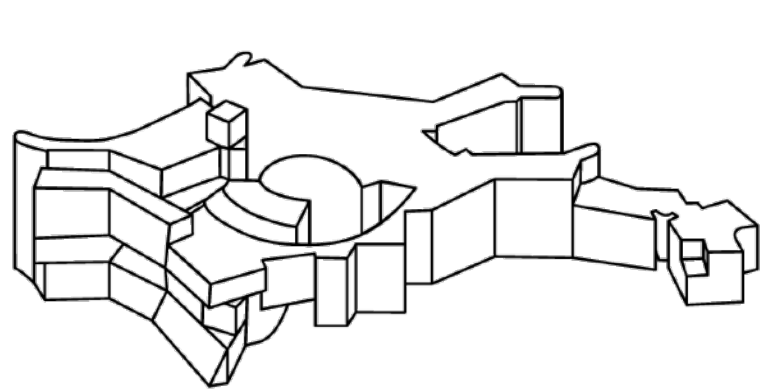


# ROBUST CMB B-MODE ANALYSIS WITH NEEDLET-ILC AND SIMULATION-BASED INFERENCE

**Adri Duivenvoorden**

**Max Planck Institute for Astrophysics**



**MAX-PLANCK-INSTITUT**  
FÜR ASTROPHYSIK



In collaboration with: Kristen Surrao, Adrian  
E. Bayer, Alexandre E. Adler, Nadia  
Dachlythra, Susanna Azzoni and J. Colin Hill

Based on 2512.16869

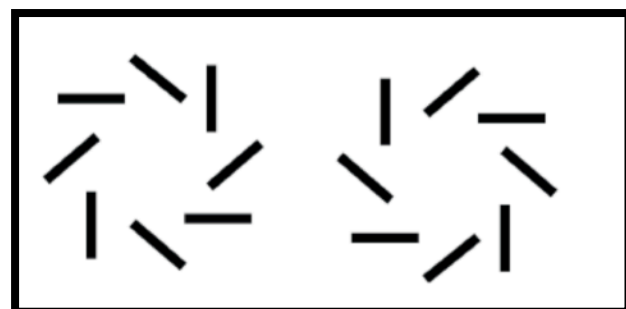
**COSMO'26**  
**08-24-2026**

# LOOKING FOR PRIMORDIAL GRAVITATIONAL WAVES

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Cosmic inflation predicts a background of primordial gravitational waves

- ▶ Parameterized by the *tensor-to-scalar ratio*  $r$
- ▶ Leaves an imprint in the CMB
- ▶ Measurable in CMB  $B$ -mode polarization if energy scale of inflation is large enough ( $\sim 10^{16}$  GeV)

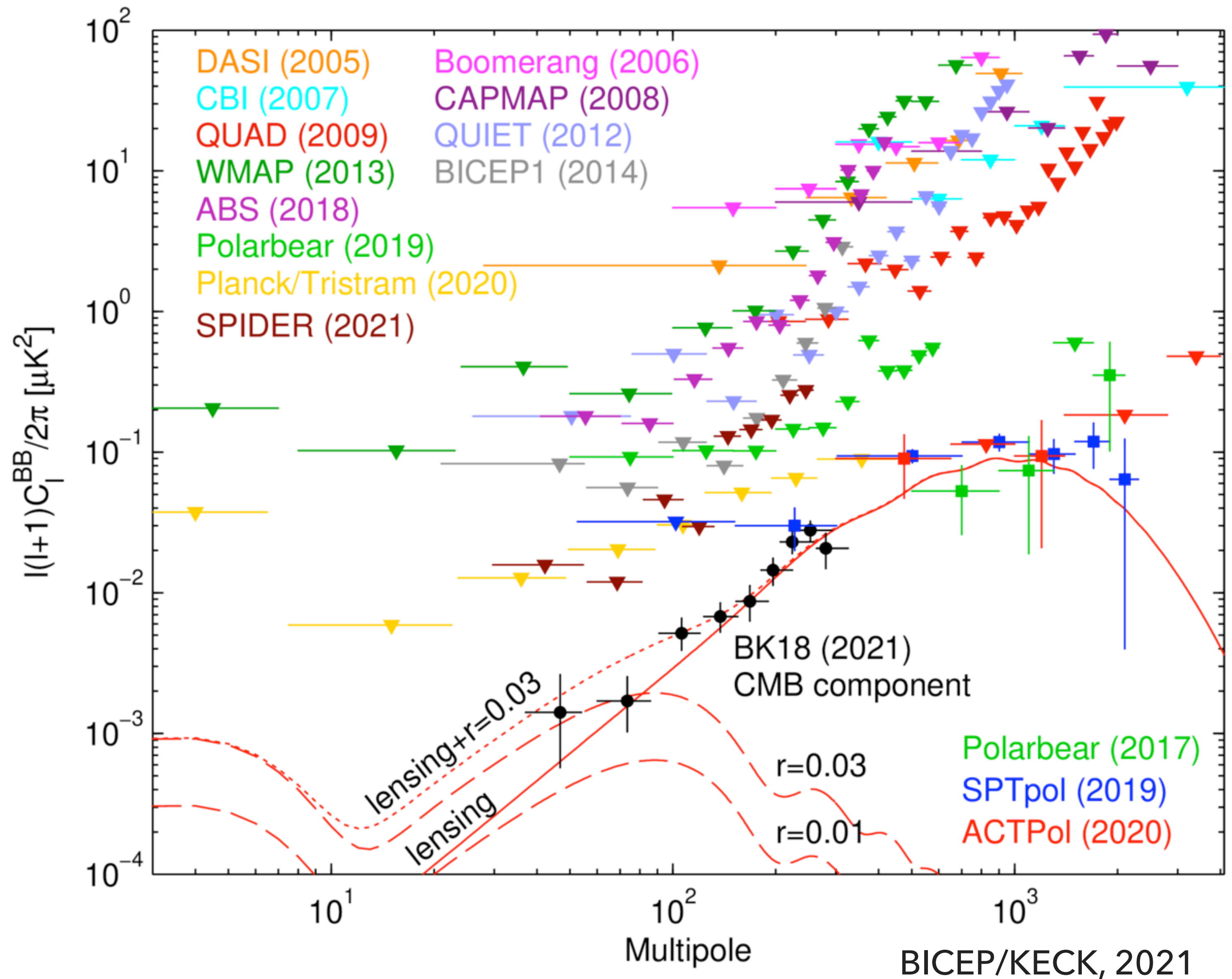
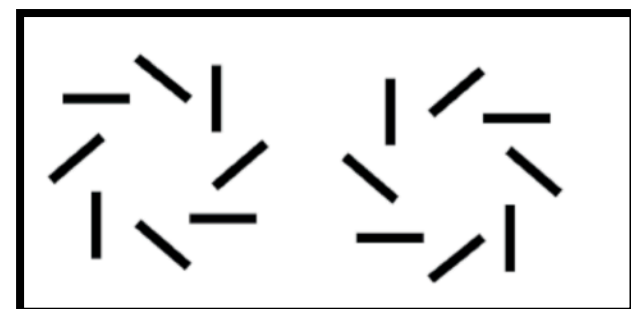


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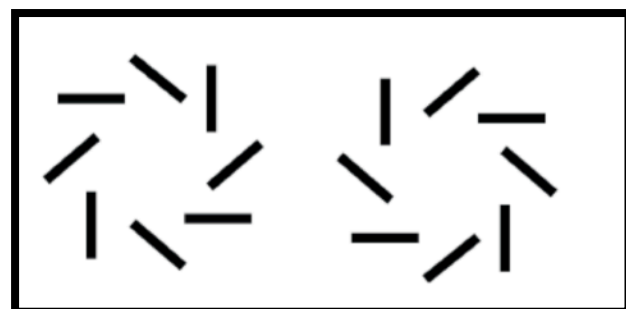


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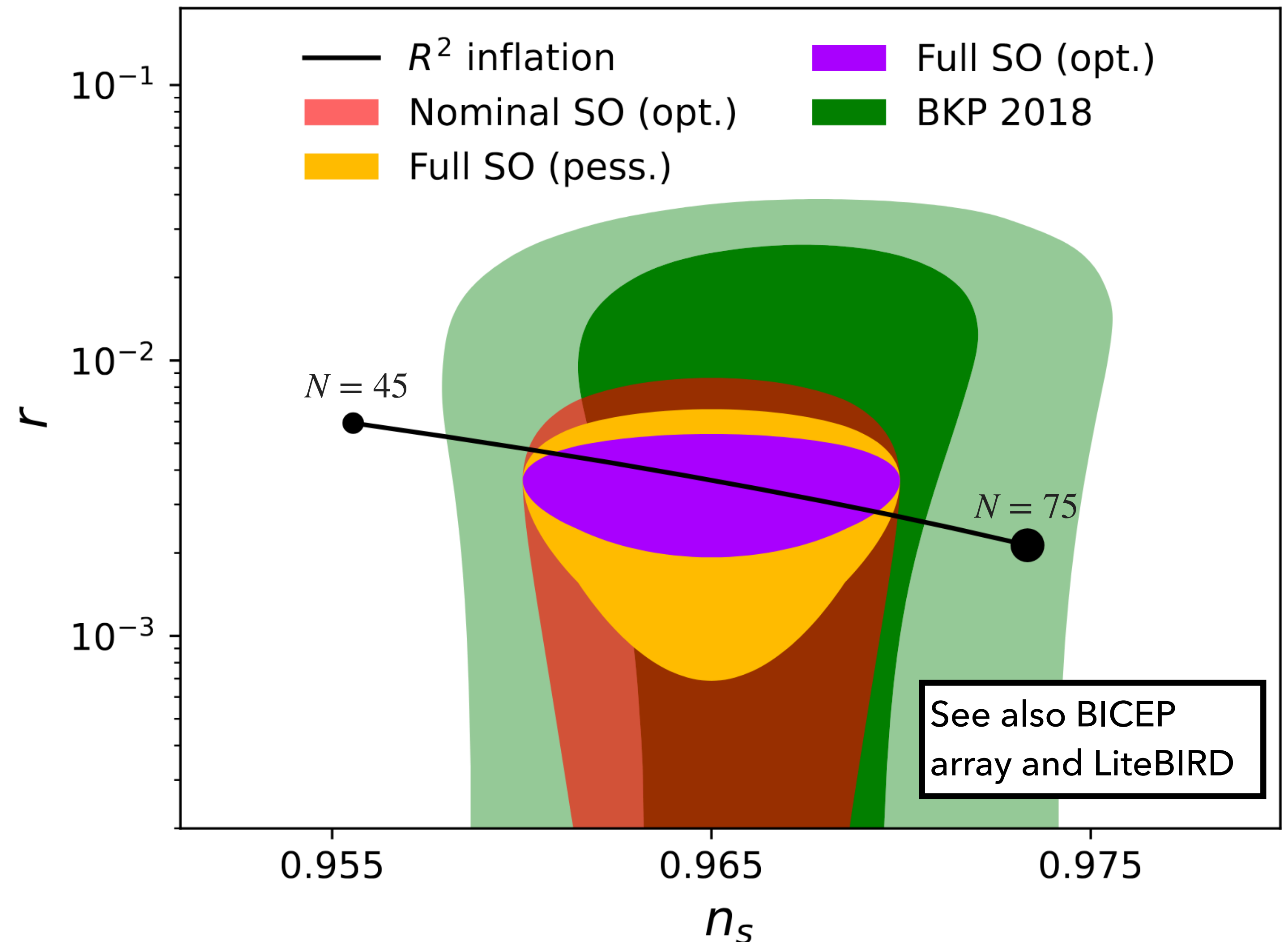
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Simons Observatory forecast



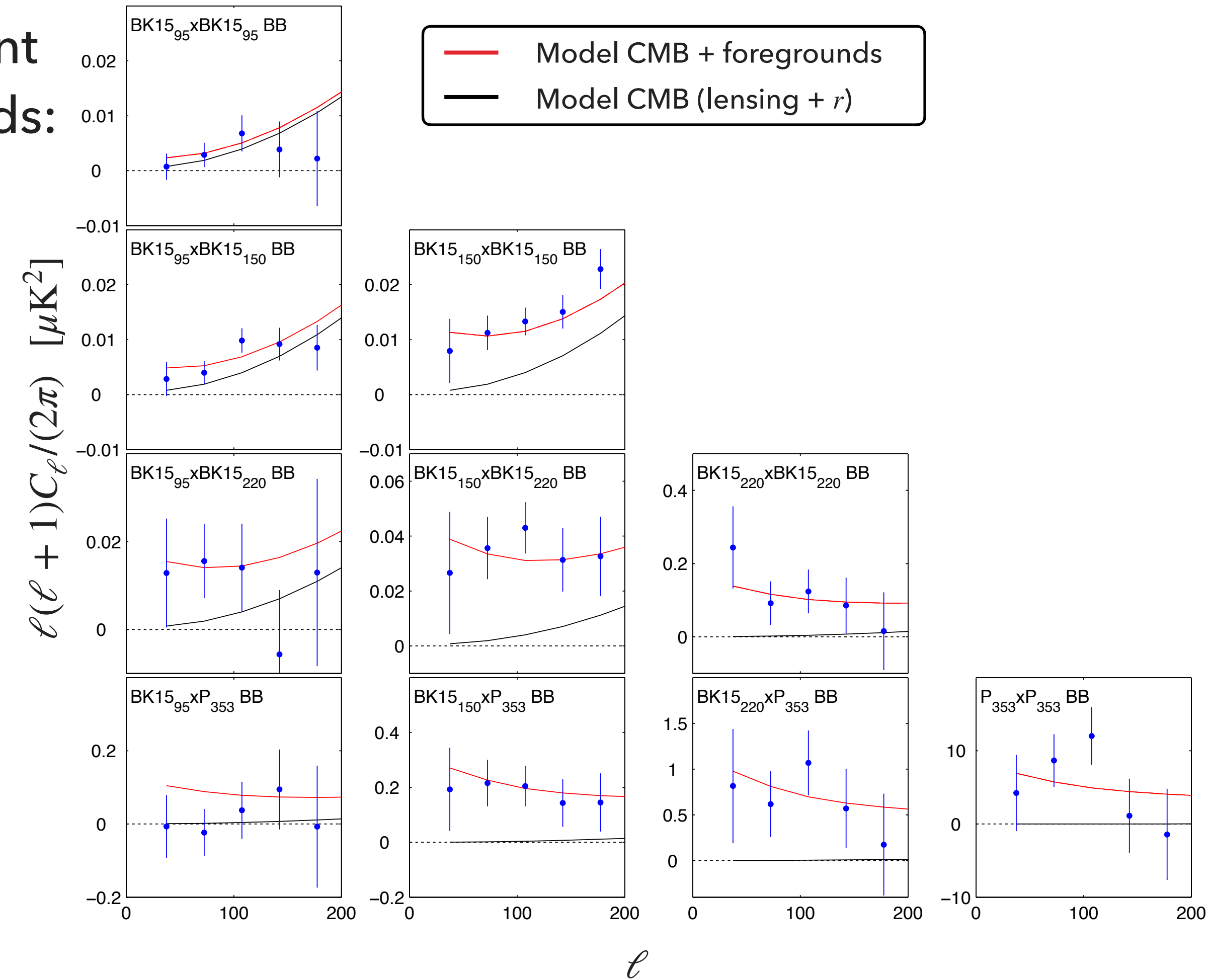
Simons Observatory et al. 2026 (2512.15833)

# SKY COMPONENT SEPARATION

4

Polarized emission from Galactic dust and synchrotron is much brighter than CMB  $B$ -modes. Requires component separation using either power-spectrum-based methods:

- ▶ “Standard” multi-frequency analysis used by e.g. BICEP/KECK, 2021
- ▶  $C_\ell$ -moments approach from Azzoni et al. 2021 (2011.11575)



BICEP/KECK, 2021

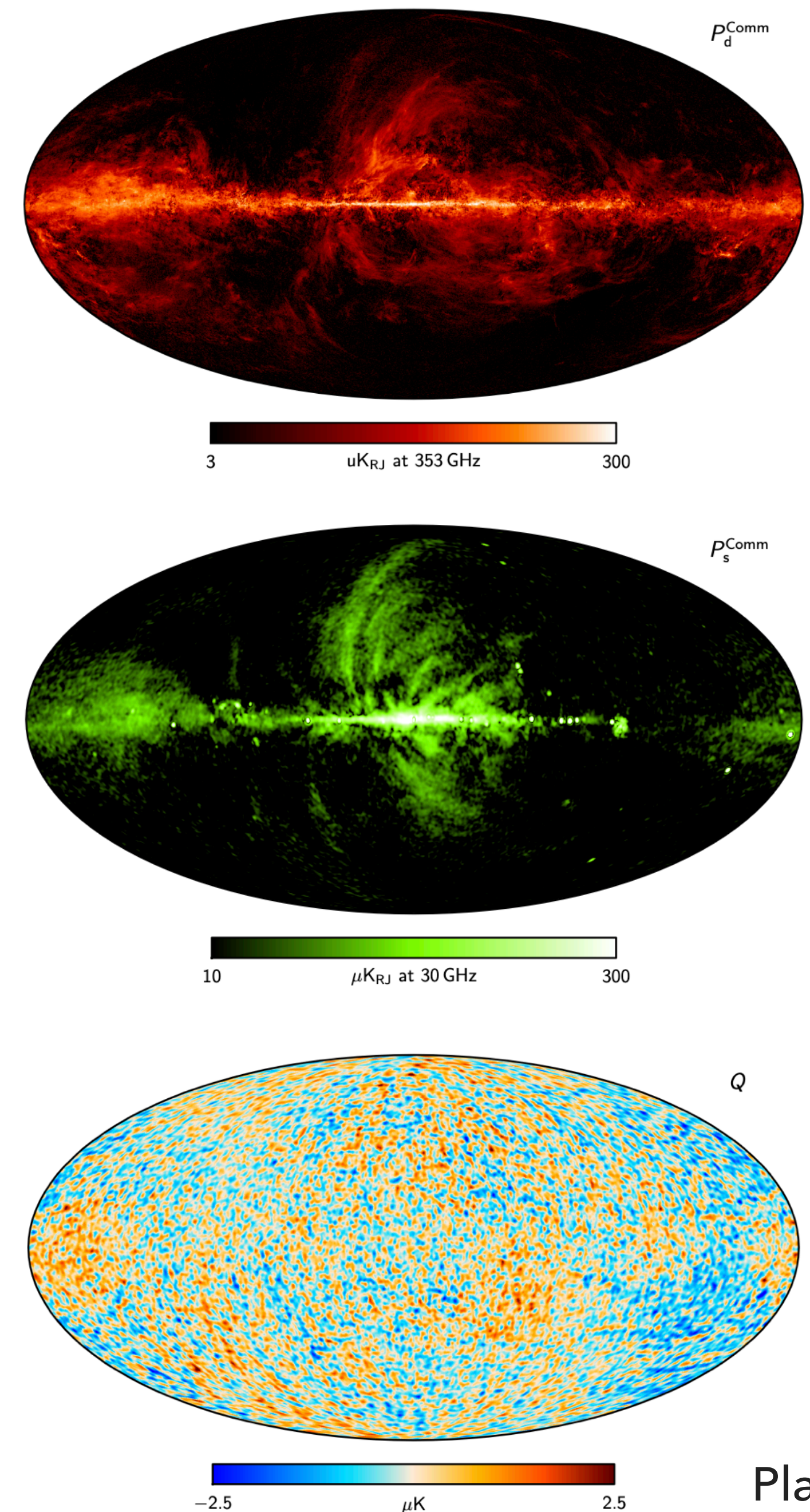
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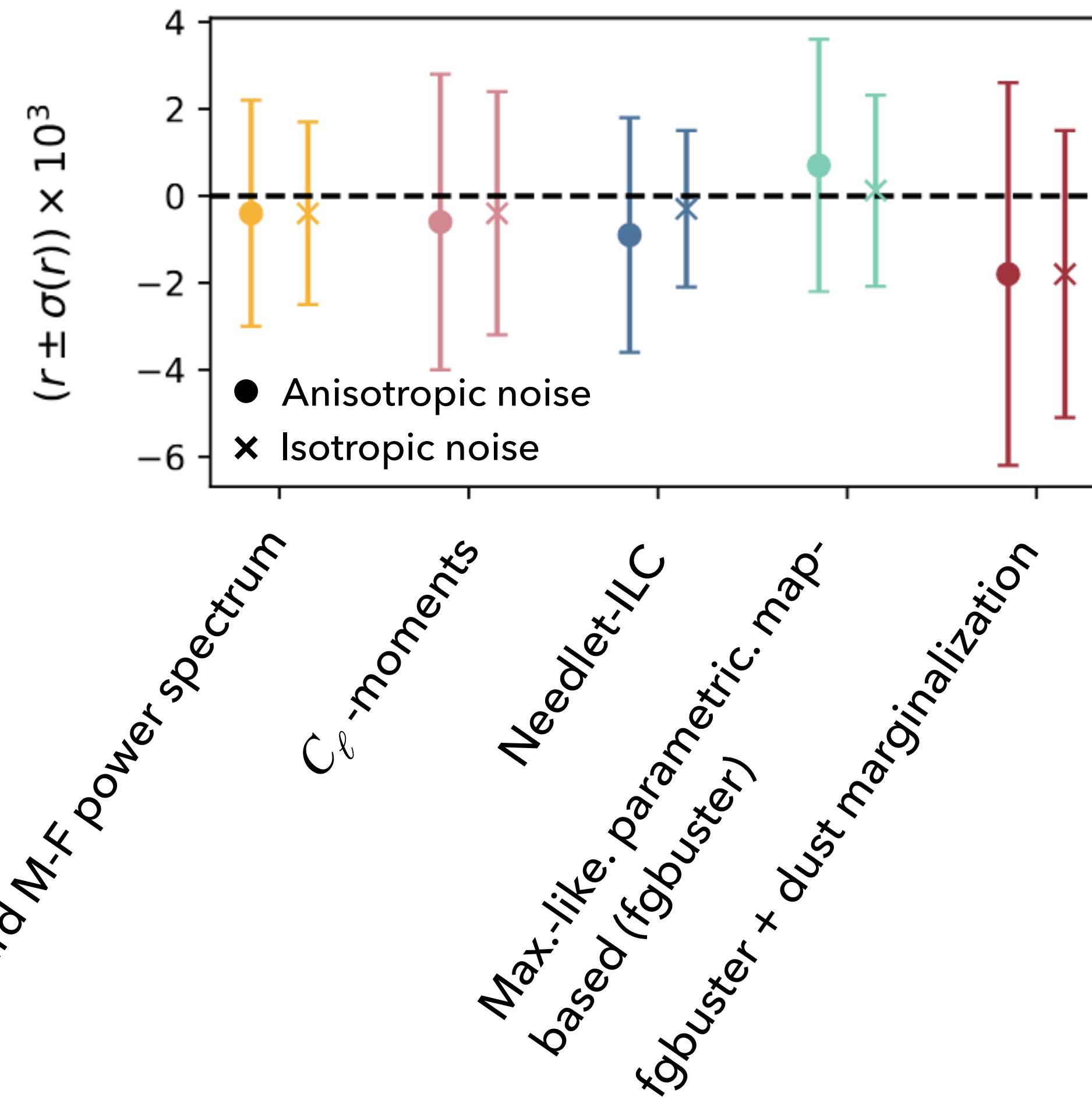
or map-based methods:

