

# The Tip of the Red Giant Branch

Theoretical Foundations & Observational Practice

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# Lecture Roadmap

## Four lectures on the tip of the red giant branch (TRGB)

### Lecture 1 — Theoretical Foundations & Observational Practice

Why can we use TRGB as a standard candle?

→ Stellar evolution: red giant branch, helium flash

How do we measure the TRGB?

→ Measuring the TRGB discontinuity; Sobel filter approach

# Lecture Roadmap (cont'd)

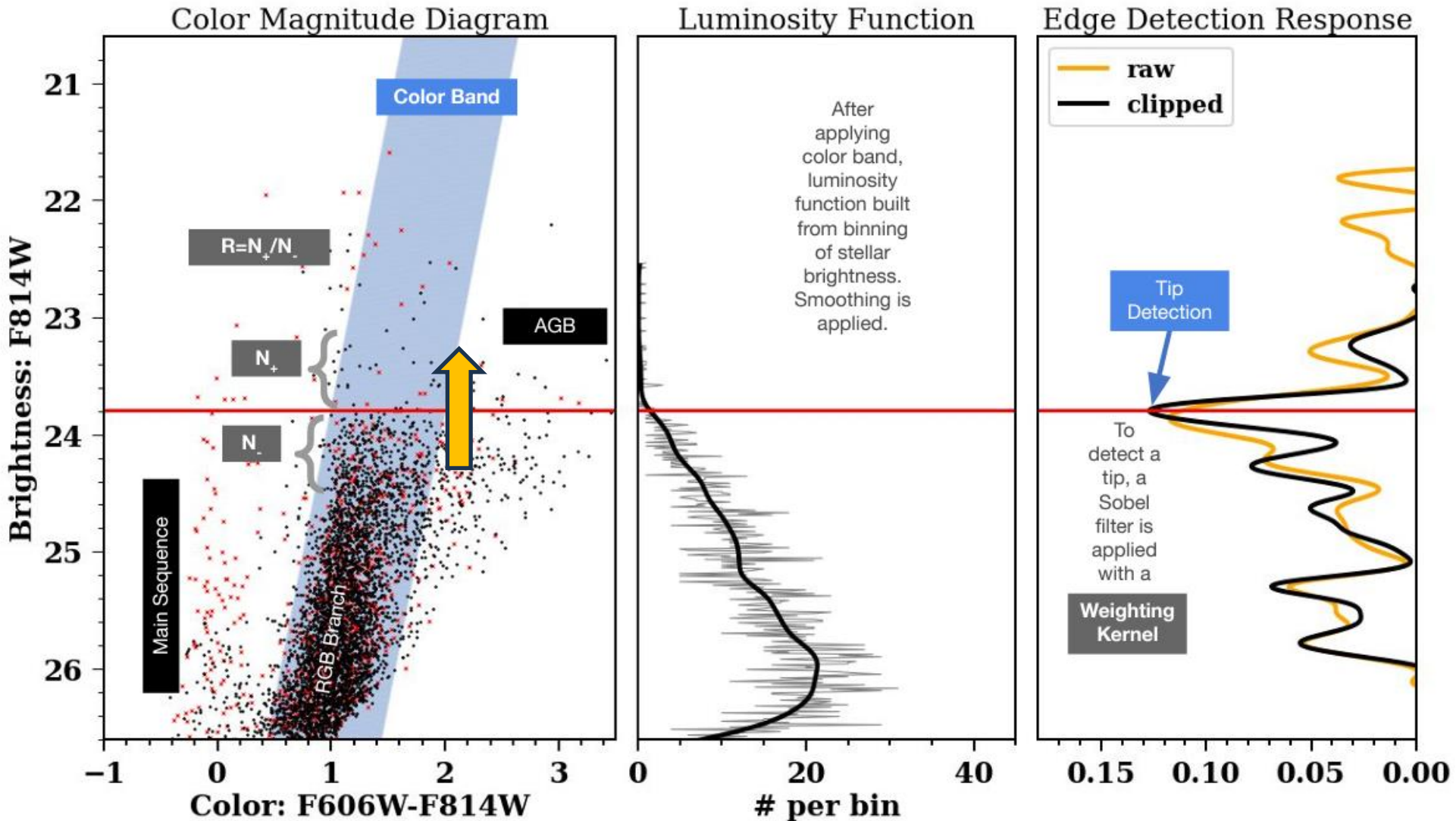
## Lecture 2 — Distances to SN Ia Host Galaxies: On the Way to $H_0$

- › Utilizing the distance ladder to measure distances to galaxies
  - SN Ia calibration
- › Systematic uncertainties
- › Recent TRGB  $H_0$  results (HST & JWST)

## Lectures 3+4 / Tutorials — CosmoVerse Data Challenge Notebook

- › Hands-on: measure TRGB from real HST (Hubble Space Telescope) photometry catalogs; TRGB measurement algorithms

# Quick Overview: TRGB, an Alternative Standard Candle to Cepheids



Section 1

# Stellar Evolution & the RGB

Why can we use the TRGB as a standard candle?

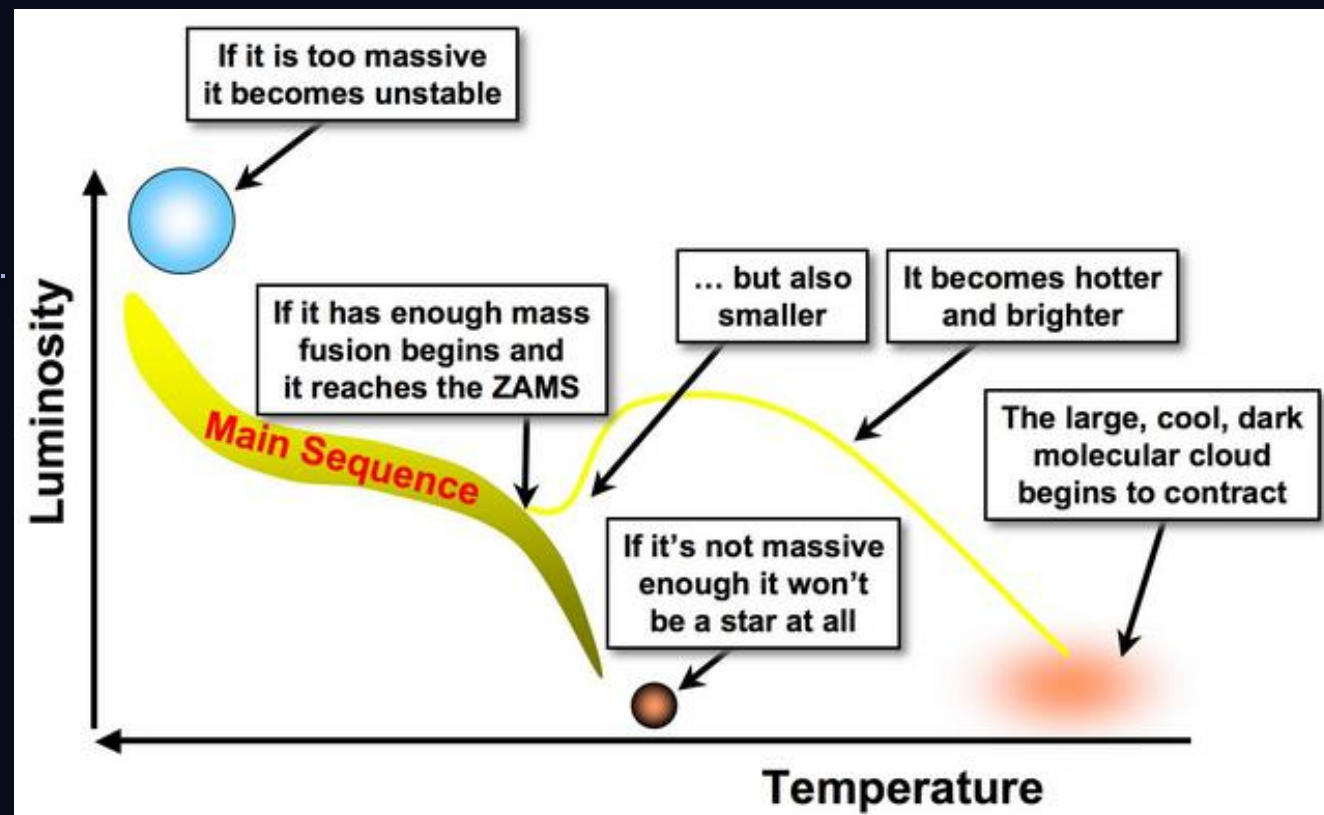
# Stellar Evolution: Main Sequence

Hydrogen core burning: the long stable phase

**Main sequence:** longest + most stable phase of their life cycle, in **hydrostatic equilibrium**

**Mass-Lifetime Correlation:**

$$\tau \sim 10^{10} (M_{\odot}/M)^{2.5} \text{ yr}$$

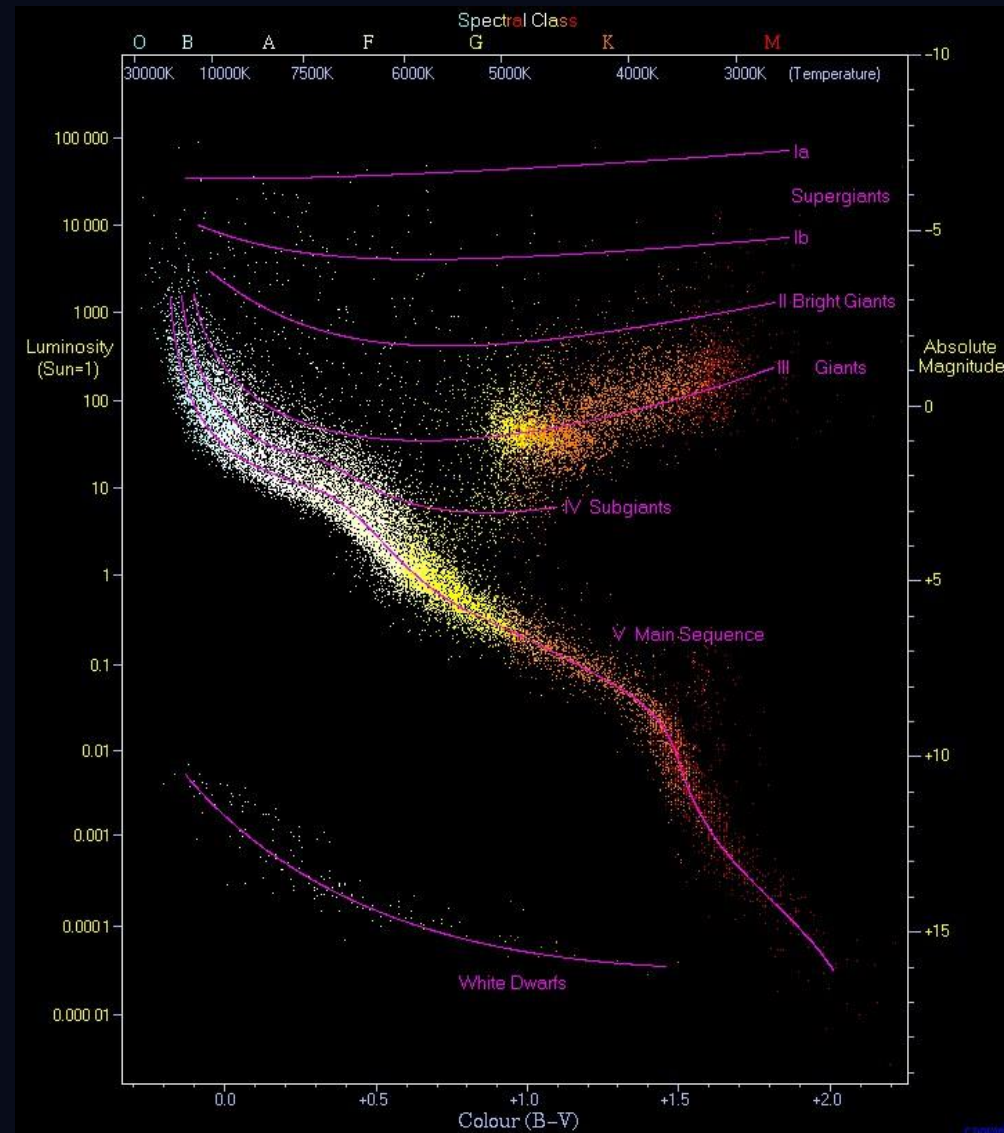


# Which Stars Produce a TRGB?

Low-mass, old stellar populations

## Properties:

- › Mass:  $M \lesssim 2 M_{\odot}$  (low enough to develop degenerate He core — next slides)
- › Age:  $> 1\text{--}2$  Gyr (time needed to evolve off main sequence)



# Which Stars Produce a TRGB? (cont'd)

Low-mass, old stellar populations

## Where to find them (generally):

- › Stellar halos of spiral/elliptical galaxies
- › Old globular clusters
- › Outer disks and dwarf satellite galaxies



