MOND: An alternative to particle dark matter

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INAF
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DI ASTROFISICA
OSSERVATORIO ASTROFISICO DI ARCETRI

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MOND = NO DM

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MOND = MilgrOmiaN Dynamics

Proposed by Moti Milgrom (1983a, b, c)

MOND is not a new idea. But it has gained "momentum" over the past 40+ years.

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

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$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla} \phi_N$ Newtonian gravitational field (from the Poisson's equation)

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3) For $a \ll a_0 \rightarrow a = \sqrt{a_0 g_N} \rightarrow \frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}}$

For a circular orbit at large radii

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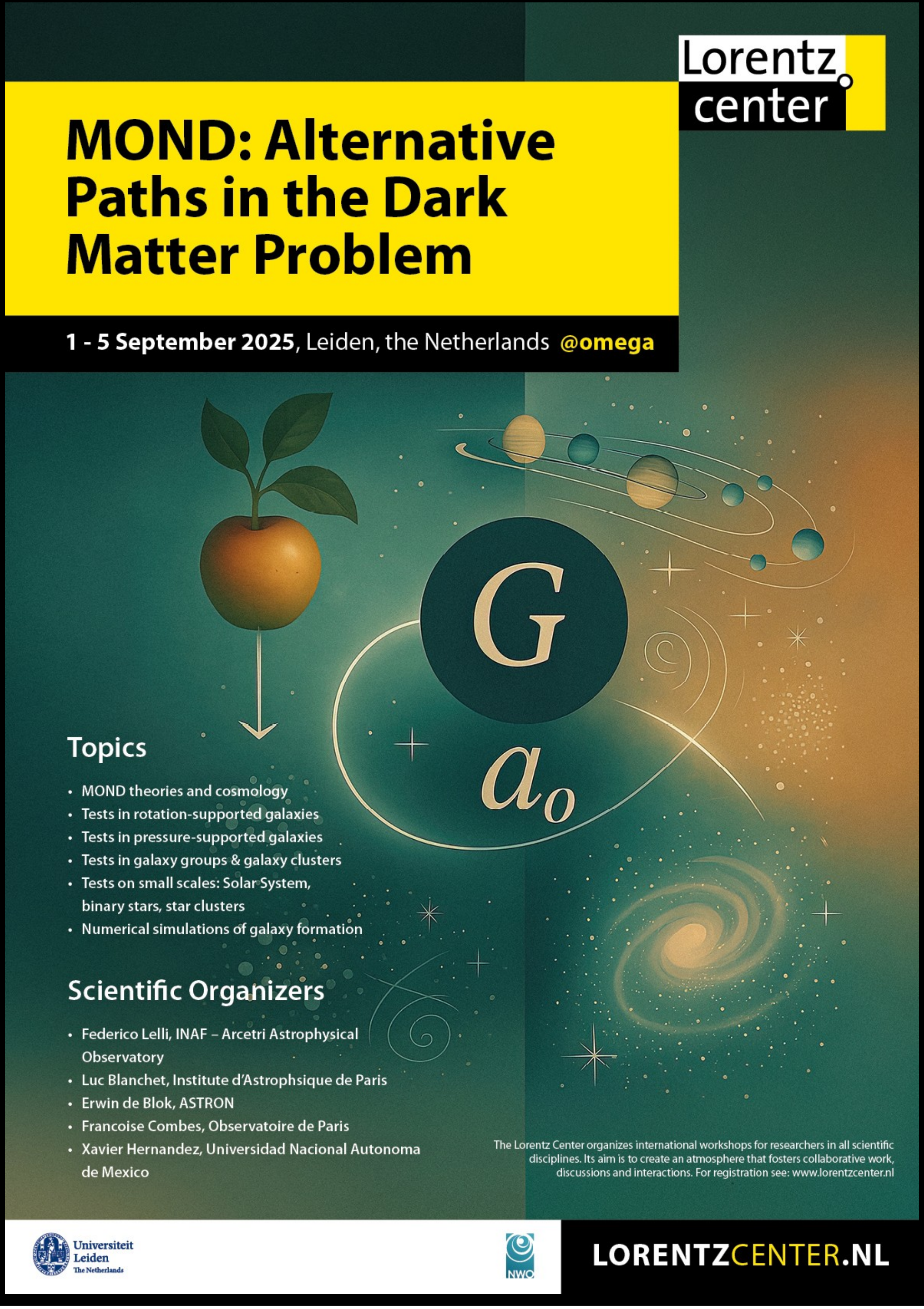
3) For $a \ll a_0 \rightarrow a = \sqrt{a_0 g_N} \rightarrow \frac{V^2}{\cancel{R}} = \sqrt{\frac{a_0 G M_b}{\cancel{R^2}}}$ Flat rotation curve at large R!

For a circular orbit at large radii

The MOND white paper

(50+ authors, 300+ pages)

Leiden (Netherlands), September 2025



The MOND research program:
A white paper on status and prospects

General coordinator: Federico Lelli

Section coordinators: Luc Blanchet, Françoise Combes, Erwin De Blok, Harry Desmond, Antonaldo Diaferio, Benoit Famaey, Jonathan Freundlich, Stacy S. McGaugh, Mordehai Milgrom, Tobias Mistele, Marcel Pawlowski

Contributors: Elena Asencio, Will Barker, Michal Bílek, Anthony Brown, Graeme Candlish, Kyu-Hyun Chae, Øyvind Christiansen, Pierfrancesco Di Cintio, Francis Duey, Amel Durakovic, Stephan Eijt, Stefano Ettori, Eanna Flanagan, Hosein Haghi, Christopher Harwey-Hawes, Akram Hasani Zoonozi, Konstantin Haubner, Aurelien Hees, Xavier Hernandez, Michael Hilker, Zichen Hua, Mark Huisjes, Mariana Júlio, Tahere Kashfi, Anastasiia Lazutkina, Steffen Mieske, Cezary Migaszewski, Giacomo Monari, Oliver Müller, Srikanth Nagesh, Roman Nagy, Luisa Ostorero, Jan Pflamm-Altenburg, Tom Richtler, Nick Samaras, Emeric Seraille, Sofia Splawska, Will Sutherland, Yong Tian, Andreea Varasteanu, Paul Visser, David M. J. Vokrouhlický, Richard Woodard

Endorsers: TBD

Content of the MOND white paper:

1. Introduction / Non-relativistic Theories
2. Tests in Galaxies
3. Tests in Galaxy Groups & Clusters
4. Tests on Small Scales
5. Relativistic Theories and Cosmology
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Tests in Galaxies, Galaxy Groups, and Galaxy Clusters

(1) $V_c(R)$ = const at large R for isolated systems

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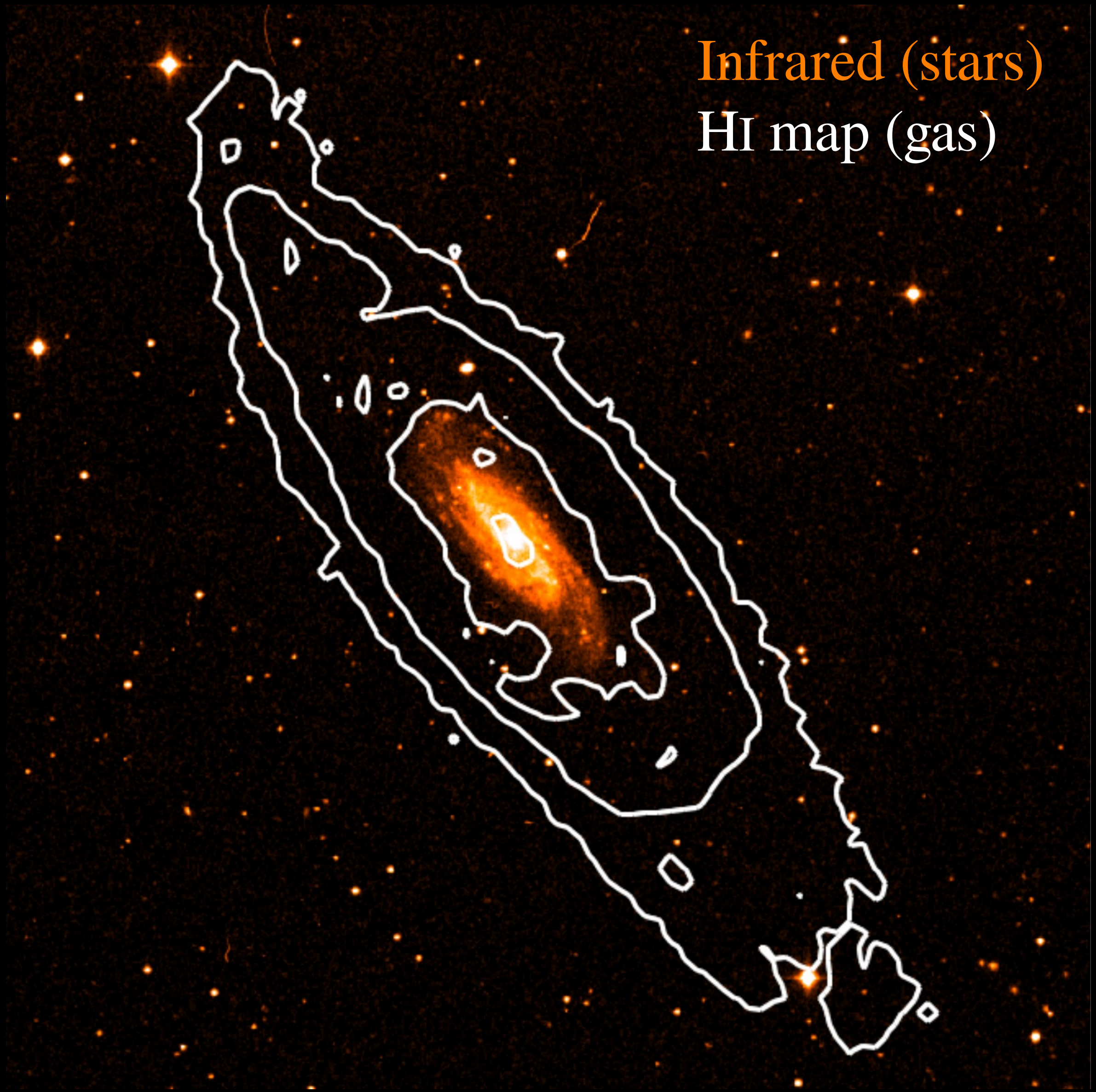

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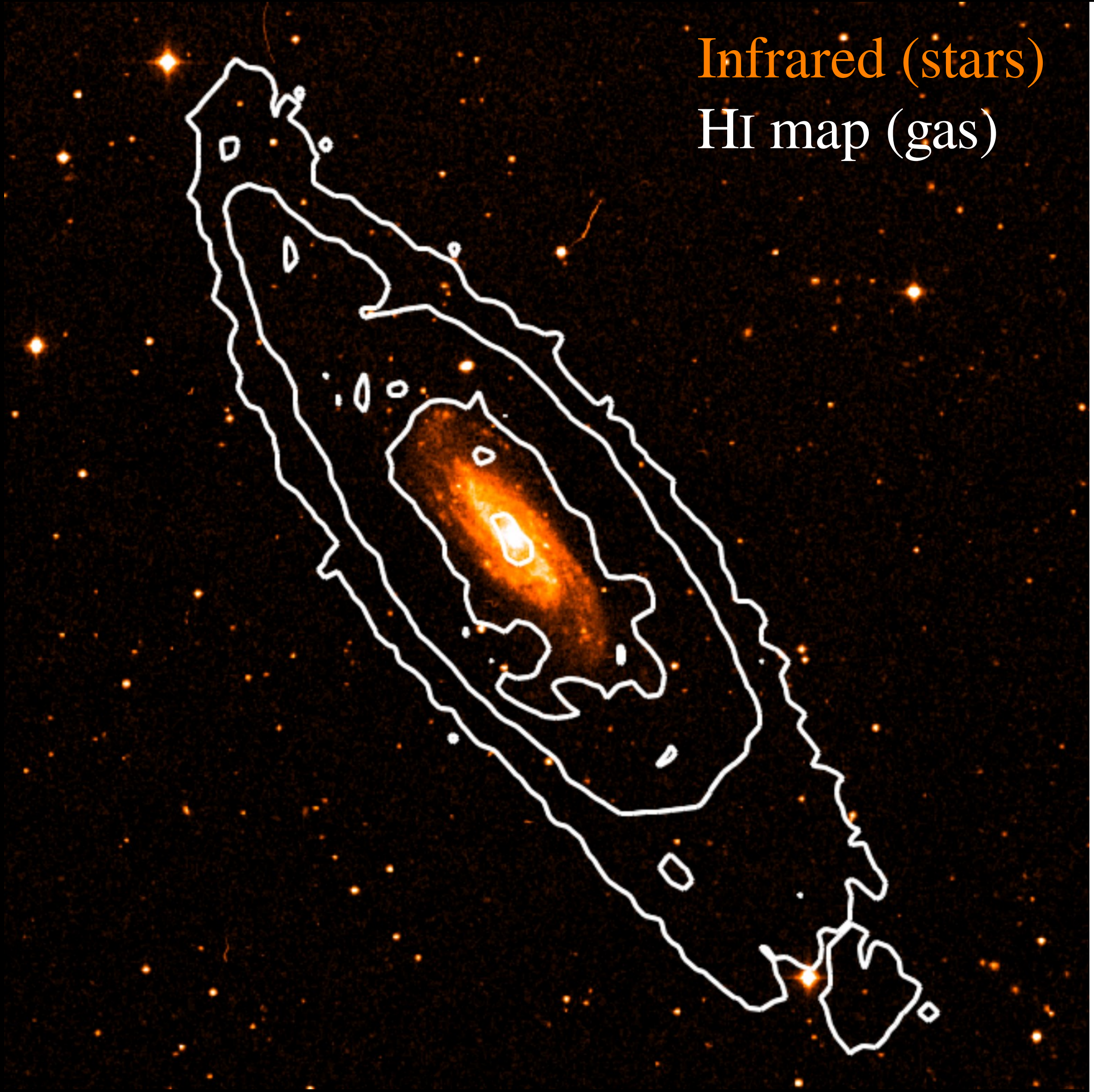

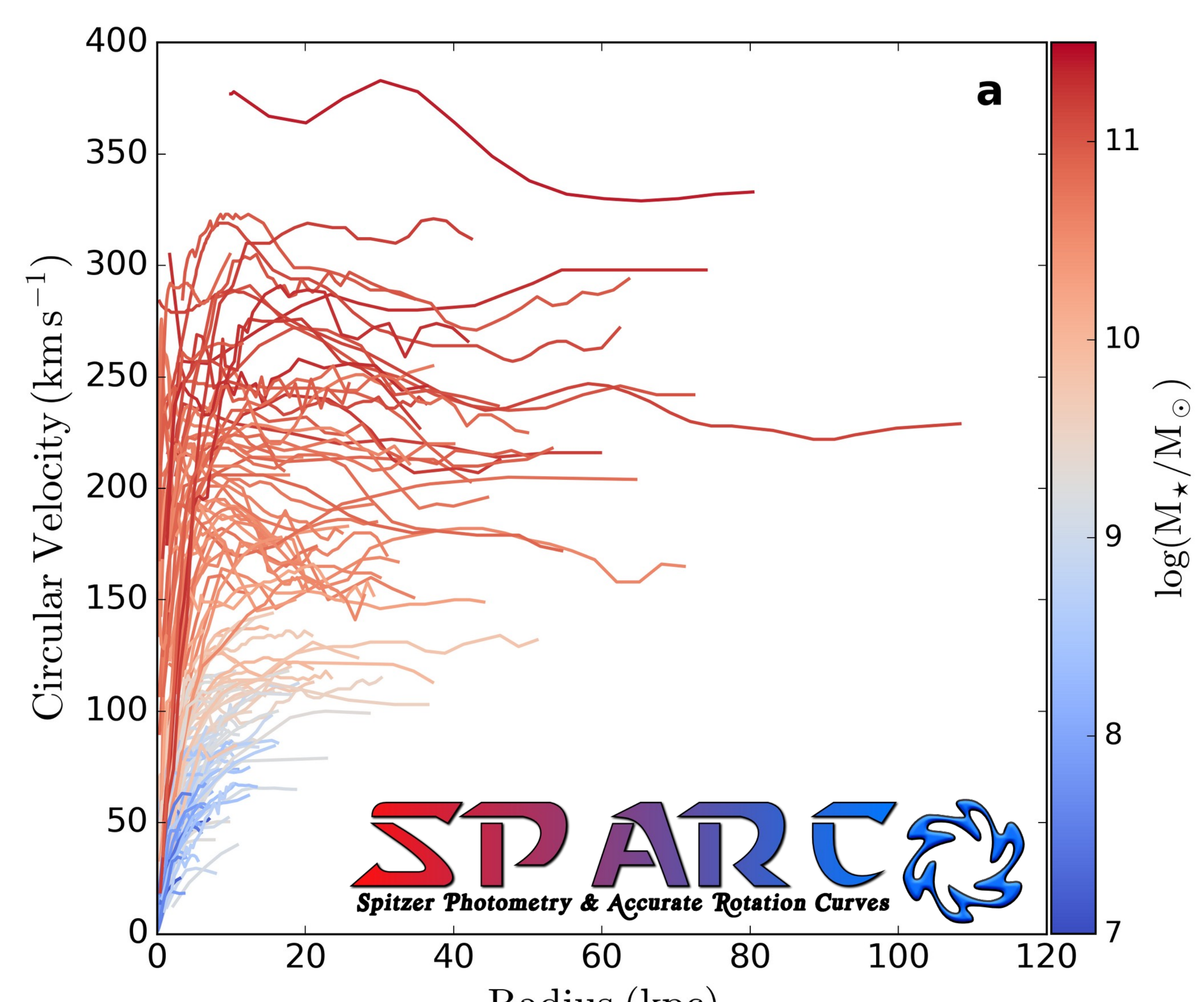

Lelli+(2016, AJ)

