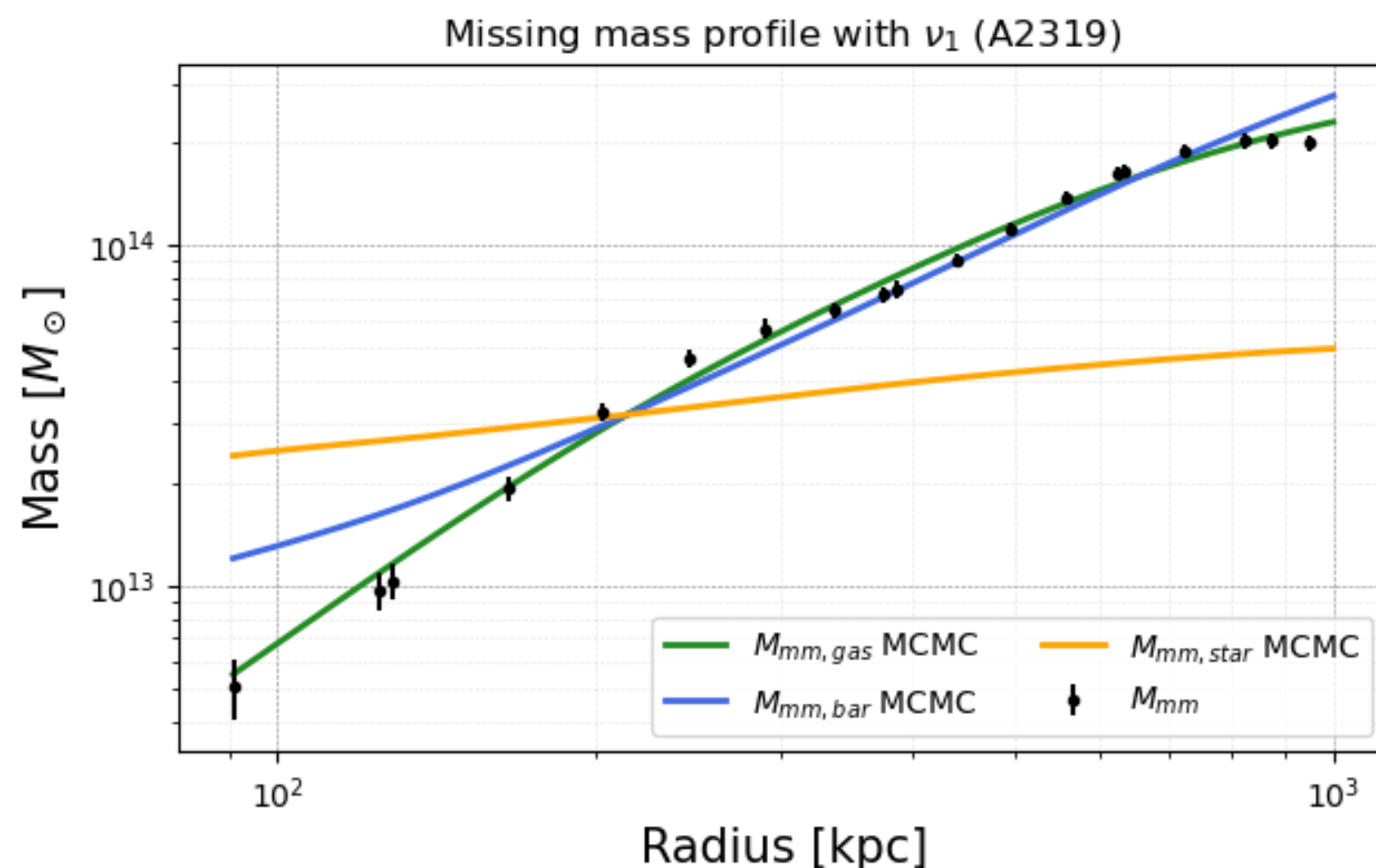


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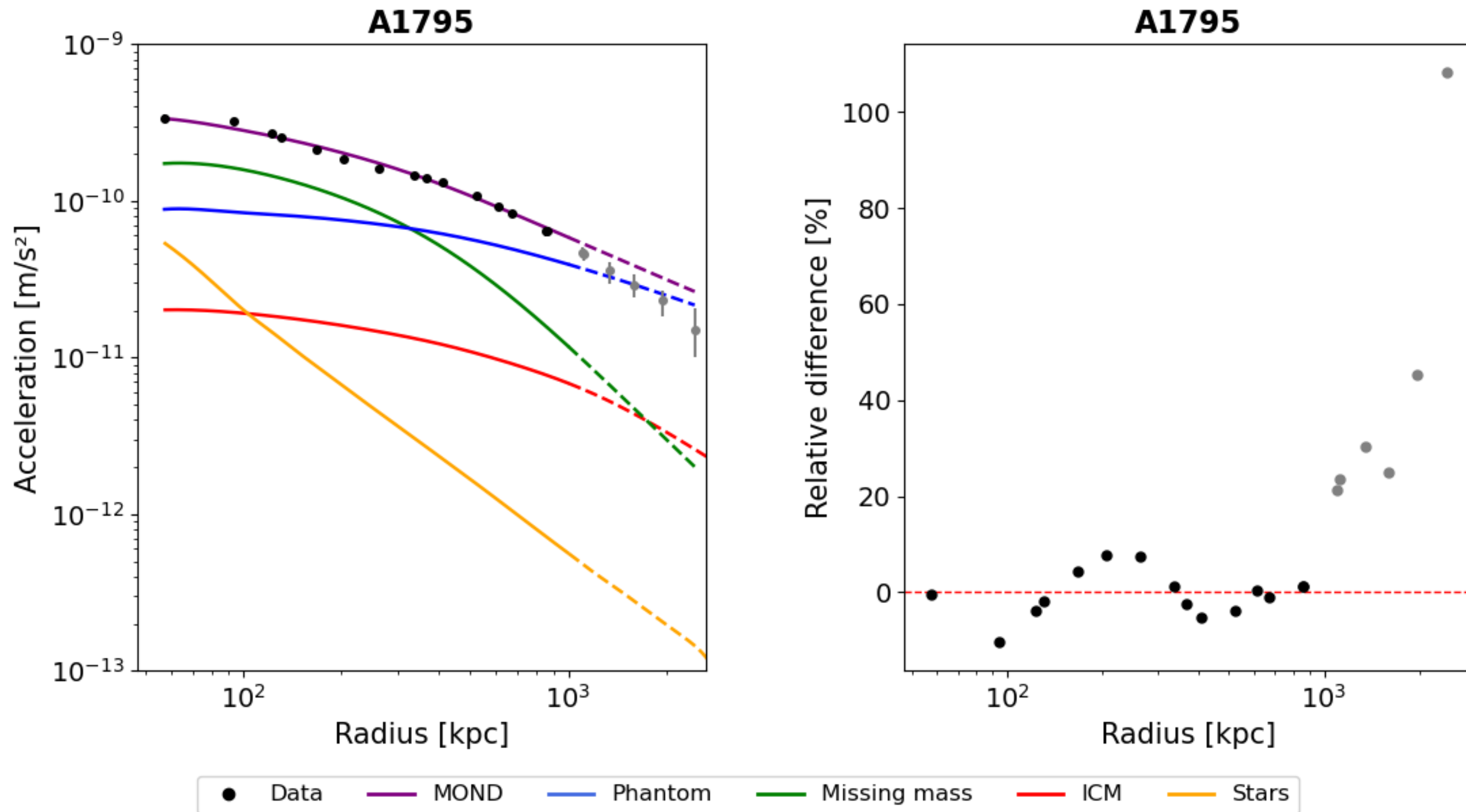
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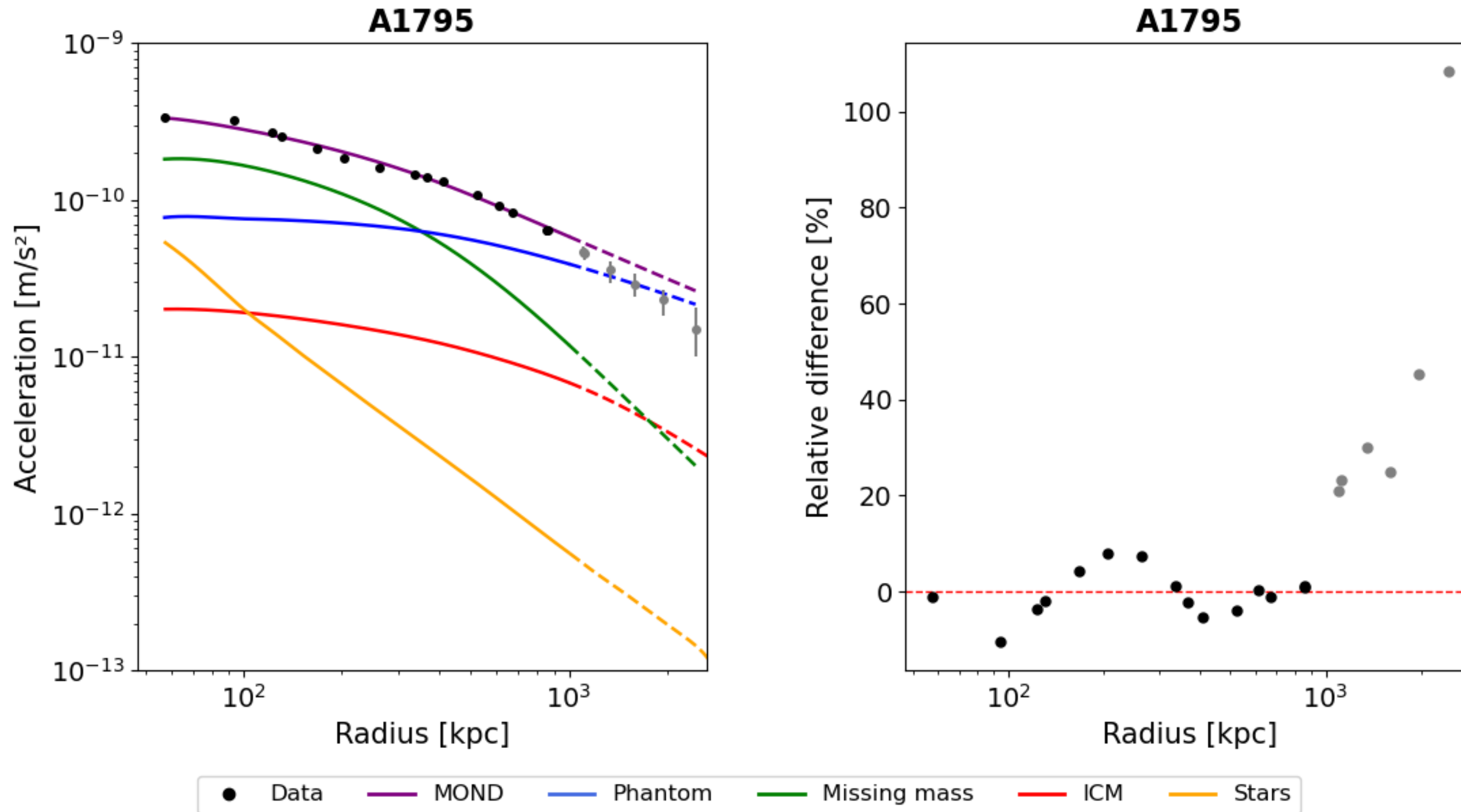
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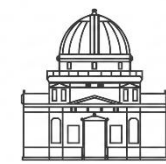


