Gravitational lensing of GWs: insights from the wave optics regime

BiCoQ conference

Martin Pijnenburg
Università degli Studi di Milano Bicocca

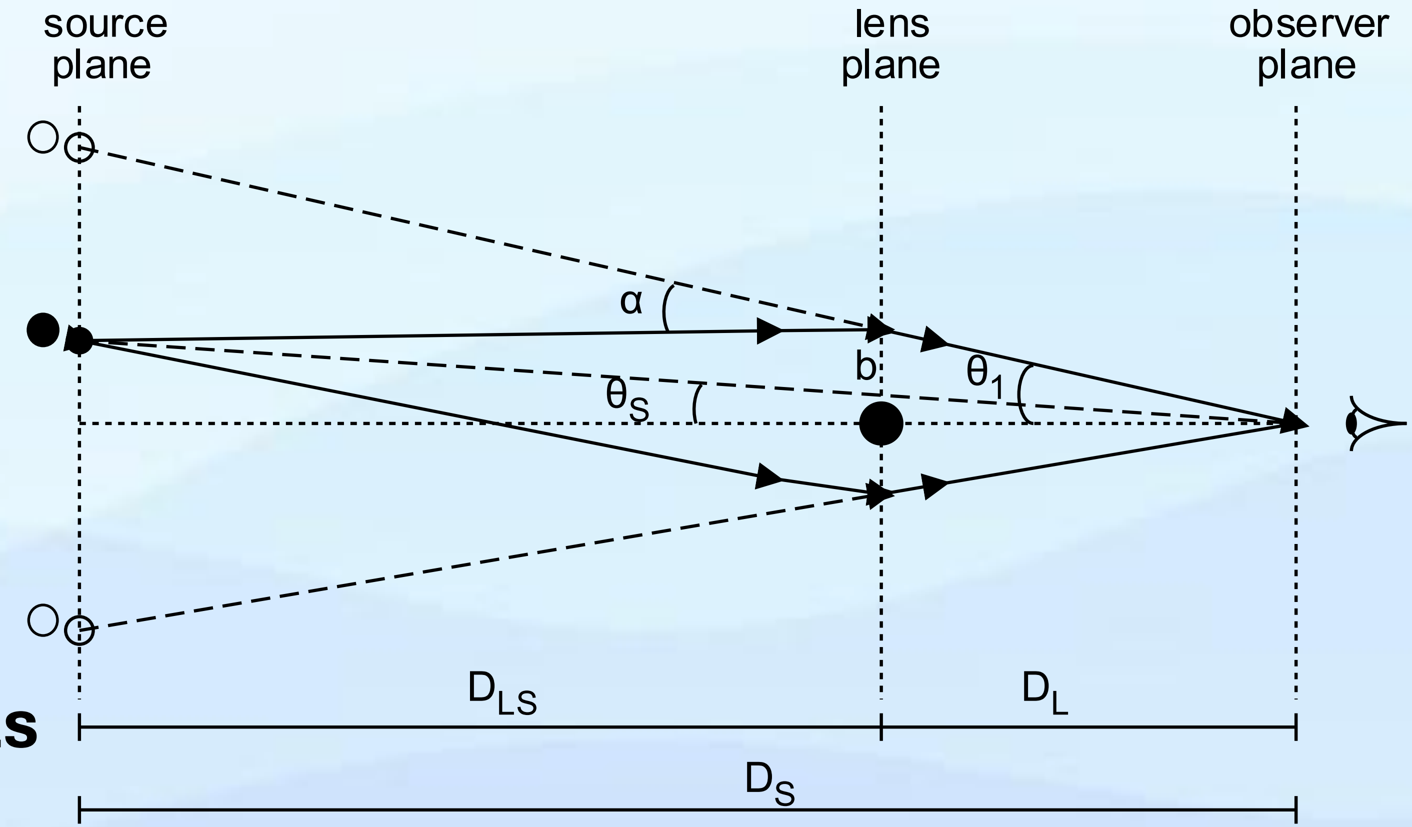
Gravitational lensing: EM waves vs gravitational waves

Null geodesics are bent by masses:

→ Usual lensing picture (deflection angle, etc.)

→ Key prediction of General Relativity

→ **Both EM and GW signals**



Gravitational lensing: EM waves vs gravitational waves

Is lensing really the same for EM and GW signals ? *Some ideas…*

Gravitational lensing: EM waves vs gravitational waves

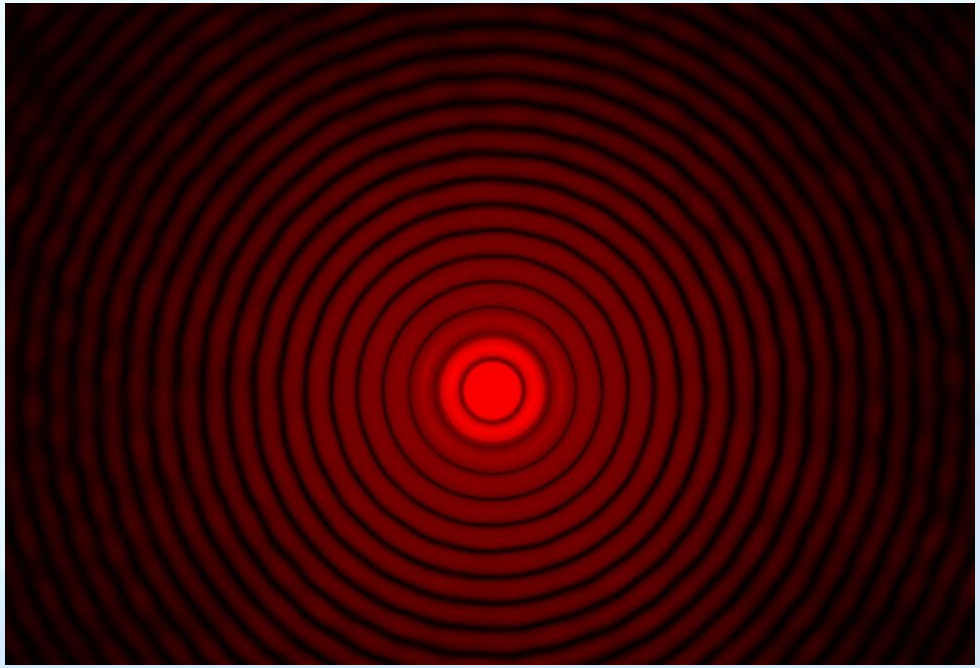
Is lensing really the same for EM and GW signals ? *Some ideas…*

EM waves	GW	
Extended sources	~ Point-like sources $\longrightarrow$ mainly magnification	
Observed λ_{wave} : $\leq \mathcal{O}(1\ \text{m})$	Observed λ_{wave} : $\leq \mathcal{O}\left(10^{7}\ \text{m}\right)$ $\leq \mathcal{O}\left(10^{10}\ \text{m}\right)$ $\leq \mathcal{O}\left(10^{16}\ \text{m}\right)$	(LIGO, ET) (LISA) (PTA)
Spin 1 (photon)	Spin 2, tensorial signal	

Gravitational lensing: EM waves vs gravitational waves

Point particle null-geodesics, known limitation in the EM case:

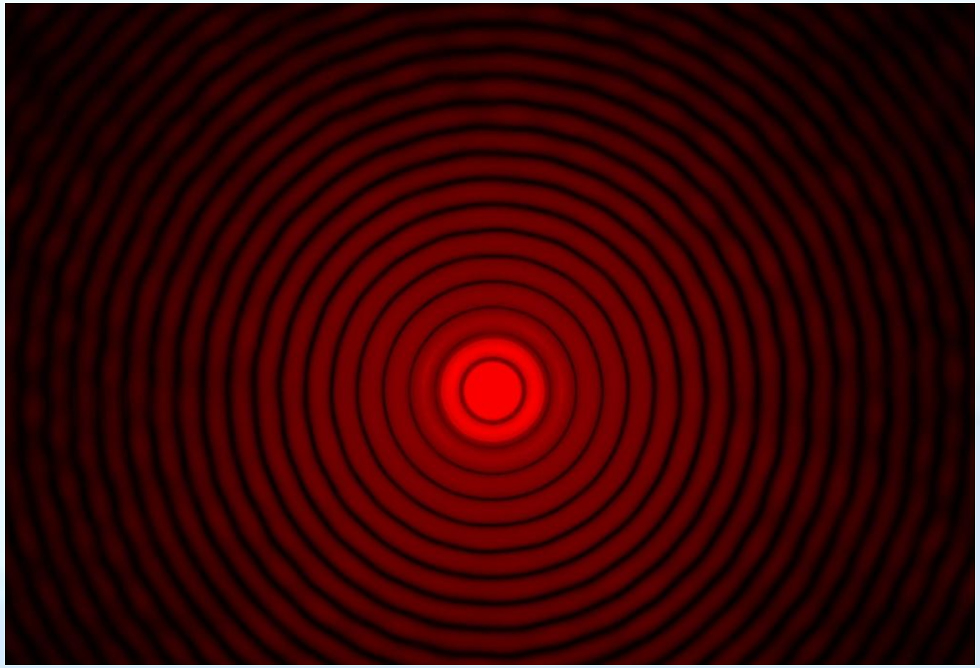
PSF (Airy disk)



Gravitational lensing: EM waves vs gravitational waves

Point particle null-geodesics, known limitation in the EM case:

PSF (Airy disk)



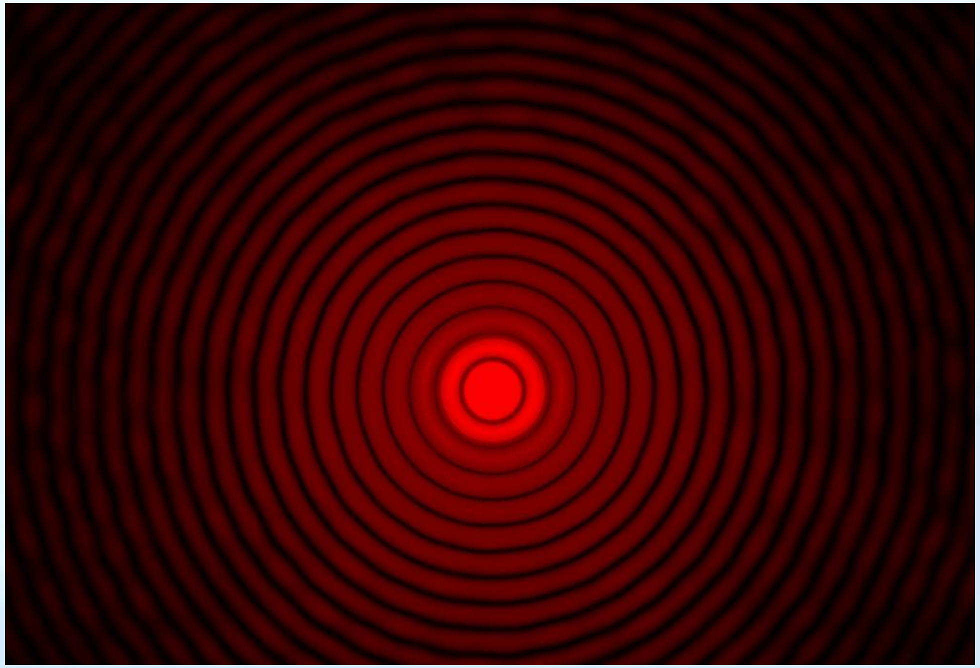
More generally : diffraction

$$\lambda_{\text{wave}} \sim \text{obstacle size}$$

Gravitational lensing:
EM waves vs gravitational waves

Point particle null-geodesics, known limitation in the EM case:

PSF (Airy disk)



More generally : diffraction

$$\lambda_{\mathrm{wave}} \sim \text{obstacle size}$$

e.g. aperture, or ...
gravitational lens ?

Gravitational lensing: EM waves vs gravitational waves

Diffraction: $\lambda_{\rm wave} \sim$ obstacle size (e.g. gravitational radius)

EM waves	GW
Observed $\lambda_{\rm wave}$: $\leq 10^{-4} M_\odot$	Observed $\lambda_{\rm wave}$: $\leq 10^{4} M_\odot$ (LIGO, ET) $\leq 10^{7} M_\odot$ (LISA) $\leq 10^{13} M_\odot$ (PTA)

Gravitational lensing: EM waves vs gravitational waves

Diffraction: $\lambda_{\mathrm{wave}} \sim$ obstacle size (e.g. gravitational radius)

EM waves	GW
Observed λ_{wave} : $\leq 10^{-4} M_\odot$	Observed λ_{wave} : $\leq 10^{4} M_\odot$ (LIGO, ET) $\leq 10^{7} M_\odot$ (LISA) $\leq 10^{13} M_\odot$ (PTA)

→ Relevant astrophysical and cosmological lenses!

Wave effects in gravitational lensing:

Ohanian1974, Bliokh+1975, Bontz+1981, Mandzos1982, Schneider+1985, Deguchi+1986, Ulmer+1995, Nakamura1998, ...

Assuming a **weak field** lensing

$$ds^2 = -(1 + 2U)\, dt^2 + (1 - 2U)\, d\mathbf{r}^2$$

Wave effects in gravitational lensing:

Ohanian1974, Bliokh+1975, Bontz+1981, Mandzos1982, Schneider+1985, Deguchi+1986, Ulmer+1995, Nakamura1998, ...

