

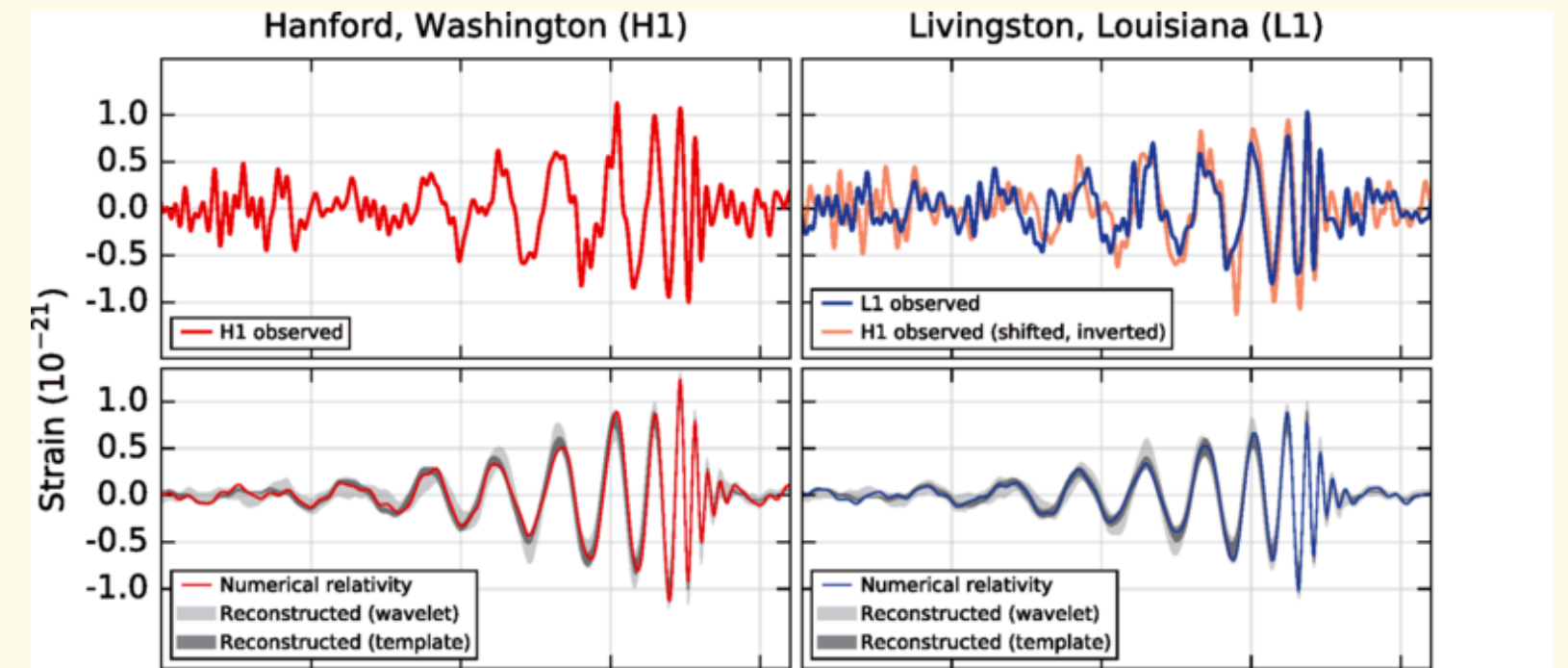
Labelling objects in binary sources of Gravitational Waves

Working on a general approach to object labelling that
does not rely on hidden assumptions

Introduction

Gravitational wave signals from binary systems of compact objects can be used to infer source properties.

B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration).
"Observation of Gravitational Waves from a Binary Black Hole Merger".
Phys. Rev. Lett. 116: 061102. DOI:10.1103/PhysRevLett.116.061102.



Introduction

Gravitational wave signals from binary systems of compact objects can be used to infer source properties. In particular, they provide information about the mass of the system.

$$\mathcal{M}_c, \eta \longrightarrow m_1, m_2$$

$m_1 [M_\odot]$	$m_2 [M_\odot]$
40.0	20.1
39.5	21.0
41.5	19.9
...	...
39.9	20.9

Introduction

The way we usually assign the *primary* and *secondary* labels to the objects is by determining which one is larger on a sample-by-sample basis.

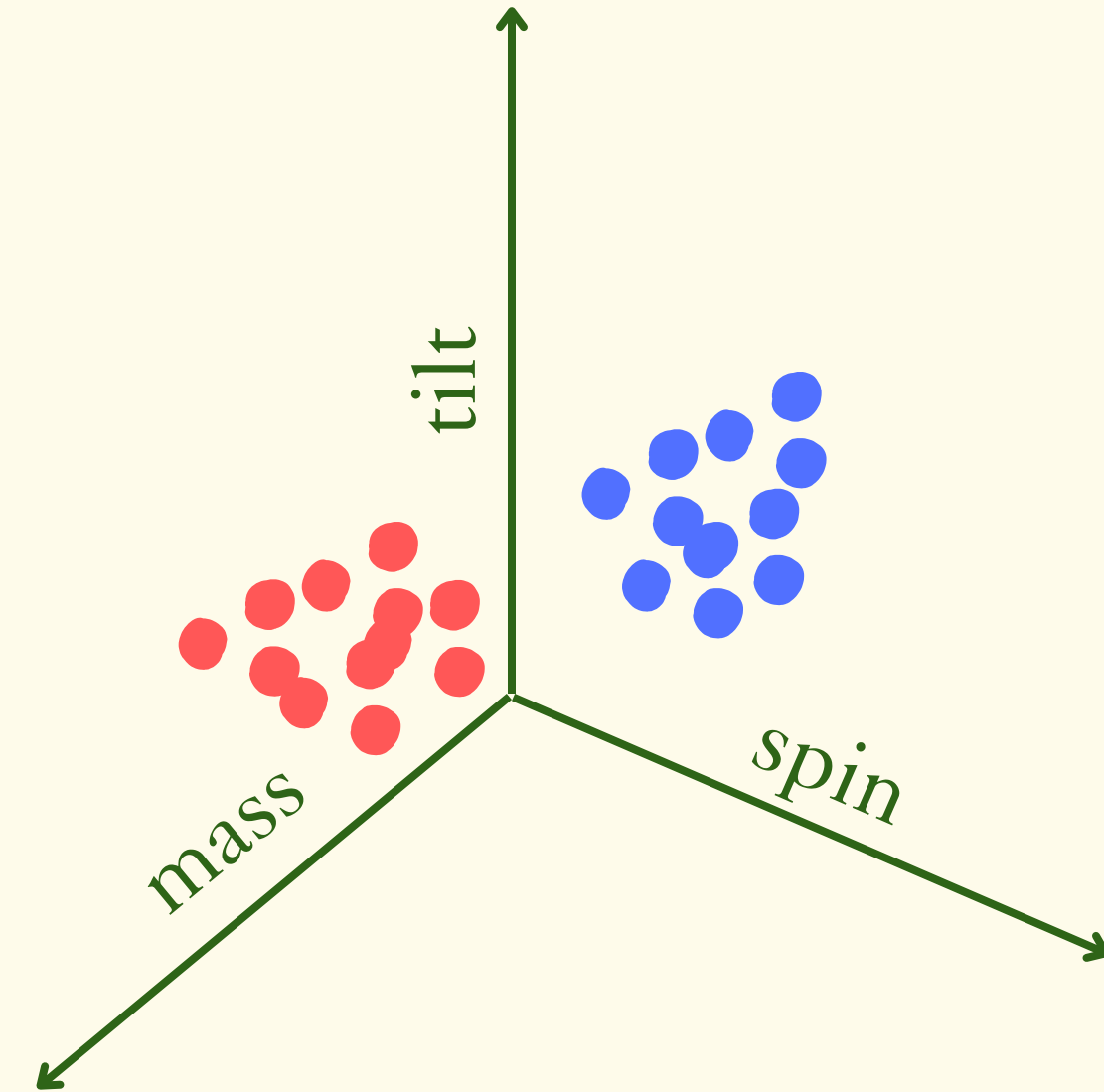
This can become tricky when the mass ratio approaches unity.



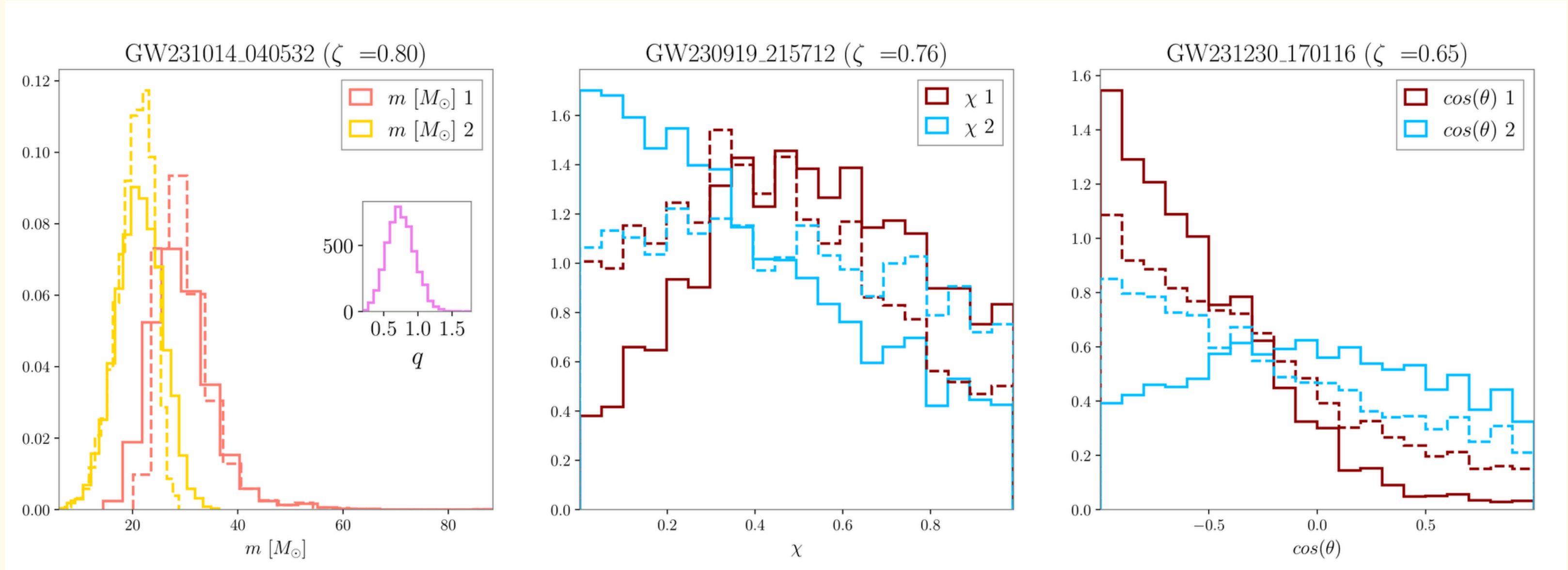
$m_1 [M_{\odot}]$	$m_2 [M_{\odot}]$
40.0	39.9
39.5	38.9
41.5	40.5
...	...
39.9	42.8