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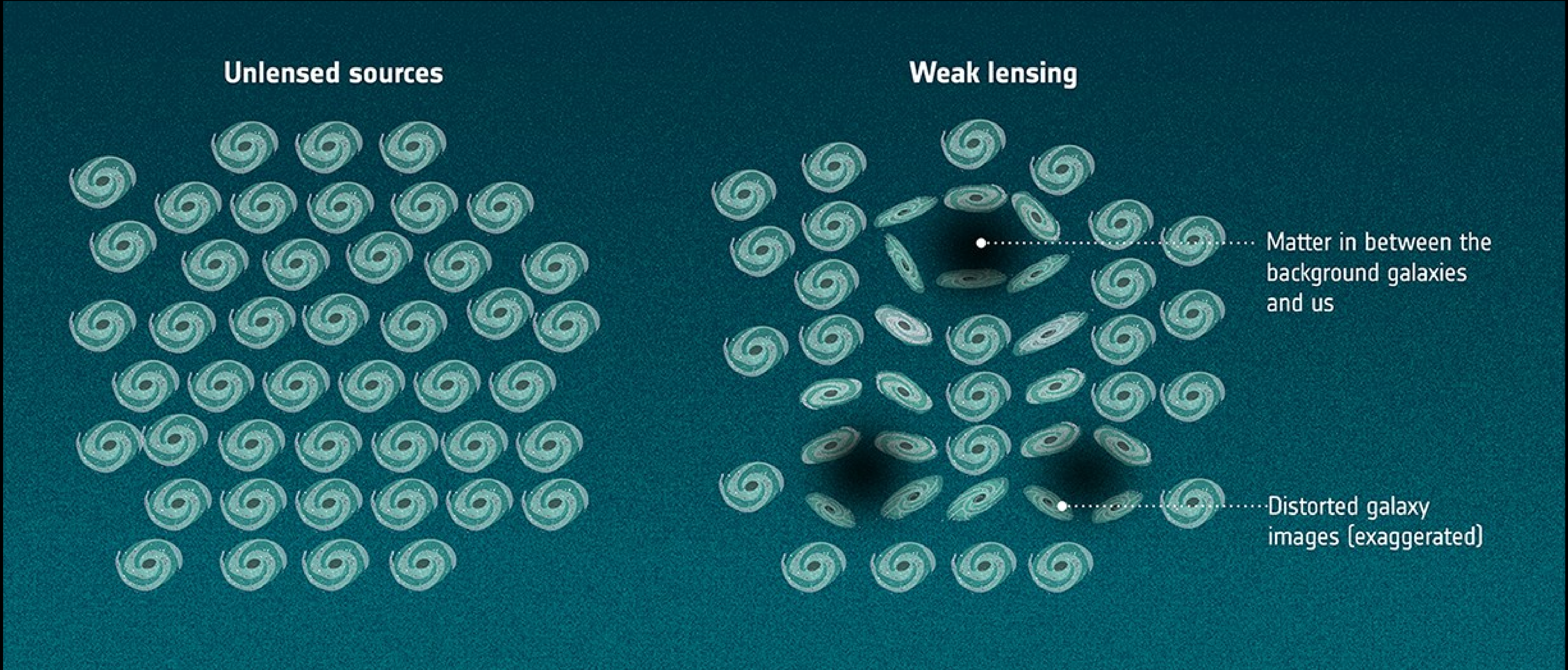

Image Credits: ESA

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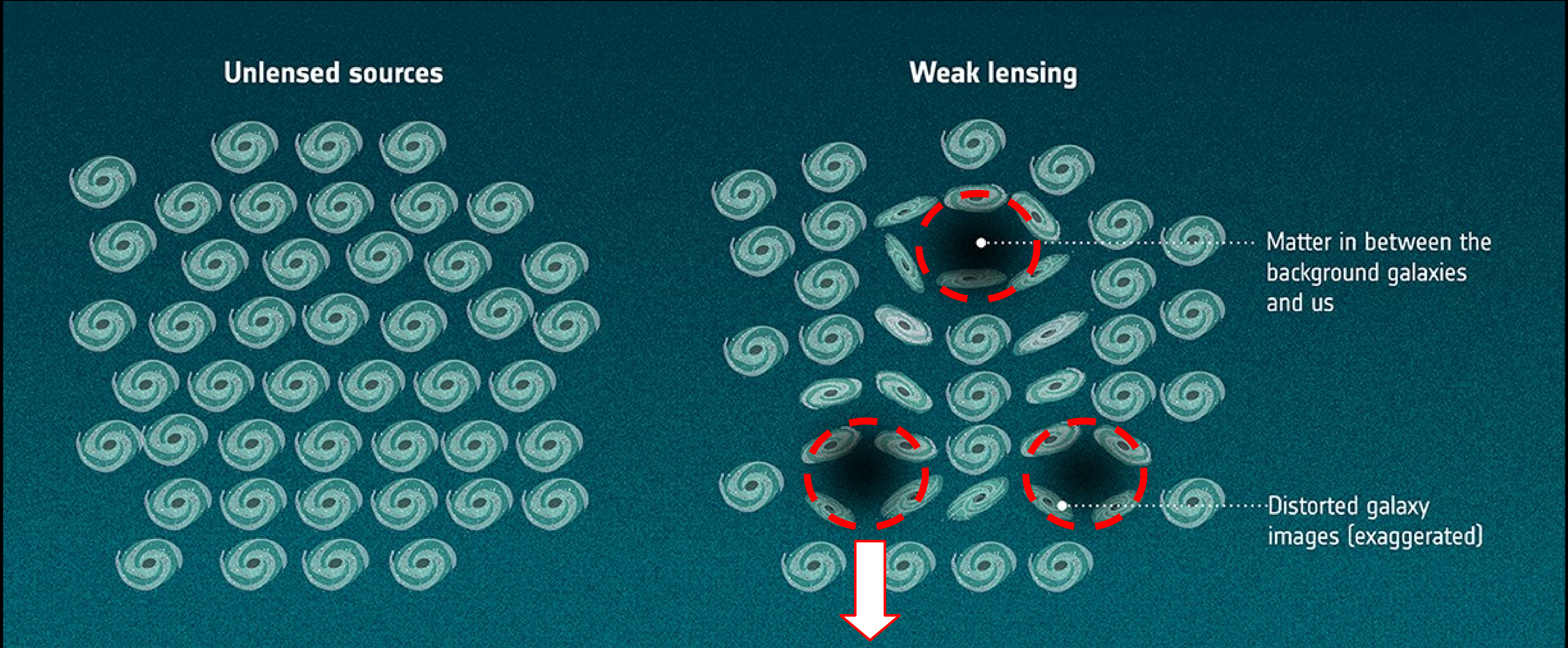

$\Delta\Sigma$ = Excess mass surface density

Image Credits: ESA

(1) $V_c(R)$ = const at large R for isolated systems → Weak Lensing

$$\frac{V_c^2}{r} = 4G \int_0^{\pi/2} \Delta\Sigma\left(\frac{r}{\sin(\theta)}\right) d\theta$$

From $\Delta\Sigma$ to $g = V_c^2/r$ in spherical symmetry

Application to isolated galaxies from KiDS

(Mistele+2024a, JCAP; Mistele+2024b, ApJ)

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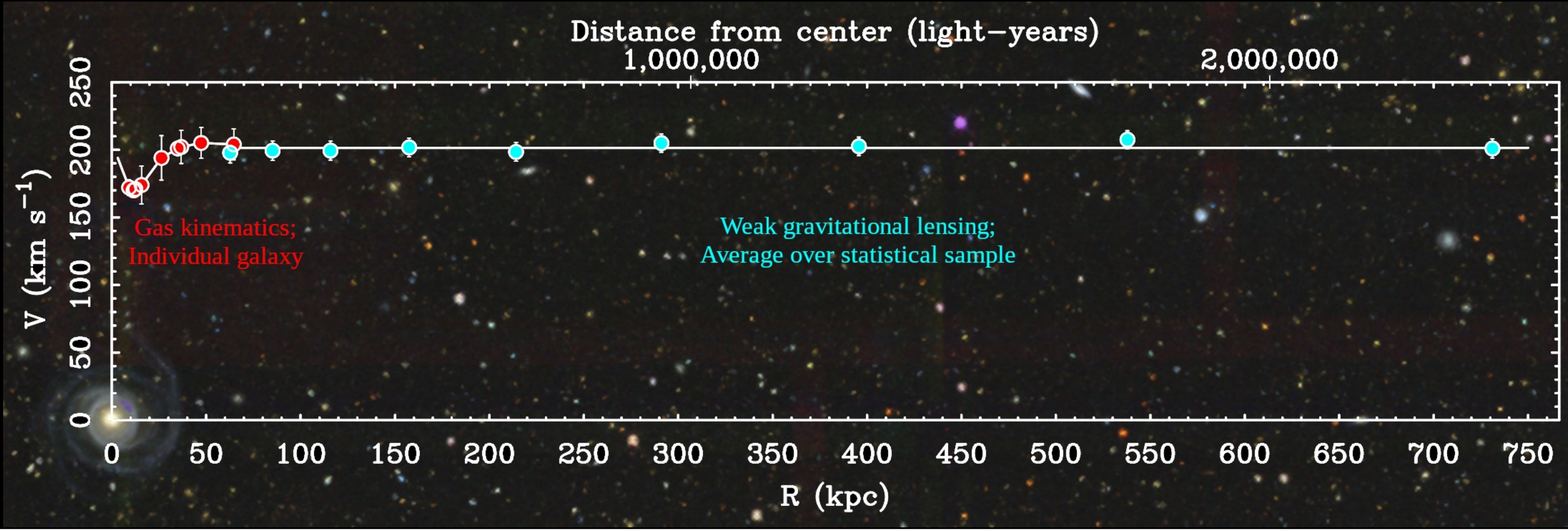

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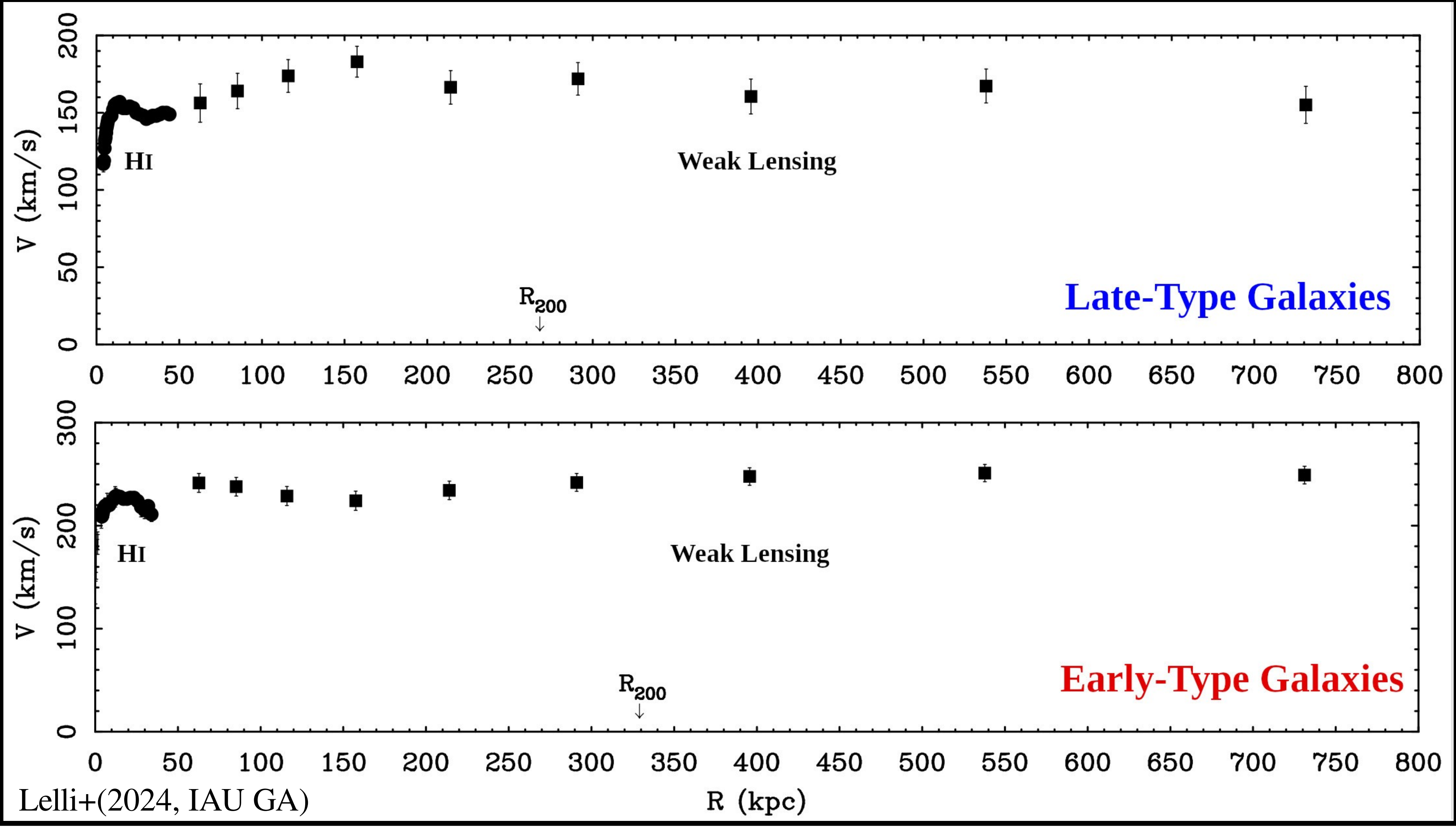

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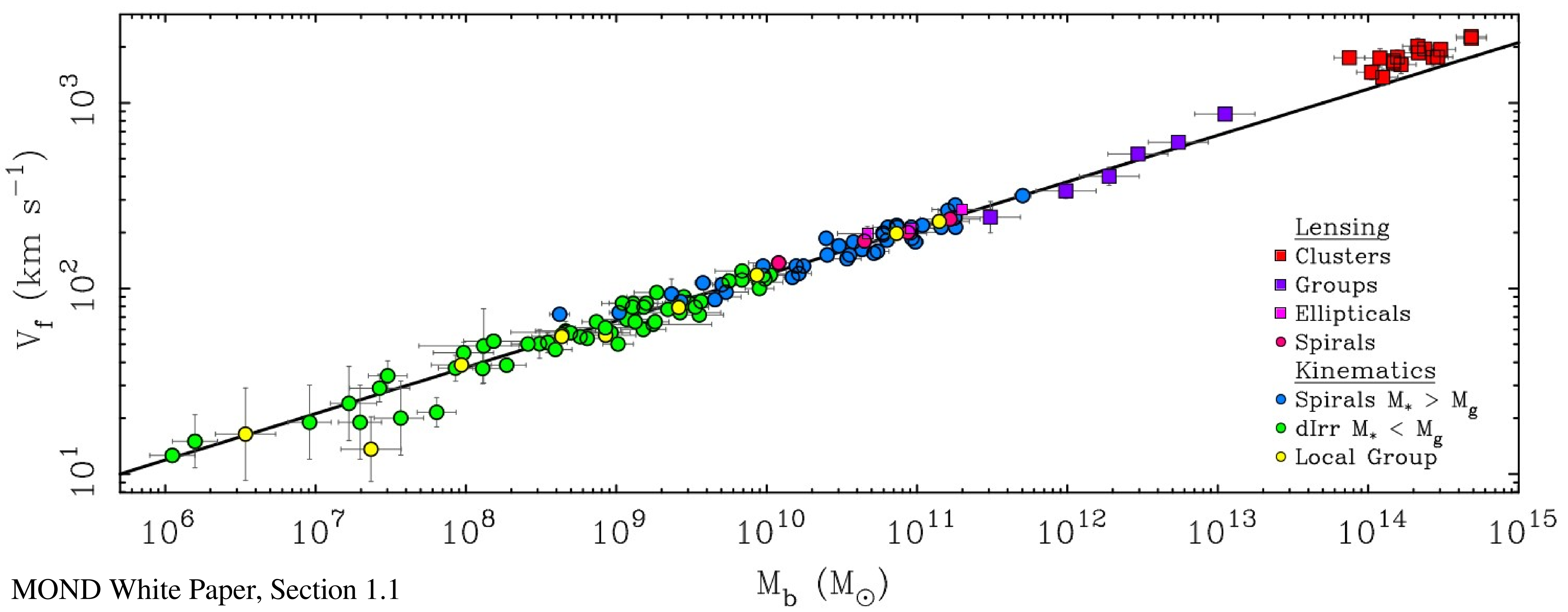
(2) $V_f^4 = a_0 G M_b$ for isolated systems

(2) $V_f^{\,4} = a_0 G M_b$ for isolated systems → Tully-Fisher Relation



MOND White Paper, Section 1.1

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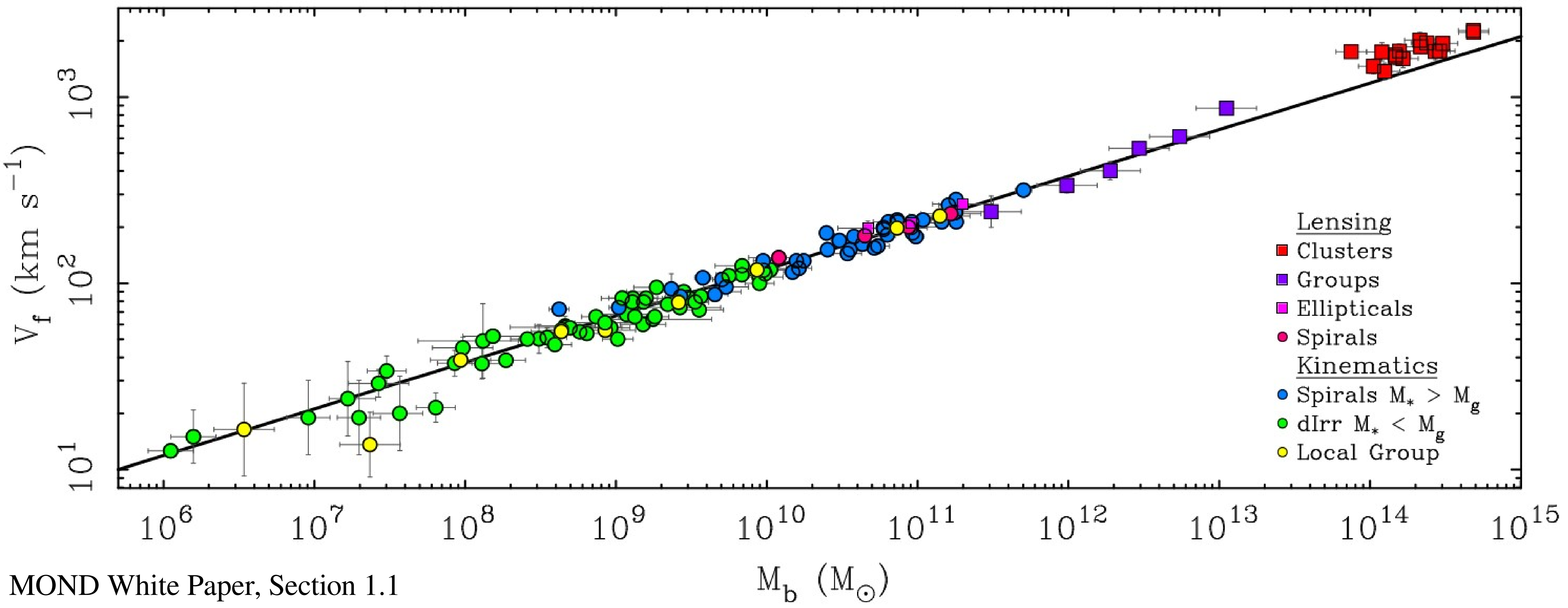

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NOT A FIT TO THE DATA!