Assuming a **weak field** lensing

$$ds^2 = -(1 + 2U)\, dt^2 + (1 - 2U)\, d\mathbf{r}^2$$

and a **scalar signal** ϕ, the propagation follows (in Fourier space) :

$$(\nabla^2 + \omega^2)\phi = 4\omega^2 U \phi, \qquad \omega = 2\pi f$$

Wave effects in gravitational lensing:

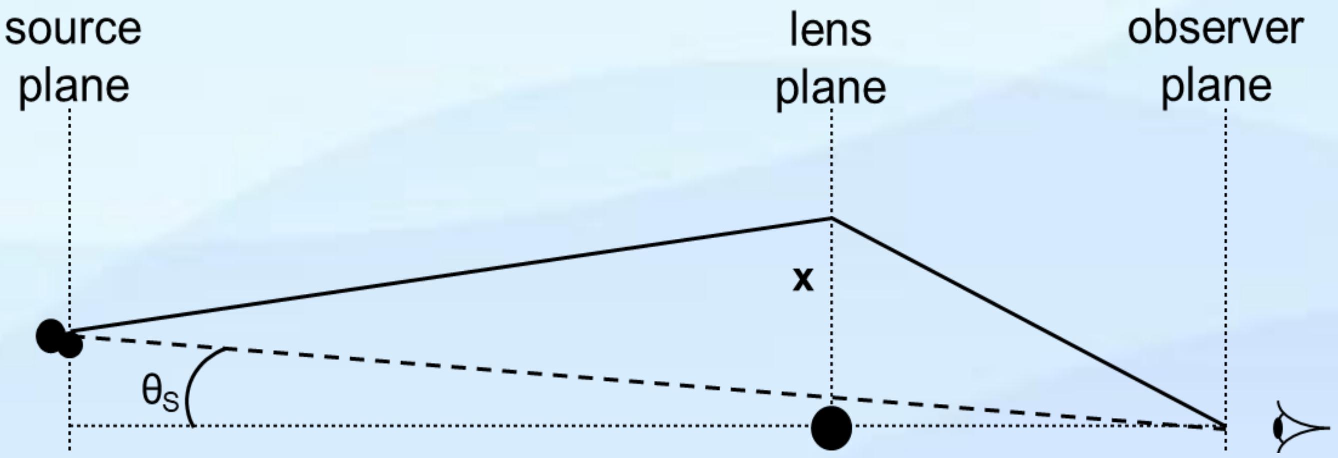
Define the signal amplification $\sqrt{\mu} = \frac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

Wave effects in gravitational lensing:

Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x} \, \exp\left(if \, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\right)$$



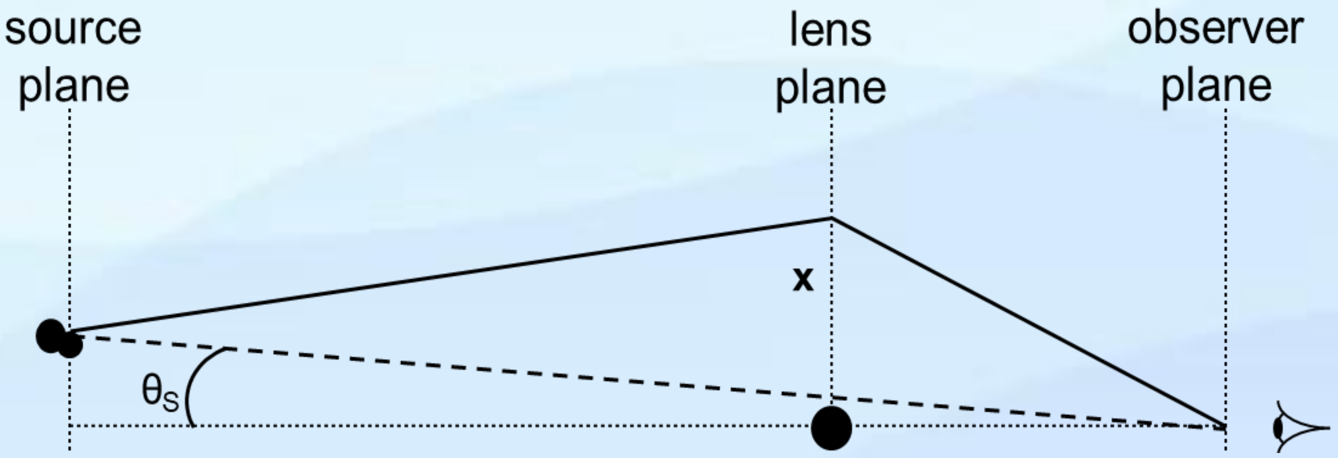
Wave effects in gravitational lensing:

Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x}\, \exp\left(i f\, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\right)$$

Geometrical + Shapiro travel time for the path through **x**



Wave effects in gravitational lensing:

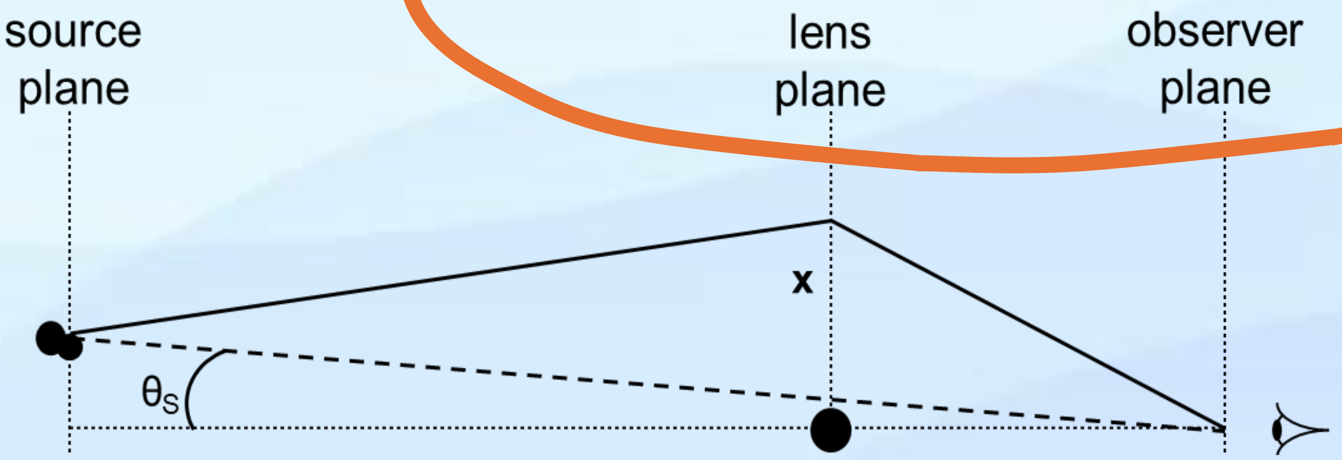
Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x} \, \exp\left(i f \, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\right)$$

Geometrical + Shapiro travel time for the path through **x**

Integrated over all hypothetical paths through the lens plane

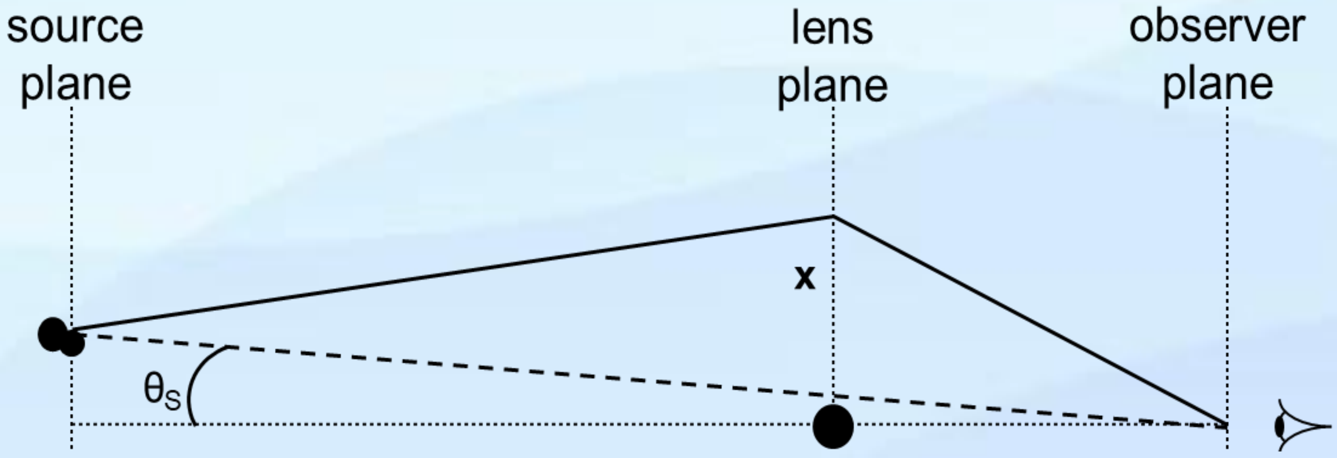


Wave effects in gravitational lensing:

Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x} \, \exp\Big(if \, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\Big)$$



- **Highly oscillatory** diffraction integral

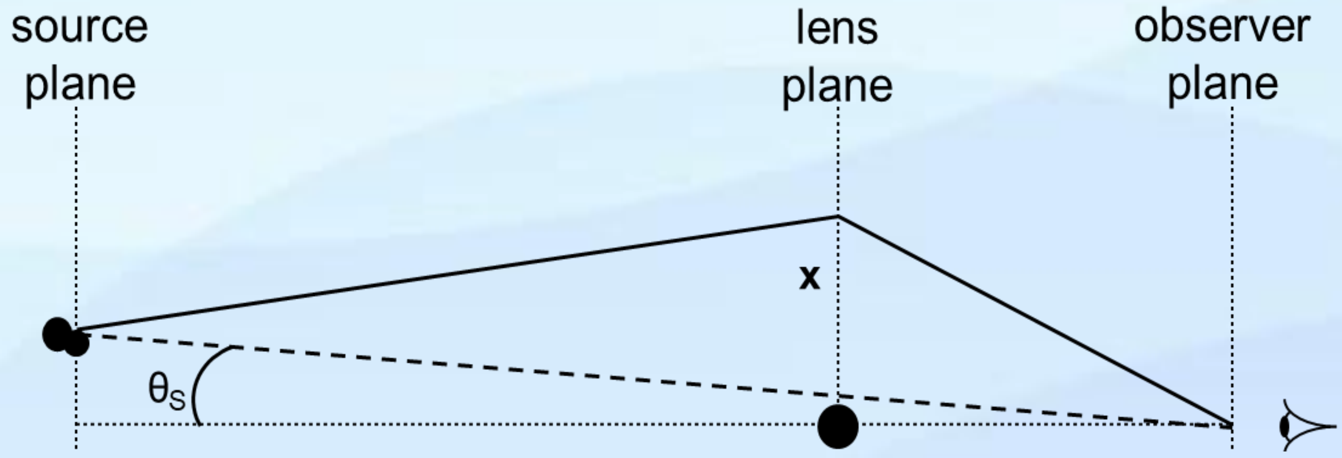
Wave effects in gravitational lensing:

Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x} \, \exp\left(i f \, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S) \right)$$

- **Highly oscillatory** diffraction integral
- **frequency dependent lensing**



Wave effects in gravitational lensing:

Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

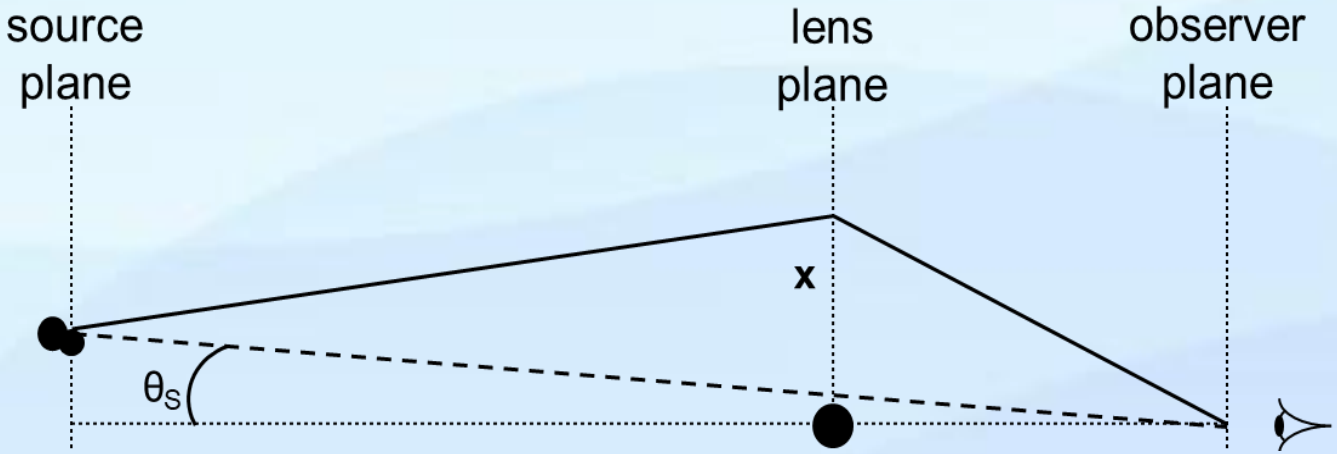
For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x} \, \exp\Big(i f \, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\Big)$$

- **Highly oscillatory** diffraction integral

- **frequency dependent lensing**

- **well defined $f \to \infty$ limit** :
only the stationary phase points contribute
i.e. $\nabla \mathrm{T}_{\text{travel}} = 0$

