Baryon Weak Decays – From Experiments to Lattice QCD, March 25-27, 2026, Cyprus

$\Xi_b \to \Xi$ form factors from lattice QCD and Standard-Model predictions for $\Xi_b \to \Xi\mu^+\mu^-$ and $\Xi_b \to \Xi\gamma$

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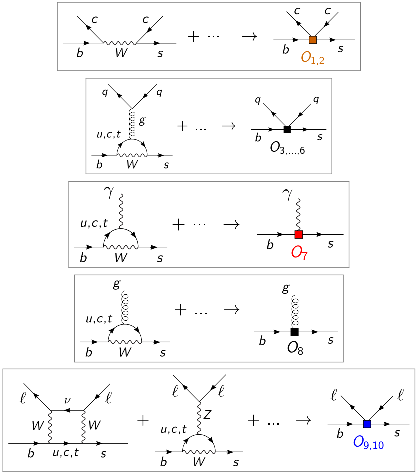
THE UNIVERSITY OF ARIZONA

Weak effective Hamiltonian for $b \to s\ell^+\ell^-$, $b \to s\gamma$

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= (e\, m_b)/(16\pi^2)\;\; \bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= e^2/(16\pi^2)\;\; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= e^2/(16\pi^2)\;\; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\dots
\end{aligned}$$



In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	...
−0.289	1.010	−0.336	4.267	−4.113	...

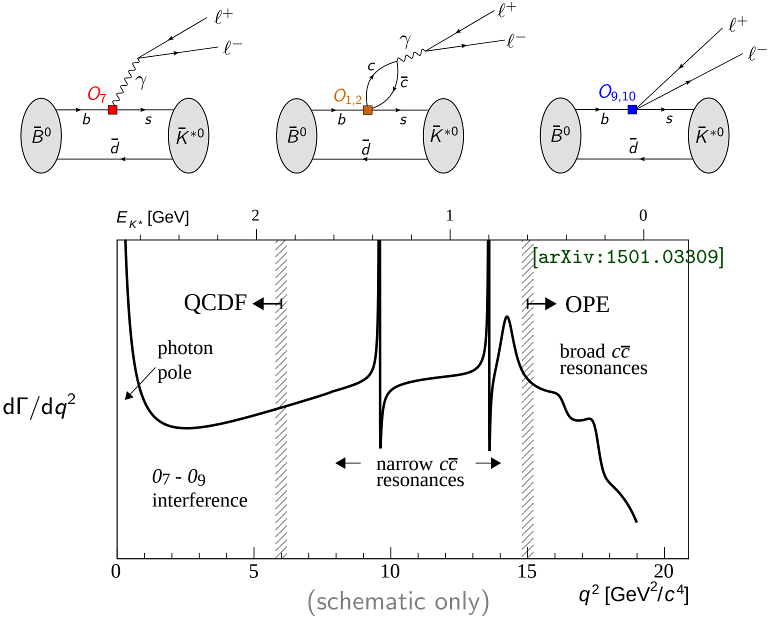
[Computed using EOS, `https://eoshep.org/`]

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$, $b \to s\gamma$ decays

For a generic decay $H_b \to H_s\ell^+\ell^-$ or $H_b \to H_s\gamma$:

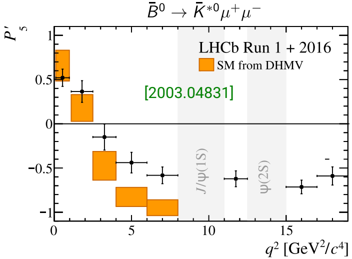
Contributions from O_7, $[O_9,\ O_{10}]$: $\langle H_s(p')|\ \bar{s}\Gamma b\ |H_b(p)\rangle$ $\to$ local form factors

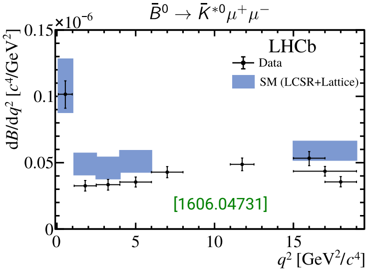
Contributions from $O_{1,\ldots,6}$, O_8: $\int \mathrm{d}^4x\ \ e^{iq\cdot x}\,\langle H_s(p')|\ \mathrm{T}\ O_i(0)\ J^{\mu}_{\mathrm{e.m.}}(x)\ |H_b(p)\rangle$ $\to$ nonlocal form factors