- ▶ Parametric models:
  - ▶ “Field-level” e.g. Commander/CosmoGlobe (e.g. BeyondPlanck 2011.05609)
  - ▶ Max.-likelihood, e.g. fgbuster (Errard++2019, 1811.00479; Rizzieri++2025, 2510.08534)
- ▶ Internal linear combination (ILC) (Bennett++ 2003 0302208)



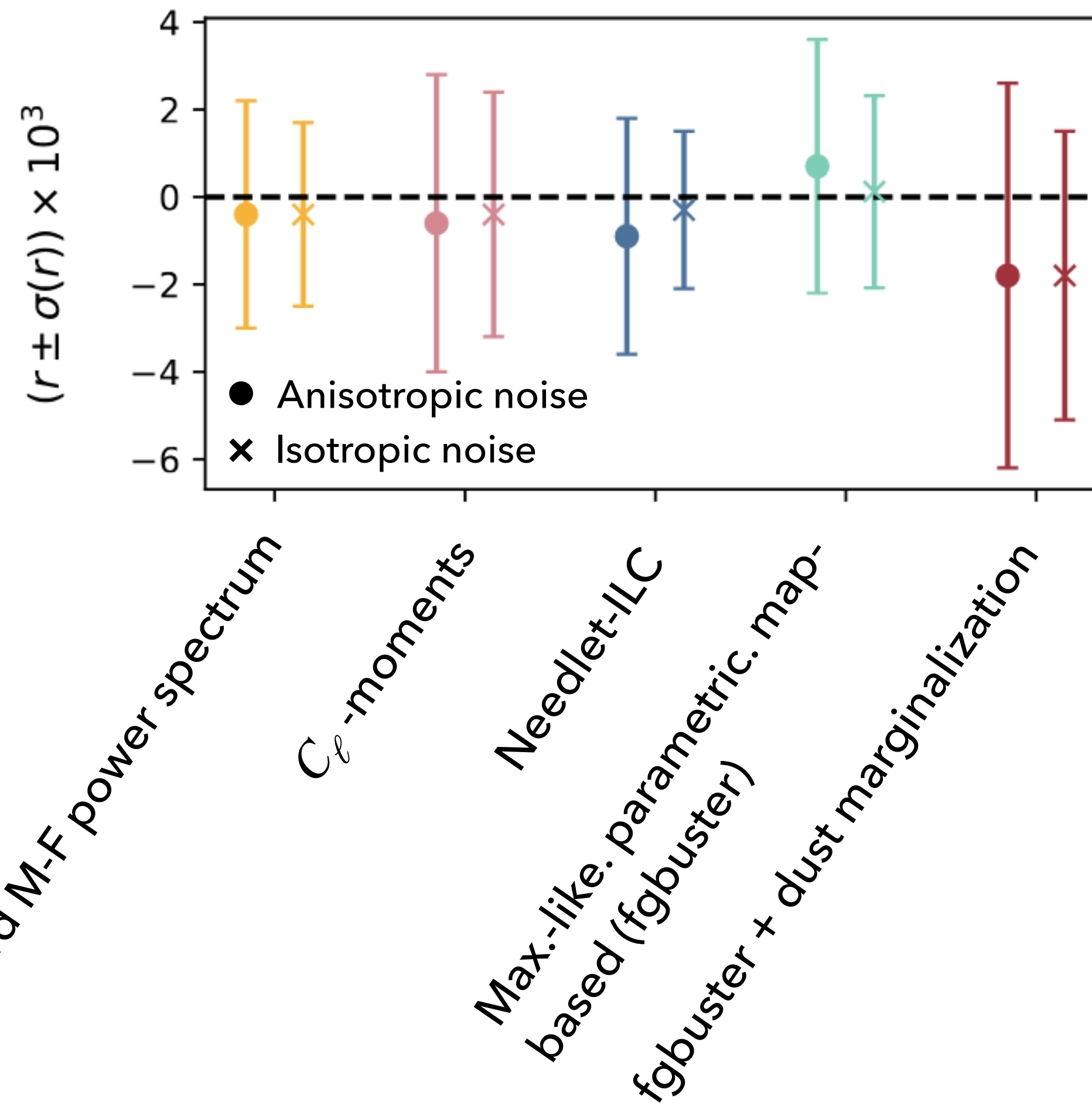
## PySM d1\_s1

(baseline white noise, optimistic  $1/f$ )



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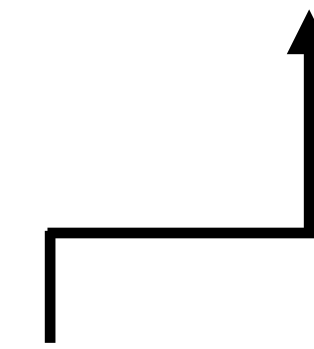


## PySM d10\_s5

(goal white noise, optimistic  $1/f$ )

|                         | $r \cdot 10^3$ |
|-------------------------|----------------|
| ▶ Standard M-F PS       | $19.2 \pm 1.9$ |
| ▶ $C_\ell$ -moments     | $2.7 \pm 2.8$  |
| ▶ Needlet-ILC           | $15.8 \pm 1.9$ |
| ▶ fgbuster              | $22.0 \pm 2.6$ |
| ▶ fgbuster + dust marg. | $-1.5 \pm 5.1$ |

$$I_{\nu}(\hat{n}) = A_d(\hat{n}) \nu^{\beta_d(\hat{n})} B_{\nu}[T_d(\hat{n})]$$

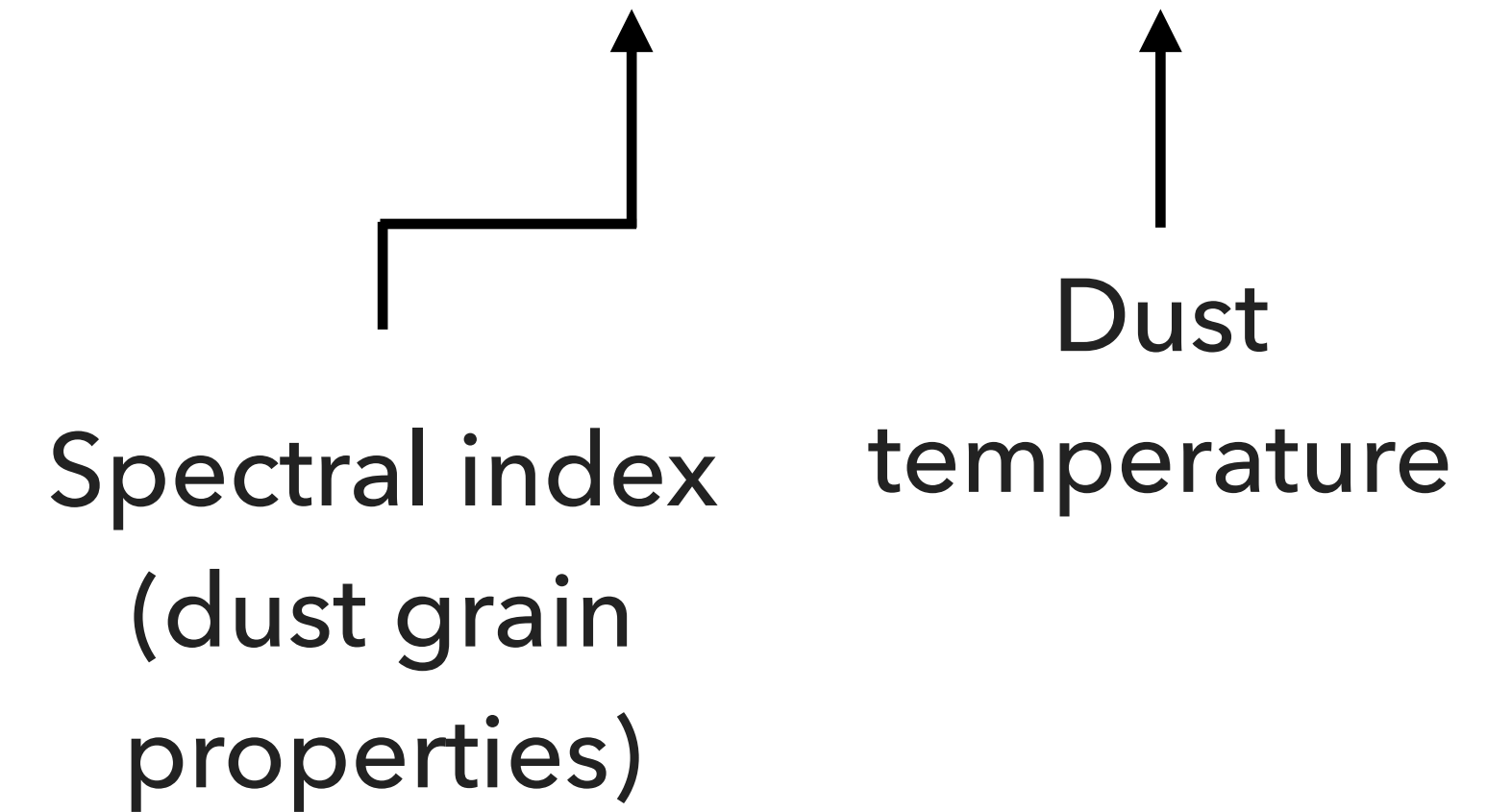


Spectral index  
(dust grain  
properties)



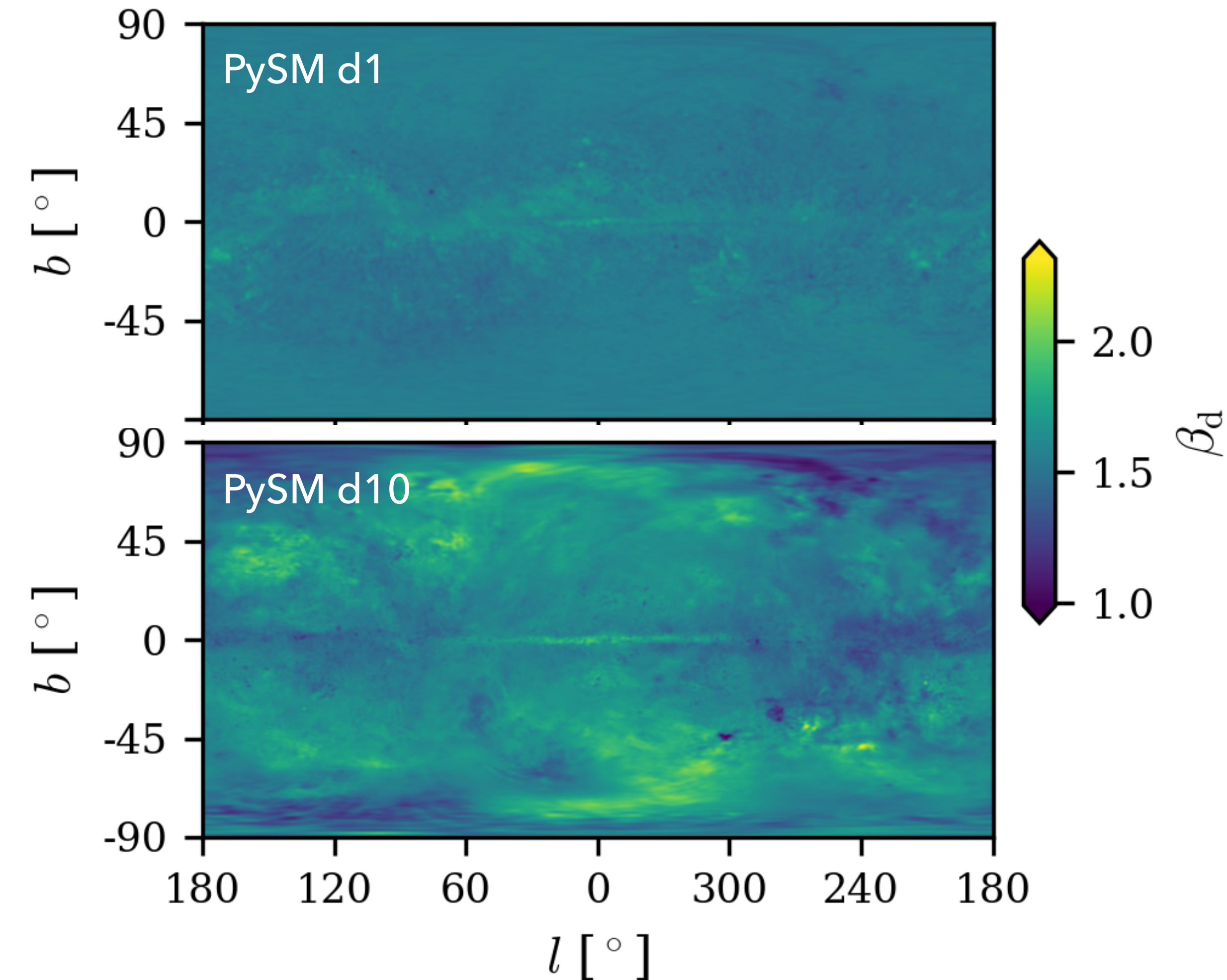
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Properties of  $\beta_d$  and  $T_d$  in polarization still largely unknown

Spectral index sky maps



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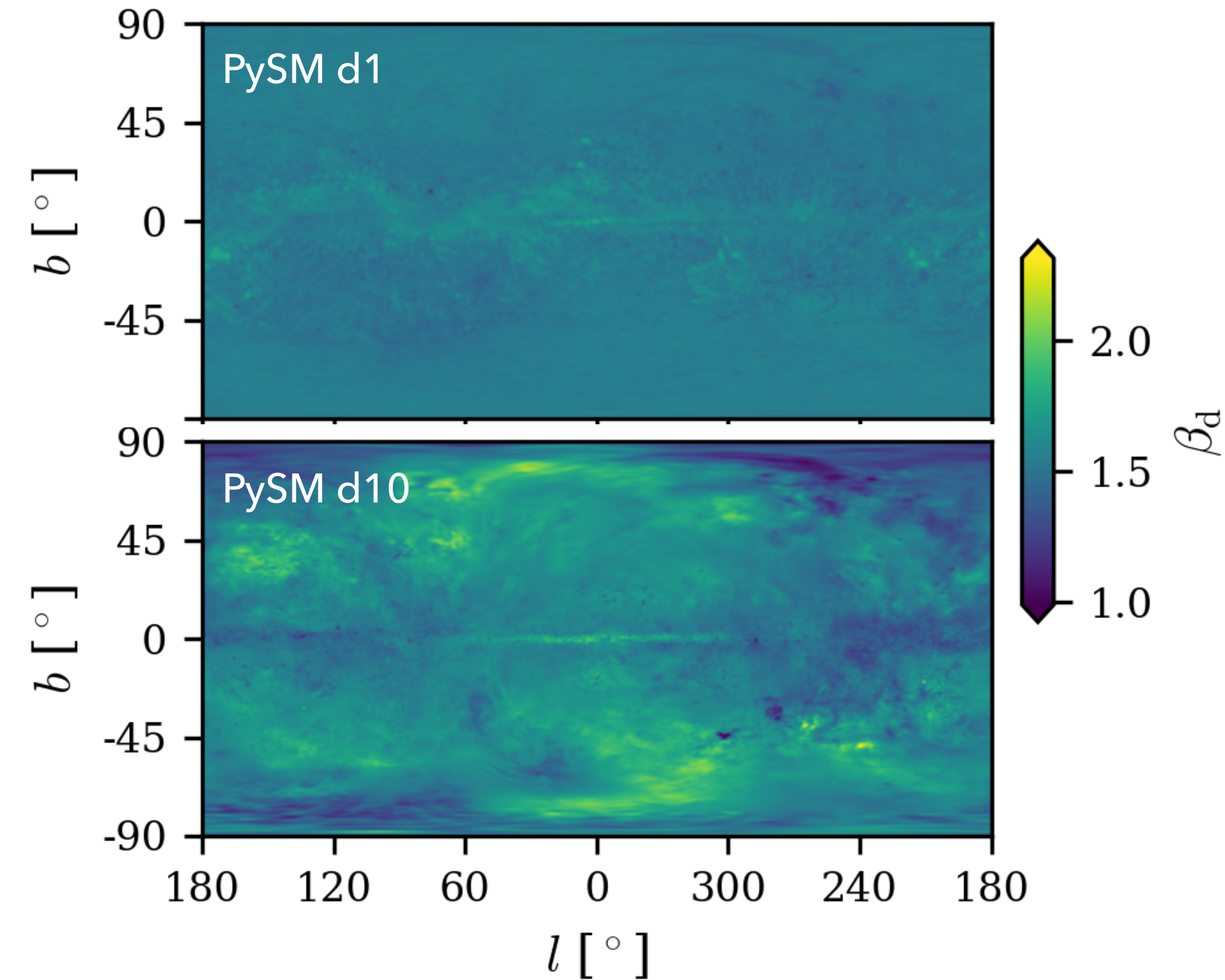
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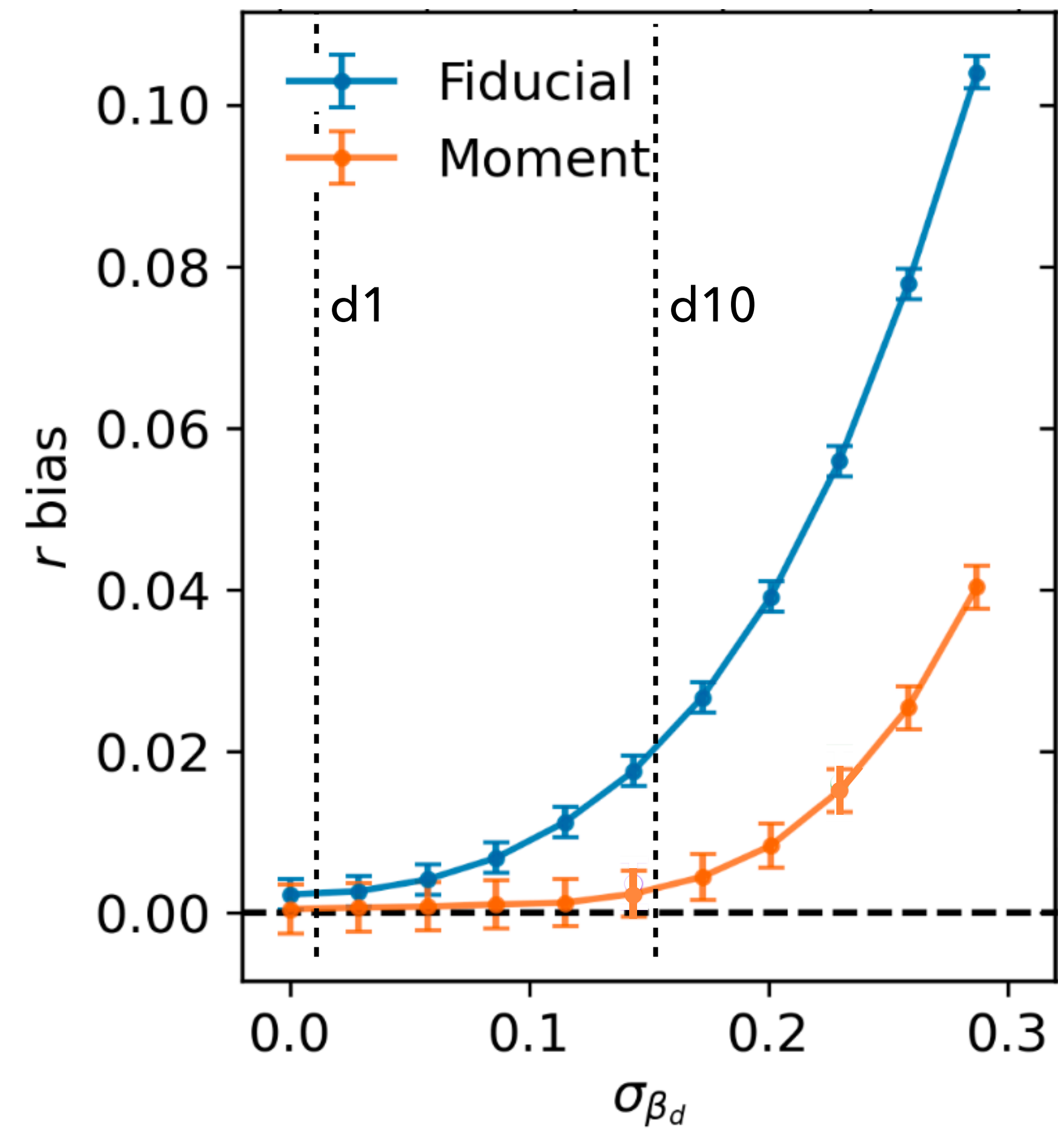
# SPATIALLY VARYING DUST SED