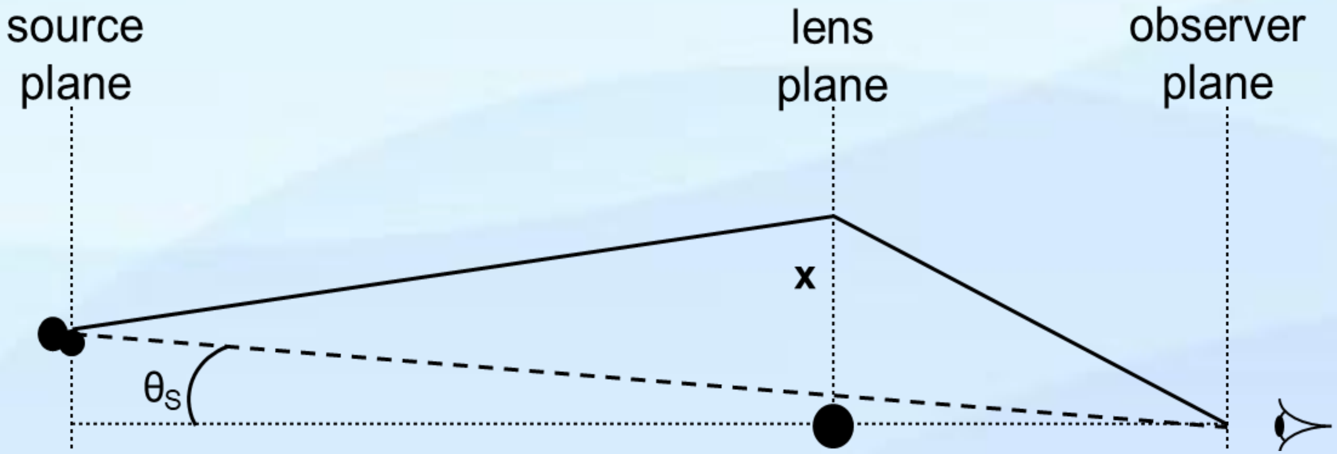


Wave effects in gravitational lensing:

Define the signal amplification $\sqrt{\mu} = \dfrac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

For **single-plane, paraxial, thin lens**, wave equation for $\sqrt{\mu}$ is solved by

$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x}\, \exp\left(if\, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\right)$$



- **Highly oscillatory** diffraction integral

- **frequency dependent lensing**

- **well defined *f* → ∞ limit** :
only the stationary phase points contribute
i.e. $\nabla \mathrm{T}_{\text{travel}} = 0$

→ **recover the geodesic limit & associated observables naturally** (Fermat principle)

Wave effects in gravitational lensing:

→ **No new physics, but different lensing phenomelogy** wrt pure null geodesics

Wave effects in gravitational lensing:

→ **No new physics, but different lensing phenomelogy** wrt pure null geodesics

State of the art numerical methods have been developed **recently**

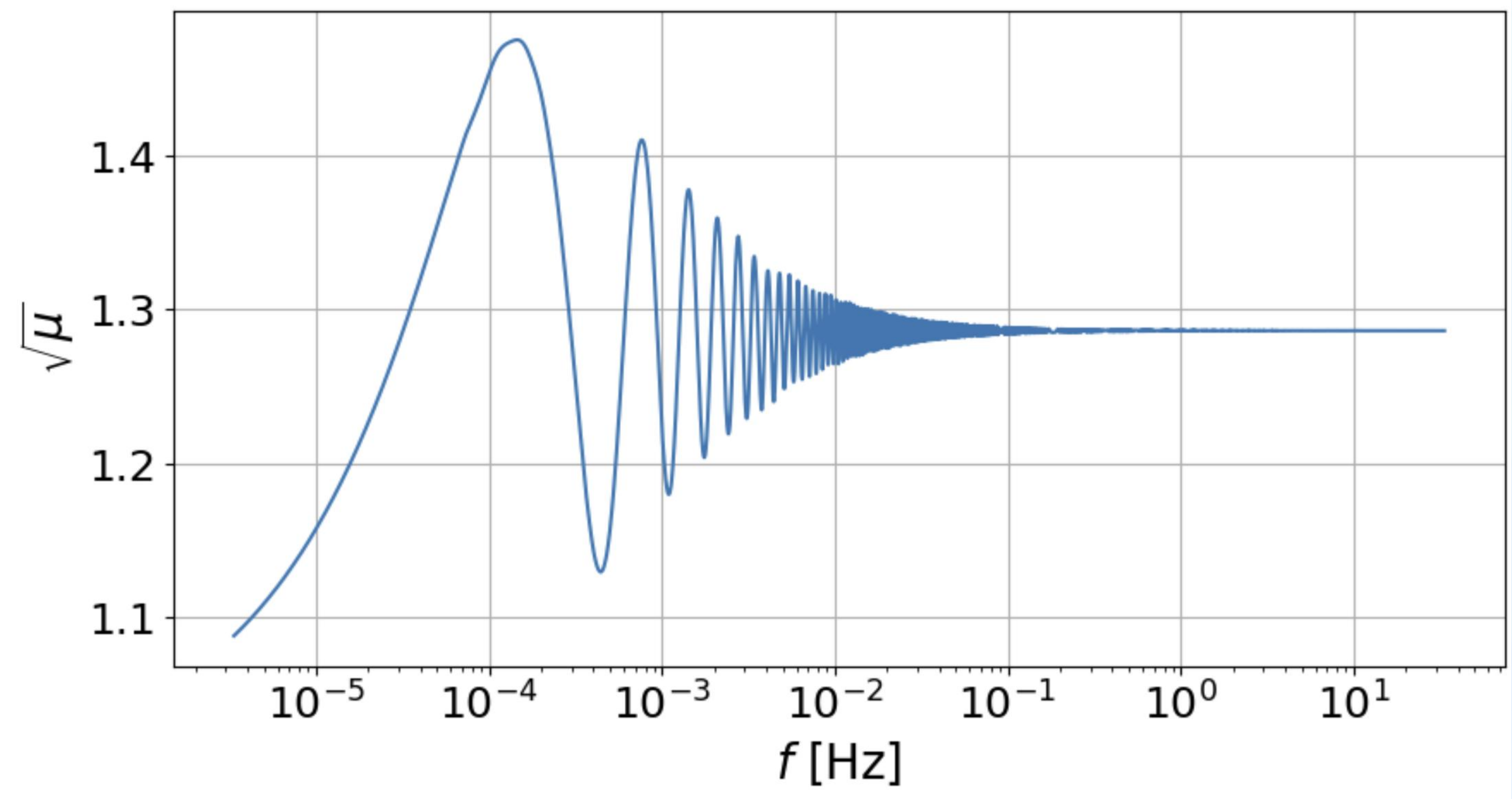
e.g. GLoW code for diffraction integral

Villarrubia-Rojo+2025

Wave effects in gravitational lensing:

Illustration example:

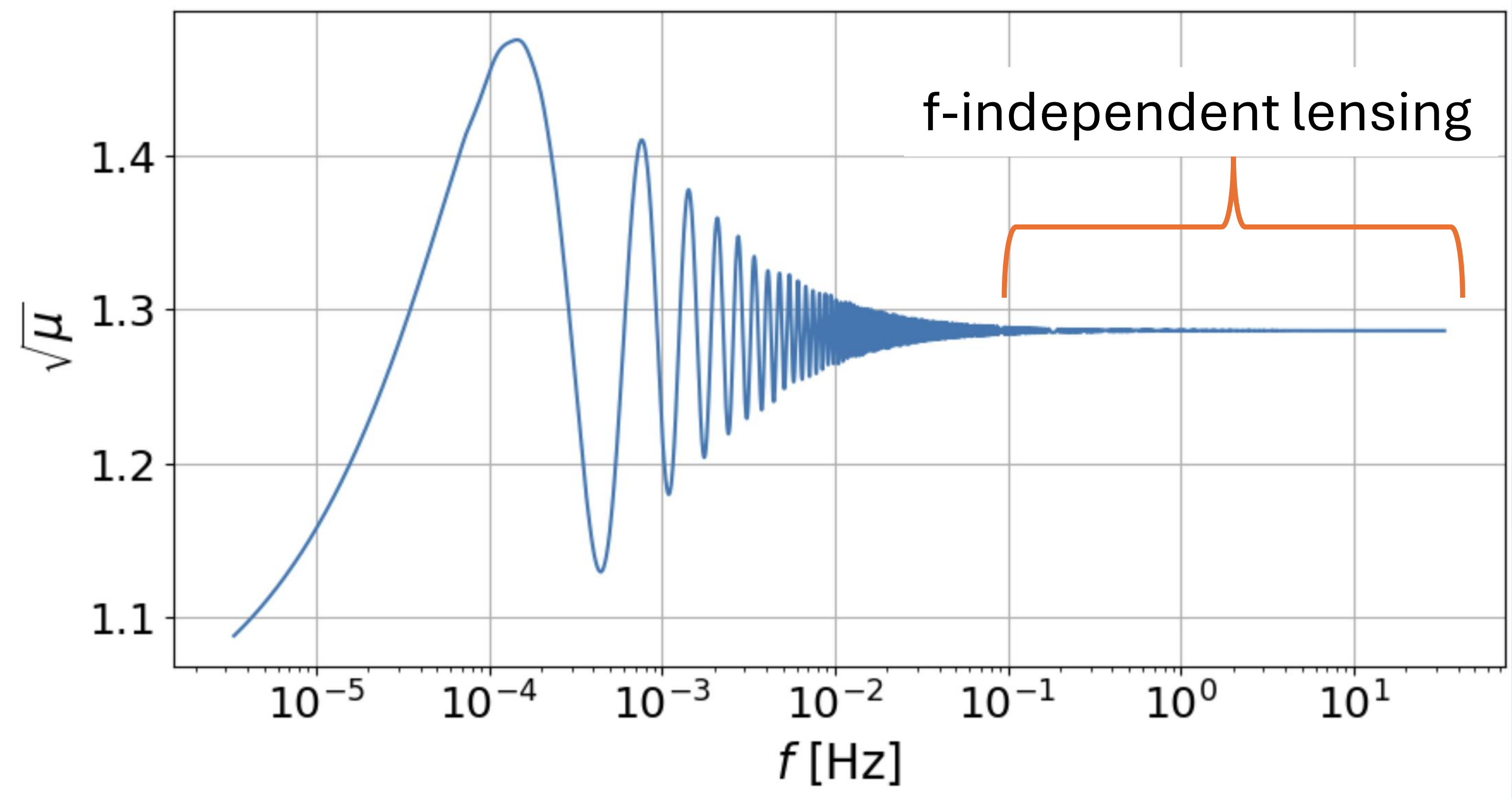
$10^9 M_\odot$ DM (sub)halo at $z = 1$ with *truncated SIS* profile, source at $z = 5$, **weak lensing** regime, "good" alignment ($\mu(f \to \infty) \simeq 1.6$)



Wave effects in gravitational lensing:

Illustration example:

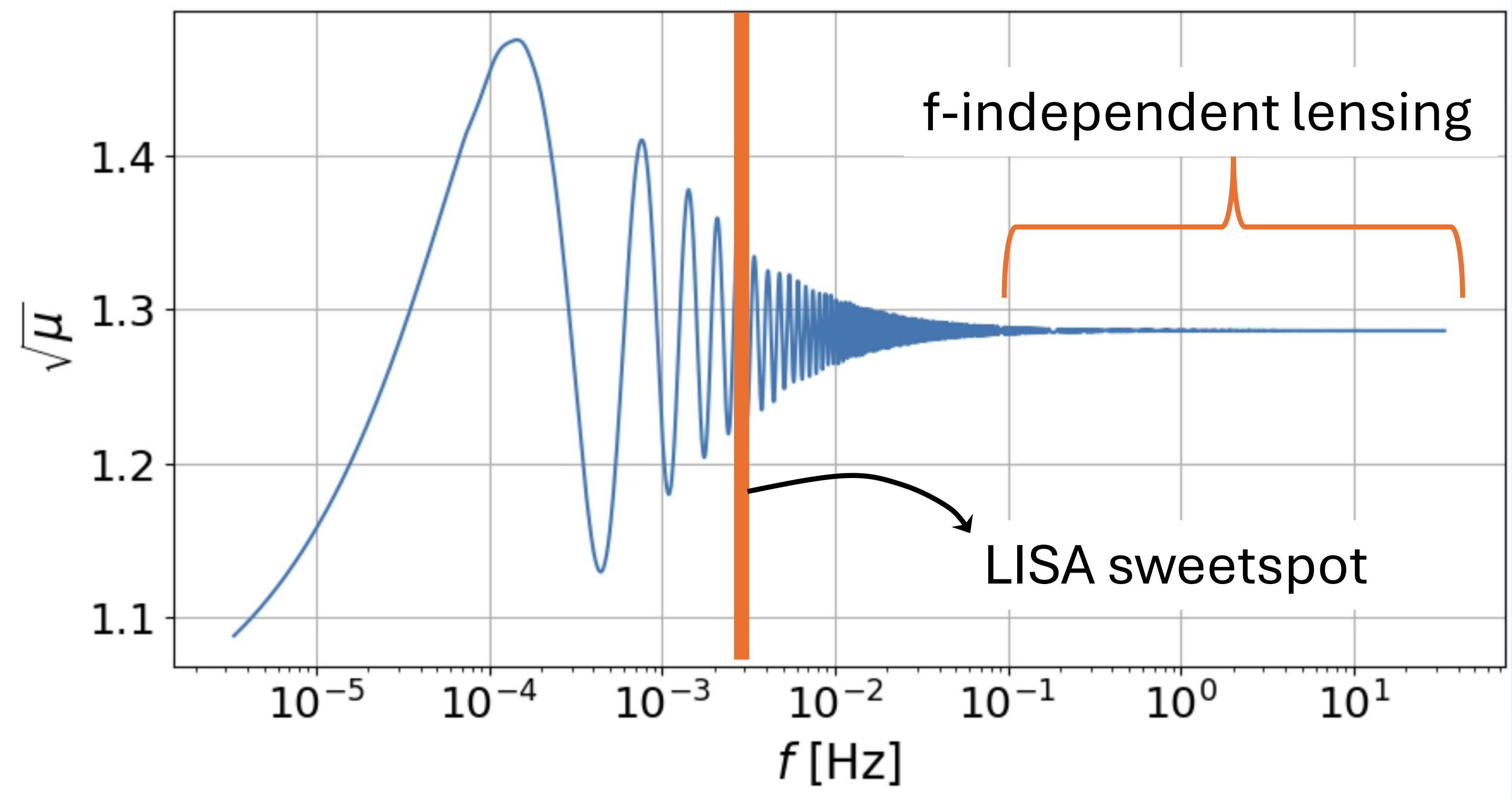
$10^9 M_\odot$ DM (sub)halo at $z = 1$ with *truncated SIS* profile, source at $z = 5$, **weak lensing** regime, "good" alignment ($\mu(f \to \infty) \simeq 1.6$)