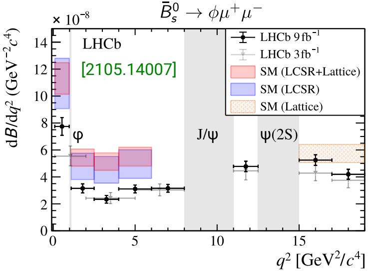


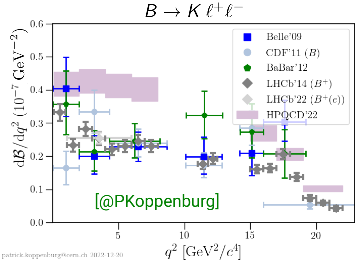
O_7
γ
ℓ^+
ℓ^-
$\bar{B}^0$
b
s
$\bar{d}$
$\bar{K}^{*0}$
$O_{1,2}$
c
$\bar{c}$
$O_{9,10}$
E_{K^*} [GeV]
2
1
0
[arXiv:1501.03309]
QCDF
OPE
photon
pole
broad $c\bar{c}$
resonances
$\mathrm{d}\Gamma/\mathrm{d}q^2$
narrow $c\bar{c}$
resonances
O_7 - O_9
interference
0
5
10
15
20
q^2 [GeV^2/c^4]
(schematic only)

Deviations from SM predictions in $b \to s\ell^+\ell^-$ angular observables and differential branching fractions

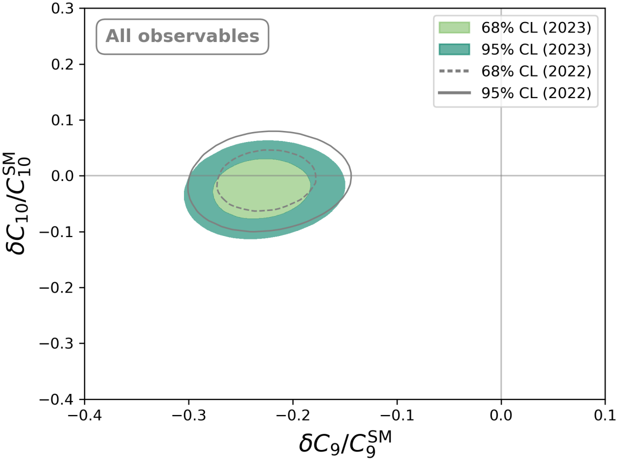






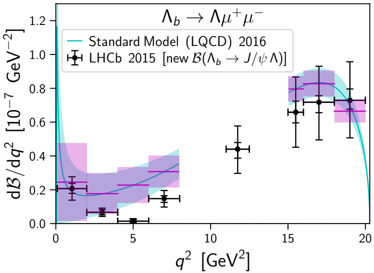


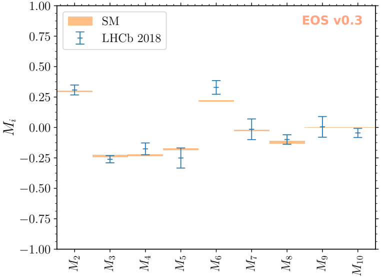
New physics in C_9?

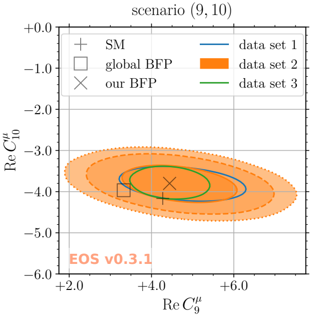


[T. Hurth, F. Mahmoudi, S. Neshatpour, 2310.05585/ PRD 2023]

$\Lambda_b \to \Lambda^{(*)}\mu^+\mu^-$



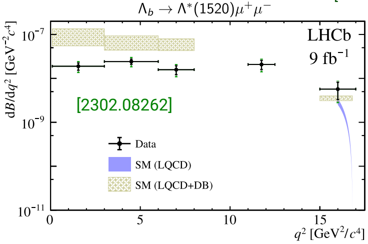




[W. Detmold, S. Meinel, 1602.01399/PRD 2016]

[T. Blake, S. Meinel, D. van Dyk, 1912.05811/PRD 2020]

Next-gen $\Lambda_b \to \Lambda$ LQCD calculation underway.

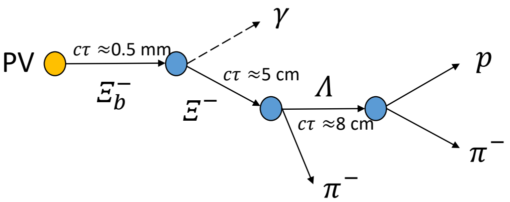


[LQCD: S. Meinel, G. Rendon, 2107.13140/PRD 2022]

What about rare decays of other b baryons?

LHCb is also analyzing Ξ_b^- decays. Low production fraction but distinctive final states.