Introduction

A new approach (Gerosa et al. 2025) seeks to place all GW posterior samples in a feature space (e.g. mass, spin and tilt of the binary source) and allow a *semi-supervised clustering algorithm* to group the samples into two clusters, which ideally identify the two objects (1 and 2). This is an *a-posteriori approach* to labelling.



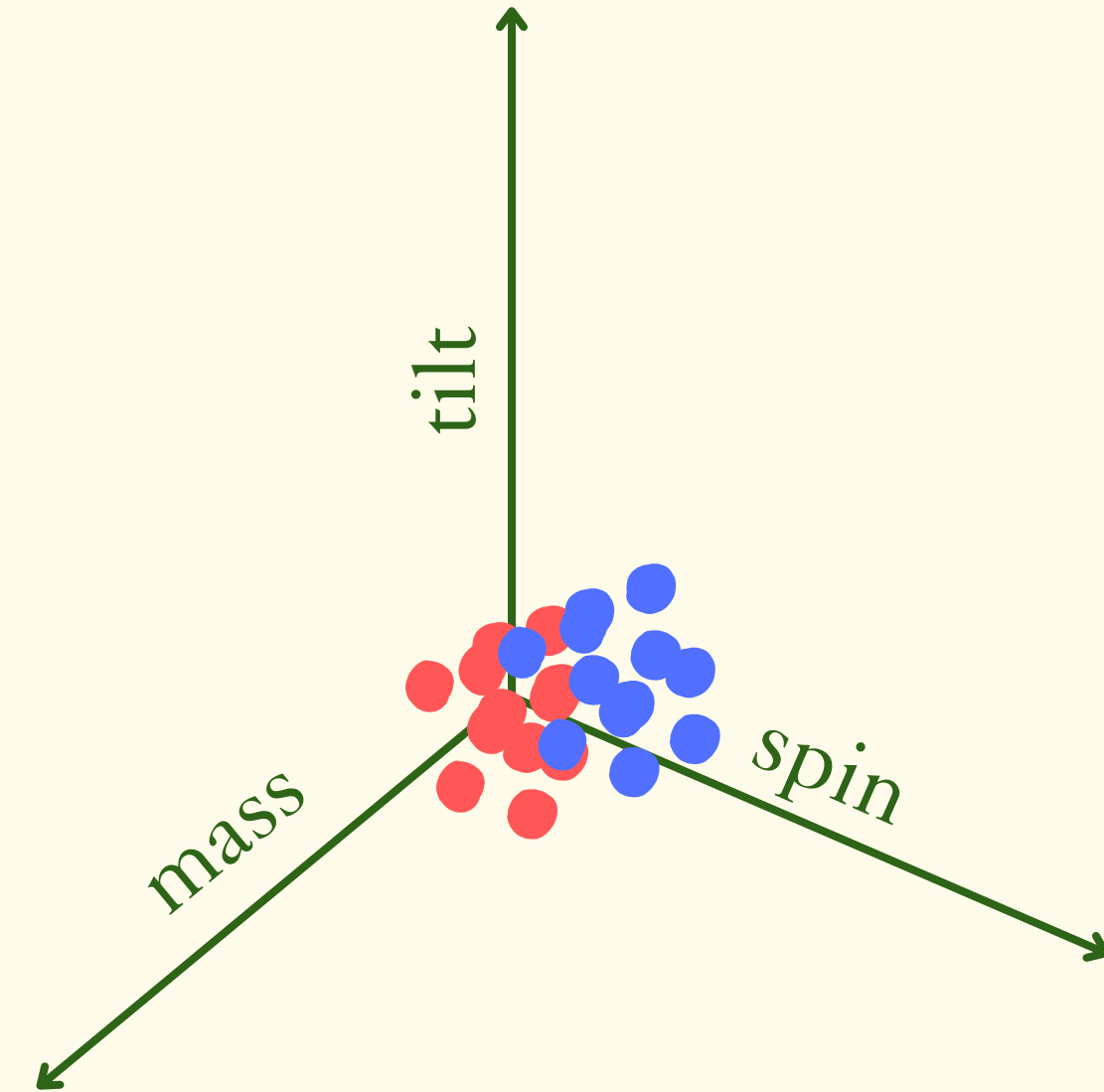
Introduction



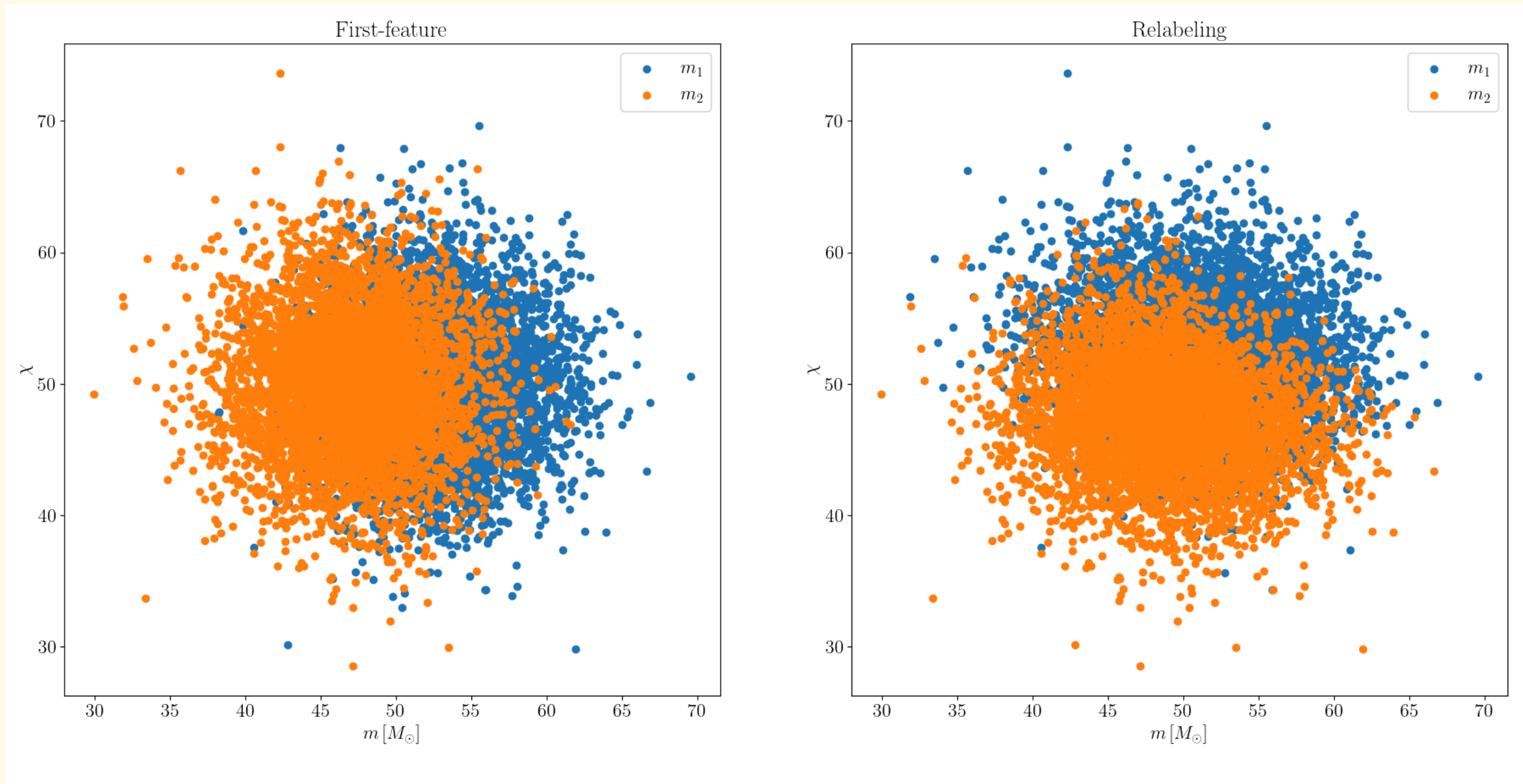
Introduction

The clustering algorithm has a tendency to identify two well-separated clusters of samples even when they are generated by the same analytical distribution.

For this reason, a refined version of this approach is proposed.

