Generalized tensión metrics for multiple cosmological datasets

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Motivation

In the cosmological context, two experiments are said to be in **tension** if the confidence intervals inferred from the data for the cosmological parameters do not agree. This may be due to:

- Errors in one of the analyses
- Unaccounted for systematic errors
- Evidence of new physics

In a cosmological context, some of the most important tensions are

- The so-called H_0 tension, where the determination of the Hubble constant using Type Ia Supernovae together with Cepheids is in tension with the CMB.
- Tension in the parameter σ_8 in analyses linked to weak lensing and CMB data.

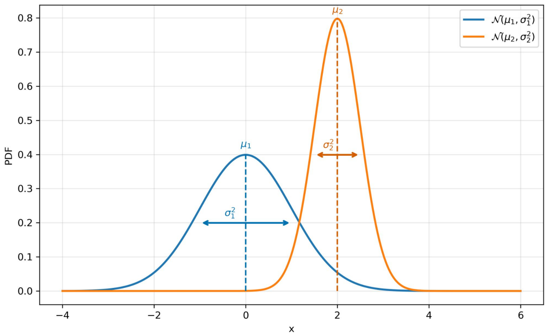
Well-known tension metrics

To quantify the inconsistency between observables, it is desirable to convert the confidence intervals for two distinct data sets into a probability measure of the tension between them.

‘Rule of Thumb’ tension metric (1D):

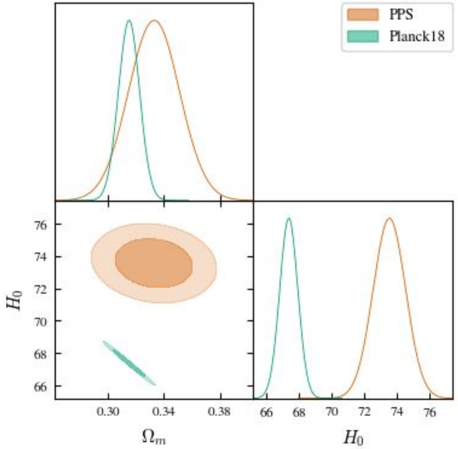
$$N_\theta = \frac{|\mu_A - \mu_B|}{\sqrt{\sigma_A^2 + \sigma_B^2}}$$

Level of tension



‘Difference in means’ tension metric (2D):

$$Q_{\rm DM} = (\vec{\bar{\theta}}_i - \vec{\bar{\theta}}_j)^T (\hat{C}_i + \hat{C}_j)(\vec{\bar{\theta}}_i - \vec{\bar{\theta}}_j)$$



Multidimensional formalism

$$\vec{r}_k = \frac{1}{\sqrt{\hat{C}_i + \hat{C}_j}} (\vec{\bar{\theta}}_i - \vec{\bar{\theta}}_j) \quad (i \neq j)$$