- $\Xi_b^- \to \Xi^- \gamma$



First result: $\mathcal{B}(\Xi_b^- \to \Xi^- \gamma) < 1.3 \times 10^{-4}$ at 95% CL [LHCb, 2108.07678/JHEP 2022]

- $\Xi_b^- \to \Xi^- \mu^+ \mu^-$

Analysis in progress [Janina Nicolini, PhD Thesis, 2024, DOI:10.17877/DE290R-24786]


$\Xi_b \to \Xi$ form factors from lattice QCD and Standard-Model predictions for $\Xi_b \to \Xi\mu^+\mu^-$ and $\Xi_b \to \Xi\gamma$ decays

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(Dated: March 18, 2026)

We present the first lattice QCD determination of the $\Xi_b \to \Xi$ vector, axial-vector, and tensor form factors, which are relevant for the theory of rare decays including $\Xi_b \to \Xi\ell^+\ell^-$ and $\Xi_b \to \Xi\gamma$. The calculation is performed with 2+1 flavors of domain-wall fermions at three different lattice spacings and pion masses in the range from approximately 430 to 230 MeV. The bottom quark is implemented using an anisotropic clover action. Three-point functions with a wide range of source-sink separations and model averaging are used to extract the ground-state contributions. We fit the dependence of the form factors on the momentum transfer, the pion mass, and the lattice spacing using modified z expansions that account for subthreshold branch cuts, and apply dispersive bounds and asymptotic-behavior constraints to achieve controlled uncertainties in the full semileptonic kinematic region. Using our form factor results, we present Standard-Model predictions for the $\Xi_b^- \to \Xi^-\gamma$ and $\Xi_b^- \to \Xi^-\mu^+\mu^-$ branching fractions and two angular observables.

[2603.18438]

Form factor definitions

Current	Form factors
Vector	$f_0,\ f_+,\ f_\perp$
Axial vector	$g_0,\ g_+,\ g_\perp$
Tensor	$h_+,\ h_\perp,\ \widetilde{h}_+,\ \widetilde{h}_\perp$

Example:

$$\begin{aligned}\langle \Xi(p',s')|\bar{s}\gamma^\mu\gamma_5 b|\Xi_b(p,s)\rangle \quad = \quad & -\overline{u}_\Xi(p',s')\,\gamma_5\Big[(m_{\Xi_b}+m_\Xi)\frac{q^\mu}{q^2}\,g_0(q^2) \\ & + \frac{m_{\Xi_b}-m_\Xi}{s_-}\left(p^\mu + p'^\mu - (m_{\Xi_b}^2 - m_\Xi^2)\frac{q^\mu}{q^2}\right) g_+(q^2) \\ & + \left(\gamma^\mu + \frac{2m_\Xi}{s_-}p^\mu - \frac{2m_{\Xi_b}}{s_-}p'^\mu\right) g_\perp(q^2)\Big] u_{\Xi_b}(p,s)\end{aligned}$$

where $q = p - p'$ and $s_\pm = (m_{\Xi_b} \pm m_\Xi)^2 - q^2$ [T. Feldmann, M. W. Y. Yip, 1111.1844/PRD 2012]

Lattice actions and ensembles

- Gluons: Iwasaki action
- u, d, s quarks: domain-wall action
- Gauge configurations generated by RBC/UKQCD

Label	$N_s^3 \times N_t$	β	$am_{u,d}^{(\text{sea,val})}$	$am_s^{(\text{sea})}$	$am_s^{(\text{val})}$	a [fm]	$m_\pi^{(\text{sea,val})}$ [MeV]	Samples
C01	$24^3 \times 64$	2.13	0.01	0.04	0.0323	≈ 0.111	≈ 430	283 ex, 2264 sl
C005	$24^3 \times 64$	2.13	0.005	0.04	0.0323	≈ 0.111	≈ 340	311 ex, 2488 sl
F004	$32^3 \times 64$	2.25	0.004	0.03	0.0248	≈ 0.083	≈ 300	251 ex, 2008 sl
F1M	$48^3 \times 96$	2.31	0.002144	0.02144	0.02217	≈ 0.073	≈ 230	113 ex, 1808 sl

Lattice actions and ensembles

- b quarks: anisotropic clover action

$$S_Q = a^4 \sum_x \bar{Q} \left[m_Q + \gamma_0 \nabla_0 - \frac{a}{2} \nabla_0^{(2)} + \nu \sum_{i=1}^{3} \left(\gamma_i \nabla_i - \frac{a}{2} \nabla_i^{(2)} \right) - c_E \frac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,j=1}^{3} \sigma_{ij} F_{ij} \right] Q$$

with m_Q, ν, $c_E = c_B$ tuned nonperturbatively using B_s energy, "speed of light," and HFS