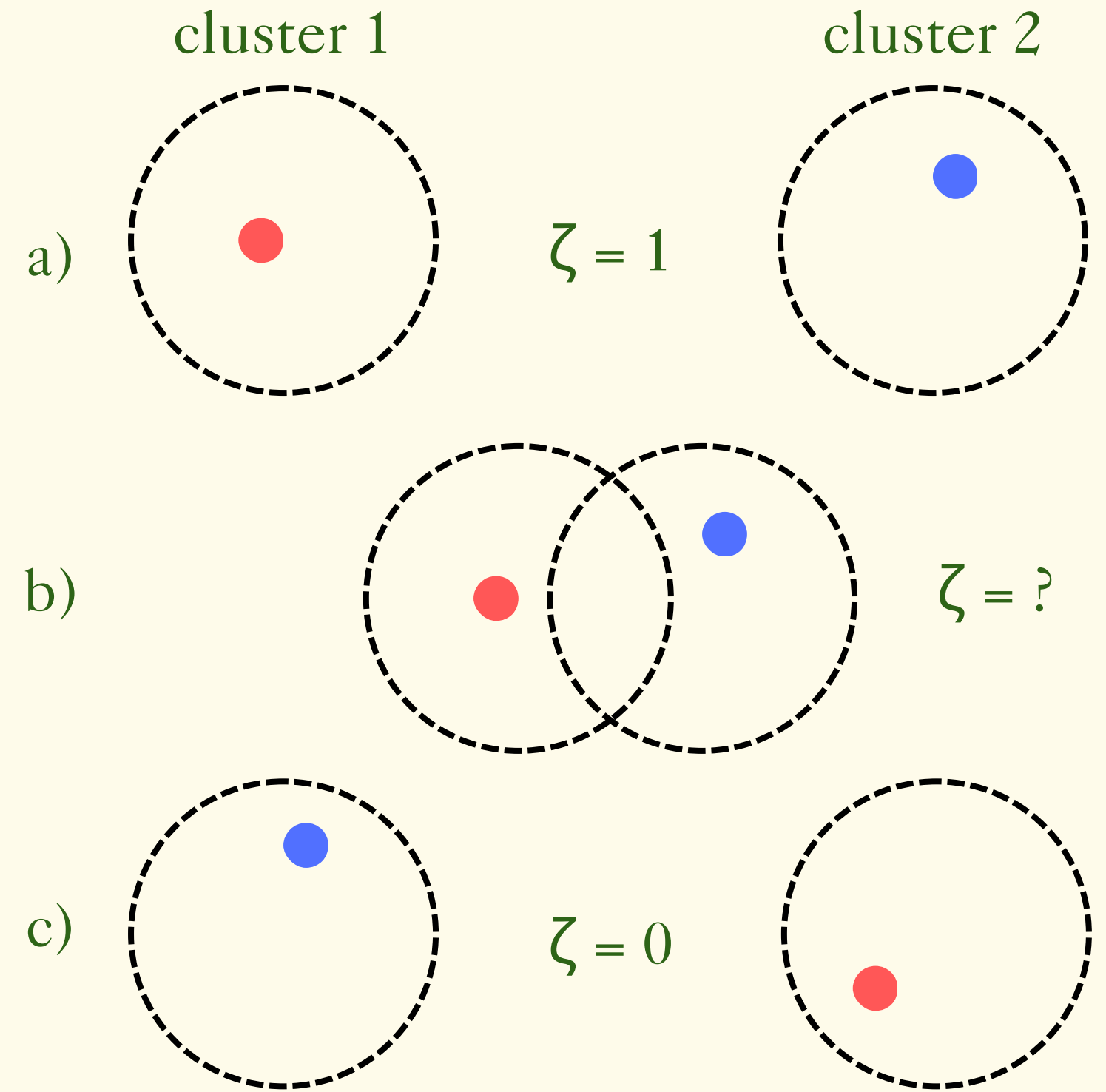


Introduction



Method

Instead of assigning a discrete identifier to each sample to tell whether it belongs to one cluster or the other, we choose to assign a *belonging probability* ζ distributed in the range $[0,1]$: if the **first item** of a given couple of points is more likely to belong to the *first cluster* (arbitrary), then ζ will be closer to 1.



Method



Log-likelihood equal to the inverse of the loss function of K-Means clustering:

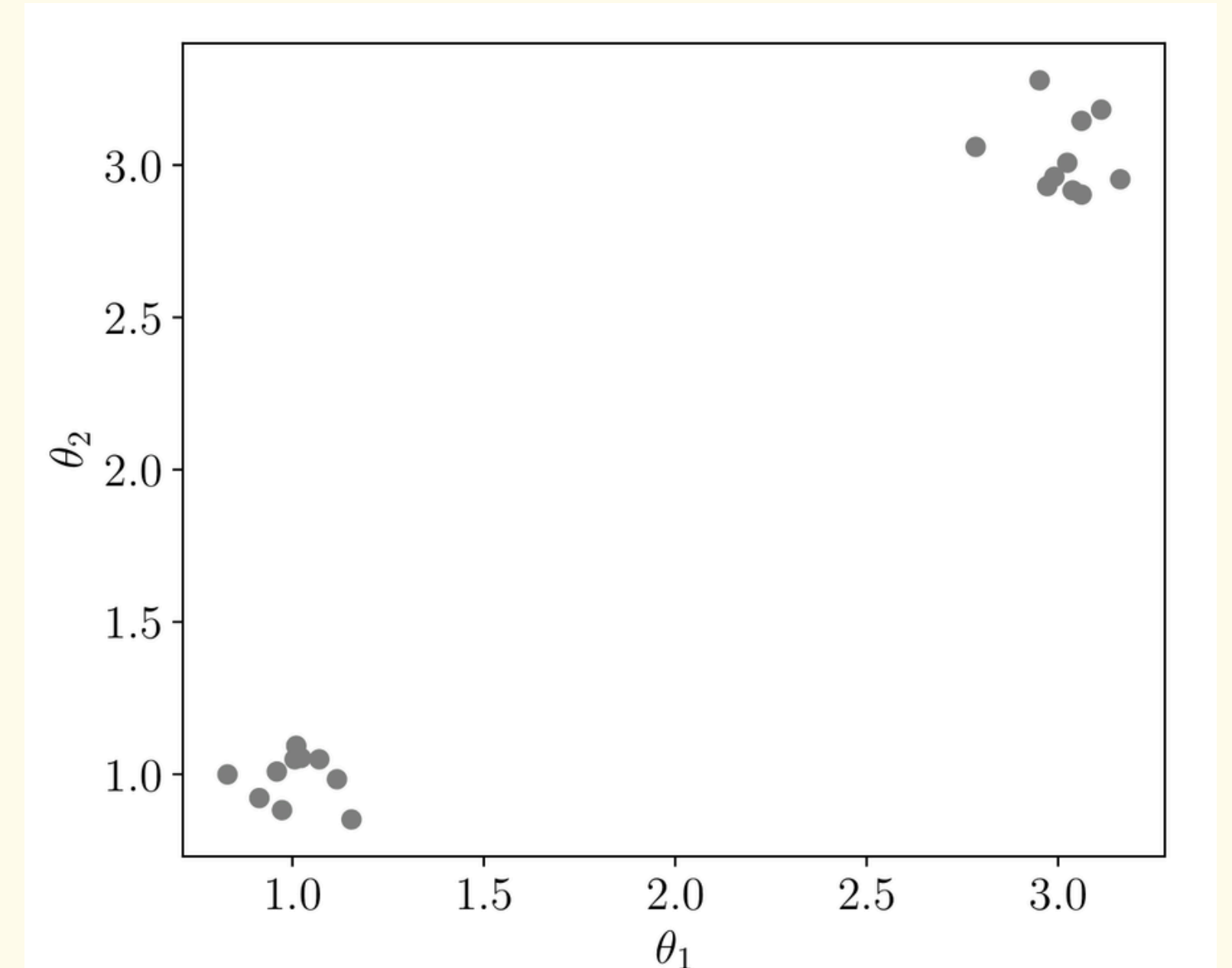
$$\ln \mathcal{L} = - \sum_{i=1}^N \left\{ \left[d(x_i, \bar{x}_1) + d(x_{i+N}, \bar{x}_2) \right] \zeta_i + \left[d(x_i, \bar{x}_2) + d(x_{i+N}, \bar{x}_1) \right] (1 - \zeta_i) \right\}$$

$$\bar{x}_1 = \frac{1}{N} \sum_{i=1}^N \left[x^i \zeta_i + x^{i+N} (1 - \zeta_i) \right]$$