7

Spectral index sky maps



Methods not fully robust against spatial variation in  $\beta_d$  or  $T_d$



Amount of spatial variation in  $\beta_d$   $\nearrow \sigma_{\beta_d}$

Liu++ 2025  
(2508.00073)

# Robust CMB B-mode analysis with Needlet-ILC and simulation-based inference

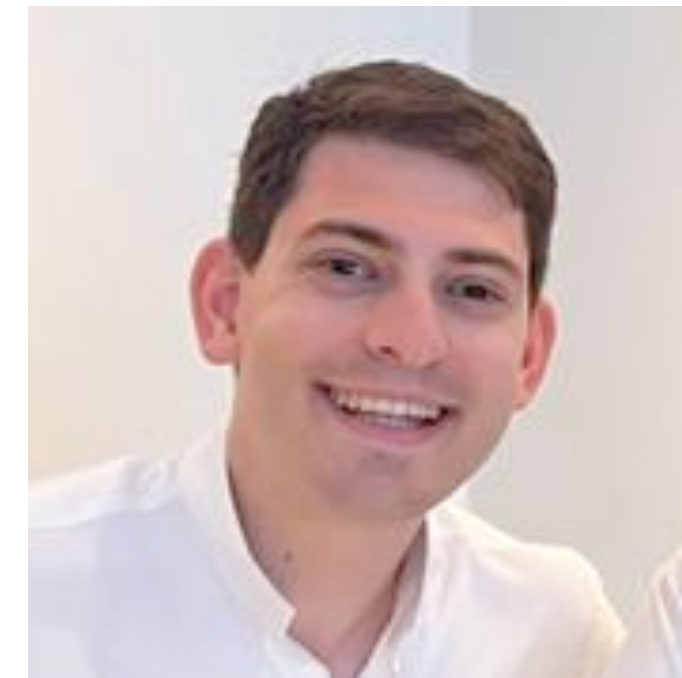
ADRIAAN J. DUIVENVOORDEN <sup>1</sup> KRISTEN SURRAO <sup>2</sup> ADRIAN E. BAYER <sup>3,4</sup> ALEXANDRE E. ADLER <sup>5,6</sup>  
NADIA DACHLYTHRA <sup>7</sup> SUSANNA AZZONI <sup>8</sup> AND J. COLIN HILL <sup>2</sup>



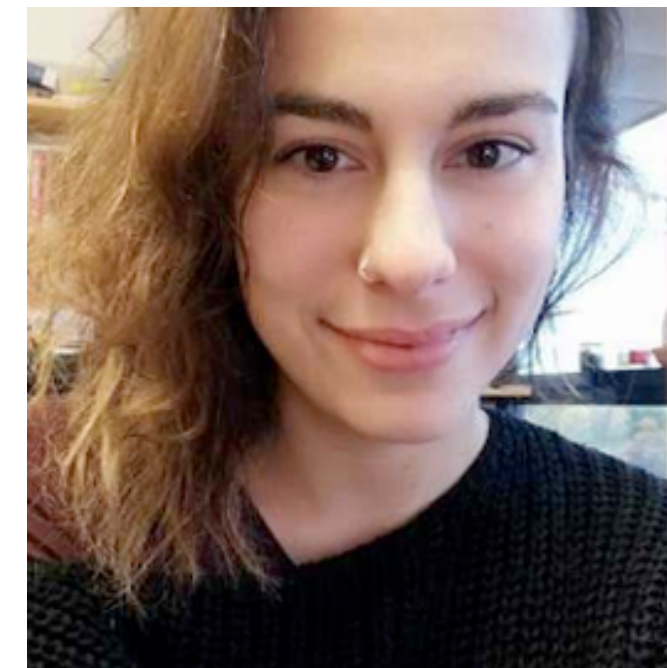
Kristen Surrao  
(Columbia)



Adrian Bayer (CCA/  
Princeton)



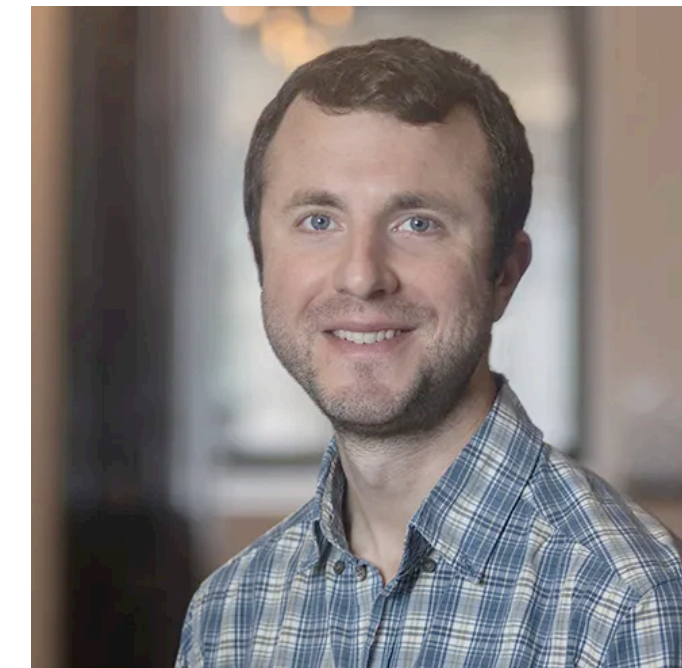
Alexandre Adler  
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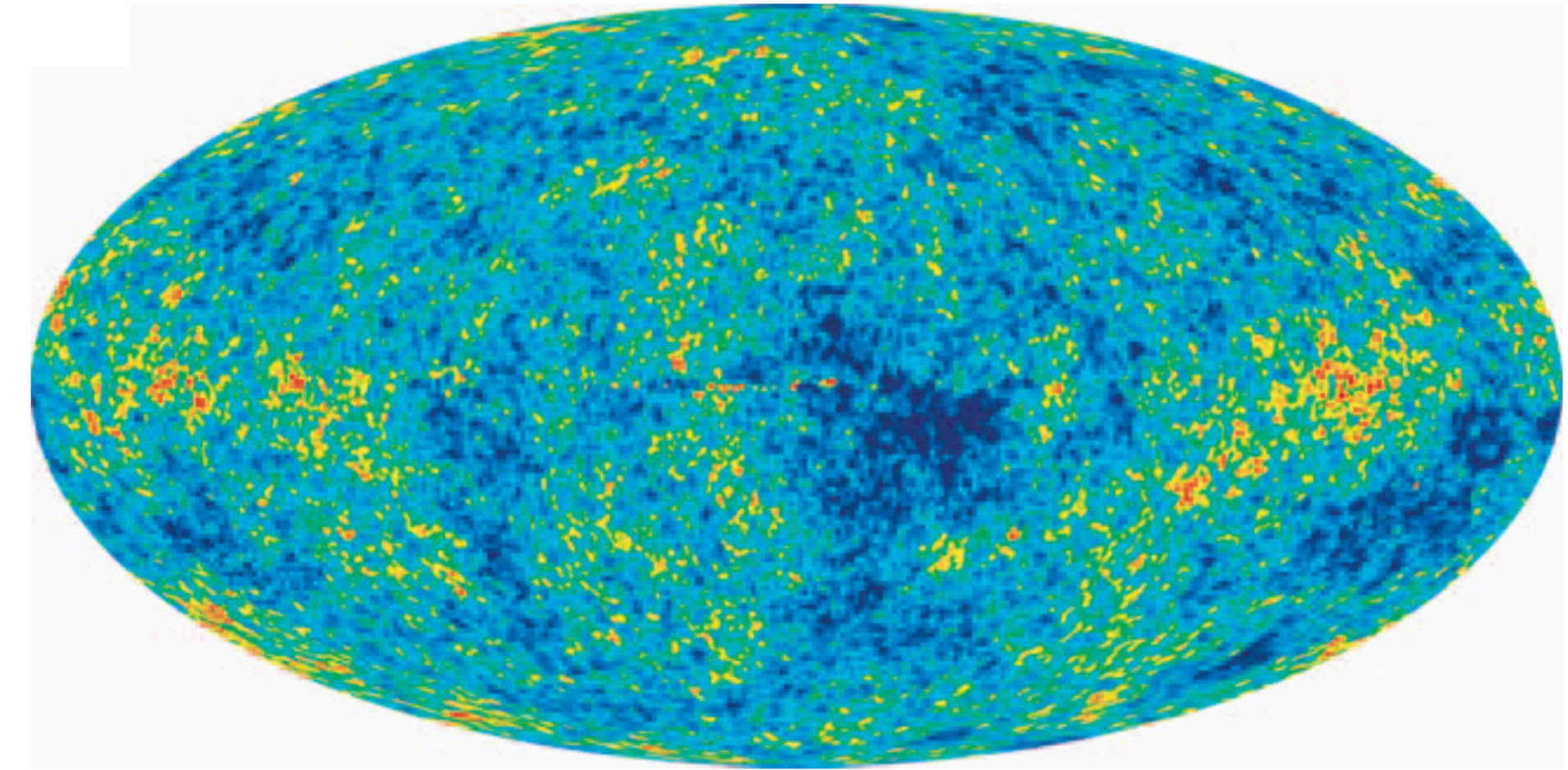


Colin Hill  
(Columbia)

arXiv: 2512.16869

Builds upon work by  
**Surrao & Hill 2024**  
(2403.02261, 2406.16811)

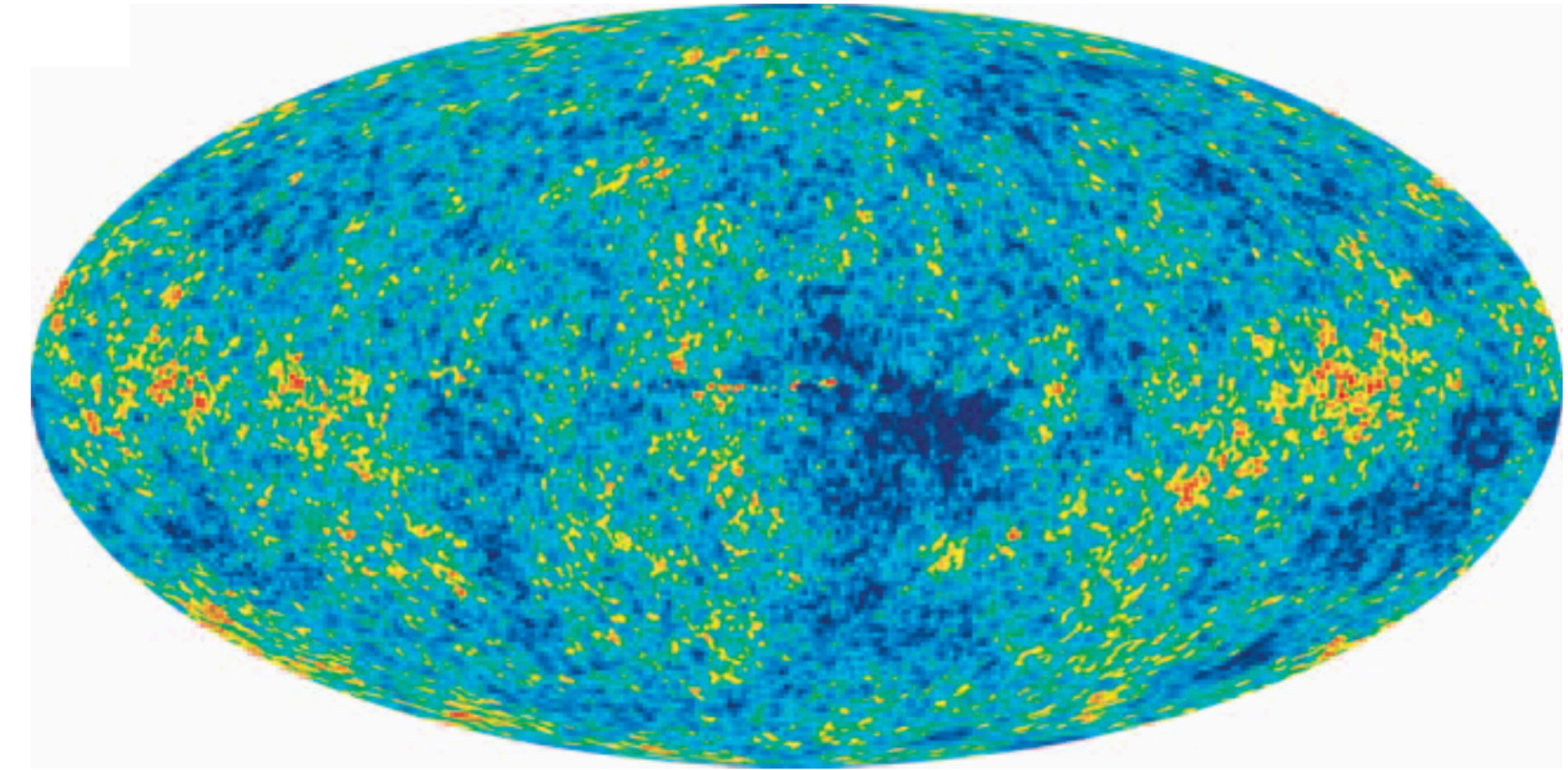
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Bennett et al. 2003 (0302208)

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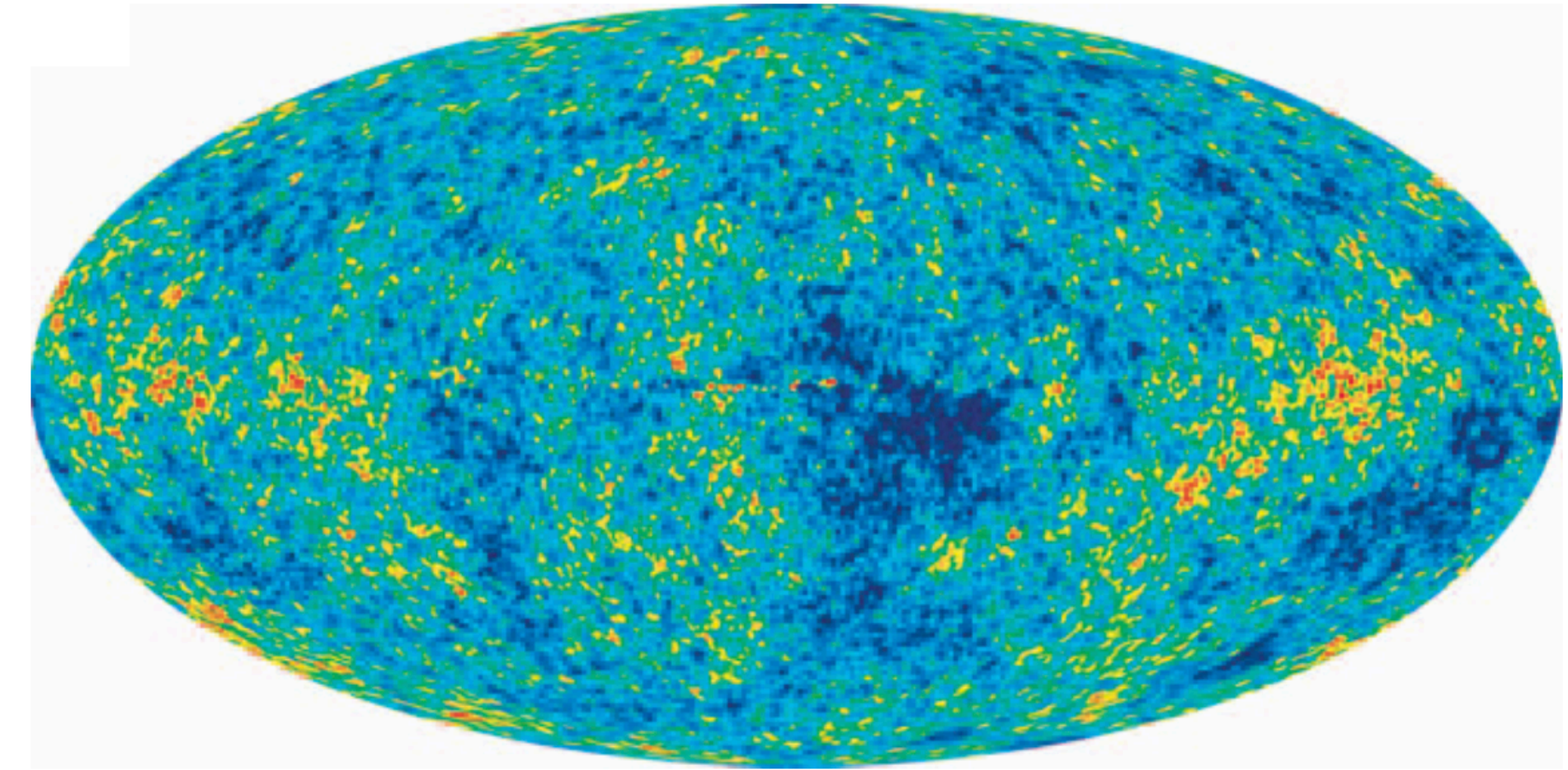