Reconstruction of the final acceleration and mass profiles with ν_1



Reconstruction of the final acceleration and mass profiles with ν_2





Conclusions and next developments

- **Dark Matter** is needed in galaxy clusters **even in MOND**.
- **Test** to see whether this “**missing mass**” follows the mass of the **gas**, the **stars** (galaxies) or the **sum of both**.
- The “**dark mass follows gas**” hypothesis **seems to be reasonable**, while the other ones fail.
- **No significant difference** between the two interpolating functions ν_1 and ν_2 for the missing mass modelling.
- A **better modelling** of the **outskirts** is still needed \longrightarrow **effects of the environment** and possible **disequilibria**.

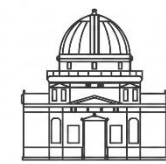


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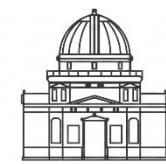
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Appendix



Introducing the External Field Effect (EFE)

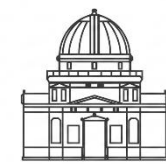
- **No objects** are **truly isolated** in the Universe.
- In **Newtonian dynamics** \rightarrow internal dynamics of a subsystem **decouples** from its mother system \rightarrow it is **independent from any external field** (Strong Equivalent Principle - SEP).
- In **MOND** \rightarrow **break SEP** because it is an **acceleration-based theory** \rightarrow what counts is the **total** gravitational acceleration, both **internal and external**.



Introducing the External Field Effect (EFE)

$$g_{MOND,EFE}(r) = g_N v \left(\frac{g_N + g_{Ne}}{a_0} \right) + g_{Ne} \left[v \left(\frac{g_N + g_{Ne}}{a_0} \right) - v \left(\frac{g_{Ne}}{a_0} \right) \right] \quad \text{Famaey & McGaugh (2012)}$$

$$g_{MOND,EFE}(r) = \begin{cases} v \left(\frac{g_N + g_{Ne}^2/3g_N}{a_0} \right) g_N & \text{if } g_N(r) \geq g_{Ne} \\ v \left(\frac{g_{Ne} + g_N^2/3g_{Ne}}{a_0} \right) g_N & \text{if } g_{Ne} \geq g_N(r) \end{cases} \quad \text{Freundlich et al. (2022)}$$



Poisson's equation for gravity

- From Gauss's law for gravity: $\oint \mathbf{g} \cdot d\mathbf{A} = -4\pi G M_{enc}$
- Applying the divergence theorem: $\int (\nabla \cdot \mathbf{g}) dV = -4\pi G \int \rho dV \longrightarrow \nabla \cdot \mathbf{g} = -4\pi G \rho$
- We can express \mathbf{g} as a scalar potential: $\mathbf{g} = -\nabla\Phi \longrightarrow \nabla^2\Phi = 4\pi G\rho$

