Exploring stability of $f(Q)$ cosmology near general relativity limit with different connections

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Motivation

- Dark matter, dark energy
- Inflation
- H_0 tension, S_8 tension

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Nonmetricity tensor:

$$Q_{\rho\mu\nu} \equiv \nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \tilde{\Gamma}^\beta{}_{\mu\rho} g_{\beta\nu} - \tilde{\Gamma}^\beta{}_{\nu\rho} g_{\mu\beta}\,. \tag{2}$$

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In connection set 2:

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Observation

The extra function in set 2 doesn't take the role of dark energy and dark matter; instead, it introduces additional dynamics into the equations of motion.

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A stability analysis is conducted approximating the equation of motion near the general relativity limit

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All perturbations smoothly approach the General Relativity limit, and a single model function accounts for all cosmic epochs of the Universe.

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Connection set 2

From the connection equation: $\ln\left(F^*_{QQ}(t)\dot{Q}_*(t)\right) = -3\int H_*(t)dt.$

In matter dominated epoch, $h(t) \sim \frac{1}{t^2}$, $r(t) \sim \frac{1}{t^3}$ $g(t) \sim c_1$,

and the model function $F(Q) = \alpha Q + \beta Q^2$

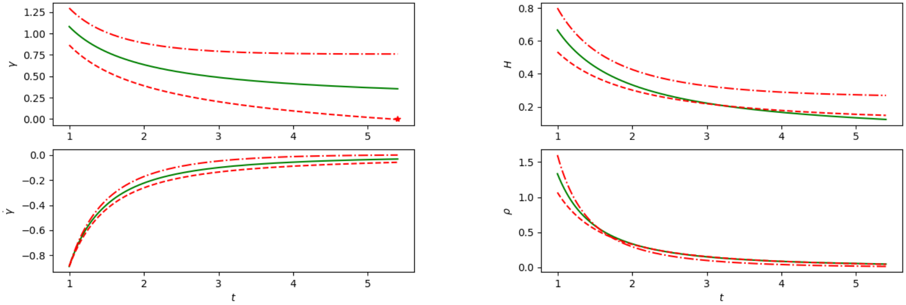
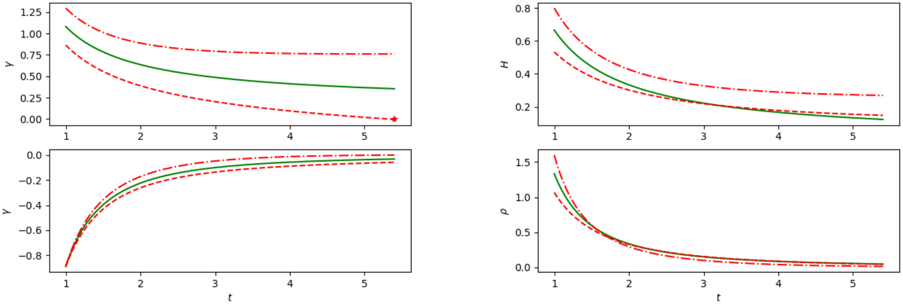


Figure: Numerical solutions of the set 2 system for the model function (**??**) with $\alpha = 1$, $\beta = 1$, $\kappa = 1$. The green solid curves depict the matter dominated solution with initial conditions matching general relativity ($t = 1$, $H = 0.666$, $\rho = 1.333$, $\gamma = 1.081$, $\dot{\gamma} = -0.888$), while the red dashed curves depict slightly perturbed configurations (of $\gamma = 1.281$ and $\gamma = 0.881$ instead). The latter solution hits a singularity by reaching $\gamma = 0$ at about $t = 5$, indicated by a star symbol.



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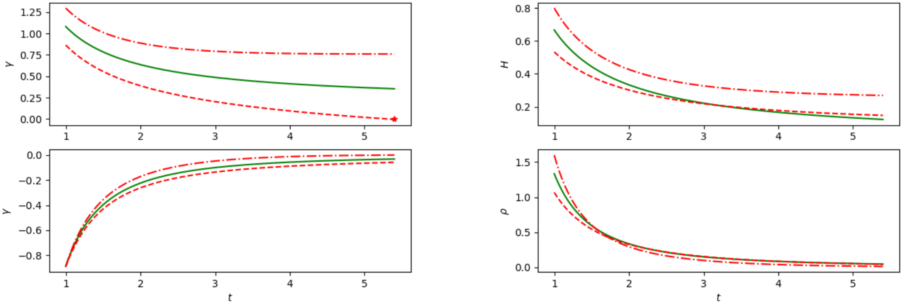
- Alternative connections are problematic already at the background level and obstruct a smooth GR limit.
- Unlike set 1, a single $f(Q)$ model cannot reproduce all cosmic epochs of the Universe; different epochs require different model choices, with stability guaranteed only during the radiation-dominated era.



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