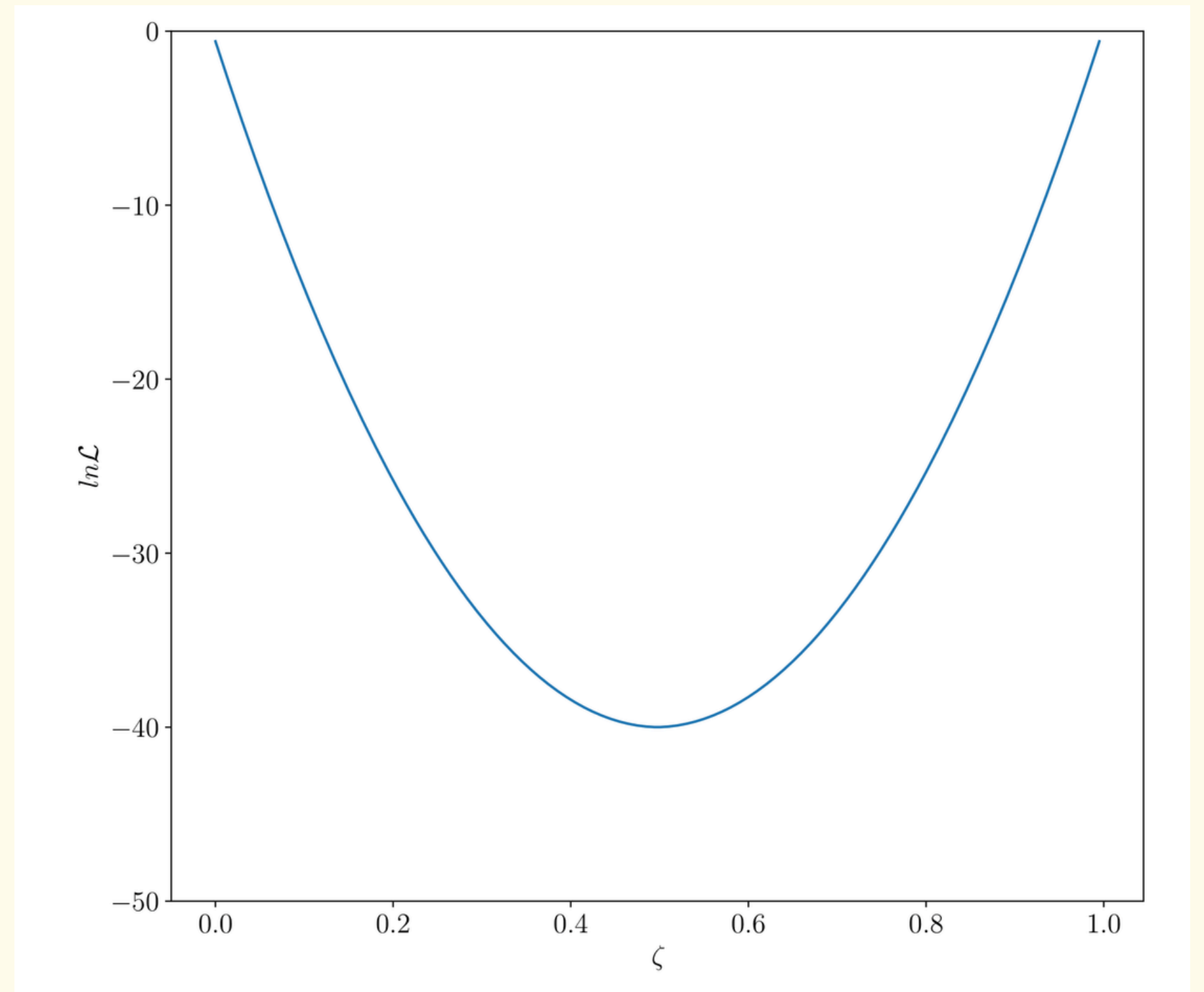
$$\bar{x}_2 = \frac{1}{N} \sum_{i=1}^N \left[x^i (1 - \zeta_i) + x^{i+N} \zeta_i \right]$$

Results

What does the ζ likelihood look like for this specific configuration of samples? 10 couples of gaussianly-distributed points, well-separated in feature space.

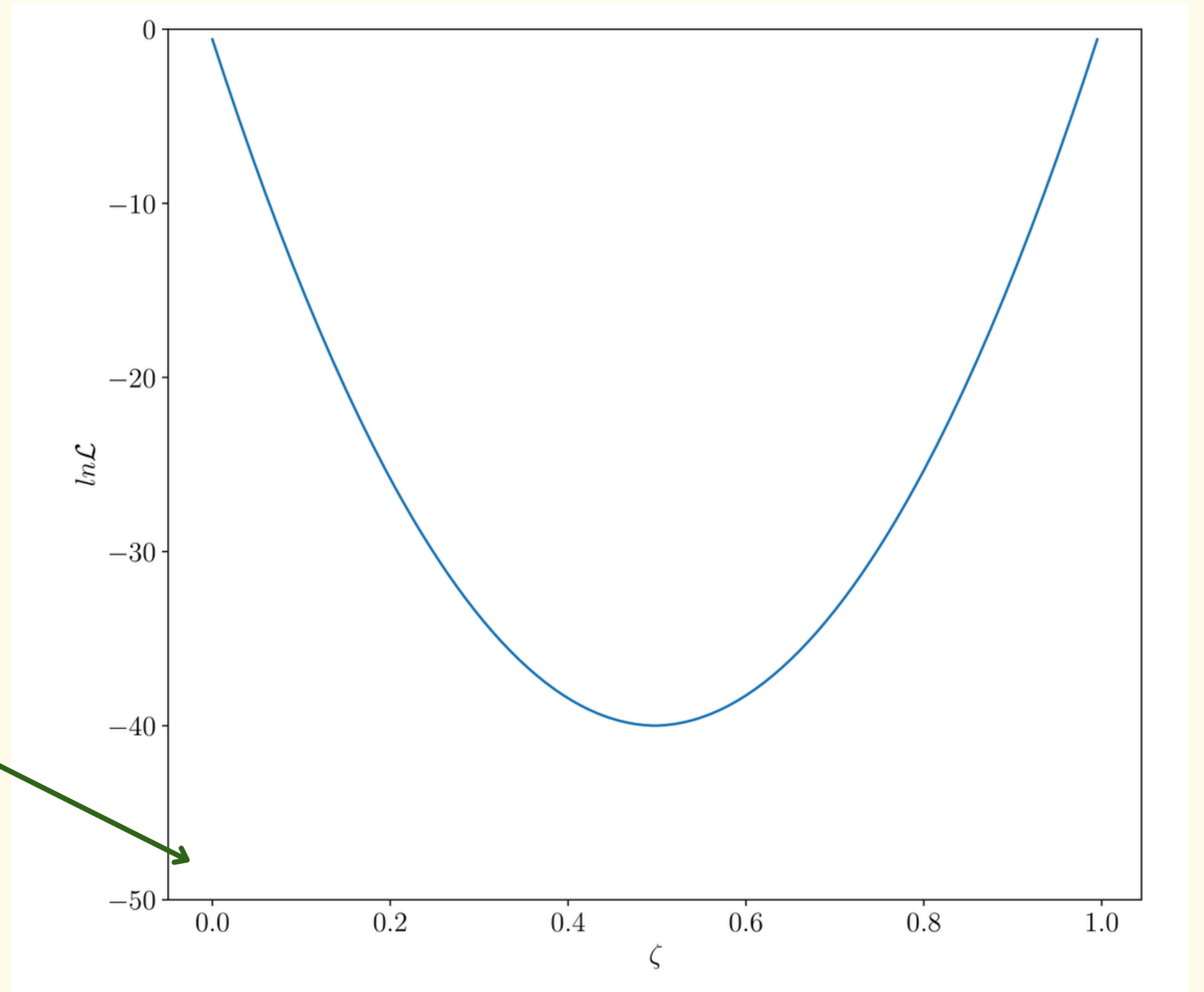
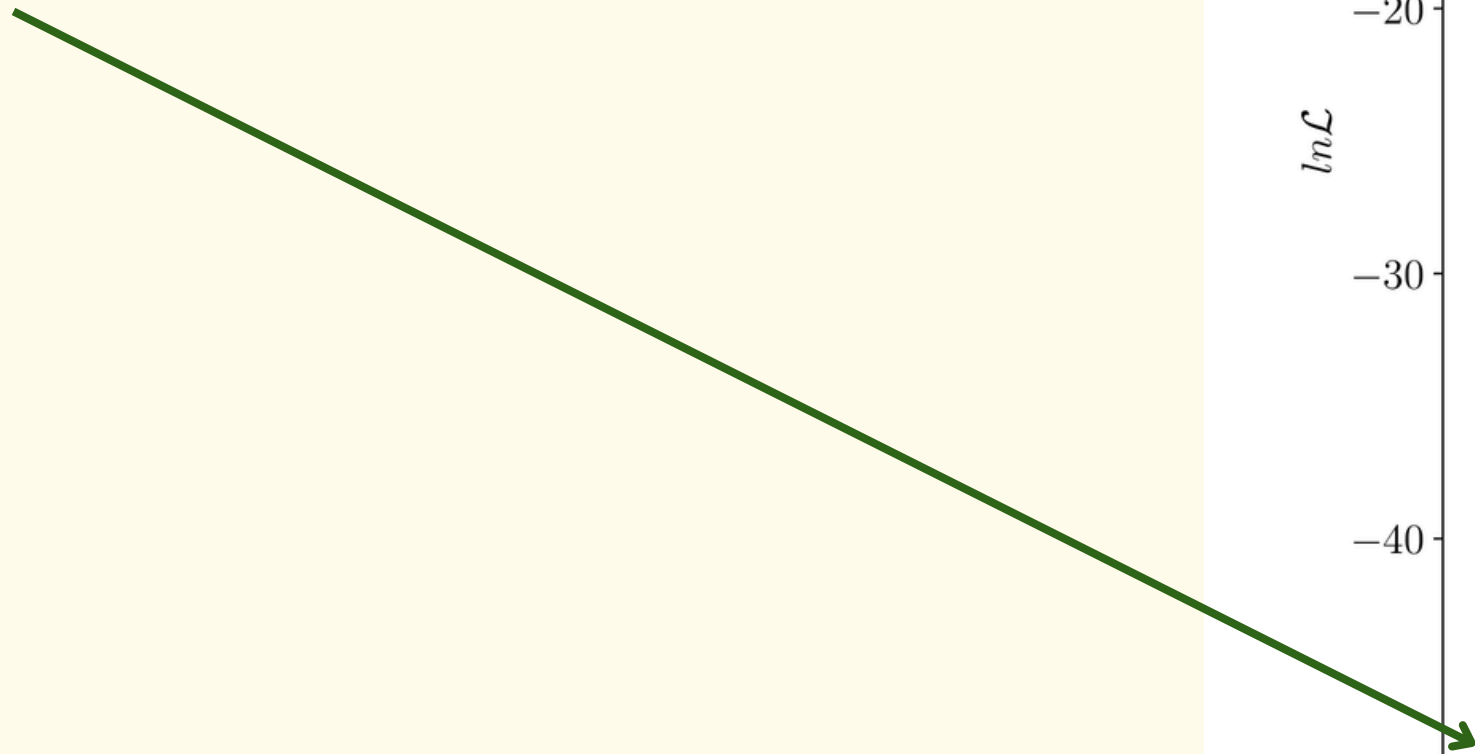