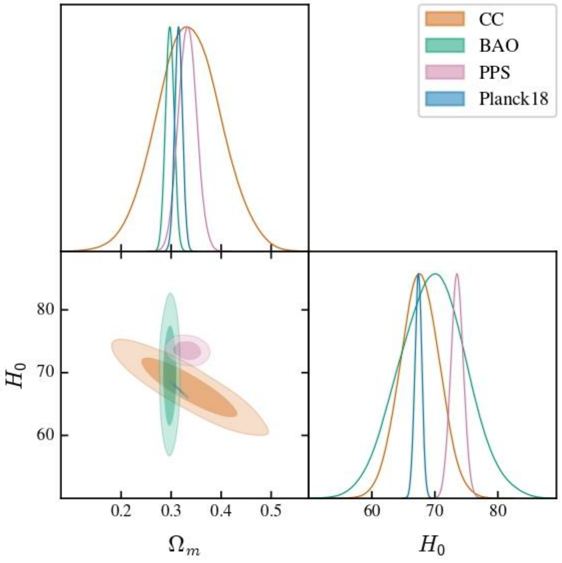
Tension vector formalism

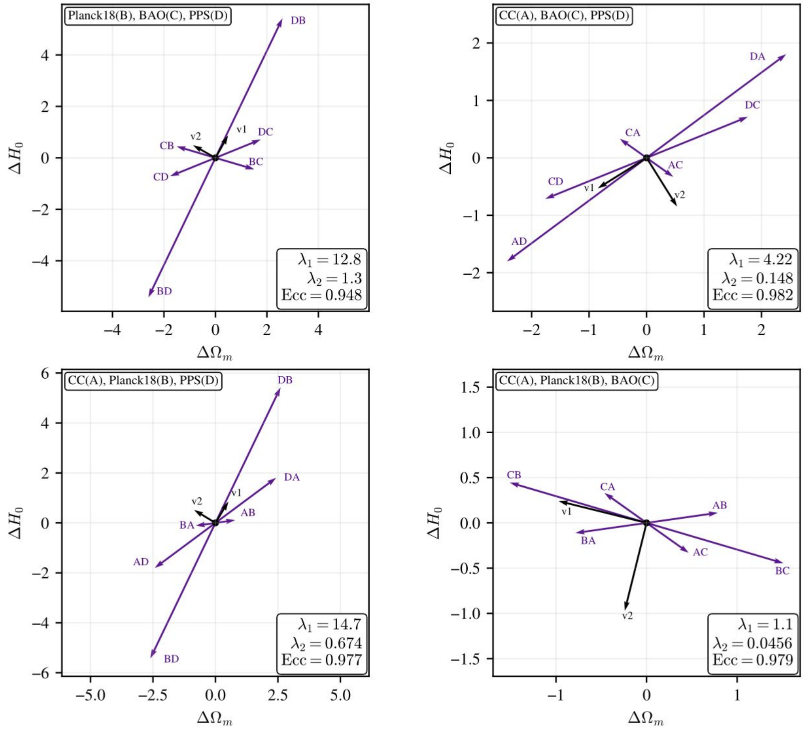
We propose a new multidimensional tension metric:

$$\mathcal{Q} \equiv \frac{1}{N_p} \sum_{k=1}^{N_p} |\vec{r}_k|^2$$

2D limit is recovered: $\left(Q_{\rm DM} = |\vec{r}_{AB}|^2\right)$

Results

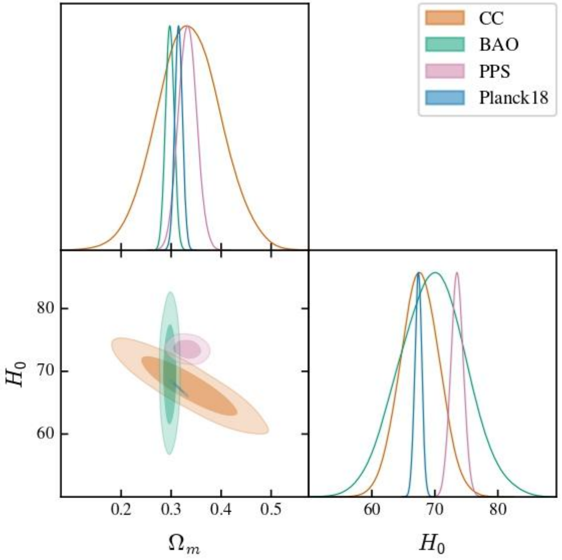




- Cosmic Chronometers **(CC)** - Moresco et al. compilation
- Baryon Acoustic Oscillations **(BAO)** - DESI DR2
- SNIa + Cepheids calibration - Pantheon+ with SH0ES calibration **(PPS)**
- Cosmic Microwave Background - **Planck 18**

We managed to quantify not only the **tension** but also the **dispersion** of the tension vectors!

