

Testing the current understanding of spin-orbit misalignment in black-hole binaries: Cluster vs Field scenario



Sofia Dossena

University of Milano-Bicocca

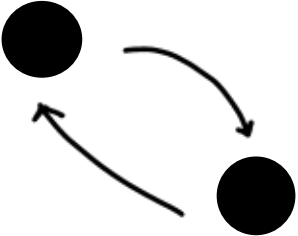
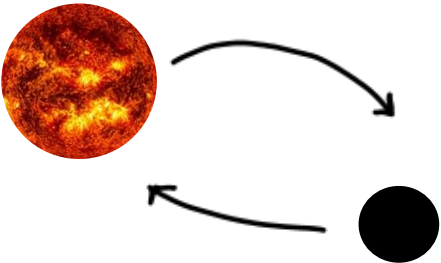
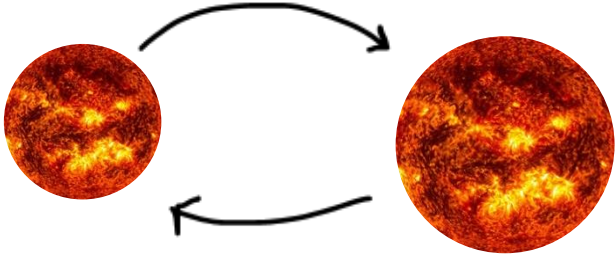
June 17th, 2026

1st BiCoQ Conference: From gravity to
particles

Where do black-hole binaries come from?

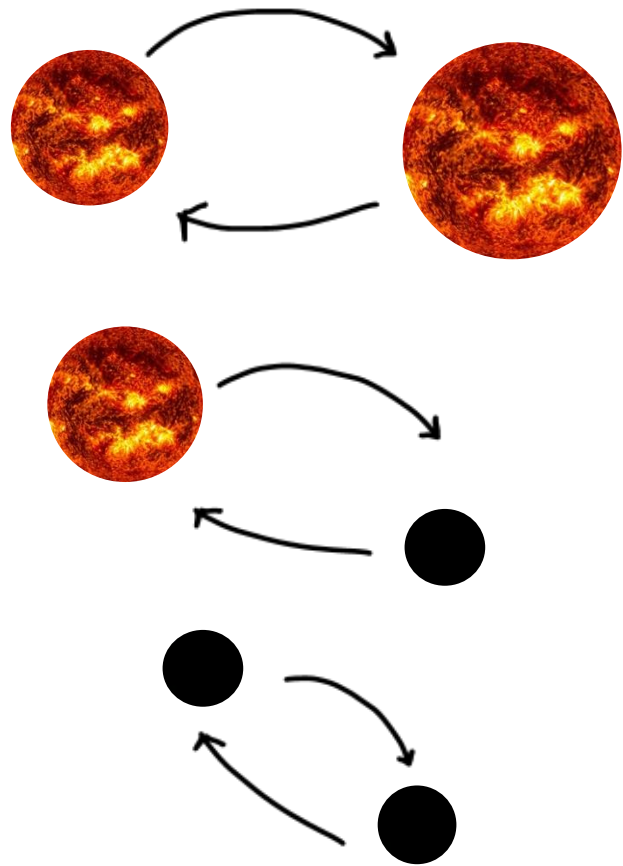
Where do black-hole binaries come from?

Field (isolated)

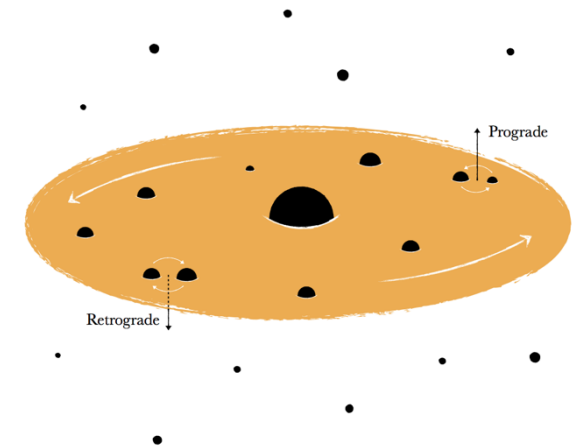
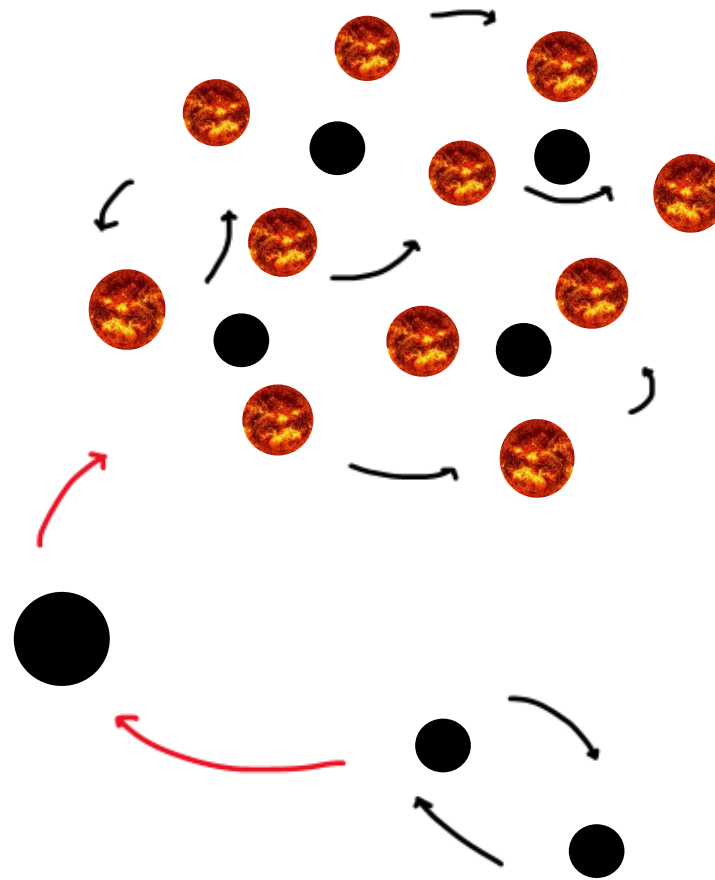


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Field (isolated)



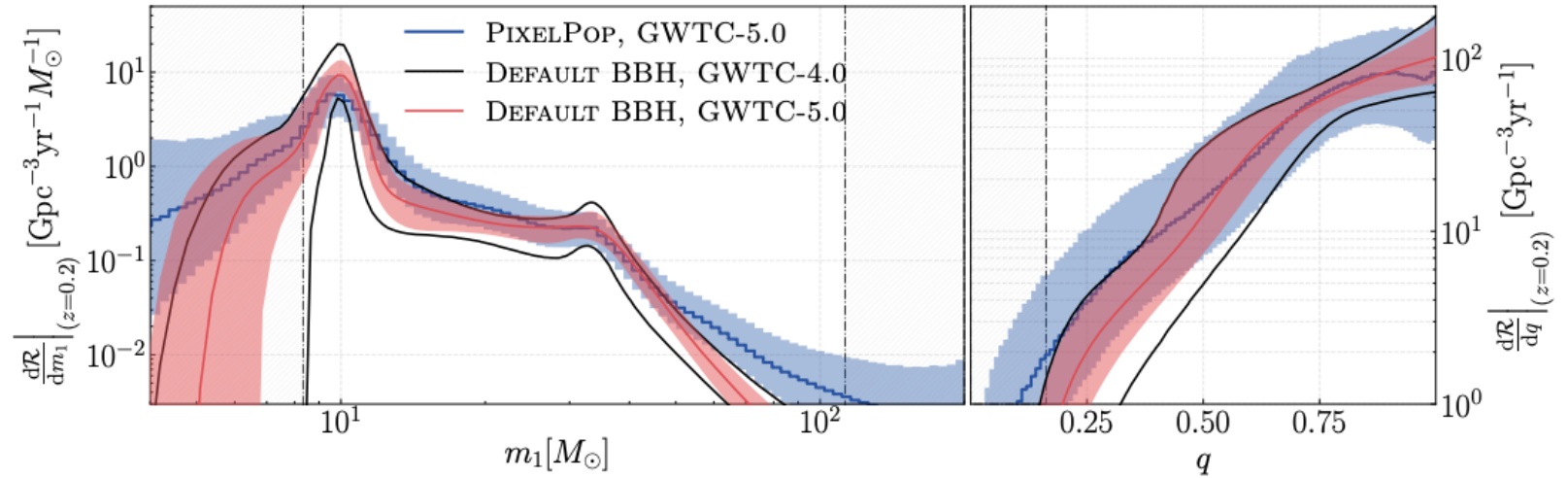
Clusters (dynamical assembling)
+
Hierarchical merging



Why are spin directions interesting?

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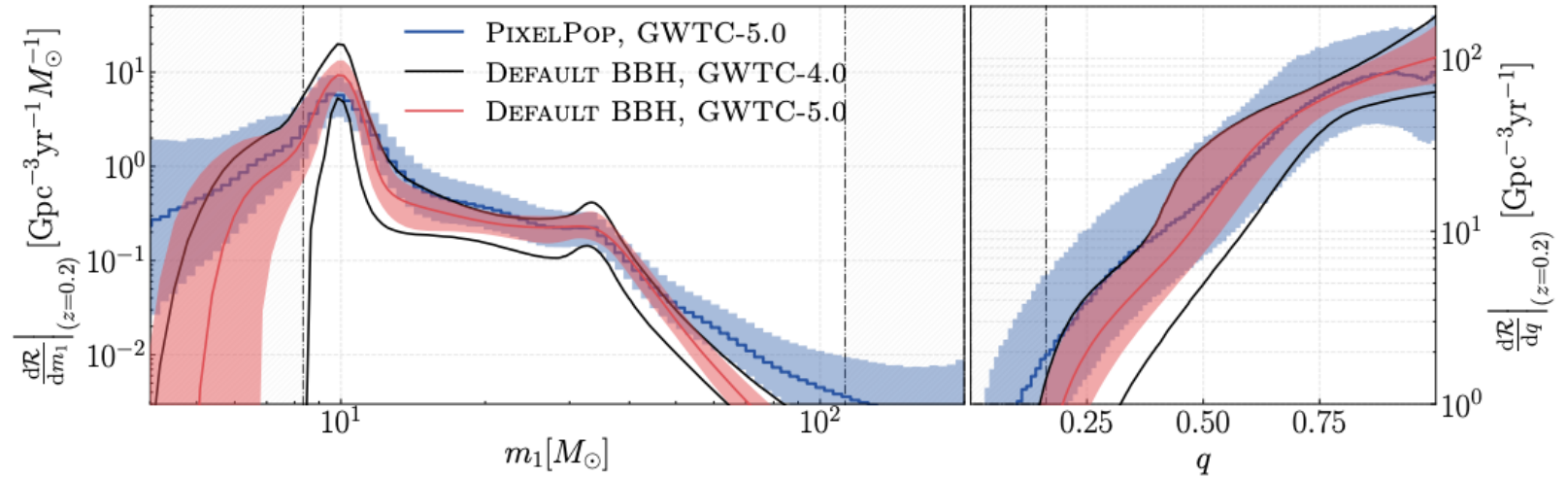
- **Masses:** they are largely degenerate with formation channels