Results



Results

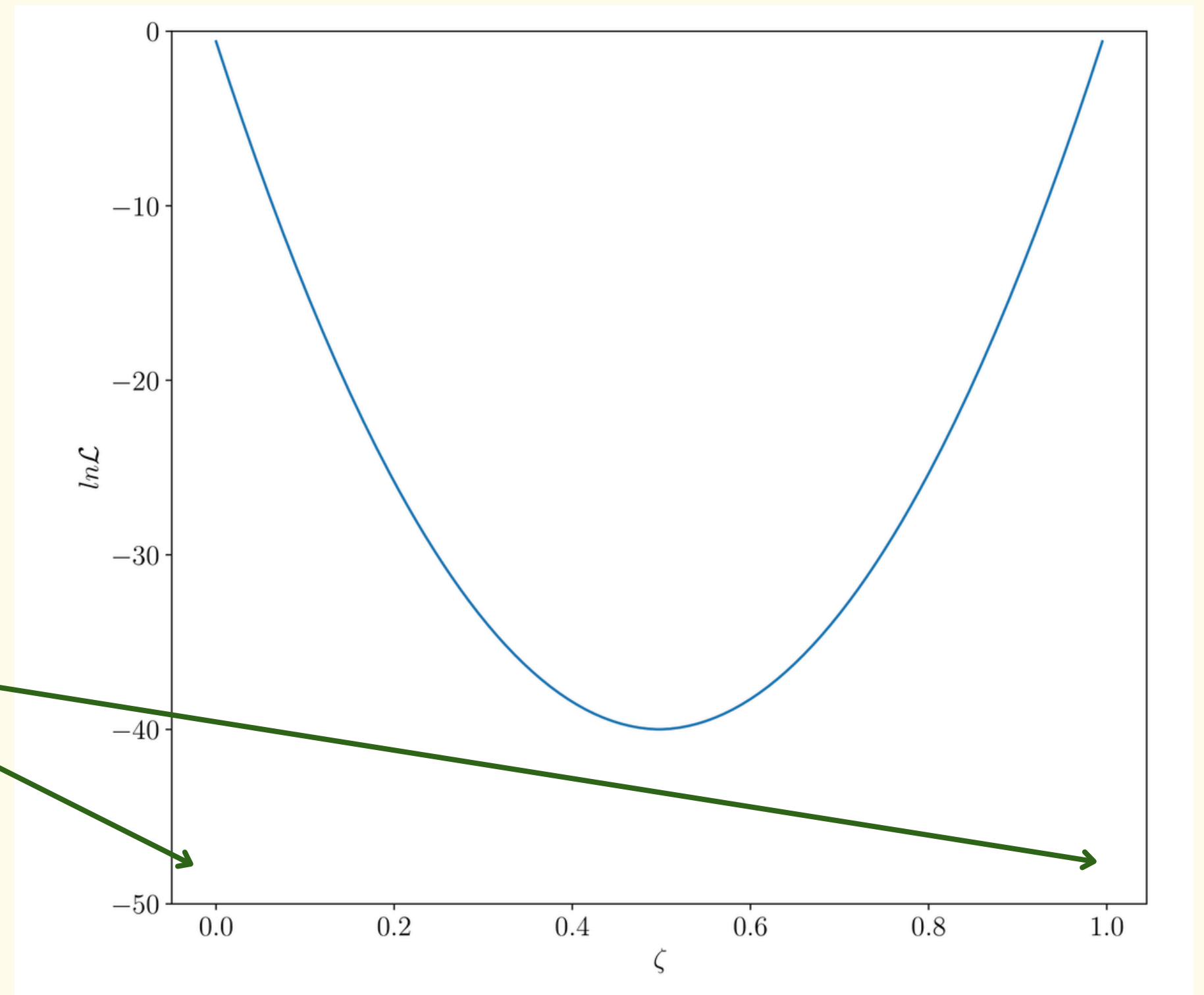
This...



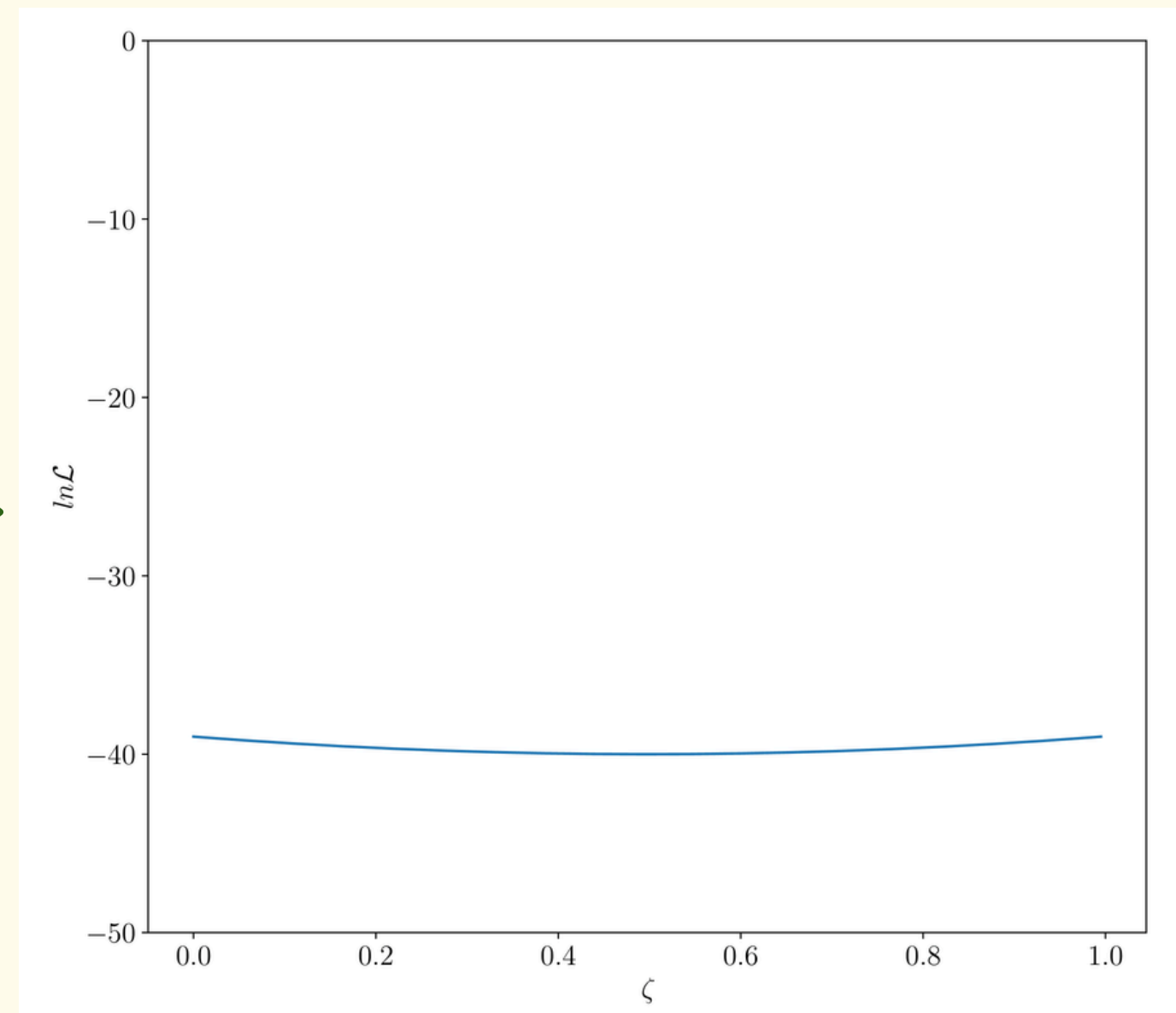
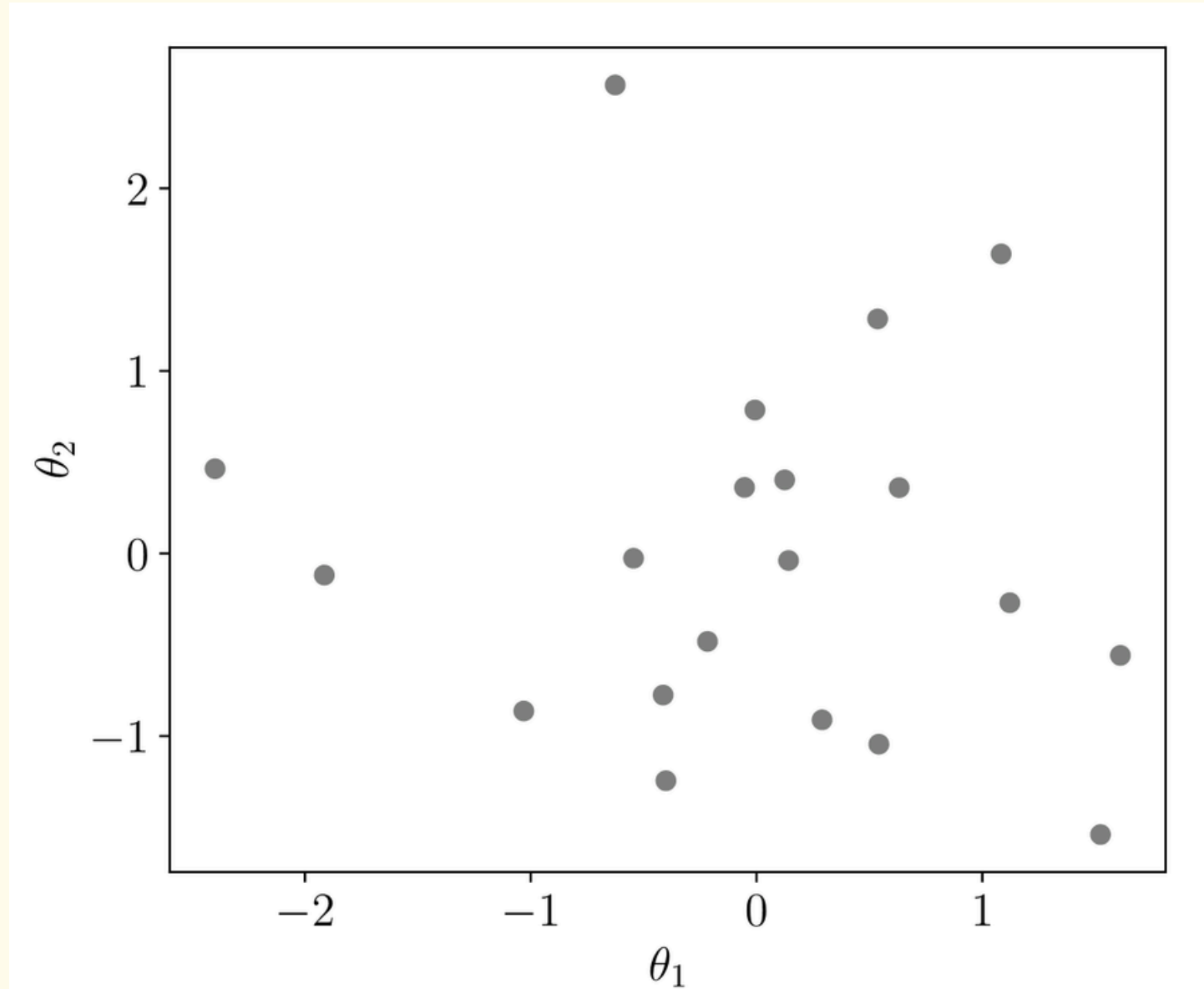
Results