Results



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Datasets	N_σ	L	$N_\sigma^{\rm eff}$
Synthetic case 1	1.51	2.13	0.991
Synthetic case 2	4.53	3.97	2.34
CC + Planck + BAO	0.429	1.48	0.555
CC + BAO + PPS	1.78	2.29	1.11
Planck + BAO + PPS	4.1	3.7	2.14
CC + Planck + PPS	4.33	3.85	2.25
CC + Planck	0.346	0.794	0.184
CC + BAO	0.189	0.569	0.0997
CC + PPS	2.57	3.03	1.64
Planck + PPS	5.68	6.02	3.86
BAO + PPS	1.39	1.9	0.833

TABLE I: Tension estimators for synthetic and real data sets: usual N_σ vs our geometrical indicator $N_\sigma^{\rm eff}$ derived from the equivalent configuration distance L. PPS: Pantheon Plus + SH0ES [11], CMB: Planck2018 [1], BAO: DESI DR2 [10].

Summary

- We propose a new estimator to evaluate tensions among multiple datasets simultaneously.
- Geometrical mapping to visualize the tension as arrows in a new vector space.
- Potential applicability to other fields where discrepancies between datasets arise (for example, particle physics).

References:

Generalized tension metrics for multiple cosmological datasets — Leizerovich, M. et al. (2025) arXiv:2512.06086

Tensions in cosmology: A discussion of statistical tools to determine inconsistencies — Leizerovich, M. et al. (2024)

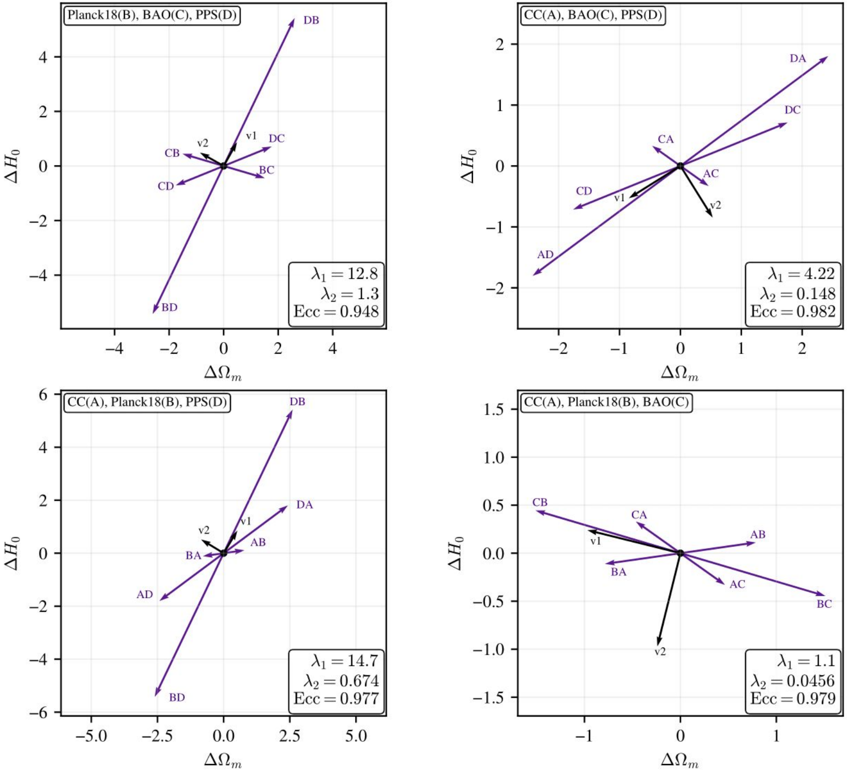
Eccentricity

$$\mathrm{Ecc} = \sqrt{1 - \frac{\lambda_{\min}}{\lambda_{\max}}}$$

→ Eigenvalues of $\mathcal{C}$

Datasets	$\{\lambda_\alpha\}$	Ecc
Synthetic case 1	$\{3.54, 0.\}$	1
Synthetic case 2	$\{16.8, 0.495\}$	0.985
CC + Planck + BAO	$\{1.1, 0.0456\}$	0.979
CC + BAO + PPS	$\{4.22, 0.148\}$	0.982
Planck + BAO + PPS	$\{1.3, 12.8\}$	0.948
CC + Planck + PPS	$\{0.674, 14.7\}$	0.977

TABLE II: Indicators of anisotropy in the tension for synthetic and real datasets