*Credit: Wikimedia Commons (CC BY-SA 2.5)*

# Which Stars Produce a TRGB? (cont'd)

Low-mass, old stellar populations

## Why halos for TRGB?

- › Low crowding: stars well separated
- › Low dust extinction
- › Dominated by old stellar population  
(no confusion with young stars)



*Credit: NASA*



# Climbing the Red Giant Branch (cont'd)

Shell burning drives expansion; the star brightens dramatically

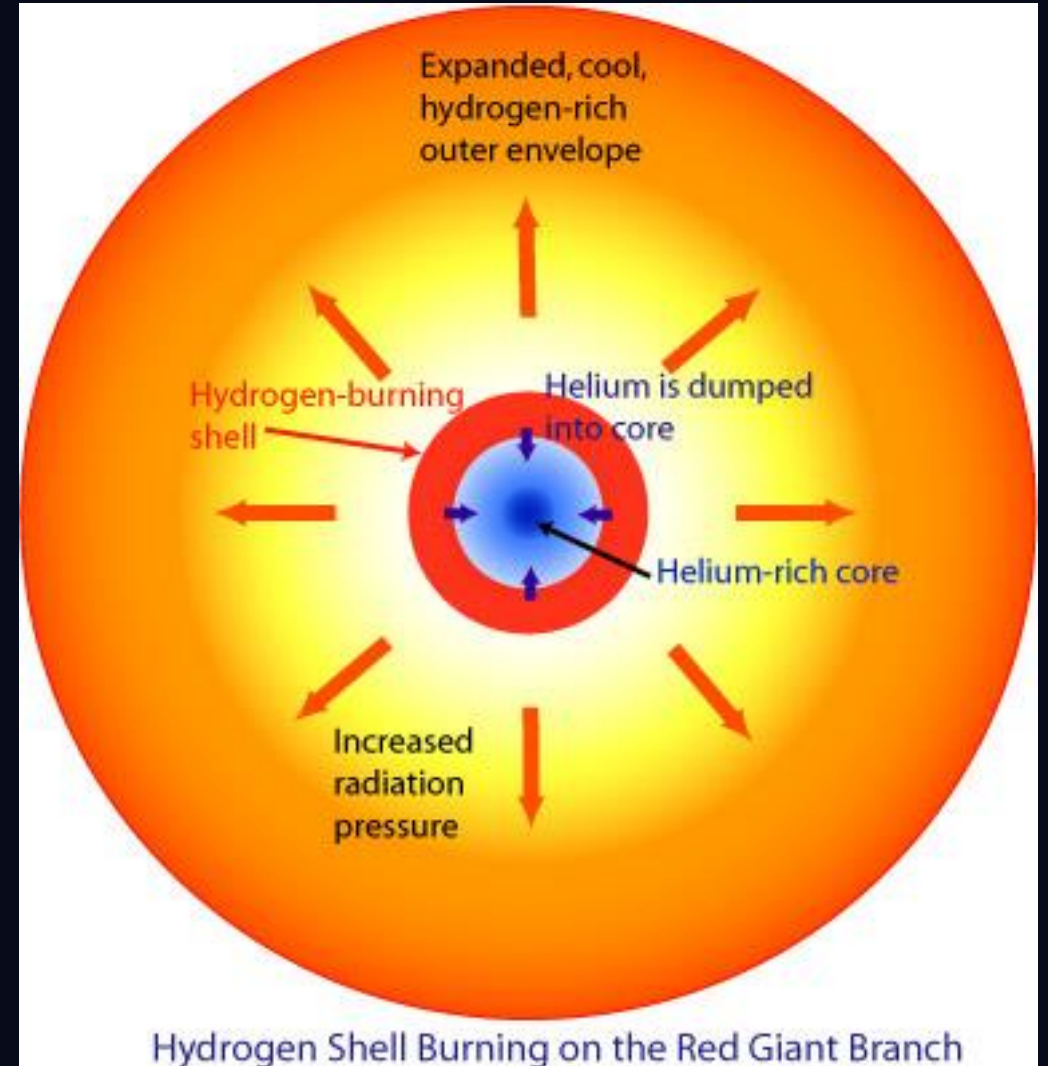
**Core mass  $M_c$  grows as H-shell deposits**

**He ash:**

- ›  $M_c$  rises from  $\sim 0.15 M_\odot \rightarrow \sim 0.50 M_\odot$
- ›  $M_c$  will determine the star's luminosity

**Observable consequences:**

- › Star appears red and very luminous  
(red giant)



# The Degenerate Helium Core

Quantum mechanics sets the stage for the helium flash

## What is electron degeneracy?

- › At high density, electrons fill all available quantum states (Pauli exclusion)
- › Degeneracy pressure:  $P_{\text{deg}}$  depends on density  $\rho$ , NOT temperature  $T$
- › Normal gas:  $P \propto \rho T$  (hotter  $\rightarrow$  higher pressure  $\rightarrow$  expansion)
- › Degenerate gas:  $P \propto \rho^{5/3}$  (temperature change  $\rightarrow$  NO pressure change)

# The Degenerate Helium Core (cont'd)

Quantum mechanics sets the stage for the helium flash

## Why this matters for the He core:

- › Core contracts → density rises → electrons become degenerate
- › T can rise (nuclear reactions begin) WITHOUT triggering expansion
- › Normal thermostat is broken — runaway possible

# Why the Degenerate Core Is So Important

The physics behind the universal tip luminosity

## Analogy: a pressure cooker with a sealed lid

› Normal star:

adding heat → pressure rises → expands → cools (stable)

› Degenerate core:

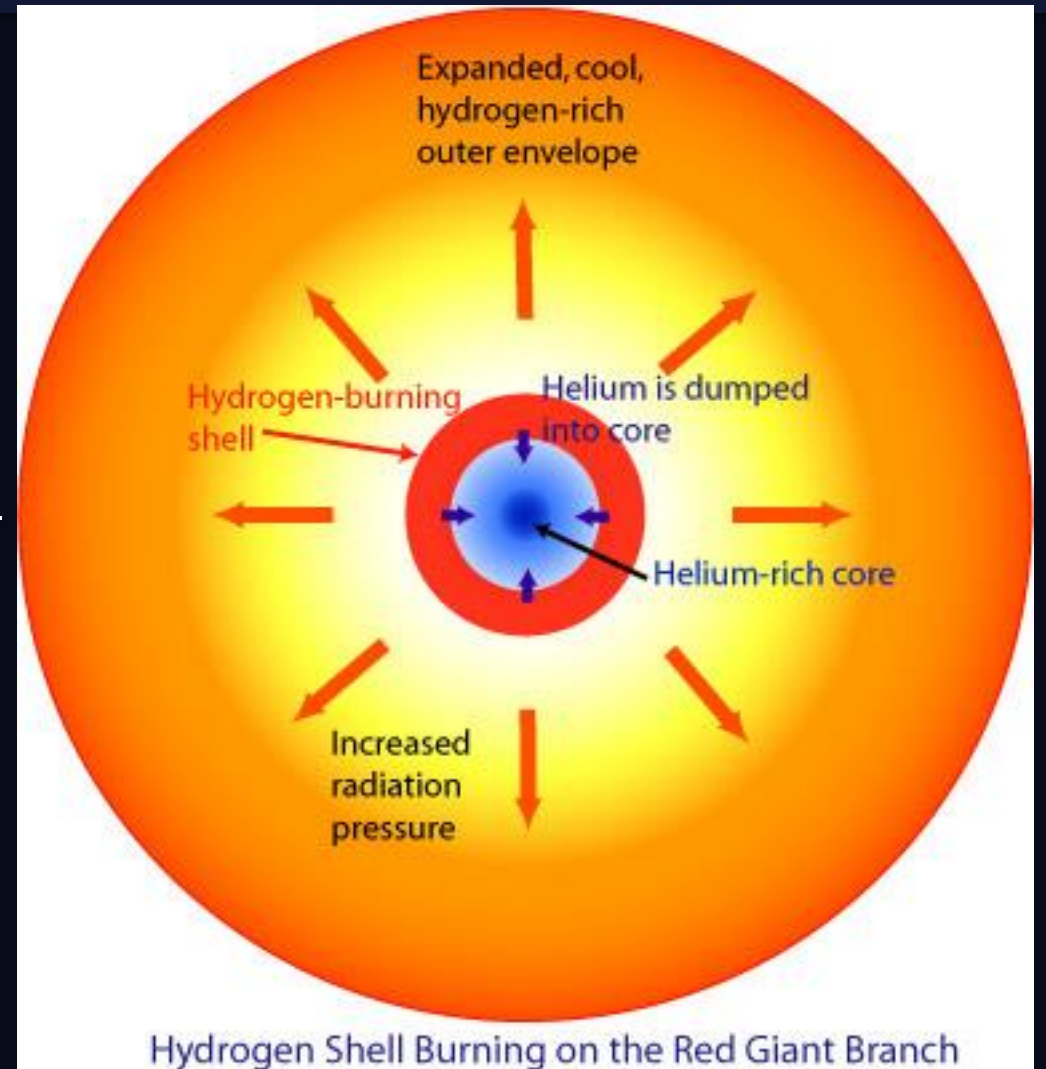
adding heat → temperature rises → more fusion

→ more heat → more fusion → runaway! (until degeneracy lifted)

**The critical core mass  $M_{c,flash} \approx 0.50 M_{\odot}$**

# Core Mass Growth Along the RGB

- ▶  $L \propto M_c^7$  — extreme sensitivity to core mass
- ▶ H-shell deposits He ash  $\rightarrow M_c$  grows slowly
- ▶ Core reaches  $M_c \approx 0.50 M_\odot$  at the RGB tip
- ▶ ★ Helium flash triggers here



Section 2

# The Helium Flash & TRGB

# The Helium Flash

A runaway thermonuclear event in the stellar core

## What happens at the flash:

- ›  $T_{\text{core}}$  reaches  $\sim 10^8$  K  $\rightarrow$  triple-alpha process ignites
- › Degenerate core: no pressure relief  $\rightarrow$  runaway heating
- ›  $L_{\text{core}}$  spikes to  $\sim 10^{11} L_{\odot}$  for hours (as bright as a galaxy!)
- › This energy lifts degeneracy, not expanding the star

# The Helium Flash (cont'd)

A runaway thermonuclear event in the stellar core

## Observable consequence:

- › The flash is NOT visible from outside  
(absorbed by envelope)
- › Star's surface luminosity actually DROPS after the flash
- › Degeneracy is lifted: core expands, shell burning drops

# The Helium Flash (cont'd)

A runaway thermonuclear event in the stellar core

## Duration:

- › The degenerate runaway itself: ~hours
- › Full thermal readjustment:  $\sim 10^5$  yr
- › Star settles onto the Horizontal Branch (He core burning)

# Why Tip Luminosity Is Nearly Universal

The key to using TRGB as a standard candle

## The flash criterion is a fixed nuclear physics threshold:

- › Triple-alpha reaction rate  $\propto T^{40}$  — extremely temperature sensitive
- › He ignition requires  $T_{\text{core}} \geq 10^8$  K regardless of stellar mass or metallicity
- › Degeneracy pressure sets the mass at which this T is achieved:  $M_{\text{c,flash}} \approx 0.50 M_{\odot}$

# Why Tip Luminosity Is Nearly Universal (cont'd)

The key to using TRGB as a standard candle

## Variations in $M_c$ , flash (theory):

- › Age (1–12 Gyr):  $\Delta M_c \approx 0.003 M_\odot \rightarrow \Delta M_I \approx 0.02$  mag (small)
- › Metallicity ([Fe/H] -2.5 to 0):  $\Delta M_I \approx 0.1$  mag in I-band (small, correctable)
- › Mass (0.6–2  $M_\odot$ ):  $\Delta M_c < 0.02 M_\odot$  (well within tolerance)