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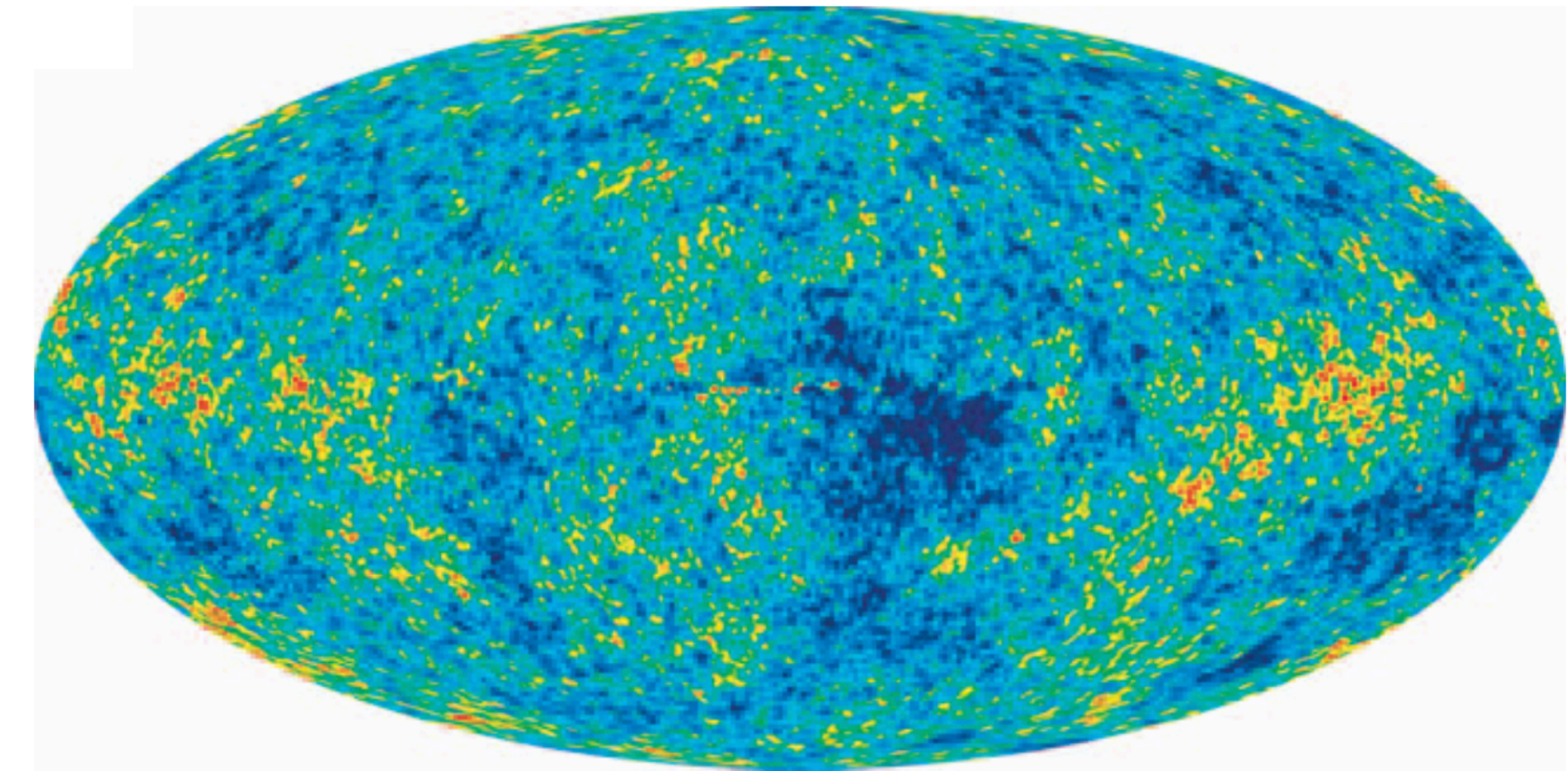


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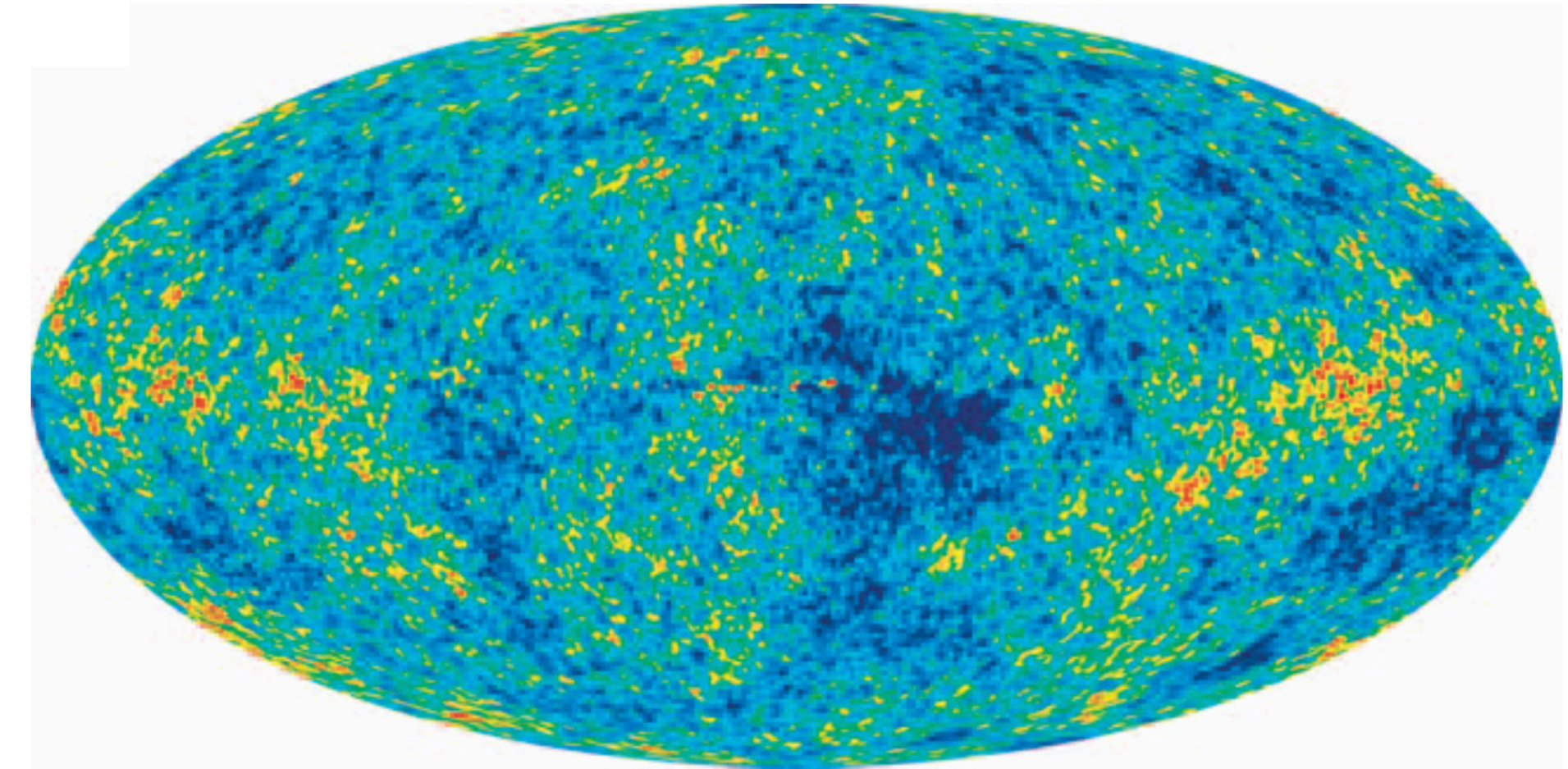


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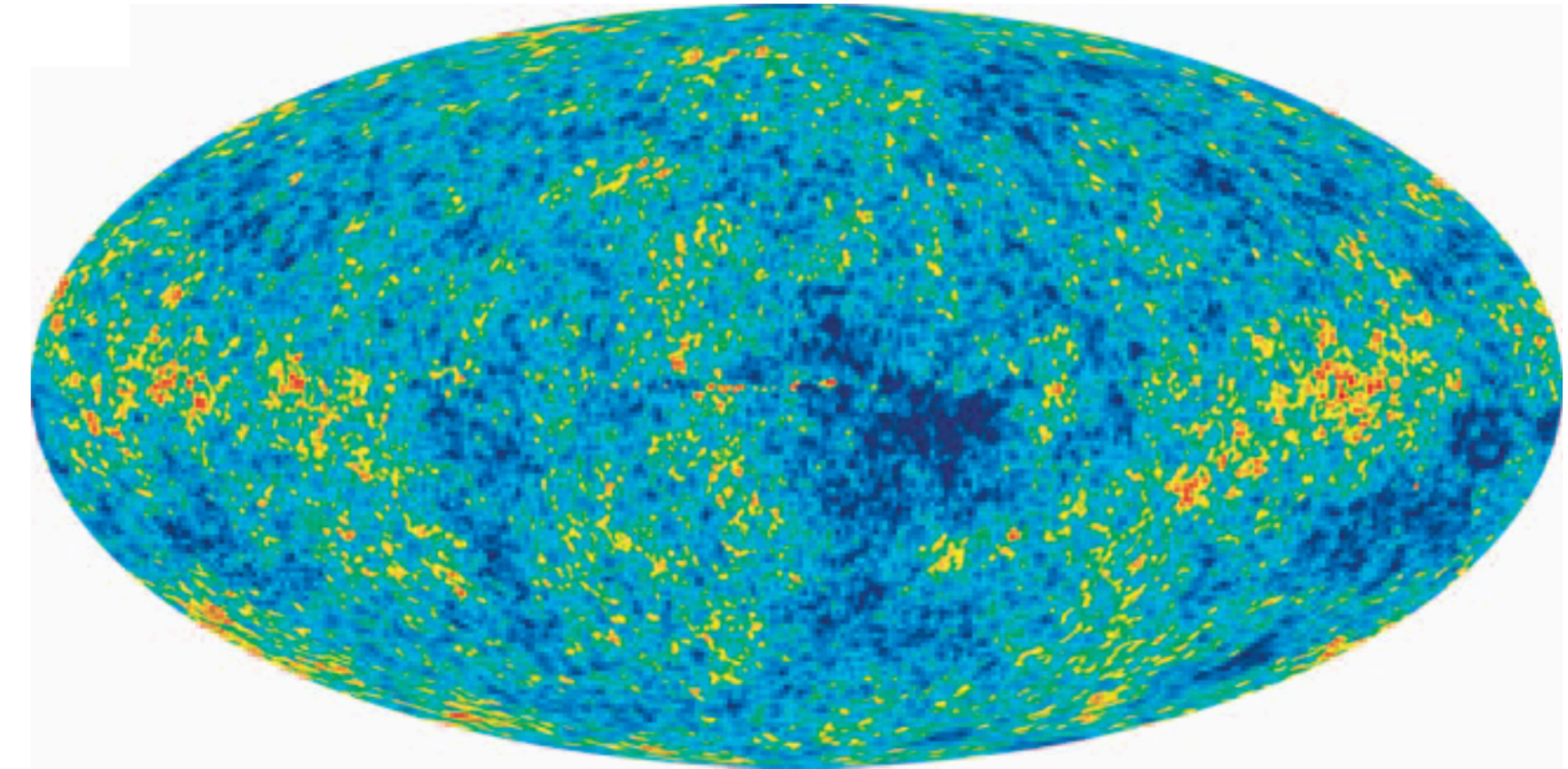


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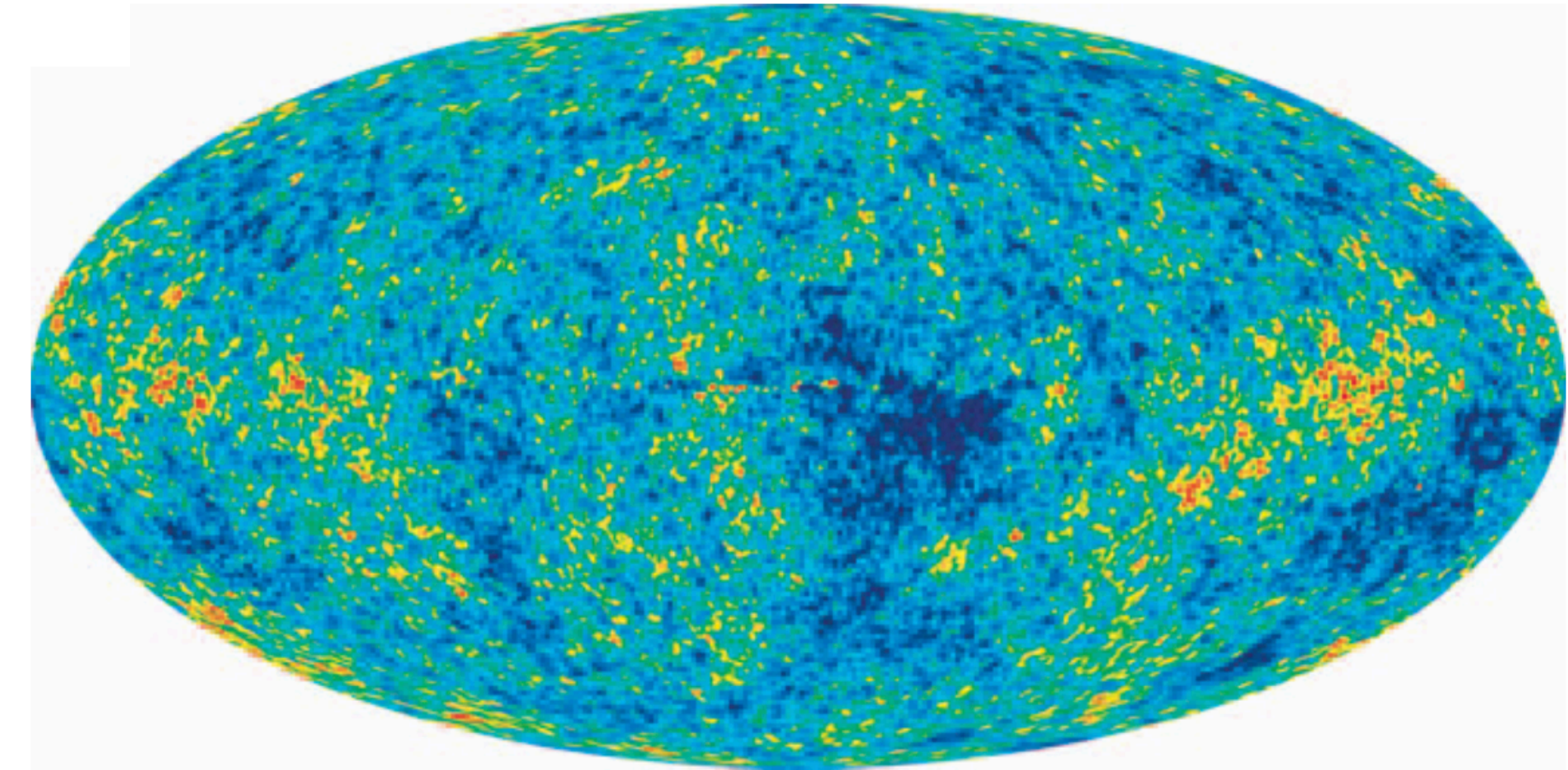


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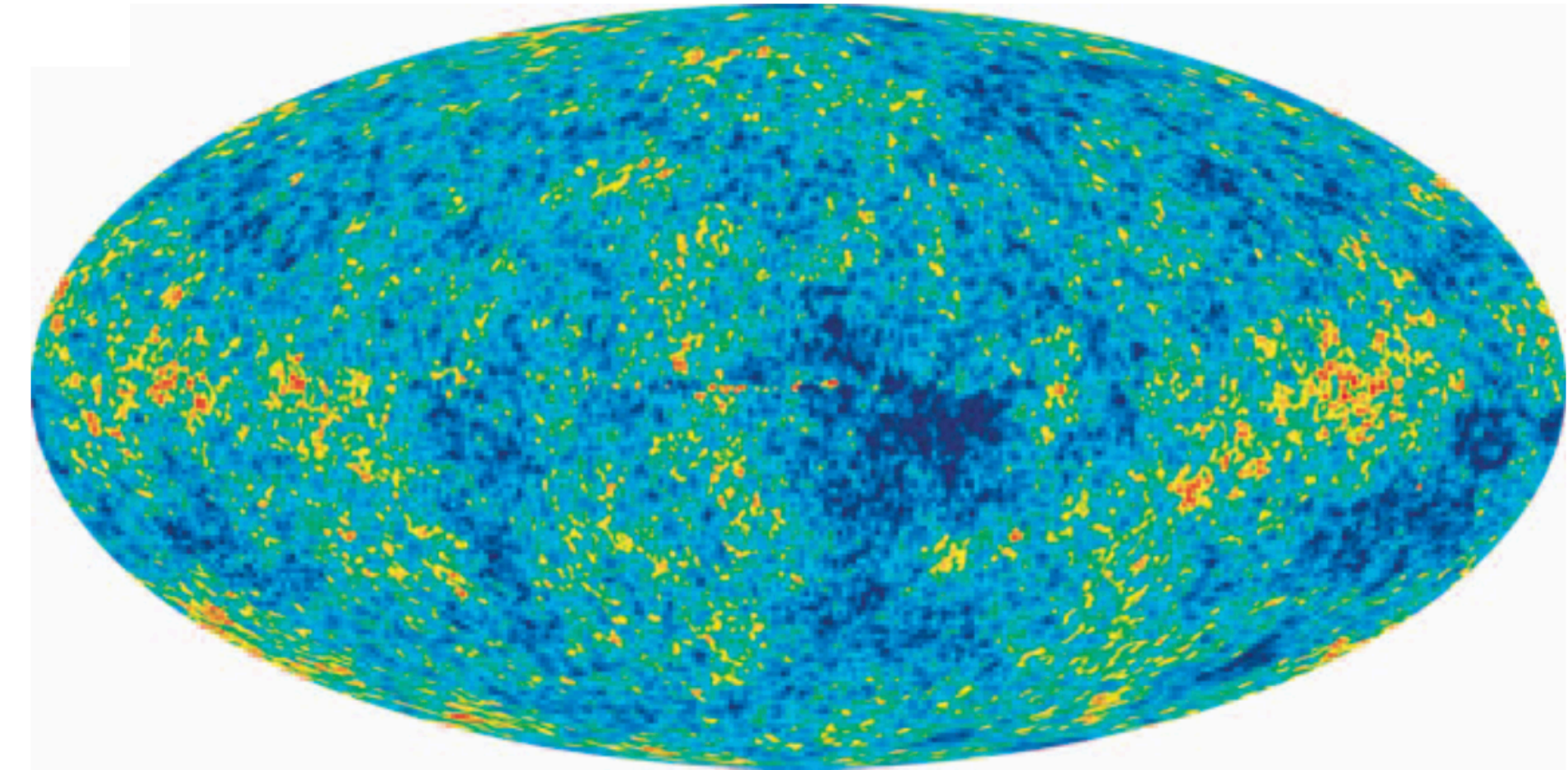


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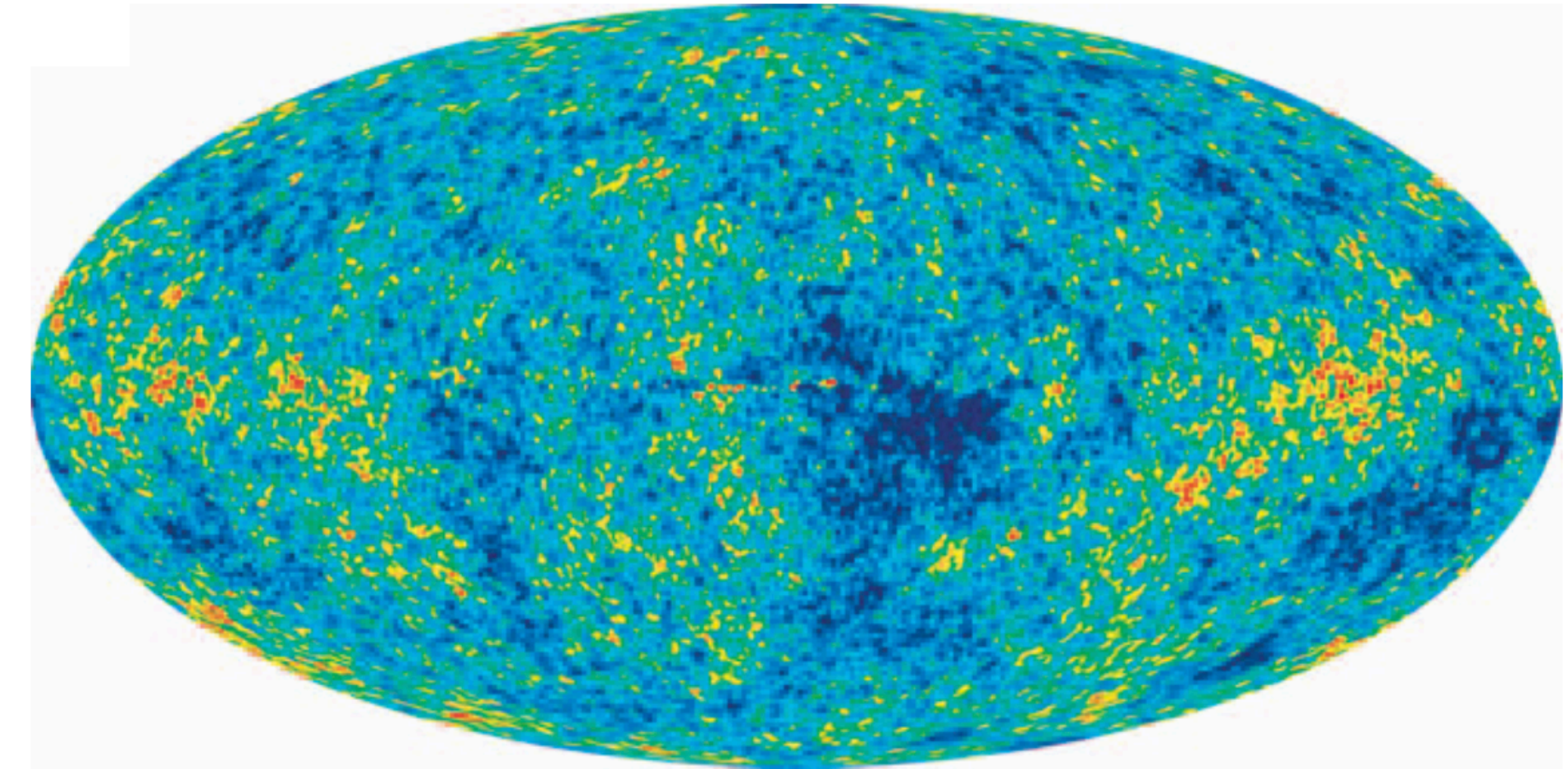


