# Tensor Stars

## Compact objects from a massive spin-2 field

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# Why Boson Stars?

**Backreacting Boson Field →Bose-Einstein condensate**

$$S_{BS} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}_s\right)$$

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### ULDM

- Bosonic fields as ULDM
- Boson stars as ULDM cores

### BH Mimickers

- Transparent to light
- Horizonless objects with BH-like signatures

### Observation

- Stellar kinematics and shadows[Pombo,2023]
- GW spectroscopy
- GW190521 → Proca star merger[Bustillo,2021]

# Why Boson Stars?

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$$S_{BS} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}_s\right) \longrightarrow$$

- Spin 0 - Scalar
- Spin 1 - Proca
- Spin 2 - Tensor

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- ULDM models extend to tensor sector
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### Nature of Spin-2

- Same spin as graviton
- Non-minimal coupling to curvature
- Probe to gravity at fundamental level

# Massive spin-2 fields in curved spacetime

### Fierz-Pauli Lagrangian (massive gravity)

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### dRGT massive gravity (de Rham, Gabadadze, Tolley)[deRham,2011]

- Ghost-free nonlinear completion of Fierz-Pauli

↓ *matter spin-2 field*

### Mazuet & Volkov[Mazuet_2018]

- Consistent description of spin-2 matter field $X_{\mu\nu}$ in curved space
- Not tied to massive gravity

# The Action and its Limit

### Kinetic term

$$h_{\mu\nu} = X_{\mu\nu} + X_{\nu\mu} \qquad L_K(h,h) = -\frac{1}{4}\nabla^\sigma h^{\mu\nu}\nabla_\mu h_{\nu\sigma} + \frac{1}{8}\nabla^\sigma h^{\mu\nu}\nabla_\sigma h_{\mu\nu} + \frac{1}{4}\nabla^\sigma h \nabla^\mu h_{\sigma\mu} - \frac{1}{8}\nabla^\mu h \nabla_\mu h$$

+

### Interaction term

$$L_M(X,X) = -\frac{1}{2}X^{\mu\nu}R^\sigma_\mu X_{\sigma\nu} + \frac{1}{2}\left(\mu^2 - \frac{R}{6}\right)\left(X_{\mu\nu}X^{\mu\nu} - X^2\right)$$

↓

### Tensor Star Action

$$S_{TS} = \frac{1}{2}\int d^4x\sqrt{-g}\left[\frac{R}{4} + L_K(h,\bar{h}) + L_M(X,\bar{X}) + \text{c.c.}\right]$$

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*flat space* →

**Fierz–Pauli (flat-space benchmark)**

$$(\Box - \mu^2)X_{\mu\nu} = 0$$

*Klein–Gordon EoMs*

$$\partial_\mu X^{\mu\nu} = 0$$

*divergence-free*

$$X^\mu_{\ \mu} = 0$$

*traceless*

# Building the Ansatz

**Spherically symmetric metric ansatz**

$$ds^2 = -H\sigma^2 dt^2 + H^{-1} dr^2 + r^2 d\Omega^2$$

**$X_{\mu\nu}$ must be:**

- *SO(3)* ***invariant***
- *Produce a* ***stationary*** *stress energy tensor*
- *Produce a* ***real*** *action*
- *Fierz-Pauli* ***compatible*** *in flat space*

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## *5 real radial functions:* $u_1, u_2, u_3, u_4, u_5$

$$X_{\mu\nu}(r,t) = u_{\mu\nu}(r) e^{i\omega t}$$

$$u_{\mu\nu}(r) = \begin{pmatrix} u_1 H\sigma^2 & iu_2\sigma & 0 & 0 \\ iu_3\sigma & u_4 H^{-1} & 0 & 0 \\ 0 & 0 & u_5 r\sqrt{H\sigma} & 0 \\ 0 & 0 & 0 & u_5 r \sin\theta \times \sqrt{H\sigma} \end{pmatrix}$$

scalar (s=0) | vector (s=1) | tensor (s=2)

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scalar (s=0) vector (s=1) tensor (s=2)