Predictions: slope = ¼; intercept = a_0G; no intrinisc scatter; no residual dependencies

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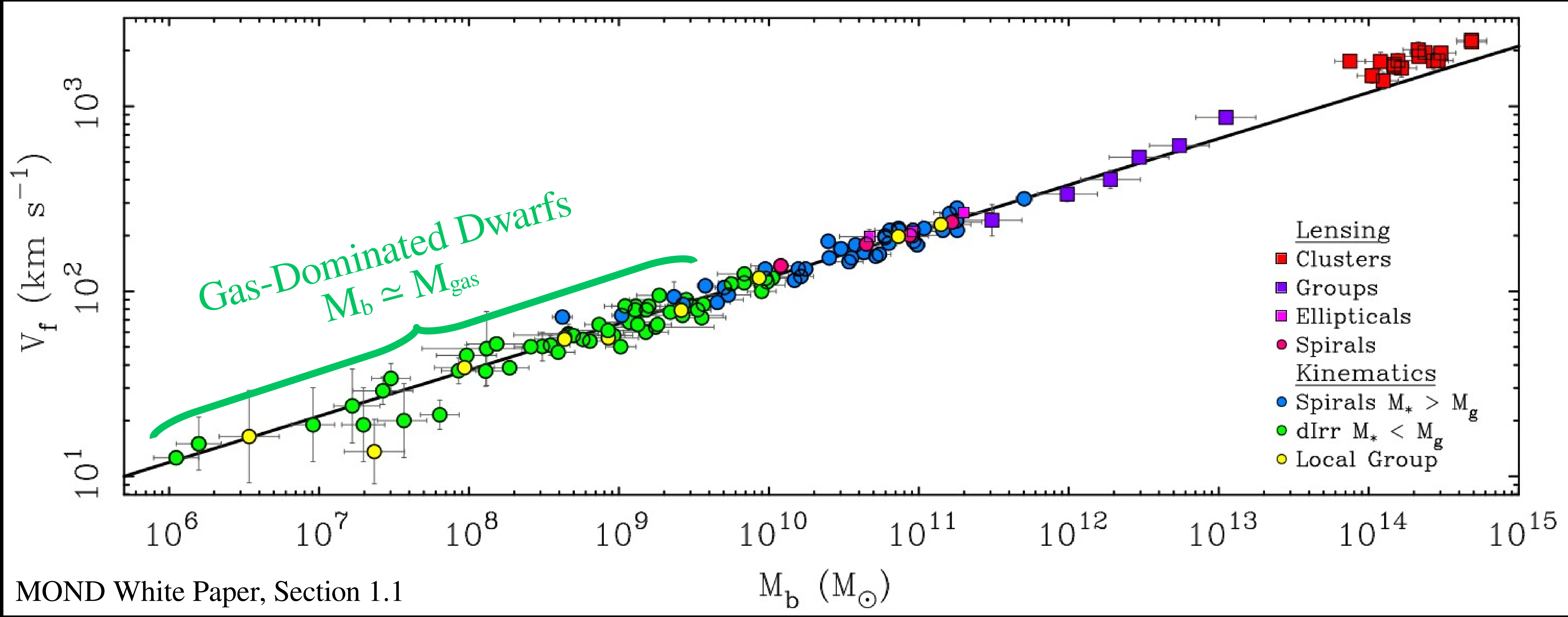

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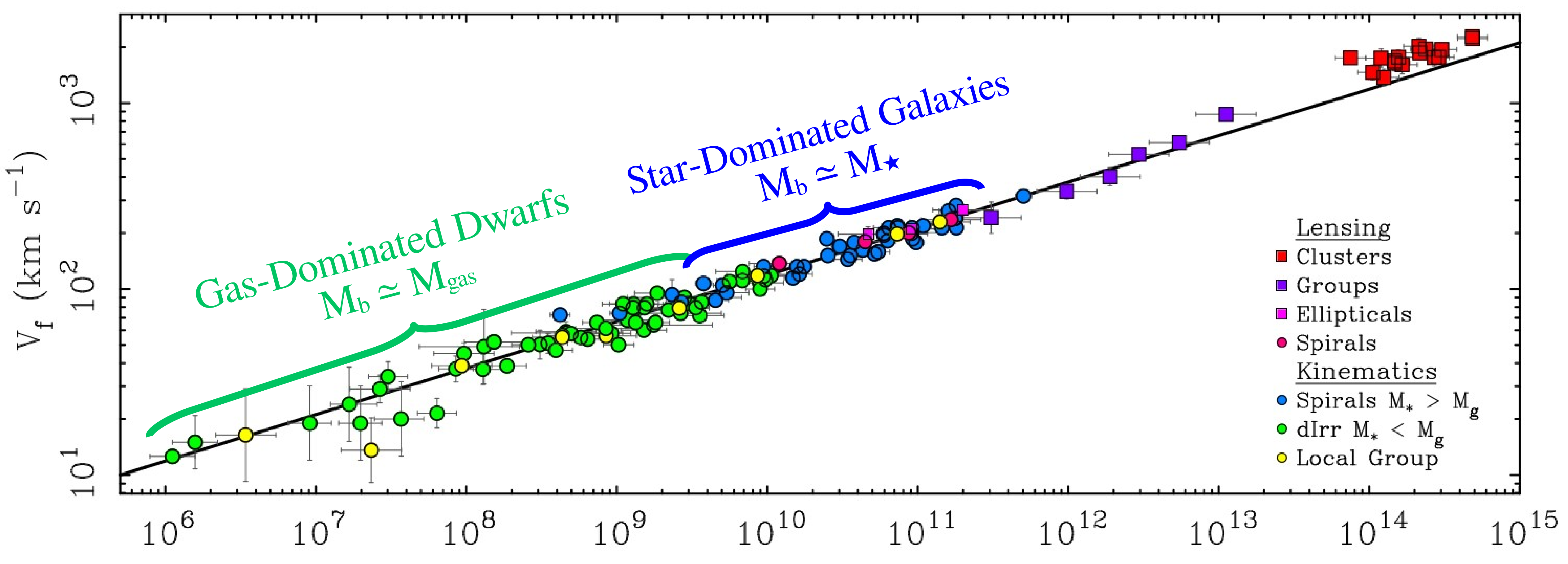

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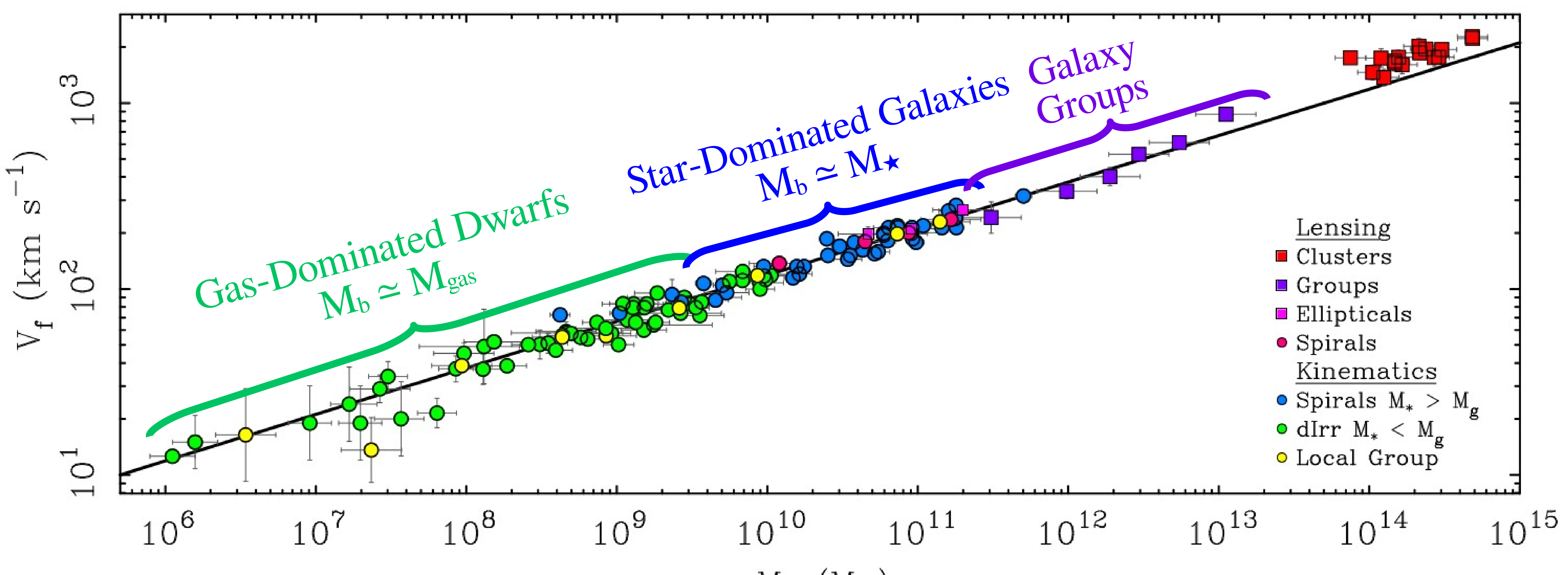

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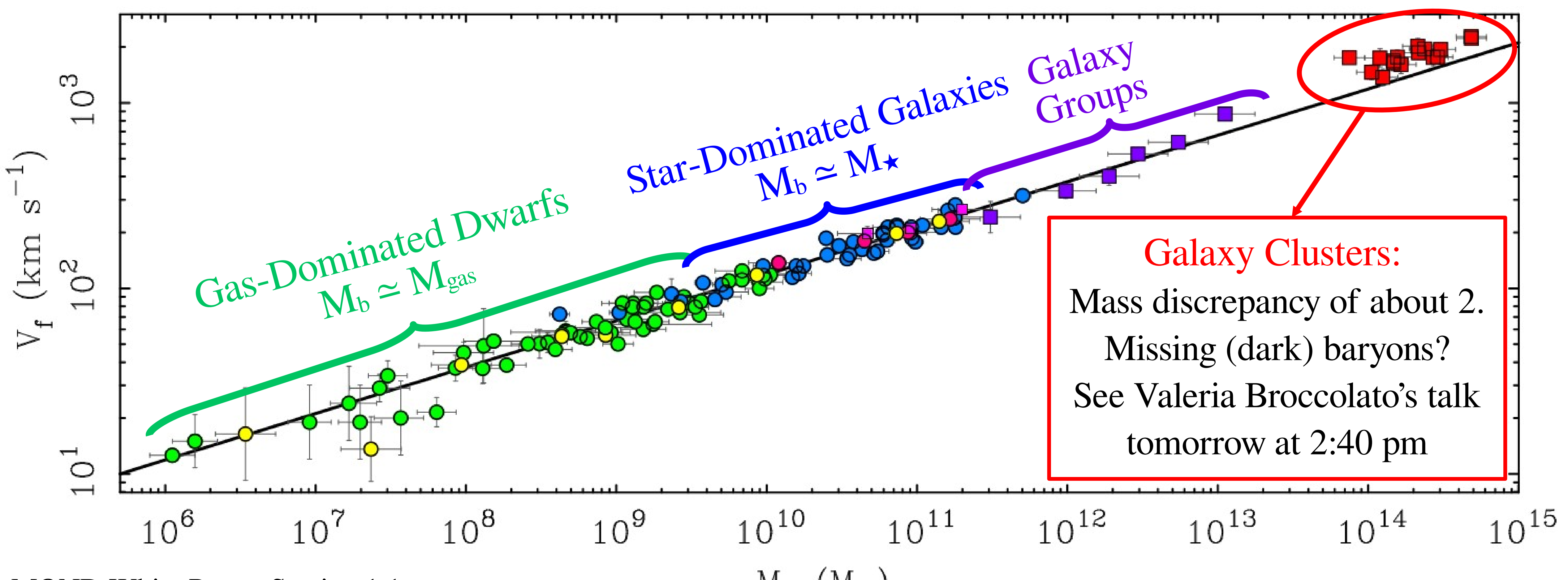

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Table 3: MOND Primary Predictions and Tests

System	Prediction	Result	References
Rotationally Supported Galaxies			
	Rotation Curves		
	Asymptotically Flat Rotation Curves	✓	[20, 42, 228, 237–243]
	Flatness Extended by Weak Lensing	✓	[43, 47]
	Detailed Rotation Curve Fits	✓	[127, 128, 131, 214, 244–246]
	Mass–Asymptotic Speed Relation		
	BTFR with slope 4: $M_b = AV_f^4$	✓	[48, 51, 228, 247–249]
	BTFR Normalization $A = (a_0 G)^{-1}$	✓	[49, 54, 126]
	No Second Parameter Dependence	✓	[50, 69, 70, 250]
	Negligible Intrinsic Scatter	✓	[49–51, 224, 248, 251, 252]
	Radial Acceleration Relation		
	Newtonian limit $a \to g_N$ for $a \gg a_0$	✓	[39, 40, 81, 83]
	DML $a \to (g_N a_0)^{1/2}$ for $a \ll a_0$	✓	[39, 40, 81–84, 86]
	Variation a function $\nu(g_N/a_0)$	✓	[40, 83, 84]
	Small Intrinsic Scatter	✓	[40, 82, 84, 129, 223, 253]
	Central Surface Density Relation		
	Surface Density Scale $\Sigma_M = a_0/(2\pi G)$	✓	[54, 92, 96, 103, 254]
	$\Sigma^0_{\rm dyn} \to \Sigma^0$ for $\Sigma^0 \gg \Sigma_M$	✓	[54, 92, 93, 97]
	$\Sigma^0_{\rm dyn} \to (4\Sigma_M \Sigma^0)^{1/2}$ for $\Sigma^0 \ll \Sigma_M$	✓	[54, 92, 93]
	Variation a function $\mathcal{S}(\Sigma^0/\Sigma_M)$	✓	[54, 92, 93, 254]
	No Second Parameter Dependence	✓	[92]
	Small Intrinsic Scatter	✓	[92]
	Maximum Disk Surface Density	✓	[29, 53, 103, 255]
Pressure Supported Galaxies			
	Weak Gravitational Lensing Masses	✓	[47, 55, 256]
	Small Discrepancies in Giant Ellipticals	✓	[113, 257–263]
	Large Discrepancies in Dwarf Spheroidals	✓	[40, 118, 123, 264]
	Newtonian virial theorem for $g_N \gg a0$	✓	[77, 112, 265, 266]
	DML virial theorem for $g_N \ll a0$	✓	[77, 112, 265, 266]
Novel Galaxy Types			
	Low Surface Brightness Galaxies	✓	[53, 79, 214, 245, 267]
	Gas Dominated Galaxies	✓	[126, 246, 268]
	Ultra Diffuse Galaxies	?	[269–273]
	Ultra Faint Dwarfs	?	[40, 120, 136, 274]
	Tidal Dwarf Galaxies	?	[275–278]
Groups of Galaxies			
	Galaxy Kinematics	✓	[5, 56–59, 112]
	Gravitational Lensing	✓	[55]
Clusters of Galaxies			
	Galaxy Kinematics	✗	[5, 90]
	Hot-Gas Hydrostatic Equilibrium	✗	[66, 67, 203–205, 279, 280]
	Gravitational Lensing	✗	[89, 281–284]
	Mass–Temperature Relation Slope	✓	[28, 285, 286]