## Net result:

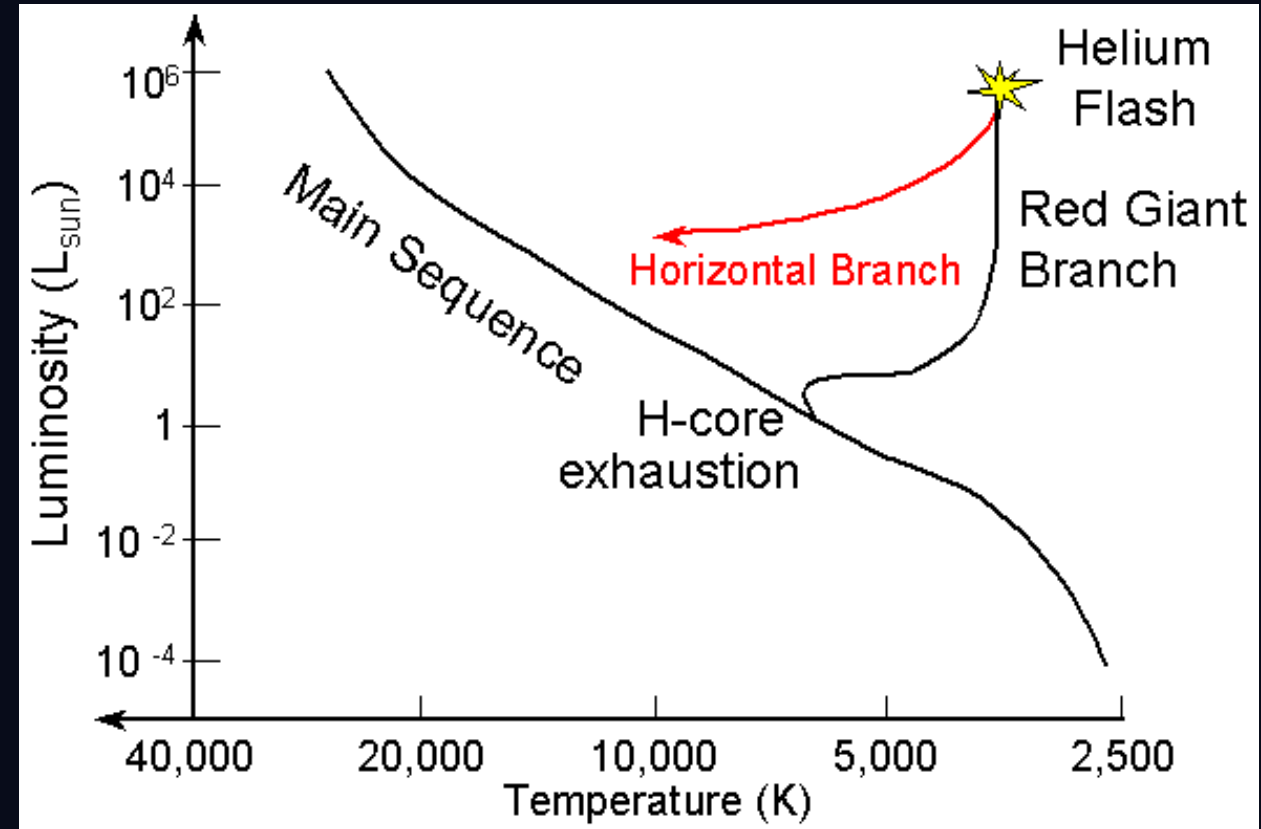
- ›  $M_{I,TRGB} \approx -4$  mag with little intrinsic scatter

# The Magic Number: $M_{c,flash} \approx 0.50 M_{\odot}$

- ▶ This single number anchors the TRGB standard candle
- ▶ Set by quantum mechanics (degeneracy)
  - + nuclear physics (triple-alpha)
- ▶ Valid for ALL low-mass stars ( $0.6-2 M_{\odot}$ ) of ANY age  $> 1$  Gyr
- ▶ Scatter:  $\pm 0.01 M_{\odot} \rightarrow \pm 0.06$  mag in  $M_I$  (dominated by metallicity)
- ▶ See i.e. Salaris & Cassisi 1997, 2005

# After the Flash — The Horizontal Branch

- ▶ He flash lifts core degeneracy  
→ star contracts
- ▶ Surface luminosity DROPS by  $\sim 1.5\text{--}2$  mag
- ▶ He core begins burning stably  
(non-degenerate)
- ▶ Star settles on Horizontal Branch



# After the Flash — The Horizontal Branch (cont'd)

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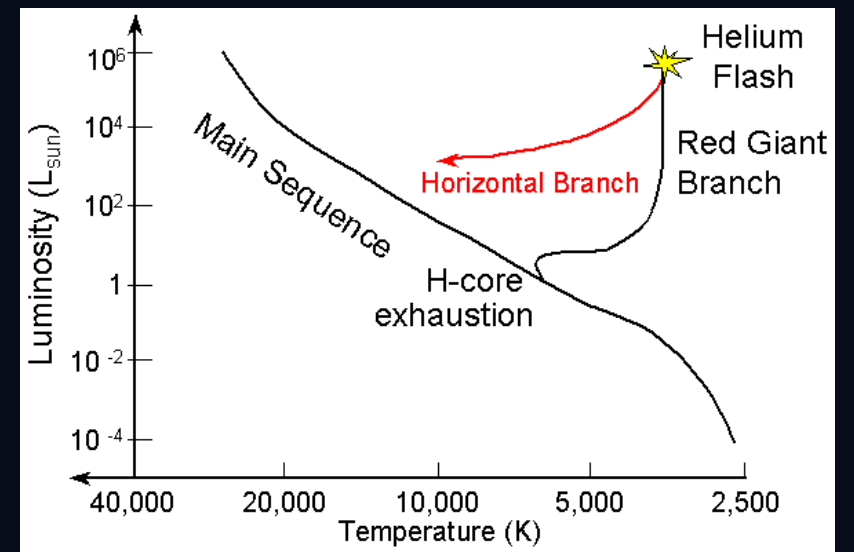
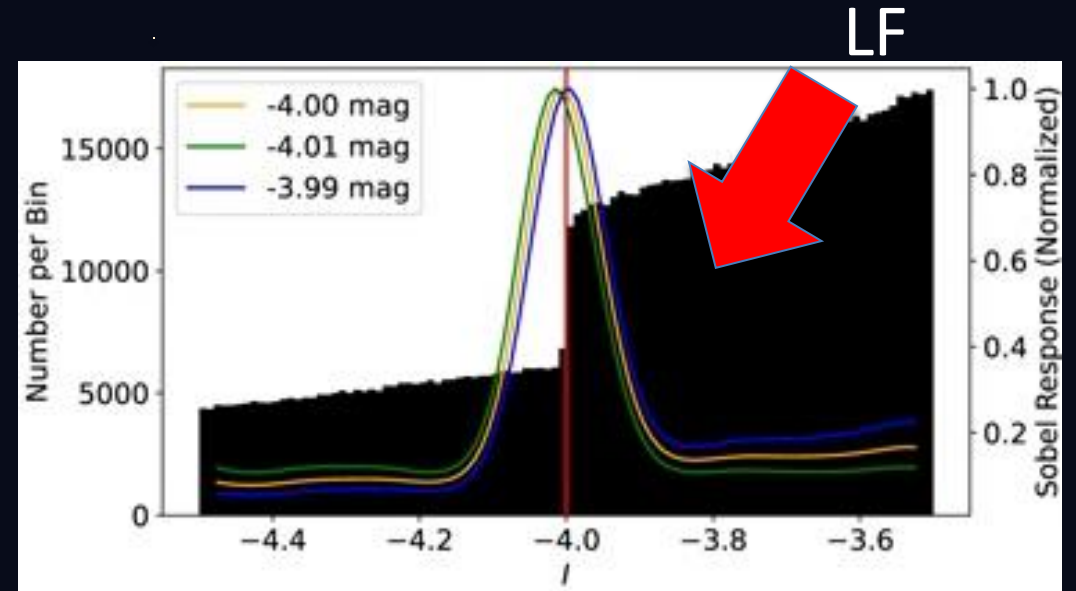
- ▶ HB appears as horizontal smear in HR diagram ▶ Duration of HB phase: ~100 Myr
  - ▶ After HB: star climbs the Asymptotic Giant Branch (AGB) — stars burning both He and H shells
  - ▶ AGB stars can be BRIGHTER than the TRGB → noise!

Section 3

# Observational Basis of the TRGB as a Standard Candle

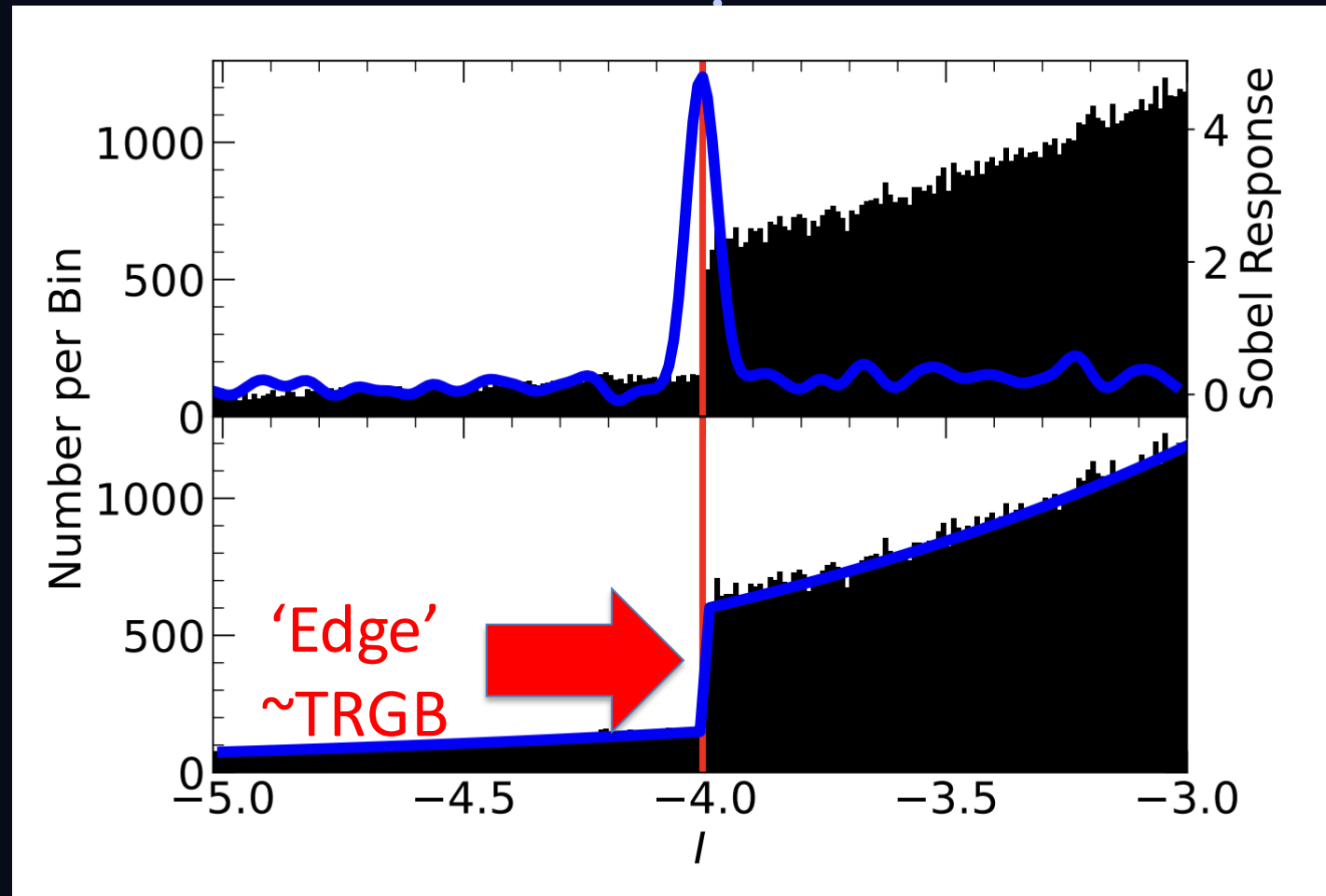
# The RGB Luminosity Function

- ▶ **Key Term:** Luminosity Function (LF): histogram of stellar brightnesses
- ▶ Below (brighter than the) TRGB: power law rise in counts (more stars brighter  $\rightarrow$  fainter direction)
- ▶ At TRGB: sudden sharp DROP — helium flash removes stars above tip