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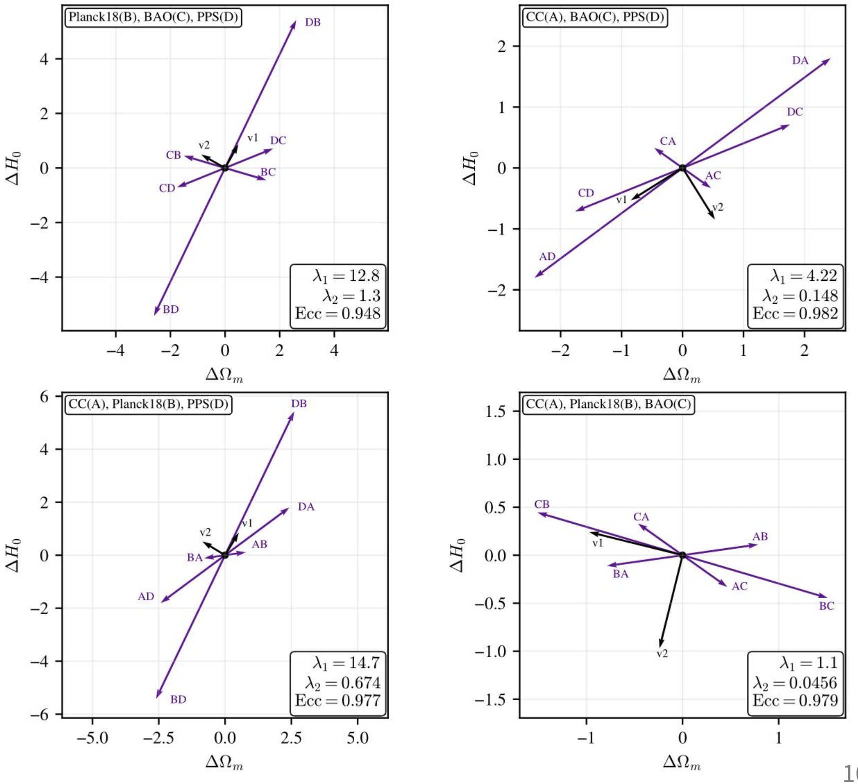
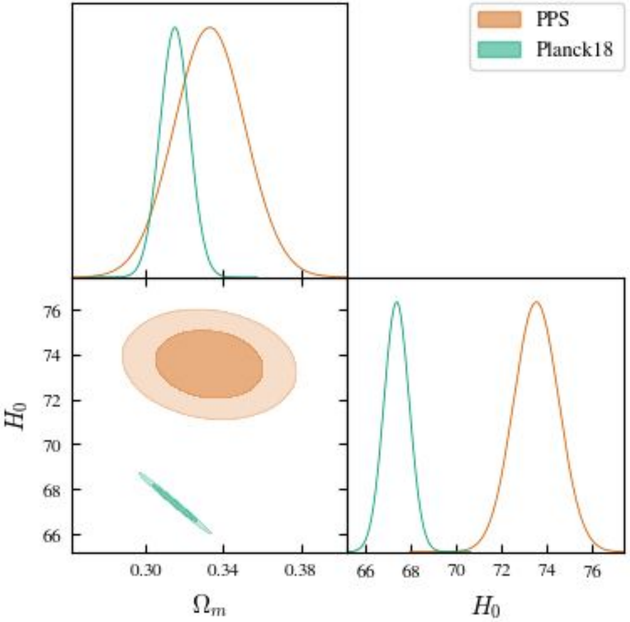
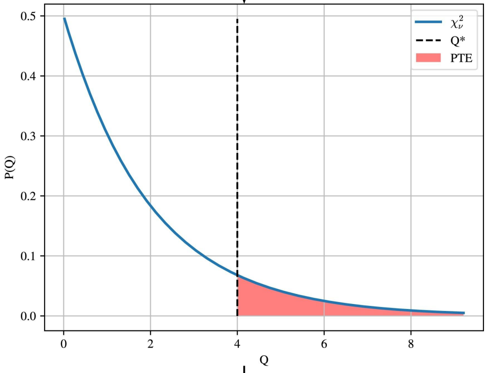