Table 4: MOND Secondary Predictions and Tests

System	Prediction	Result	References
Binary Stars			
	Wide Binary Orbits	?	[287–299]
	Wide Binary Extent	✓	[300]
Systems Subject to Disruption			
	Asymmetry of Star Cluster Tidal Tails	✓	[301–303]
	Dwarf Galaxies in Galaxy Clusters	✓	[304]
Rotationally Supported Galaxies			
	Stellar Population Mass-to-Light Ratios	✓	[39, 245, 305]
	Stellar Bars and Spiral Arms	✓	[29, 106, 257, 306, 307]
	Vertical Dynamics	?	[29, 53, 308–310]
Binary & Interacting Galaxies			
	Pairwise Velocities	✓	[5, 13, 311, 312]
	Dynamical Friction	?	[105, 313–315]
	Tidal Tail Morphology	✓	[315]
	Polar Rings	✓	[316]
	Planes of Satellites	?	[317, 318]
Clusters of Galaxies			
	Collision Velocities	✓	[177, 319, 320]
	Bulk Velocities	✓	[321, 322]
	Early Formation	✓	[151, 161, 177]
Structure Formation			
	Galaxy Power Spectrum	✓	[6, 147–149]
	Baryon Acoustic Oscillation	?	[323]
	Large, Mostly Empty Voids	✓	[28, 179]
	Massive Galaxies at $z \gtrsim 10$	✓	[141, 151]
	Early Emergence of Cosmic Web	✓	[178, 324, 325]
	Early Reionization	?	[144]
	Dark Ages & Cosmic Dawn	?	[145]
Cosmology			
	Geometry	?	[198–200]
	Expansion History	?	[198–200]
	Background Radiation	✓	[198–200]
	Big Bang Nucleosynthesis	✓	[141]
External Field Effect			
	EFE in Dwarf Satellite Galaxies	?	[118–120, 123, 136, 326]
	EFE from Large Scale Structure	?	[44–46, 206, 327]
Solar System	AQUAL/QUMOND EFE Quadrupole		
	Cassini Spacecraft	✗	[328–332]
	Trans-Neptunian Objects	✓	[333, 334]
	Long Period Comets	✗	[335]

Cosmology & Relativistic Theories

Cosmology with modified Newtonian dynamics (MOND)

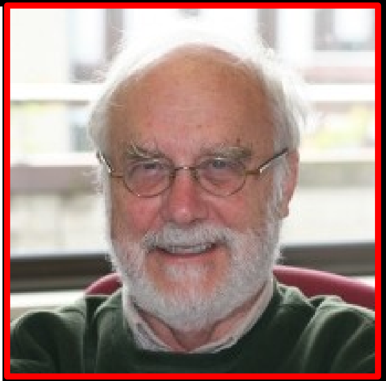
R. H. Sanders

Kapteyn Astronomical Institute, Groningen, The Netherlands

Accepted 1998 January 13. Received 1997 December 22; in original form 1997 October 17

ABSTRACT

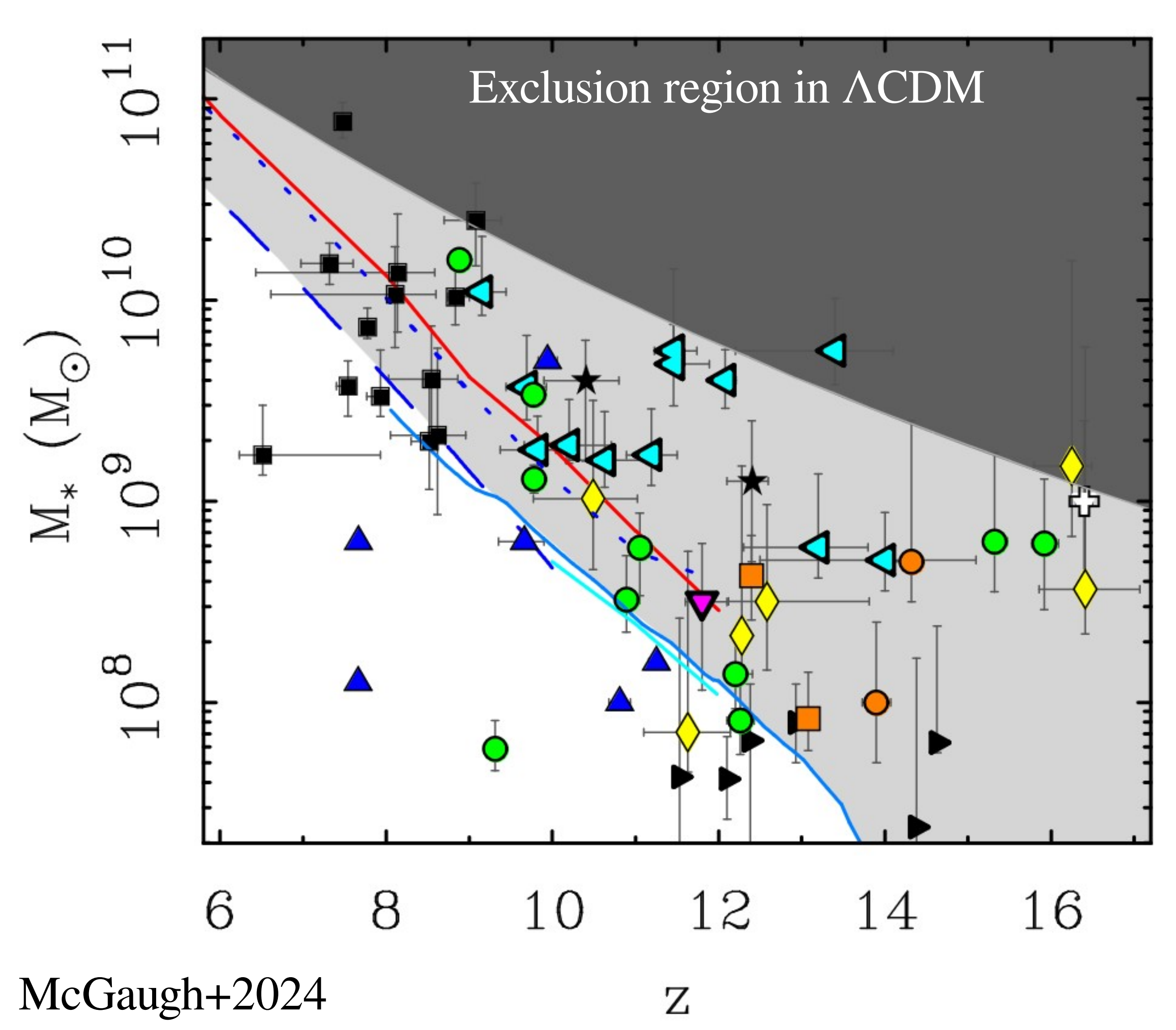
It is well known that the application of Newtonian dynamics to an expanding spherical region leads to the correct relativistic expression (the Friedmann equation) for the evolution of the cosmic scalefactor. Here, the cosmological implications of Milgrom's modified Newtonian dynamics (MOND) are considered by means of a similar procedure. Earlier work by Felten demonstrated that in a region dominated by modified dynamics the expansion cannot be uniform (separations cannot be expressed in terms of a scalefactor) and that any such region will eventually recollapse regardless of the initial expansion velocity and mean density. Here I show that, because of the acceleration threshold for the MOND phenomenology, a region dominated by MOND will have a finite size which, in the earlier Universe ($z > 3$), is smaller than the horizon scale. Therefore, uniform expansion and homogeneity on the horizon scale are consistent with MOND-dominated non-uniform expansion and the development of inhomogeneities on smaller scales. In the radiation-dominated era, the amplitude of MOND-induced inhomogeneities is much smaller than that implied by observations of the cosmic background radiation, and the thermal and dynamical history of the Universe is identical to that of the standard big bang model. In particular, the standard results for primordial nucleosynthesis are retained. When matter first dominates the energy density of the Universe, the cosmology diverges from that of the standard model. Objects of galaxy mass are the first virialized objects to form (by $z = 10$), and larger structure develops rapidly. At present, the Universe would be inhomogeneous out to a substantial fraction of the Hubble radius.



Sanders (1998, MNRAS, 296)

Massive galaxies at z >10 is the new normal!

Compilation of JWST observations

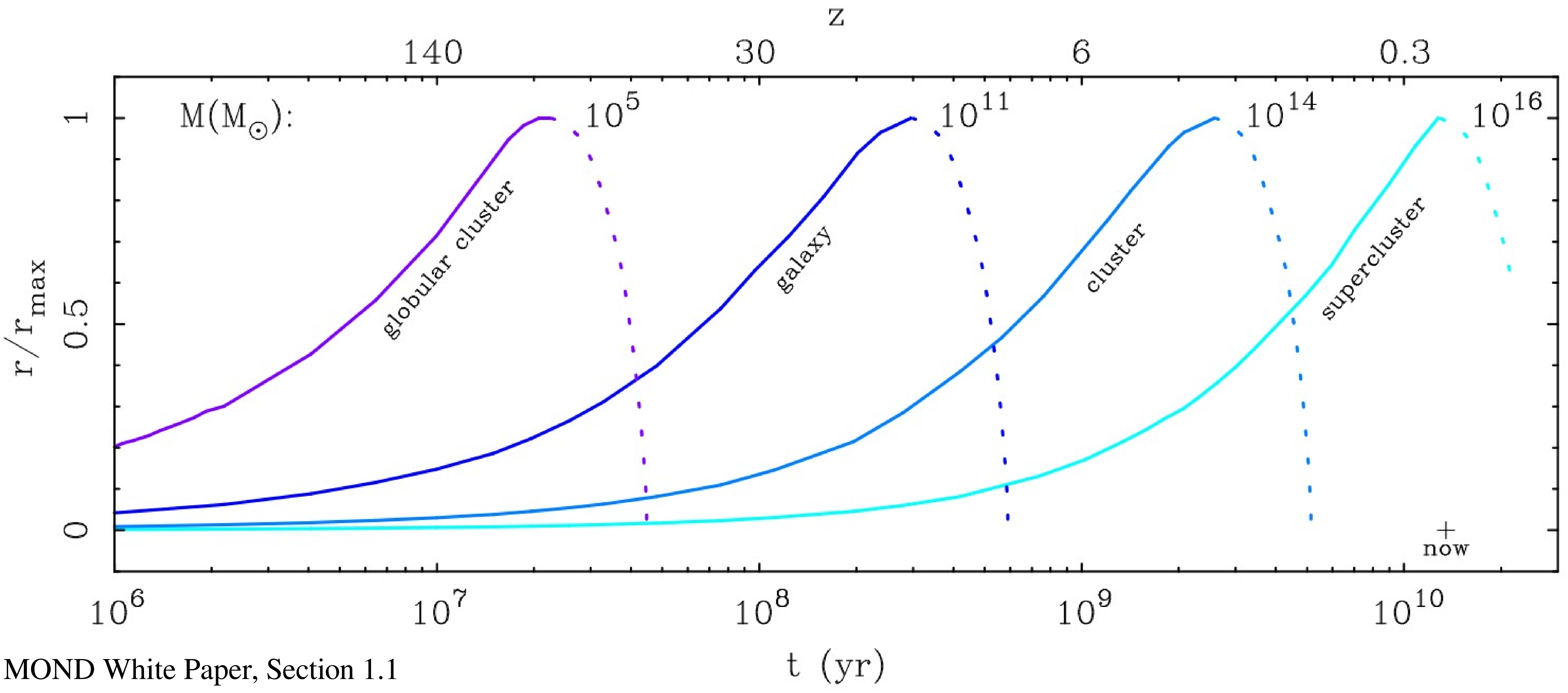


Galaxy formation & evolution happens earlier and faster than anticipated in ΛCDM

- Bright, massive galaxies at $z \gtrsim 10$
- Passive galaxies (without star formation) at $z \simeq 4$ and inferred formation at $z \gtrsim 10$
- Regularly rotating galaxy disks at $z \gtrsim 5$
- Disk structures (bars, spiral arms) at $z \gtrsim 3$
- Massive galaxy clusters at $z \gtrsim 3$ and protocluster candidates at $z \gtrsim 6$
- Lα emission at $z \gtrsim 10$ (early reionization)

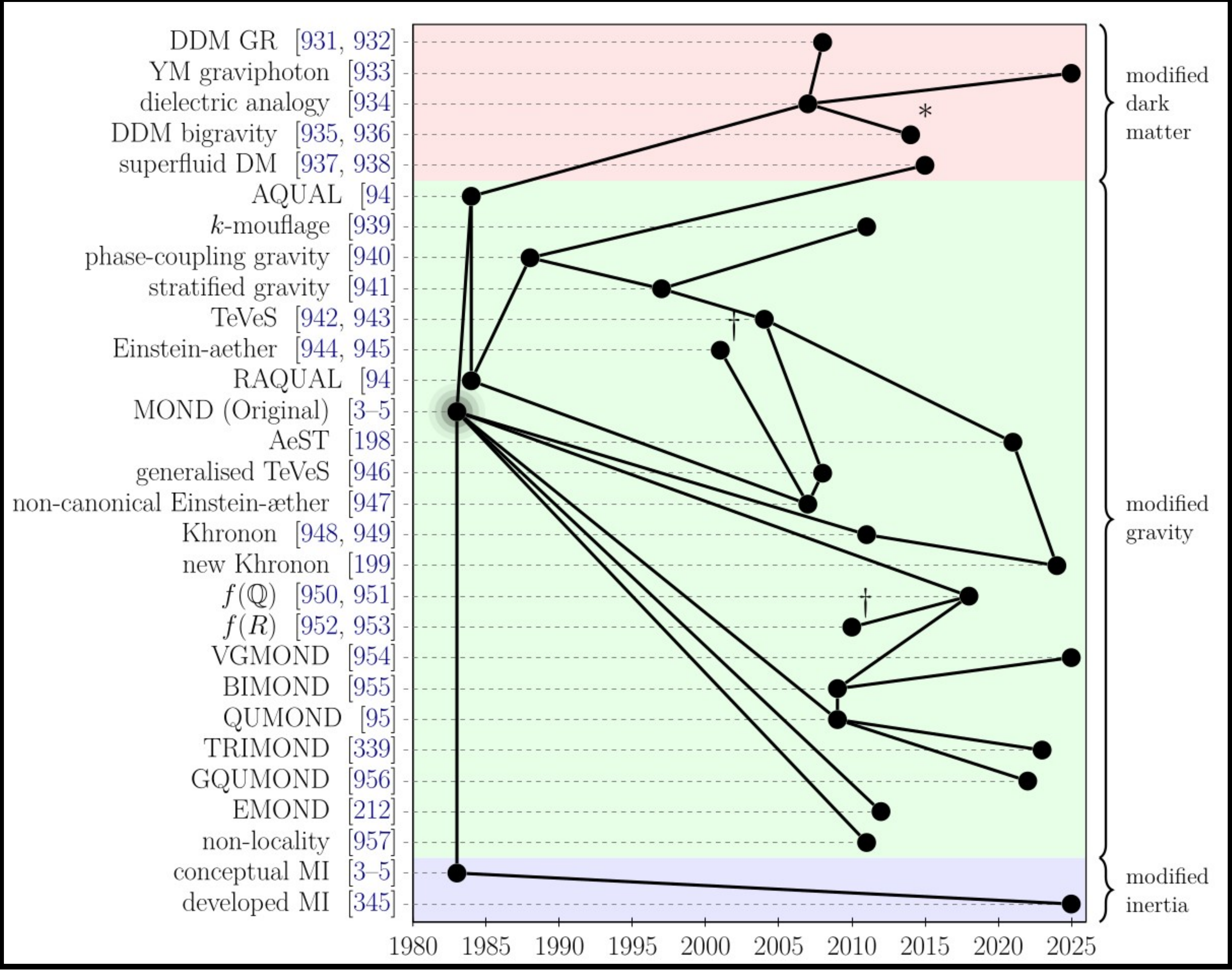
(see McGaugh, Schombert, Lelli, Franck 2024, ApJ)

MOND predictions for structure formation (Sanders 1998)