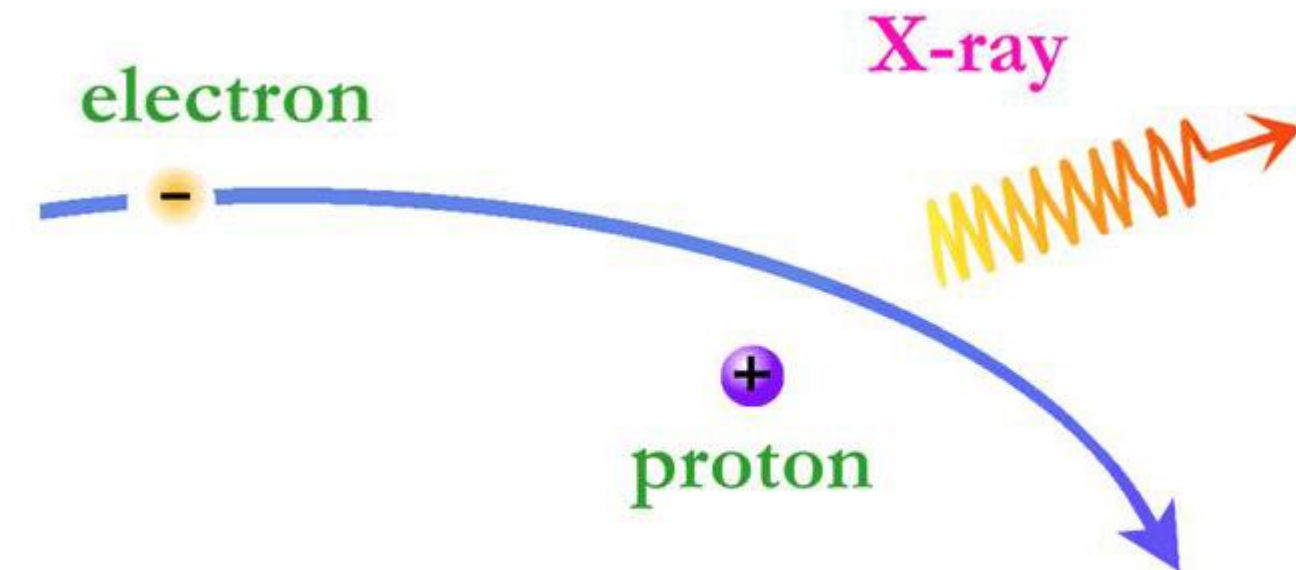
Thermal bremsstrahlung in the ICM

- Temperatures for the ICM: $\sim 10^8$ K
- Density for the ICM: $10^{-1}\text{cm}^{-3} - 10^{-4}\text{cm}^{-3}$
- Emissivity equation: $\epsilon_{ff} \propto Z^2 n_i n_e T^{1/2}$