Wave effects in gravitational lensing:

Illustration example:

$10^9 M_\odot$ DM (sub)halo at $z = 1$ with *truncated SIS* profile, source at $z = 5$, **weak lensing** regime, "good" alignment ($\mu(f \to \infty) \simeq 1.6$)



Wave effects in gravitational lensing:

A few results (*not mine !*) from the literature:

- Including lensing analyses is **necessary** for next gen. interferometers
  Gupta+2025

Wave effects in gravitational lensing:

A few results (*not mine !*) from the literature:

- Including lensing analyses is **necessary** for next gen. interferometers
  Gupta+2025

- LISA observation prospects of **diffraction** : ***expected detection*** to ***low chances of detection*** depending on study
  e.g. Savastano+ 2023, Brando+2024

Wave effects in gravitational lensing:

A few results (*not mine !*) from the literature:

- Including lensing analyses is **necessary** for next gen. interferometers
  Gupta+2025

- LISA observation prospects of **diffraction** : ***expected detection*** to ***low chances of detection*** depending on study
  e.g. Savastano+ 2023, Brando+2024

- Very sensitive to the **abundance** and **profile** of the low-mass end (e.g. $< 10^{11} M_\odot$) DM halos : potential to distinguish DM models (?)
  Savastano+ 2023, Brando+2024, Singh+2025

Beyond initial assumptions:

This framework relied on **weak fields potentials, scalar signals…**

… But **GW** are not scalars!

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0$$

Beyond initial assumptions:

This framework relied on **weak fields potentials, scalar signals...**

... But **GW** are not scalars!

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0$$

... And what if the lens is **strong field** (e.g. a **black hole**)?

Expect even **richer phenomenology !**

Beyond initial assumptions:

This framework relied on **weak fields potentials, scalar signals…**

… But **GW** are not scalars! $$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0$$

… And what if the lens is **strong field** (e.g. a **black hole**)?

Expect even **richer phenomenology !**

… but a practical and generic framework for diffractive gravitational lensing is missing

Tensorial lensing by a strong field:

If the lens has high symmetry e.g. Schwarzschild BH:

Black hole perturbation theory provides approximate analytical results for long wavelength $R_{\rm Schw}/\lambda_{\rm wave} < 1$

Tensorial lensing by a strong field:

If the lens has high symmetry e.g. Schwarzschild BH:

Black hole perturbation theory provides approximate analytical results for long wavelength $R_{\mathrm{Schw}}/\lambda_{\mathrm{wave}} < 1$

Solving black perturbation theory as a wave scattering problem:

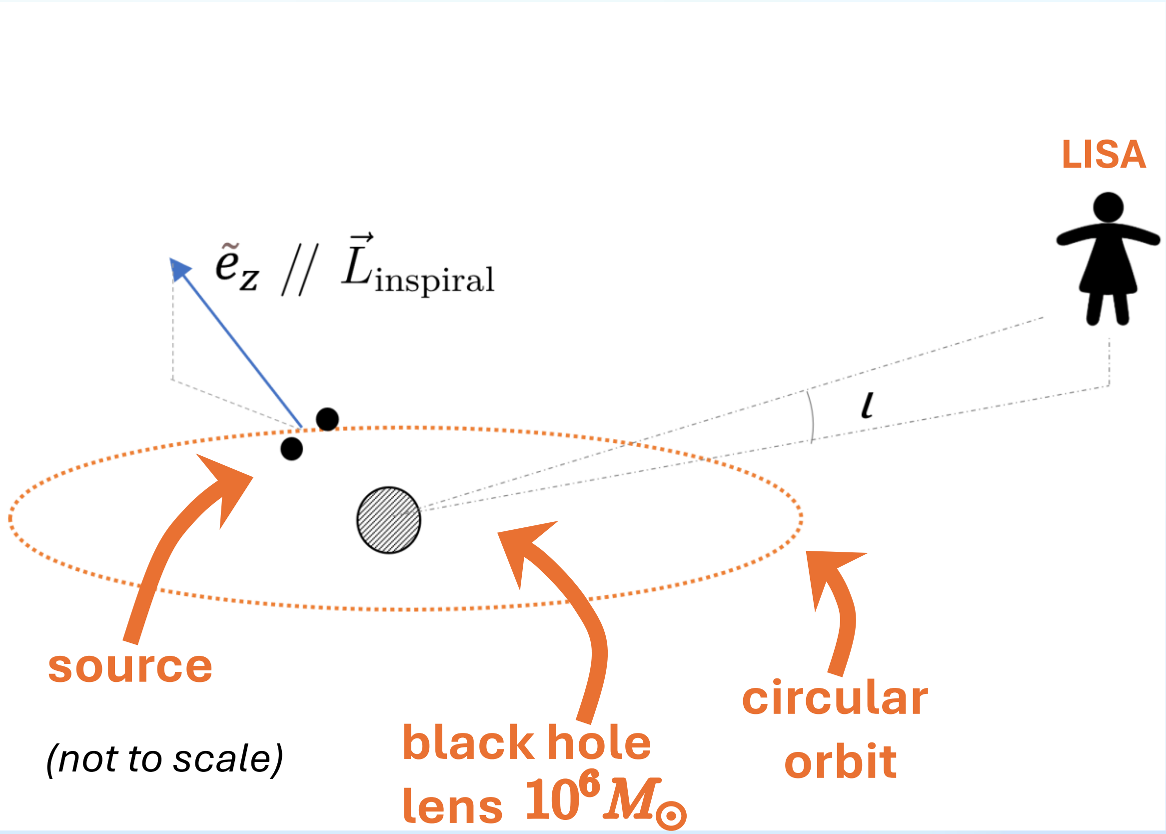
$\rightarrow$ **lensing by a BH depends on the helicity of the signal**

$\rightarrow$ **non-trivial effects on polarisations,** the polarisation content is **not** preserved by lensing Pijnenburg+, 2024,

(historical works: Matzner, Peters, De Logi, Kovacs, Chrzanowski, Futterman, ...)

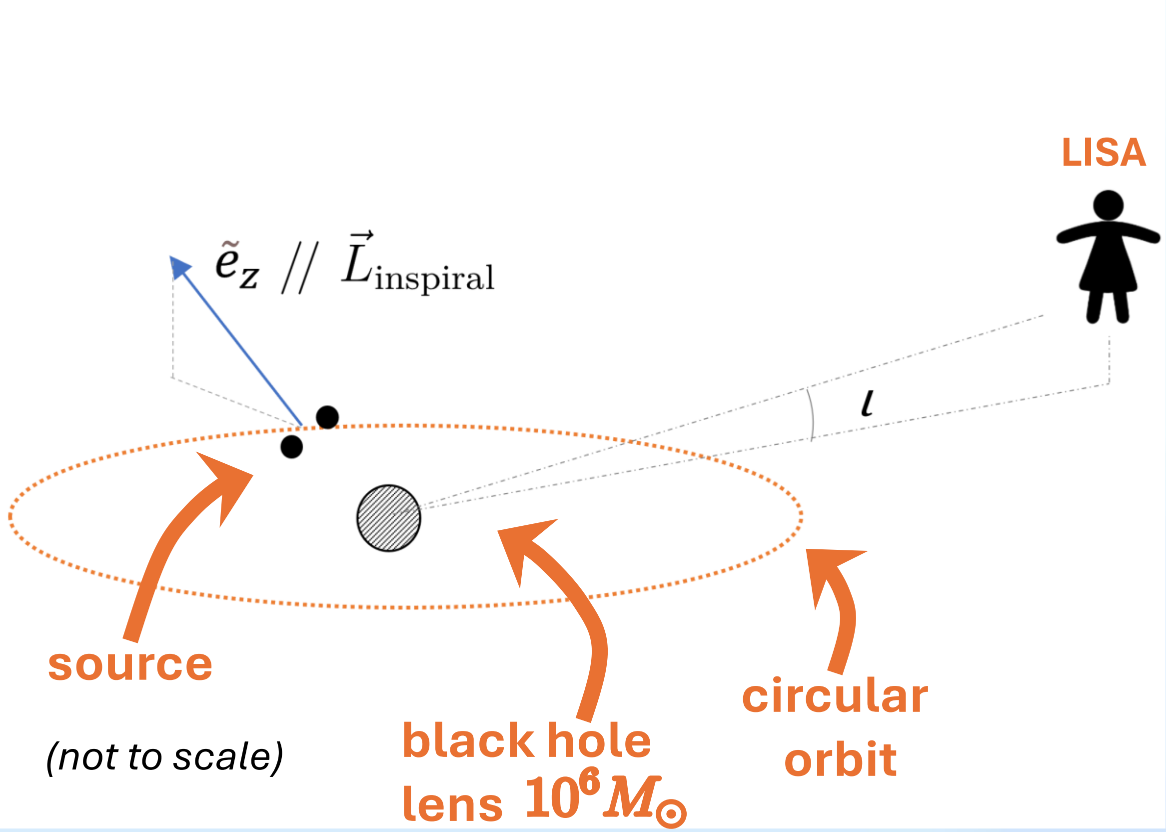
Wave optics lensing in triple systems: towards a phenomenology

Example: consider a microlensing-like **dynamical** setup



Wave optics lensing in triple systems: towards a phenomenology

Example: consider a microlensing-like **dynamical** setup



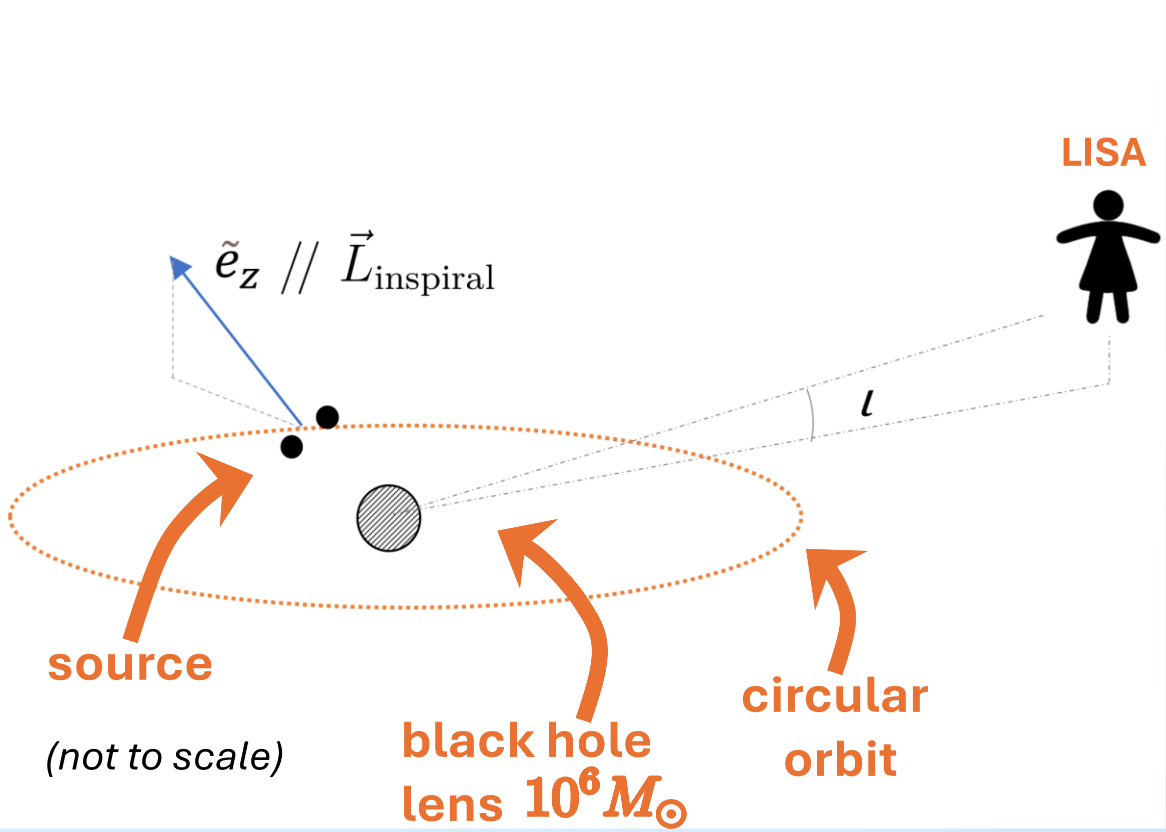
Has been avocated for e.g GW190521

But abundance of such systems remains essentially **unknown**

Distribution of low-end supermassive/intermediate mass black holes ?