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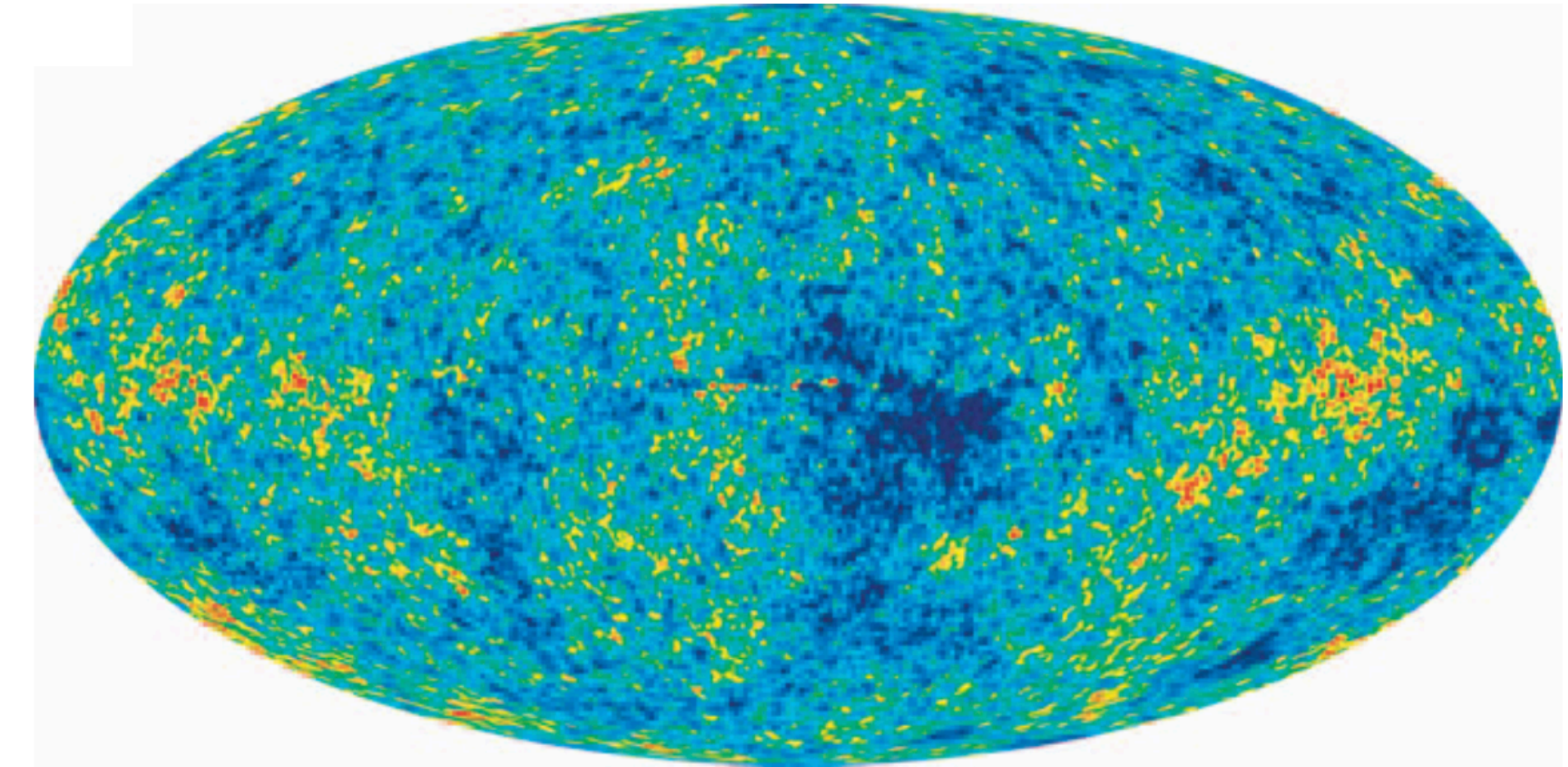


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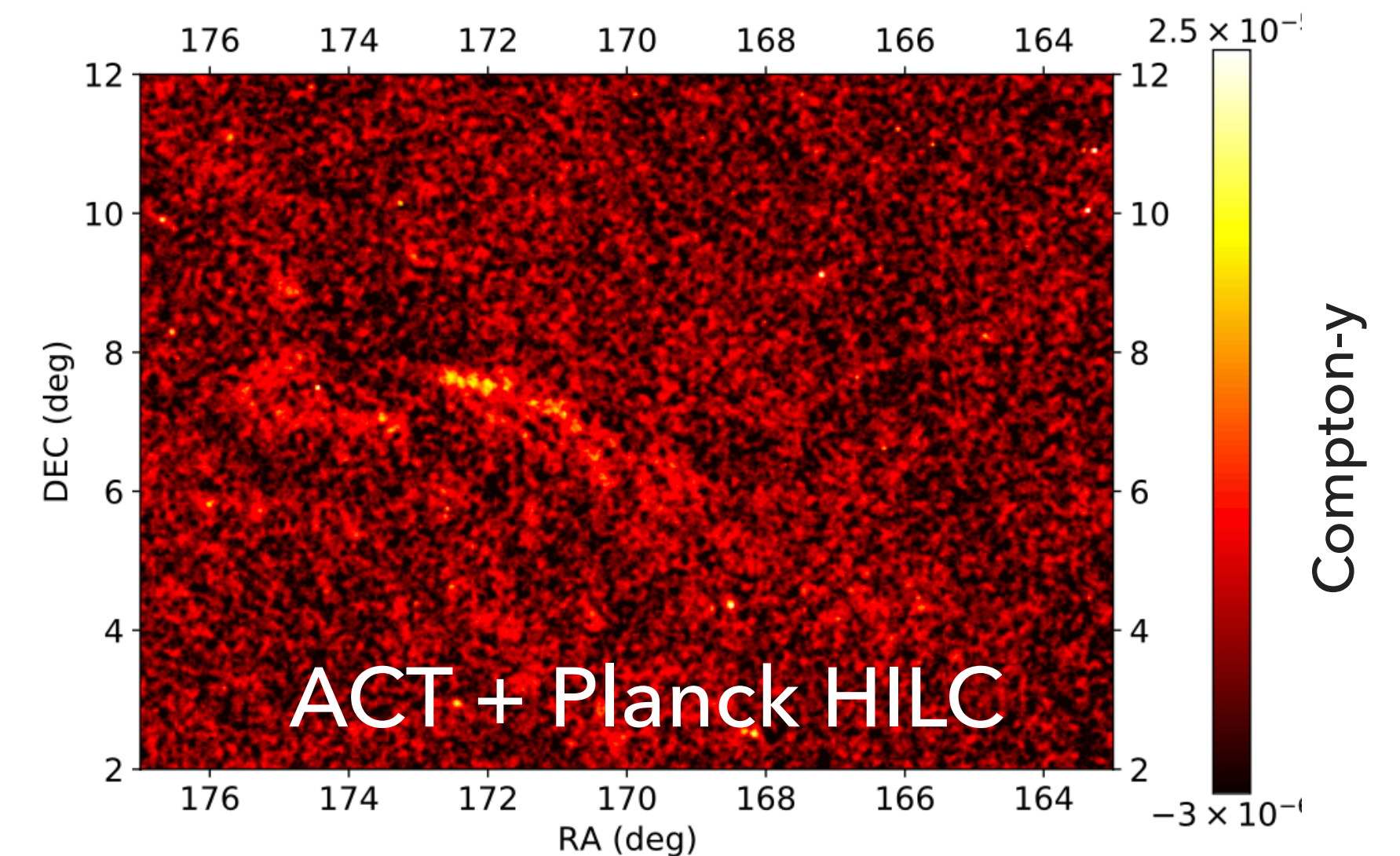
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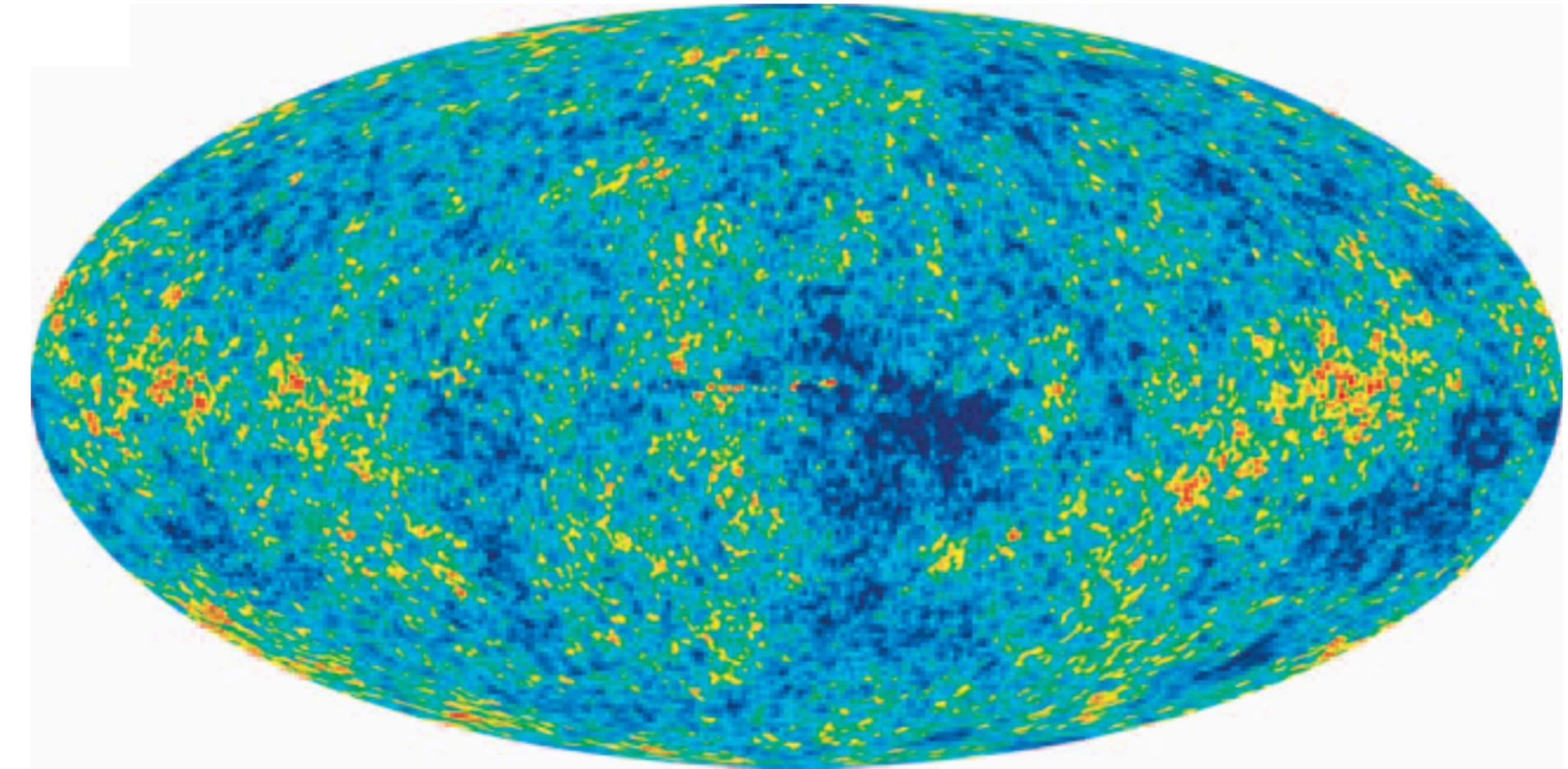


Coulton, Madhavacheril, AJD, Hill++ (2307.01258)

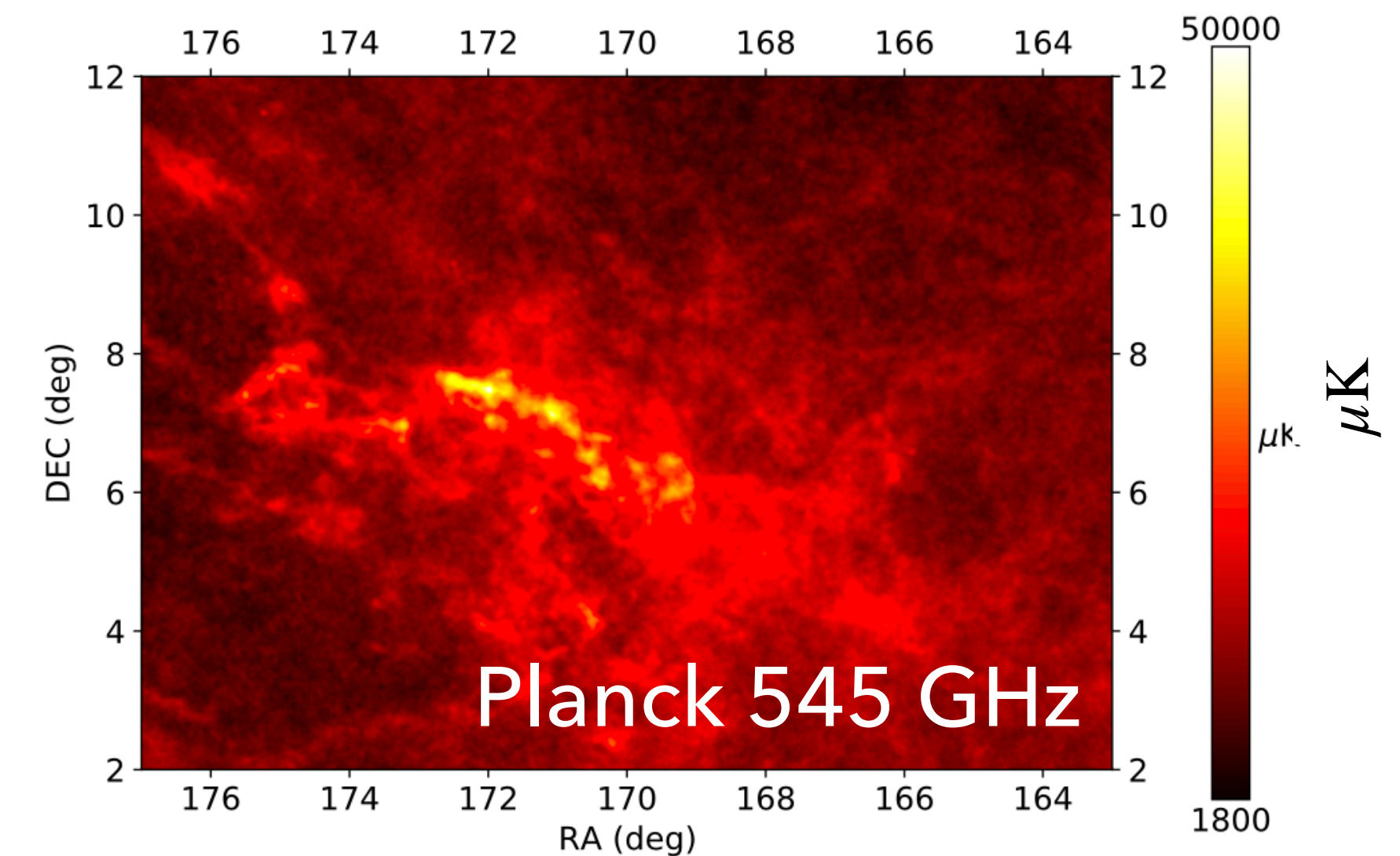
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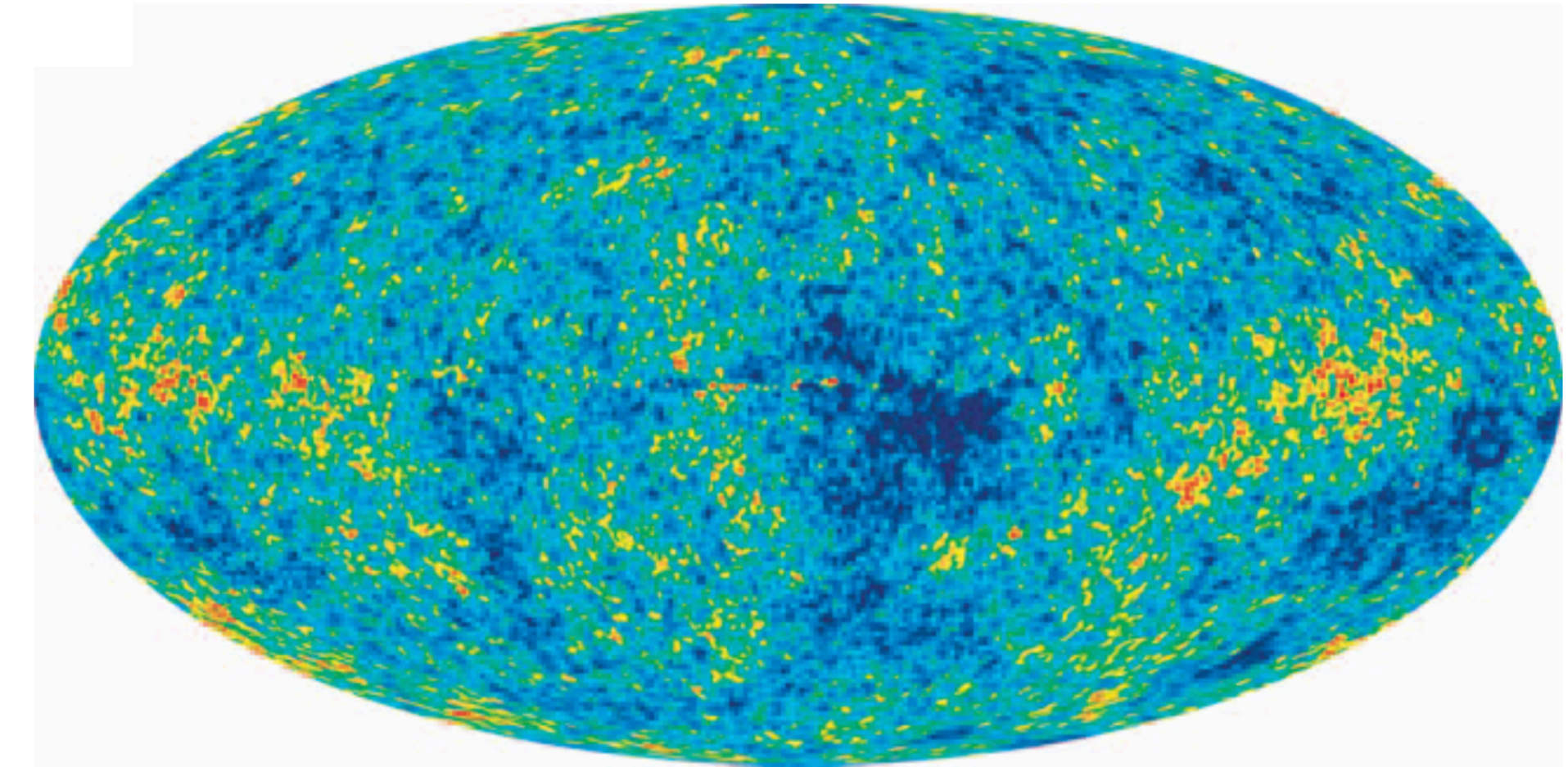