Difference in Mean (DM) tension metric



2 datasets tension on a 2D shared parameter space

$$Q_{\mathrm{DM}} = (\vec{\bar{\theta}}_i - \vec{\bar{\theta}}_j)^T (\hat{C}_i + \hat{C}_j)(\vec{\bar{\theta}}_i - \vec{\bar{\theta}}_j)$$



Distribution of Q_{DM} under the hypothesis of no tension

$$Q_{\mathrm{DM}}\big|_{H0} \sim \chi^2_D$$

$$N_\sigma = \sqrt{2} \cdot \mathrm{Erf}^{-1}\left(1 - \mathrm{PTE}\right)$$

Difference in Mean (DM) tension metric

2D tension metric

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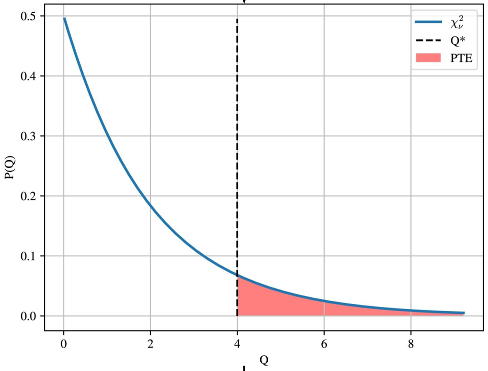

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$$N_\sigma = \sqrt{2} \cdot \mathrm{Erf}^{-1}\,(1 - \mathrm{PTE})$$

1D gaussian analogy!

Quantifying Cosmological tensions

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'Rule of Thumb' tension metric (1D):

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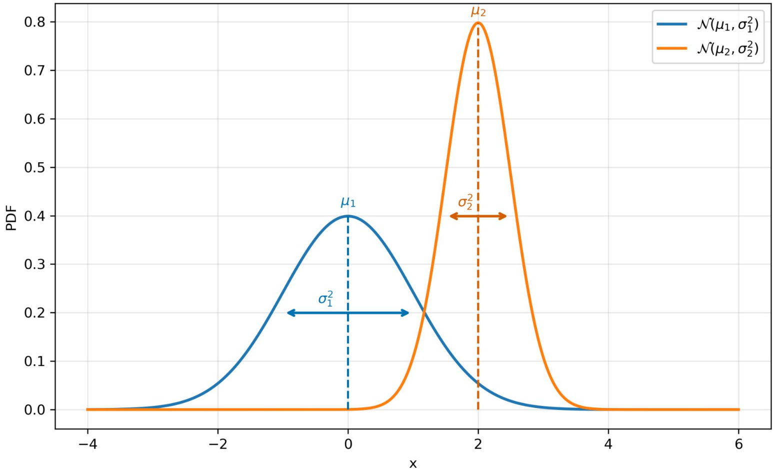

Cosmological tensions

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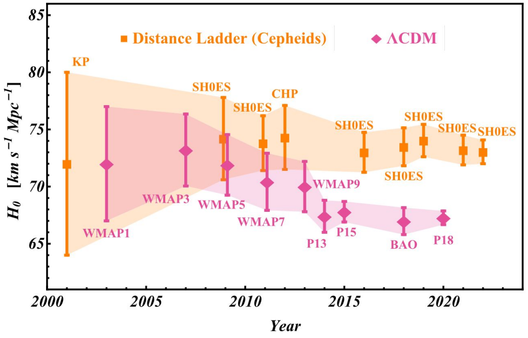
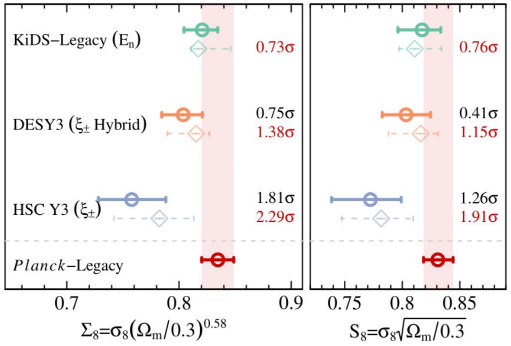