Wave optics lensing in triple systems: towards a phenomenology

Example: consider a microlensing-like **dynamical** setup



Stokes parameters of GWs:

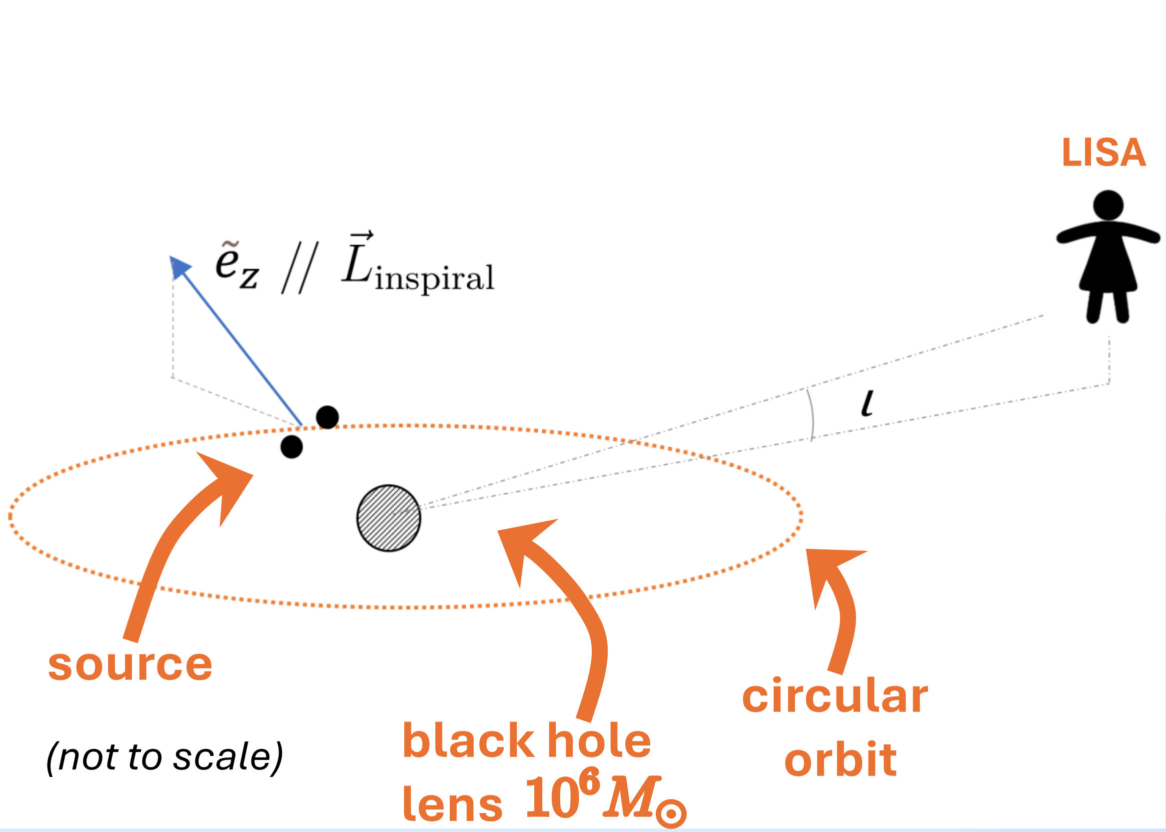
I : total intensity

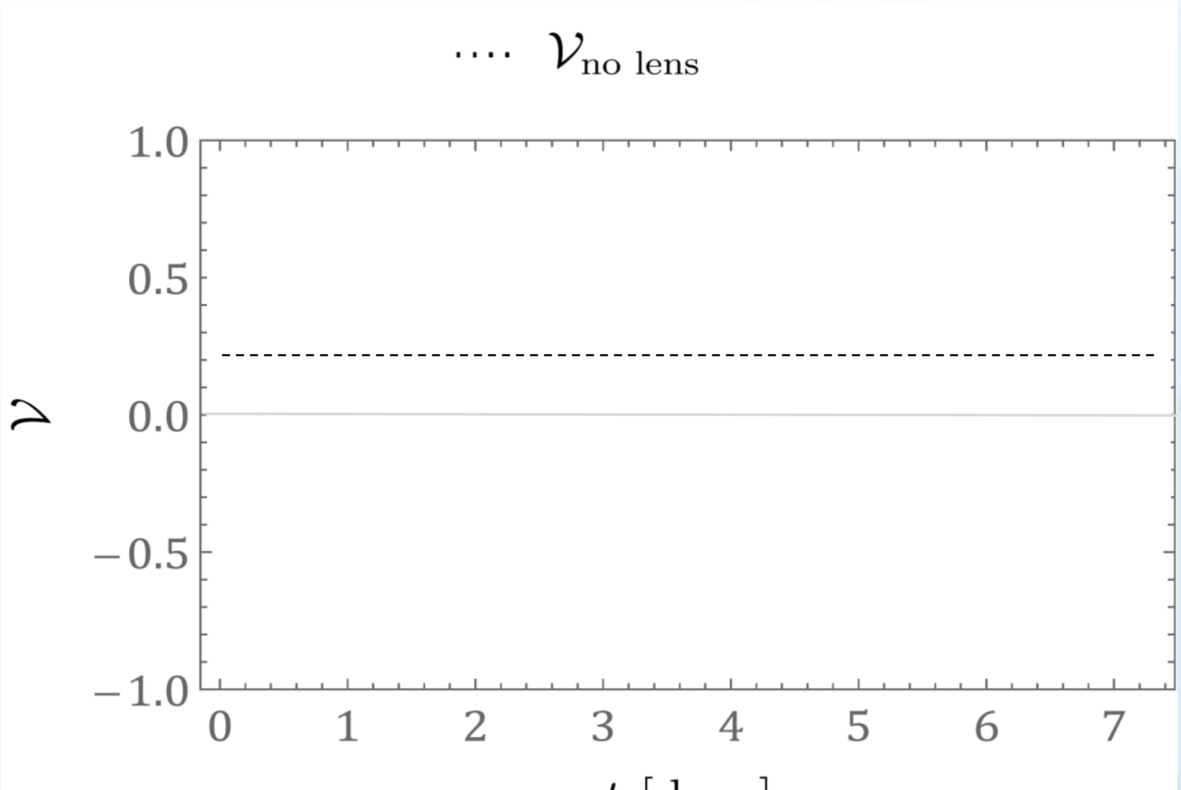
V : circularly-polarized intensity

$\mathcal{V} = V/I$: circularly polarized fraction of the intensity

Wave optics lensing in triple systems: towards a phenomenology

Example: consider a microlensing-like **dynamical** setup

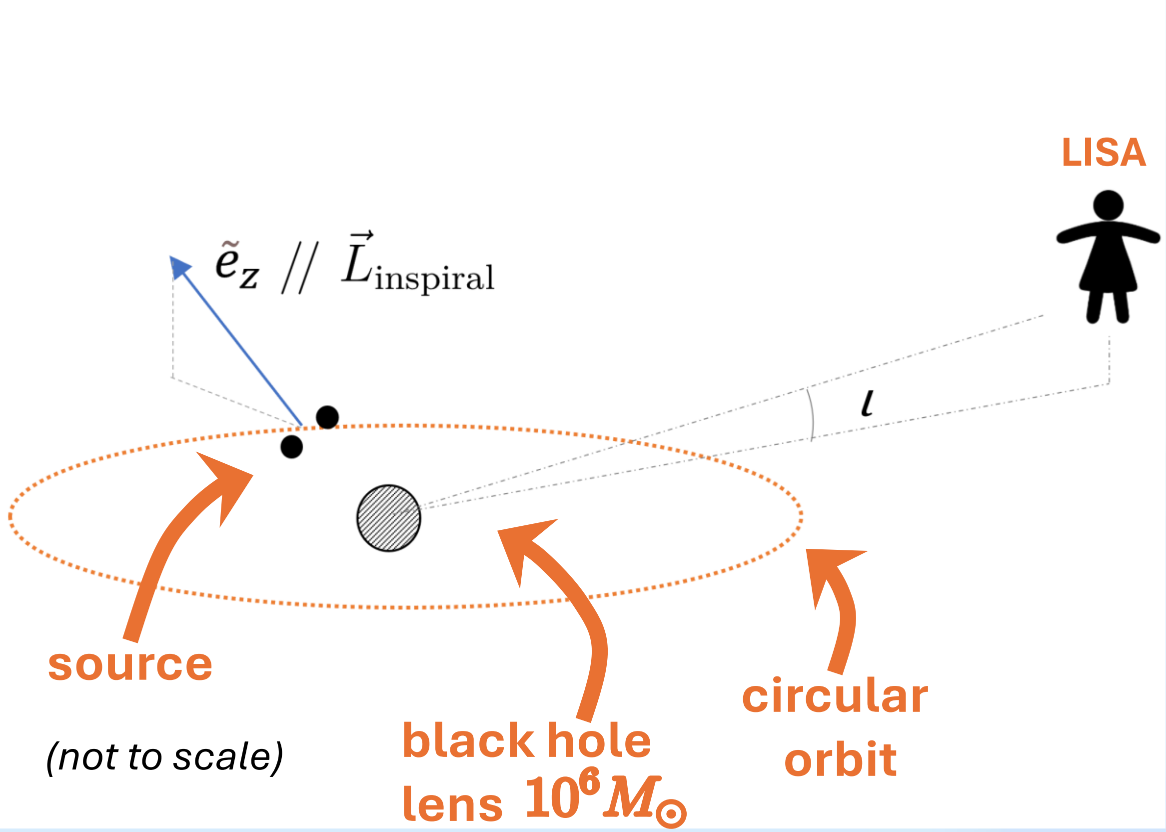


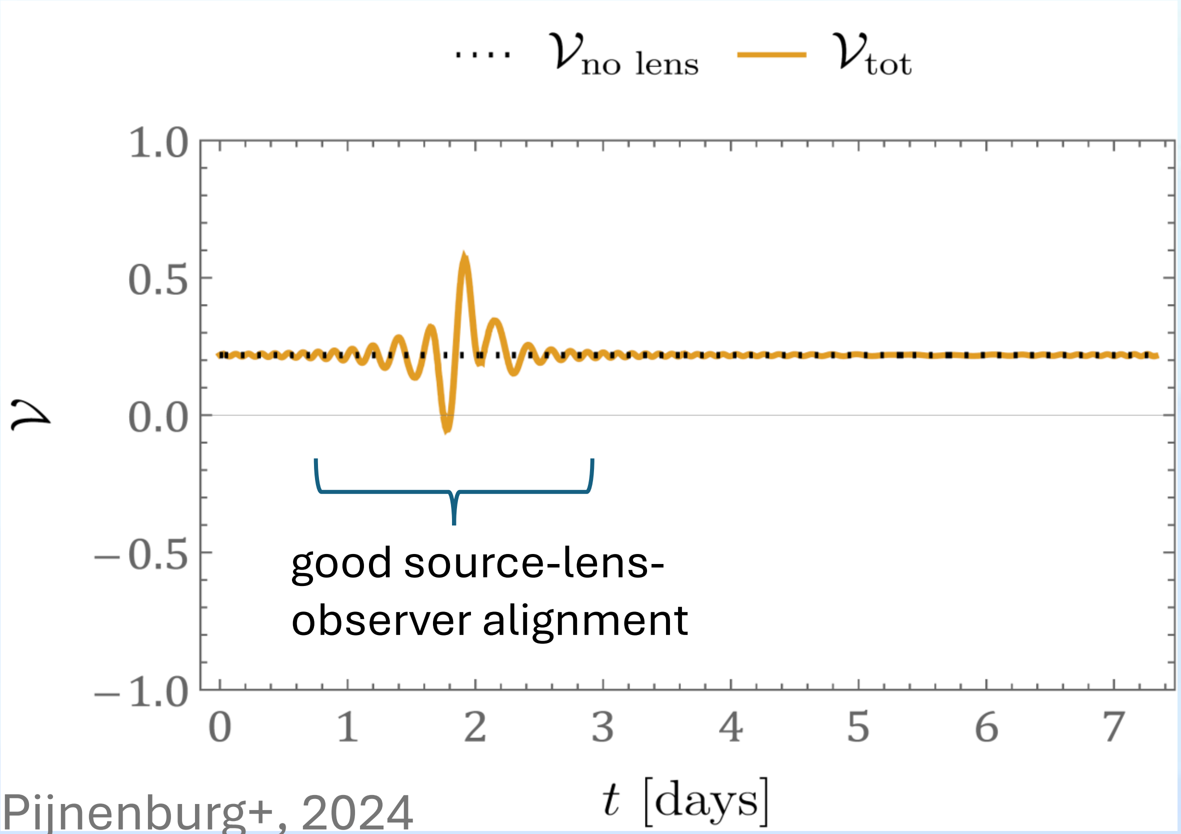


Pijnenburg+, 2024

Wave optics lensing in triple systems: towards a phenomenology

Example: consider a microlensing-like **dynamical** setup





Pijnenburg+, 2024

Conclusions

- **Wave optics** is a regime in the range of future GW observations
- Relevant lenses include **DM (sub)halos, supermassive black holes**, ...
- **Without new physics, new lensing phenomenology** wrt EM case
  - Frequency-dependent magnification
  - Changes in the polarisation/helicity content (BH lens)

Thanks for your attention !