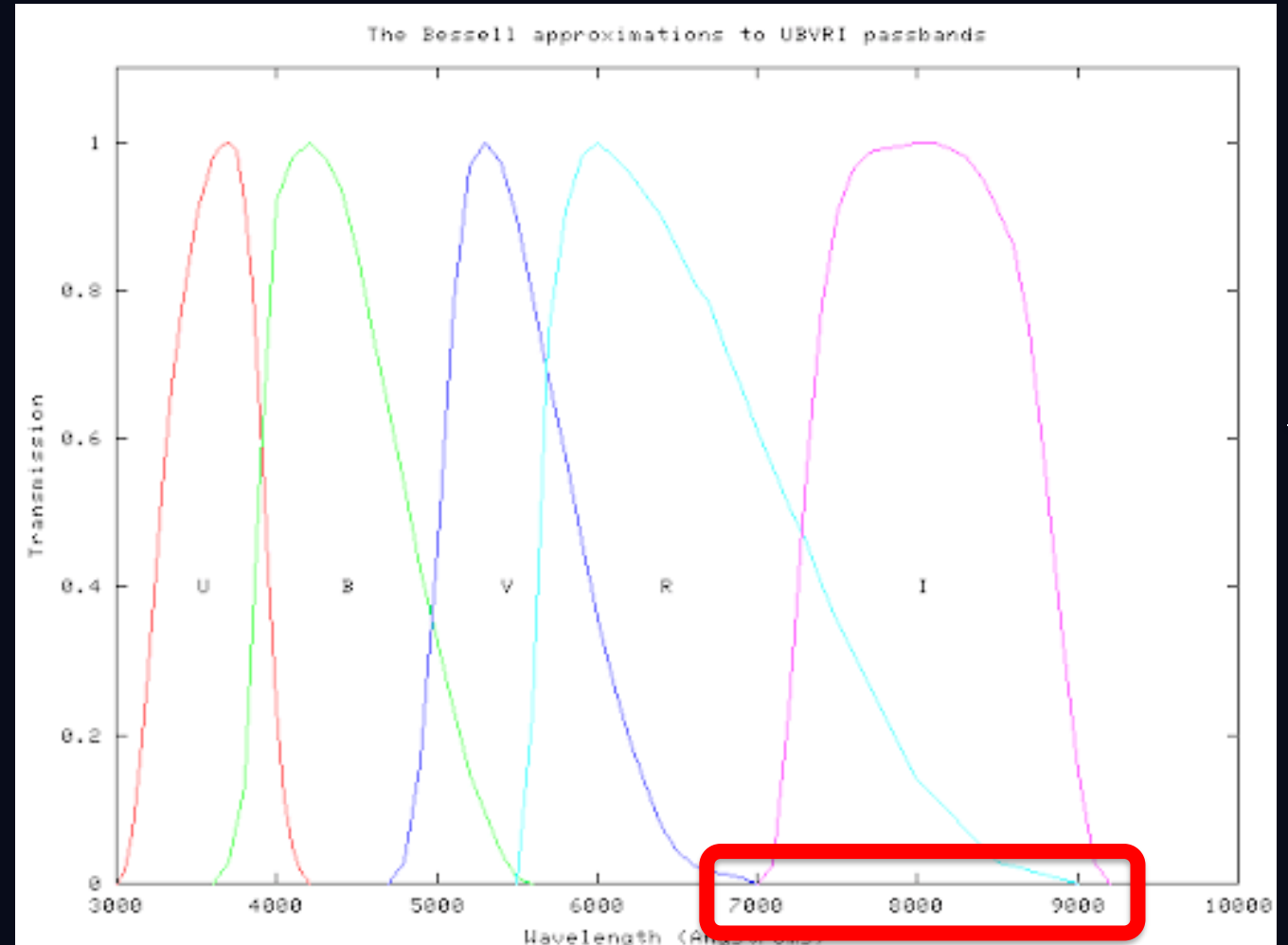
# The RGB Luminosity Function (cont'd)

Goal: find the magnitude of the discontinuity (the edge)



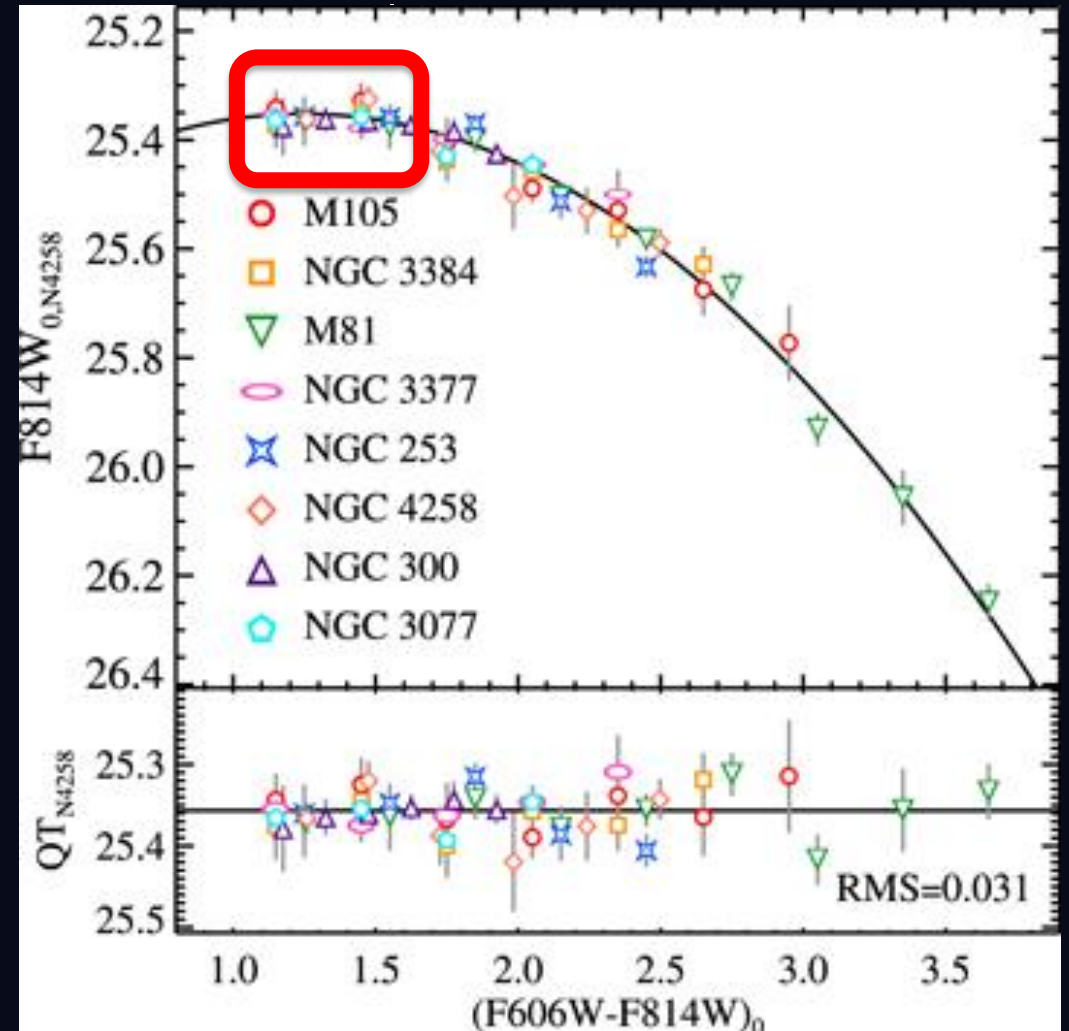
# Why the I-Band?

- ▶ I-band:  $\sim 700\text{--}900$  nm
- ▶ F814W  $\rightarrow$  HST filter
- ▶ F090W  $\rightarrow$  JWST filter



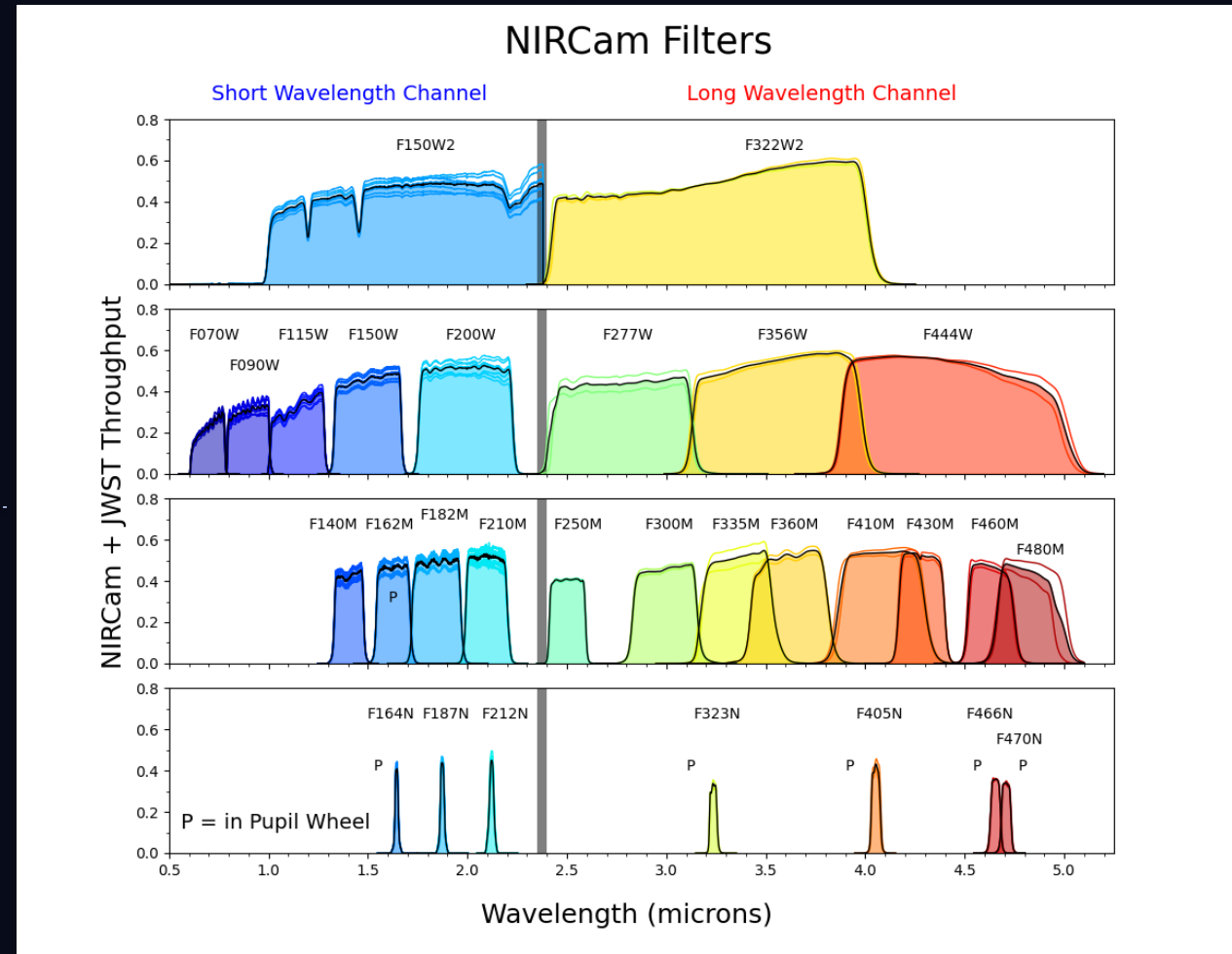
# Why the I-Band? HST Filters

- ▶ TRGB has known metallicity dependence (traced by color; line blanketing)
  - Blue absorption (metals); red emission
- ▶ Nearly-flat TRGB in blue regime  
e.g.  $< F606W - F814W < 1.5 \text{ mag (HST)}$
- ▶ Past studies have measured how TRGB changes w/ color  
(e.g. Rizzi+07, Jang+17, Newman+24)
- ▶ Data challenge uses Jang+17 calibration



# Use of JWST filters

- ▶ F090W (JWST) ~ I-band; can also take advantage of this low color sensitivity
- ▶ Excellent opportunity to crosscheck HST F814W TRGB measurements (e.g. Li+24, Anand+24, Freedman+25)

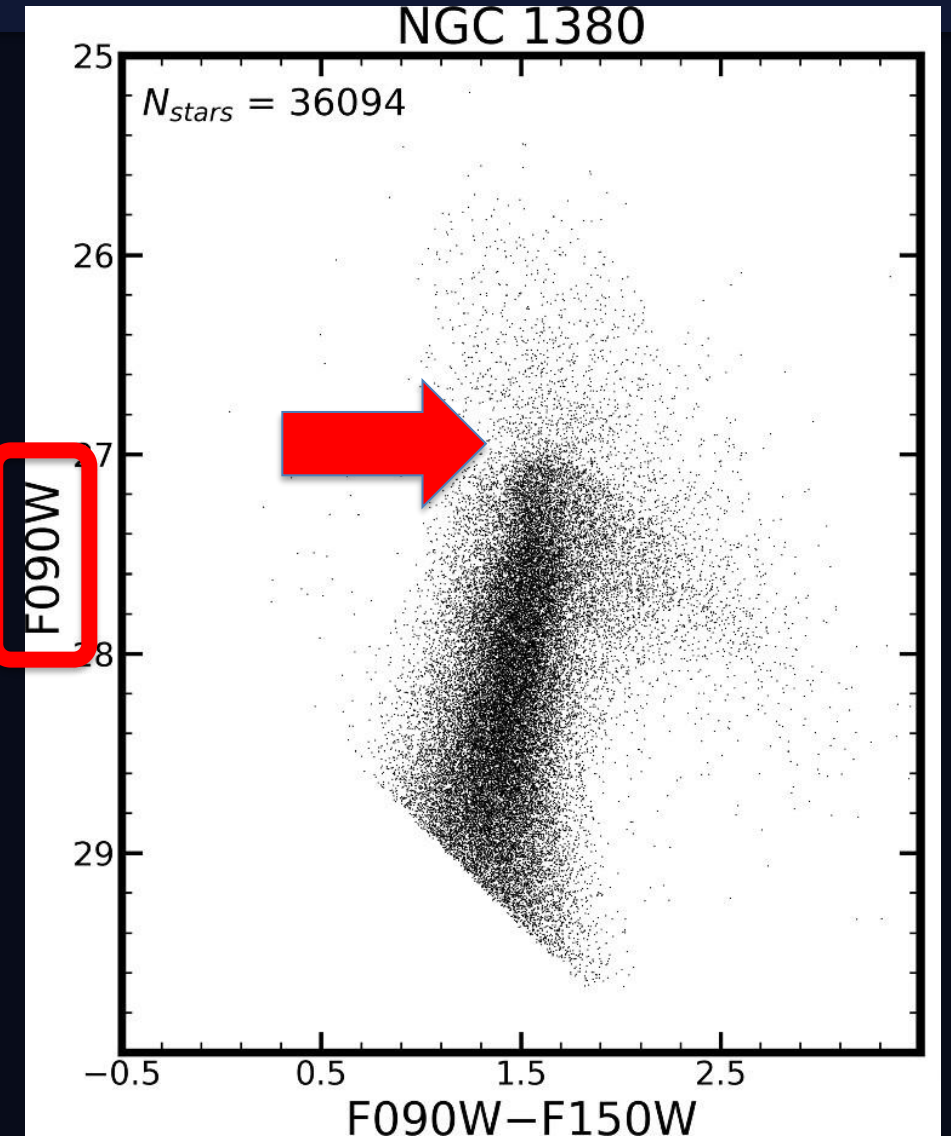


# Use of JWST filters

- ▶ Same concept, flat region before color correction is needed
- ▶ For simplicity, many TRGB measurements restricted to this area; color rectification possible (e.g. see Newman+24)

