

Inconsistencies of Tsallis Cosmology

within horizon thermodynamics and holographic scenarios

Cai-Kim thermodynamic gravity · Holographic dark energy · BBN & CMB viability tests

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Overview

Motivation

Nonextensive entropy formalisms have been widely proposed to go beyond Λ CDM. **Tsallis entropy** introduces a nonextensivity parameter δ , recovering Bekenstein-Hawking at $\delta = 1$.

Key question: Are Tsallis-based cosmologies actually viable across the full expansion history?

Two frameworks studied

- **Cai-Kim:** first law of thermodynamics on the apparent horizon
- **Tsallis HDE:** holographic dark energy with Hubble or GO cutoff

Central finding

Even **infinitesimally small** departures from $\delta = 1$ produce **pathological behavior** in the early Universe:

- Negative or complex dark energy density
 - Divergent equation of state w_{DE}
 - Excessive early dark energy fraction
- Violating BBN & CMB constraints.

Viable cosmology emerges **only** in the limit $\delta = 1$, collapsing both frameworks to Λ CDM.

Clausius relation

Apparent horizon: $R_A = 1/H$ (flat FLRW)

Hawking temperature: $T_h = 1/2\pi R_A$
(Cai, Cao and Hu 2009)

Tsallis entropy: $S_T = \frac{\tilde{\alpha}}{4G} A^\delta$

First law: $\delta Q = T_h dS_T$

integrate

Modified Friedmann equations

Modified Friedmann equations

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{\text{DE}})$$

Effective dark energy density:

$$\rho_{\text{DE}} = \frac{3}{8\pi G} \left[H^2 \left(1 - \frac{\alpha\delta}{2-\delta} H^{2(1-\delta)} \right) + \frac{\Lambda}{3} \right]$$

Tsallis correction $\propto H^{2(1-\delta)}$

Equation of state:

$$w_{\text{DE}} = -1 - \frac{2\dot{H}[1 - \delta\alpha H^{2(1-\delta)}]}{3H^2 \left[1 - \frac{\delta\alpha}{2-\delta} H^{2(1-\delta)} \right] + \Lambda}$$

$\delta = 1 \Rightarrow \rho_{\text{DE}} = \Lambda/8\pi G, w_{\text{DE}} = -1: \Lambda\text{CDM}.$

Cai-Kim Framework

Analytical solution and viability conditions

Closed-form dark energy parameter

$$1 - \Omega_{\text{DE}} = H_{(mr)}^2 \left\{ \frac{2 - \delta}{\alpha \delta} \left[\frac{\Lambda}{3} + H_{(mr)}^2 \right] \right\}^{\frac{1}{\delta - 2}}$$

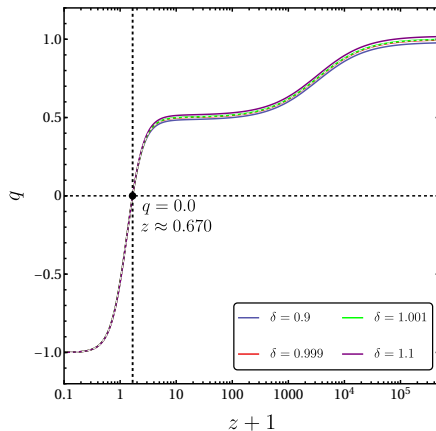
where $H_{(mr)}^2 \equiv H_0^2 \{ \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 \}$

Acceleration condition ($q < 0$ today):

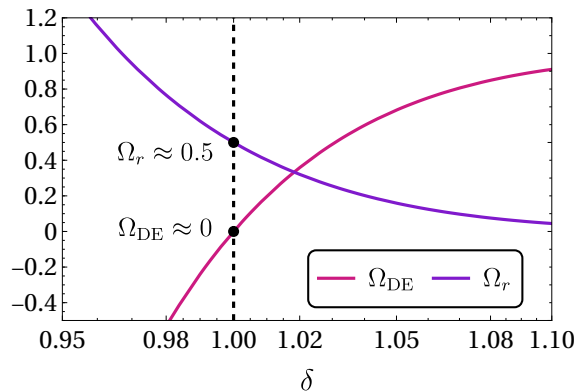
$$0.45 \lesssim \delta < 2 \quad (\hat{\alpha} = 1, \Omega_{m0} = 0.3)$$

But late-time acceleration $\neq$ early-Universe viability!