Ensemble	am_Q	ν	$c_{E,B}$	aE_{B_s}	E_{B_s} [GeV]	$c^2_{B_s}$	$aE_{B_s^*} - aE_{B_s}$	$E_{B_s^*} - E_{B_s}$ [MeV]
C005	7.42	2.92	4.86	3.00324(81)	5.360(15)	0.9184(89)	0.02602(52)	46.44(94)
C005	7.471	2.929	4.92	3.00801(81)	5.369(15)	0.9184(89)	0.02616(53)	46.69(95)
C005	7.9	2.929	4.92	3.05463(85)	5.452(15)	0.8895(91)	0.02482(53)	44.29(96)
C005	7.471	2.929	6	2.97872(77)	5.316(15)	0.9399(85)	0.03165(58)	56.5(1.1)
C005	7.471	3.3	4.92	3.03049(75)	5.409(15)	1.0186(89)	0.02665(48)	47.56(86)
C005	7.3258	3.1918	4.9625	3.00695(75)	5.367(15)	1.0013(88)	0.02719(49)	48.52(88)
F004	3.485	1.76	3.06	2.25010(82)	5.363(19)	0.855(12)	0.02053(62)	48.9(1.5)
F004	3.3	1.76	3.06	2.20958(79)	5.266(19)	0.877(12)	0.02148(61)	51.2(1.5)
F004	3.485	1.76	3.7	2.21551(78)	5.280(19)	0.877(12)	0.02468(68)	58.8(1.6)
F004	3.485	2.2	3.06	2.29690(70)	5.474(20)	1.047(12)	0.02126(51)	50.7(1.2)
F004	3.2823	2.0600	2.7960	2.25175(71)	5.366(19)	1.004(12)	0.02046(51)	48.8(1.2)
F1M	2.4538	1.7563	2.6522	1.97455(85)	5.347(20)	0.960(16)	0.01876(79)	50.8(2.2)
F1M	2.55	1.7563	2.6522	2.00012(88)	5.416(20)	0.943(17)	0.01821(81)	49.3(2.2)
F1M	2.4538	2	2.6522	2.00698(75)	5.435(20)	1.089(16)	0.01926(70)	52.2(1.9)
F1M	2.4538	1.7563	3.1	1.94472(81)	5.266(20)	0.983(16)	0.02151(83)	58.3(2.3)
F1M	2.3867	1.8323	2.4262	1.98009(81)	5.362(20)	1.003(16)	0.01799(73)	48.7(2.0)

[2309.01821]

Renormalization and improvement of weak currents

We use the "mostly nonperturbative method" to renormalize the currents,

$$J_\Gamma = \sqrt{Z_V^{qq}\, Z_V^{QQ}}\ \rho_\Gamma \left[\bar{q}\,\Gamma\,Q + \mathcal{O}(a)\right],$$

where

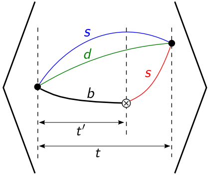
■ = nonperturbative renormalization factors of $\bar{q}\gamma^0 q$ and $\bar{Q}\gamma^0 Q$ determined using charge conservation,
■ = one-loop residual-matching and $O(a)$-improvement coefficients, computed by Christoph Lehner.

For tensor currents, ρ_Γ and $\mathcal{O}(a)$ terms at tree level only.

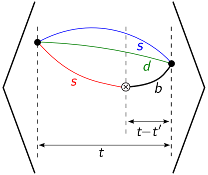
Extraction of the form factors from correlation functions

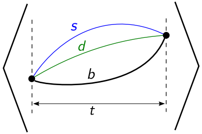
Each form factor f is obtained from the large-t limit (t = source-sink separation) of

$R_f = \sqrt{\quad}$ (kinematic factors) $\times$

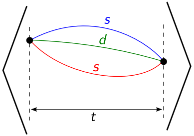


$\times$





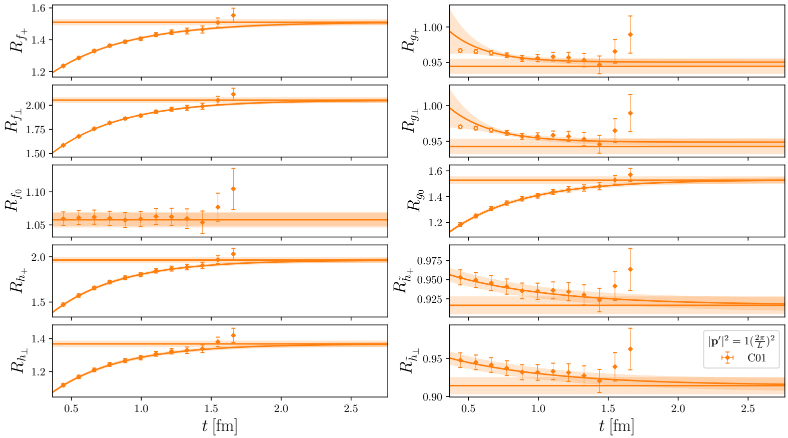
$\times$



where we are setting $t' = t/2$.

Extraction of the form factors from correlation functions

$R_f(t) \to f$ for large t. We use fits of the form $R_f(t) = f + A\, e^{-\Delta E\, t}$, where f, A, ΔE are fit parameters.