$$P_{gas}(r) = \frac{k_B}{\kappa m_p} \rho_{gas}(r) T_{gas}(r)$$

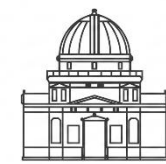


$$g_{obs}(r) = -\frac{1}{\rho_{gas}(r)} \frac{dP_{gas}(r)}{dr}$$



Credits: Chandra X-ray observatory

MOND in galaxies

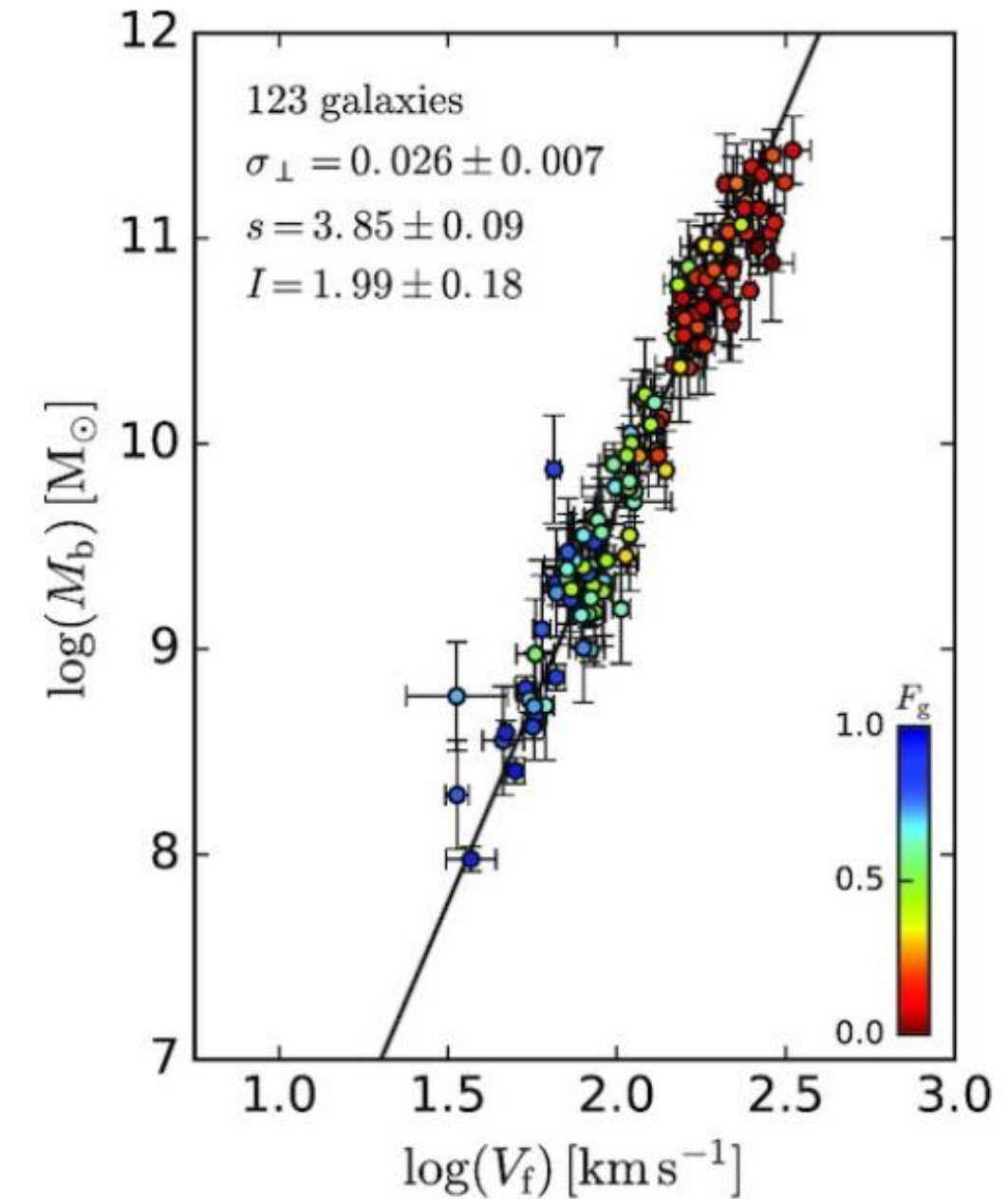


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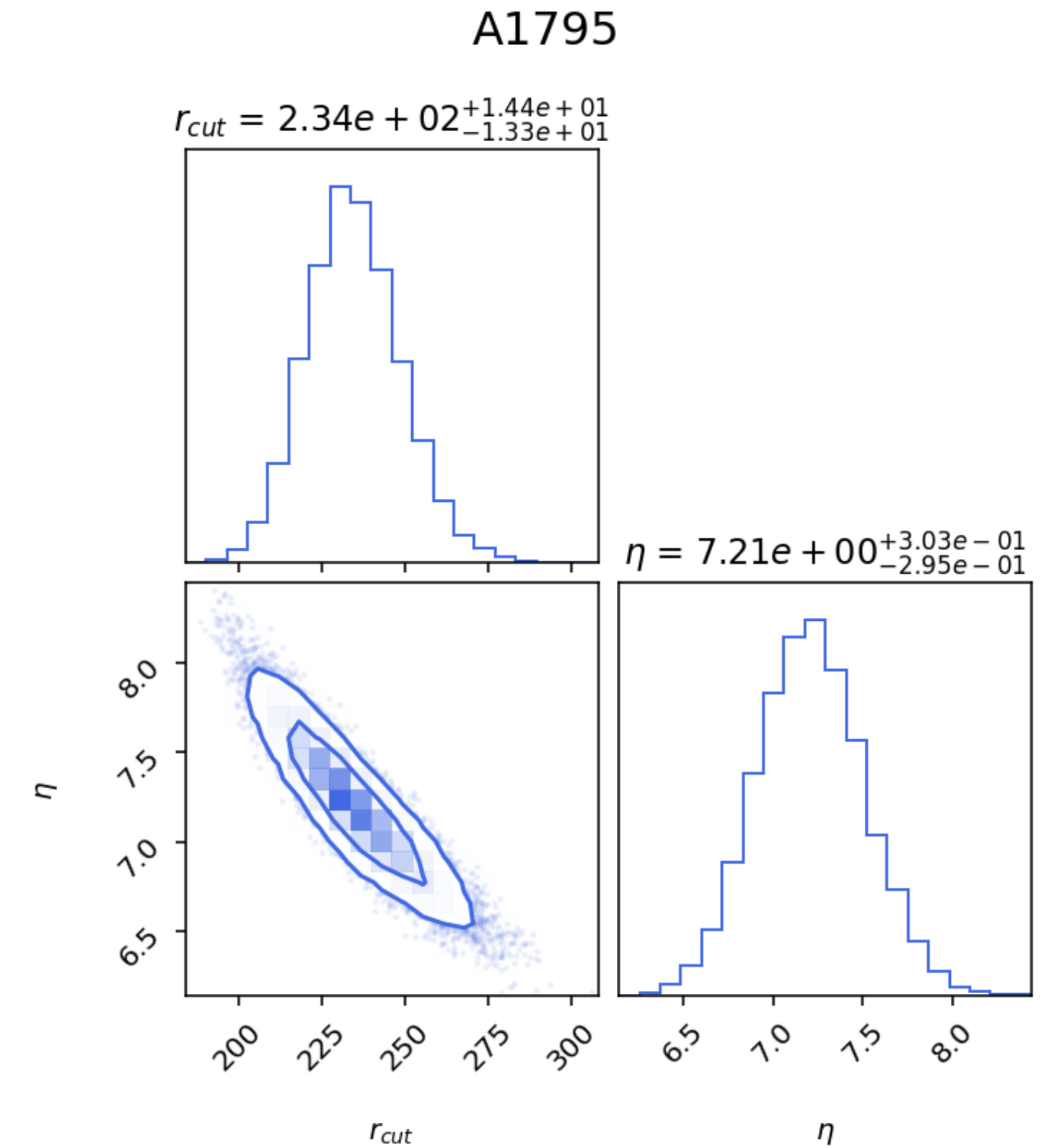
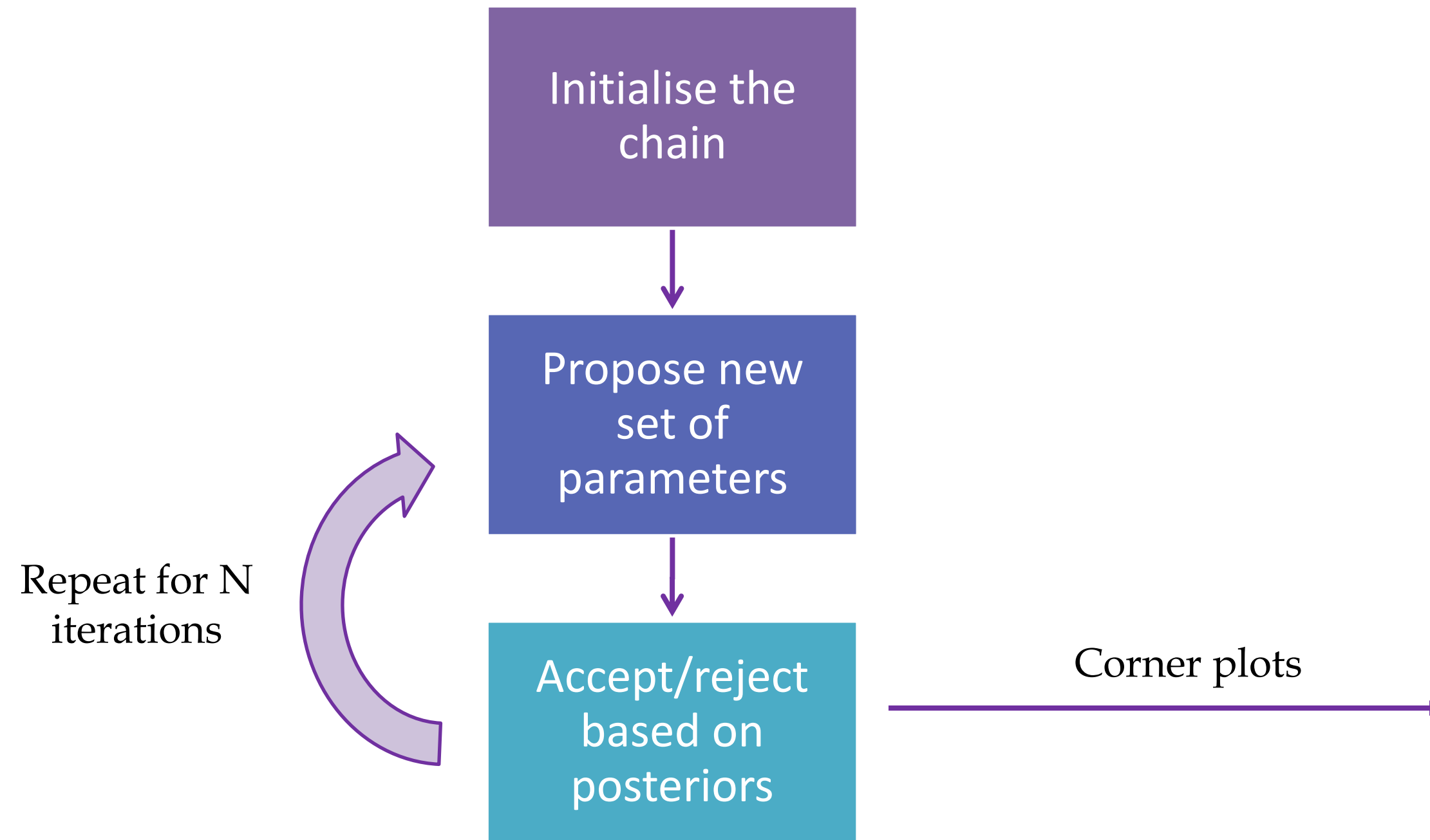