MOND White Paper, Section 1.1

Taxonomy of MOND theories



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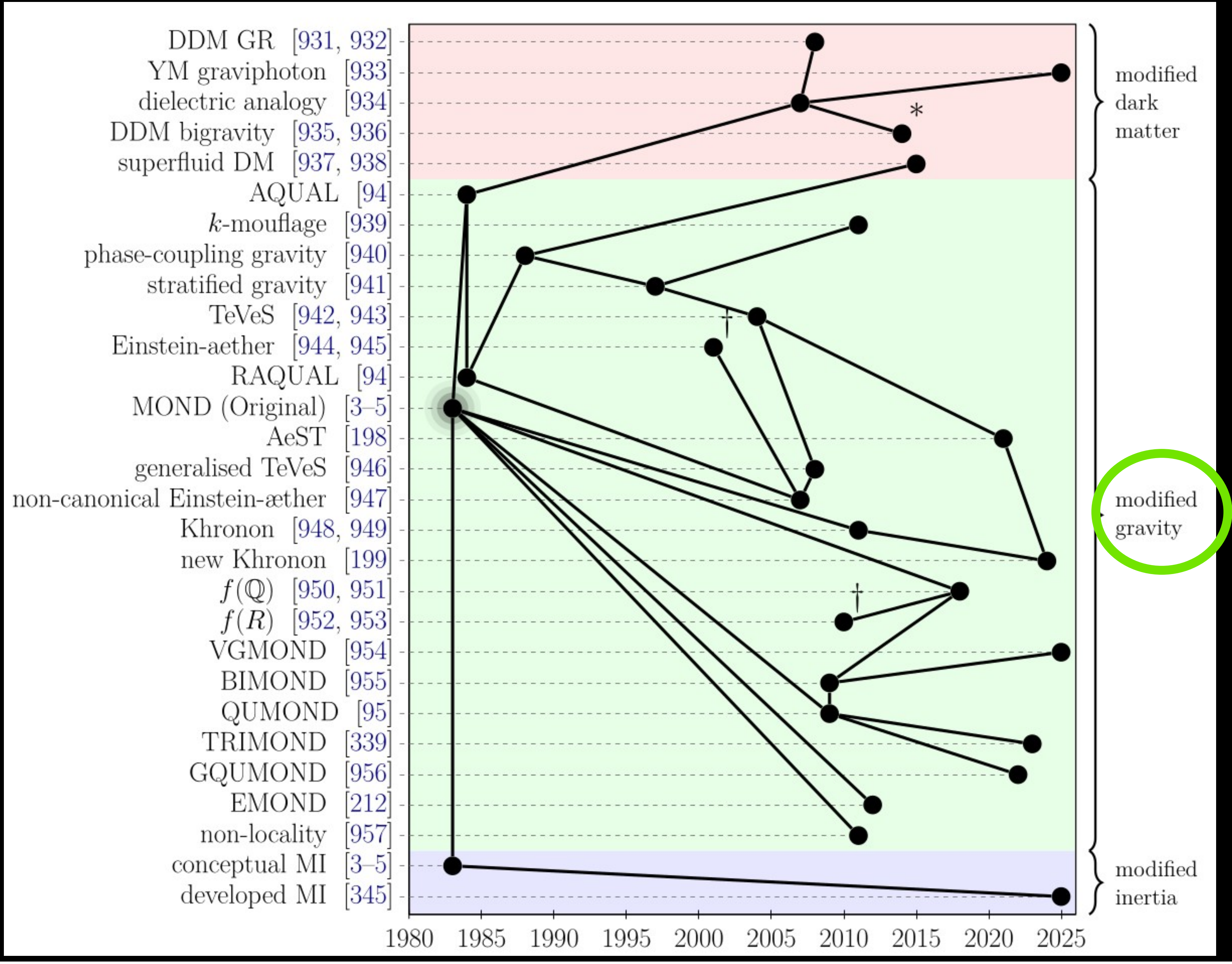
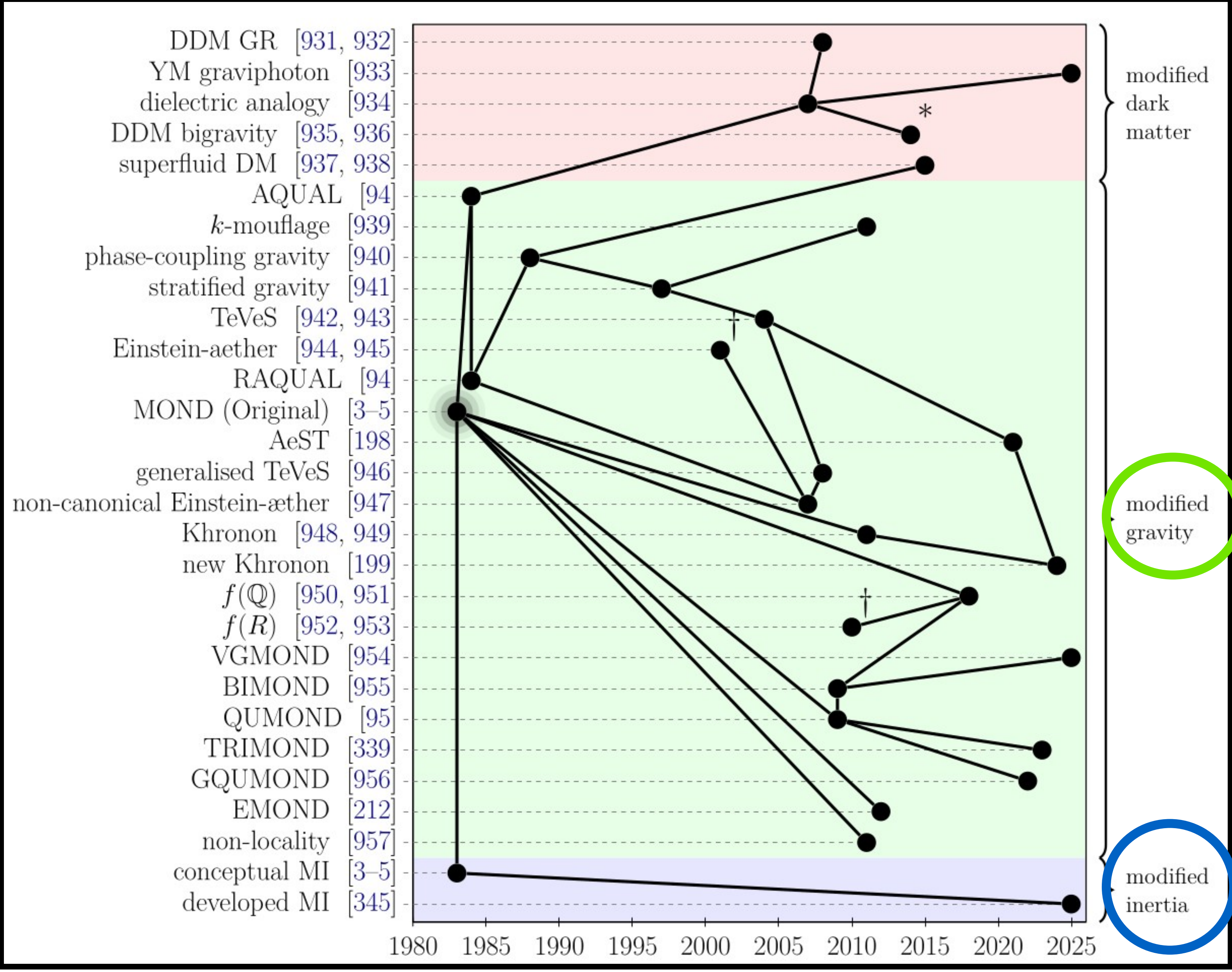


Modify E-H gravitational action of GR. In the stationary non-relativistic limit, they give a modified Poisson's equation.

Taxonomy of MOND theories



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Modify 2nd law of Newton (F=ma) in the non-relativistic limit.
No proper relativistic version yet.

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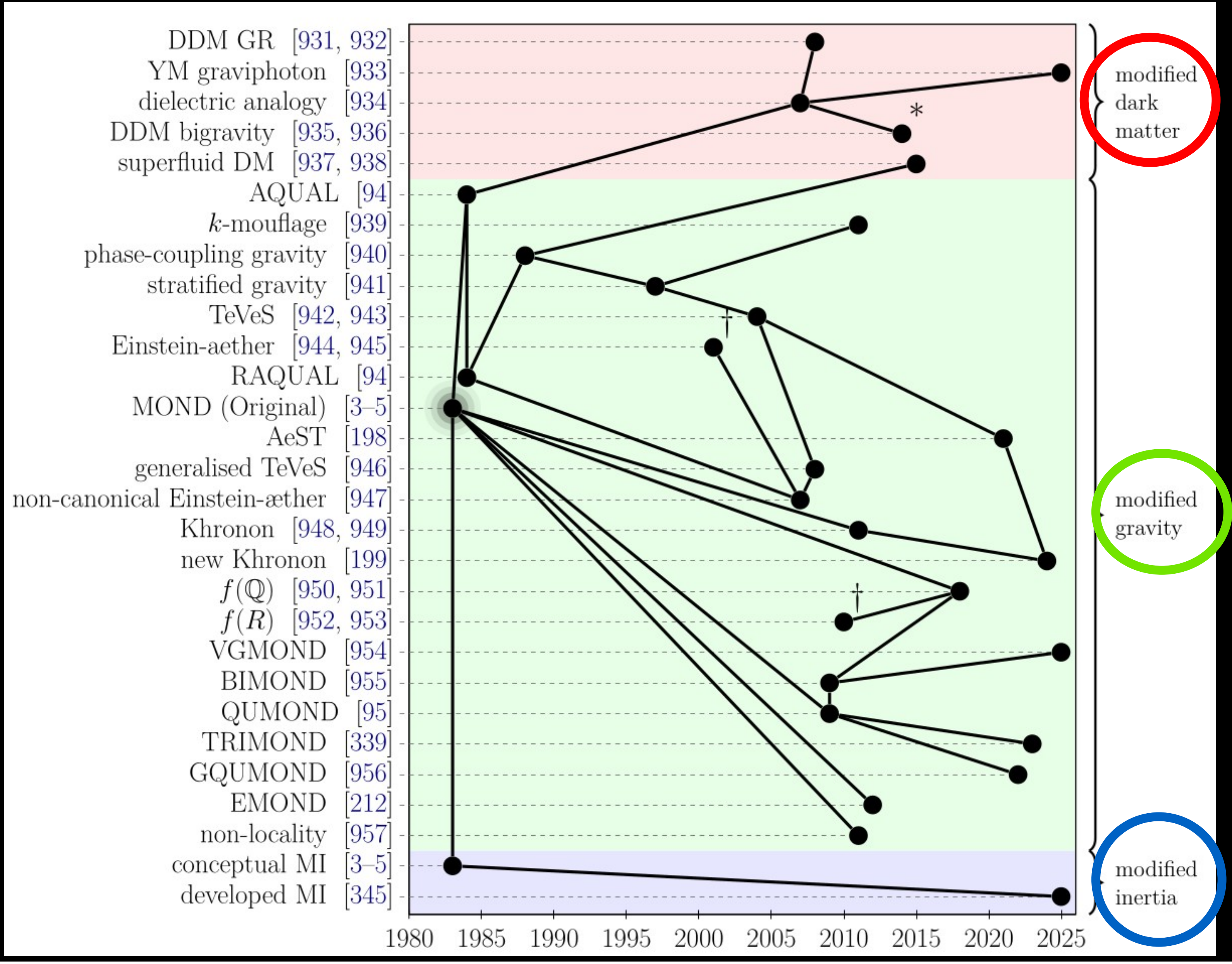

Hybrid approach: non-baryonic DM particles that interact with baryons to give a MOND-like behaviour on galaxy scales, while preserving ΛCDM cosmology on large scales

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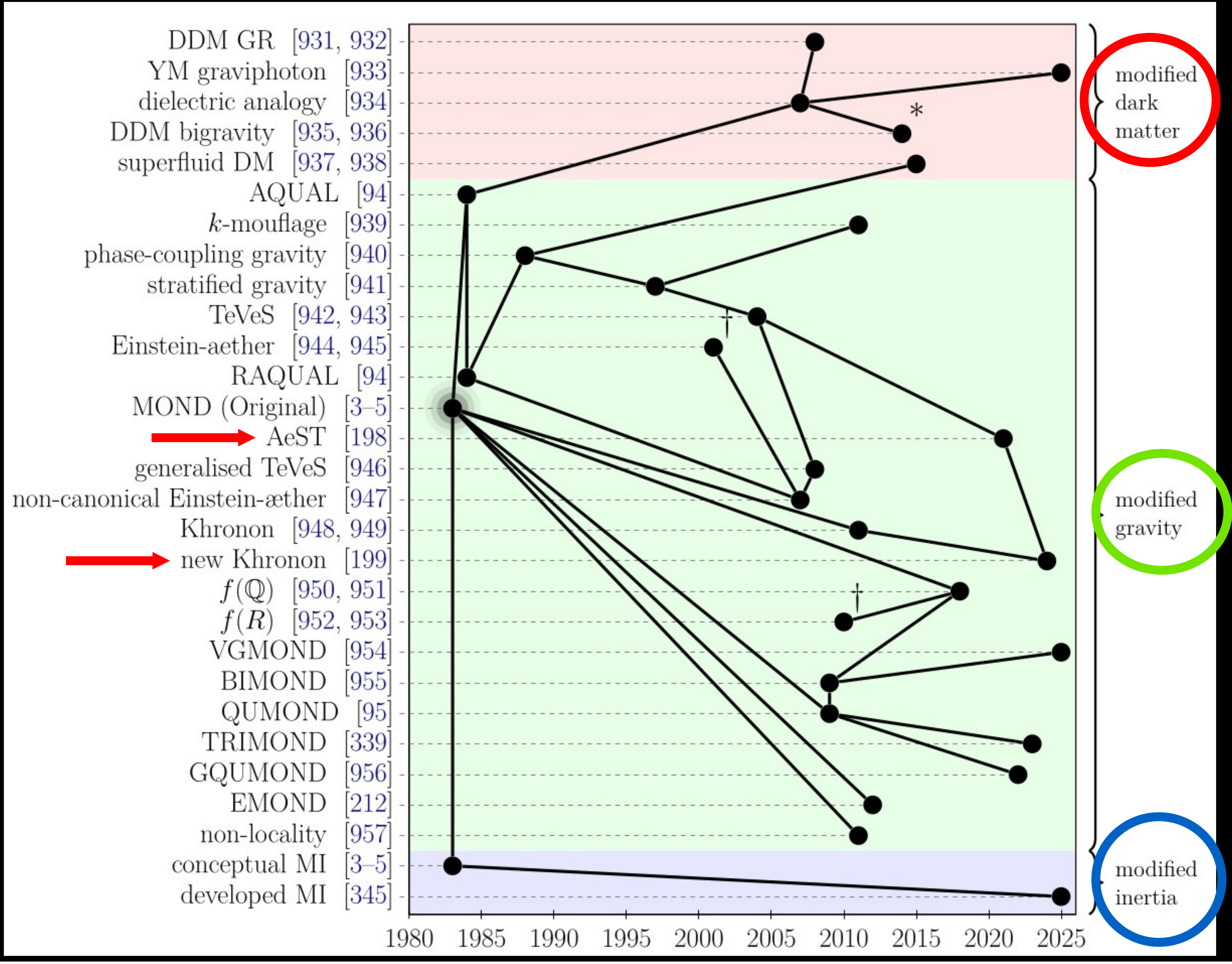