We use a generalized version of the Akaike information criterion to average over fits with different t_{min} [W. Jay and E. Neil, 2008.01069/PRD 2021]. Horizontal bands = AIC averages of f

Extraction of the form factors from correlation functions

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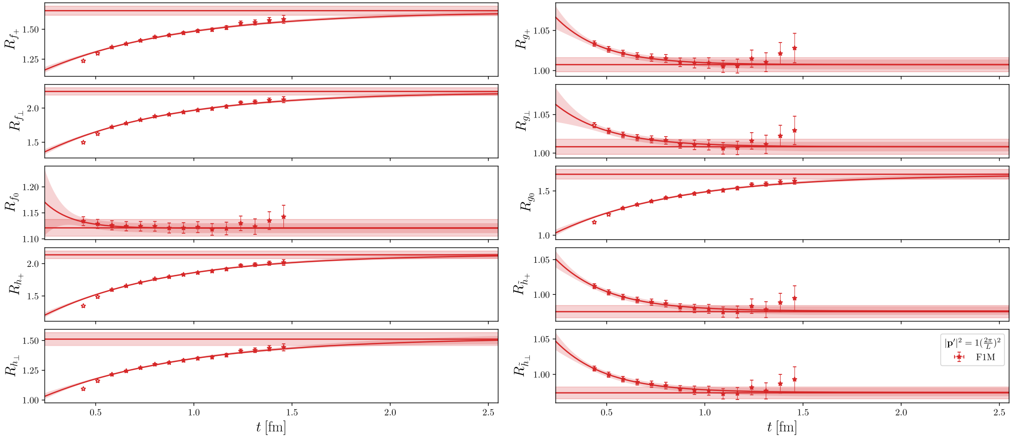
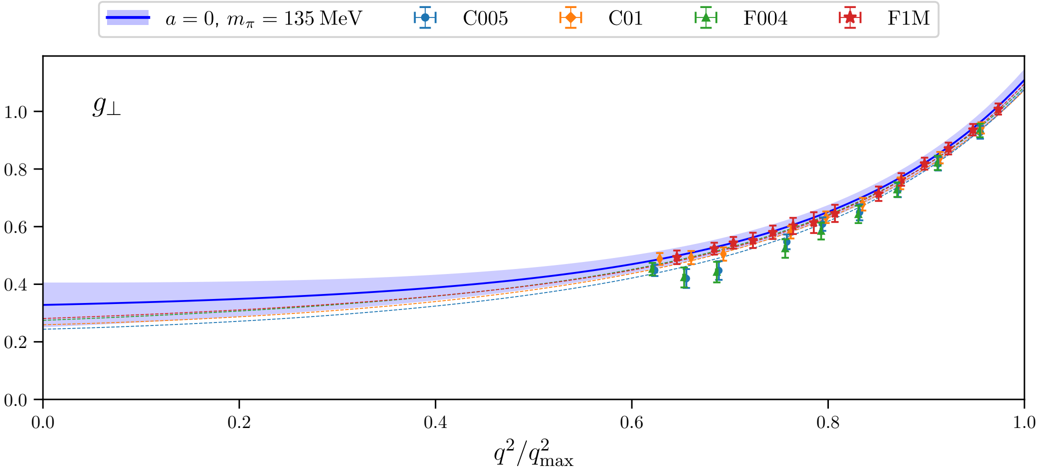


We use a generalized version of the Akaike information criterion to average over fits with different $t_{\min}$ [W. Jay and E. Neil, 2008.01069/PRD 2021]. Horizontal bands = AIC averages of f

Chiral-continuum-kinematic extrapolations

Example:



The fit is done using a modified z expansion of order z^8 with a dispersive bound on the coefficients and additional constraints ensuring reasonable asymptotic behavior for $q^2 \to -\infty$.

Dispersive bounds

[T. Blake, S. Meinel, M. Rahimi, D. van Dyk, 2205.06041/PRD 2023]

An example of a dispersive bound is

$$\underbrace{\chi_V^{J=1}(Q^2)\Big|_{\text{OPE}}}_{\text{perturbation theory}} \geq \int_{t_{\text{th}}}^{\infty} \mathrm{d}t \, \frac{1}{24\pi^2} \frac{\sqrt{\lambda(m_{\Xi_b}^2, m_{\Xi}^2, t)}}{t^2 (t - Q^2)^3} s_-(t) \left((m_{\Xi_b} + m_{\Xi})^2 |f_+(t)|^2 + 2t |f_\perp(t)|^2 \right),$$

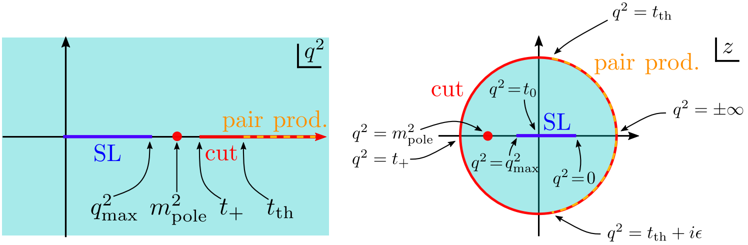
where $t_{\text{th}} = (m_{\Xi_b} + m_{\Xi})^2$, $\quad \chi_V^{J=1}(Q^2) = \frac{1}{2!} \left(\frac{\mathrm{d}}{\mathrm{d}Q^2} \right)^2 \Pi_V^{J=1}(Q^2)$, $\quad$ and we use $Q^2 = 0$.

NB: 2205.06041 uses different names for the form factors, such as $(f_0^V, f_\perp^V, f_t^V)$ instead of $(f_+, f_\perp, f_0)$, but the definitions are identical.