$q(z)$ for various δ



Density parameters Ω_{DE} and Ω_r at $z \approx 3200$ as functions of δ



Pathologies at matter-radiation equality

$\delta < 1$:

Radiation overshoots: $\Omega_r \gtrsim 1$. Compensated by **negative** $\Omega_{\text{DE}} \Rightarrow$ unphysical.

$\delta > 1$:

Dark energy **prematurely dominates**. Standard radiation $\rightarrow$ matter transition erased.

BBN bound ($\Omega_{\text{DE}}|_{z \sim 3200} < 0.045$):

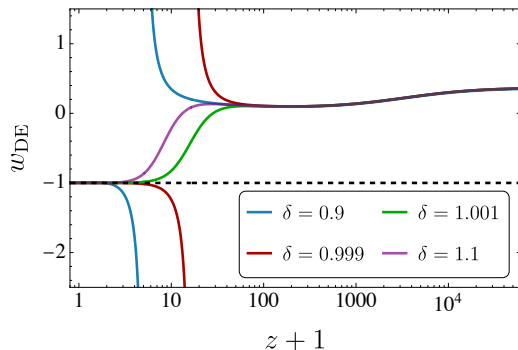
$$1.00 \leq \delta < 1.00038$$

Essentially indistinguishable from $\delta = 1$!

Cai-Kim Framework

| Dark energy equation of state

$w_{\text{DE}}(z)$ for various δ



Late-time convergence (misleading)

All solutions converge to $w_{\text{DE}} \rightarrow -1$ (de Sitter) at low z . Earlier studies (*Lymperis & Saridakis 2018, Nojiri et al. 2019*) up to $z \sim 2-3$ **missed** the divergence entirely.

High- z catastrophe

For $\delta < 1$: denominator of w_{DE} vanishes at some intermediate $z \Rightarrow$ **pole**.

For $\delta > 1$: Ω_{DE} grows excessively, already excluded by BBN.

Lesson: Supernovae-only fits ($z \lesssim 2$) cannot diagnose these pathologies. Full expansion history analysis is essential.

Modified expansion law

$$\frac{\alpha\delta}{2-\delta} H^{2(2-\delta)} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} \equiv H_{\Lambda}^2$$

Expanding around $\delta = 1$ ($\hat{\alpha} = 1$):

$$H^2 \approx H_{\Lambda}^2 + \delta H_{\Lambda,1}^2 (\delta - 1) + \delta H_{\Lambda,2}^2 (\delta - 1)^2$$

where the correction grows as:

$$\delta H_{\Lambda,1}^2 \sim H_{\Lambda}^2 \ln \frac{H_{\Lambda}^2}{H_0^2}$$

At high z : $\ln(H_{\Lambda}^2/H_0^2) \propto \ln(1+z)^4 \rightarrow \infty$

Impact on radiation era

$$\Omega_r \propto \frac{\rho_r}{H_{\Lambda}^2} \left[1 + \frac{\delta H_{\Lambda,1}^2}{H_{\Lambda}^2} (\delta - 1) + \dots \right]^{-1}$$

Correction factor $\propto \ln(1+z)^4$:

- $\delta < 1$: $\Omega_r > 1$ at high $z \Rightarrow \Omega_{DE} < 0$
- $\delta > 1$: excess early dark energy, violates BBN

Key insight: Tsallis corrections are *not* small perturbations around Λ CDM. They grow **non-perturbatively** toward the past, invalidating the model for any $\delta \neq 1$.