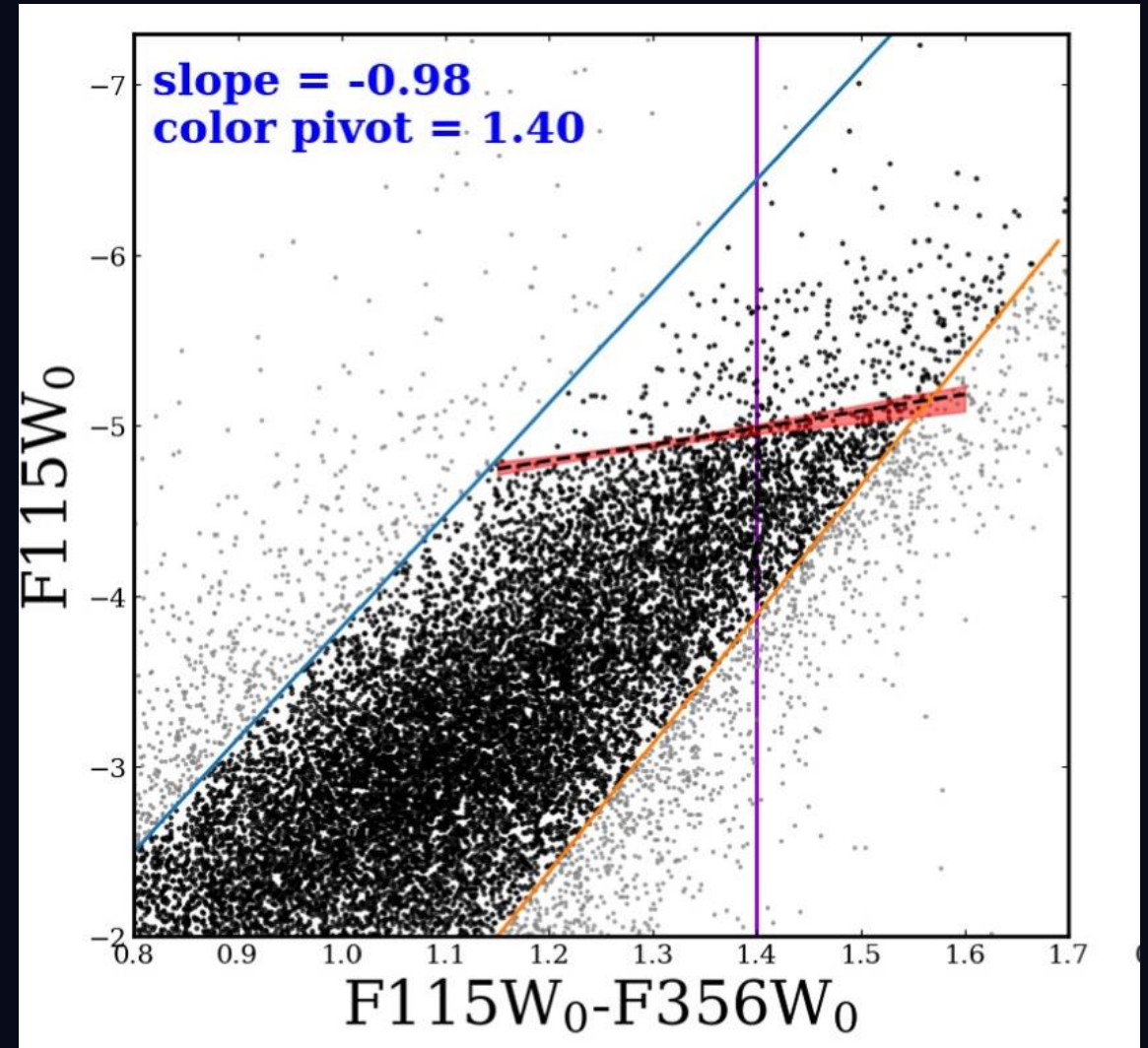
JWST  
Filter

F090W



# Use of JWST filters

- ▶ Sticking to blue I-band: Simple, but fewer stars
- ▶ Another approach: use all stars regardless of color
- ▶ Calibrate slope to reference color and rectify (e.g. Madore, Newman+24)

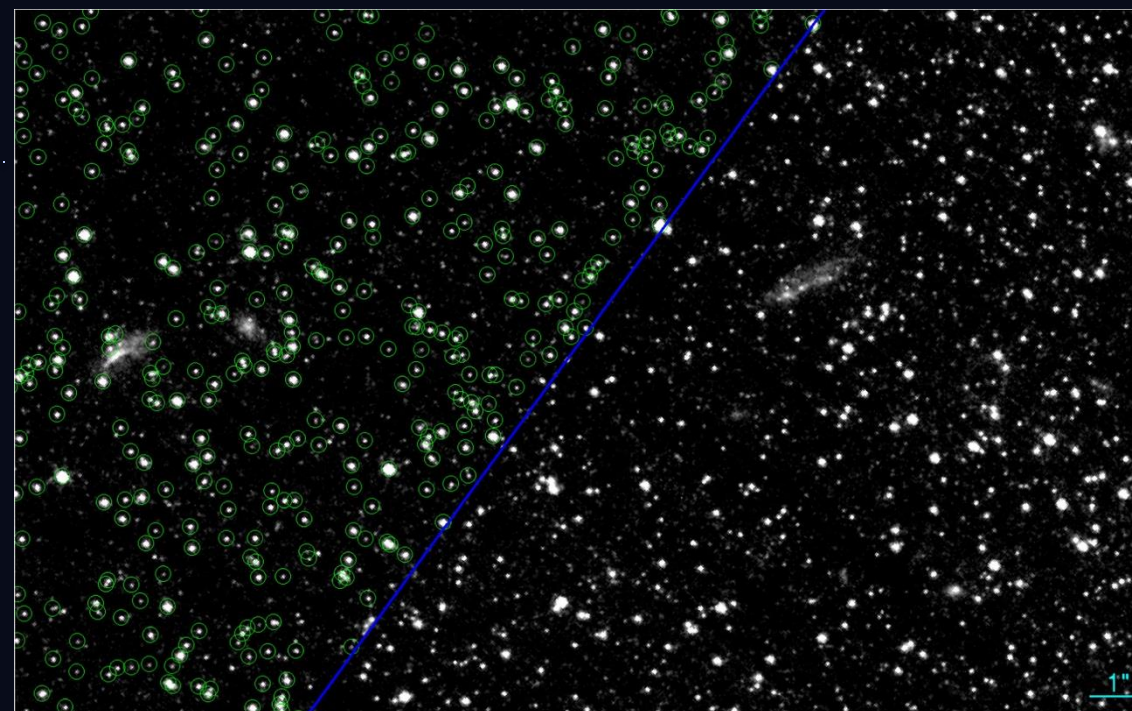


# Choosing the Right Target Field

Where to point the telescope for a clean TRGB measurement

## Ideal field properties:

- › Stellar halo of target galaxy
- › Away from bright foreground stars (avoid diffraction spikes)
- › Low foreground dust:  $E(B-V)$   
 $E(B-V)$  = color excess = reddening  
by dust between us and target



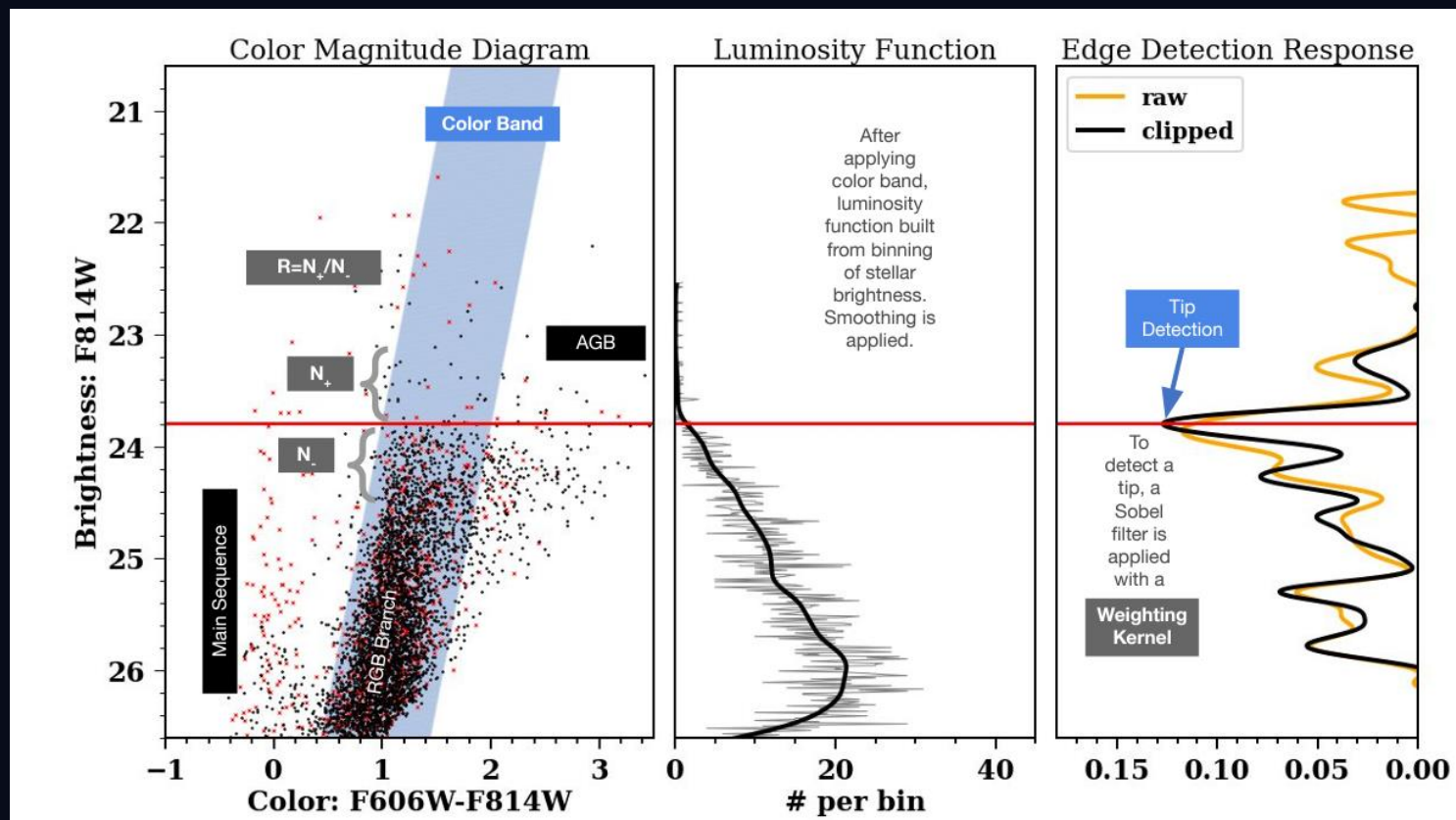
Credit: Anand+24

# Choosing the Right Target Field

Where to point the telescope for a clean TRGB measurement

## Robustness metrics:

- ›  $\geq \sim 400$  RGB stars between TRGB and TRGB+1 (fainter; Madore+09)
- › Contrast ratio  $> 3$  (CATs; Wu+23, Li+23, Scolnic+23)