This...

...is as good as this.

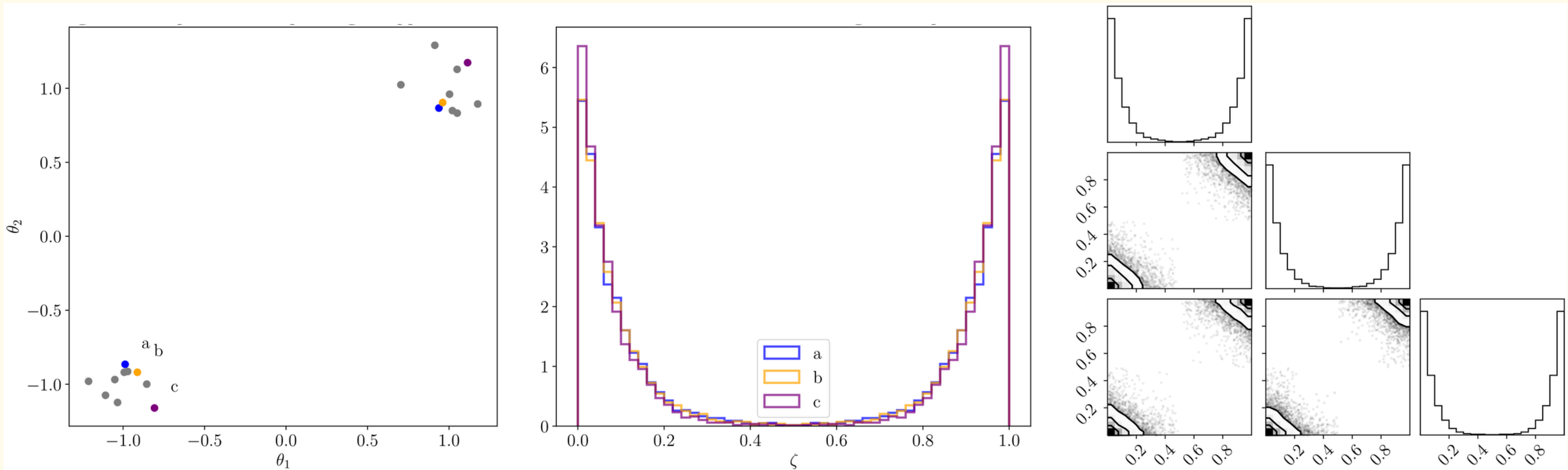


Results



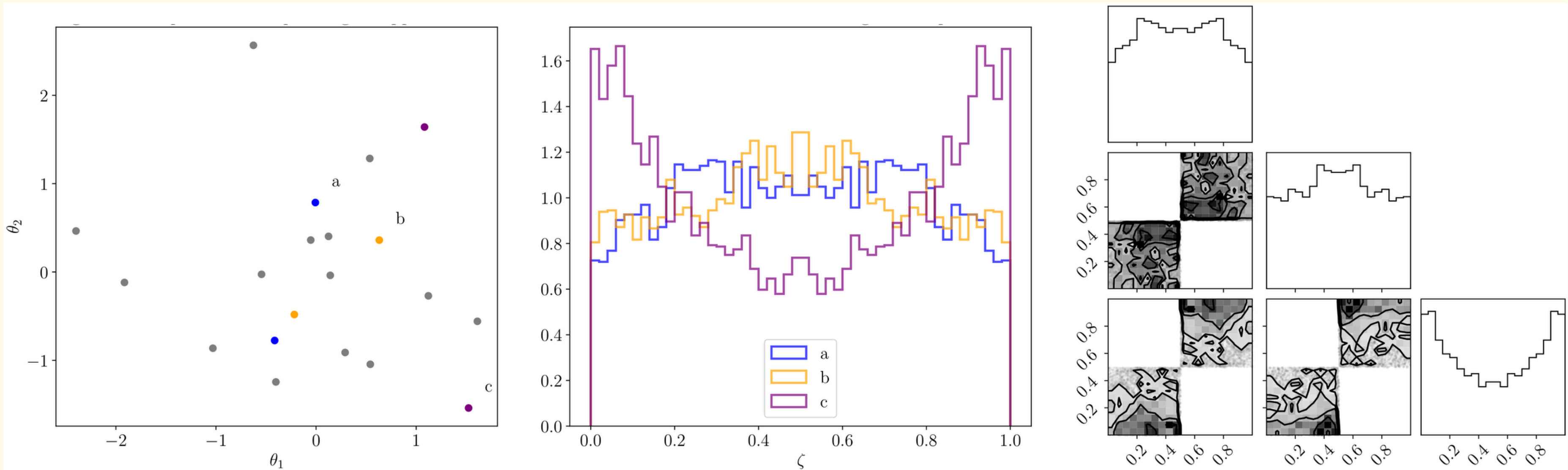
Results - Eryn

Karnesis, Nikolaos and Katz, Michael L. and Korsakova, Natalia and Gair, Jonathan R. and Stergioulas, Nikolaos, *Eryn : A multi-purpose sampler for Bayesian inference* (2023)



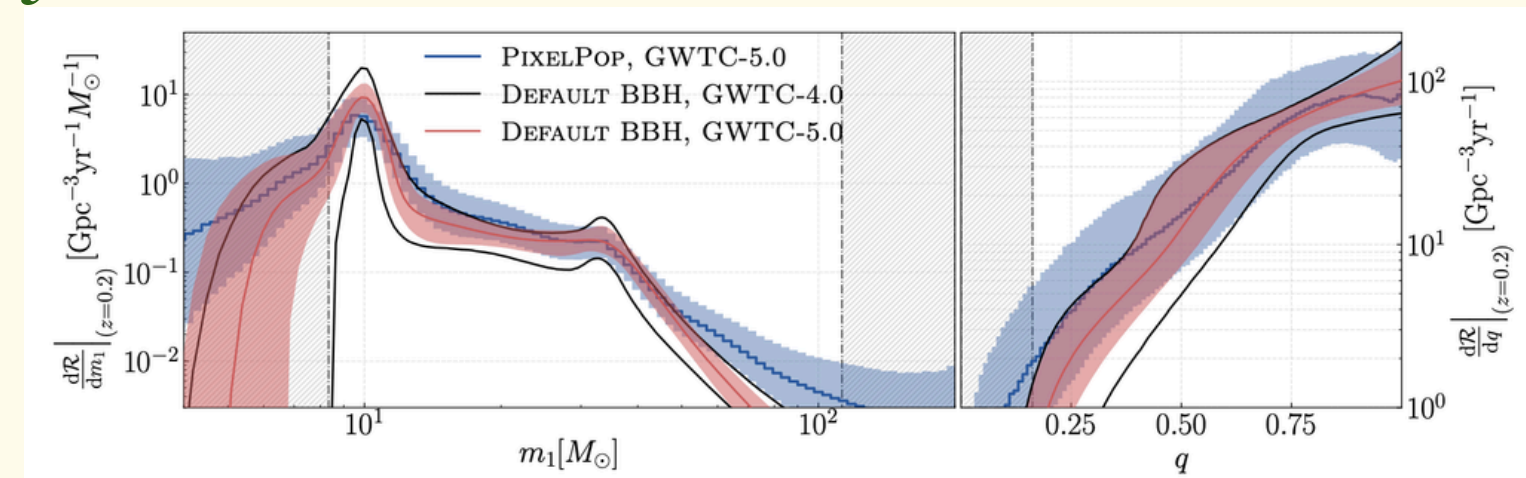
Results - Eryn

Karnesis, Nikolaos and Katz, Michael L. and Korsakova, Natalia and Gair, Jonathan R. and Stergioulas, Nikolaos, *Eryn : A multi-purpose sampler for Bayesian inference* (2023)



