Warped accretion disks and black-hole spins in gravitational-wave astronomy

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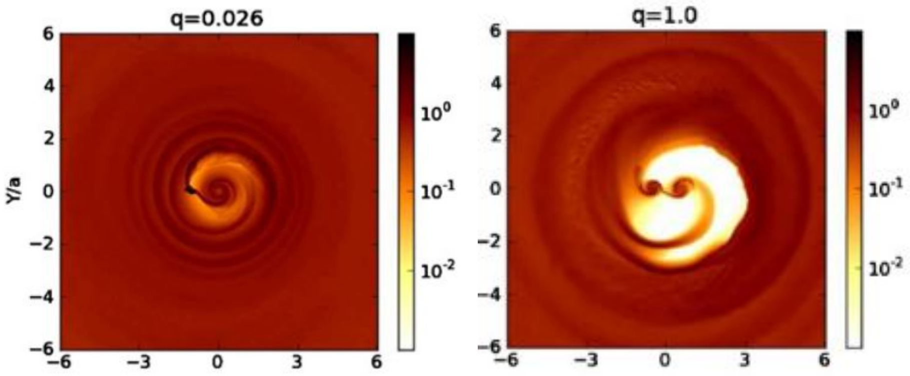
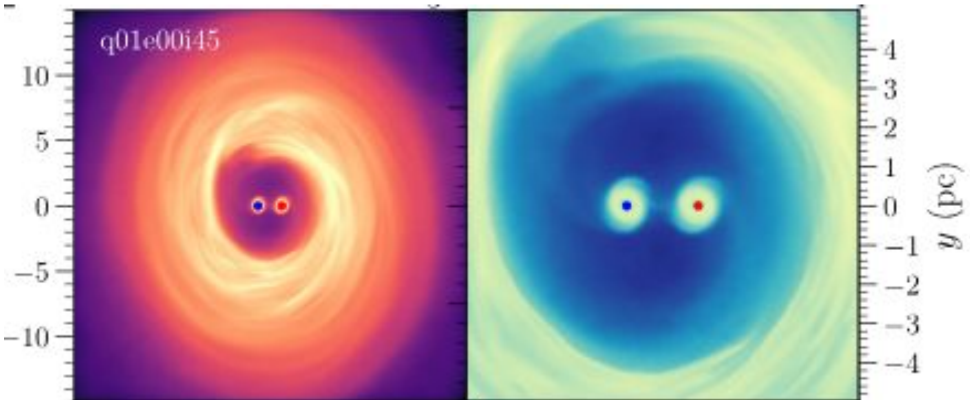
Supermassive Black Hole Binaries

1. Galaxy merger
2. Dynamical friction
3. Star scattering
4. **Gas-driven phase**
5. GW-driven inspiral
6. Merger

Farris et al 2014





Bourne et al. 2024

Bardeen - Petterson effect

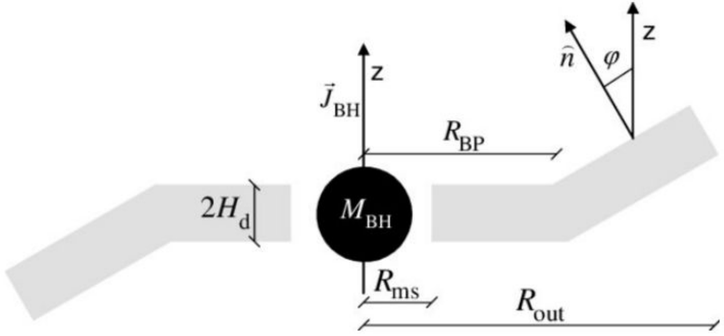
When J_{BH}, L_{disc} misaligned :

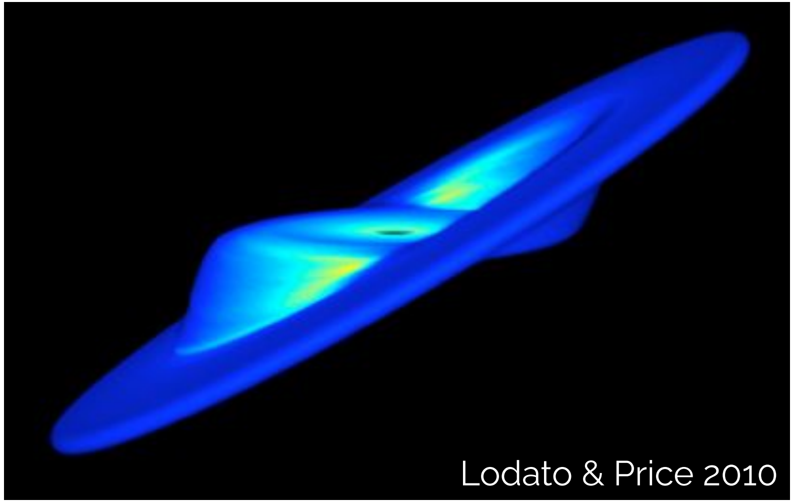
→ Lense-Thirring precession aligns inner disc

$$\frac{\partial \mathbf{L}_{\mathrm{disc}}}{\partial t} = \frac{2G}{c^2} \frac{\mathbf{J}_{\mathrm{BH}} \times \mathbf{L}_{\mathrm{disc}}}{R^3}$$