What about the PTA band ?

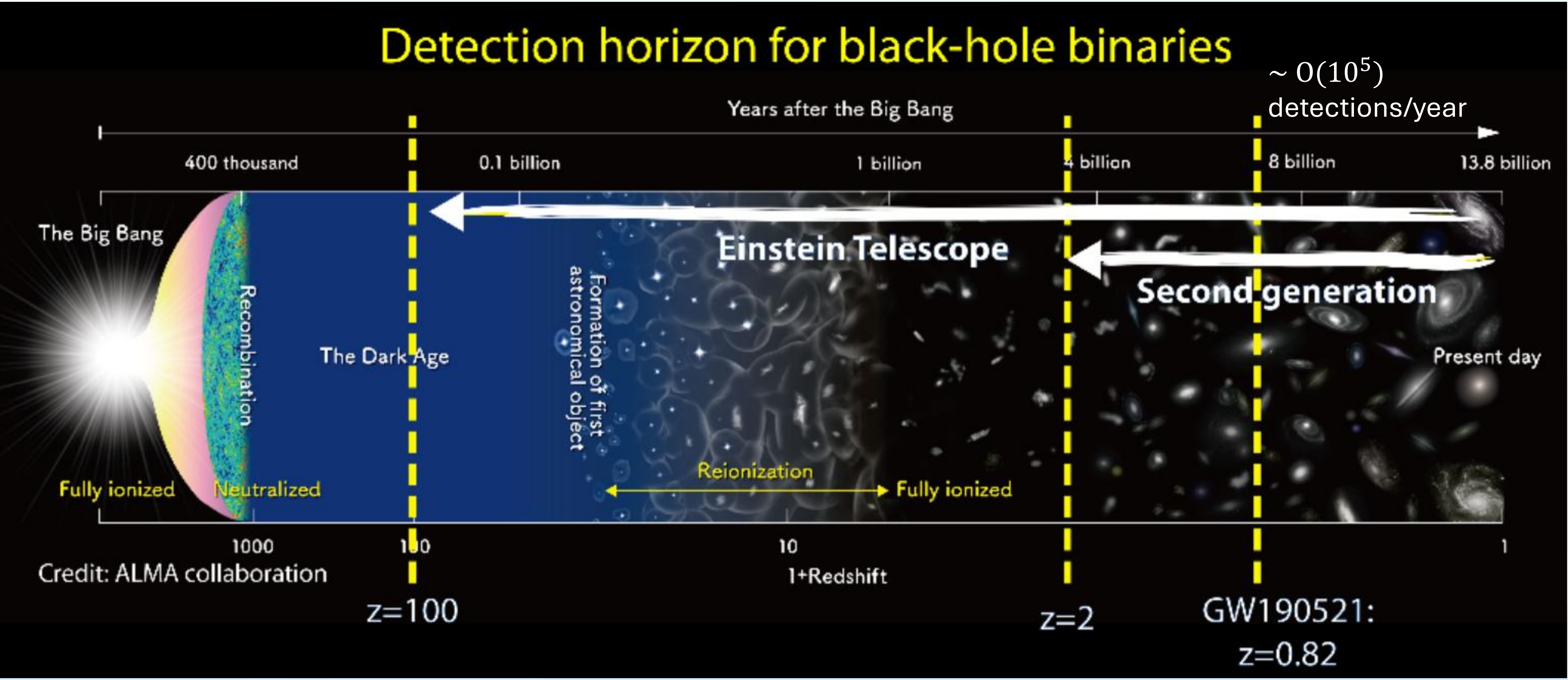
nHz GW signals can in principle be sensitive to big diffractive effects by **halo-sized structures** ...

... but for **GW backgrounds**, the presented diffractive description does not apply due to **the lack of coherence in the signal**

... and **individually resolvable sources** are expected to be **rare**

→ lensing effects are **non trivial** but a priori **hard to exploit**

Einstein Telescope, Cosmic Explorer



Wave optics lensing in triple systems: towards a phenomenology

- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$

Wave optics lensing in triple systems: towards a phenomenology

- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$

- AGN lens mass : $M = 1.2 \times 10^6 M_\odot \implies \boxed{\omega M = 0.11}$

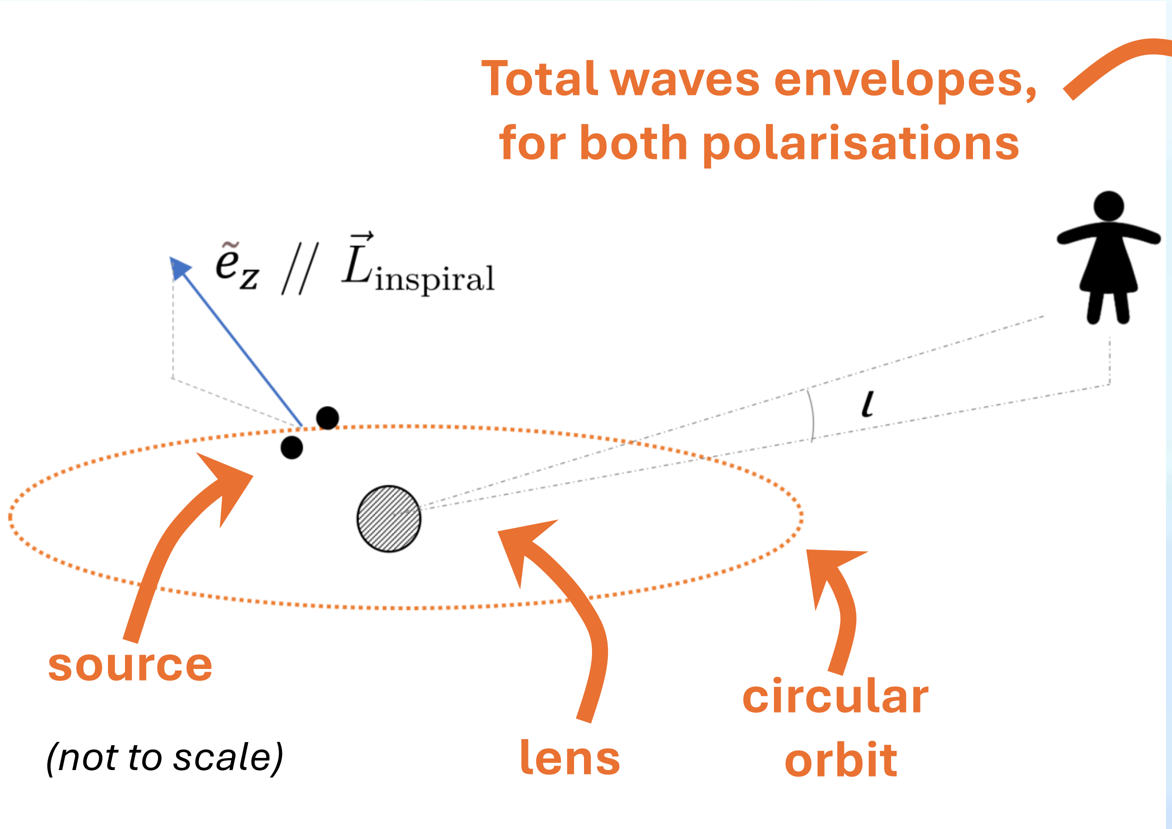
Wave optics lensing in triple systems: towards a phenomenology

- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$
- AGN lens mass : $M = 1.2 \times 10^{6} M_{\odot} \implies \boxed{\omega M = 0.11}$
- Source-Lens distance (disk migration trap) : $d_{SL} = 700M$

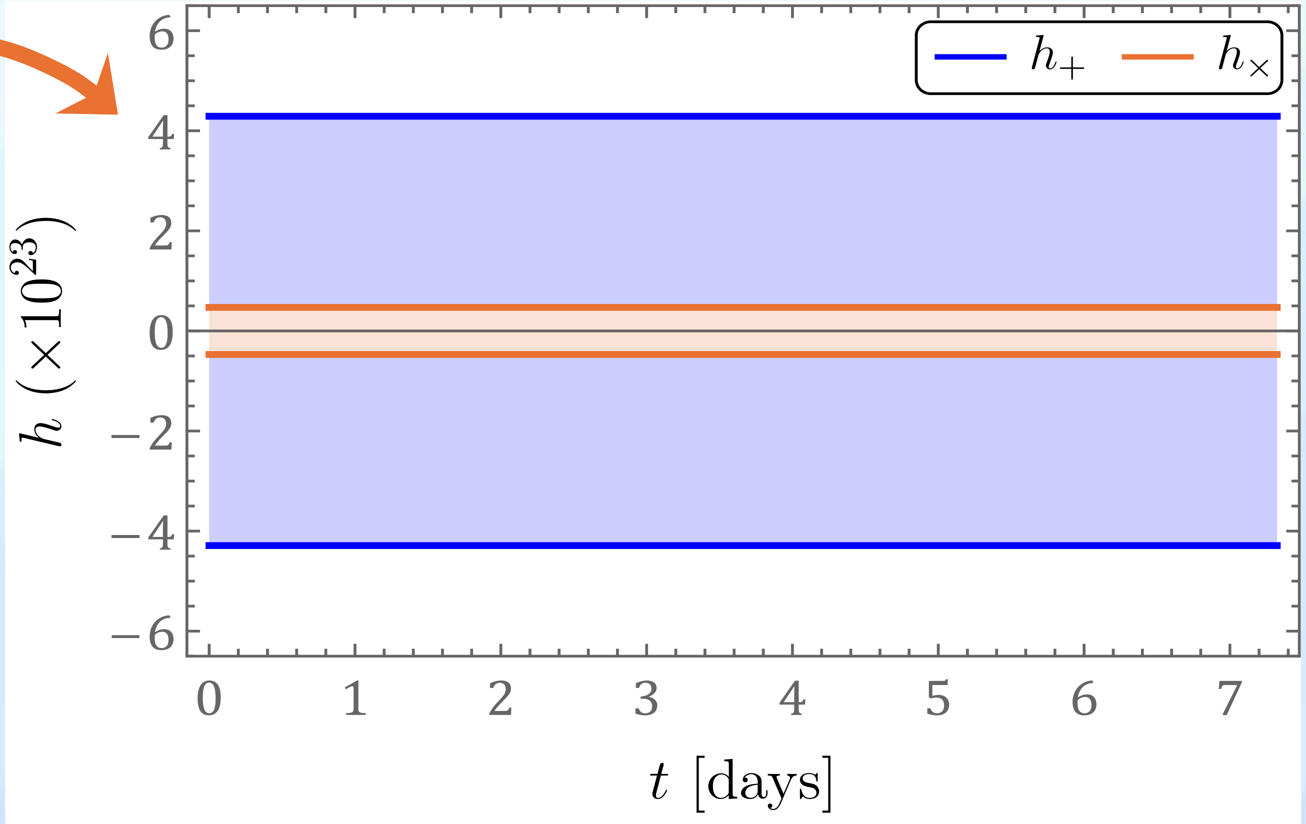
Wave optics lensing in triple systems: towards a phenomenology

- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$
- AGN lens mass : $M = 1.2 \times 10^6 M_\odot \implies \boxed{\omega M = 0.11}$
- Source-Lens distance (disk migration trap) : $d_{SL} = 700M$
- GW190521-inspired heavy source : $m_1 = 120 M_\odot, m_2 = 71 M_\odot$