Dispersive bounds

The vector form factors have a branch cut starting at $t_+ = (m_B + m_K)^2$. We define the variable

$$z(t; t_0, t_+) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}\,.$$



Then the dispersive bound can be written as

$$1 \geq \int_{-\alpha_{\Xi_b\Xi}}^{+\alpha_{\Xi_b\Xi}} \mathrm{d}\alpha \, \left(|\phi_{f_+}(z)|^2 |f_+(z)|^2 + |\phi_{f_\perp}(z)|^2 |f_\perp(z)|^2 \right)_{z=e^{i\alpha}}$$

where the ϕ's are the *outer functions* and

$$\alpha_{\Xi_b\Xi} = \arg z((m_{\Xi_b} + m_{\Xi})^2, t_0, t_+) \approx 1.43829 \qquad [\text{for } t_0 = q^2_{\rm max} = (m_{\Xi_b} - m_{\Xi})^2].$$

Dispersive bounds

We use BGL-type z-expansions to describe the continuum form factors,

$$f(q^2) = \frac{1}{\mathcal{P}_f(q^2)\,\phi_f(z)} \sum_n a_{f,n}\, z^n,$$

where $\mathcal{P}_f(q^2) = z(q^2; m^2_{\mathrm{pole},f}, t_+)$ is the Blaschke factor. If we had $t_{\mathrm{th}} = t_+$, so that the dispersive integral would go over the entire unit circle, we would have $1 \geq \sum_n (a^2_{f_+,n} + a^2_{f_\perp,n})$. Instead, because the integral only goes over a smaller arc, we have

$$1 \geq \sum_{n,n'} a_{f_+,n} \langle z^n | z^{n'} \rangle a_{f_+,n'} + \sum_{n,n'} a_{f_\perp,n} \langle z^n | z^{n'} \rangle a_{f_\perp,n'},$$

where [J.M. Flynn, A. Jüttner, J.T. Tsang, 2303.11285/JHEP 2023]

$$\langle z^n | z^{n'} \rangle = \begin{cases} \dfrac{\sin(\alpha_{\Xi_b\Xi}(n-n'))}{\pi(n-n')}, & n \neq n', \\ \dfrac{\alpha_{\Xi_b\Xi}}{\pi}, & n = n'. \end{cases}$$

This is equivalent to the orthogonal-polynomial expansion in [T. Blake, S. Meinel, M. Rahimi, D. van Dyk, 2205.06041/PRD 2023]

Asymptotic behavior

A possible problem with BGL z expansion is that $\frac{1}{\mathcal{P}_f(q^2)\,\phi_f(z)}$ diverges for $q^2 \to -\infty$ ($z \to 1$), while $\sum_n a_{f,n}\, z^n$ remains finite. This allows the form factor to diverge, which contradicts the expectation from perturbative QCD.

One possible solution is to use a BCL z expansion with $\frac{1}{1 - q^2/m^2_{\text{pole},f}}$ instead of $\frac{1}{\mathcal{P}_f(q^2)\,\phi_f(z)}$, but then the dispersive bounds become more complicated.

We instead solve the problem by imposing the sum rules
[J.M. Flynn, A. Jüttner, J.T. Tsang, 2303.11285/JHEP 2023]

$$\left(\frac{\mathrm{d}}{\mathrm{d}z}\right)^m \sum_{n=0}^{N+M} a_{f,n} z^n \Bigg|_{z=1} = 0 \qquad \forall m \in \{0, 1, ..., M-1\}$$

with $M = 4$ for f_+, g_+, $h_\perp$, and $\tilde{h}_\perp$ and $M = 3$ for the other form factors, which, given our outer functions, ensures fall-off at least as fast as $1/(-q^2)$ for $q^2 \to -\infty$. We solve for the highest M coefficients in terms of the lower coefficients.