To-do list

- Interpretation of the results and implementation of a criterion to determine when two black holes are indistinguishable.
- Analysis of real Gravitational Wave events (large and small mass ratios).
- Study of the population distributions in mass, spin and cosine of the tilt (*what changes when we label compact objects in a binary this way?*)

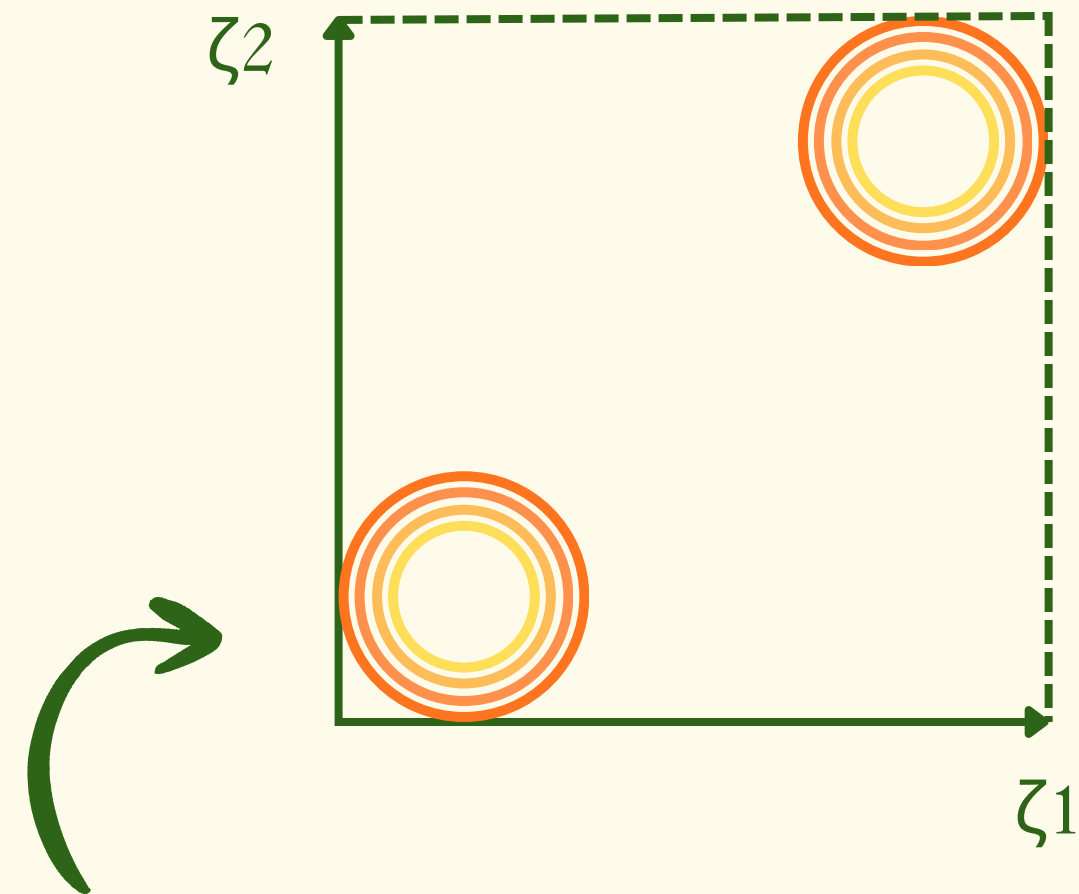


Thank you for your attention!

Method

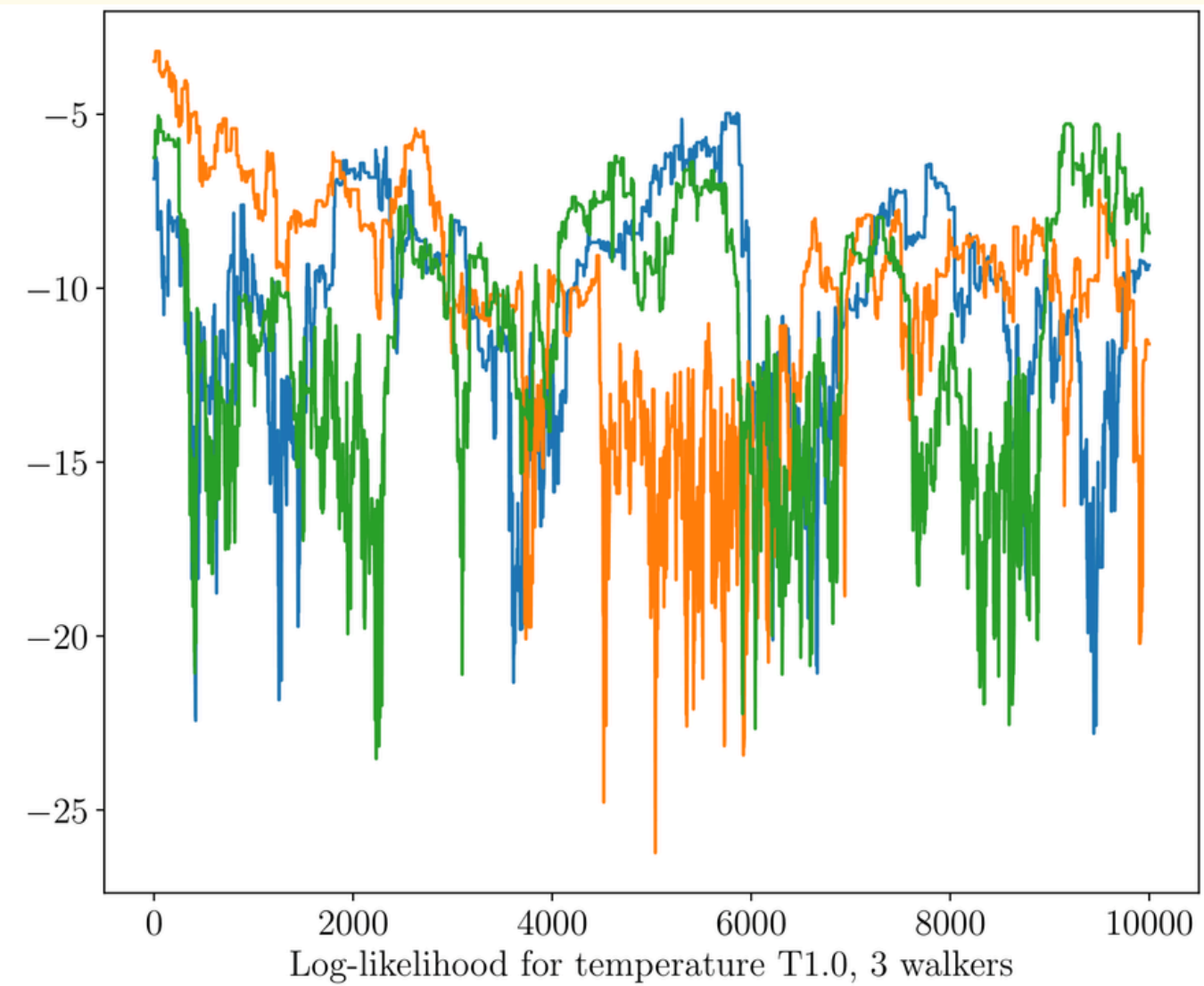
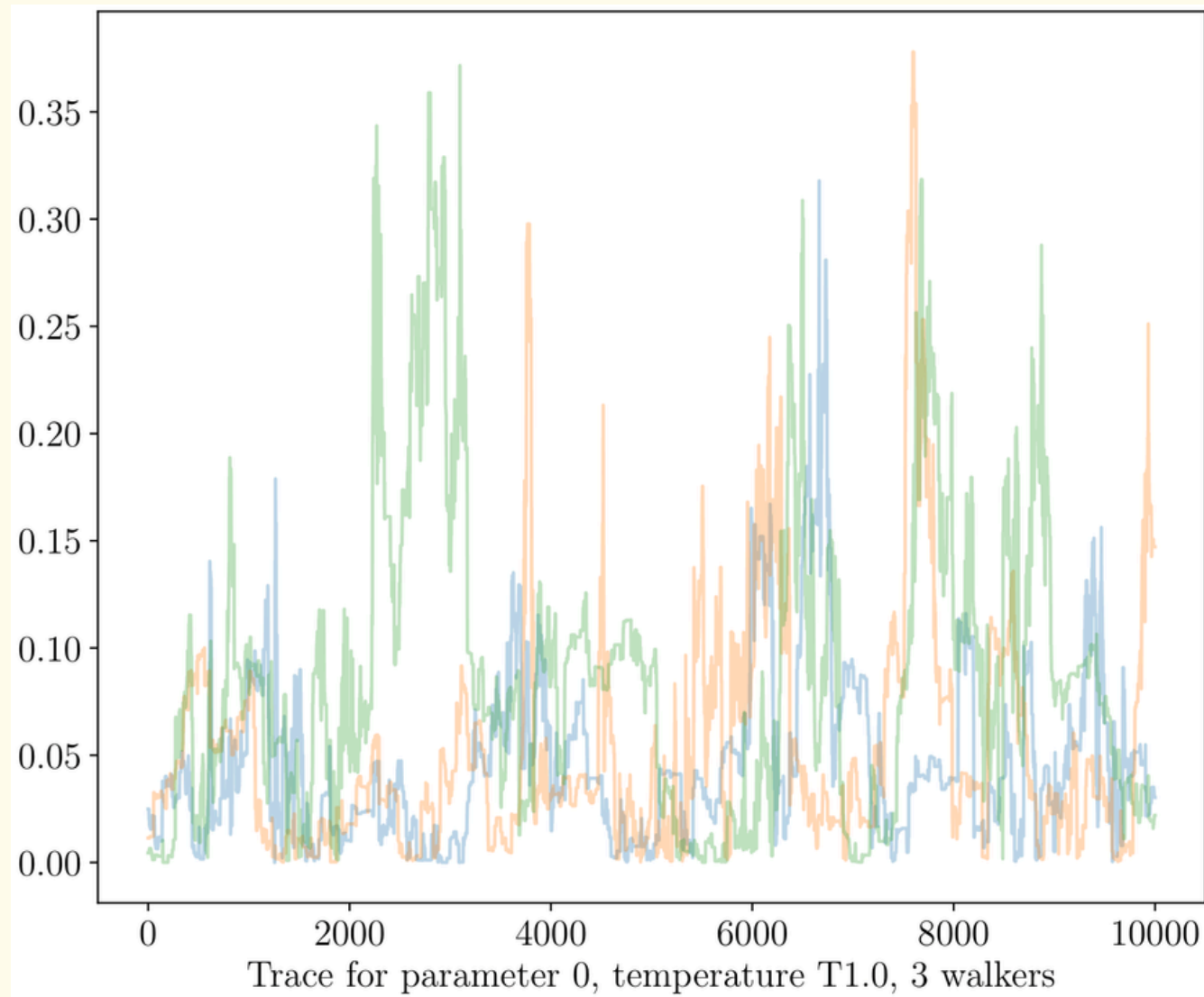
We derive ζ values for each data point by sampling an N-dimensional ζ posterior.