H_0 tension

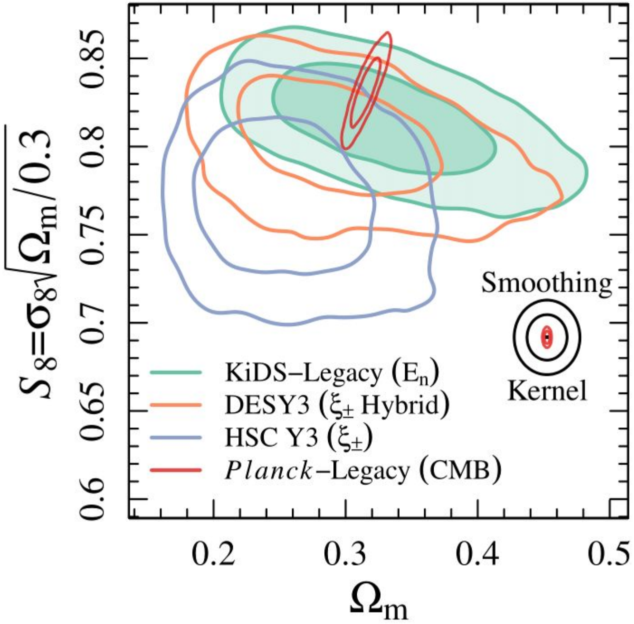
Perivolaropoulos et al. (2022)



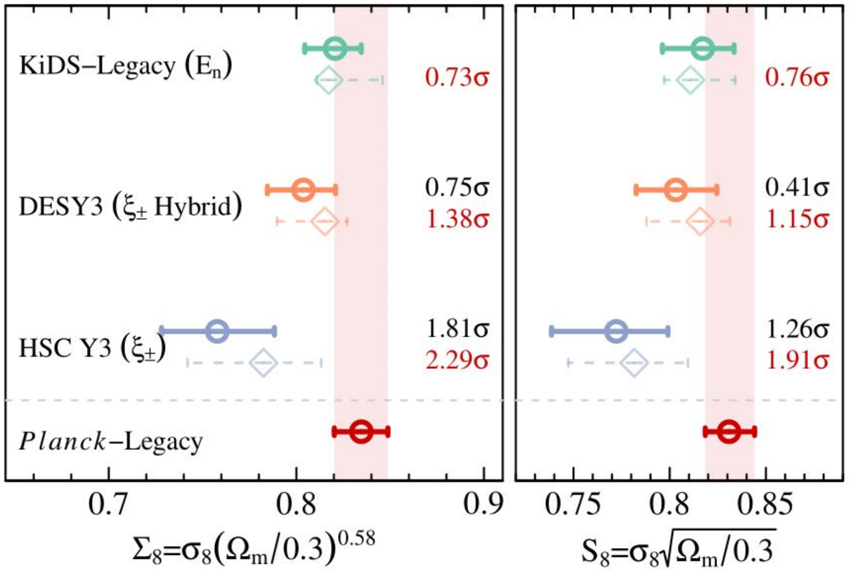
σ_8 tension

Wright et al. (2025)

Why exploring multidimensional tension?



2D shared parameter space



1D marginalizations

Extracted from 'KiDS-Legacy: Cosmological constraints from cosmic shear with the complete Kilo-Degree Survey'. Wright et al. (2025)

Many particles formalism (geometrical interpretation)

$$\vec{r}_k = \frac{1}{\sqrt{\hat{C}_i + \hat{C}_j}} (\vec{\bar{\theta}}_i - \vec{\bar{\theta}}_j) \quad (i \neq j)$$

Tension vector

$$\left(Q_{\mathrm{DM}} = |\vec{r}_{AB}|^2\right)$$

The number of ordered tension vectors, Np, is twice the number of pairs of datasets:

$$\vec{r}_{ij} = -\vec{r}_{ji} \qquad N_p = 2\binom{N}{2} = N \cdot (N-1)$$

We therefore adopt a symmetric construction to avoid ordering ambiguities.