→ Outer disc stays misaligned and brings the system to co / counteralignment

Caproni et al. 2005





Lodato & Price 2010

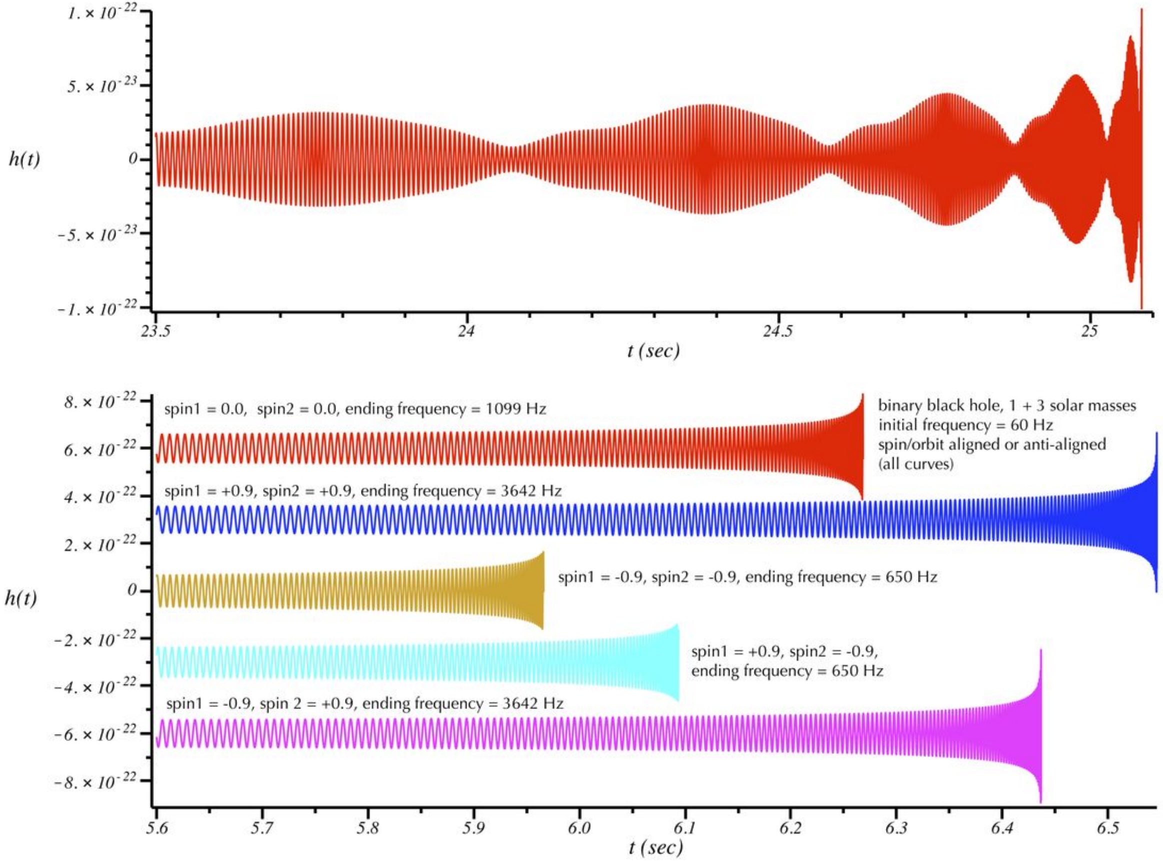
Why spin alignment matters

Binary spin alignment affects GWs!

But also:

- BH recoils (Gerosa & Sesana 2015)
- subgrid models for simulations of the circumbinary disc (Fiacconi et al. 2018, Bourne et al. 2024)

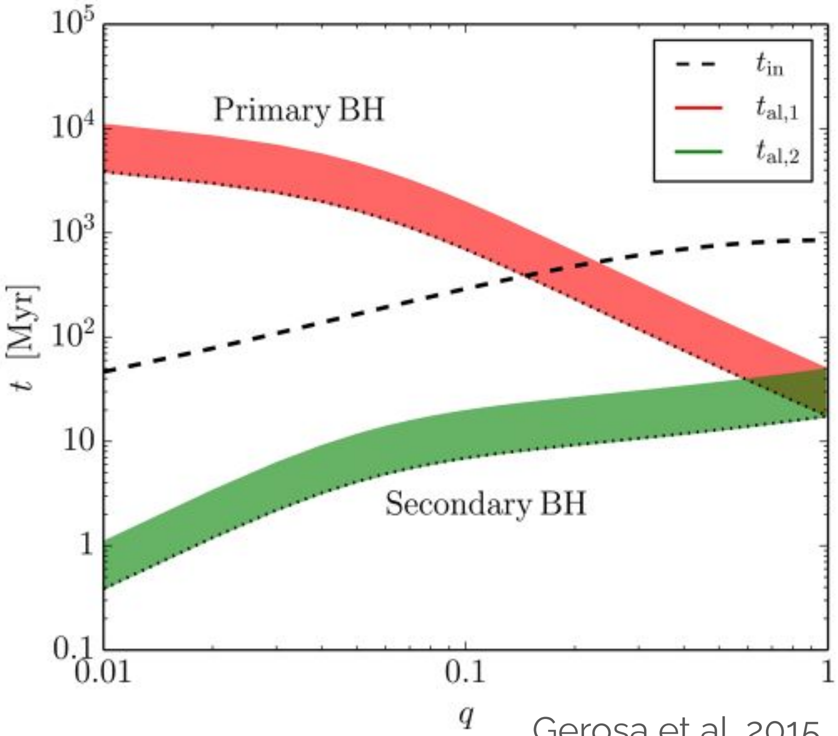
M. Favata



Can the binary align in time?

Scheuer & Feiler 1996
Natarajan & Pringle 1998
Martin et. al 2007
Lodato & Gerosa 2013

- $t_{align} < t_{inspiral}$ ← but this depends on mass ratio q
- **Differential accretion**: smaller secondaries orbit closer to circumbinary disc and accrete more gas
- Beyond timescales: can we measure the residual misalignments?



Gerosa et al. 2015

1D Boundary Value Problem

Pringle 1992
Scheur & Feiler 1996
Martin et al. 2007, 2009
Tremaine & Davis 2014

Mass conservation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(\nu_1 \Sigma R^{1/2}\right)\right] + \frac{1}{R}\frac{\partial}{\partial R}\left[\nu_2 \Sigma R^2 \left|\frac{\partial \hat{\boldsymbol{L}}}{\partial R}\right|^2\right]$$

Angular momentum conservation

$$\frac{\partial \boldsymbol{L}}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[\frac{R^{1/2}}{\Sigma}\frac{\partial}{\partial R}\left(\nu_1 \Sigma R^{1/2}\right)\boldsymbol{L}\right] + \frac{1}{R}\frac{\partial}{\partial R}\left[\left(\nu_2 R^2 \left|\frac{\partial \hat{\boldsymbol{L}}}{\partial R}\right|^2 - \frac{3}{2}\nu_1\right)\boldsymbol{L}\right] + \frac{1}{R}\frac{\partial}{\partial R}\left(\frac{1}{2}\nu_2 R \boldsymbol{L}\frac{\partial \hat{\boldsymbol{L}}}{\partial R}\right) + \frac{\partial}{\partial R}\left(\nu_3 R \boldsymbol{L} \times \frac{\partial \hat{\boldsymbol{L}}}{\partial R}\right) + \frac{2G}{c^2}\frac{\boldsymbol{J} \times \boldsymbol{L}}{R^3} + \frac{3GM_\star \Sigma R^2}{4R_\star^3}\left(\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{L}}_\star\right)\left(\hat{\boldsymbol{L}} \times \hat{\boldsymbol{L}}_\star\right)$$

Gerosa et al. 2020

- viscosities are **non-linear** (Ogilvie 1999, Ogilvie & Latter 2013)
- assume $J_{disc} >> J_{BH}$

  ... isn't this too unrealistic?

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Lense-Thirring

Companion

Gerosa et al. 2020

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Infinite-angular-momentum assumption

Why?

- in gas-rich galaxies circumbinary discs act as a gas reservoir → disc outer boundary is kept constant → BH coaligns with disc
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- no middle ground for finite-angular-momentum discs
- Bourne et al. 2024 find $\eta \lesssim 1$
- King et al. 2005: for $\eta < 2$ system counteraligns

Finite-angular-momentum discs

(Isolated BH only)

Instead of fixing disc shape ($\hat{\mathbf{L}}_{\rm disc}(r_{\rm max})$), local BVP

↓

Fix $J_{disc}(r_{max})$,
non-local BVP!

$$\mathbf{J}_d(r_{\rm max}) = \int_{r_{\rm ISCO}}^{r_{\rm max}} 2\pi r \mathbf{L}(r) dr$$