# Coupled Nonlinear ODEs

### System structure

Fields: $H, \sigma, u_1, u_2, u_3, u_4, u_5$

↓

*Nonlinear coupled system on domain $r \in [0, \infty)$*

↓

*Numerical challenge*

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*Numerical challenge*

### Key equations for antisymmetric part $W = u_2 - u_3$

**1. Algebraic constraint:**

$$\omega W \left(u_4 - \sqrt{H}\sigma u_5'\right) = H u_3^2 \sigma'$$

*$\sigma' = 0$ (flat) ⇒ $W = 0$ ⇒ Symmetric field*

*$\sigma' \neq 0$ (curved) ⇒ $W$ activated by curvature*

**2. Homogeneous dynamical equation:**

$$W\left[r\omega u_4 \sigma' + 2\omega\sqrt{H} u_5 \sigma\sigma' + 2rH\sigma\sigma' u_3' + \frac{u_3}{2H}\frac{d}{dr}\left(rH^2 \frac{d(\sigma^2)}{dr}\right)\right] - W'[rHu_3\sigma\sigma'] = 0$$

*$W(r_0) = 0$ ⇒ $W \equiv 0$ everywhere*

# Asymptotic Boundary Conditions

$$g_{\mu\nu} \longrightarrow \eta_{\mu\nu}$$

$$r \to \infty$$

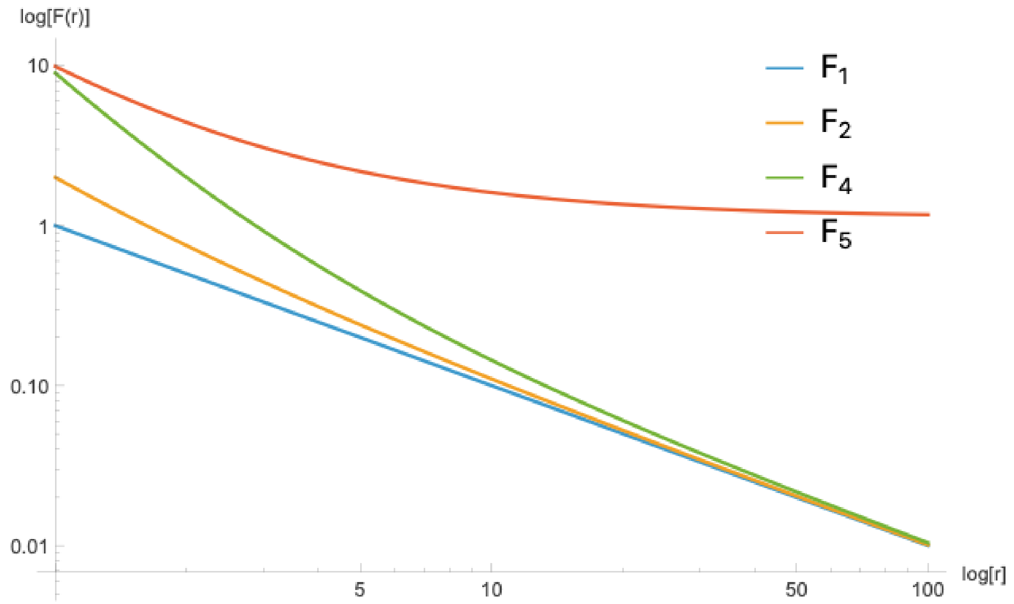
$$S_{TS} \longrightarrow S_{FP}$$

*Tensor Star action* → *Fierz-Pauli action*

$$(\square - \mu^2)X_{\mu\nu} = 0$$

**Field behaviour at infinity**

$$u_i(r) \sim F_i(r)e^{-\kappa r}, \ \kappa = \sqrt{\mu^2 - \omega^2}$$



# Frobenius Expansion at the Origin

**Power series expansion**

$$u_1 = \alpha_0 + \alpha_2 r^2 + \cdots$$

$$u_2 = \beta_1 r + \beta_3 r^3 + \cdots$$

$$u_3 = \gamma_1 r + \gamma_3 r^3 + \cdots$$

$$u_4 = \delta_0 + \delta_2 r^2 + \cdots$$

$$u_5 = \eta_1 r + \eta_3 r^3 + \cdots$$

$$H = 1 + H_2 r^2 + \cdots$$

$$\sigma = s_0 + s_2 r^2 + \cdots$$

**Free parameters: $\alpha_0$, $\eta_1$, $\beta_1$**

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Origin $\leftrightarrow X_{\mu\nu} = 0$