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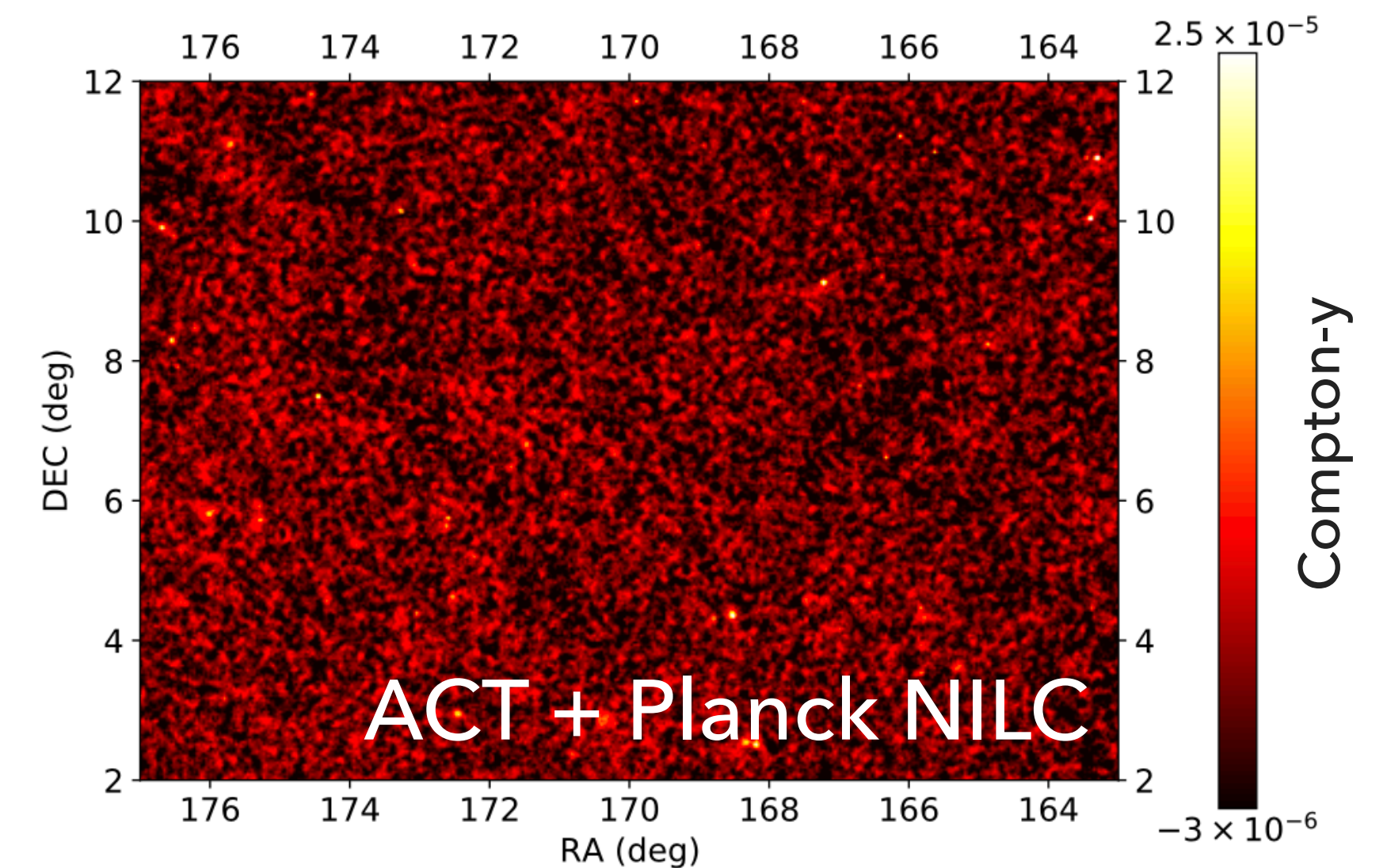
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- ▶ Foreground contamination is not quantified and error propagation becomes ad hoc
- ▶ **So, instead of fully relying on the ILC estimate, try to use it as a data compressor for simulation-based inference**

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$$\{\vec{\theta}_i, \vec{x}_i\}_{i=1}^{N_{\text{sim}}}$$

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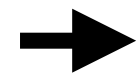
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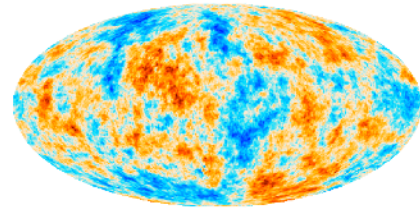
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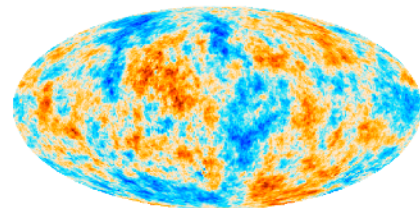
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$$\vec{\phi}_i \sim P(\vec{\phi} | \vec{\theta}_i)$$

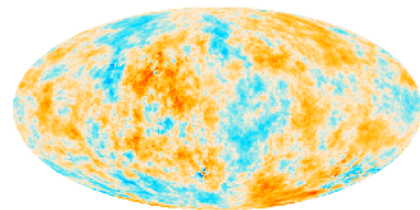
23 GHz



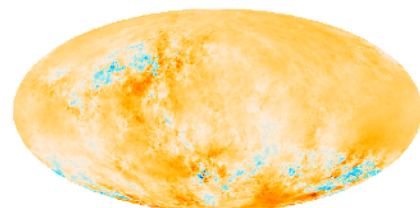
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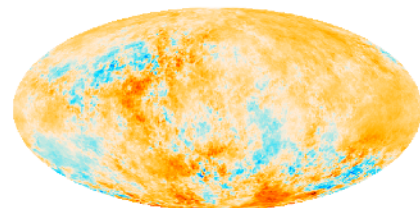
39 GHz



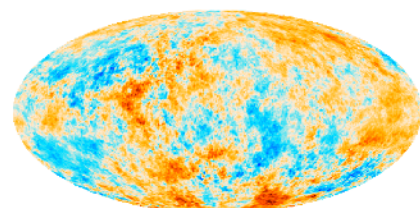
93 GHz



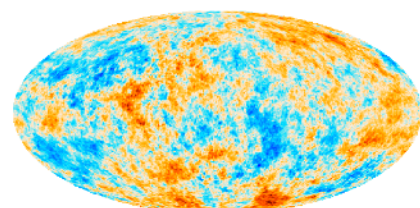
145 GHz



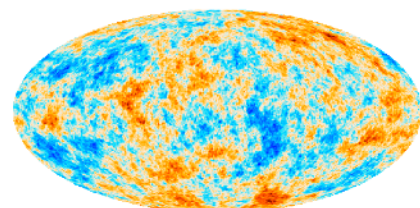
225 GHz



280 GHz



353 GHz



Draw sky realization

# GENERATING TRAINING SET

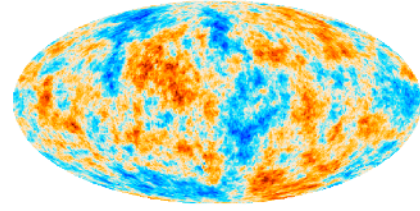
$$\{\vec{\theta}_i, \vec{x}_i\}_{i=1}^{N_{\text{sim}}}$$

$$\vec{\theta}_i \sim P(\vec{\theta})$$

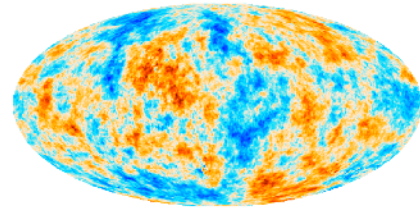
Draw from prior

$$\vec{\phi}_i \sim P(\vec{\phi} | \vec{\theta}_i)$$

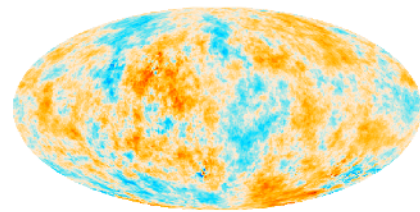
23 GHz



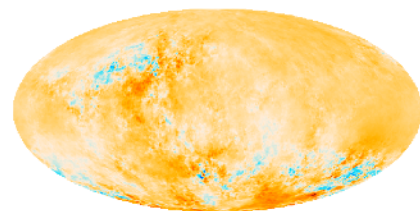
27 GHz



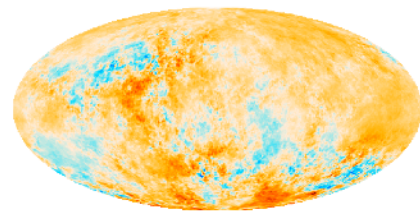
39 GHz



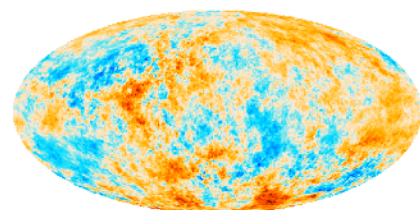
93 GHz



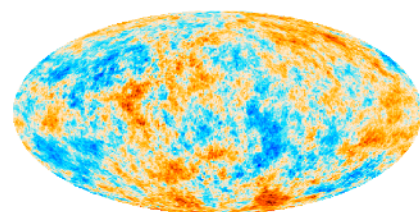
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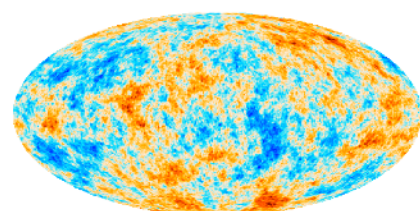
225 GHz



280 GHz



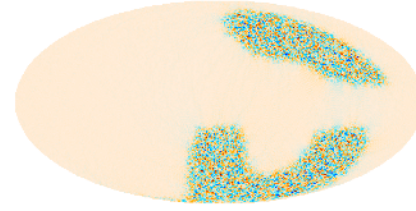
353 GHz



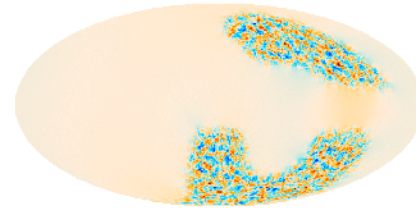
Draw sky realization

$$\vec{d}_i \sim P(\vec{d} | \vec{\phi}_i)$$

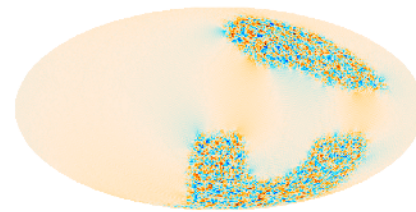
23 GHz



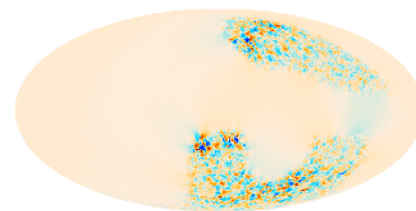
27 GHz



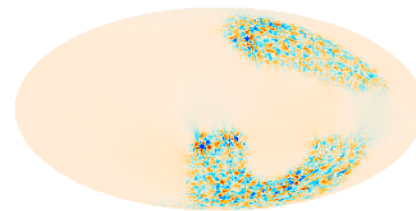
39 GHz



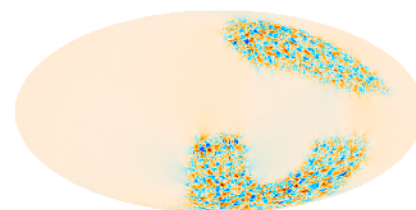
93 GHz



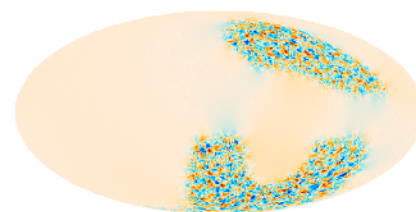
145 GHz



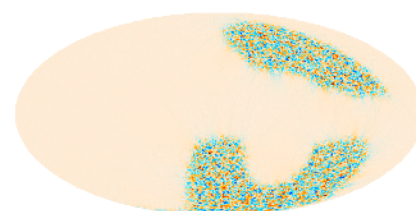
225 GHz



280 GHz



353 GHz



Convolve with beam, filter, add noise, mask

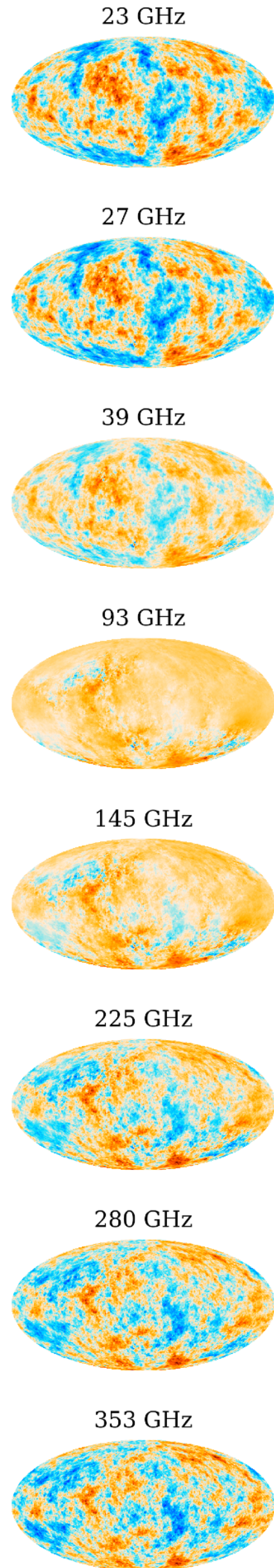
# GENERATING TRAINING SET

$$\{\vec{\theta}_i, \vec{x}_i\}_{i=1}^{N_{\text{sim}}}$$

$$\vec{\theta}_i \sim P(\vec{\theta})$$

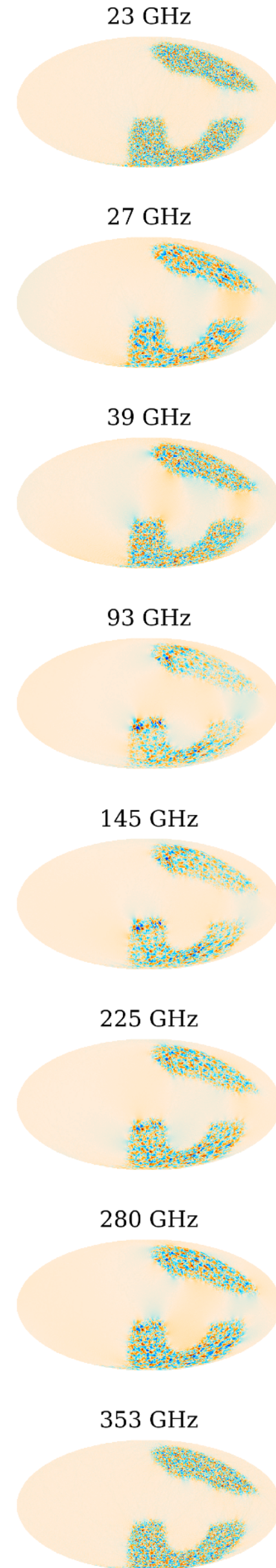
Draw from prior

$$\vec{\phi}_i \sim P(\vec{\phi} | \vec{\theta}_i)$$



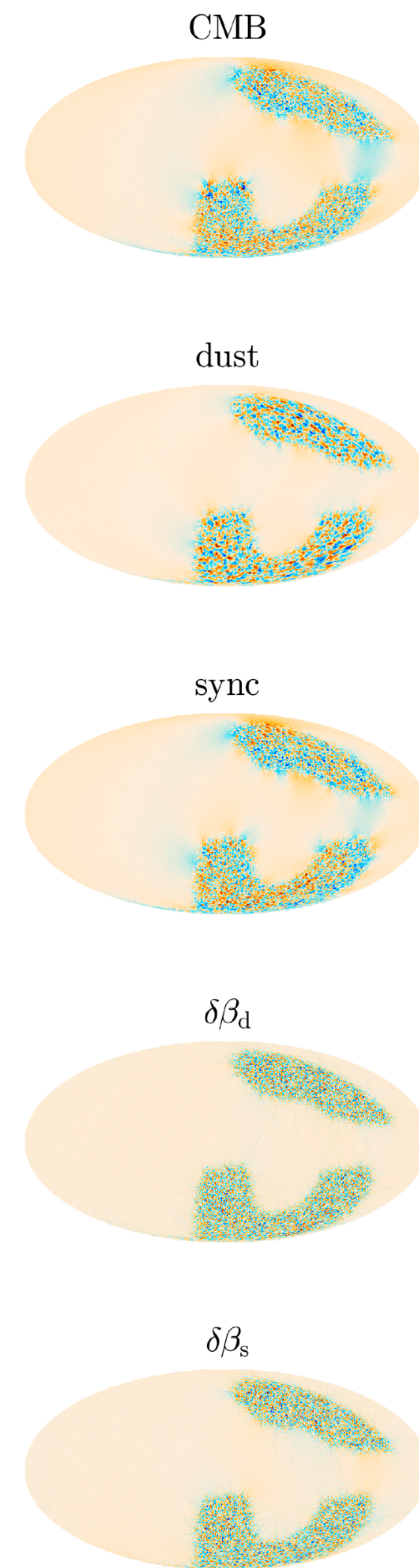
Draw sky realization

$$\vec{d}_i \sim P(\vec{d} | \vec{\phi}_i)$$



Convolve with beam, filter, add noise, mask

$$\vec{x}_i = f(\vec{d}_i)$$



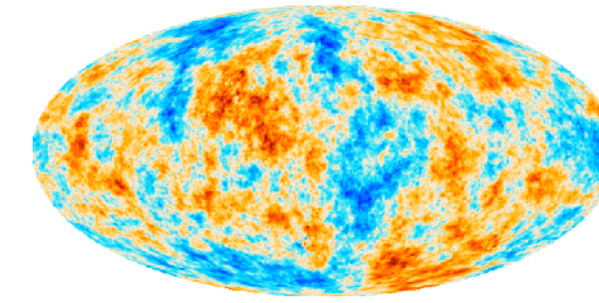
$$\left\{ \begin{array}{c} \hat{C}_\ell^{\text{CMB} \times \text{CMB}} \\ \hat{C}_\ell^{\text{CMB} \times \text{dust}} \\ \vdots \\ \hat{C}_\ell^{\text{sync} \times \delta\beta_s} \\ \hat{C}_\ell^{\delta\beta_s \times \delta\beta_s} \end{array} \right\}$$