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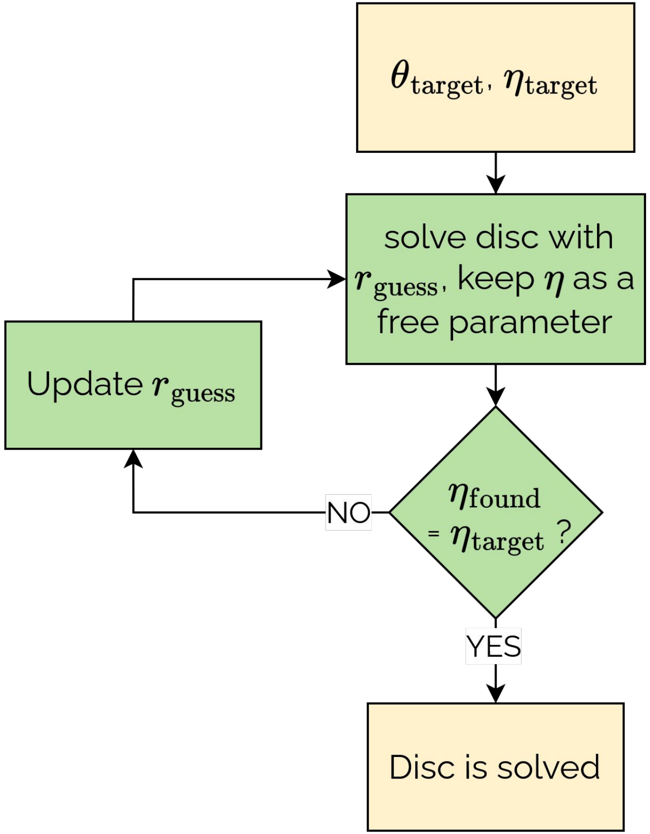
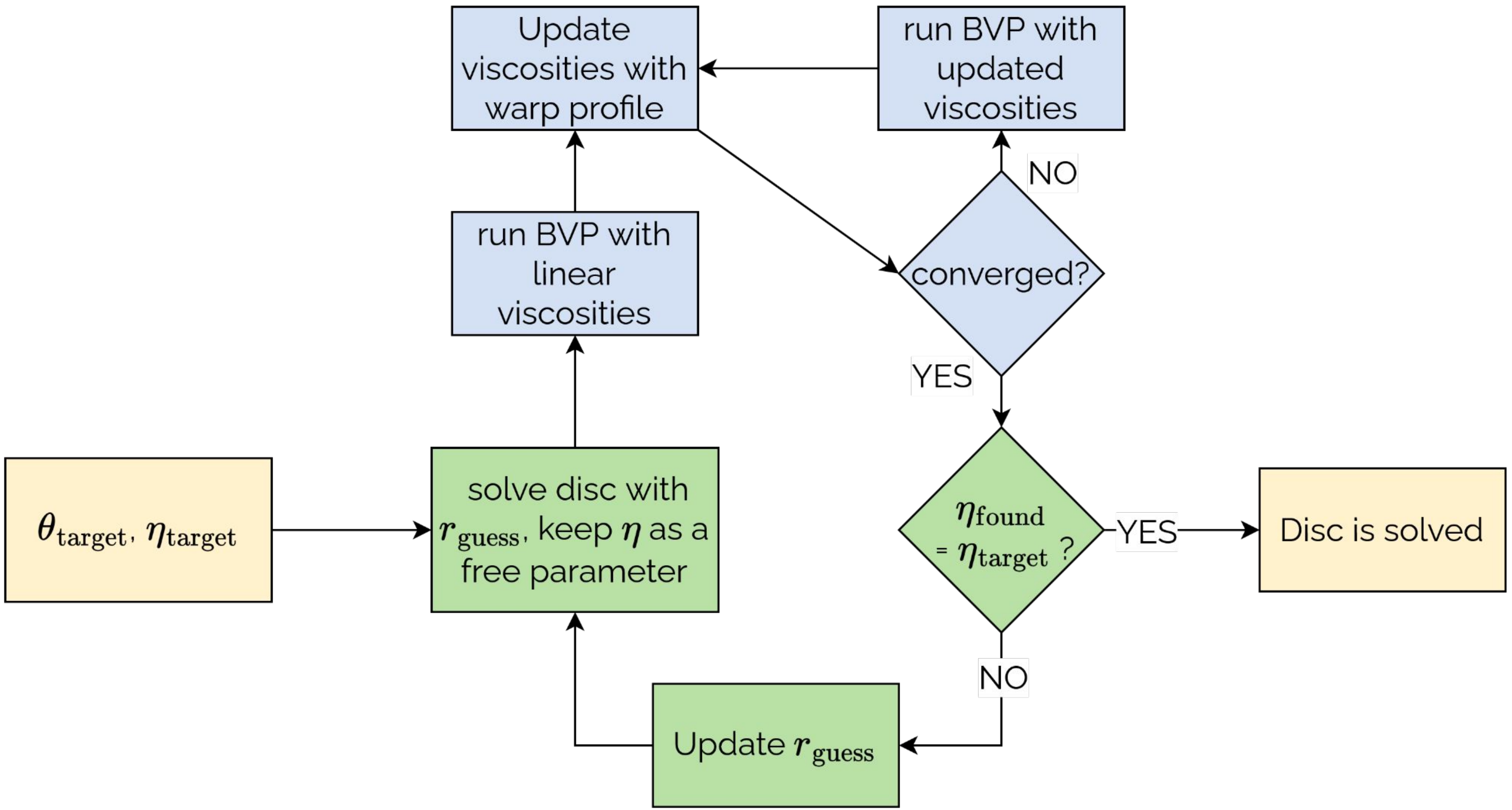
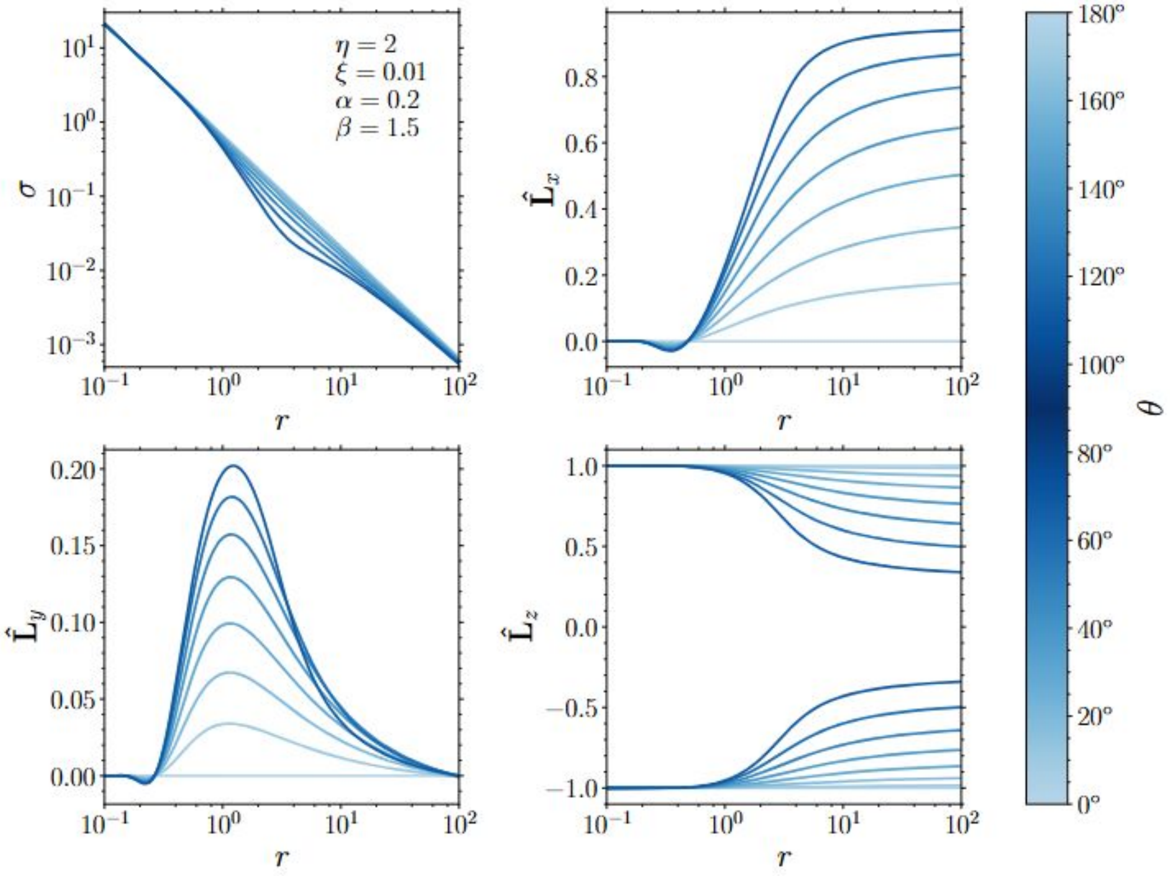