GWTC-5.0, LIGO-Virgo-KAGRA collaboration, 2026

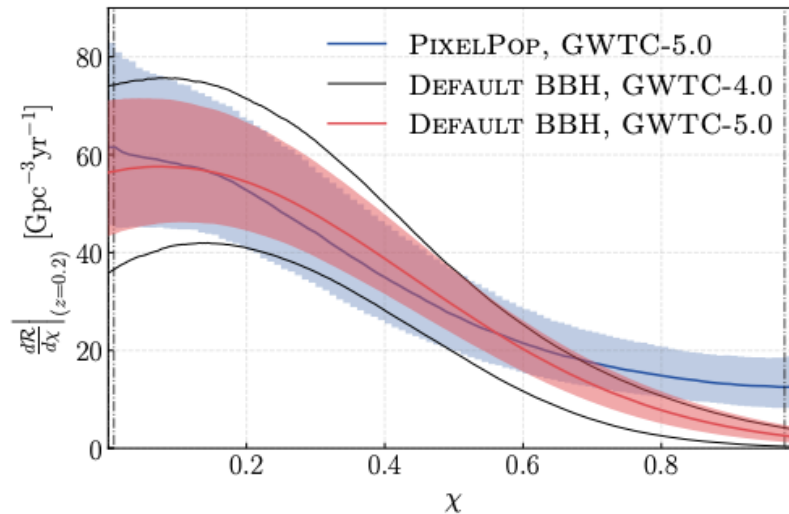
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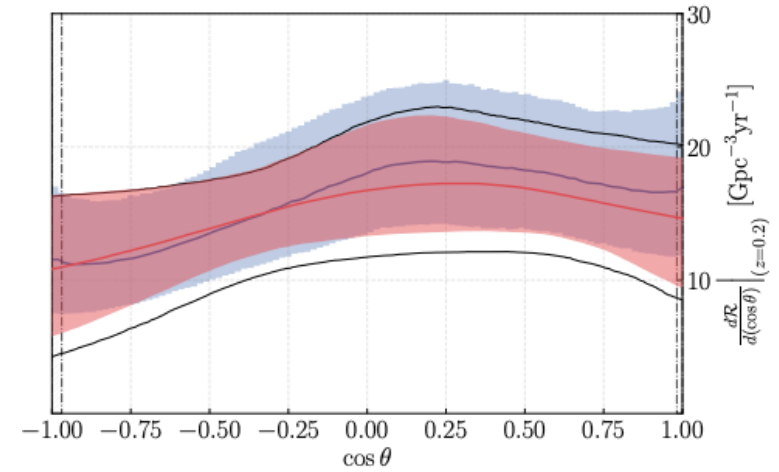
GWTC-5.0, LIGO-Virgo-KAGRA collaboration, 2026

- **Spins magnitude:**
 - rotation of the progenitor
 - dynamics of collapse
 - accretion mechanisms



Why are spin directions interesting?

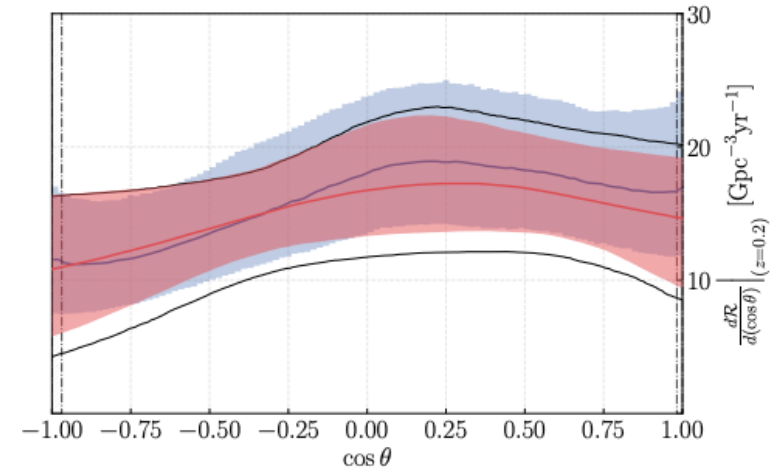
- **Spin directions:**
They are measured with higher uncertainty



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CLUSTER

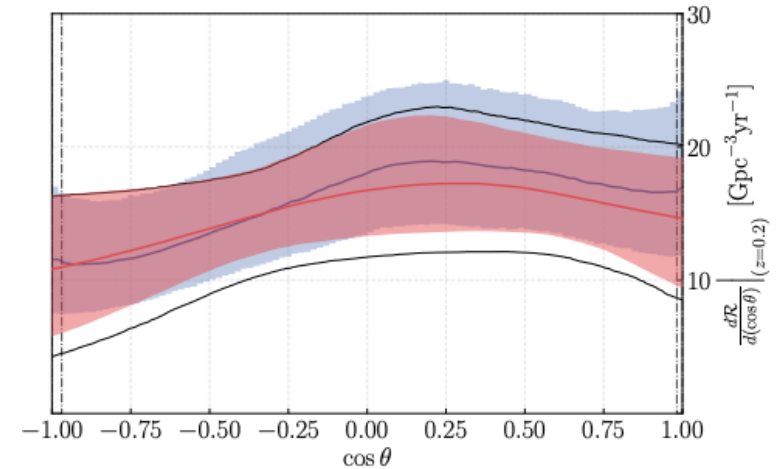
- Multiple body interactions
- Random pairing

FIELD

- Originating from the same proto-stellar disc
- Tidal forces
- Accretion mechanism

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GWTC-5.0, LIGO-Virgo-KAGRA collaboration, 2026

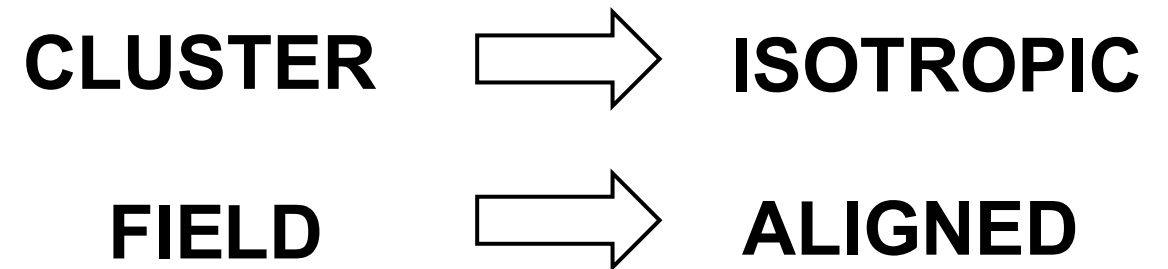
CLUSTER

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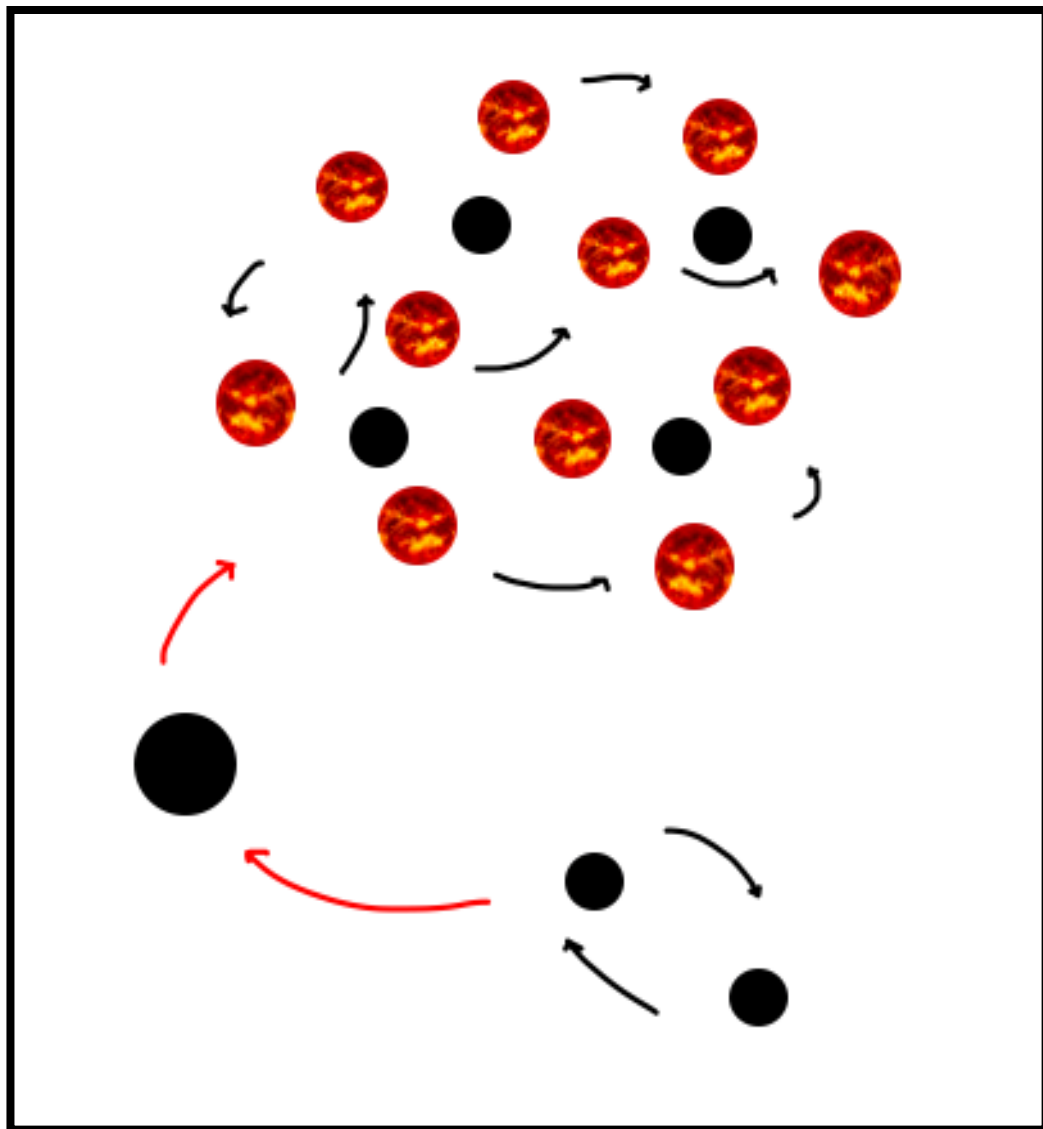
FIELD