15 spectra,  
165 bins total

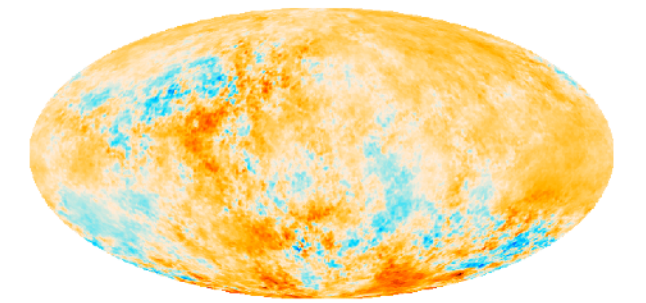
# SIMULATIONS, FORWARD MODEL

12

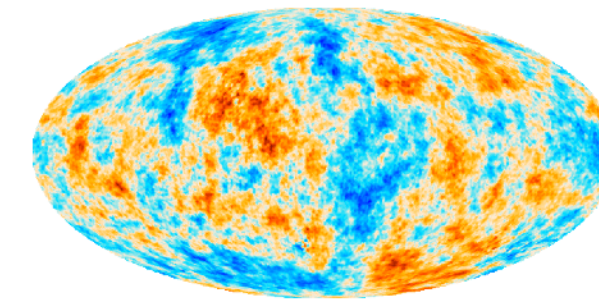
23 GHz



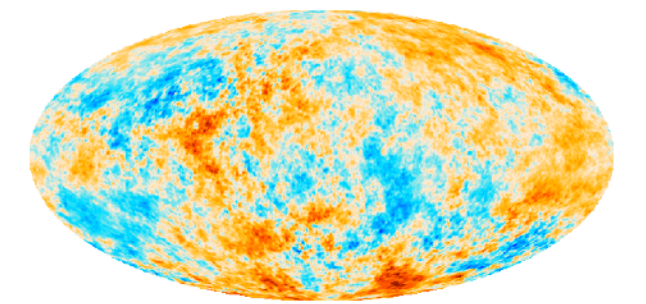
145 GHz



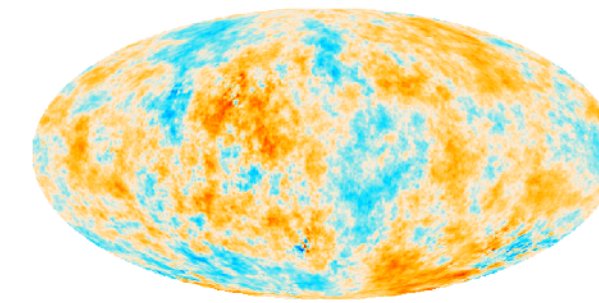
27 GHz



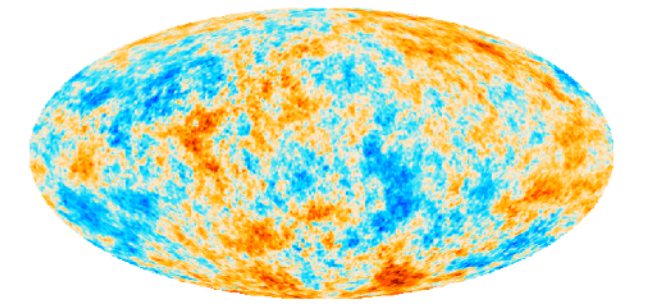
225 GHz



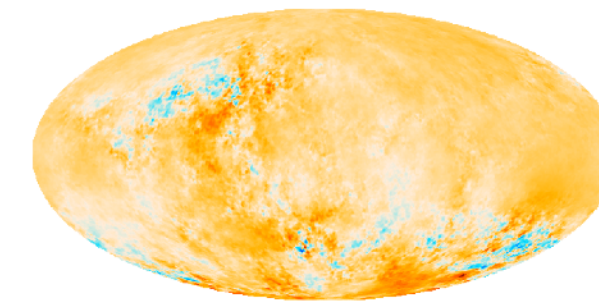
39 GHz



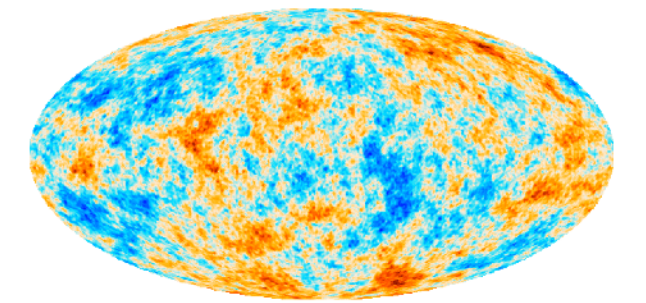
280 GHz



93 GHz

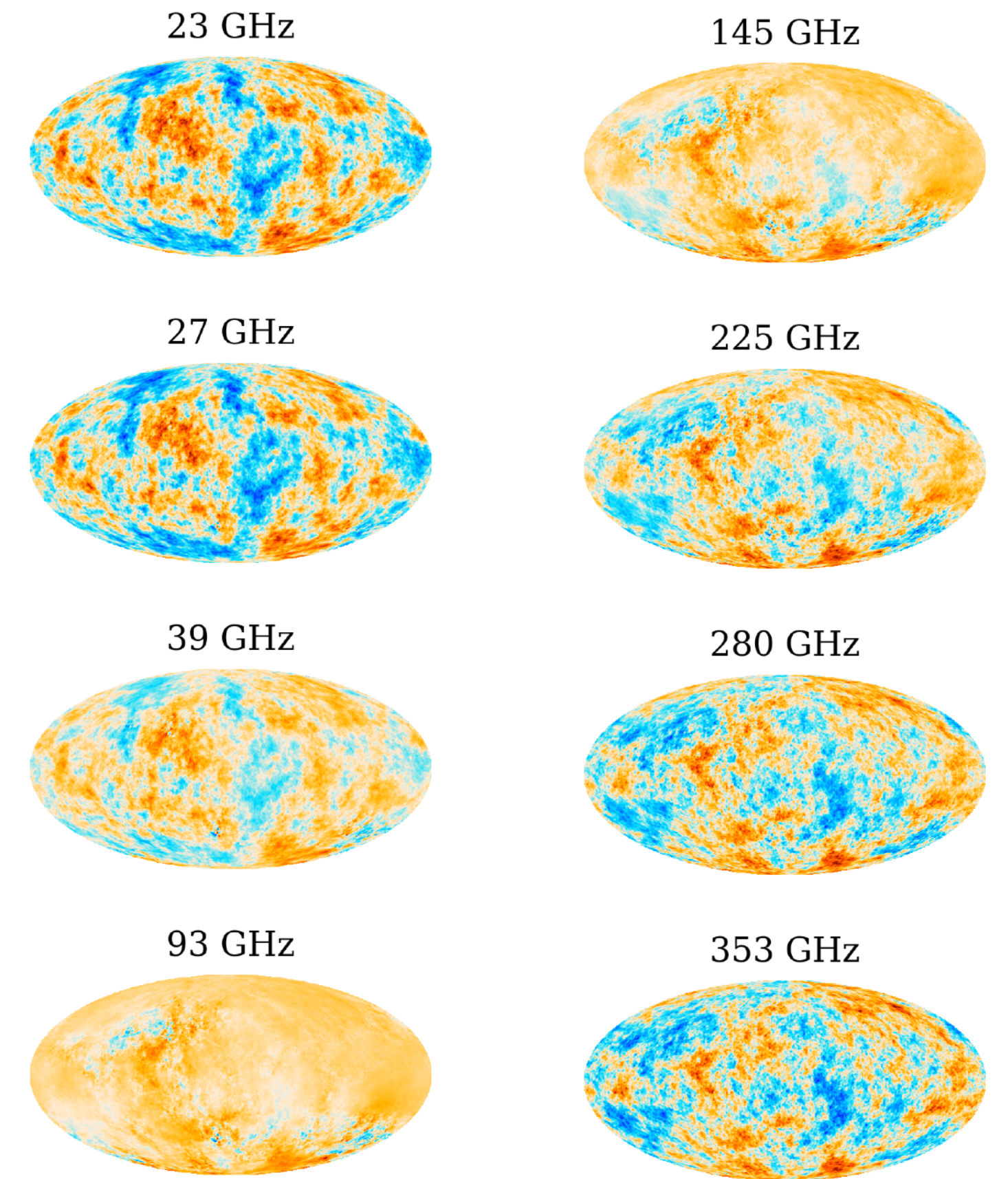


353 GHz



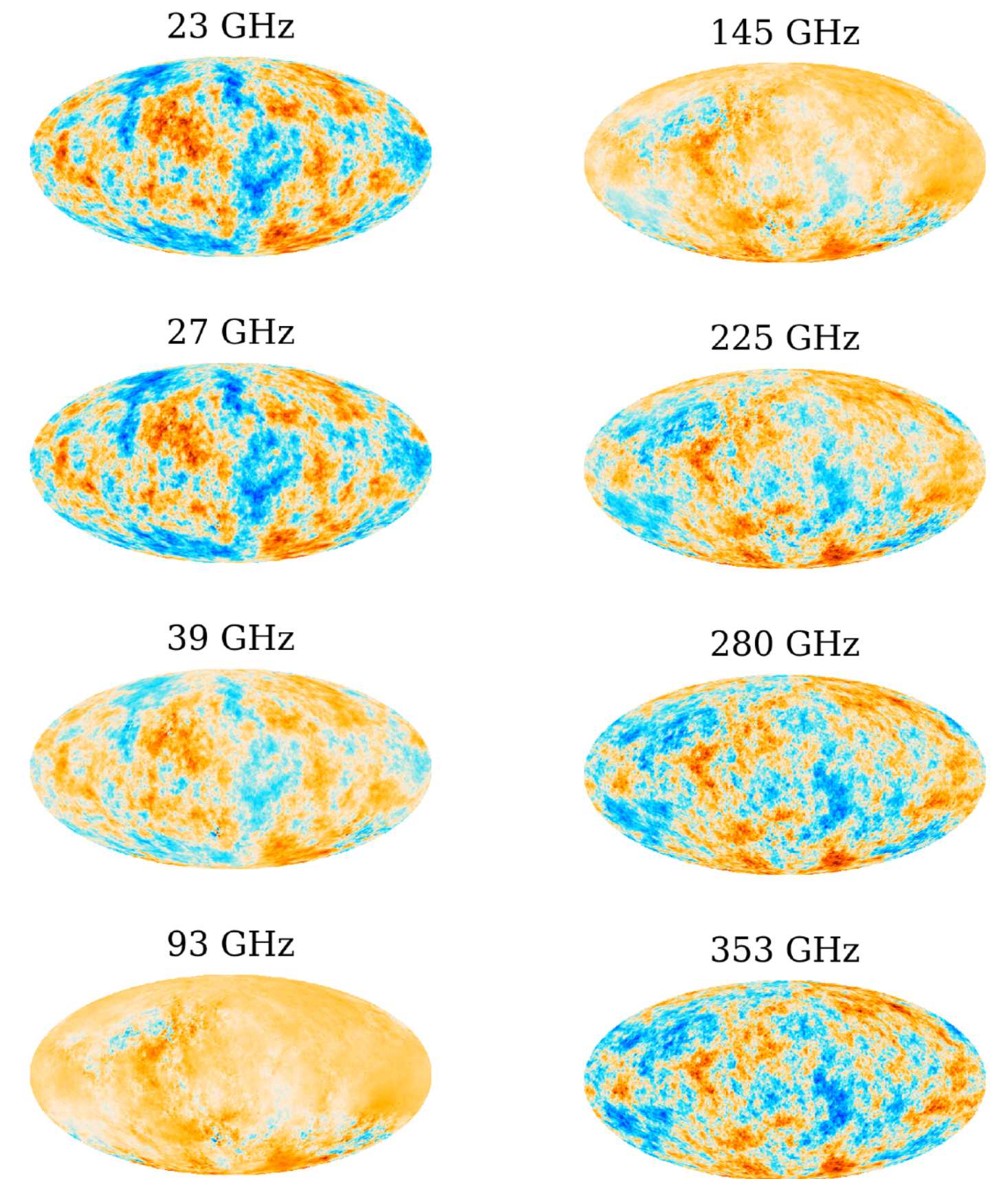
# SIMULATIONS, FORWARD MODEL

$$\vec{\theta} \in \underbrace{\{r, A_{\text{lens}}\}}_{\text{Cosmo}}, \underbrace{\{A_{\text{d}}, \alpha_{\text{d}}, \beta_{\text{d}}, B_{\text{d}}, \gamma_{\text{d}}, A_{\text{s}}, \alpha_{\text{s}}, \beta_{\text{s}}, B_{\text{s}}, \gamma_{\text{s}}, \rho_{\text{ds}}\}}_{\text{Foregrounds}}$$



# SIMULATIONS, FORWARD MODEL

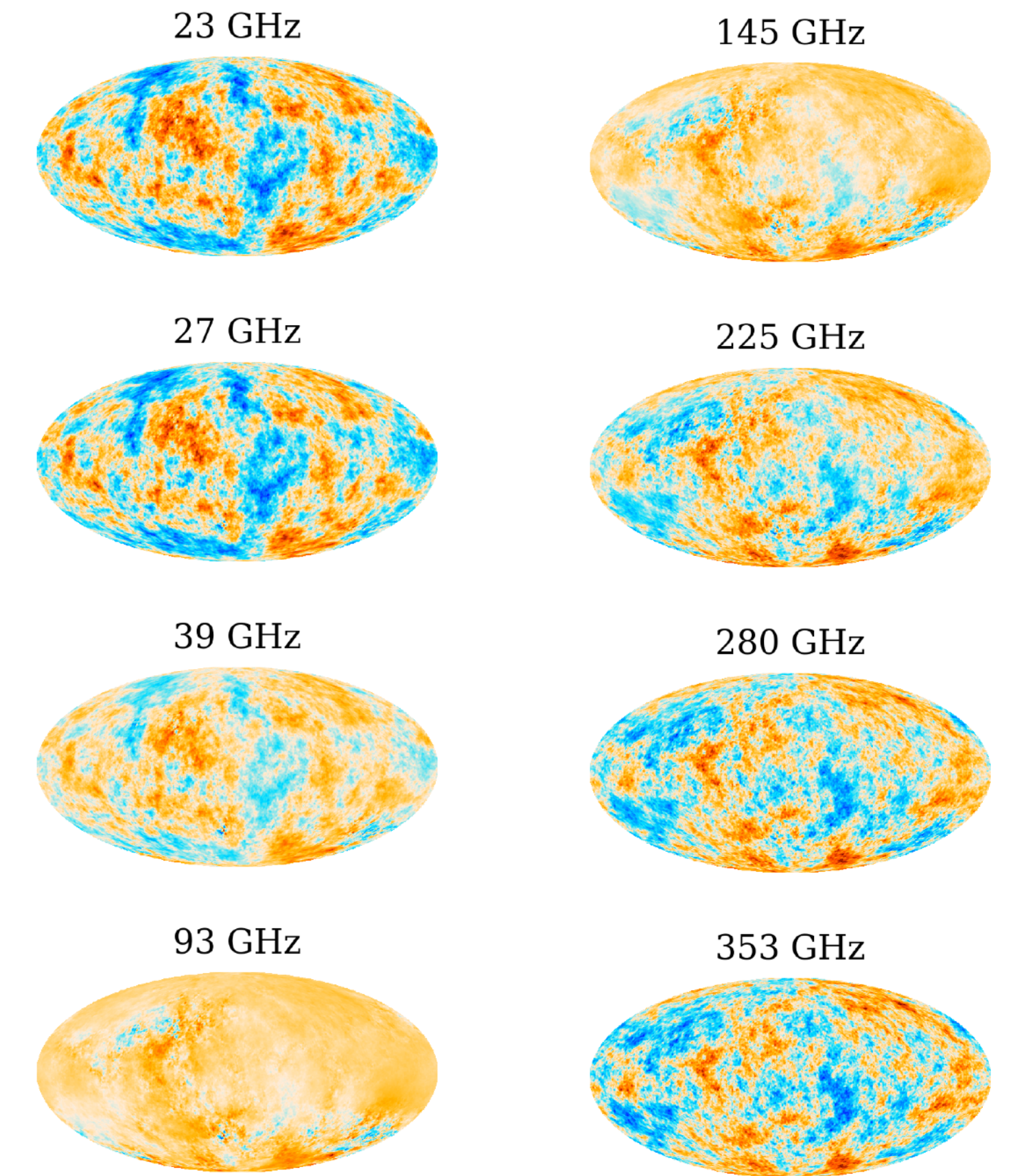
$$\vec{\theta} \in \underbrace{\{r, A_{\text{lens}}\}}_{\text{Cosmo}} \underbrace{\{A_{\text{d}}, \alpha_{\text{d}}, \beta_{\text{d}}, B_{\text{d}}, \gamma_{\text{d}}, A_{\text{s}}, \alpha_{\text{s}}, \beta_{\text{s}}, B_{\text{s}}, \gamma_{\text{s}}, \rho_{\text{ds}}\}}_{\text{Foregrounds}}$$



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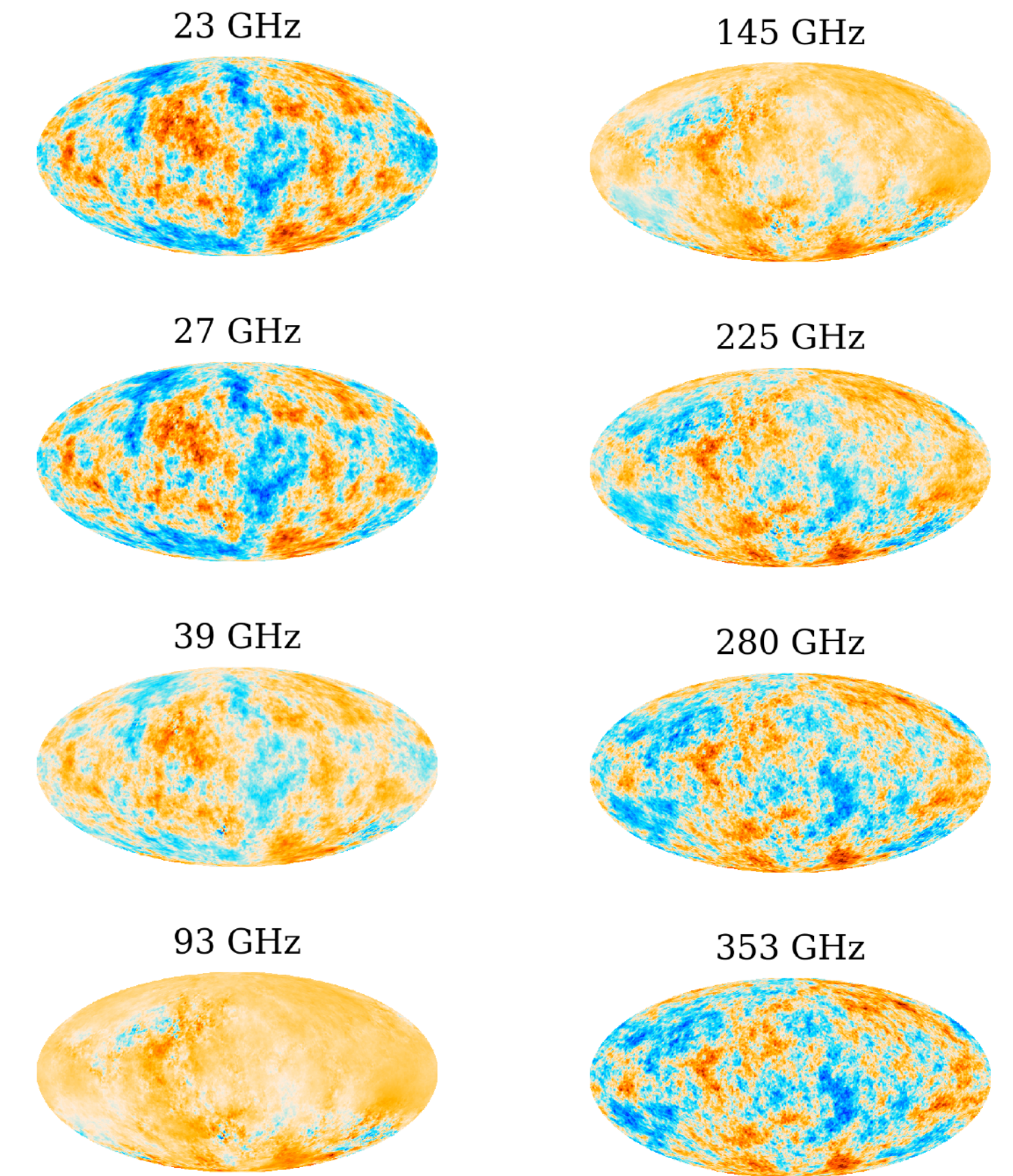
- ▶ Statistically isotropic Gaussian dust amplitudes



# SIMULATIONS, FORWARD MODEL

$$\vec{\theta} \in \underbrace{\{r, A_{\text{lens}}\}}_{\text{Cosmo}} \underbrace{\{A_{\text{d}}, \alpha_{\text{d}}, \beta_{\text{d}}, B_{\text{d}}, \gamma_{\text{d}}, A_{\text{s}}, \alpha_{\text{s}}, \beta_{\text{s}}, B_{\text{s}}, \gamma_{\text{s}}, \rho_{\text{ds}}\}}_{\text{Foregrounds}}$$