Lattice-spacing and pion-mass dependence

We fit the lattice data using

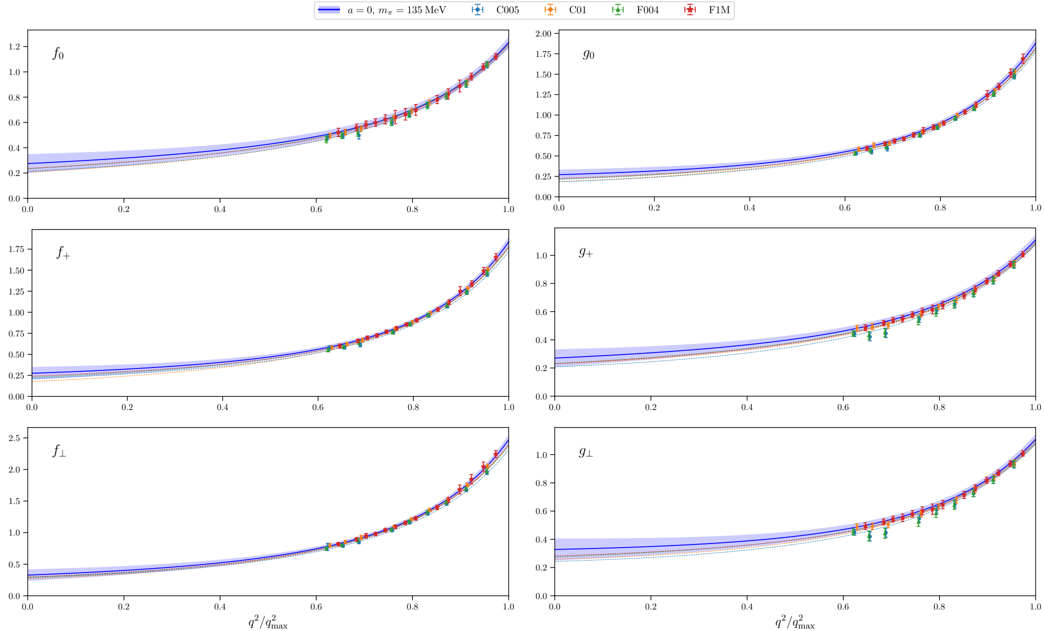
$$
\begin{aligned}
f(q^2) \quad = \quad & \frac{1}{\mathcal{P}_f(q^2)\phi_f(q^2)}\left[a_{f,0}\left(1 + c_0^f\,\frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{\Lambda_\chi^2} + \tilde{c}_0^f\,\frac{m_\pi^3 - m_{\pi,\mathrm{phys}}^3}{\Lambda_\chi^3}\right)\right. \\
& \quad + a_{f,1}\left(1 + c_1^f\,\frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{\Lambda_\chi^2} + \tilde{c}_1^f\,\frac{m_\pi^3 - m_{\pi,\mathrm{phys}}^3}{\Lambda_\chi^3}\right) z(q^2, q^2_{\mathrm{max}}, t_+^f) \\
& \quad \left. + \sum_{n=2}^{N+M} a_{f,n}\,[z(q^2, q^2_{\mathrm{max}}, t_+^f)]^n\right] \\
& \times\left[1 + b^f\, a^2|\mathbf{p}'|^2 + d^f\, a^2\Lambda_{\mathrm{had}}^2 + \tilde{b}^f\, a^4|\mathbf{p}'|^4 + \hat{d}^f\, a^3\Lambda_{\mathrm{had}}^3 + \tilde{d}^f\, a^4\Lambda_{\mathrm{had}}^4 + j^f\, a^4|\mathbf{p}'|^2\Lambda_{\mathrm{had}}^2\right],
\end{aligned}
$$

where the higher-order parameters $\tilde{c}_0^f$, $\tilde{c}_1^f$, $\tilde{b}^f$, $\hat{d}^f$, $\tilde{d}^f$, and j^f, which are not needed to describe the data, were constrained with Gaussian priors to be not unnaturally large.

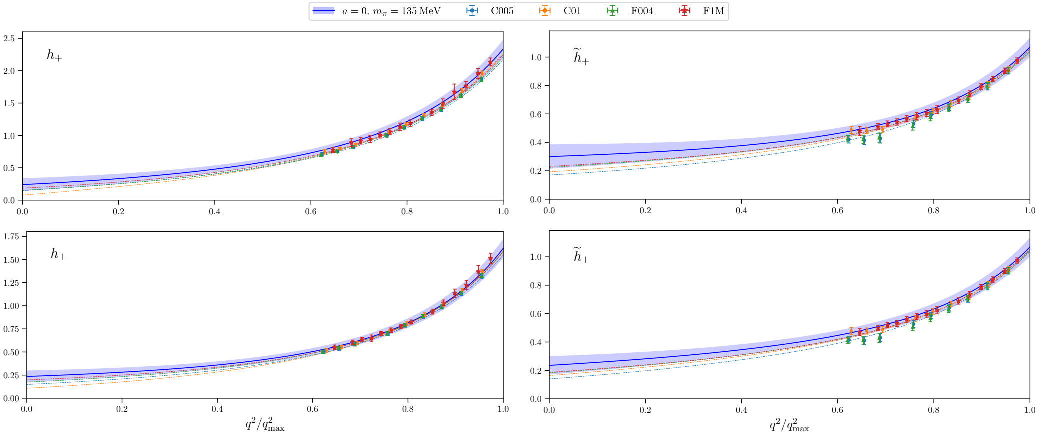
Statistical approach

We use the algorithm of [J.M. Flynn, A. Jüttner, J.T. Tsang, 2303.11285/JHEP 2023], which involves an initial fit with a Gaussian prior on the dispersive-bound function, followed by random sampling and an accept-reject step to apply the dispersive bound and reweight to remove the initial prior.

We increase the order of the z expansion until the central values and uncertainties of the form factors stabilize in the full kinematic range. We found that we need $N = 5$ (and recall that there are an additional $M = 3$ or $M = 4$ higher powers, so we go up to z^8 or z^9).