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- **Baryonic Tully-Fisher Relation (BTFR):** $M_b \propto V_f^4$
- Explanation for the **rotational curves:** no «fine-tuning» required
- The **Radial Acceleration Relation (RAR)**



Credits: [Cesare, V. \(2023\)](#)

MCMC - how does it work?



Comparison between models: ν_1 vs ν_2

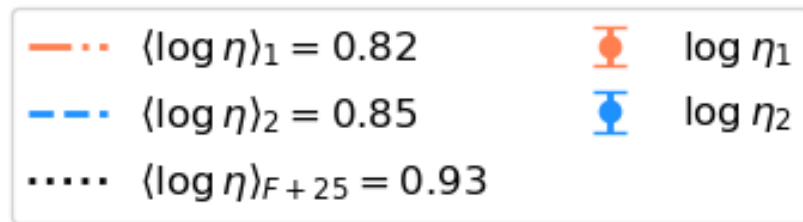
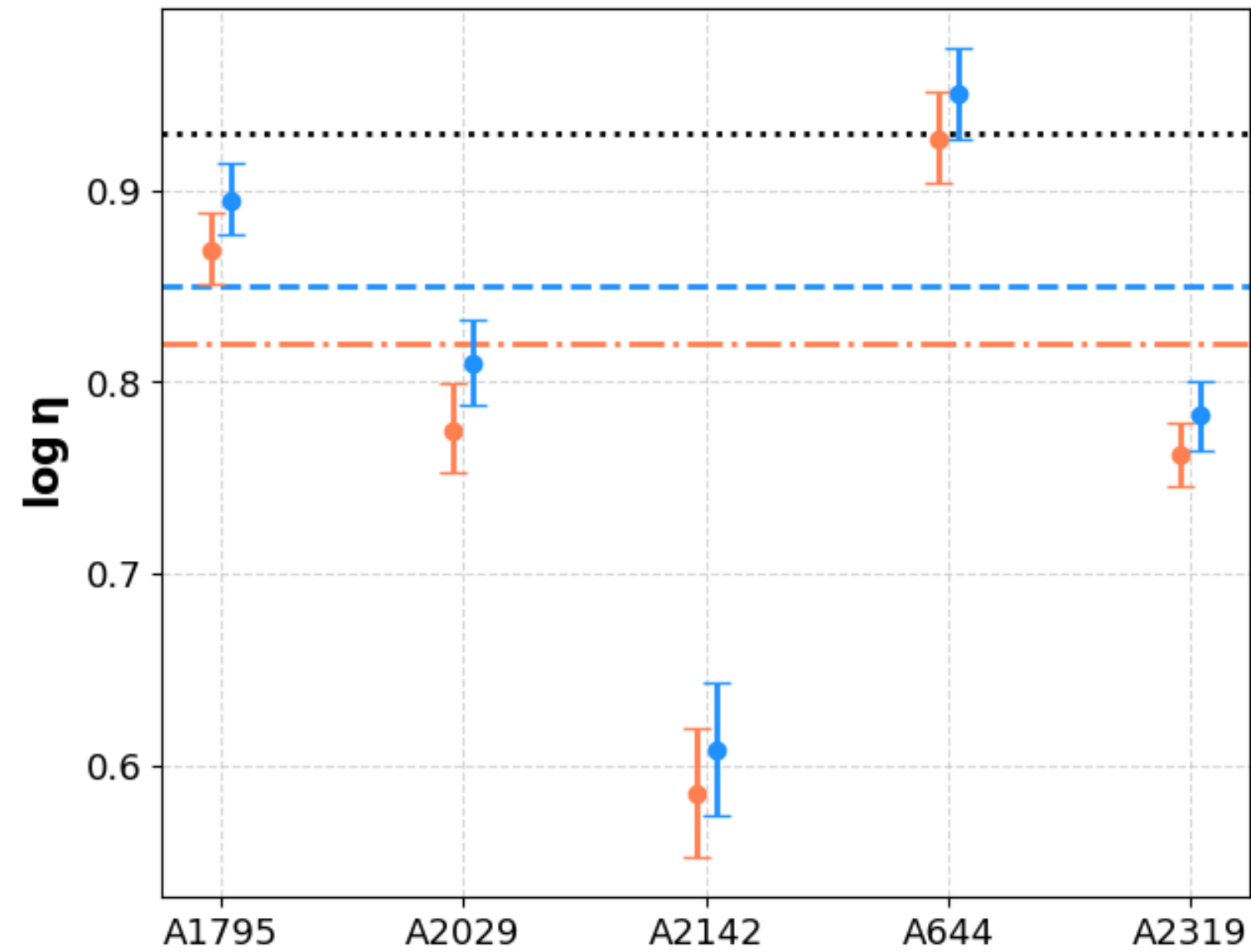