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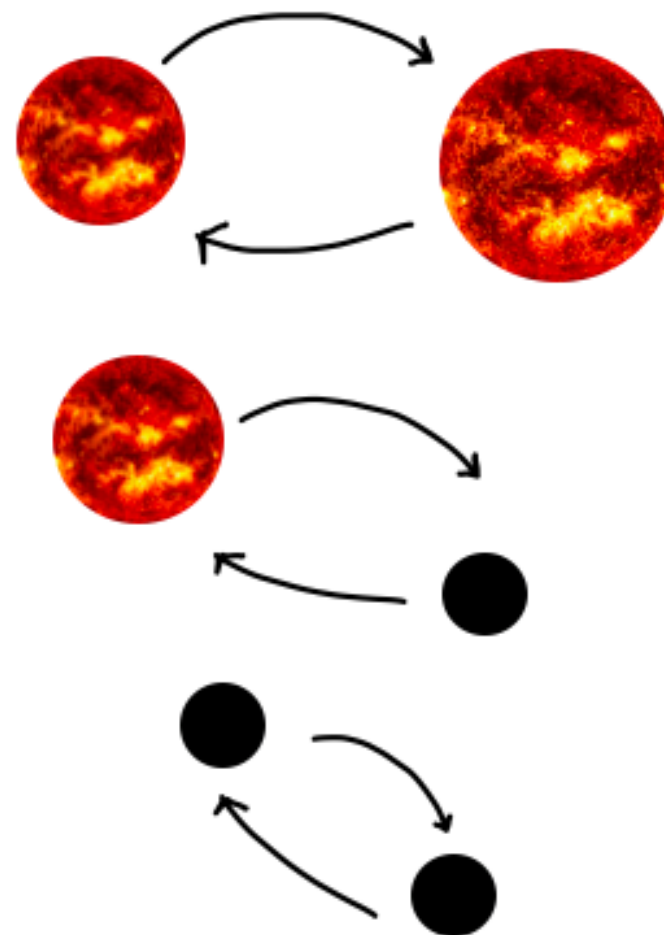
ZERO-ORDER IDEA



CLUSTER



FIELD



Vs

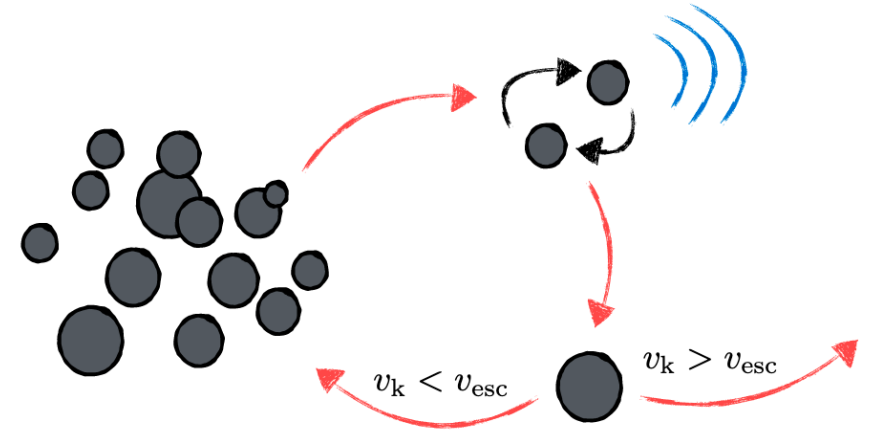
Modification to QCLUSTER: LCLUSTER

Gerosa, Mould, 2023

Modification to QLUSTER: LCLUSTER

Gerosa, Mould, 2023

Collection of BH seeds with masses distributed according to $\rho(m) \propto m^\gamma$ and dimensionless spins distributed uniformly.

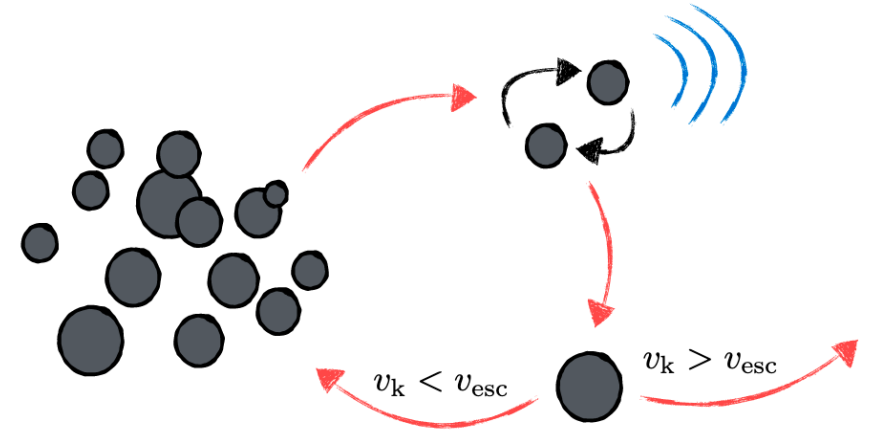


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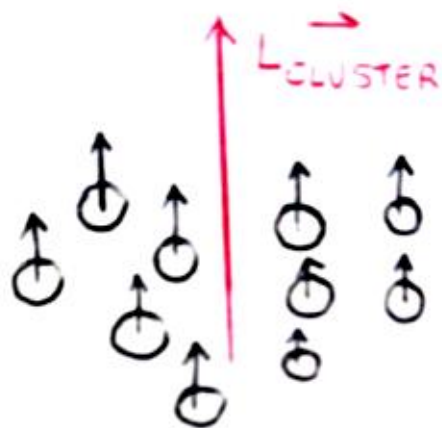
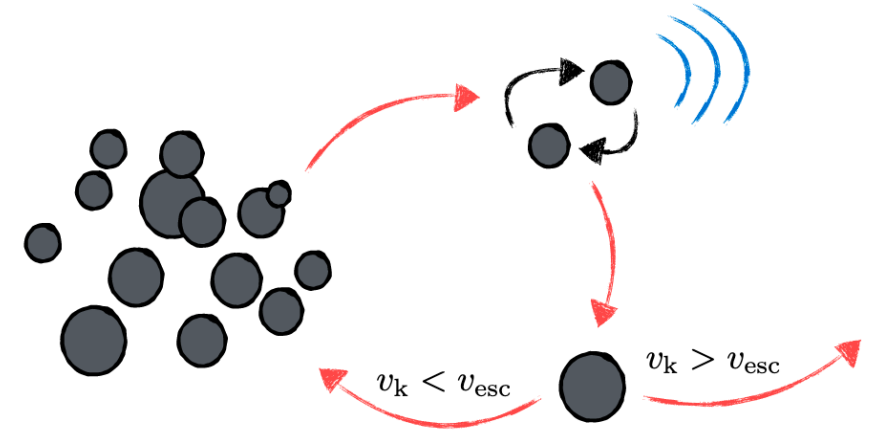


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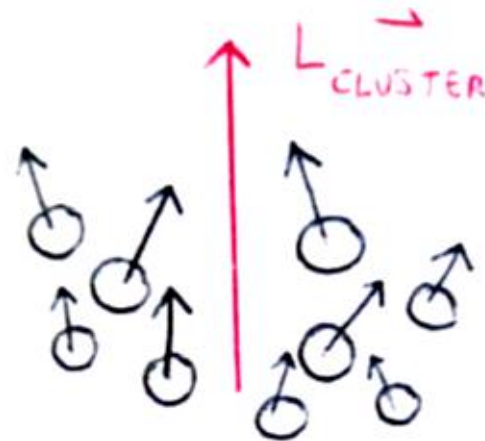
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Collection of BH seeds with masses distributed according to $p(m) \propto m^\gamma$ and dimensionless spins distributed uniformly.

- **Escape speed**
- θ_{\max} characterises the degree of alignment of the system



$$\theta_{\max} = 0$$



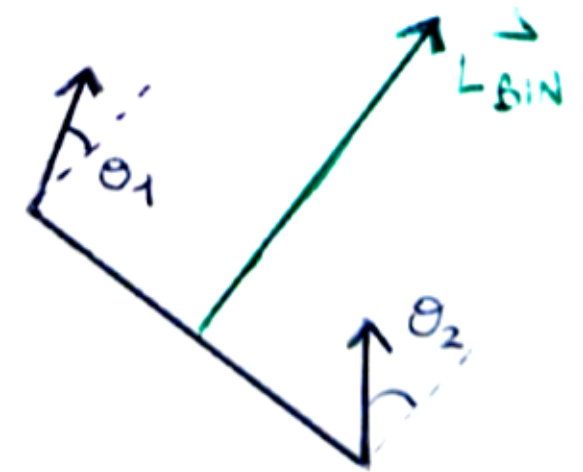
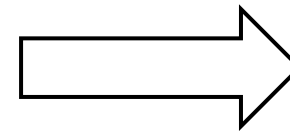
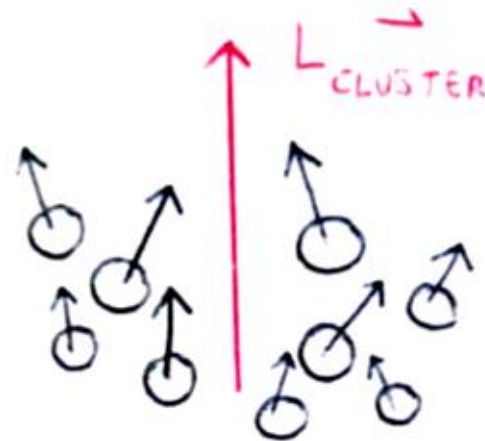
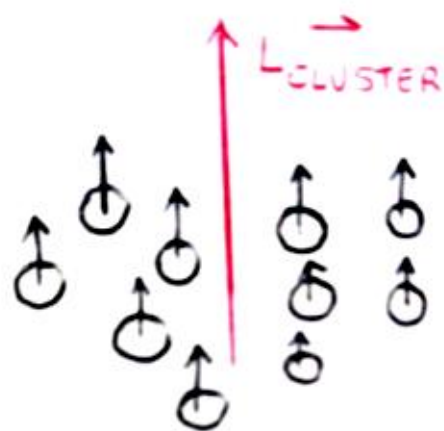
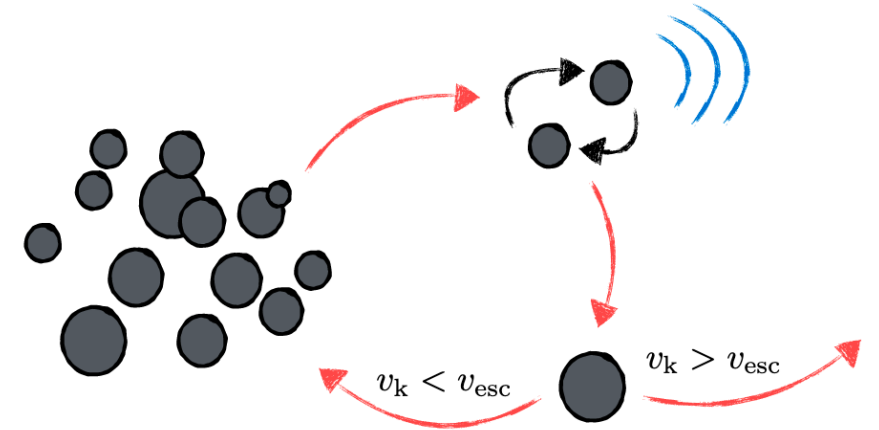
$$\theta_{\max} \neq 0$$

Modification to QCLUSTER: LCLUSTER

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Collection of BH seeds with masses distributed according to $\rho(m) \propto m^\gamma$ and dimensionless spins distributed uniformly.