We do so by using ?



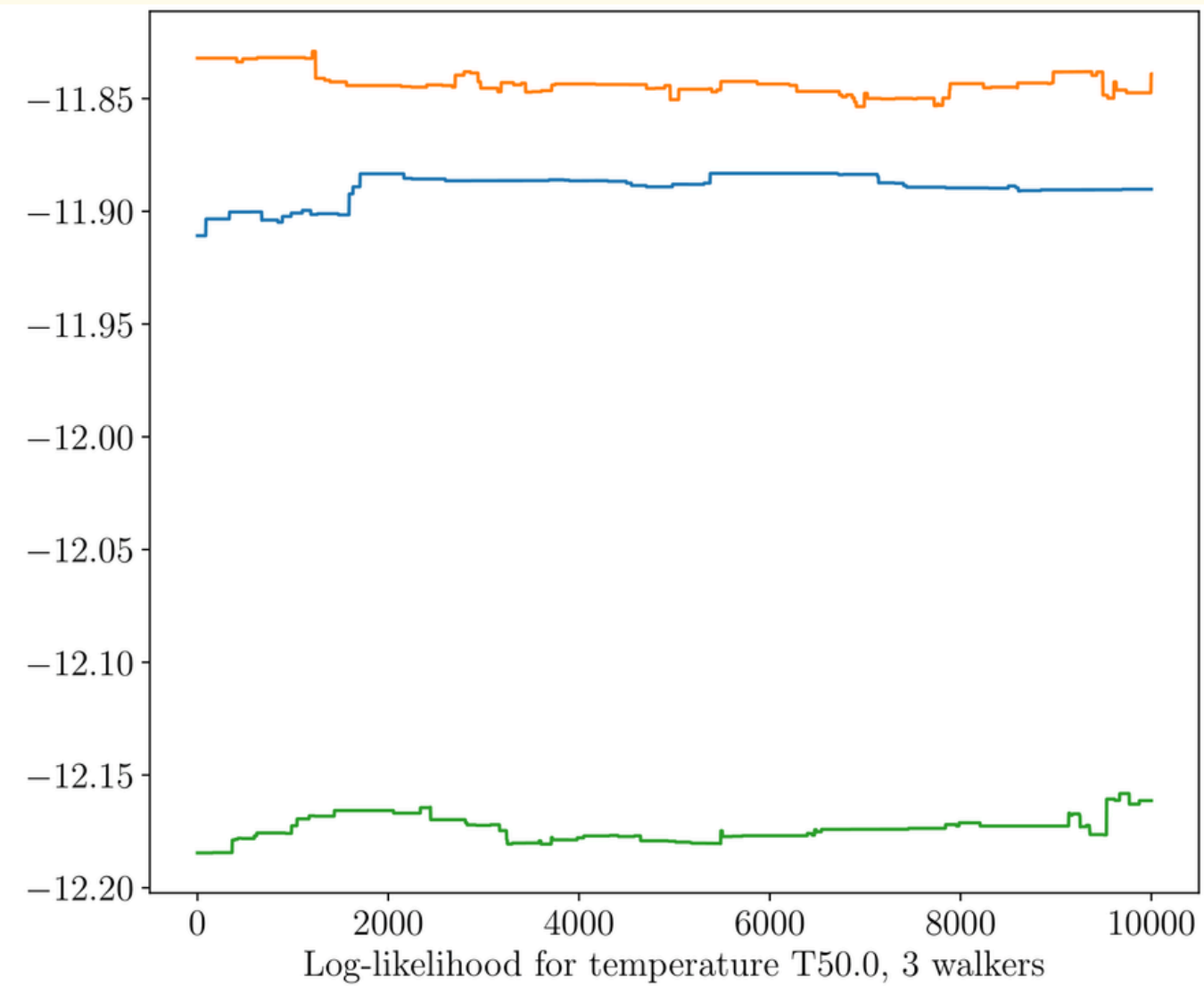
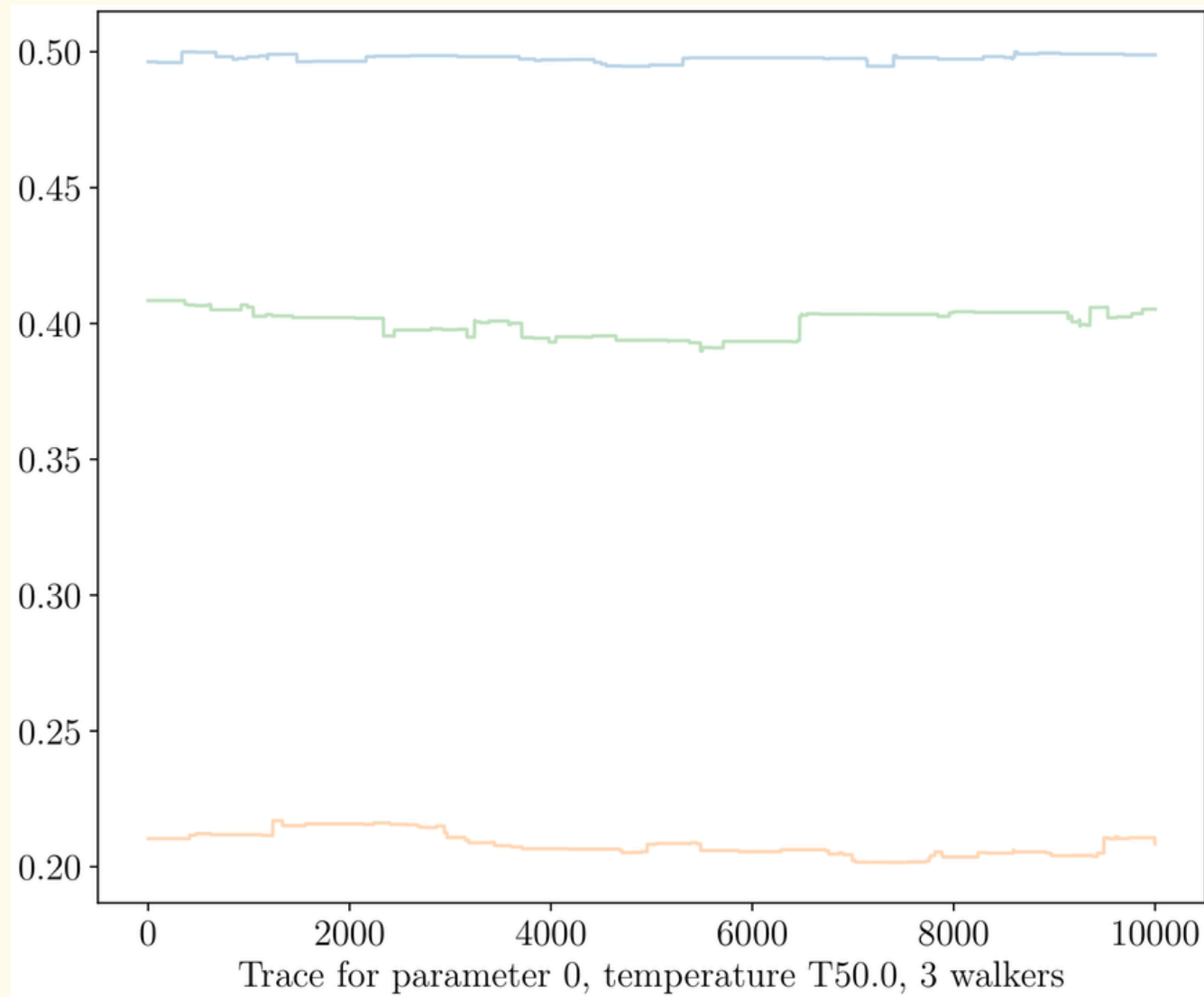
Example case of two data points and the distribution of the respective probabilities ζ_1 and ζ_2

Results - issues



10 couples

Results - issues



200 couples