$$\vec{R}_{CM} = \frac{\sum_{k=1}^{N_p} m_k \vec{r_k}}{\sum_{k=1}^{N_p} m_k} = \frac{\sum_{k=1}^{N_p} \vec{r_k}}{N_p} = 0$$

Center of mass of tension vectors

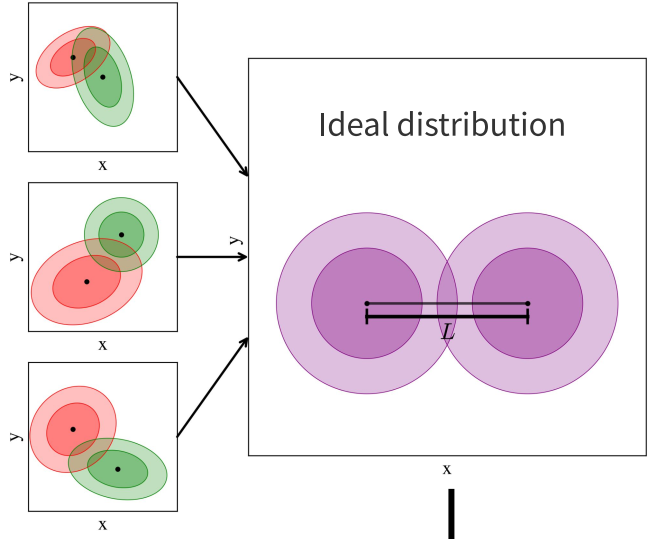
Great, we now have a new estimator that quantifies the tension among three datasets.

Analogously to Q_{DM} , this estimator gives us a PTE and therefore an N_σ.

But how should we interpret these numbers?

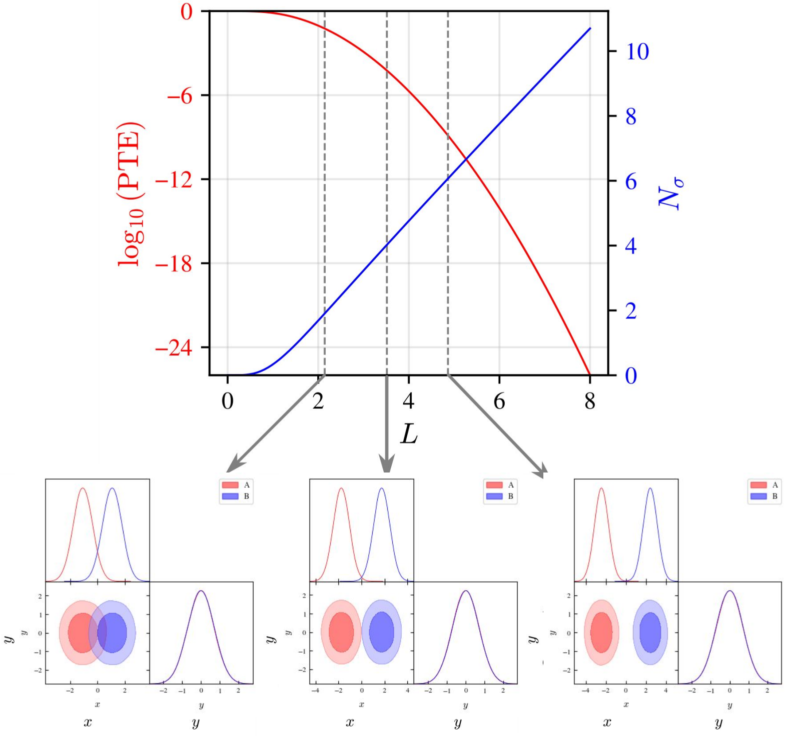
Is there a way to map any three–dataset configuration into a reference geometry where tension becomes intuitive?