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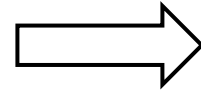


The orbital angular momentum of the binary is isotropically distributed

Joint distribution of the polar angles

Joint distribution of the polar angles

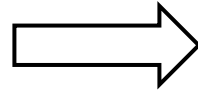
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Projection relative to the orbital angular momentum of the binary system

Joint distribution of the polar angles

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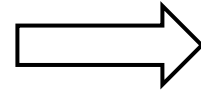
Projection relative to the orbital angular momentum of the binary system

$$\cos(\Theta_1) = \cos(\phi_1 - \beta) \sin(\alpha) \sin(\theta_1) + \cos(\alpha) \cos(\theta_1)$$

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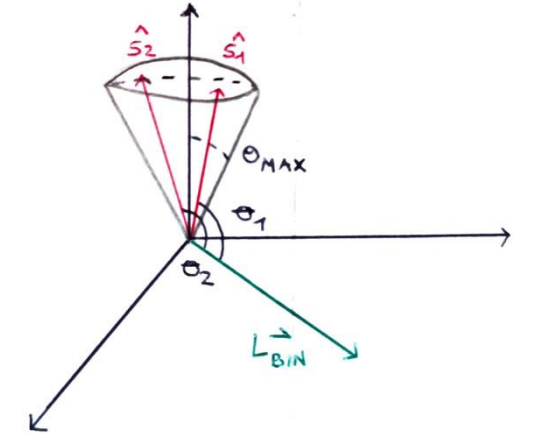
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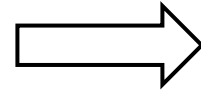
$$P(\cos \Theta_1, \cos \Theta_2) = \frac{1}{4} \sum_{l=0}^{\infty} (2l + 1) \mu_l^2(\theta_{\max}) P_l(\cos \Theta_1) P_l(\cos \Theta_2)$$

$$\mu_l(\theta_{\max}) = \frac{1}{1 - \cos \theta_{\max}} \int_{\cos \theta_{\max}}^1 P_l(t) dt$$



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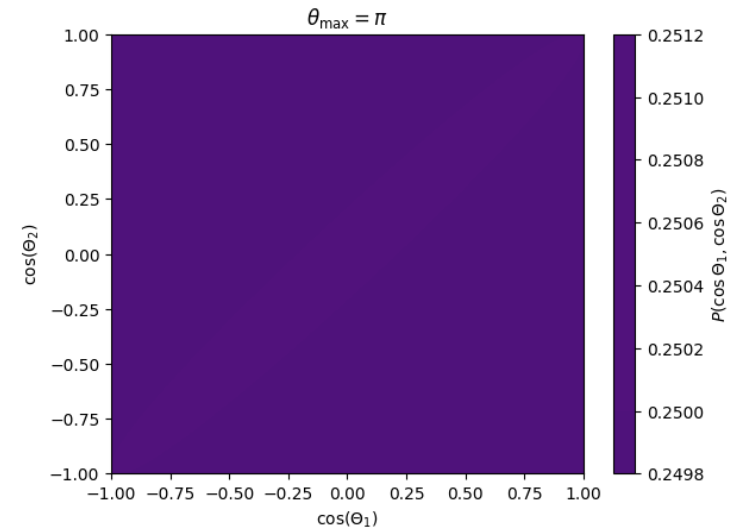
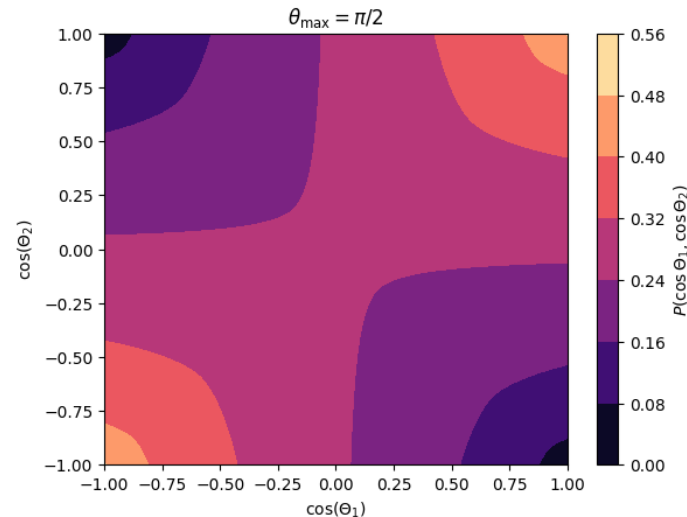
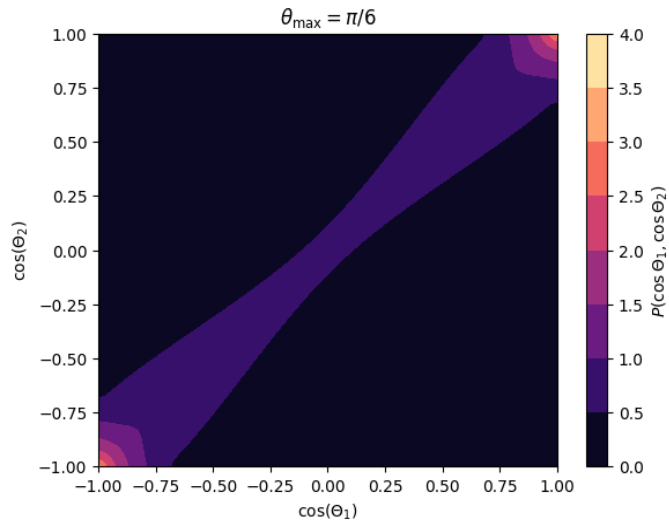
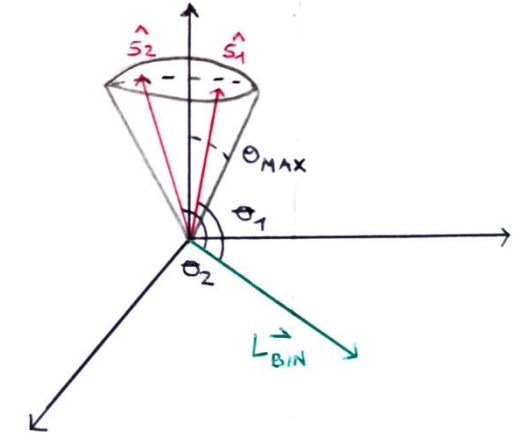
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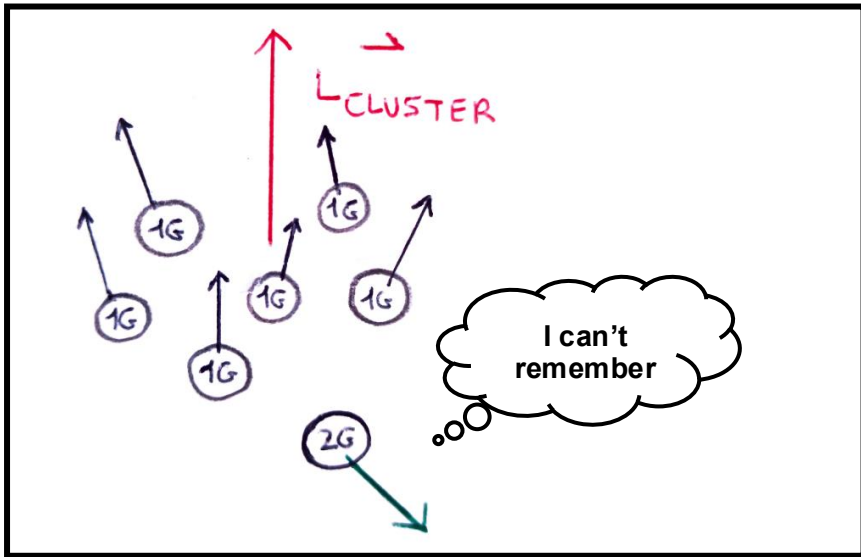
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HIERARCHICAL MERGING: What about spin memory?

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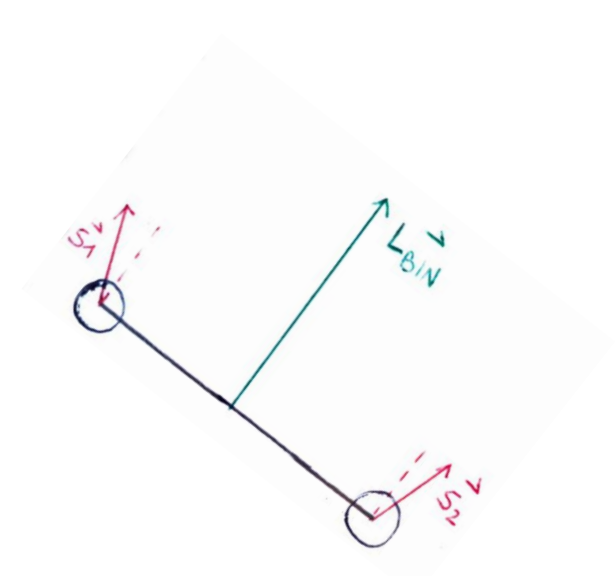
$$\vec{J} = \vec{L}_B + \vec{S}_1 + \vec{S}_2$$

The direction of the total angular momentum is approximately conserved:

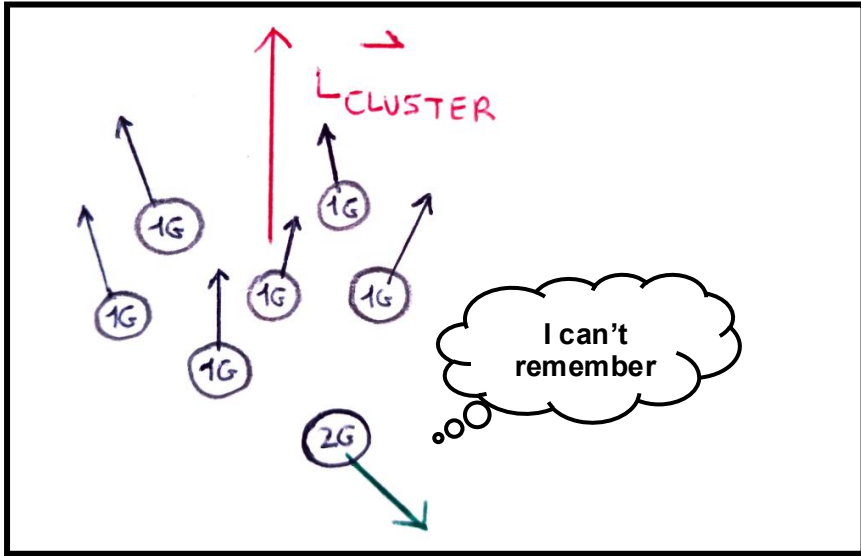
$$\hat{J}_{\text{formation}} = \hat{J}_{\text{plunge}} \quad \text{Zhao, Kesden, Gerosa, 2017}$$

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At formation, the total angular momentum is dominated by the orbital angular momentum -> isotropic



HIERARCHICAL MERGING: What about spin memory?