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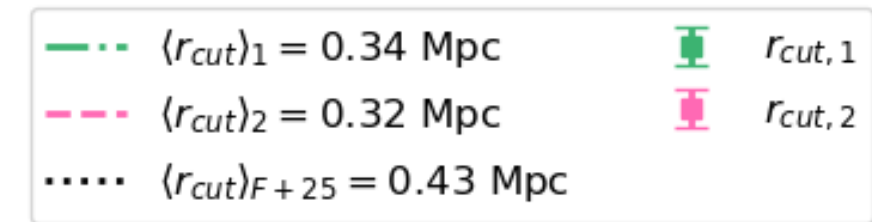


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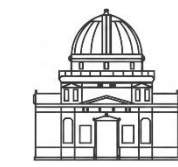
Best fit $\log \eta$



Best fit r_{cut}



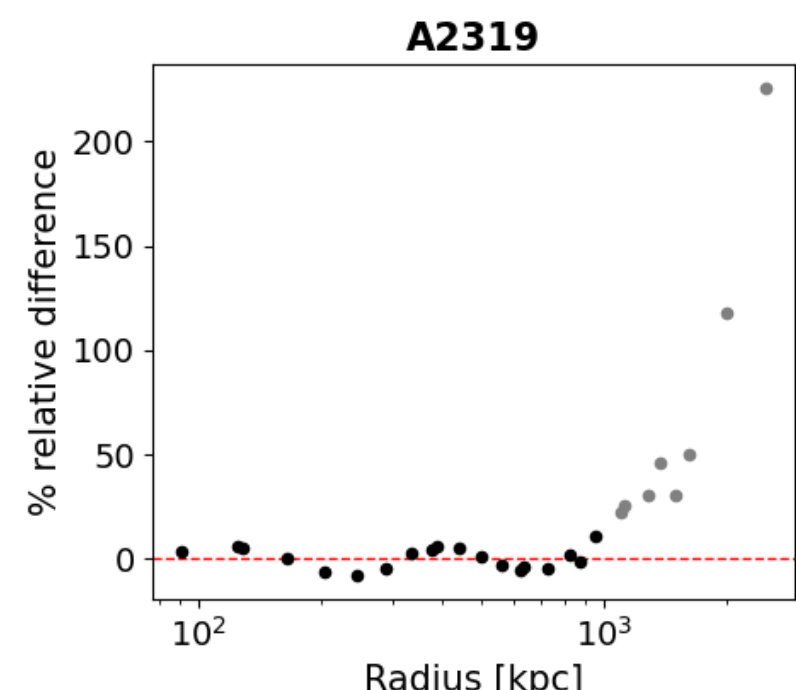