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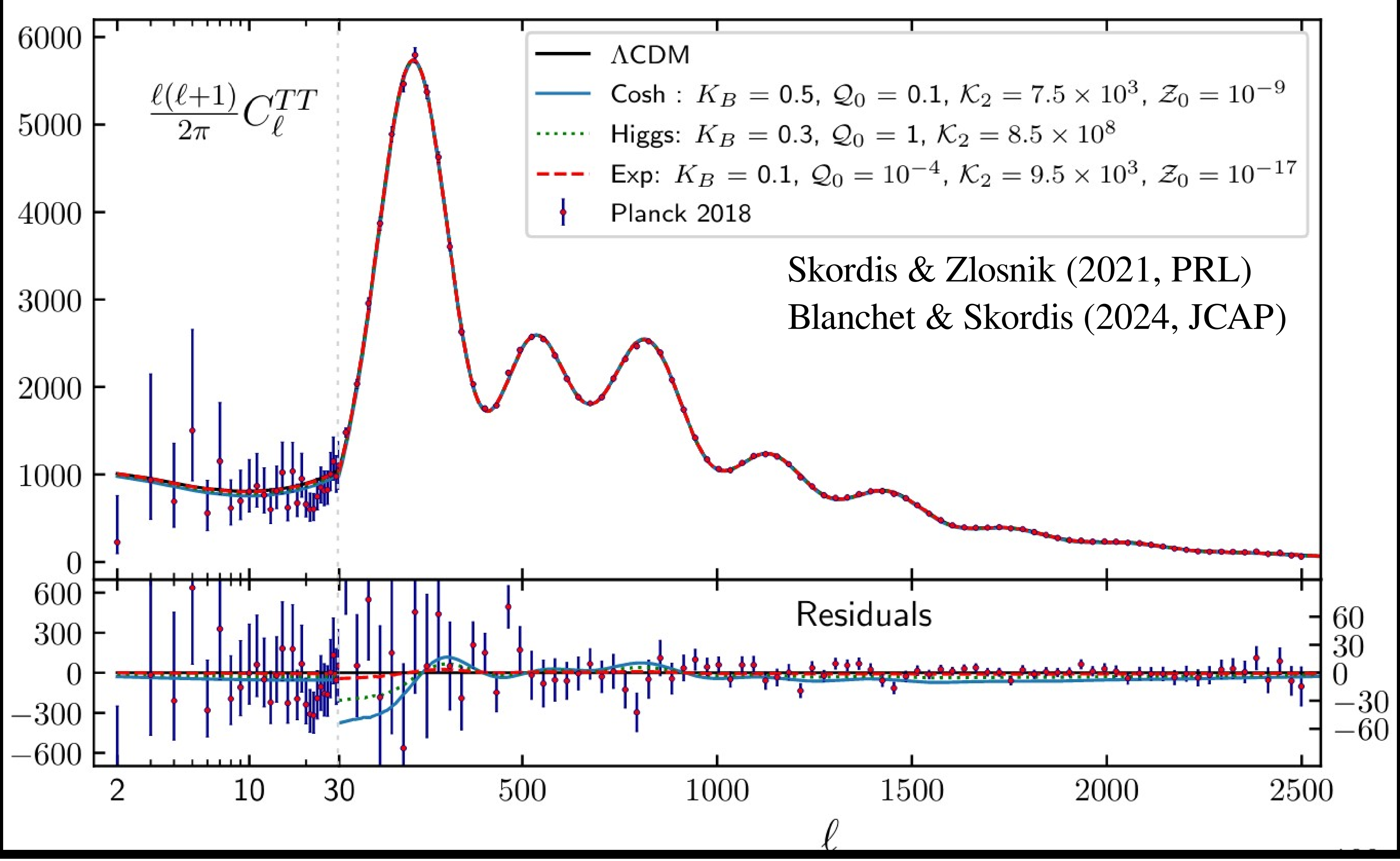
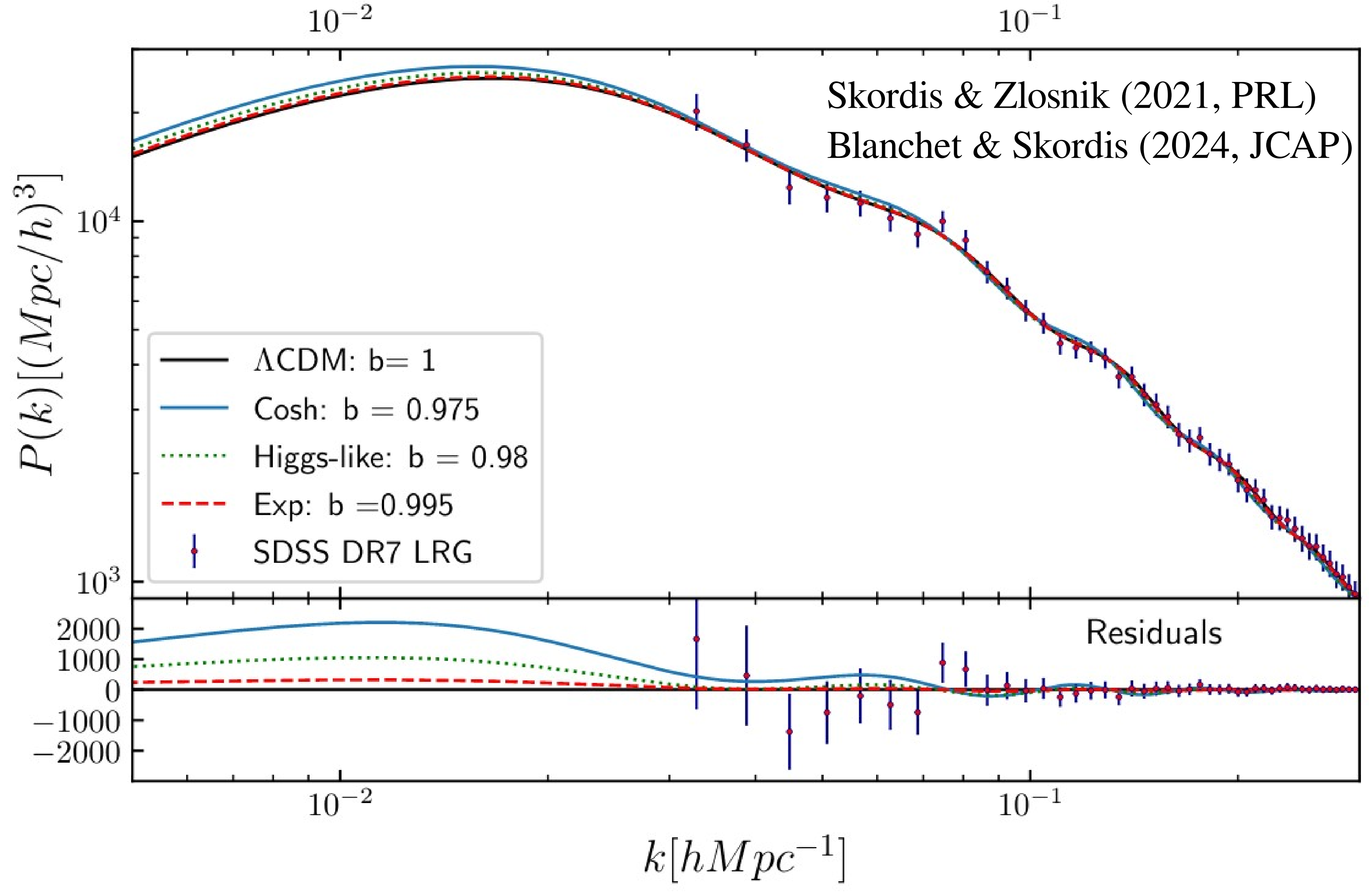
CMB Power Spectrum (from AeST or Khronon)

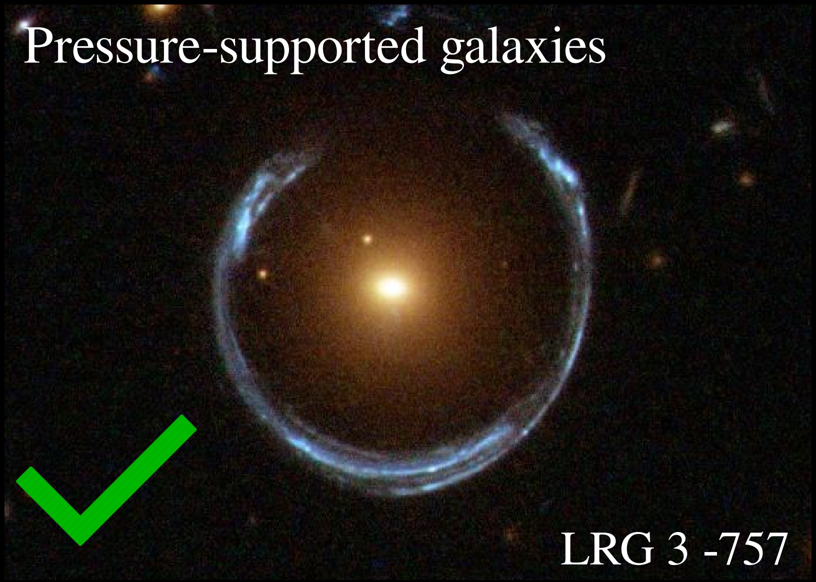


Linear Matter Power Spectrum (from AeST or Khronon)

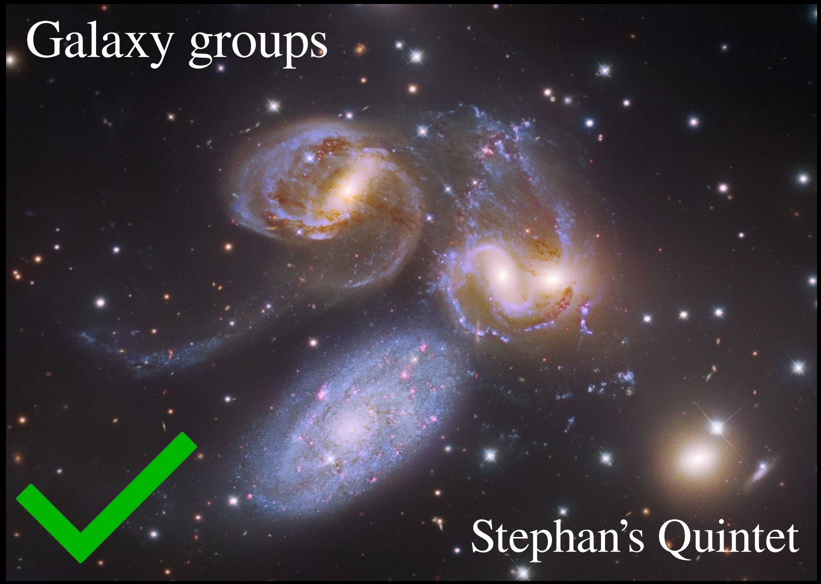


Status of MOND at Various Scales

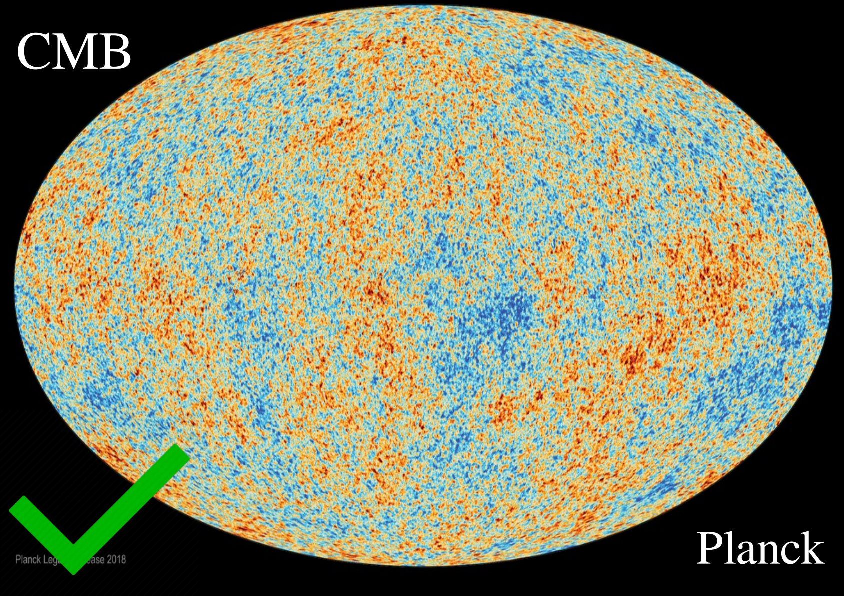
Galaxy Scales (~1-100 kpc)

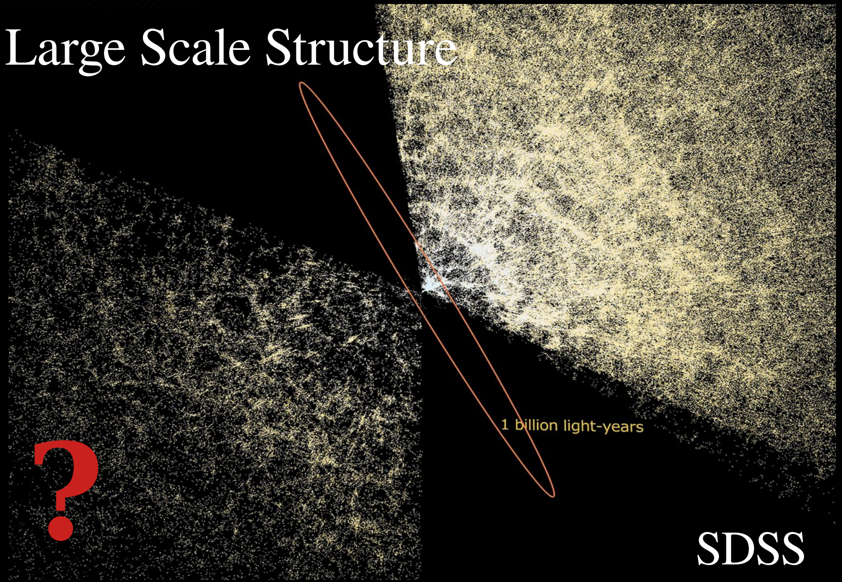


Groups/Clusters Scales (~1-2 Mpc)



Cosmological Scales (>100 Mpc)





More Slides

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BBAROLO
3D Based Analysis of Rotating Objects via Line Observations

Machine Learning

GALDYN-MOD

- Next-generation software for 3D dynamical modeling
- Mock-observed simulations to compare to observations

GALDYN-HIGH

- 15 galaxies at $z \simeq 4$-5: [CII] & CO data + JWST images
- DM halos & dynamical laws as a function of cosmic time

GALDYN-LOW

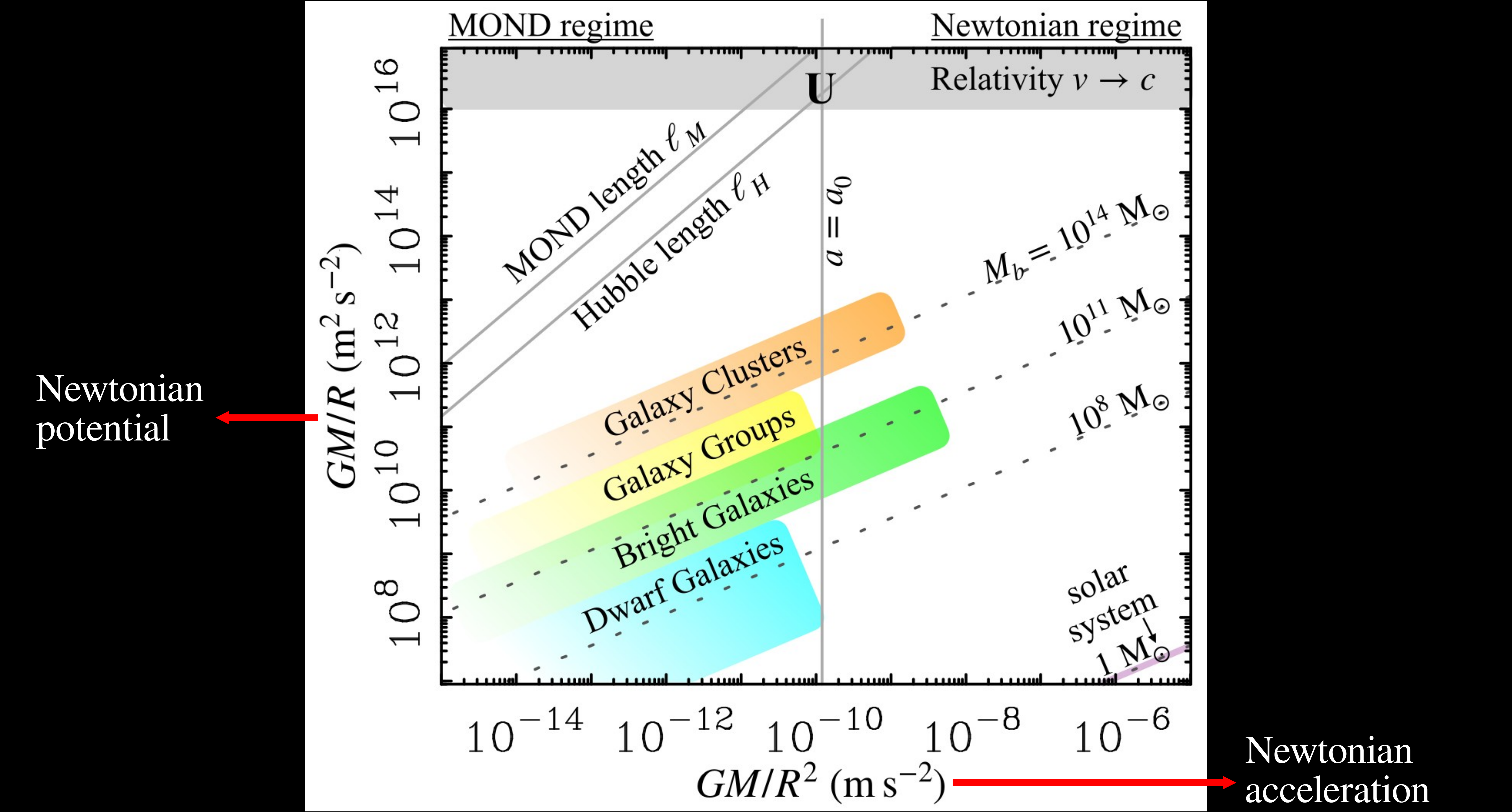
- About 28,000 galaxies at $z \simeq 0$: HI data + near-infrared images
- DM halos & dynamical laws as a function of environment

ALMA · Very Large Array New Mexico · James Webb Space Telescope JWST NASA • ESA • CSA · HUBBLE 25 · SKAO pathfinders · esa euclid · WISE

FIS

Consolidator Grant
(~1.5 M€)
PI: Federico Lelli

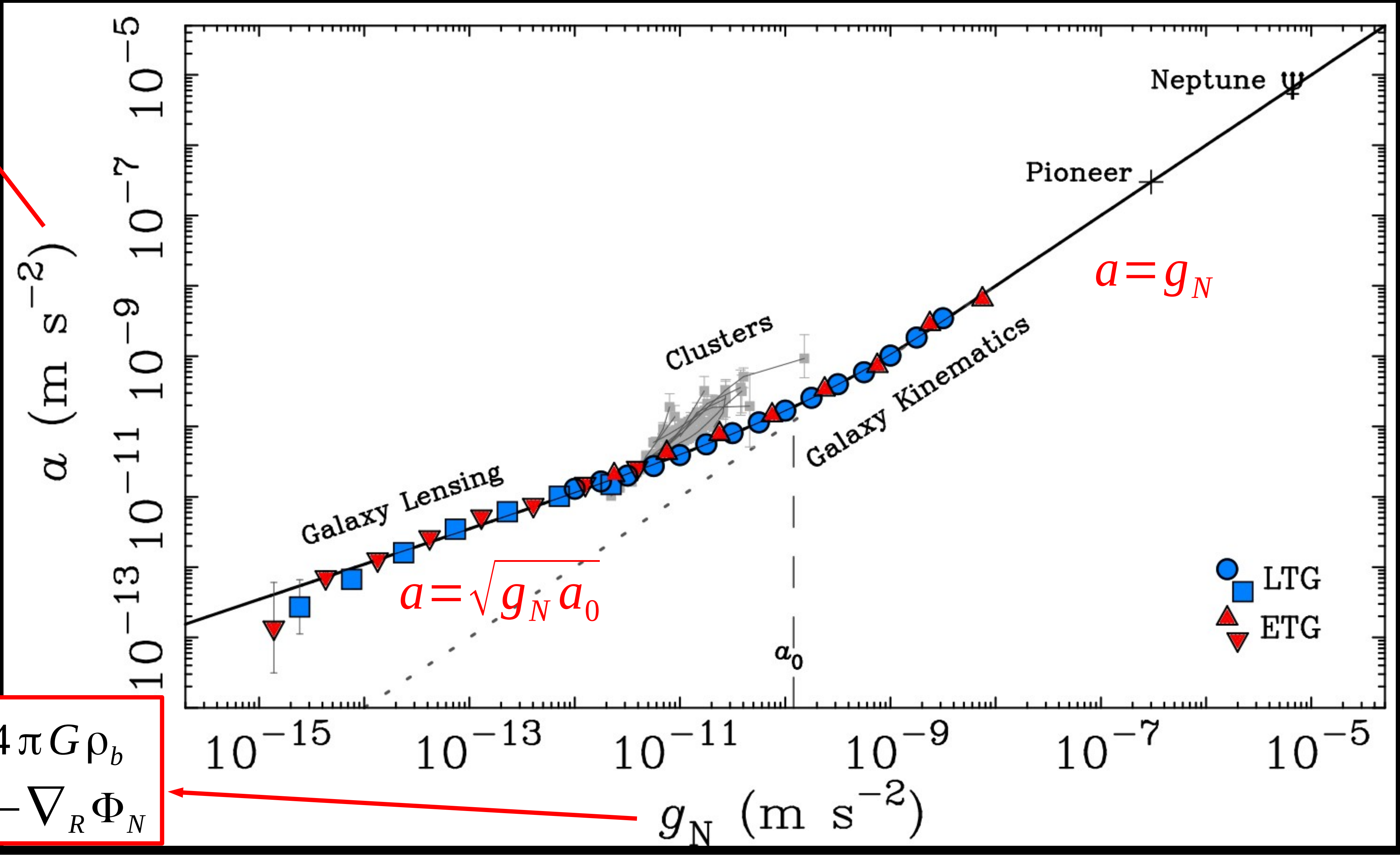
MUR
Ministero dell'Università e della Ricerca