Holographic Dark Energy

| Tsallis HDE with Hubble horizon cutoff

Setup

Holographic principle with Tsallis entropy
 $S_T = \gamma A^\delta$ and IR cutoff $L_{\text{IR}} = 1/H$:

$$\rho_{\text{HDE}} = B H^{2(2-\delta)}$$

Continuity equation:

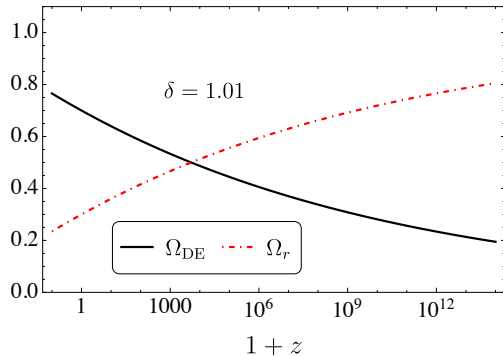
$$\dot{\rho}_{\text{HDE}} + 3H\rho_{\text{HDE}}(1 + w_{\text{HDE}}) = 0$$

Evolution of Ω_{HDE} :

$$\frac{d\Omega_{\text{HDE}}}{d \ln a} = \frac{4(\delta - 1)\Omega_{\text{HDE}}(1 - \Omega_{\text{HDE}})}{1 - (2 - \delta)\Omega_{\text{HDE}}}$$

Radiation-era pathology

Hubble Horizon cutoff



For $\delta = 1.01$, even at $z = 10^{14}$: $\Omega_{\text{HDE}} \sim 20\%$
Standard radiation era is **impossible**.

Holographic Dark Energy

| Tsallis HDE with Granda-Oliveros cutoff

GO cutoff (avoids causality problem)

$$L_{\text{IR}}^{(\text{GO})} = (\tilde{\alpha}H^2 + \tilde{\beta}\dot{H})^{-1/2}$$

$$\rho_{\text{HDE}} = 3m_P^2(\alpha H^2 + \beta\dot{H})^{2-\delta}$$

Perturbative expansion around $\delta = 1$:

$$\frac{\Omega_{\text{HDE}}}{H_{(r)}^2} = (\alpha - 2\beta) - (\alpha - 2\beta)(\delta - 1) \ln \left[\frac{H_0^2 \Omega_{r0}}{a^4} (\alpha - 2\beta) \right] + \mathcal{O}[(\delta - 1)^2]$$

Problems

Always large at high z

Correction $\propto \ln(1+z)^4$ for $\delta \neq 1$. Only $\delta = 1$ keeps it constant — i.e. no dynamics.

Fine-tuning dilemma

$\alpha \gtrsim 2\beta$: radiation era OK, but **significant DE during matter domination**.

$2\alpha \gtrsim 3\beta$: matter era OK, but HDE non-negligible during radiation.

No choice of α, β avoids both pathologies.

Extra theoretical burden beyond what Tsallis entropy was meant to solve.

Conclusions

| Summary and implications

What we showed

- Cai-Kim: $\delta < 1 \Rightarrow$ negative DE density;
 $\delta > 1 \Rightarrow$ excess early DE
- HDE (Hubble): $\sim 20\%$ dark energy at $z = 10^{14}$ even for $\delta \approx 1$
- HDE (GO): no choice of parameters avoids all pathologies
- Perturbative analysis: logarithmic growth — not a small perturbation!

Viable Cai-Kim cosmology requires:

$$1.00 \leq \delta < 1.00038$$

Indistinguishable from $\delta = 1$ (Λ CDM)

Physical interpretation

Tsallis corrections $\propto H^{2(2-\delta)}$ are negligible at low z (where H is small) but grow **uncontrollably** toward the past. This is a **fundamental structural incompatibility** with the standard thermal history.

Broader lesson

Dynamical consistency and cosmological viability tests (BBN, CMB, radiation era) are **essential** when assessing nonextensive entropy formalisms.

Future work: Apply same tests to Kaniadakis, Rényi, Barrow, and related proposals.

Tsallis cosmology is viable *only* at
 $\delta = 1 \implies \text{exact } \Lambda\text{CDM}$



Scan for paper

2 Frameworks

Cai-Kim · HDE

3 Pathologies

$\rho < 0 \cdot w \rightarrow \infty \cdot \text{EDE}$

1 Conclusion

$\delta \rightarrow 1$ in all cases

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Thank you!

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