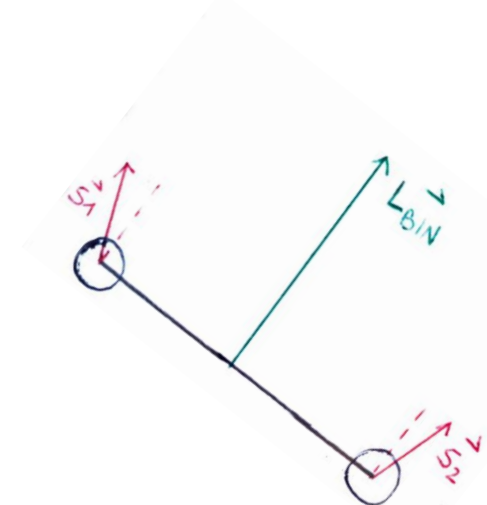
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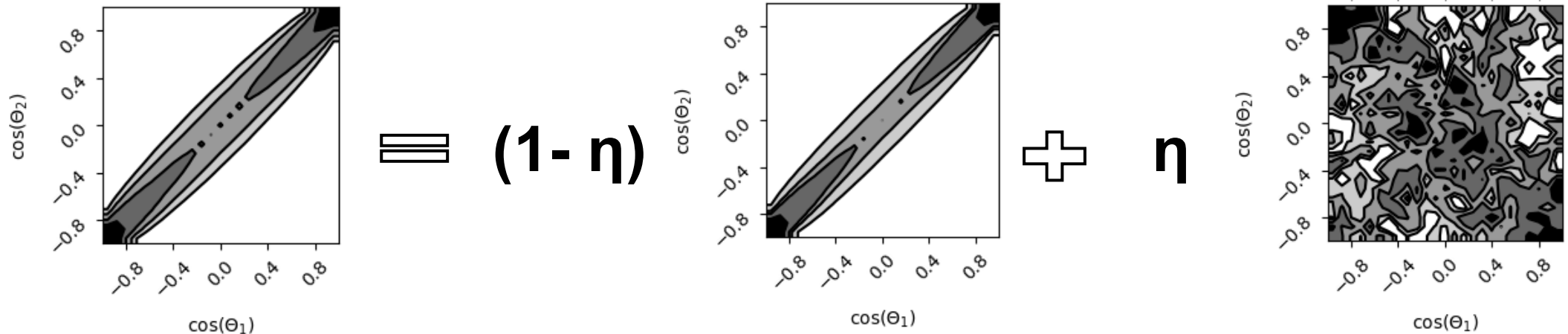
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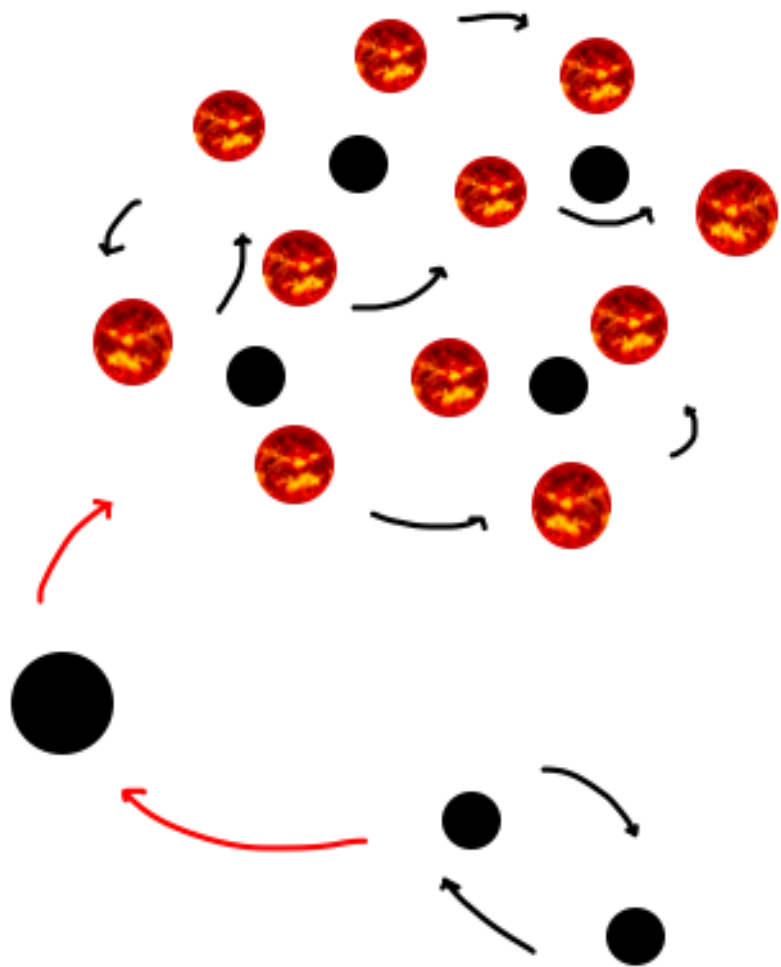
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η = the fraction of hierarchical mergers in the cluster:

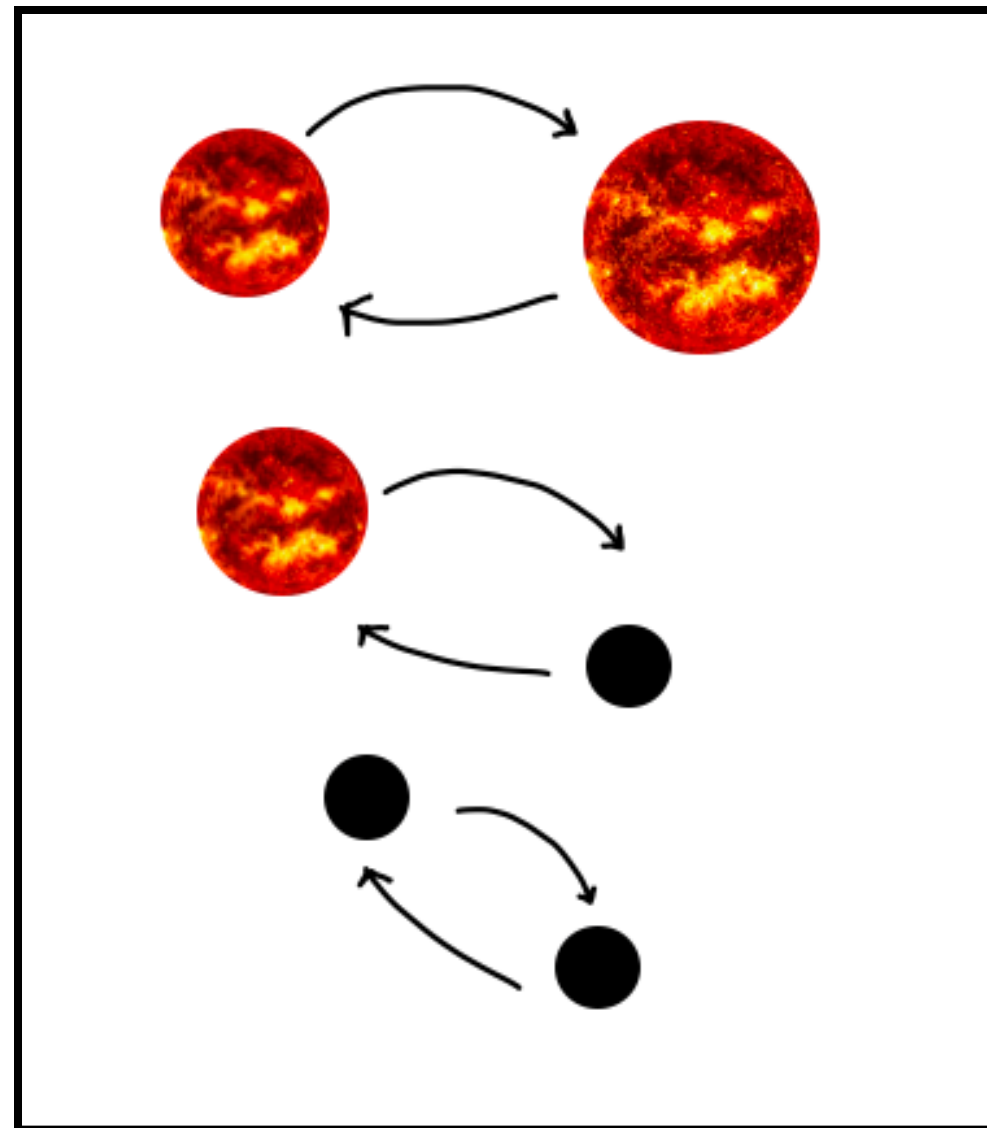


CLUSTER



Vs

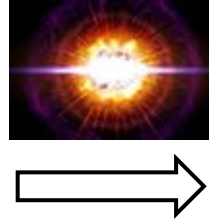
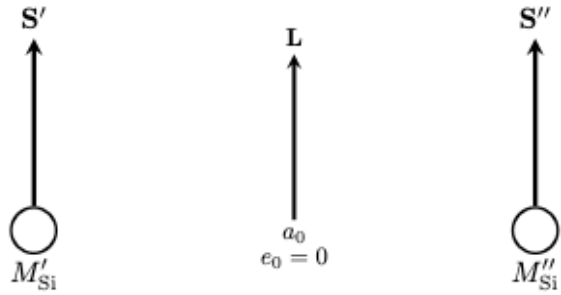
FIELD



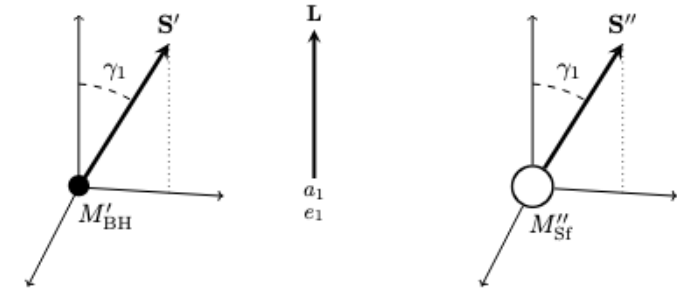
FIELD BINARIES

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Main sequence binary

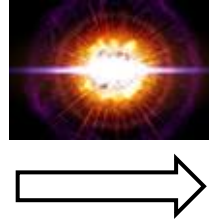
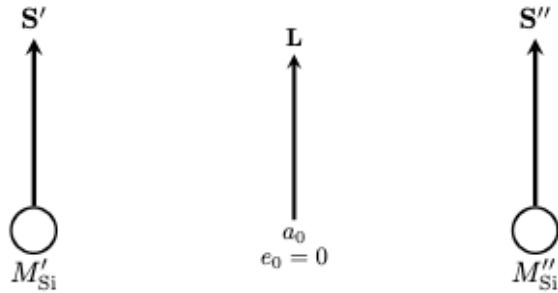