$a = 0,\ m_\pi = 135$ MeV
C005
C01
F004
F1M
f_0
g_0
f_+
g_+
$f_\perp$
$g_\perp$
q^2/q^2_{max}
q^2/q^2_{max}



$a = 0,\ m_\pi = 135$ MeV
C005
C01
F004
F1M
h_+
$\widetilde{h}_+$
$h_\perp$
$\widetilde{h}_\perp$
$q^2/q^2_{\rm max}$
$q^2/q^2_{\rm max}$

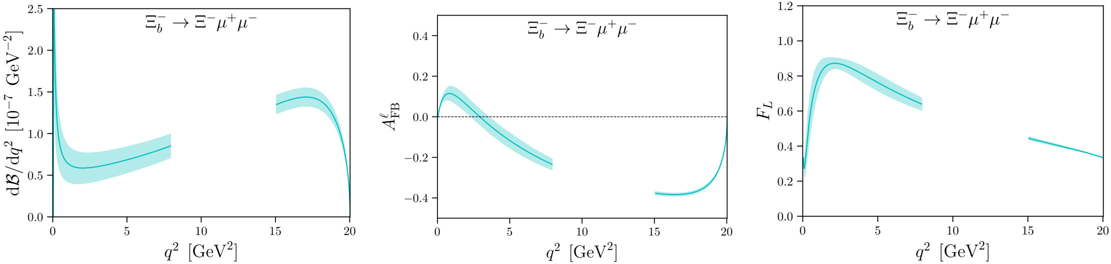
Calculation of $\Xi_b^- \to \Xi^- \mu^+ \mu^-$ and $\Xi_b^- \to \Xi^- \gamma$ observables

We limit our predictions to the kinematic regions $q^2 < 8\,\text{GeV}^2$ and $q^2 > 15\,\text{GeV}^2$ to avoid the effects of narrow charmonium resonances, and use the approximation in which all nonlocal hadronic matrix elements are reduced to local matrix elements through the effective Wilson coefficients

$$
\begin{aligned}
C_7^{\text{eff}}(q^2) &= C_7 - \frac{1}{3}\left[C_3 + \frac{4}{3}\,C_4 + 20\,C_5 + \frac{80}{3}\,C_6\right] - \frac{\alpha_s}{4\pi}\left[(C_1 - 6\,C_2)\,F_{1,c}^{(7)}(q^2) + C_8\,F_8^{(7)}(q^2)\right], \\
C_9^{\text{eff}}(q^2) &= C_9 + \frac{4}{3}\,C_3 + \frac{64}{9}\,C_5 + \frac{64}{27}\,C_6 + h(0,q^2)\left(-\frac{1}{2}\,C_3 - \frac{2}{3}\,C_4 - 8\,C_5 - \frac{32}{3}\,C_6\right) \\
&\quad + h(m_b,q^2)\left(-\frac{7}{2}\,C_3 - \frac{2}{3}\,C_4 - 38\,C_5 - \frac{32}{3}\,C_6\right) + h(m_c,q^2)\left(\frac{4}{3}\,C_1 + C_2 + 6\,C_3 + 60\,C_5\right) \\
&\quad - \frac{\alpha_s}{4\pi}\left[C_1\,F_{1,c}^{(9)}(q^2) + C_2\,F_{2,c}^{(9)}(q^2) + C_8\,F_8^{(9)}(q^2)\right].
\end{aligned}
$$

See [W. Detmold, S. Meinel, 1602.01399/PRD 2016] and references therein

$\Xi_b^- \to \Xi^- \mu^+ \mu^-$ Standard-Model predictions



$\Xi_b^- \to \Xi^-\gamma$ Standard-Model predictions

	Method	$h_\perp(0) = \widetilde{h}_\perp(0)$	$\mathcal{B}(\Xi_b^- \to \Xi^-\gamma)$
LHCb, 2021 [87]	Experiment		$< 1.3 \times 10^{-4}$
This work	Lattice QCD	0.235 ± 0.065	$(2.9 \pm 1.6) \times 10^{-5}$
Rui *et al.*, 2025 [85]	pQCD approach	$0.338^{+0.015+0.009+0.024}_{-0.000-0.000-0.015}$	
Aliev *et al.*, 2023 [92]	Light-cone sum rules	0.31 ± 0.04	$(4.8 \pm 1.3) \times 10^{-5}$
Davydov *et al.*, 2022 [91]	Relativistic quark model	0.144 ± 0.007	$(0.95 \pm 0.15) \times 10^{-5}$
Geng *et al.*, 2022 [90]	Light-front quark model	0.143	$(1.1 \pm 0.1) \times 10^{-5}$
Olamaei *et al.*, 2021 [89]	Light-cone sum rules [81]		$(1.08^{+0.63}_{-0.49}) \times 10^{-5}$
Wang *et al.*, 2021 [88]	Flavor-$SU(3)$ symmetry		$(1.23 \pm 0.64) \times 10^{-5}$
Liu *et al.*, 2011 [93]	Light-cone sum rules	≈ 0.6	$(3.03 \pm 0.10) \times 10^{-4}$

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