# Choosing the Right Target Field

Where to point the telescope for a clean TRGB measurement

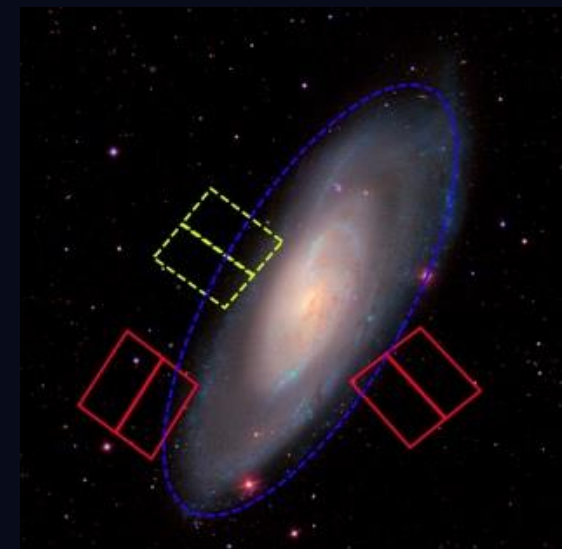
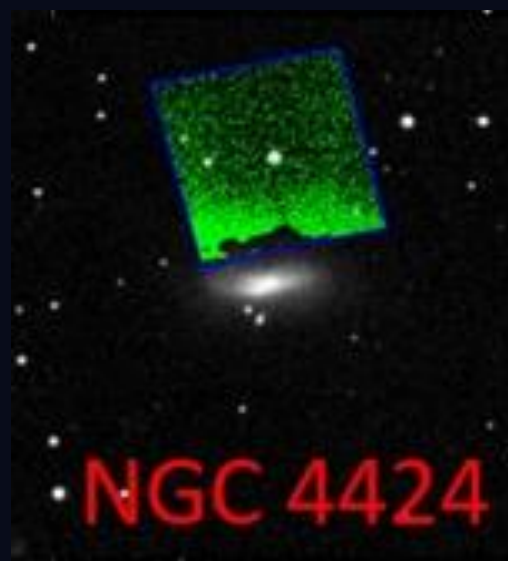
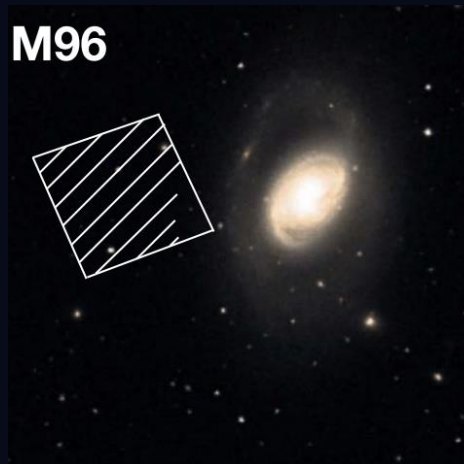
Spatial clipping: different approaches

› By eye (e.g. Freedman+19)

› R25 ellipse (e.g. Anand+22)

→ B-mag 25<sup>th</sup> isophote

› Density clipping (e.g. CATS)

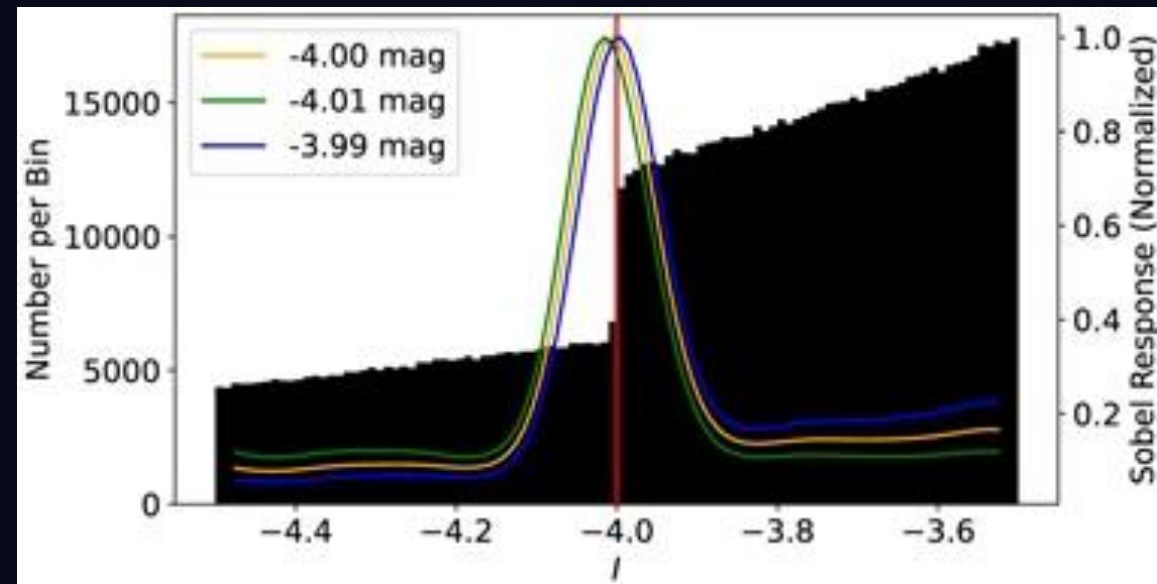


# The Problem — Finding the Edge

Poisson statistics make the sharp edge noisy

## The ideal TRGB LF:

→ Sharp step function in the luminosity function at  $m = m_{\text{TRGB}}$



Li+23

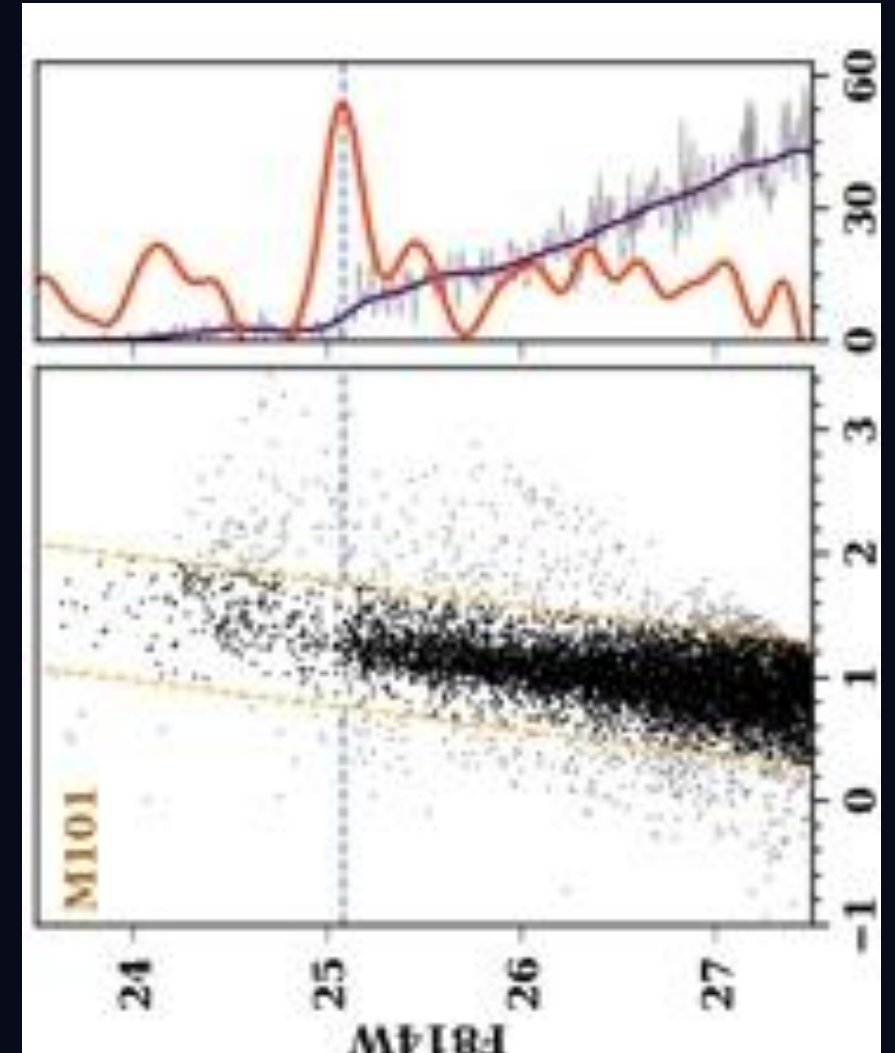
$$g(m) = \begin{cases} 10^{a(m-m_{\text{TRGB}})} , & \text{if } m - m_{\text{TRGB}} > 0 , \\ 10^{b(m-m_{\text{TRGB}})-c} , & \text{if } m - m_{\text{TRGB}} < 0 . \end{cases}$$

# The Problem — Finding the Edge

Poisson statistics make the sharp edge noisy

## Real data complications:

- › Finite star counts: Poisson noise  $\propto \sqrt{N}$  in each magnitude bin
- › Photometric uncertainties: stars scatter across the true edge
- › AGB contamination: AGB stars appear above the TRGB
- › Incompleteness: stars near the limiting magnitude are missed



# GLOESS Smoothing

Suppressing Poisson noise before edge detection

## Why smooth?

- › Near the TRGB, star counts are low  $\rightarrow$  Poisson fluctuations ( $\sim\sqrt{N}$ ) are large
- › A raw luminosity function is too noisy for derivative-based edge detection

## What is GLOESS?

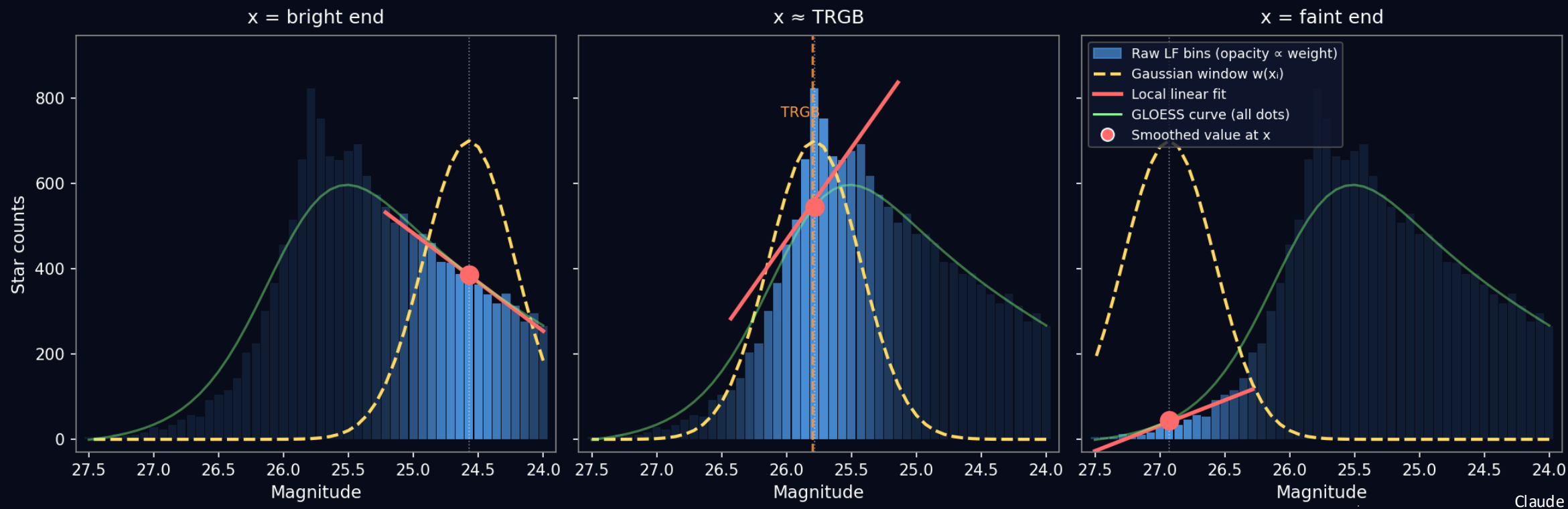
- › Gaussian-windowed LOcal regrESSion (Monson+04, Persson+17)
  - $\rightarrow$  smooth the LF while preserving sharp gradients
- › Each output bin gets a weighted local linear fit using a Gaussian window of width  $\tau$ :

$$w_{ij} = \exp[ -(x_i - x_j)^2 / 2\tau^2 ] \quad \rightarrow \quad \hat{N}_i = a + b \cdot x_i$$