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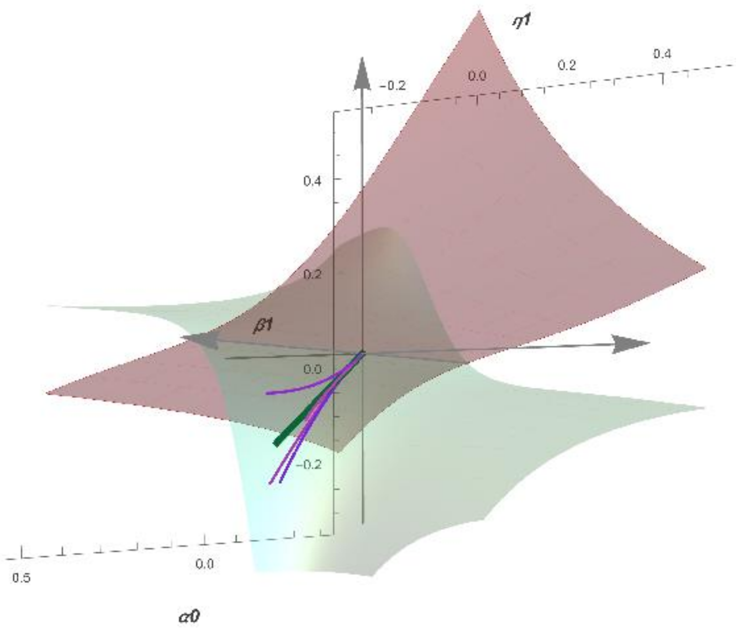
*Approaching the origin along the Fierz-Pauli direction*

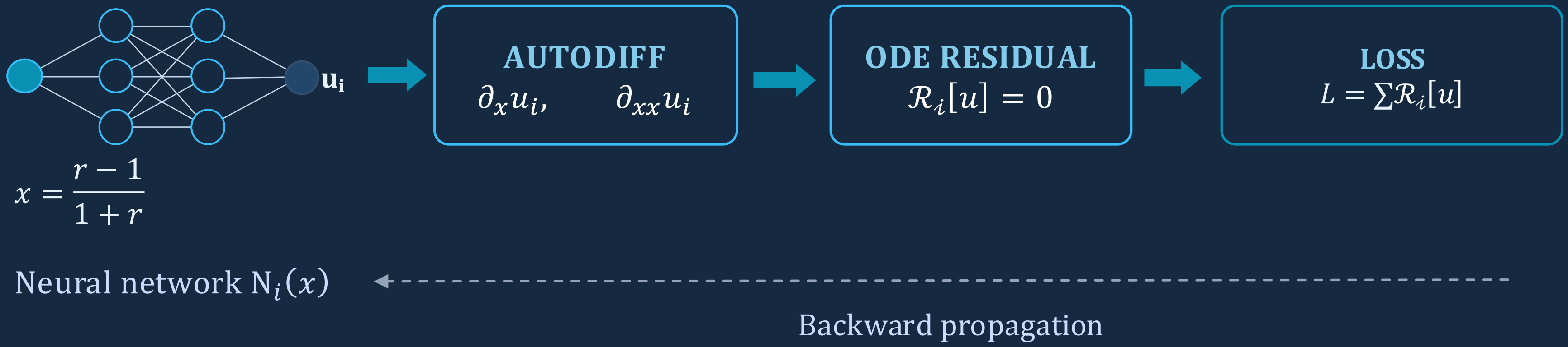
↓

$X_{\mu\nu} \to 0 \wedge g_{\mu\nu} \to \eta_{\mu\nu}$ *(weak-field solution)*



# Physics-Informed Neural Networks

**FORWARD PASS → PHYSICS-INFORMED LOSS → BACKPROP**



# Outlook

## Short term

- Characterize numerical solutions
- Comparison with scalar and Proca stars

## Medium term

- Stability analysis
- Non-spherical configurations

## Long term

- Quasi-normal modes
- Spinning tensor star
- Self-interacting models
- Gravitational imprints detection

# Conclusions

## PHYSICAL NOVELTY

**Natural direct coupling to curvature**

- *Not required in spin-0 / spin-1 boson stars*

**Unique probe of General Relativity**

- *Since tensor stars directly interact with curvature*

↓

**Peculiar observational signatures?**

- *Possibly within reach of next-gen GW detectors*

## STRUCTURAL FEATURES

**Tensor field ansatz**

- *Curvature coupling implies a non-symmetric tensor*

**Privileged direction in parameter space**

- *A unique direction connects weak-field solutions to the vacuum*

# Thank you for the attention!

Compact objects from a massive spin-2 field

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