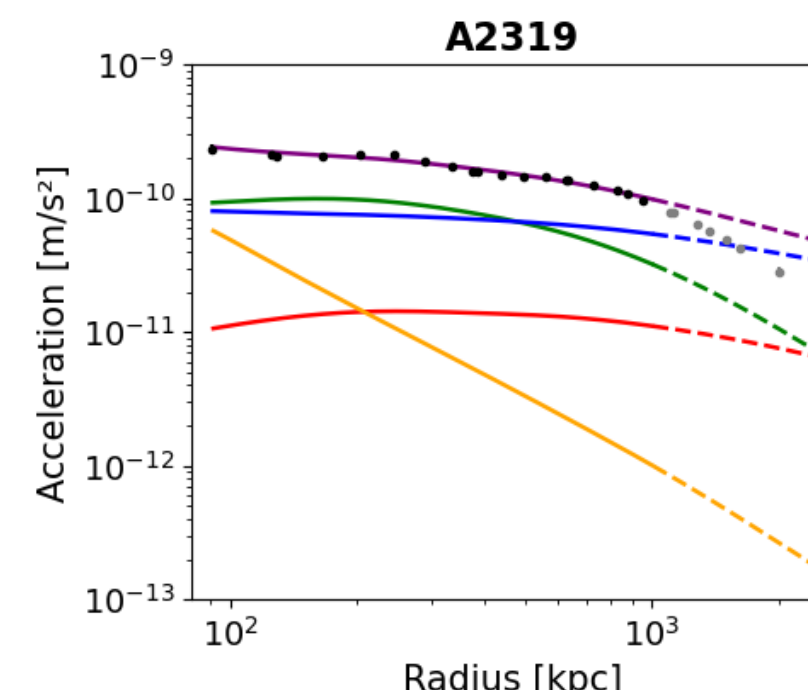
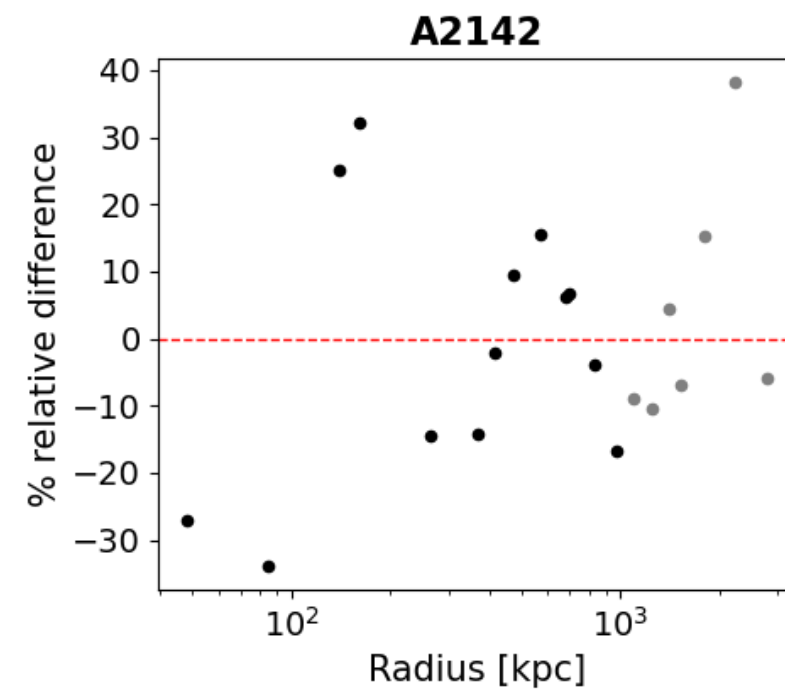
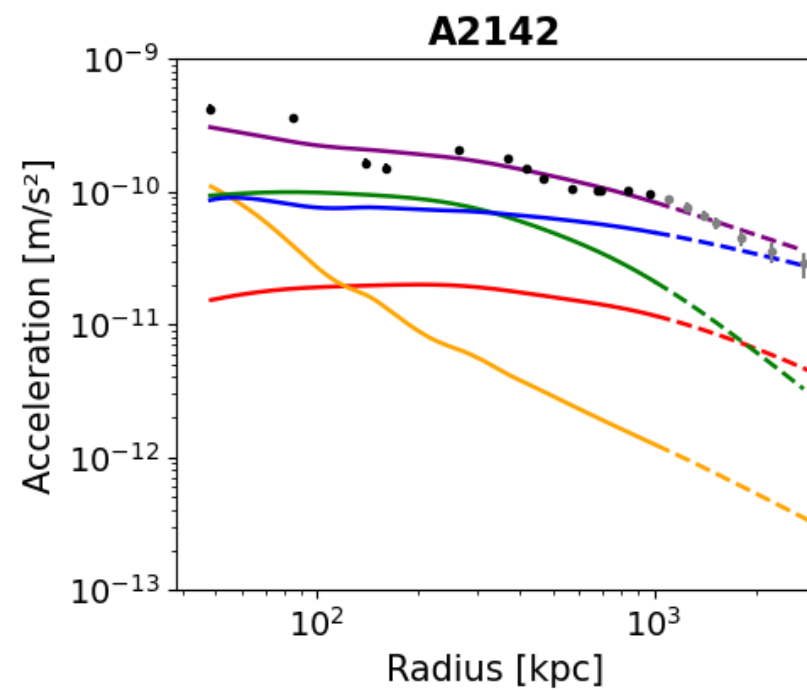
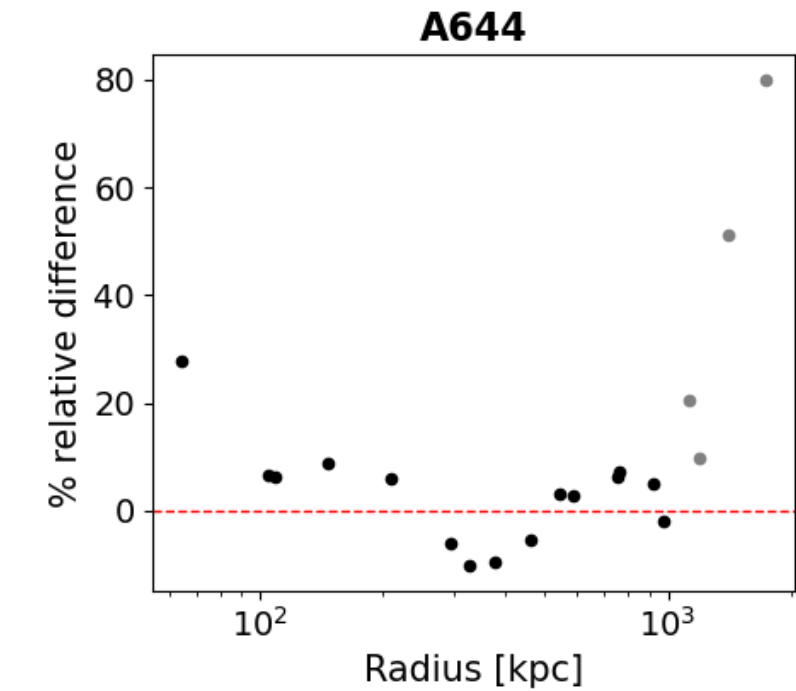
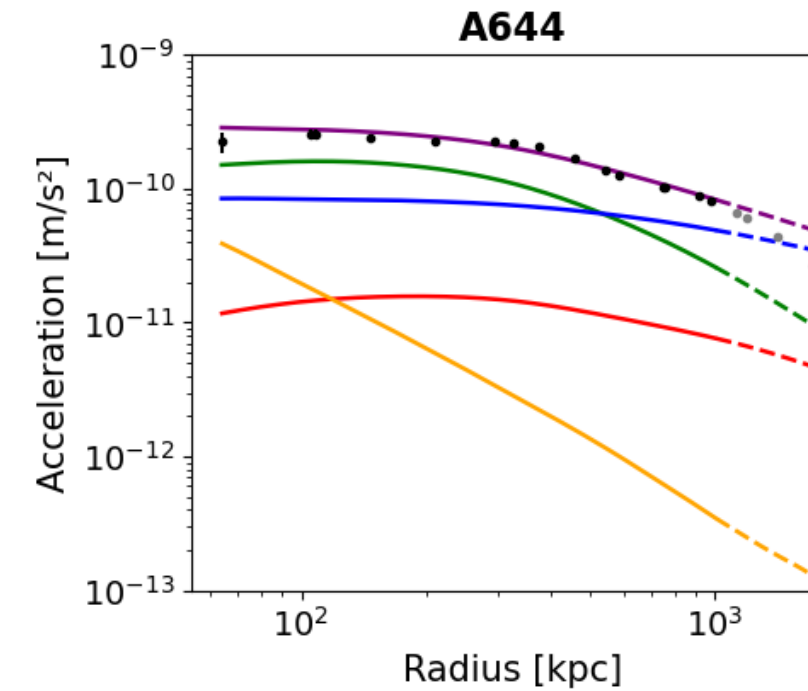
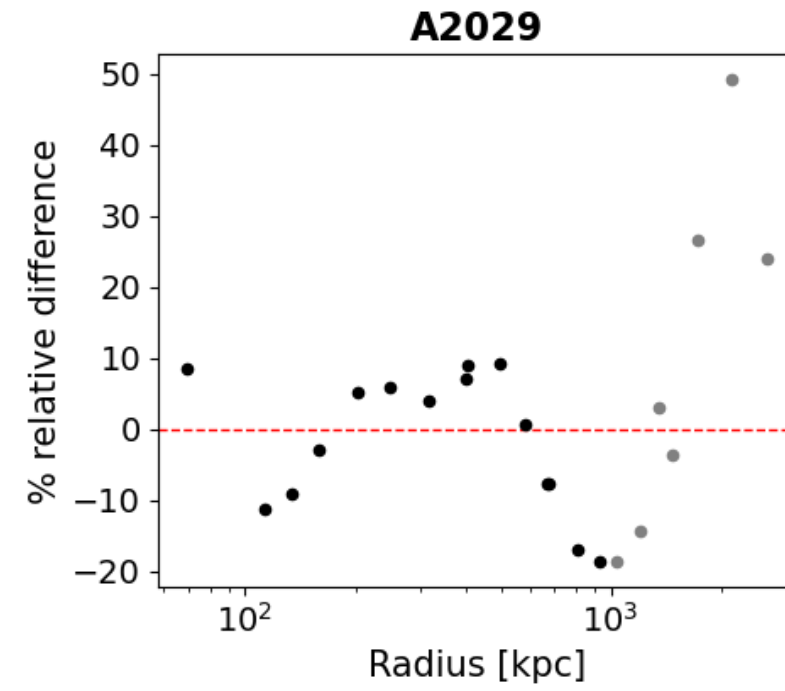
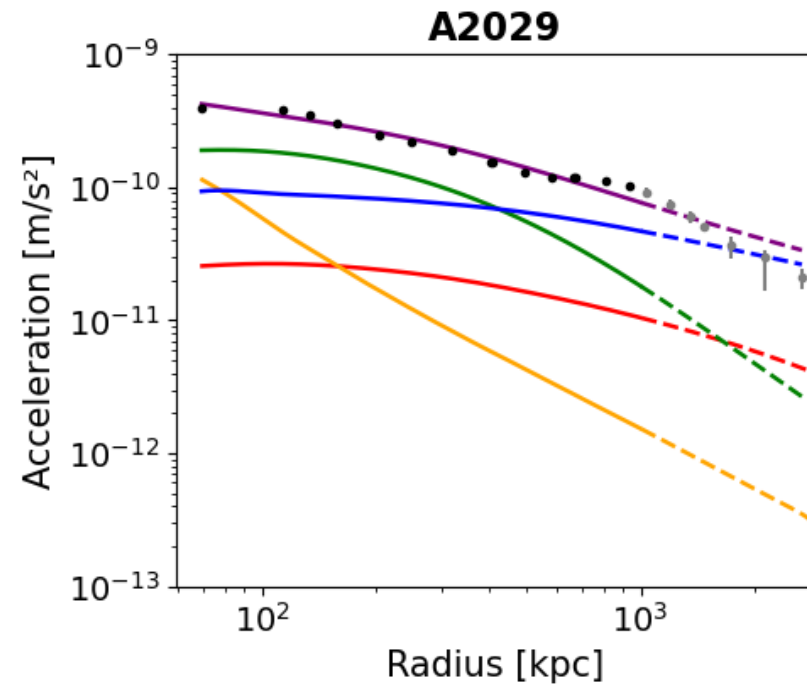
Other results with v_1