Wave optics lensing in triple systems: towards a phenomenology

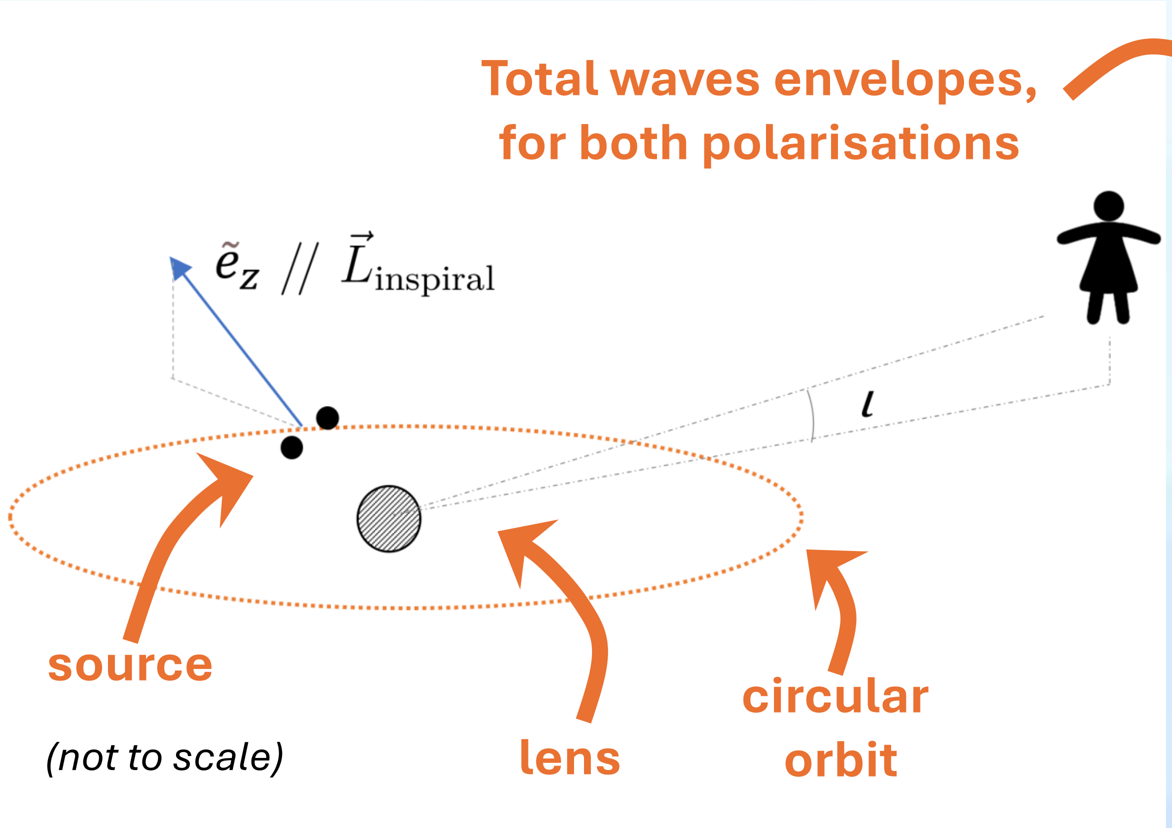


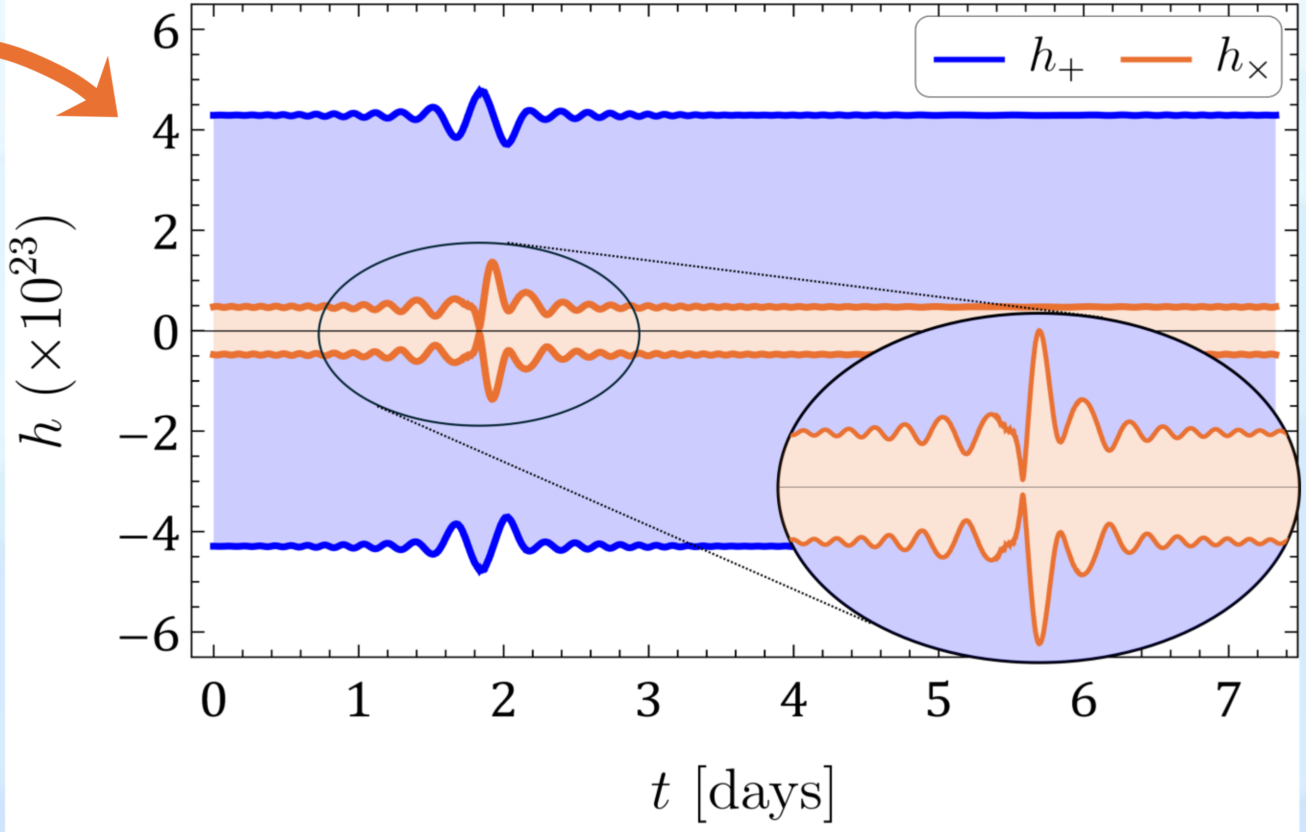
Inspiral phase *without lens*



Wave optics lensing in triple systems: towards a phenomenology

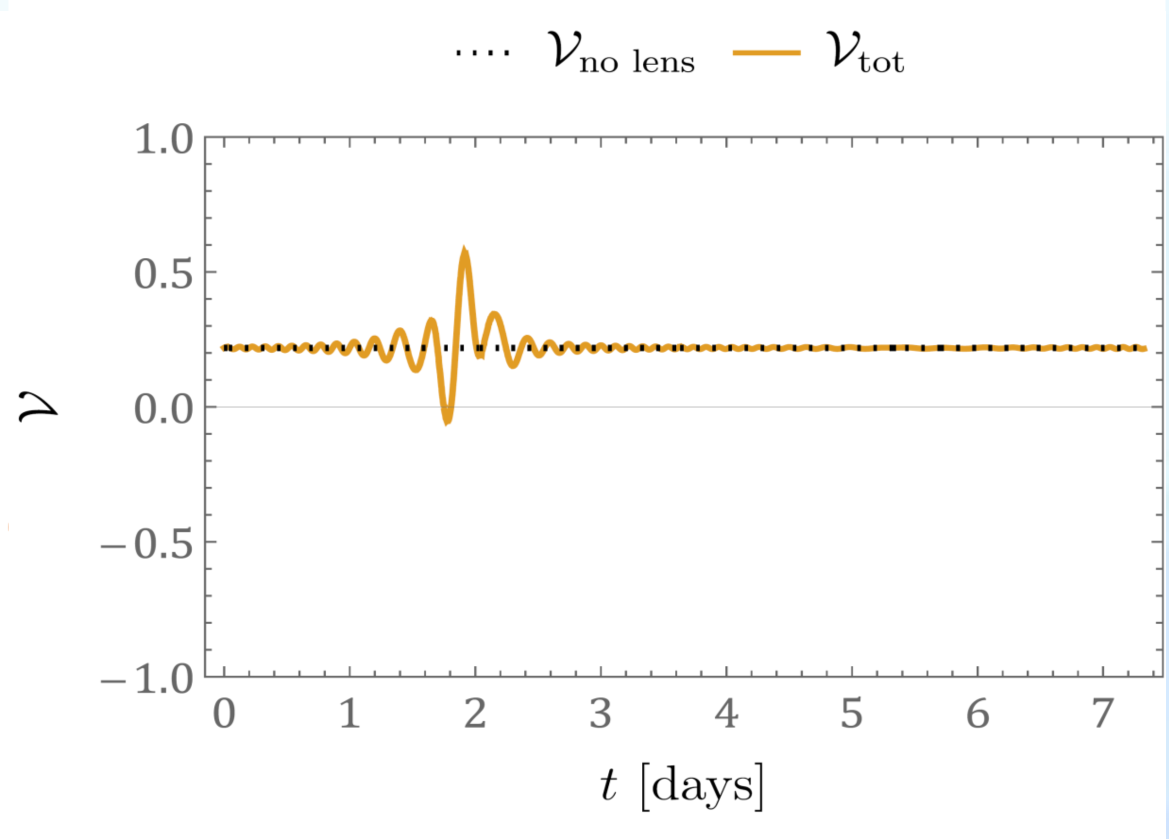
Toy GW190521-inspired source, in LISA- « optimal » wave optics

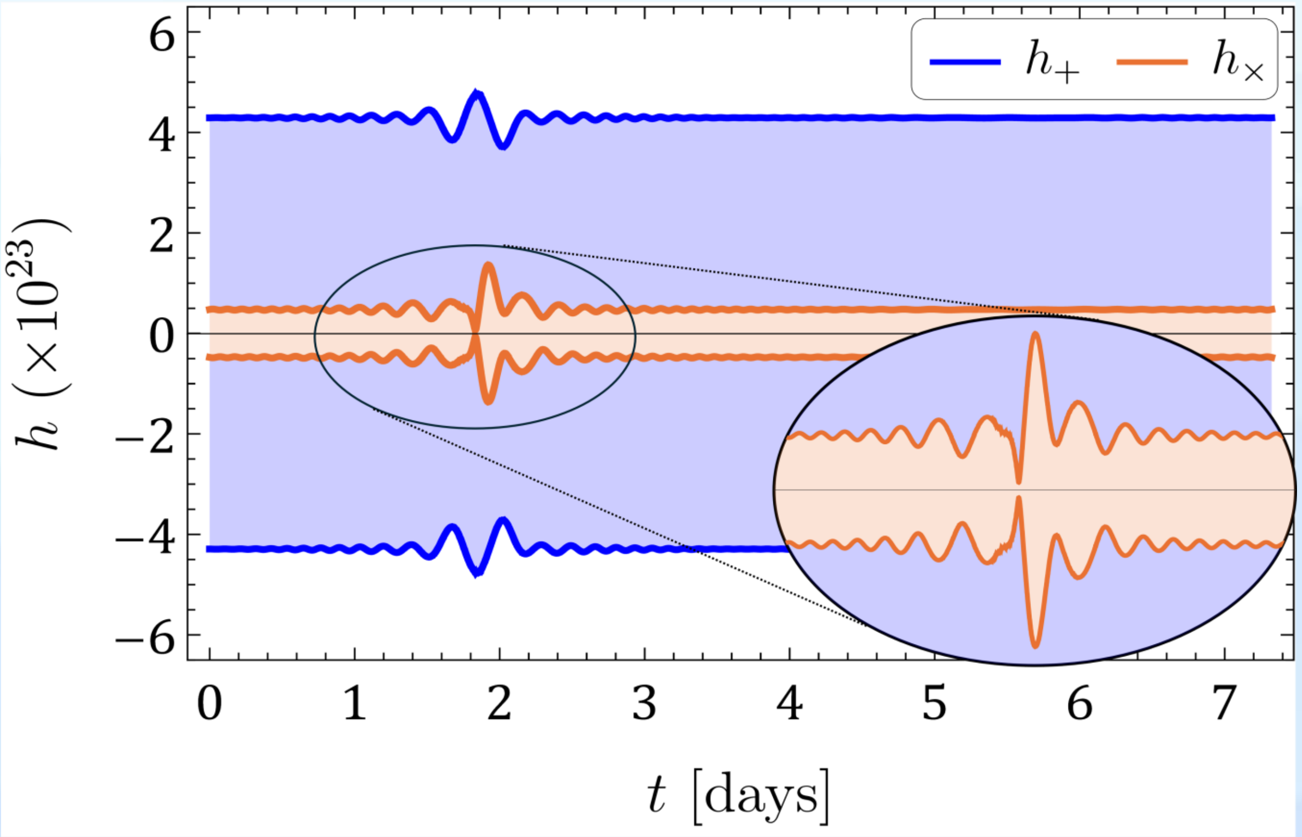




Wave optics lensing in triple systems: towards a phenomenology

Toy GW190521-inspired source, in LISA- « optimal » wave optics





GW lensing: wave optics

Start with : $\mathrm{d}s^2 = (\bar{g}_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$

E.g. gauge fixing : $h^{\nu}_{\mu;\nu} = 0, \quad h^{\mu}{}_{\mu} = 0$

$\longrightarrow$ Wave equation :

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0, \quad \text{with} \quad h_{\mu\nu;\alpha}{}^{;\alpha} \equiv \Box h_{\mu\nu}.$$

GW lensing: wave optics

BH lenses, historical works, at the formal level :

- Matzner (1968)
- Peters (1976)
- Chrzanowski *et al.* (1976)
- De Logi, Kovacs (1977)
- Futterman *et al.* (1988)

…

More recently: Dolan (2018)

GW lensing: wave optics

Reference work for phenomenology :

Wave effects in gravitational lensing of gravitational waves from chirping binaries

Ryuichi Takahashi (Kyoto U.), Takashi Nakamura (Kyoto U.)

May, 2003

28 pages

Published in: *Astrophys.J.* 595 (2003) 1039-1051

e-Print: astro-ph/0305055 [astro-ph]

DOI: 10.1086/377430

View in: ADS Abstract Service

pdf cite claim reference search 253 citations

GW lensing: wave optics

Reference work for phenomenology :

Wave effects in gravitational lensing of gravitational waves from chirping binaries

Ryuichi Takahashi (Kyoto U.), Takashi Nakamura (Kyoto U.)