MOND regime
Newtonian regime
Relativity $v \to c$
U
MOND length ℓ_M
Hubble length ℓ_H
$a = a_0$
$M_b = 10^{14}$ M$_\odot$
10^{11} M$_\odot$
10^{8} M$_\odot$
Galaxy Clusters
Galaxy Groups
Bright Galaxies
Dwarf Galaxies
solar system
1 M$_\odot$
10^{8}
10^{10}
10^{12}
10^{14}
10^{16}
GM/R (m^2 s^{-2})
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
GM/R^2 (m s^{-2})
Newtonian potential
Newtonian acceleration

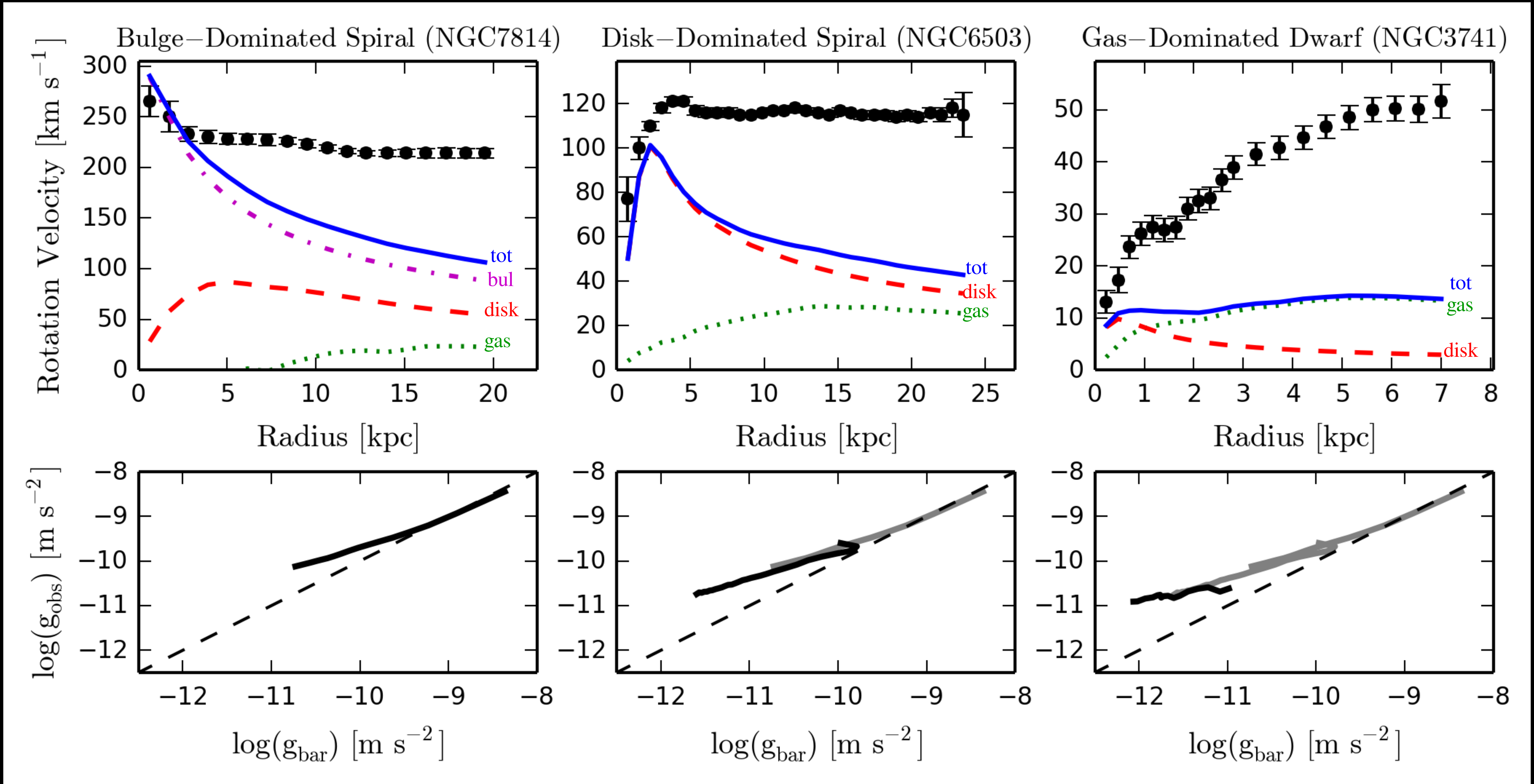
Radial Acceleration Relation: $a = \nu(g_N/a_0)g_N$

$$\frac{V_c^2(R)}{R}$$

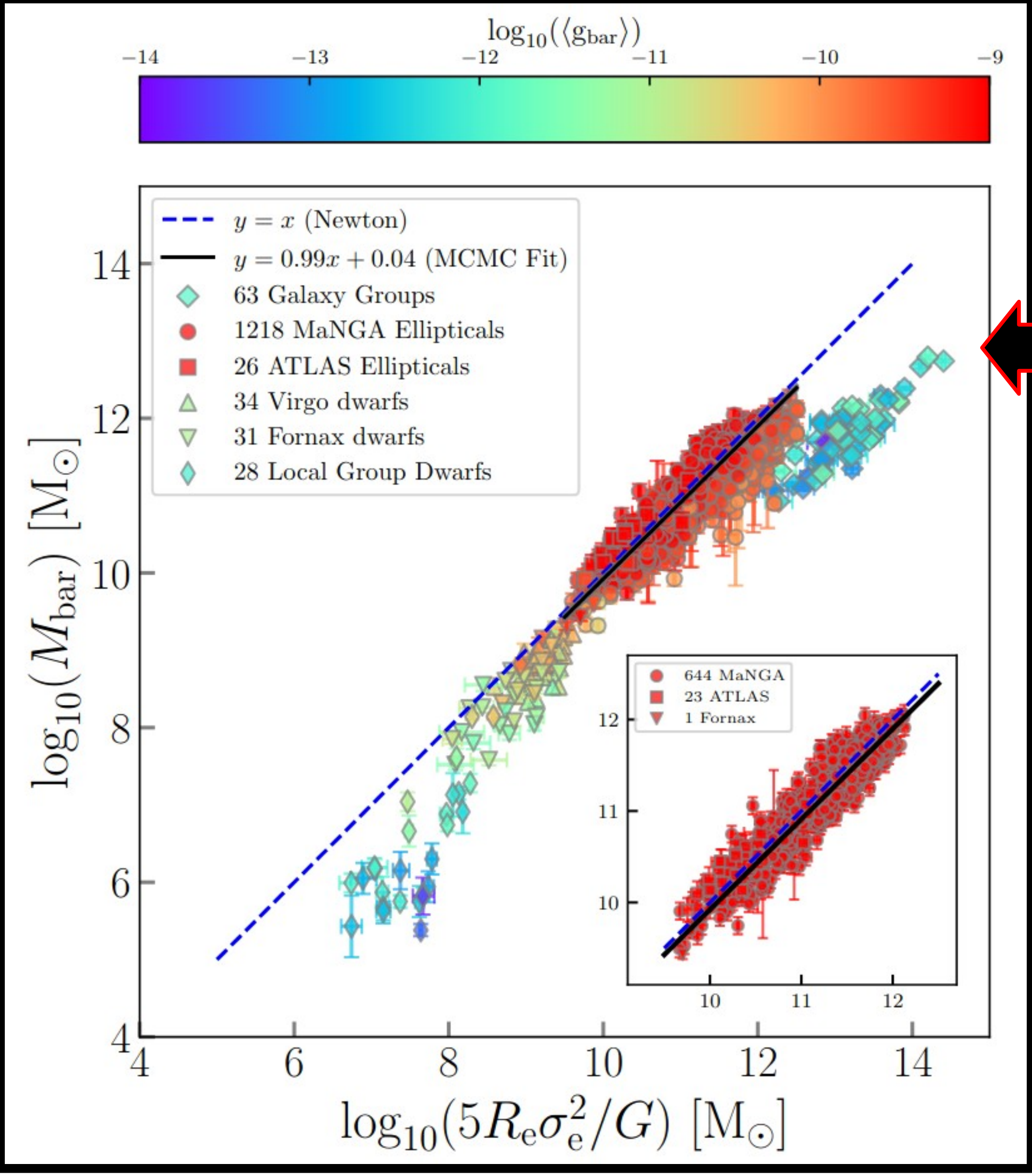


$$\nabla^2 \Phi_N = 4\pi G \rho_b$$
$$g_N(R) = -\nabla_R \Phi_N$$

Galaxies lie on the same RAR *despite* their diversity



Two Virial theorems for different acceleration regimes:

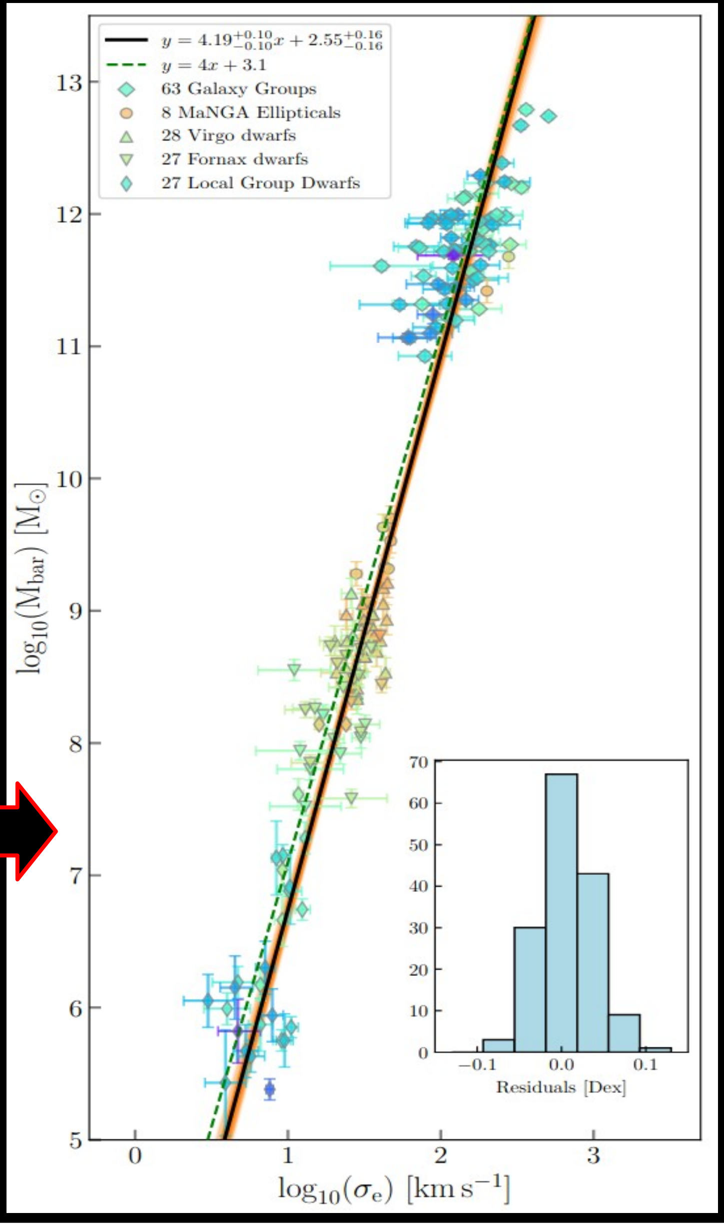


For $g_b \gg a_0$: Newton

$$\sigma_V^2 = \frac{G\,M_b}{5\,R_e}$$

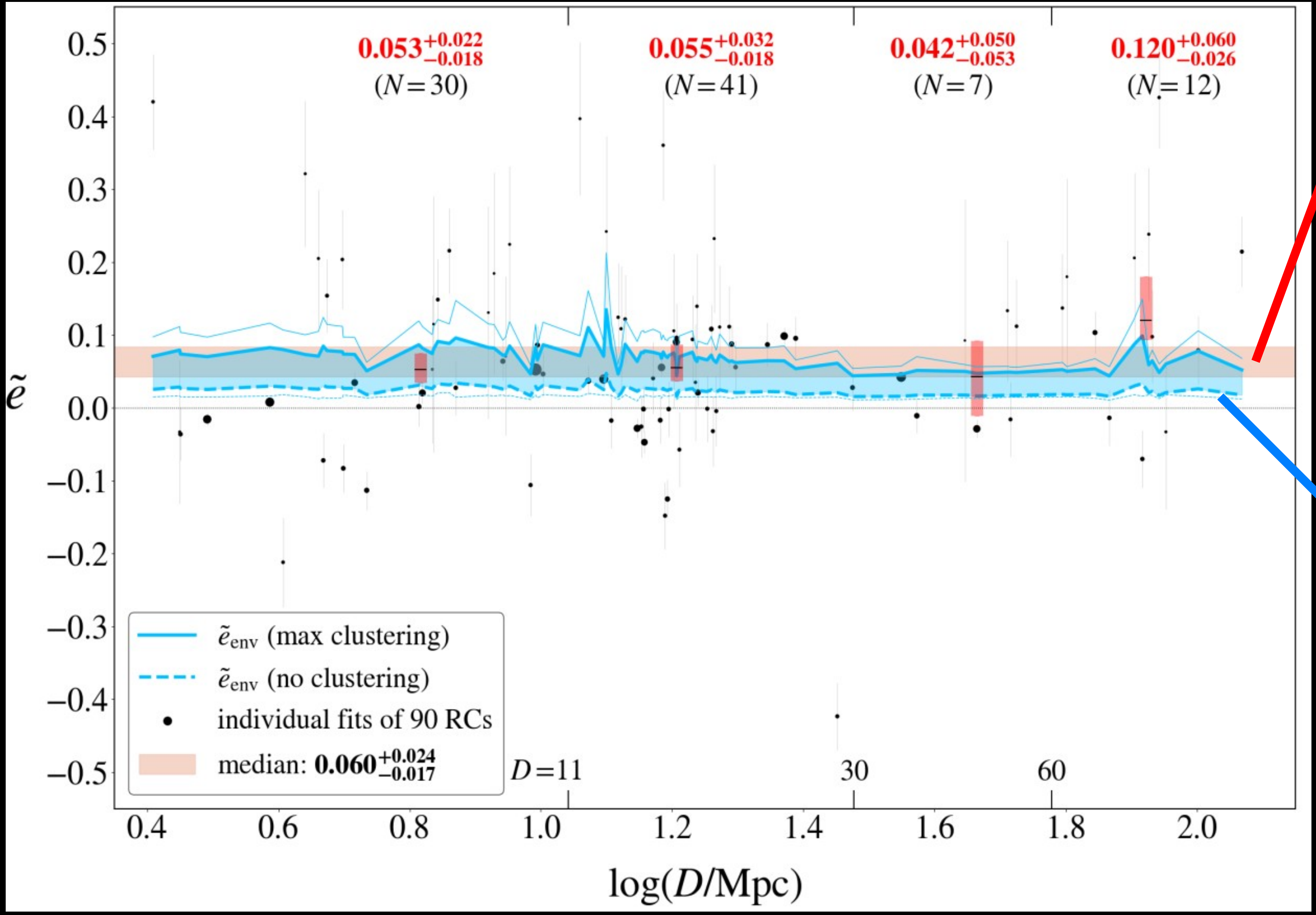
For $g_b \ll a_0$: MOND

$$\sigma_V^4 = \frac{a_0\,G}{c}\,M_b$$



Statistical approach: EFE>0 at >4σ and agrees with LSS

$$\frac{g_{ext}}{a_0}$$



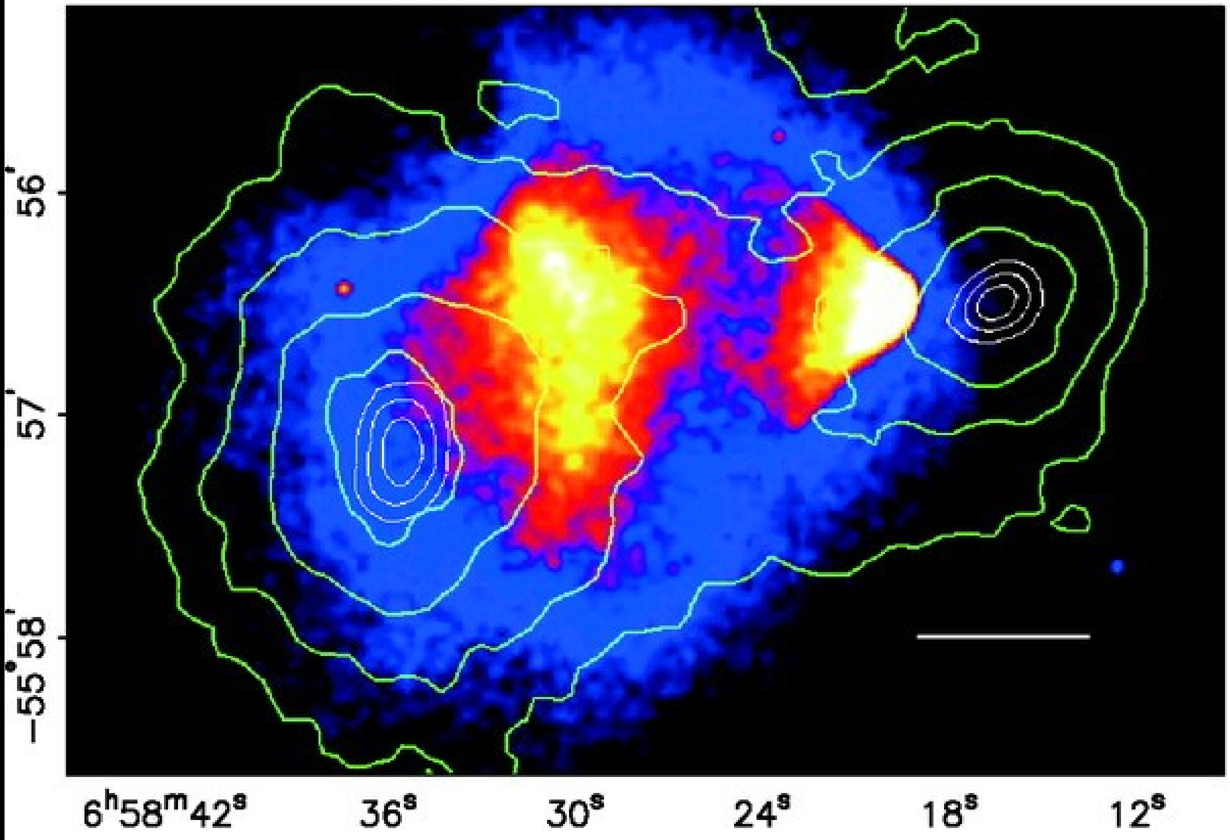
From Rotation Curve Fits

From Baryon Large Scale Structure

Chae+2020, 2021

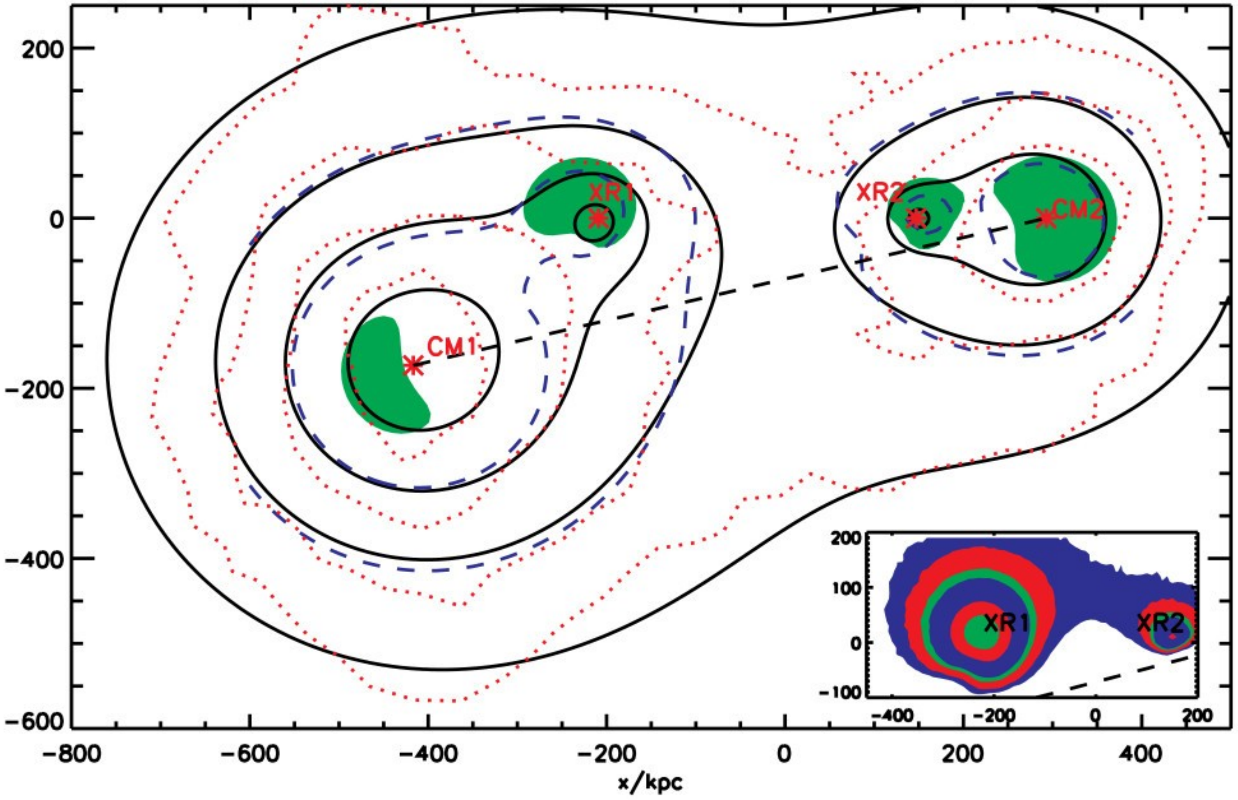
The bullet cluster in MOND: missing mass must be collisionless

OBSERVATIONS (Clowe+2004, ApJ)



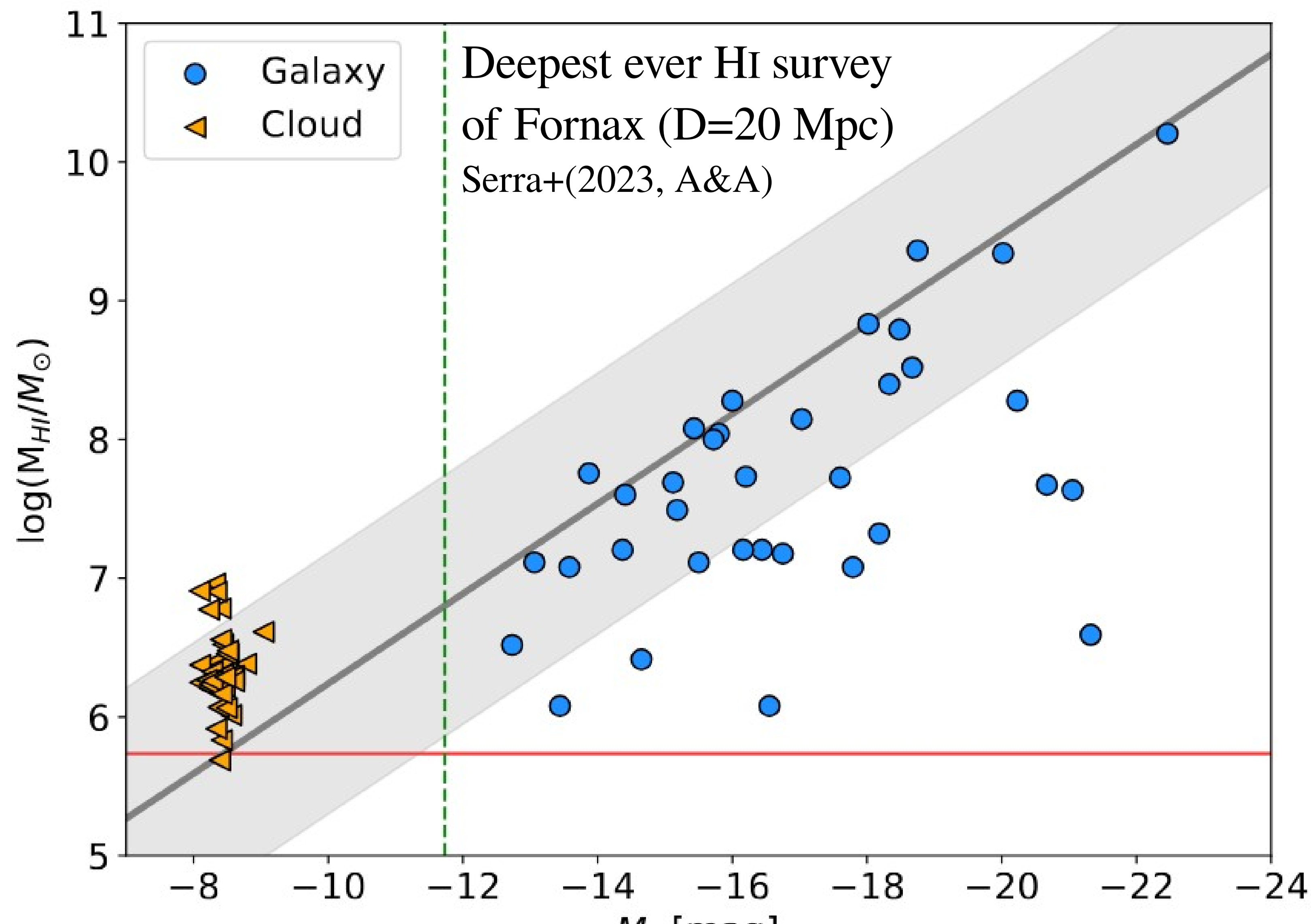
Green: Observed lensing map (total mass)
Blue/**Red**/**Yellow**: X-ray emission (hot gas)

MOND (Angus+2006, MNRAS; Angus+2007, ApJ)



Red: Observed lensing convergence map
Black: TeVeS model with 2eV neutrinos
Blue: total surface densities (baryons+υ)

Missing mass in clusters: compact clouds of cold gas?



Kleiner+(2025,A&A)

To solve the problem:

$M_{tot} \simeq M_{ICM} \simeq 10^{13\text{-}14}\,M_{\odot}$

Many ($\sim 10^{10}$) tiny clouds?

If they are in pressure eq:

$$T_{hot}\,\rho_{hot} = T_{cloud}\,\rho_{cloud}$$

For $T_{cloud} \simeq 10^4$ K (HI gas)

→ $R_{cloud} < 50$ pc

→ filling factor $\sim 10^{-4}$

→ nearly impossible to detect in absorption

MOND – Cosmology Connection?

Two numerical coincidences (Milgrom 1983a, ApJ; Milgrom 1999, PhLA):

$$a_0 \simeq \frac{H_0 \cdot c}{2\pi}$$

H_0 = Hubble constant → maybe a_0(t) ~ H(t) ?

$$a_0 \simeq \frac{c^2 \sqrt{\Lambda/3}}{2\pi}$$

Λ = Cosmological constant → relation to Dark Energy?

IF this numerology has some deeper, fundamental meaning:

Either the state of the Universe at large enters in local dynamics, or

the same parameters enters both Cosmology (Λ) and local dynamics (a_0).

Let's start from the Newtonian non-relativistic Action

$$S = \int dt\, L = \int dt \left(L_{matter} + L_{gravity} + L_{coupling} \right) = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Change this for modified inertia

Change this for modified gravity

Changing this modify both

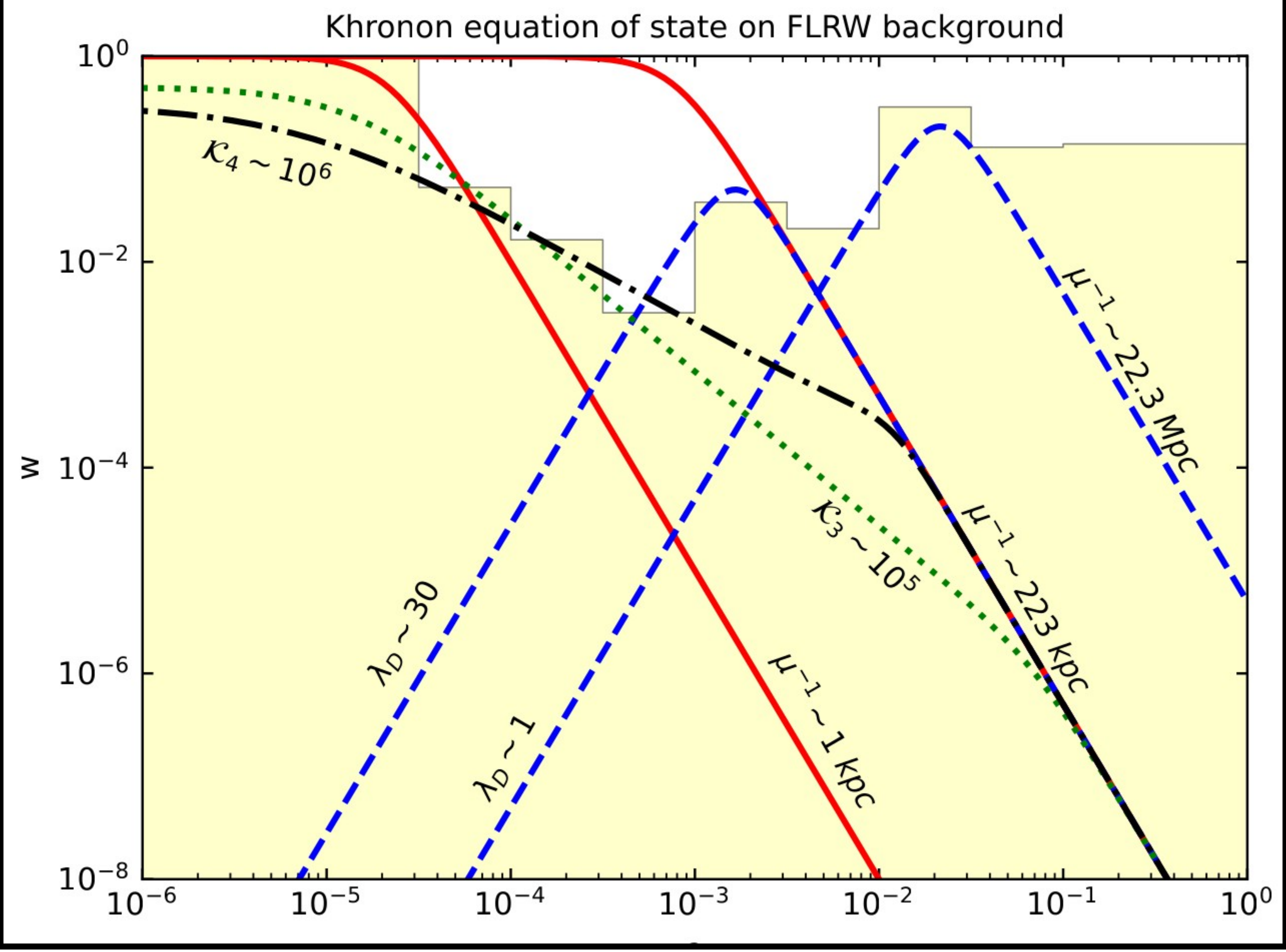
Principle of Least Action:

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla^2 \Phi = 4\pi G \rho$$ Poisson's equation

$$\frac{\delta S}{\delta \vec{x}} = 0 \rightarrow \vec{a} = -\vec{\nabla}\Phi$$ Newton's 2nd Law

CMB constraints on the evolution of the Khronon scalar field

$$\overline{P_K} = w\,\overline{\rho_K}$$



MOND white paper
Section 5.4.4

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (universally coupled to matter fields Ψ)

Scalar $\tau \rightarrow$ Khronon field (units of time; drive a foliation in 3D space-like hypersurfaces)

$$n_\mu = -\frac{c}{Q}\nabla_\mu \tau \qquad Q = c\sqrt{-g^{\mu\nu}\nabla_\mu\tau\nabla_\nu\tau}$$

Dimensionless unit timelike ($n^\mu n_\mu = -1$) vector, pointing in the future ($n^0 > 0$).

$$A_\mu = c^2 n^\nu \nabla_\nu n_\mu$$

Covariant spacelike acceleration

$$Y = \frac{A_\mu A^\mu}{c^4}$$

Squared "acceleration" with units of $1/L^2$

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\left[R - 2I(Y) + 2K(Q)\right] + S_m[\Psi, g]$$

- R: Ricci Scalar (as in GR)
- $I(Y)$: Free function to get MOND
- $K(Q)$: Free function to get cosmology
- S_m: Matter Action

Khronon Field Equations (Blanchet & Skordis 2024)

$$G^{\mu\nu} + (I - K)\, g^{\mu\nu} - \frac{2}{c^4}\frac{dI}{dY} A^{\mu} A^{\nu} + \left[\frac{2}{c^4}\nabla_{\rho}\left(\frac{dI}{dY} A^{\rho}\right) - Q\frac{dK}{dQ}\right] n^{\mu} n^{\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

$$T_{\tau}^{\mu\nu} \stackrel{\text{def}}{=} \frac{c^4}{8\pi G}\left\langle -(I - K)\, g^{\mu\nu} + \frac{2}{c^4}\frac{dI}{dY} A^{\mu} A^{\nu} - \left[\frac{2}{c^4}\nabla_{\rho}\left(\frac{dI}{dY} A^{\rho}\right) - Q\frac{dK}{dQ}\right] n^{\mu} n^{\nu}\right\rangle$$

The Khronon stress-energy tensor contains $g_{\mu\nu}$ explicitly and implicitly in Y and Q.

This is very different from adding DM to GR!