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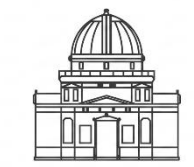
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● Data — MOND — Phantom — Missing mass — ICM — Stars

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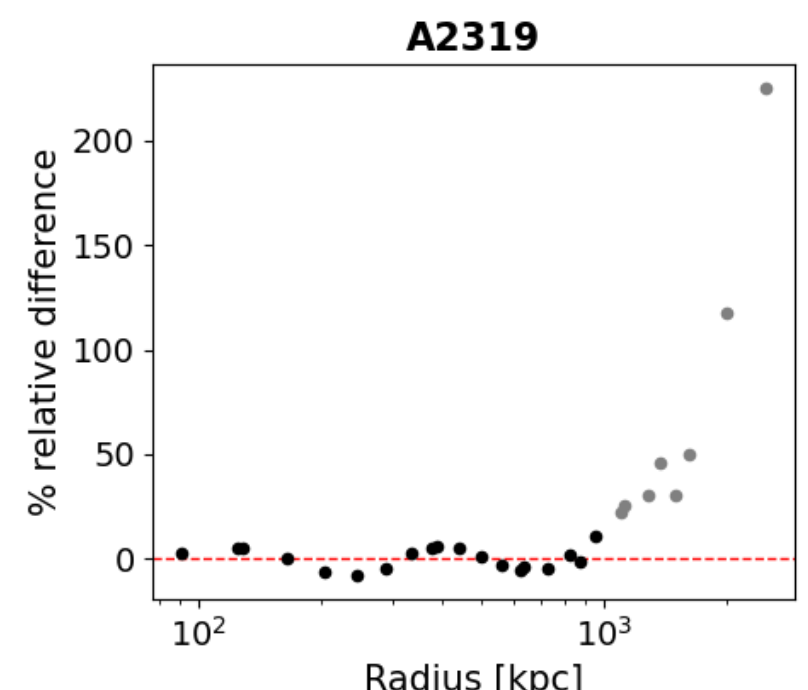
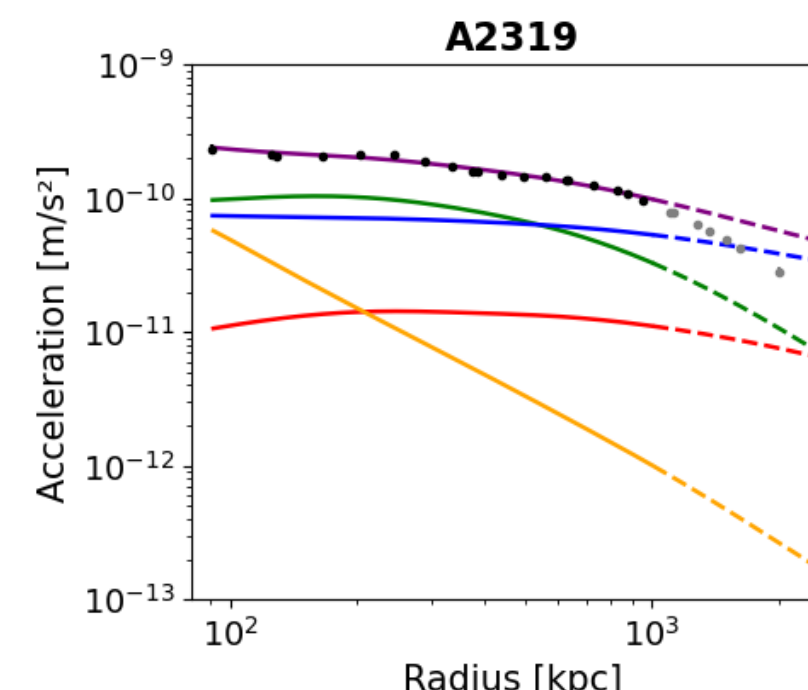
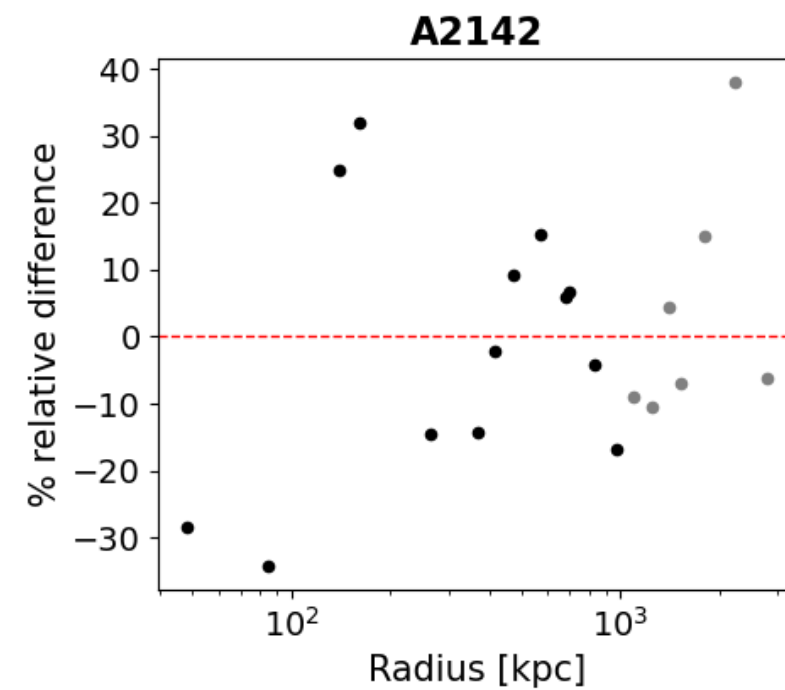
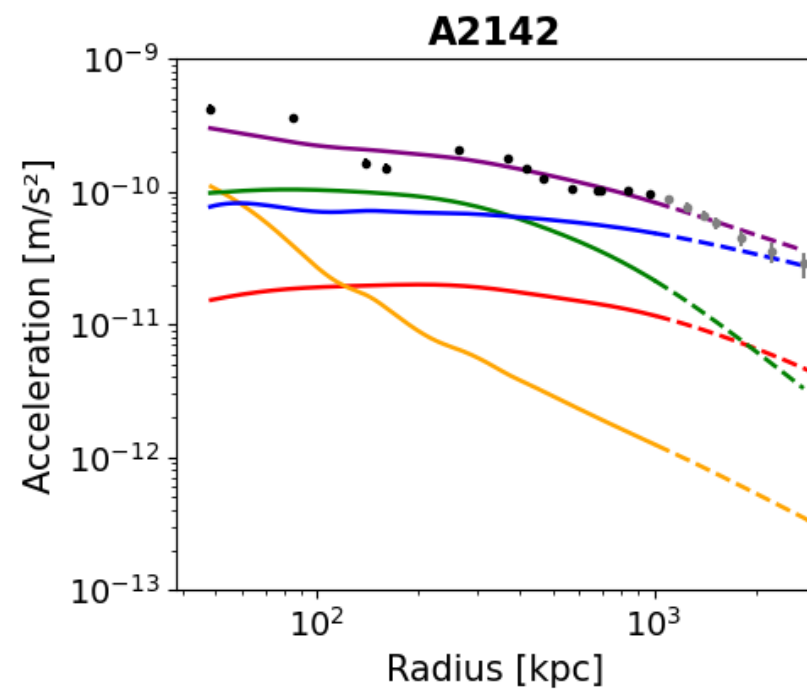
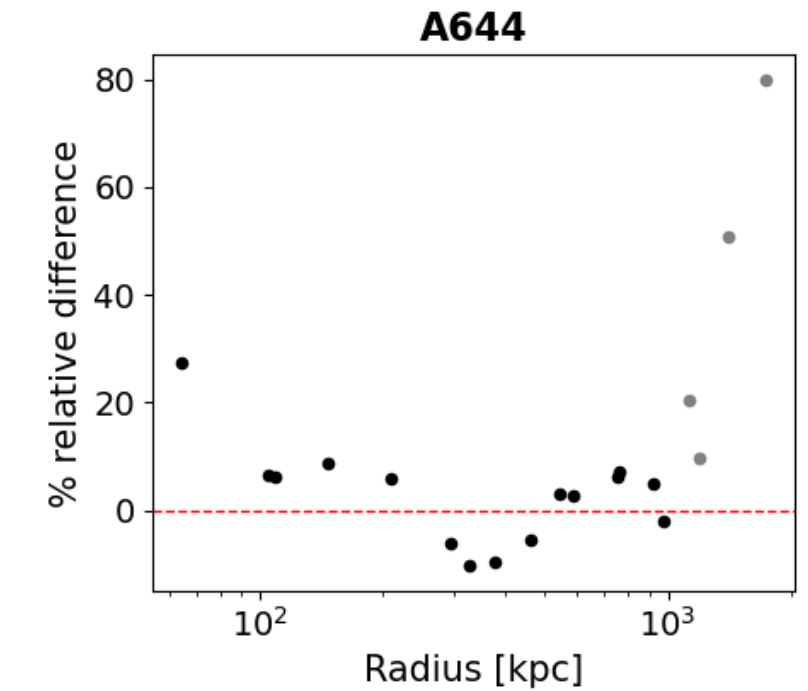
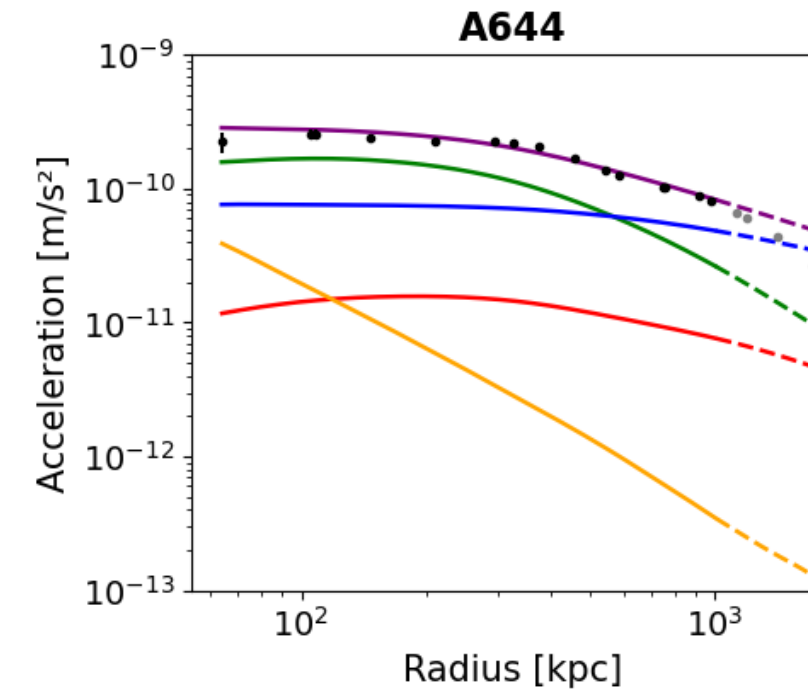
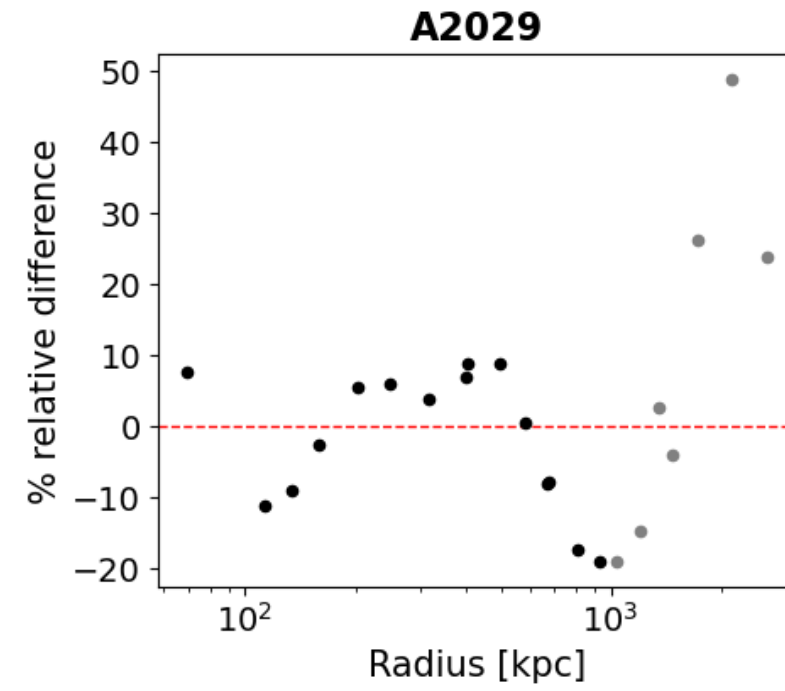
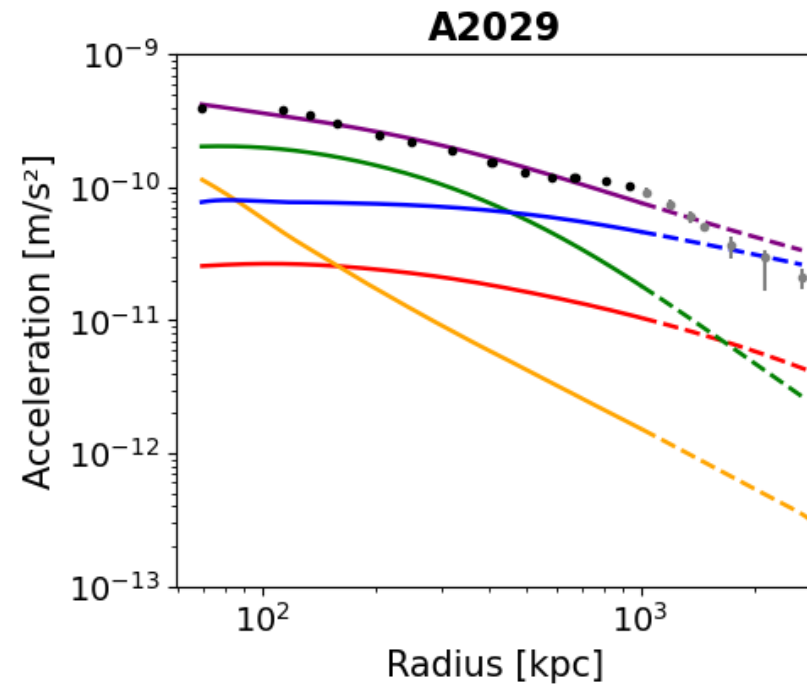
Other results with v_2



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