May, 2003

28 pages

Published in: *Astrophys.J.* 595 (2003) 1039-1051

e-Print: astro-ph/0305055 [astro-ph]

DOI: 10.1086/377430

View in: ADS Abstract Service

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0$$

Assume $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Solve for ϕ

Tensorial wave optics

In *Pijnenburg, et al., 2024*, we treat lensing by a Schwarzschild BH

avoiding the assumption $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Tensorial wave optics

In *Pijnenburg, et al.*, *2024*, we treat lensing by a Schwarzschild BH

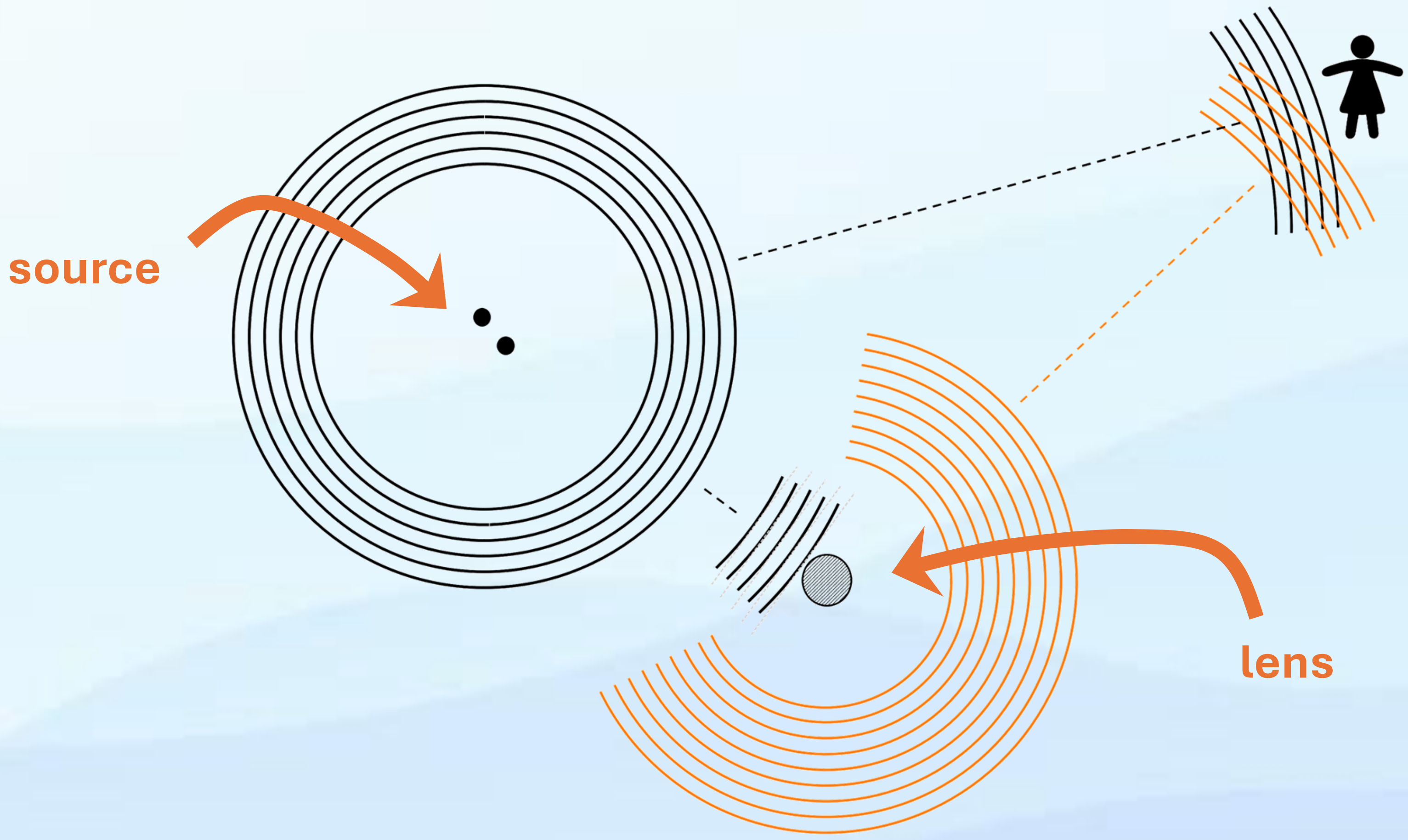
avoiding the assumption $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Rather use : - black hole perturbation theory (BHPT)
- (quantum) waves scattering (e.g. phase shifts)

since the equations are quantum like (RW, Zerilli)

to keep track of the full polarisation structure analytically

Tensorial wave optics



Polarisation

Quantifying the signal polarisation content $\mathcal{V} \in [-1, 1]$:

$$\mathcal{V} \equiv \frac{2\mathrm{Im}[\tilde{h}_+ \tilde{h}_\times^*]}{|\tilde{h}_+|^2 + |\tilde{h}_\times|^2} = V/I \qquad \text{in terms of the Stokes parameters } V, I\,.$$

$$= \frac{|\tilde{h}^{(2)}|^2 - |\tilde{h}^{(-2)}|^2}{|\tilde{h}^{(2)}|^2 + |\tilde{h}^{(-2)}|^2}$$

constant in geometric optics and scalar wave optics

Polarisation

Quantifying the signal polarisation content $\mathcal{V} \in [-1, 1]$:

$$\mathcal{V} \equiv \frac{2\mathrm{Im}[\tilde{h}_+ \tilde{h}_\times^*]}{|\tilde{h}_+|^2 + |\tilde{h}_\times|^2} = V/I \qquad \text{in terms of the Stokes parameters } V, I\,.$$

$$= \frac{|\tilde{h}^{(2)}|^2 - |\tilde{h}^{(-2)}|^2}{|\tilde{h}^{(2)}|^2 + |\tilde{h}^{(-2)}|^2}$$

constant in geometric optics and scalar wave optics

in general **not constant** in tensorial wave optics for $\lambda_{GW} \gg \frac{2GM_{\mathrm{lens}}}{c^2}$

Tensorial wave optics : BHPT

Project $h_{\mu\nu}$ on basis functions on the sphere with even (Y) and odd (X) parity :

$$h_{rr} = \sum_{\ell m} h_{rr}^{\ell m} \; Y^{\ell m}, \qquad \text{(radial)}$$

$$h_{rA} = \sum_{\ell m} h_r^{\ell m} \; X_A^{\ell m} + j_r^{\ell m} \; Y_A^{\ell m}, \quad A = \theta, \phi, \qquad \text{(radial/angular)}$$

$$h_{AB} = \sum_{\ell m} h_2^{\ell m} X_{AB}^{\ell m} + r^2 G^{\ell m} Y_{AB}^{\ell m} + r^2 K^{\ell m} \Omega_{AB} Y^{\ell m}, \quad A, B = \theta, \phi, \qquad \text{(angular)}$$

Tensorial wave optics : BHPT

From metric multipoles, define two **gauge invariant** master functions:

$$\Psi_{\text{odd}}^{\ell m} = \frac{2r}{(\ell-1)(\ell+2)}\left(\frac{\partial}{\partial r}\hat{h}_t^{\ell m} - \frac{\partial}{\partial t}\hat{h}_r^{\ell m} - \frac{2}{r}\hat{h}_t^{\ell m}\right)$$

$$r^{-1}\Psi_{\text{even}}^{\ell m} \propto \hat{K}^{\ell m} + \frac{2(1-2M/r)}{(\ell-1)(\ell+2)+6M/r}\left((1-2M/r)\hat{h}_{rr}^{\ell m} - r\frac{\partial}{\partial r}\hat{K}^{\ell m}\right)$$

Martel, Poisson. *Physical Review. D* 71.10 (2005)

Tensorial wave optics : BHPT

$\Psi_\bullet^{\ell m}$ obey Zerilli & Regge-Wheeler equations, $\bullet = \text{even, odd}$

$$\frac{\mathrm{d}^2\Psi_\bullet}{\mathrm{d}r_*^2} + \left(\omega^2 - V_\bullet\right)\Psi_\bullet = 0, \qquad \text{with } r_*(r) = r - 2M\ln\left(\frac{r}{2M} - 1\right)$$

Schrödinger-like, for given potentials $V_\bullet(\ell, r, M)$

Poisson, Sasaki. *Physical Review D* 51.10 (1995)

Tensorial wave optics : BHPT

$\Psi_\bullet^{\ell m}$ obey Zerilli & Regge-Wheeler equations, $\quad \bullet = \text{even, odd}$

$$\frac{\mathrm{d}^2\Psi_\bullet}{\mathrm{d}r_*^2} + \left(\omega^2 - V_\bullet\right)\Psi_\bullet = 0, \qquad \text{with } r_*(r) = r - 2M\ln\left(\frac{r}{2M} - 1\right)$$

Schrödinger-like, for given potentials $V_\bullet(\ell, r, M)$

For the scattering problem :

Asymptotic solutions **for $\boldsymbol{\omega M \ll 1}$** are known, expect $\Psi_\bullet^{\ell m} \sim \Psi_\bullet^{\text{plane}} + \Psi_\bullet^{\text{sph}}$

Poisson, Sasaki. *Physical Review D* 51.10 (1995)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

→

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

↓

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

←

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

Tensorial wave optics : BHPT

« Observables »

Assume initial

$h_{\mu\nu}^{\text{source}}$

(absence of lens)

Derive corresponding

$\Psi_{\bullet}^{\text{source}}$

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Derive final

$h_{\mu\nu}$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

→

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

↓

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

←

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

→

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

↓

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

←

Derive final

$$h_{\mu\nu}$$

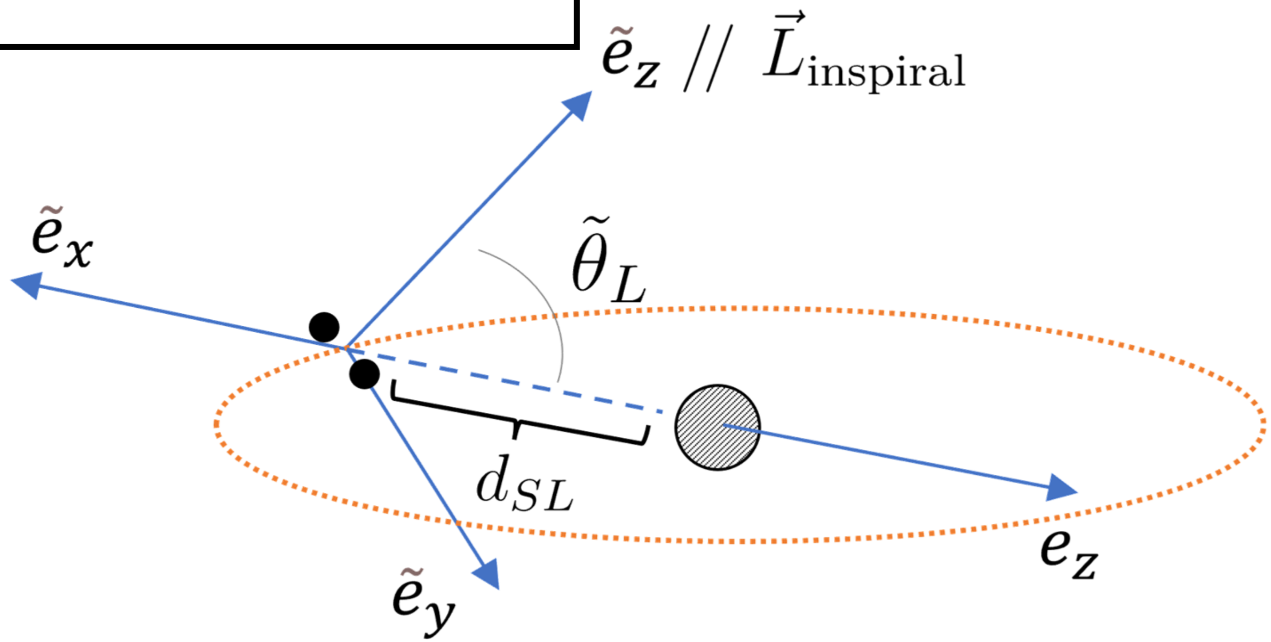
(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\rm source}$$

(absence of lens)



TT gauge, propagation along e_z :

$$h_{ij}^{\rm source} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}$$

$$h_+ = \frac{A_{\rm in}}{\tilde{r}} \frac{1+\cos^2\tilde{\theta}_L}{2} \cos[\omega(t-\tilde{r}) - 2\tilde{\phi}_L]$$

$$h_\times = \frac{A_{\rm in}}{\tilde{r}} \cos\tilde{\theta}_L \sin[\omega(t-\tilde{r}) - 2\tilde{\phi}_L]$$

(locally plane wave)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$h_{\mu\nu}^{\text{source}}$

(absence of lens)

Derive corresponding

$\Psi_{\bullet}^{\text{source}}$

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Derive final

$h_{\mu\nu}$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$h_{\mu\nu}^{\text{source}}$

(absence of lens)

Derive corresponding

$\Psi_{\bullet}^{\text{source}}$

Using scattering phase shifts

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Derive final

$h_{\mu\nu}$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

→

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

↓

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

←

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

→

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

↓

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

←

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

Tensorial wave optics : BHPT

Technicality : in principle, should sum $\lim_{r\to\infty} \sum_{\ell m} \Psi_{\bullet}^{\ell m}$

In practice : $\sum_{\ell m} \lim_{r\to\infty} \Psi_{\bullet}^{\ell m}$

Derive final

$h_{\mu\nu}$

(incl. lensing)

←

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Tensorial wave optics : BHPT

Technicality : in principle, should sum $\lim_{r\to\infty} \sum_{\ell m} \Psi_{\bullet}^{\ell m}$

In practice : $\sum_{\ell m} \lim_{r\to\infty} \Psi_{\bullet}^{\ell m}$ ... **diverges** analytically & numerically

Derive final

$h_{\mu\nu}$

(incl. lensing)

← **non standard summation methods**

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\mathrm{source}} + \Psi_{\bullet}^{\mathrm{lensed}}$