# GLOESS Smoothing

Suppressing Poisson noise before edge detection

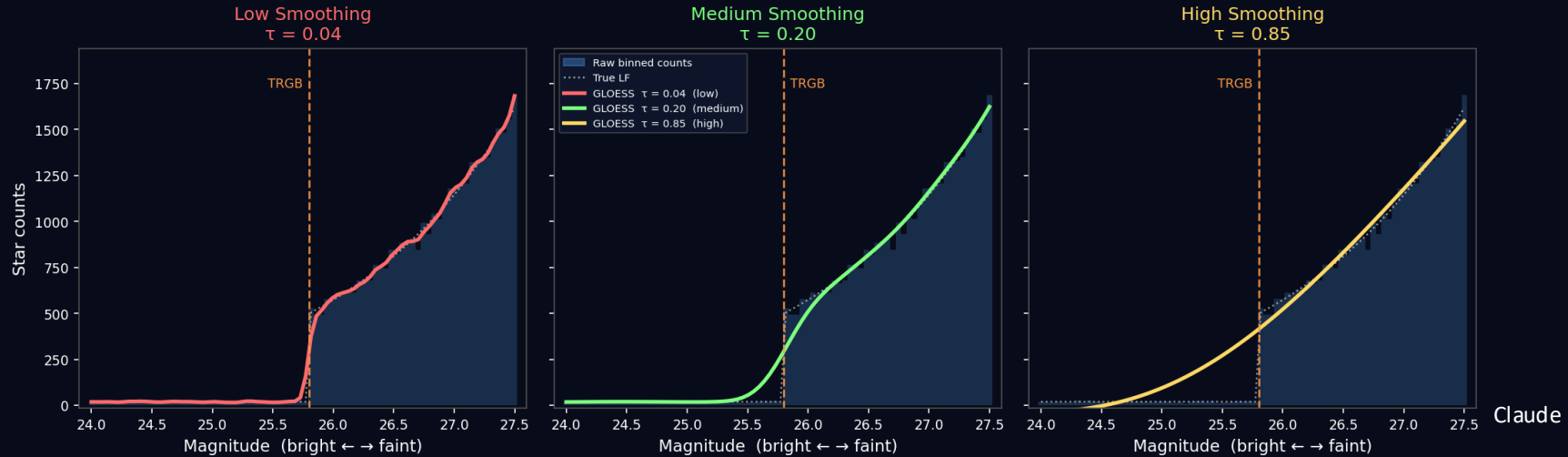
## GLOESS: Gaussian-Windowed Local Linear Smoothing



# GLOESS Smoothing

Suppressing Poisson noise before edge detection

Effect of Smoothing Scale  $\tau$  on GLOESS — RGB Luminosity Function  
Smoothing values are exaggerated for illustration purposes only



**Key parameter  $\tau$  (smoothing bandwidth in magnitudes):**

- › Too small ( $\tau \approx 0.05$ ): LF still noisy  $\rightarrow$  false edge detections
- › Too large ( $\tau \approx 0.30$ ): TRGB edge blurred  $\rightarrow$  distance biased faintward
  - › Typical working range: 0.05 – 0.20 mag

**Best practice: run multiple  $\tau$  values  $\rightarrow$  TRGB should be stable**

# The Sobel Filter — Edge Detection

- ▶ Adopted from 1D image edge detection (Sobel operator in computer vision).
- ▶ Most common:  $[-1, 0, +1]$ , used by modern implementations
- ▶ Variants exist e.g.  $[-2, -1, 0, +1, +2]$  (Sakai+96, etc.)

Original Image



Sobel Edge Detection



Towardsdatascience.com

$$S(m) = \phi(m+1) - \phi(m-1)$$

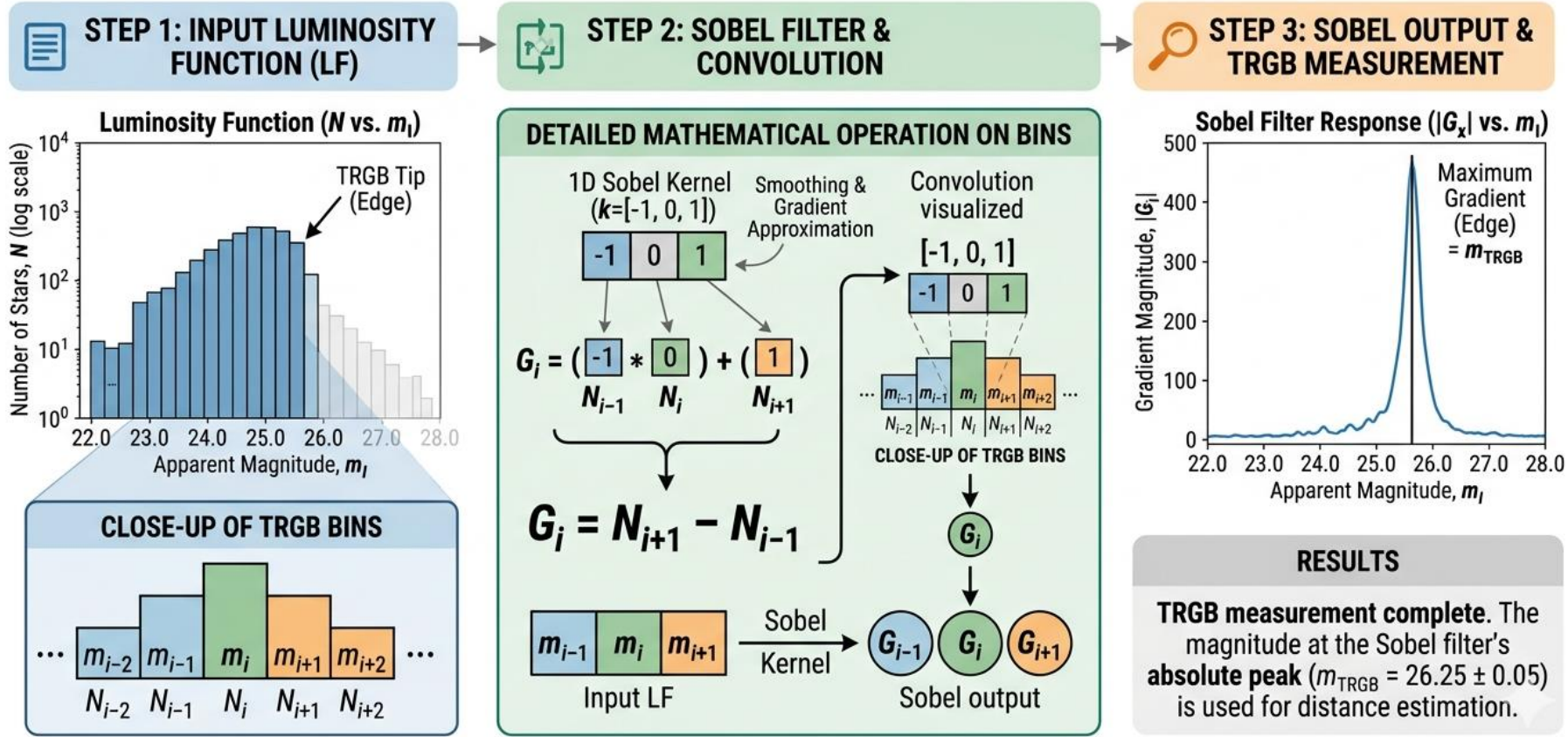
# The Sobel Filter — Edge Detection

$$S(m) = \phi(m+1) - \phi(m-1)$$

- ▶  $S(m)$  = Sobel response at magnitude bin  $m$
- ▶  $\phi(m)$  = star count in magnitude bin  $m$  (from the luminosity function)
- ▶ Peak of  $S(m)$  locates the TRGB:  $m_{\text{tip}} = \text{argmax}[S(m)]$
- ▶ Bins above vs below the edge: compares "above-edge" to "below-edge" counts

# Sobel Detection — Worked Example

## MEASURING TRGB WITH SOBEL FILTER: LF EDGE DETECTION



# Sobel Weighting Schemes

**Sobel response:**  $\phi_i = f(N_{\{i+1\}}) - f(N_{\{i-1\}})$

- › Three-point finite difference on the smoothed LF
- › TRGB = magnitude where  $\phi_i$  is maximized

**Three weighting schemes:**

› **Simple:**  $\phi_i = N_{\{i+1\}} - N_{\{i-1\}}$

raw finite difference — no noise correction

› **Poisson:**  $\phi_i = \sqrt{N_{\{i+1\}}} - \sqrt{N_{\{i-1\}}}$

variance-stabilizing transform; compresses dynamic range

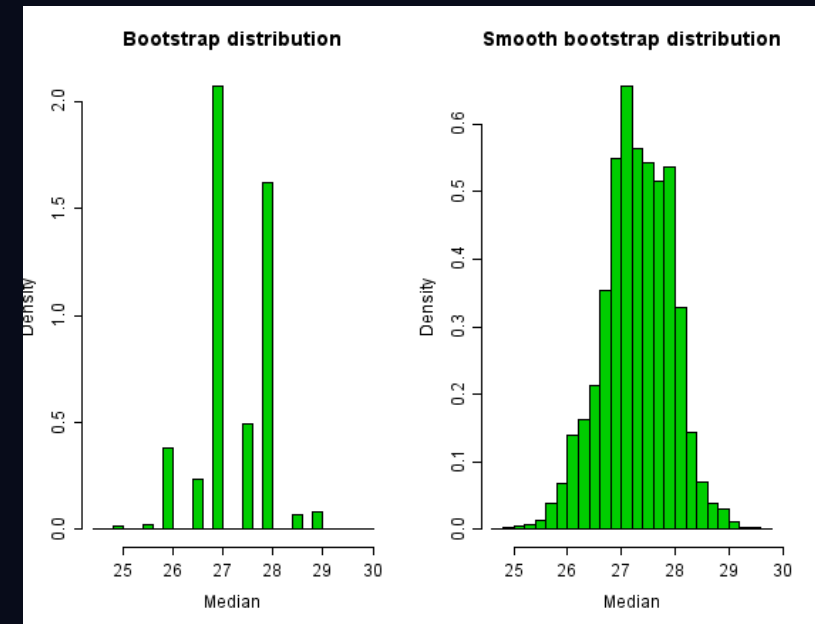
› **Hatt:**  $\phi_i = (N_{\{i+1\}} - N_{\{i-1\}}) / \sqrt{N_{\{i+1\}} + N_{\{i-1\}}}$

$\approx$  signal-to-noise of the gradient  $\rightarrow$  down-weights low-count noise-dominated bins  
(see Hatt et al. 2017)