1st Supernova explosion

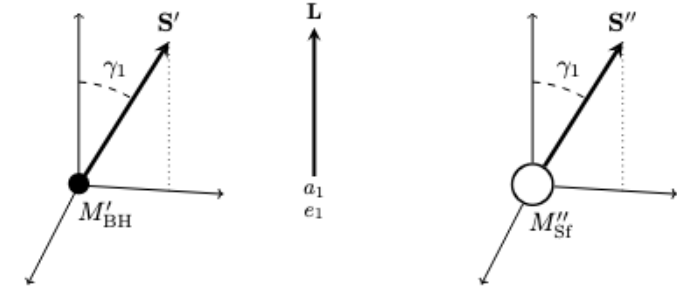


FIELD BINARIES

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Assumptions:

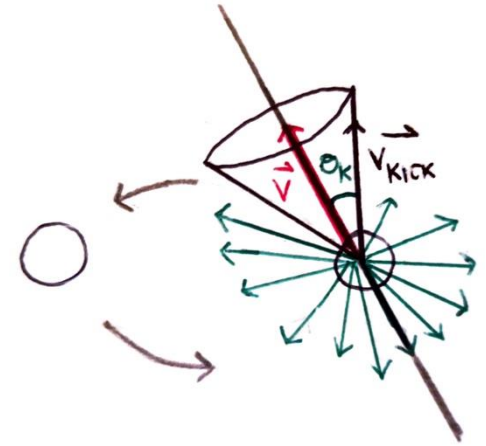
1. Initially aligned stars
2. Initial circular orbit
3. Neglecting the tilt imparted by the second SN

$$u_k = \frac{v_{kick}}{v}$$

$$\cos(\gamma) = \frac{1 + u_k \cos \theta_k}{\sqrt{(1 + u_k \cos \theta_k)^2 + u_k^2 \sin^2 \theta_k \cos^2 \phi_k}}$$

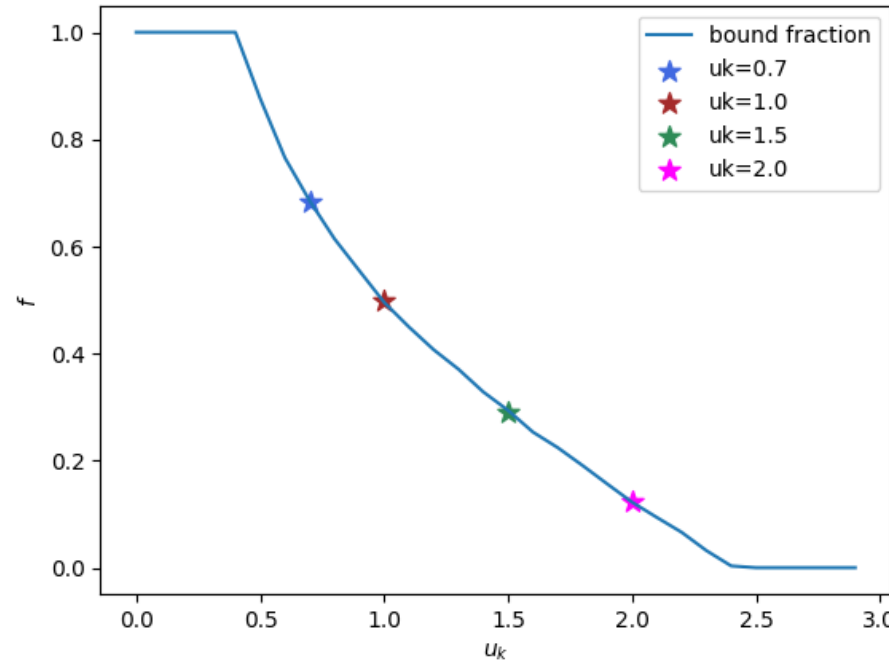
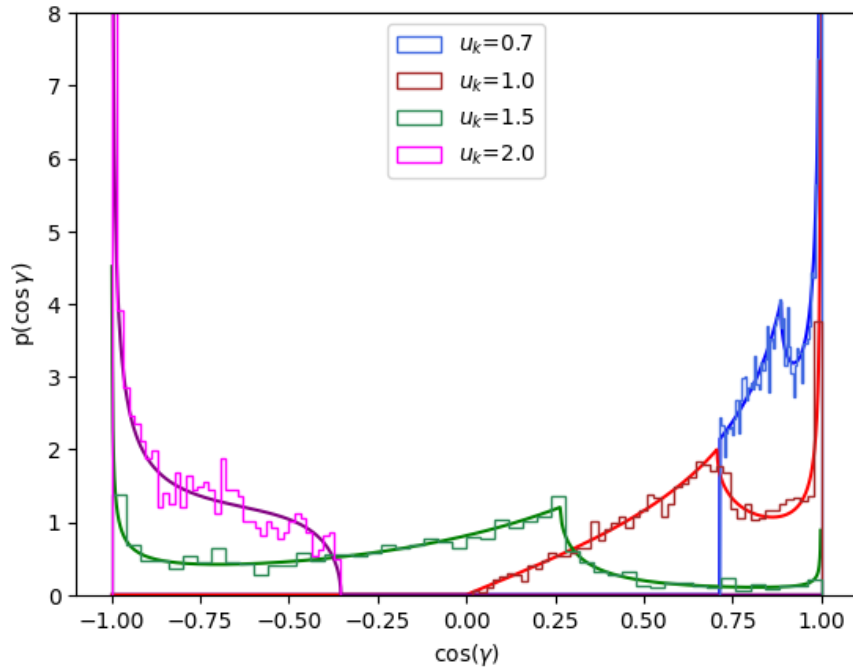
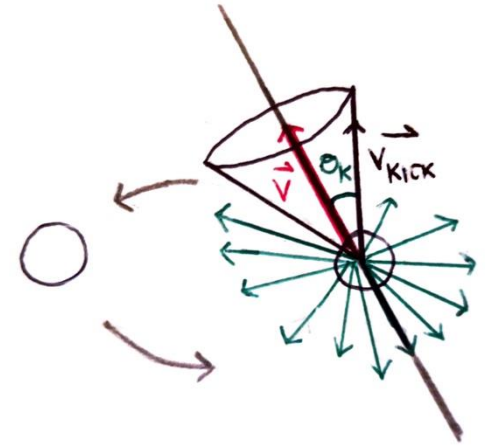
Distribution of the spin-orbit misalignment after 1st SN explosion

$$p(\cos \gamma; u_k) \Big|_{\cos \theta_k < \frac{2\beta - 1 - u_k^2}{2u_k}} = \begin{cases} \frac{4[A(\beta) - A(\alpha)]}{\pi(1 + 2u_k - u_k^2) \cos \gamma \sqrt{1 - \cos^2 \gamma}}, & I = [\alpha, \beta] \neq \emptyset, \\ 0, & I_y = \emptyset, \end{cases}$$



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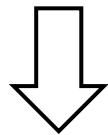


Limits on the tilt:

$$\cos \gamma \in \left[\sqrt{1 - u_k^2}, 1 \right] \quad \text{if } u_k \leq 1$$

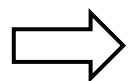
$$\cos \gamma \in \left[-1, 1 \right] \quad \text{if } 1 < u_k \leq \sqrt{3}$$

$$\cos \gamma \in \left[-1, \frac{3 - u_k^2}{2\sqrt{2}} \right] \quad \text{if } \sqrt{3} < u_k < 1 + \sqrt{2}$$



u_k distribution

Low kicks to keep systems bound

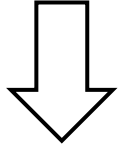


Preferentially aligned spins

Effects of the post-Newtonian evolution

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The detection occurs \sim Myr after formation

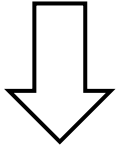


Spin precession changes the initial configuration

The correlation between spins changes

Effects of the post-Newtonian evolution

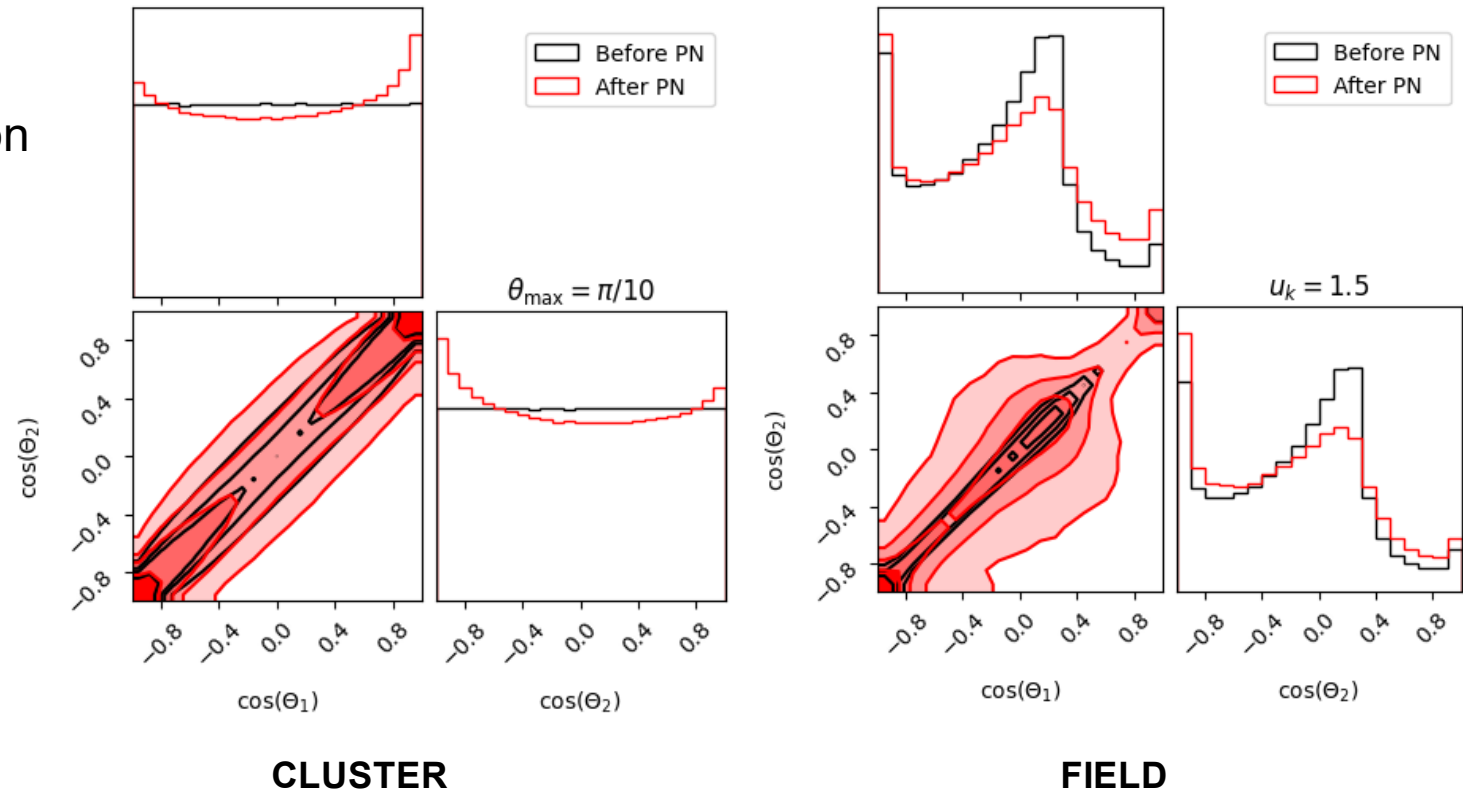
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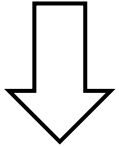
The correlation between spins changes