Non-linear viscosities

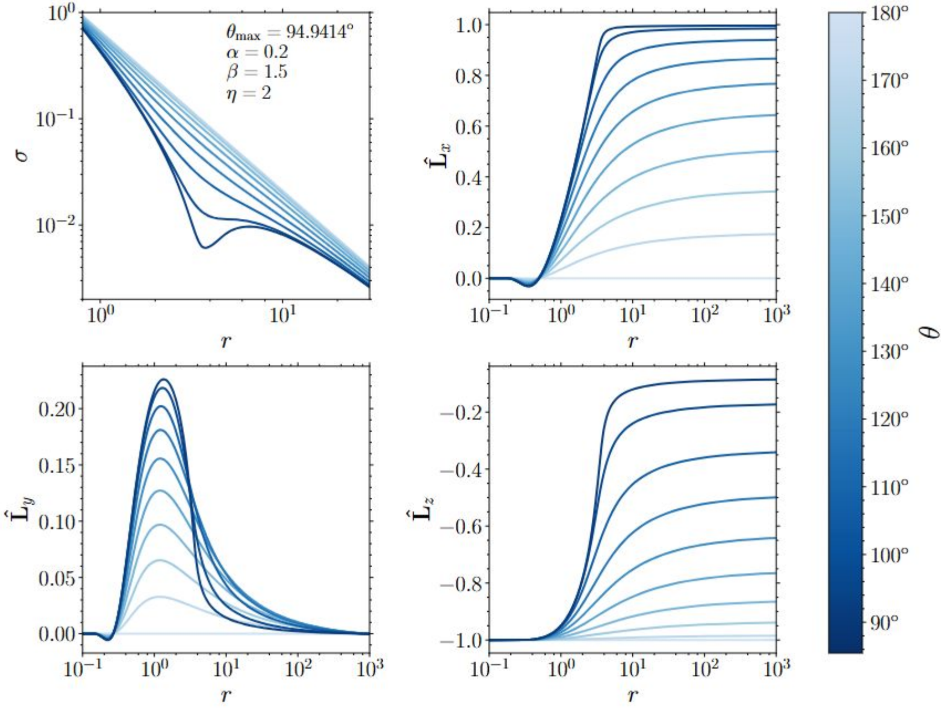


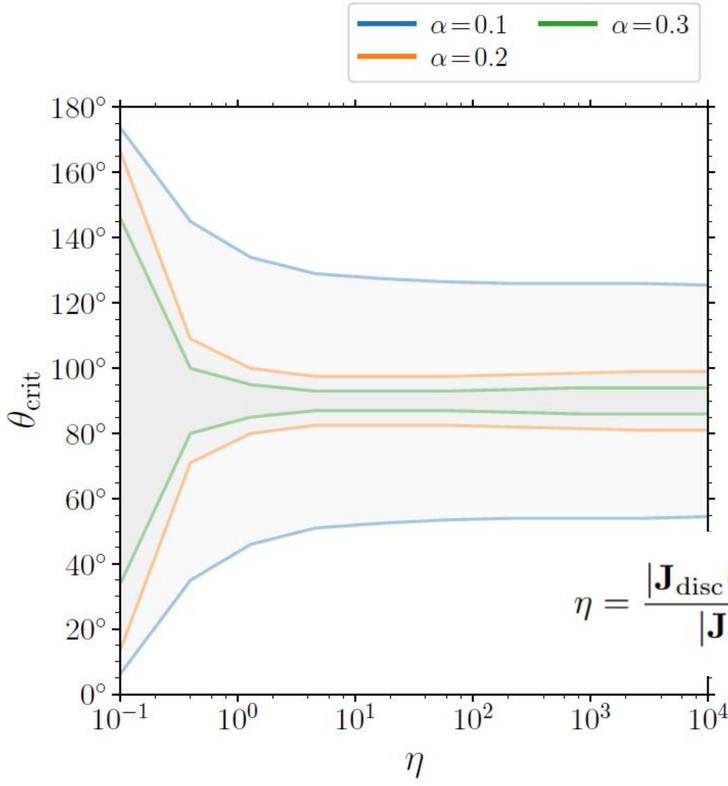
Disc shape

(Isolated BH only)



Critical obliquity



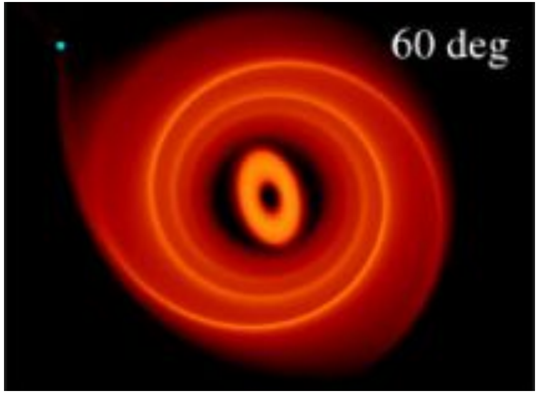


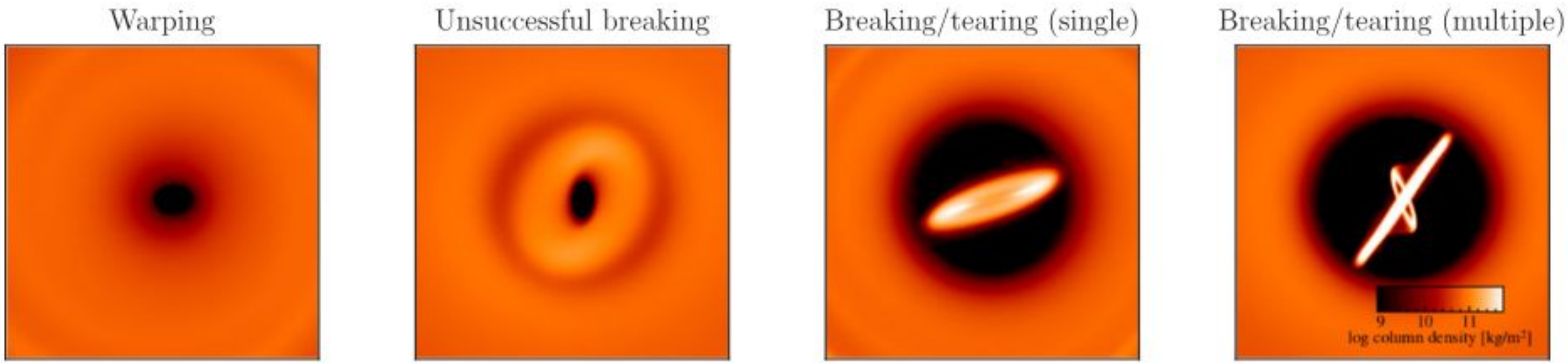
$$\eta = \frac{|\mathbf{J}_{\mathrm{disc}}(r_{\max})|}{|\mathbf{J}_{\mathrm{BH}}|}$$

Critical obliquity

Nealon et al 2022:

- critical obliquity corresponds to disc breaking,
- disc breaking hinders (and can even prevent) disc-BH alignment,
- discs show additional phenomena (e.g. spiral arms) not captured by 1D approximations.