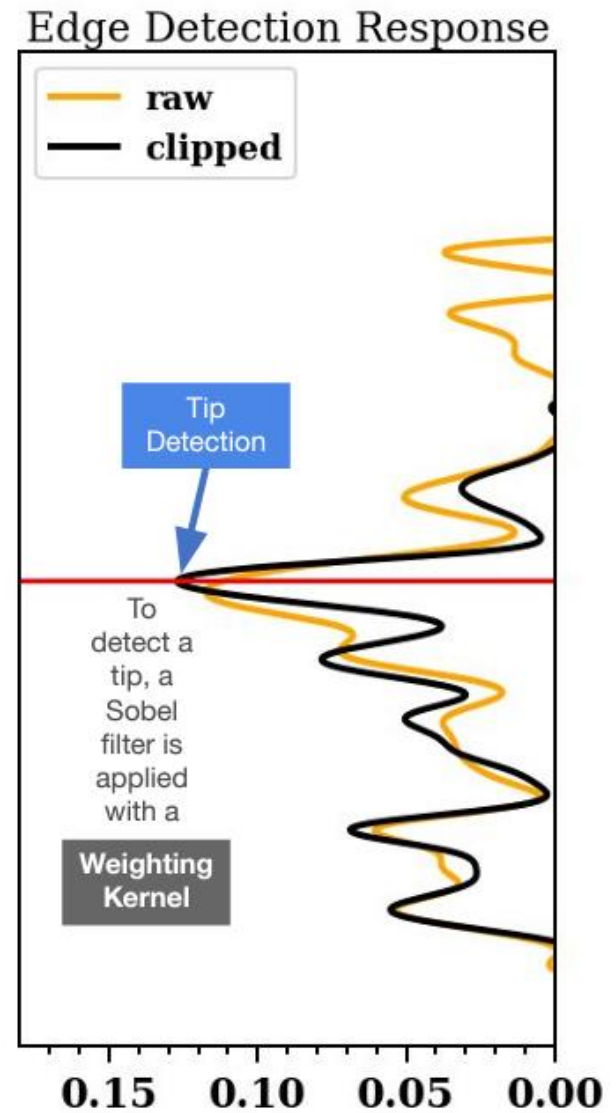
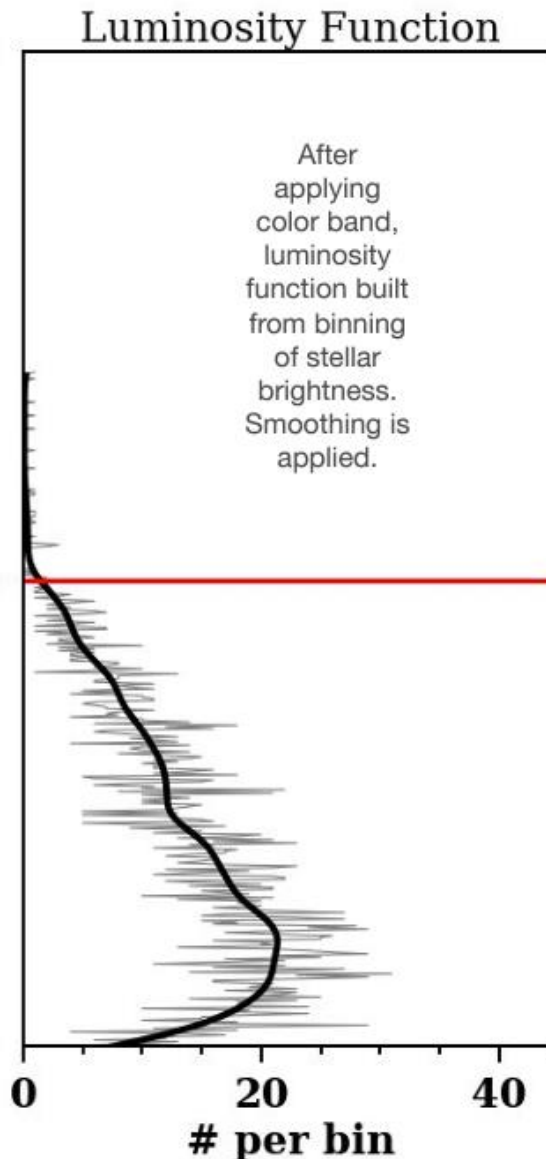
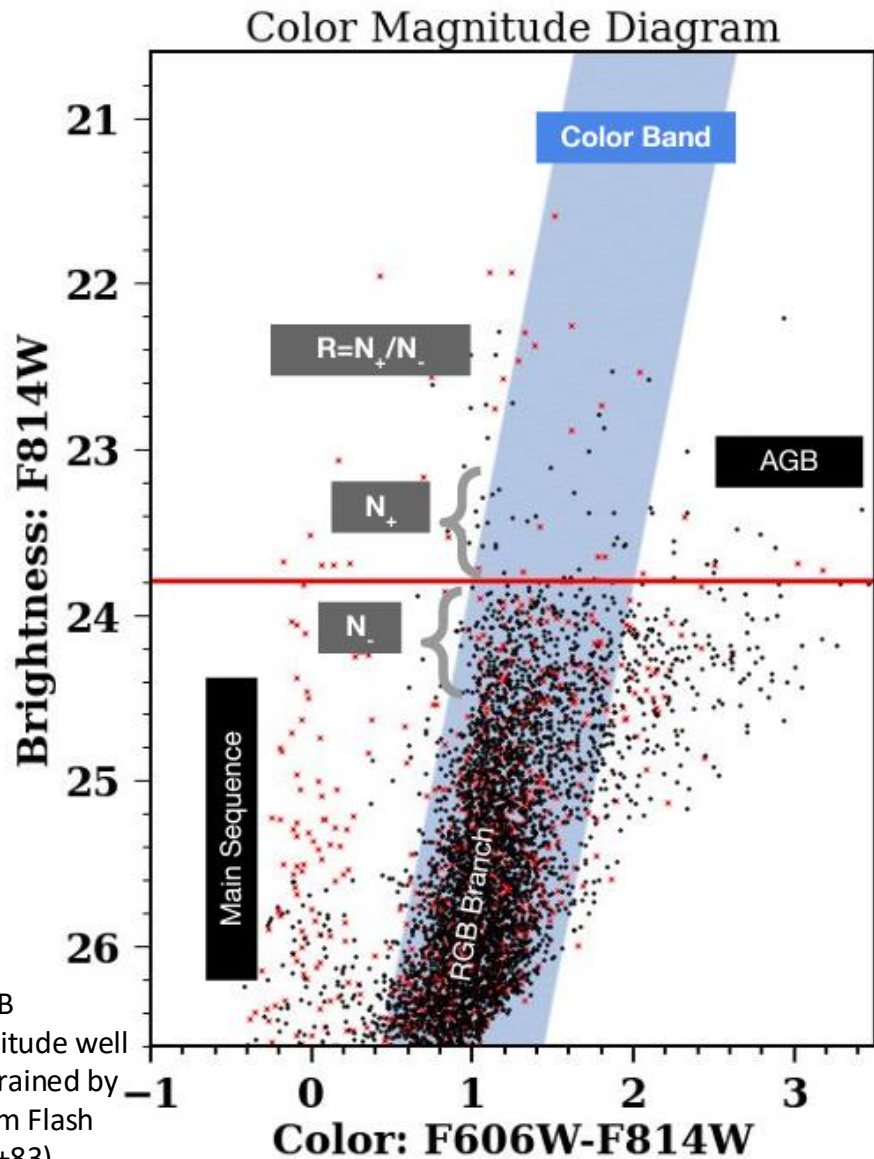
 Anderson et al. 2024: Hatt weighting can introduce a systematic bias  $\rightarrow$  test sensitivity

# Bootstrap Resampling — Uncertainty Estimation

- ▶ **Goal** — Estimate statistical uncertainty on the TRGB magnitude; no closed-form expression exists
- ▶ **Method** — Draw  $N$  stars with replacement from full CMD  $\rightarrow$  re-run full pipeline (color cut  $\rightarrow$  GLOESS  $\rightarrow$  Sobel); repeat 500–1000 $\times$
- ▶ **Result** — Median  $\pm$  MAD- $\sigma$  ( $= 1.4826 \times \text{MAD}$ ), robust to outliers and multimodality
- ▶ **Key rule** — Resample the full CMD (not just the RGB selection) to propagate selection uncertainty
- ▶ **Warning** — Multimodal histogram = unstable detection; do NOT sigma-clip



# Wrapping Up the Sobel Filter Approach



\*TRGB magnitude well constrained by Helium Flash (Iben+83)

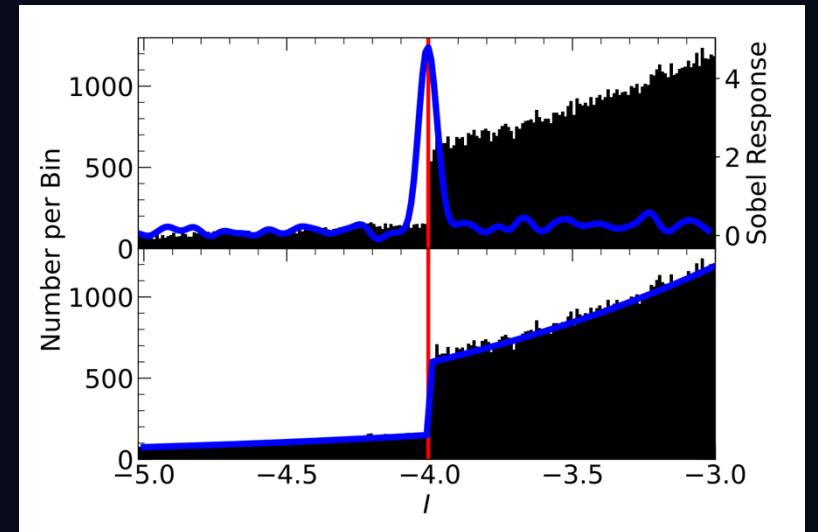
# Model-based methods

- ▶ TRGB luminosity function modeled w/ broken power law:

$$\psi(m) = \begin{cases} 10^{a(m-m_{TRGB})+b} & m \geq m_{TRGB} \\ 10^{c(m-m_{TRGB})} & m < m_{TRGB} \end{cases}$$

- ▶ Fit to TRGB luminosity function using maximum likelihood estimation, least squares fitting (e.g. Méndez et al. 2002, Makarov et al. 2006, Wu+14, Anand+22)

Question for you: Advantages? Disadvantages?



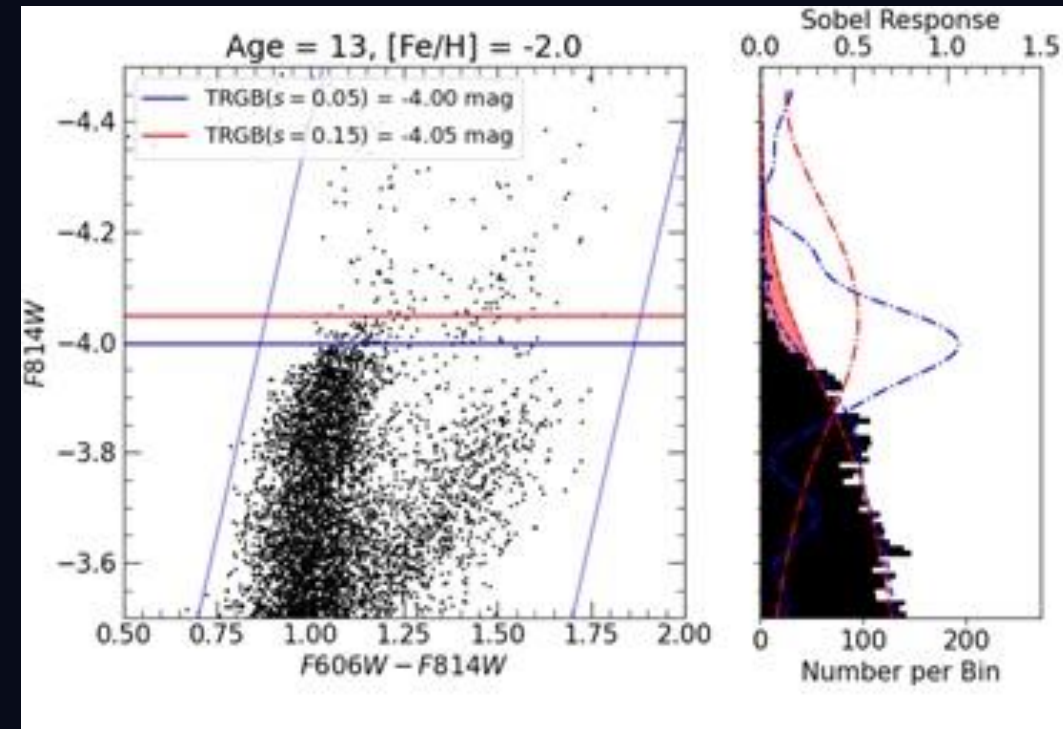
# Method Comparison: Sobel Filter

## Advantages:

- **Model-Independent:** Free from rigid parametric assumptions about underlying stellar profile.
- **Rapid computation:** Faster to run compared to maximum likelihood estimation

## Disadvantages:

- **Smoothing bias**
- **More sensitive to low count degradation**



# Method Comparison: Model Fit

## Advantages:

- Eliminates need for smoothing, binning (loss of info)
- Can incorporate completeness, bias, etc. into model

## Disadvantages:

- Can be expensive to run (maximum likelihood)
- Difficult to model mixed populations (TRGB; see CATs)

# Sobel vs MLE — Best Practice

Comparing the two edge-detection approaches

## When they agree ( $\lesssim 0.02$ mag): good sign

- › Both methods find the same sharp discontinuity
- › Report the average; use difference as a systematics check

## When they disagree ( $> 0.05$ mag): investigate

**Data challenge uses only Sobel filter method for simplicity**

# Full Distance Measurement Pipeline (Preview)

9 steps from raw HST data to distance modulus

**Step 1: Download HST FLT frames from MAST (Mikulski Archive for Space Telescopes — the HST data archive) archive**

**Step 2: Run DOLPHOT PSF photometry on FLT frames**

**Step 3: Quality cuts (S/N, sharpness, crowding) --> Data Challenge starts here**

**Step 4: Build CMD → select RGB stars by color and magnitude**

**Step 5: Foreground extinction correction**

# Full Distance Measurement Pipeline (cont'd)

9 steps from raw data to distance modulus

**Step 6: Build luminosity function (histogram of brightnesses)**

**Step 7: Apply GLOESS smoothing → run Sobel filter**

**Step 8:  $m_{\text{tip}}$  + dust correction →  $\mu = m_{\text{tip},0} - M_I(\text{TRGB})$**

**Step 9: Calculate statistical & systematic uncertainties**

# Looking Ahead — Lecture 2

TRGB in SN Ia Host Galaxies

## What we have now:

- › Theoretical justification for using the TRGB as a standard candle
  - Helium Flash; constrained core mass
- › Observational practice for measuring the TRGB
  - Sobel filter measurement

# Looking Ahead — Lecture 2 (cont'd)

TRGB in SN Ia Host Galaxies & the Path to  $H_0$

## Lecture 2:

- › TRGB systematics
- › How to measure distances to SN Ia host galaxies
- › Recent TRGB  $H_0$  results
- › How this all relates to the CosmoVerse Data Challenge

# Lecture 1 Complete

Next: Lecture 2 — TRGB in SN Ia Host Galaxies

Questions?