Post-Newtonian evolution obtained through **PRECESSION**



Effects of the post-Newtonian evolution

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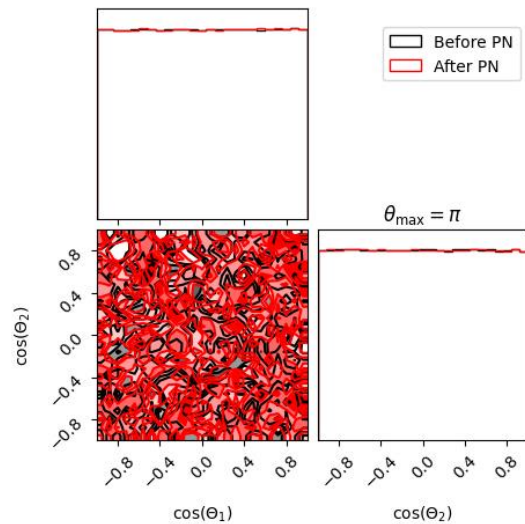


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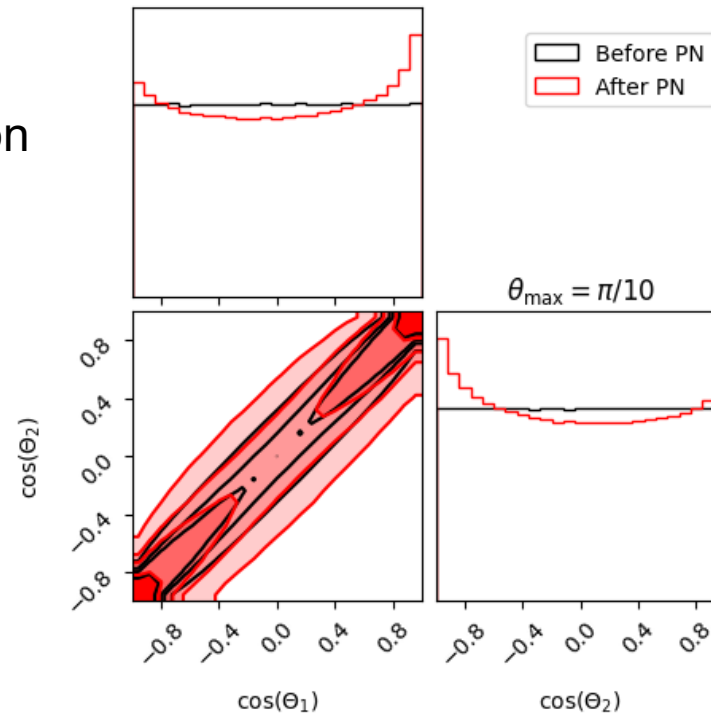
The correlation between spins changes

With good approximation
an isotropic distribution remains isotropic

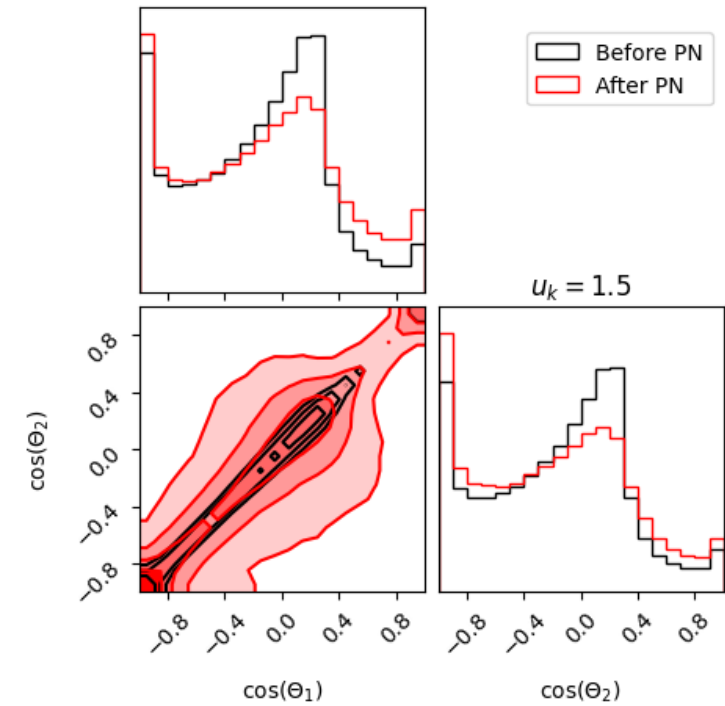
Gerosa et al., 2015



Post-Newtonian evolution obtained through PRECESSION



CLUSTER



FIELD

FUTURE DIRECTIONS:

- How can post-Newtonian evolution be considered?
- Selection effects
- Let's apply this to real data!



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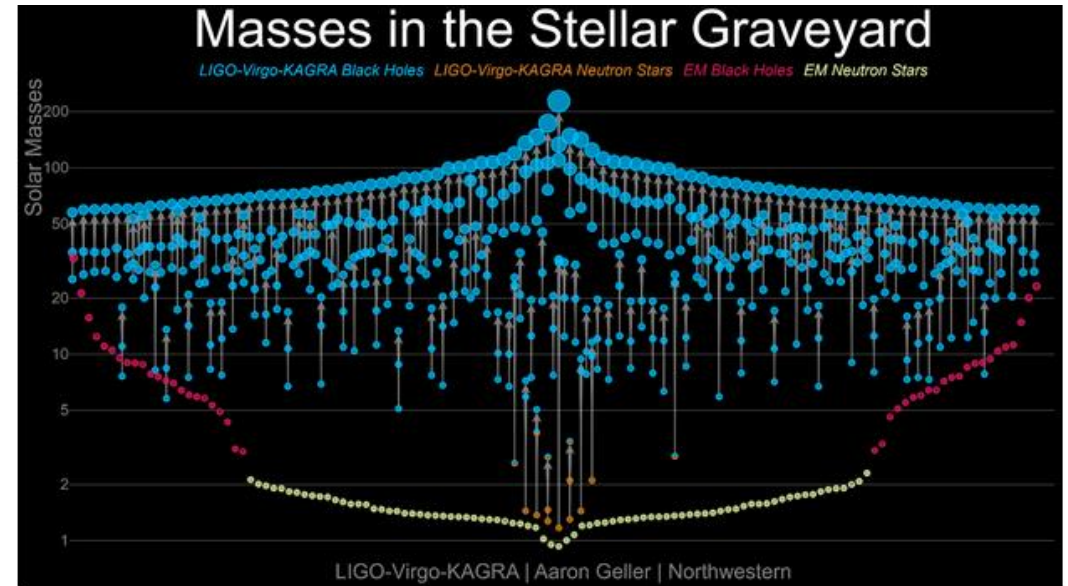
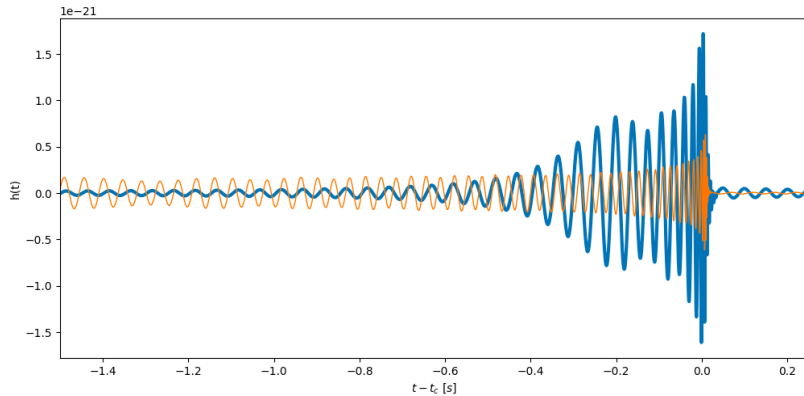


**THANK YOU FOR
YOUR ATTENTION**

BACKUP SLIDES

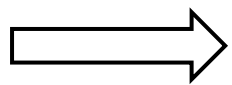
Why are spin directions interesting?

- **Masses:** degenerate with formation channels, higher masses in hierarchical merging → no single signature



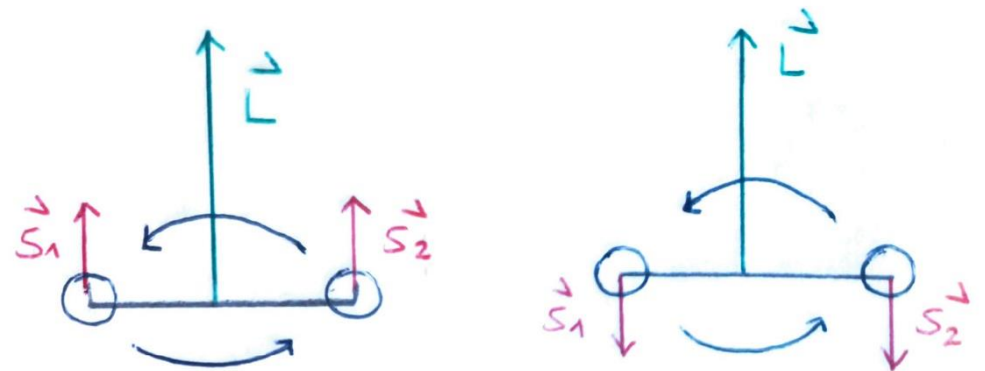
- **Spins magnitude:** rotation of the progenitor, dynamics of collapse, accretion mechanisms

- **Spins directions!**



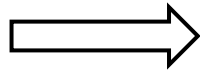
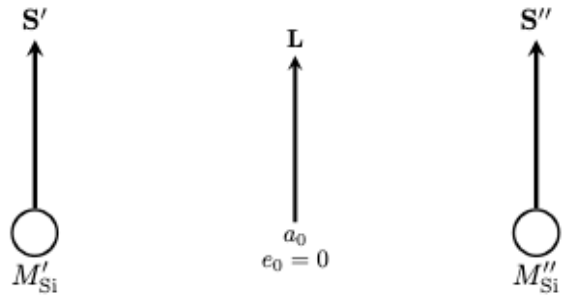
They contain information about the formation environment