Time evolution

From angular momentum conservation:

$$\frac{\mathrm{d}}{\mathrm{dt}}\eta = -\hat{\mathbf{J}}_{\mathrm{disc}} \cdot \frac{\mathrm{d}\hat{\mathbf{J}}_{\mathrm{BH}}}{\mathrm{dt}}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\cos\theta = -\frac{\mathrm{d}\,\eta}{\mathrm{dt}}\left(1 + \frac{\cos\theta}{\eta}\right)$$

Time evolution

From angular momentum conservation:

always negative!

$$\boxed{\frac{\mathrm{d}}{\mathrm{dt}}\eta} = -\hat{\mathbf{J}}_{\mathrm{disc}} \cdot \frac{\mathrm{d}\hat{\mathbf{J}}_{\mathrm{BH}}}{\mathrm{dt}}$$

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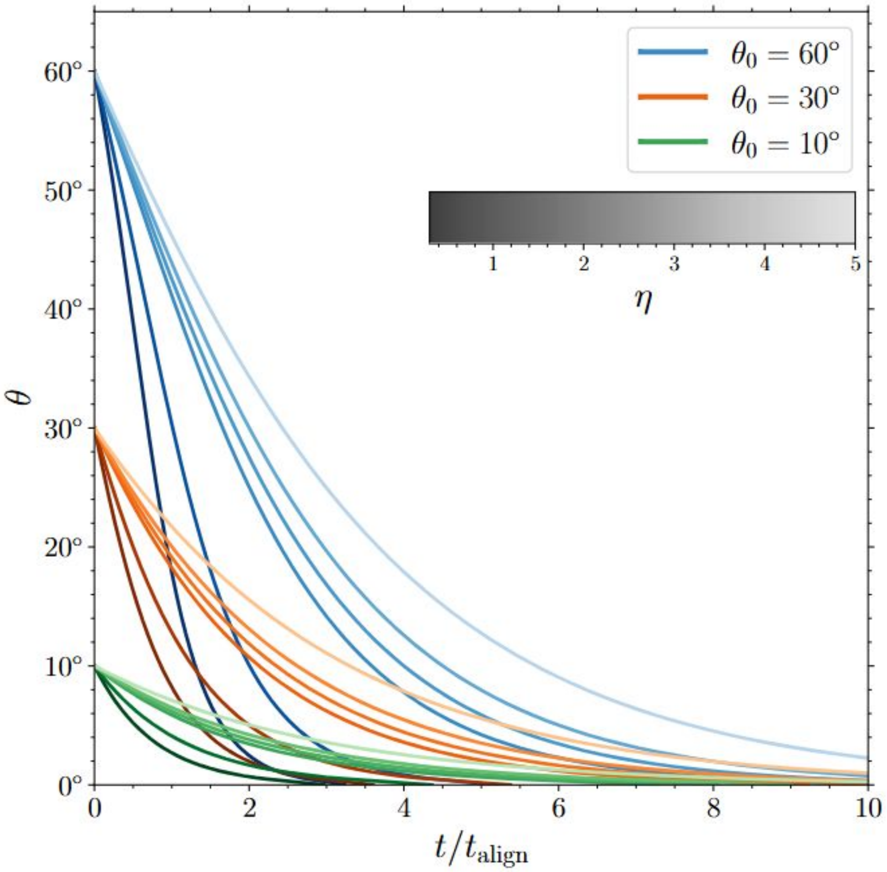
If θ (t=0) < 90˚, discs will always **coalign**!



Time evolution

Given θ, η at t=0:

1. **cos θ > - η/2**

   disc tries to coalign → critical obliquity

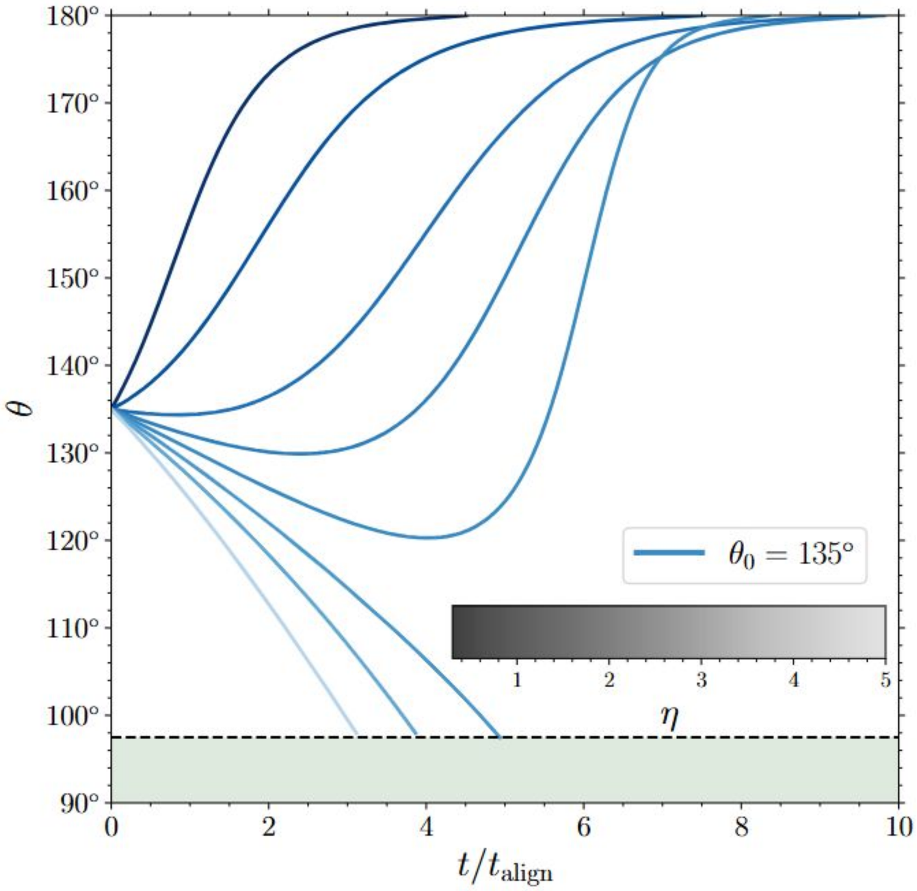
2. **cos θ < - η**

   disc counteraligns

3. **- η < cos θ < - η/2**

   tries to coalign, reaches turning point → counteraligns

Exactly as predicted in King et al. 2005



Next steps

- Add a companion and study the **binary** spin alignment
- Collaboration with Cambridge cosmology group (Bourne et al 2024): can we add non-linear warps in hydrodynamical simulations of the circumbinary disc?