Effective tension level mapping ($N_\sigma^{\rm eff}$) N=2



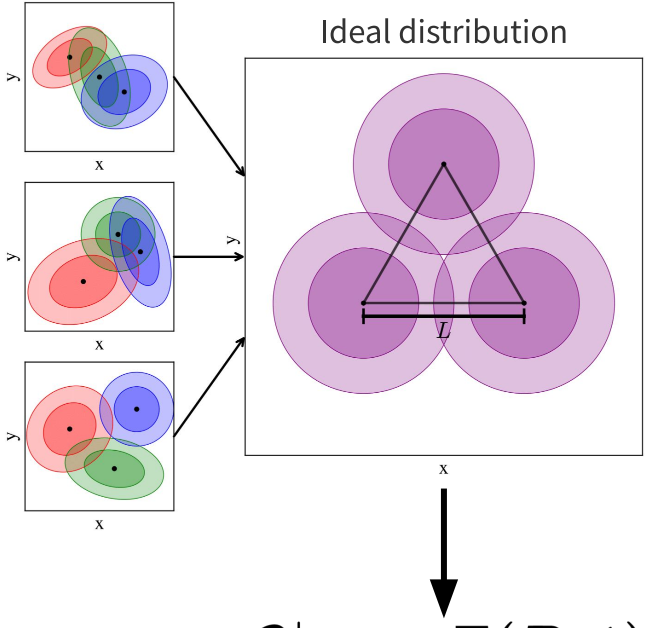
$$Q_{\rm DM}|_{H0} \sim \chi^2_D$$

Analytic null Hypothesis!



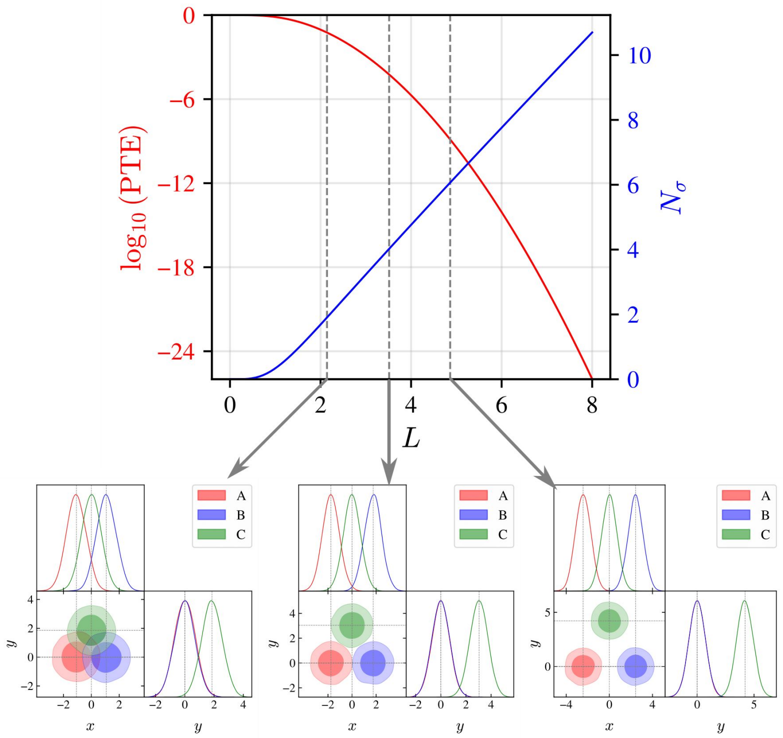
$N_\sigma^{\rm eff} = 1$ $N_\sigma^{\rm eff} = 2$ $N_\sigma^{\rm eff} = 3$

Equilateral triangles mapping (N_σ^{eff}) N=3



$$\mathcal{Q}|_{H0} \sim \Gamma(D, 1)$$

Analytic null Hypothesis!



$N_\sigma^{\mathrm{eff}} = 1$ $N_\sigma^{\mathrm{eff}} = 2$ $N_\sigma^{\mathrm{eff}} = 3$

Summary

- New estimator to evaluate tensions among multiple datasets simultaneously.
- Geometric framework to interpret tensions (two and three datasets) in a 2D shared parameter space.
- $N_{\sigma}^{\mathrm{eff}}$ generalizes the interpretation beyond the conventional N_{σ} (based on a 1D Gaussian analogy).
- Potential applicability to other fields where discrepancies between datasets arise (for example, particle physics).