Orbital hang up effect:

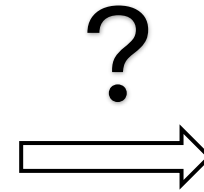
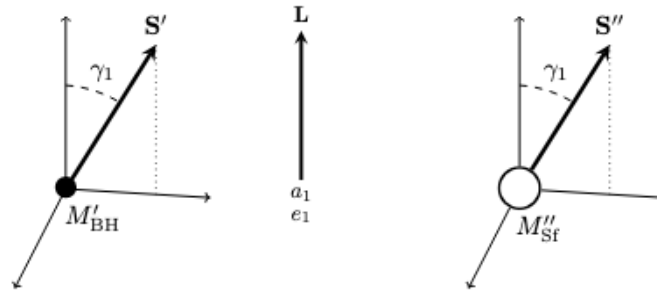


FIELD BINARIES

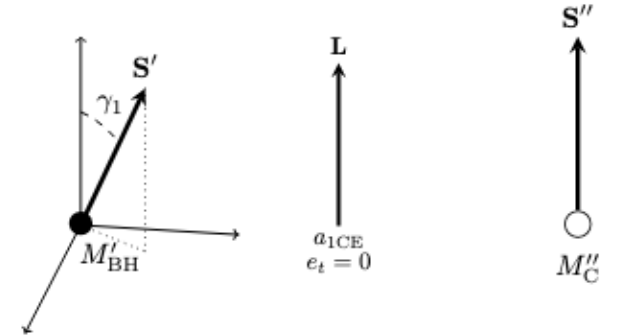
Main sequence binary



1st Supernova explosion



Tides



Assumptions:

1. Initially aligned stars
2. Initial circular orbit
3. Neglecting the tilt imparted by the second SN

$$\cos(\gamma) = \frac{1 + u_k \cos \theta_k}{\sqrt{(1 + u_k \cos \theta_k)^2 + u_k^2 \sin^2 \theta_k \cos^2 \phi_k}}$$

$$\begin{aligned} \vec{l}_i &= \vec{r} \times \vec{v} \\ \vec{l}_f &= \vec{r} \times (\vec{v} + \vec{v}_{\text{kick}}) \\ \cos(\gamma) &= \vec{l}_i \cdot \vec{l}_f \end{aligned}$$

$$u_k = \frac{v_{\text{kick}}}{v}$$

FIELD BINARIES

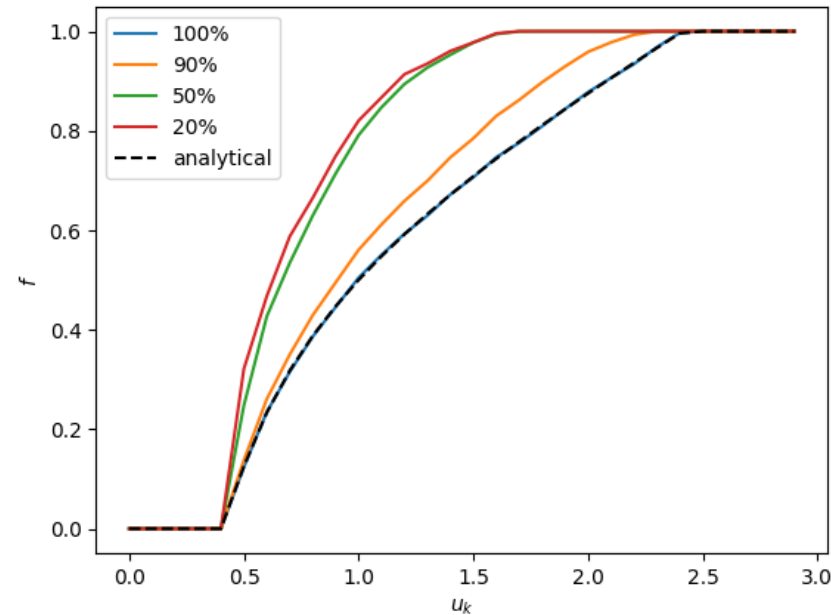
The binary survives the supernova explosion if the orbit remains close:

$$2\beta - 1 - u_k^2 - 2u_k \cos \theta_k > 0 \quad \text{with} \quad \beta = \frac{M_f}{M_i}$$

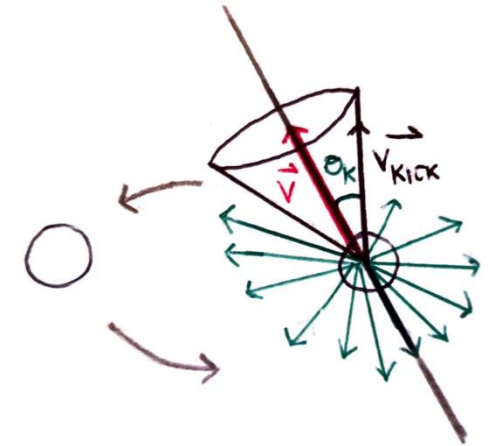
$$\cos \theta_k < \frac{2\beta - 1 - u_k^2}{2u_k} \quad \text{No dependency on } \phi_k$$

Assuming a uniform spatial distribution of the supernova kick, the fraction of unbound orbits is:

$$f = \frac{1 - \frac{2\beta - 1 - u_k^2}{2u_k}}{2}$$



$$e < 1$$

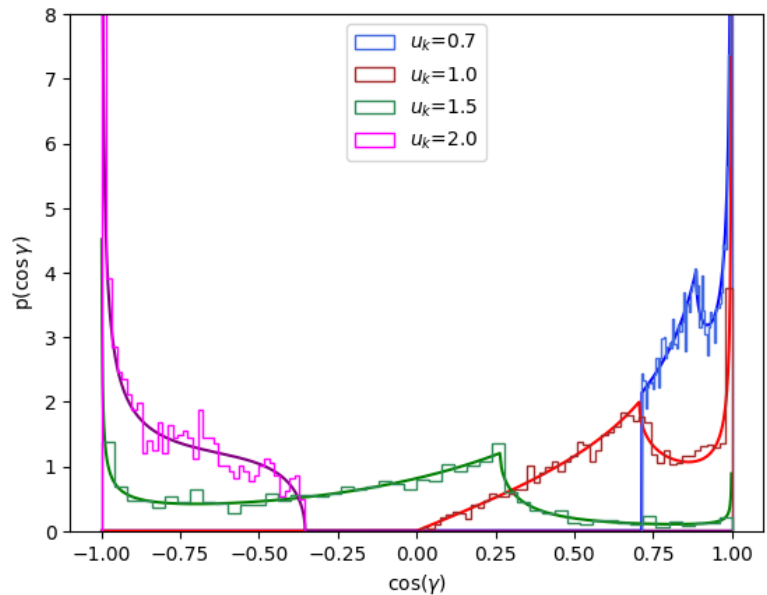


Distribution of the spin-orbit misalignment after SN explosion

$$p(\cos \gamma; u_k) \Big|_{\cos \theta_k < \frac{2\beta - 1 - u_k^2}{2u_k}} = \begin{cases} \frac{4 [A(\beta) - A(\alpha)]}{\pi (1 + 2u_k - u_k^2) \cos \gamma \sqrt{1 - \cos^2 \gamma}}, & I = [\alpha, \beta] \neq \emptyset, \\ 0, & I_y = \emptyset, \end{cases} \quad \alpha = \begin{cases} \max(1 - u_k, 0, \cos^2 \gamma - R), & \text{if } \cos \gamma > 0, \\ \max(1 - u_k, \cos^2 \gamma - R), & \text{if } \cos \gamma \leq 0, \end{cases}$$

$$A(x) = \cos^2 \gamma \cdot \arcsin\left(\frac{x - \cos^2 \gamma}{R}\right) - \sqrt{R^2 - (x - \cos^2 \gamma)^2}. \quad \beta = \begin{cases} \min\left(\frac{3 - u_k^2}{2}, \cos^2 \gamma + R\right), & \text{if } \cos \gamma > 0, \\ \min\left(\frac{3 - u_k^2}{2}, 0, \cos^2 \gamma + R\right), & \text{if } \cos \gamma \leq 0, \end{cases}$$

$$R = |\cos \gamma| \sqrt{u_k^2 + \cos^2 \gamma - 1}.$$

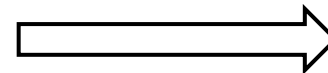


Limits on the tilt:

$$\cos \gamma \in \left[\sqrt{1 - u_k^2}, 1 \right] \quad \text{if } u_k \leq 1$$

$$\cos \gamma \in \left[-1, 1 \right] \quad \text{if } 1 < u_k \leq \sqrt{3}$$

$$\cos \gamma \in \left[-1, \frac{3 - u_k^2}{2\sqrt{2}} \right] \quad \text{if } \sqrt{3} < u_k < 1 + \sqrt{2}$$



u_k distribution

How are u_k 's distributed?

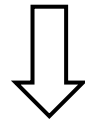
$$u_k = v_{\text{kick}} \sqrt{\frac{a_i}{GM_i}}$$

v_{kick} is circled in orange with a downward arrow.
The square root term is circled in green with a rightward arrow pointing to "Stellar evolution".

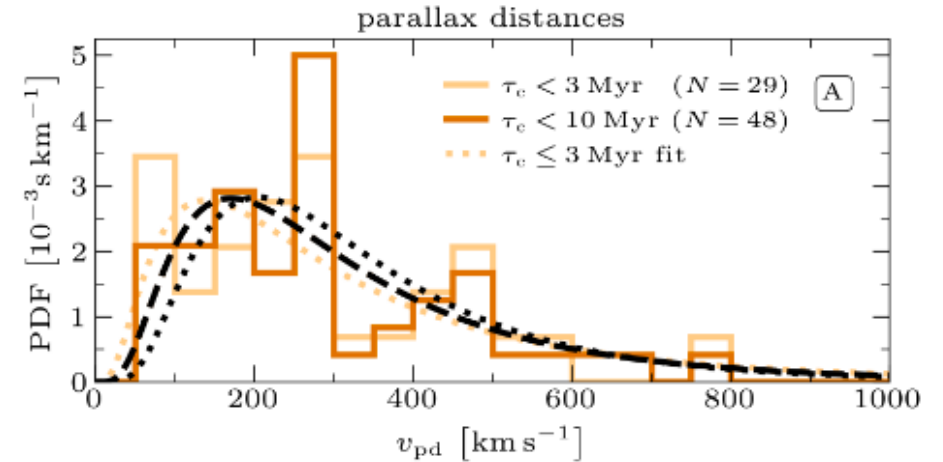
Neutron stars: kick velocity from the proper motion of young isolated pulsars.

Black holes? FALLBACK? $v_{\text{kick}} = (1 - f_{\text{FB}})v_{\text{NS}}$

Low kicks to keep systems bound



Preferentially aligned spins



Paul Disberg and Ilya Mandel, 2025.



Supernova remnant G272.2-03.2 from Chandra