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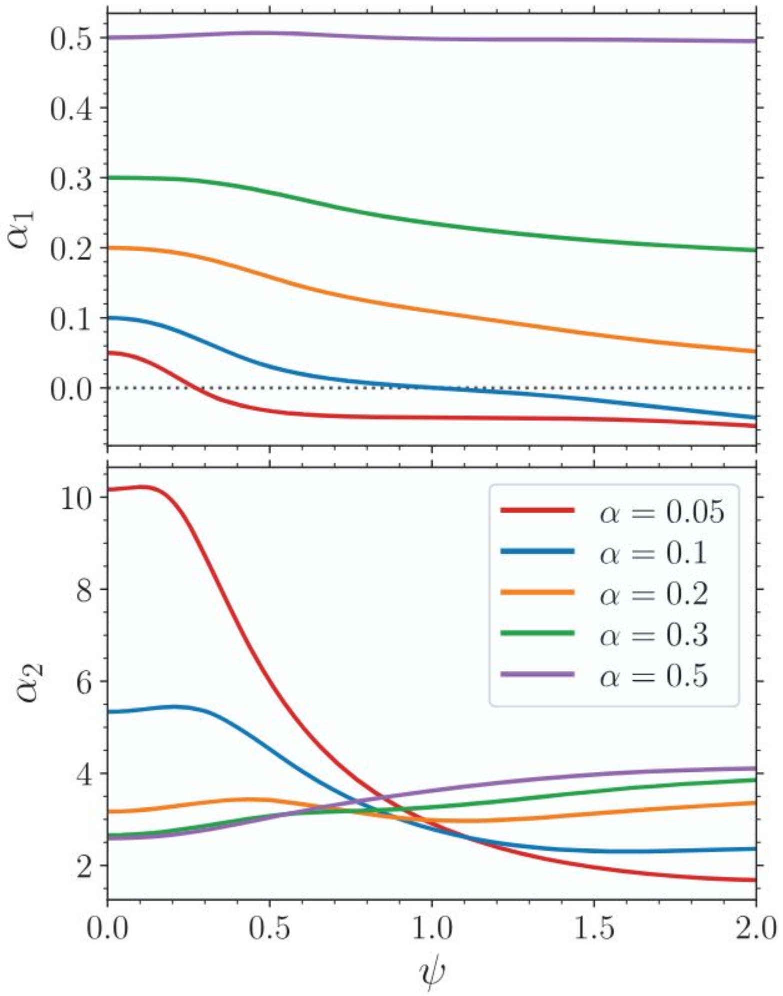
Thank you!

Conclusions

- Relaxed the infinite-angular-momentum assumption
- Developed a numerical scheme which finds self-consistently both the disc size and the non-linear viscosities
- Found the critical obliquity regime
- Evolved the system in time and confirmed the possibility of counteralignment cases

$$\nu_1 = \nu_0 \left(\frac{R}{R_0} \right)^{\beta} \alpha_1(\alpha, \psi),$$

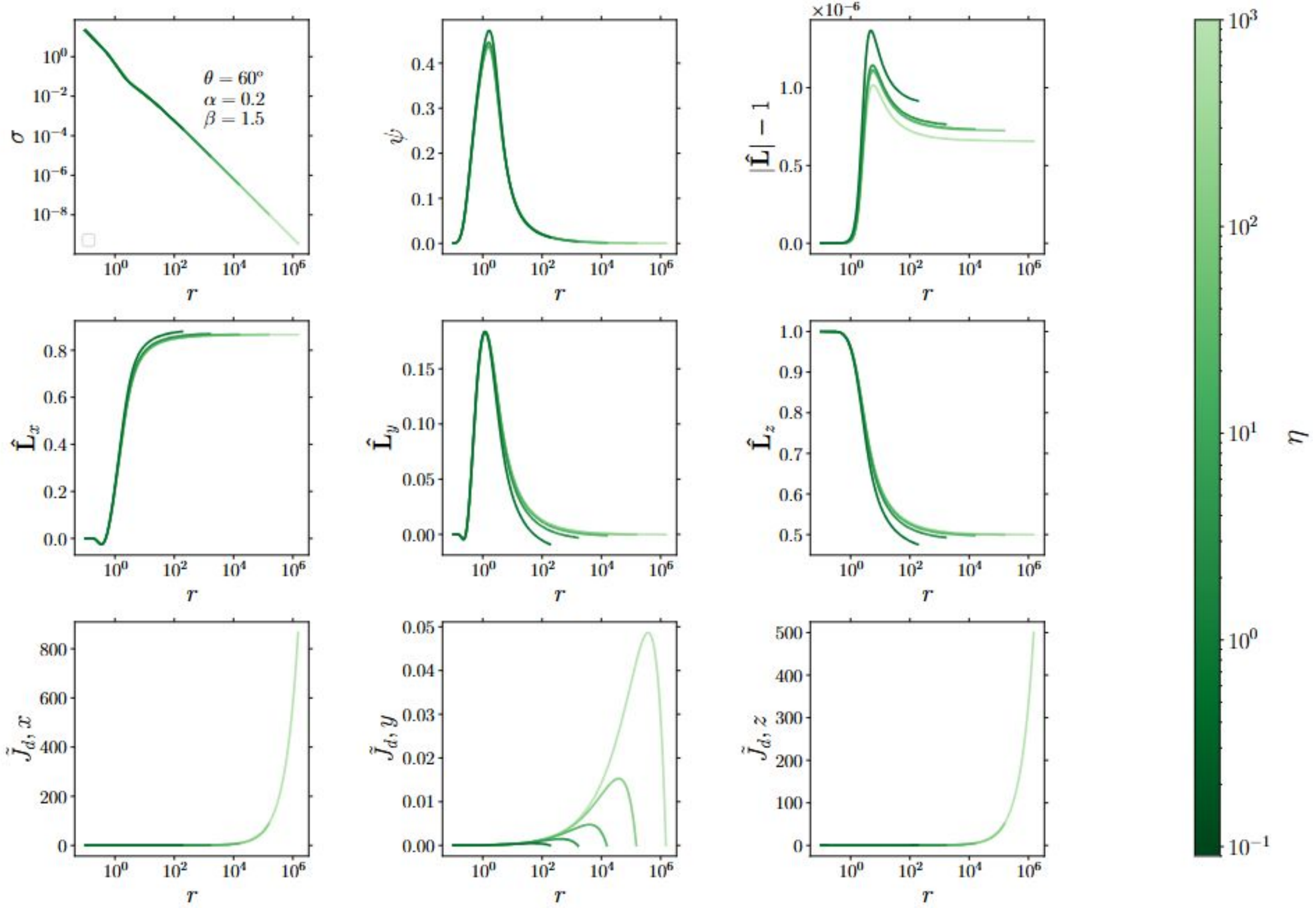
$$\nu_2 = \nu_0 \left(\frac{R}{R_0} \right)^{\beta} \alpha_2(\alpha, \psi),$$



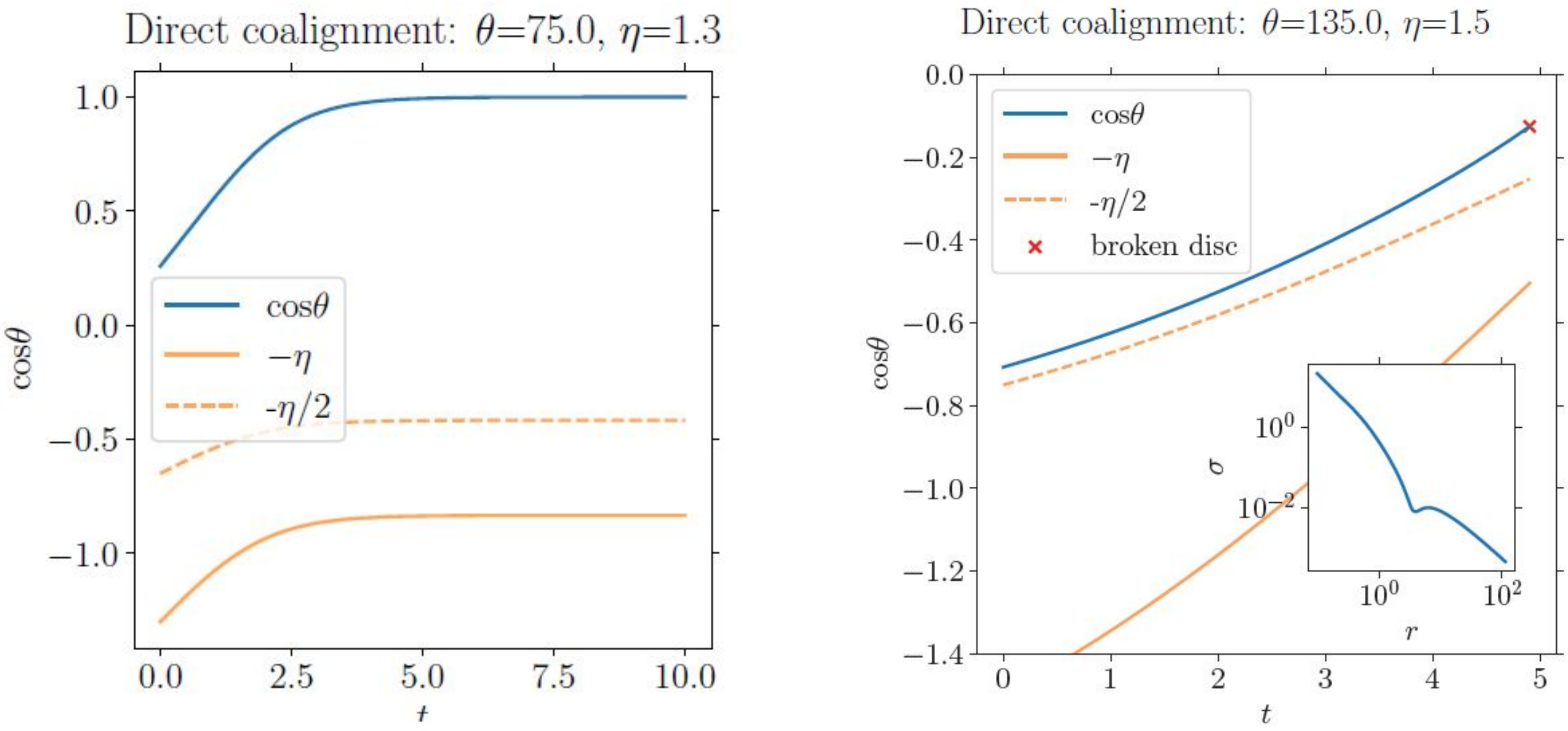
$$\frac{\partial \sigma}{\partial r} = -\left(\beta + \frac{1}{2}\right)\frac{\sigma}{r} - \frac{\zeta \sigma \psi^2}{3r}\frac{\tilde{\alpha}_2(\alpha,\psi)}{\tilde{\alpha}_1(\alpha,\psi)}$$
$$+ \frac{r^{-\beta-1}}{3\tilde{\alpha}_1(\alpha,\psi)} - \frac{\sigma}{\tilde{\alpha}_1(\alpha,\psi)}\frac{\partial \tilde{\alpha}_1(\alpha,\psi)}{\partial r}\,, \qquad (67)$$

$$\frac{\partial^2 \hat{\mathbf{L}}}{\partial r^2} = \frac{\partial \hat{\mathbf{L}}}{\partial r}\Bigg[-\frac{2r^{-\beta-1}}{\zeta\,\tilde{\alpha}_2(\alpha,\psi)\sigma} + \frac{3}{\zeta r}\frac{\tilde{\alpha}_1(\alpha,\psi)}{\tilde{\alpha}_2(\alpha,\psi)} - \left(\beta + \frac{3}{2}\right)\frac{1}{r}$$
$$- \frac{1}{\sigma}\frac{\partial \sigma}{\partial r} - \frac{1}{\tilde{\alpha}_2(\alpha,\psi)}\frac{\partial \tilde{\alpha}_2(\alpha,\psi)}{\partial r}\Bigg] - \frac{\psi^2}{r^2}\hat{\mathbf{L}}$$
$$- \left(\frac{R_{\mathrm{LT}}}{R_0}\right)\frac{r^{-\beta-3}}{\tilde{\alpha}_2(\alpha,\psi)}\left(\hat{\mathbf{J}}_h \times \hat{\mathbf{L}}\right)$$
$$- \left(\frac{R_{\mathrm{tid}}}{R_0}\right)^{-7/2}\frac{r^{-\beta+3/2}}{\tilde{\alpha}_2(\alpha,\psi)}\left(\hat{\mathbf{L}}\cdot\hat{\mathbf{L}}_\star\right)\left(\hat{\mathbf{L}}\times\hat{\mathbf{L}}_\star\right). \qquad (68)$$

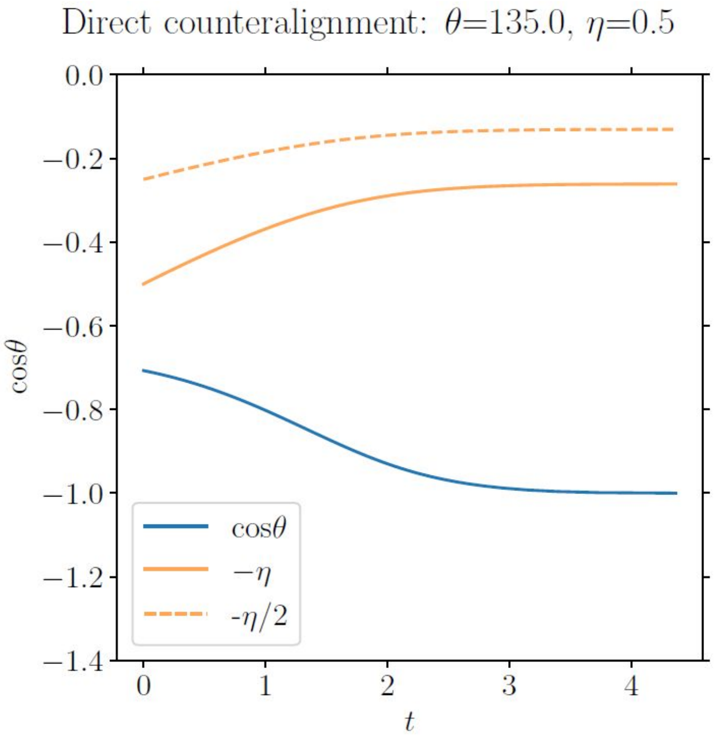
$$\frac{\partial \tilde{\mathbf{J}}_d}{\partial r} = \left(\frac{R_{\mathrm{c}}}{R_{\mathrm{LT}}}\right)^{-5/2}\sigma(r) r^{3/2}\hat{\mathbf{L}}(r)\,. \qquad (69)$$

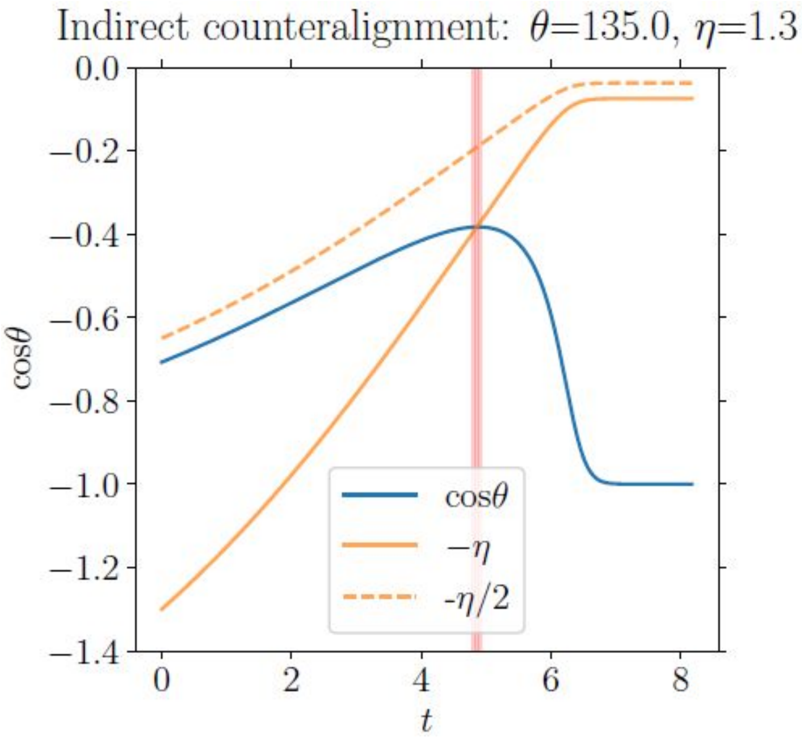


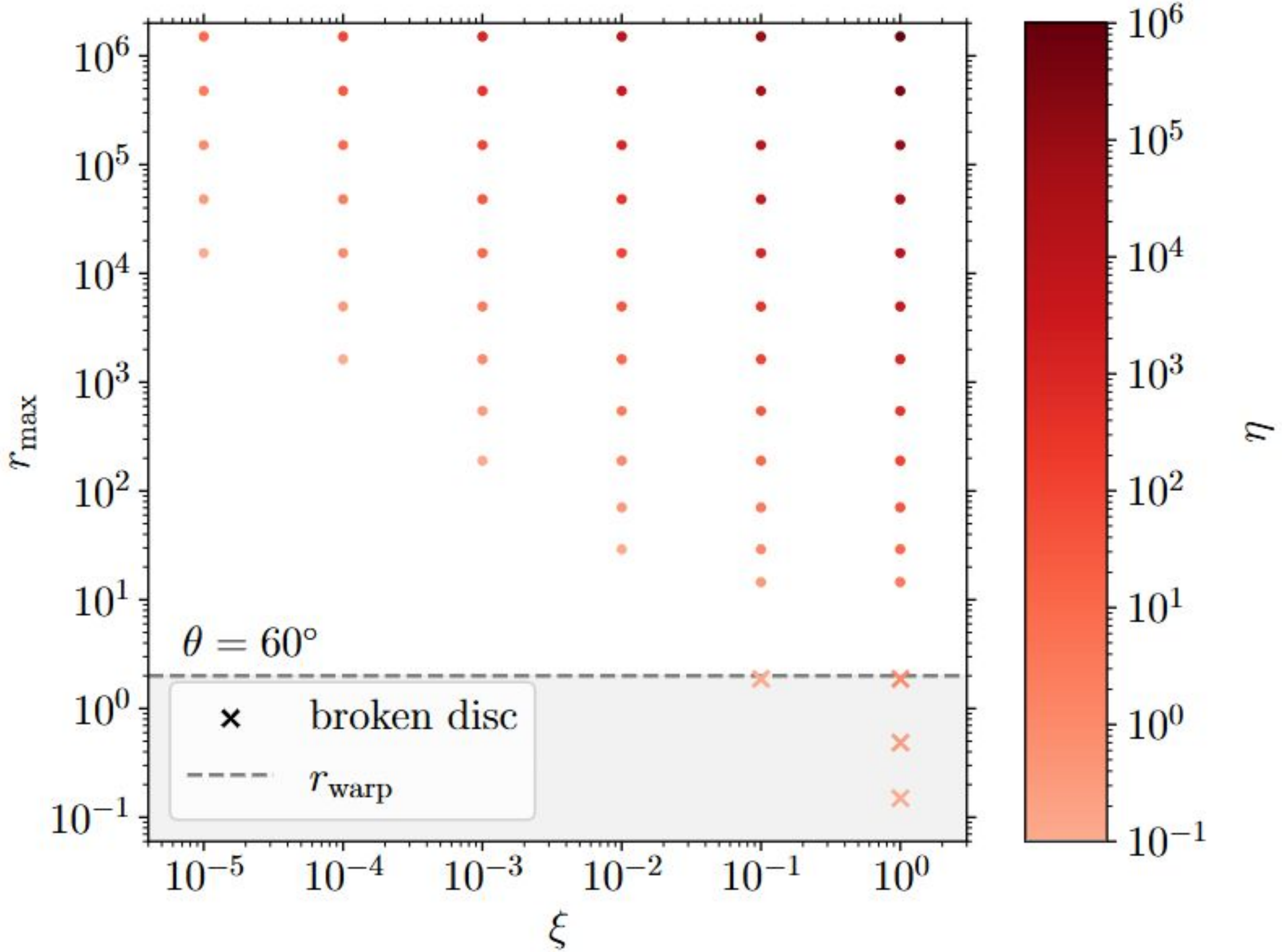
$\theta = 60^o$
$\alpha = 0.2$
$\beta = 1.5$
σ
ψ
$|\hat{\mathbf{L}}| - 1$
$\times 10^{-6}$
$\hat{\mathbf{L}}_x$
$\hat{\mathbf{L}}_y$
$\hat{\mathbf{L}}_z$
$\tilde{J}_d, x$
$\tilde{J}_d, y$
$\tilde{J}_d, z$
r
η



Direct coalignment: θ=75.0, η=1.3
$\cos\theta$
t
$\cos\theta$
$-\eta$
$-\eta/2$
Direct coalignment: θ=135.0, η=1.5
$\cos\theta$
t
$\cos\theta$
$-\eta$
$-\eta/2$
broken disc
σ
r


Direct counteralignment: θ=135.0, η=0.5
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
cosθ
0
1
2
3
4
t
cosθ
$-\eta$
-η/2


Indirect counteralignment: θ=135.0, η=1.3
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
cosθ
0
2
4
6
8
t
cosθ
$-\eta$
-η/2


$\theta = 60^\circ$
broken disc
$r_{\rm warp}$
$r_{\rm max}$
ξ
η