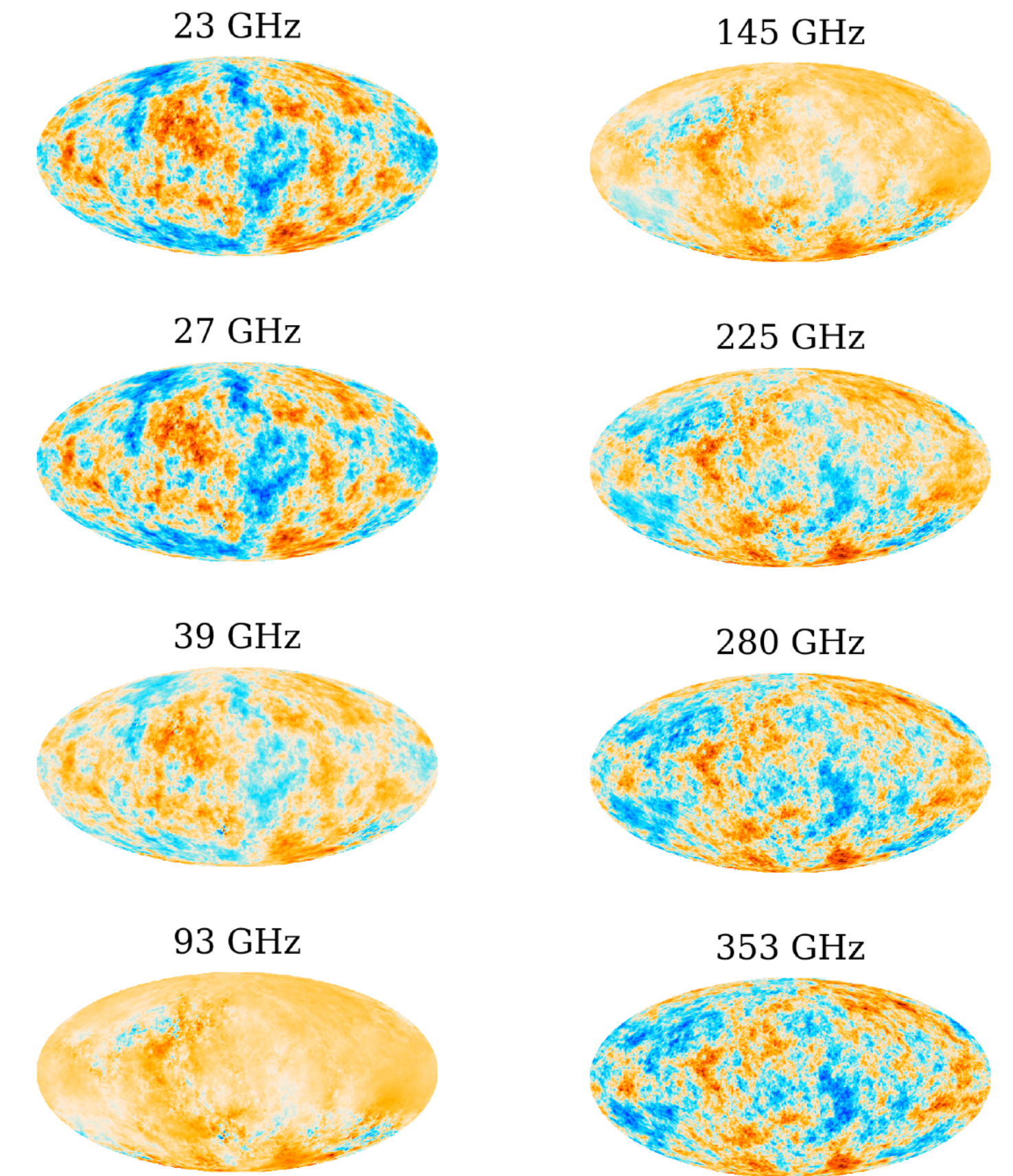
- ▶ Statistically isotropic Gaussian dust amplitudes
- ▶ Statistically isotropic Gaussian  $\beta_{\text{d}}$  field



# SIMULATIONS, FORWARD MODEL

$$\vec{\theta} \in \underbrace{\{r, A_{\text{lens}}\}}_{\text{Cosmo}}, \underbrace{\{A_{\text{d}}, \alpha_{\text{d}}, \beta_{\text{d}}, B_{\text{d}}, \gamma_{\text{d}}, A_{\text{s}}, \alpha_{\text{s}}, \beta_{\text{s}}, B_{\text{s}}, \gamma_{\text{s}}, \rho_{\text{ds}}\}}_{\text{Foregrounds}}$$

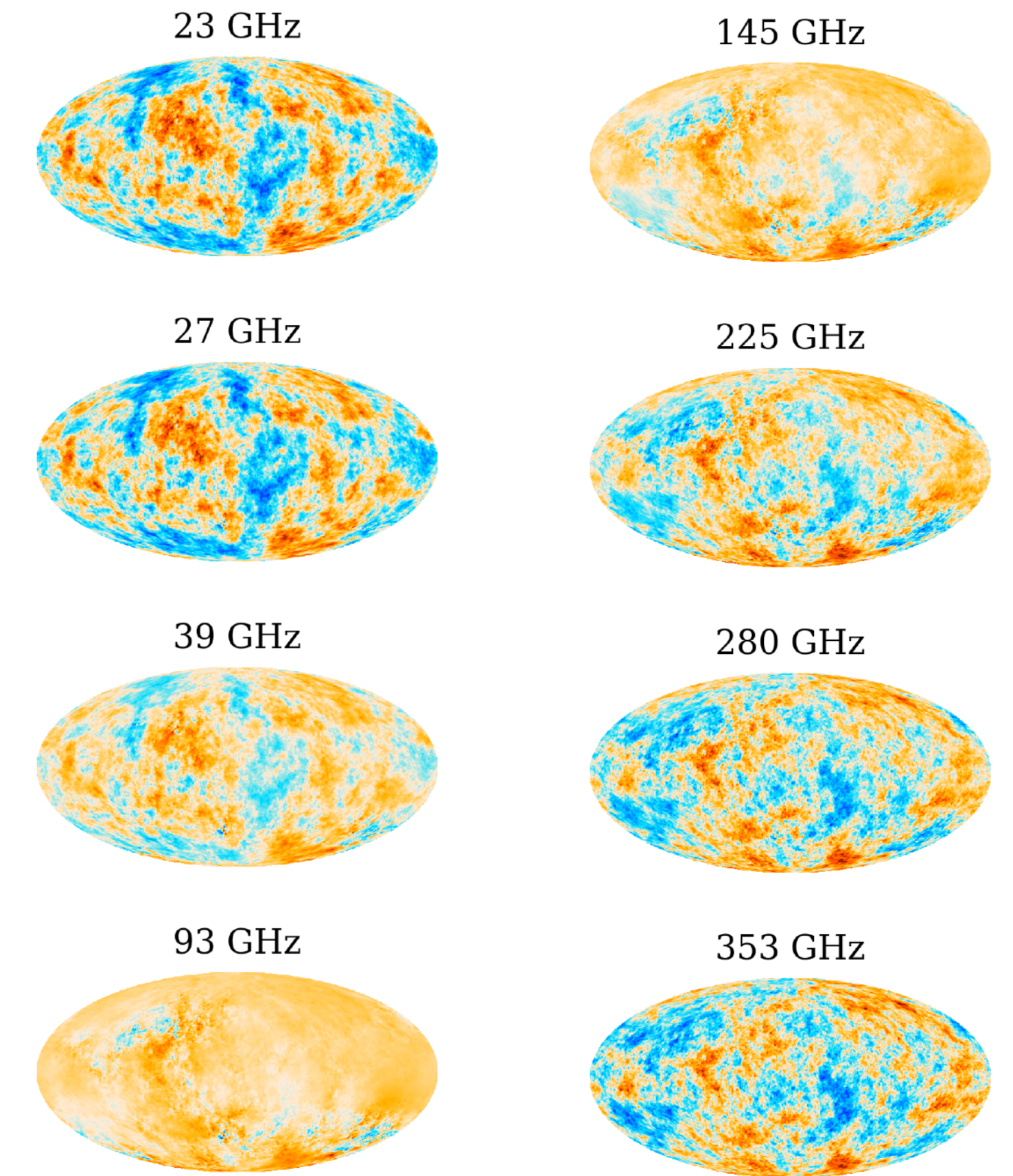
- ▶ Statistically isotropic Gaussian dust amplitudes
- ▶ Statistically isotropic Gaussian  $\beta_{\text{d}}$  field
- ▶ Same for synchrotron, see Azzoni++ 2021 (2011.11575)



# SIMULATIONS, FORWARD MODEL

$$\vec{\theta} \in \underbrace{\{r, A_{\text{lens}}\}}_{\text{Cosmo}} \underbrace{\{A_{\text{d}}, \alpha_{\text{d}}, \beta_{\text{d}}, B_{\text{d}}, \gamma_{\text{d}}, A_{\text{s}}, \alpha_{\text{s}}, \beta_{\text{s}}, B_{\text{s}}, \gamma_{\text{s}}, \rho_{\text{ds}}\}}_{\text{Foregrounds}}$$

- ▶ Statistically isotropic Gaussian dust amplitudes
- ▶ Statistically isotropic Gaussian  $\beta_{\text{d}}$  field
- ▶ Same for synchrotron, see Azzoni++ 2021 (2011.11575)
- ▶ Dust-synchrotron cross-correlation coefficient  $\rho_{\text{ds}}$

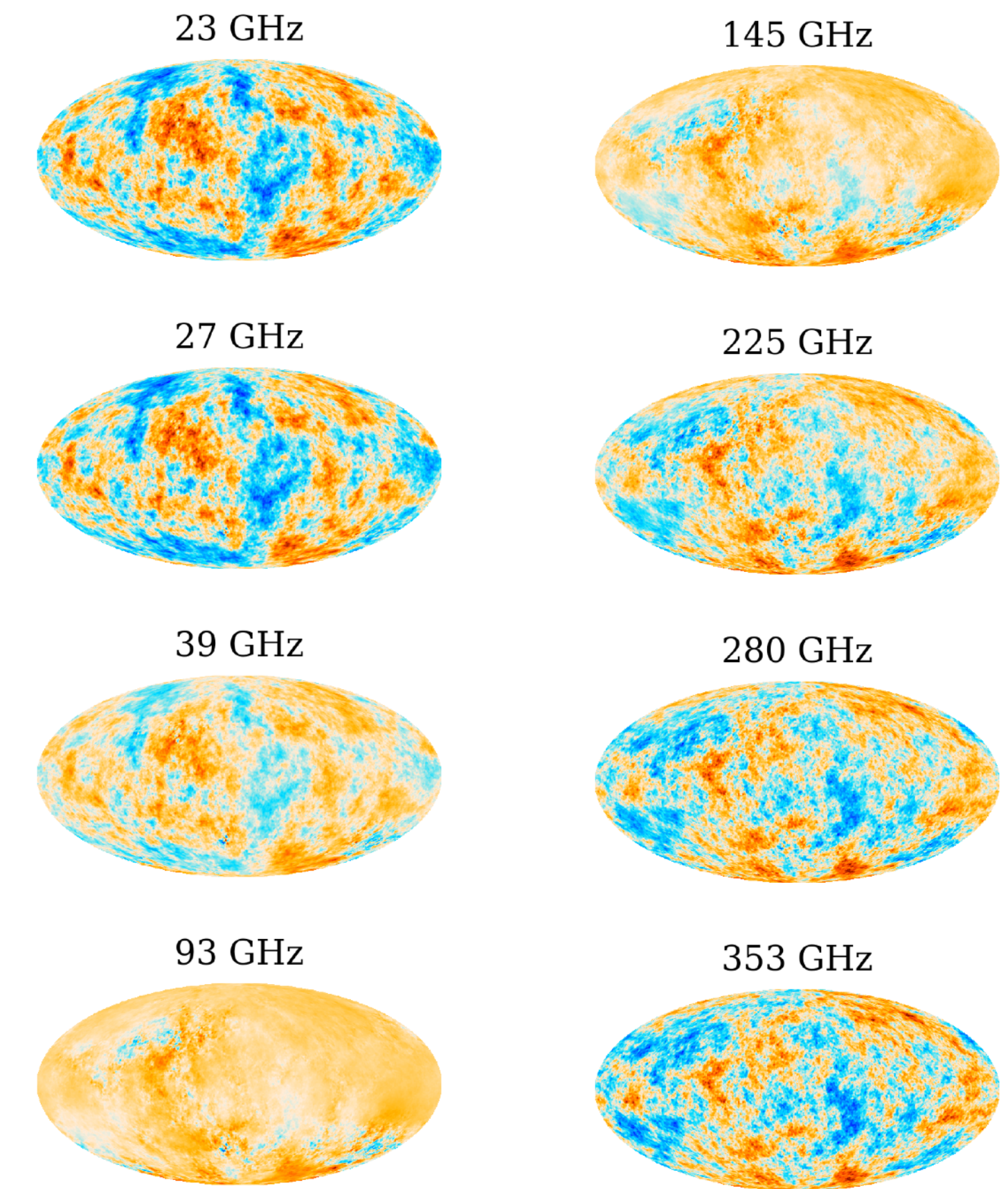


# SIMULATIONS, FORWARD MODEL

12

$$\vec{\theta} \in \underbrace{\{r, A_{\text{lens}}\}}_{\text{Cosmo}} \underbrace{\{A_{\text{d}}, \alpha_{\text{d}}, \beta_{\text{d}}, B_{\text{d}}, \gamma_{\text{d}}, A_{\text{s}}, \alpha_{\text{s}}, \beta_{\text{s}}, B_{\text{s}}, \gamma_{\text{s}}, \rho_{\text{ds}}\}}_{\text{Foregrounds}}$$

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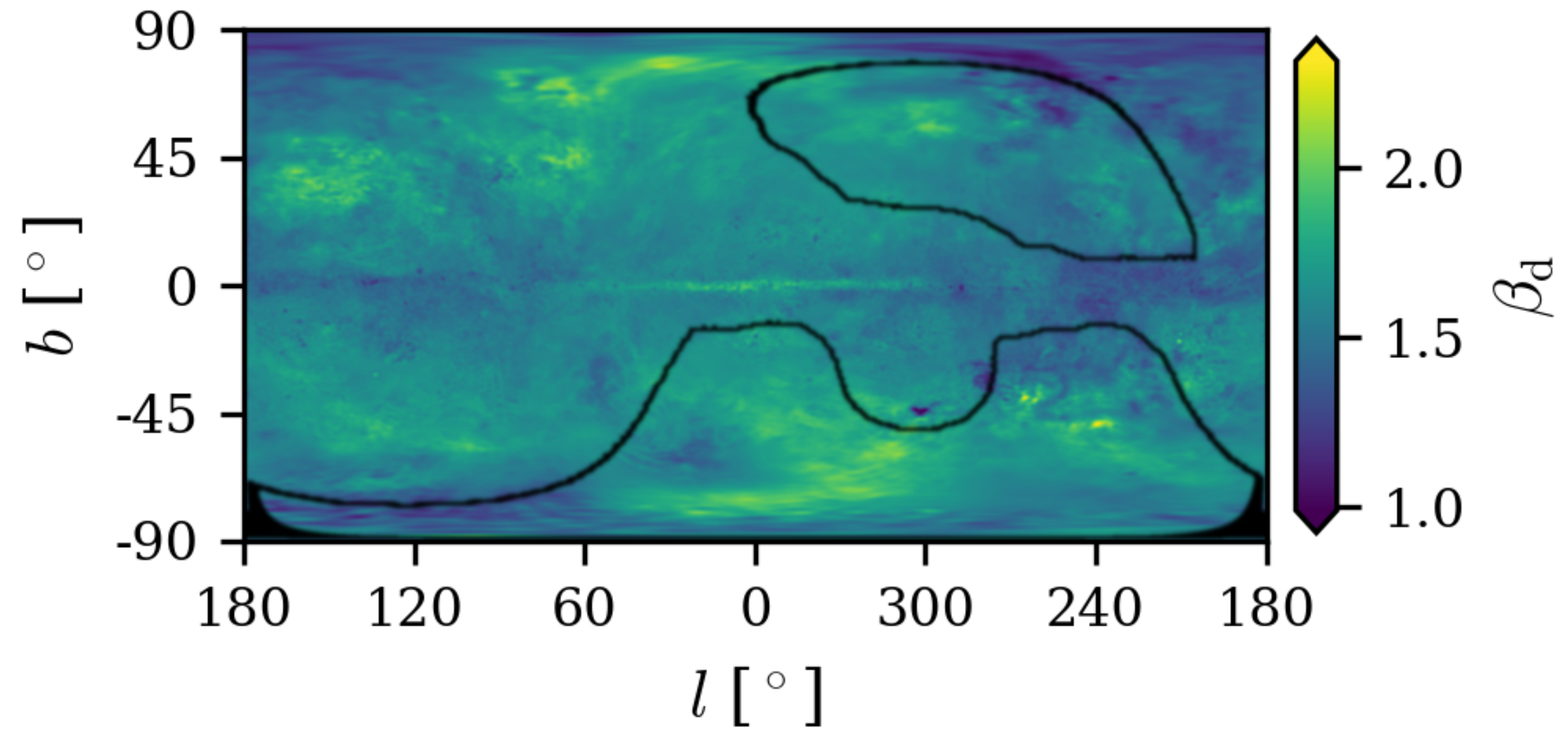


**Central question paper:** Given the lack of accurate simulations of Galactic CMB foregrounds, can we get away with such a simplified foreground model? (Using SBI with tailored data compression)

- ▶ Neural posterior estimation  
(Papamakarios, Murray, 1605.06376) with  
normalizing flow to learn  
analytically intractable NILC  
likelihood
- ▶ 13 parameters, 165 data points  
(NILC cross-spectrum bins)

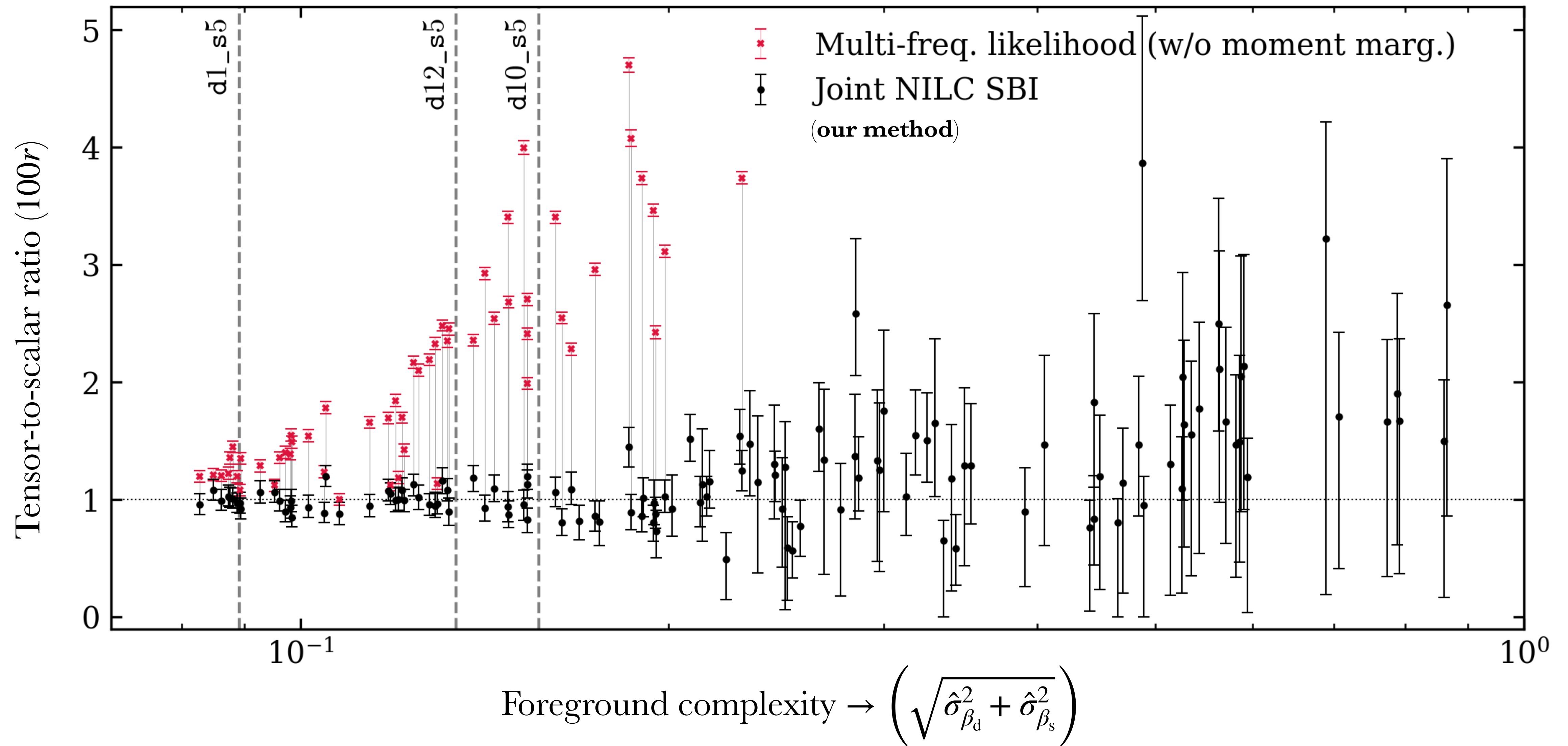
# SBI SETUP

- ▶ Neural posterior estimation  
(Papamakarios, Murray, 1605.06376) with  
normalizing flow to learn  
analytically intractable NILC  
likelihood
- ▶ 13 parameters, 165 data points  
(NILC cross-spectrum bins)
- ▶  $f_{\text{sky}} = 21\%$  :  $\sim 2\times$  area of SO forecasts,  $7\times$  CMB-S4 forecasts: more  
challenging due to increased  $\beta$  variation
- ▶ (idealized) Simons Observatory noise (*goal, optimistic*  $\ell_{\text{knee}}$ ) + *WMAP* K and  
*Planck* 353 bands. SO noise **not** rescaled for increased  $f_{\text{sky}}$



# COMPARISON TO STANDARD MULTI-FREQ POWER SPECTRUM APPROACH

14



# TESTS ON (OUT-OF-DISTRIBUTION) PYSM MODELS

15

The Pan-Experiment Galactic  
Science Group, 2025 (2502.20452)

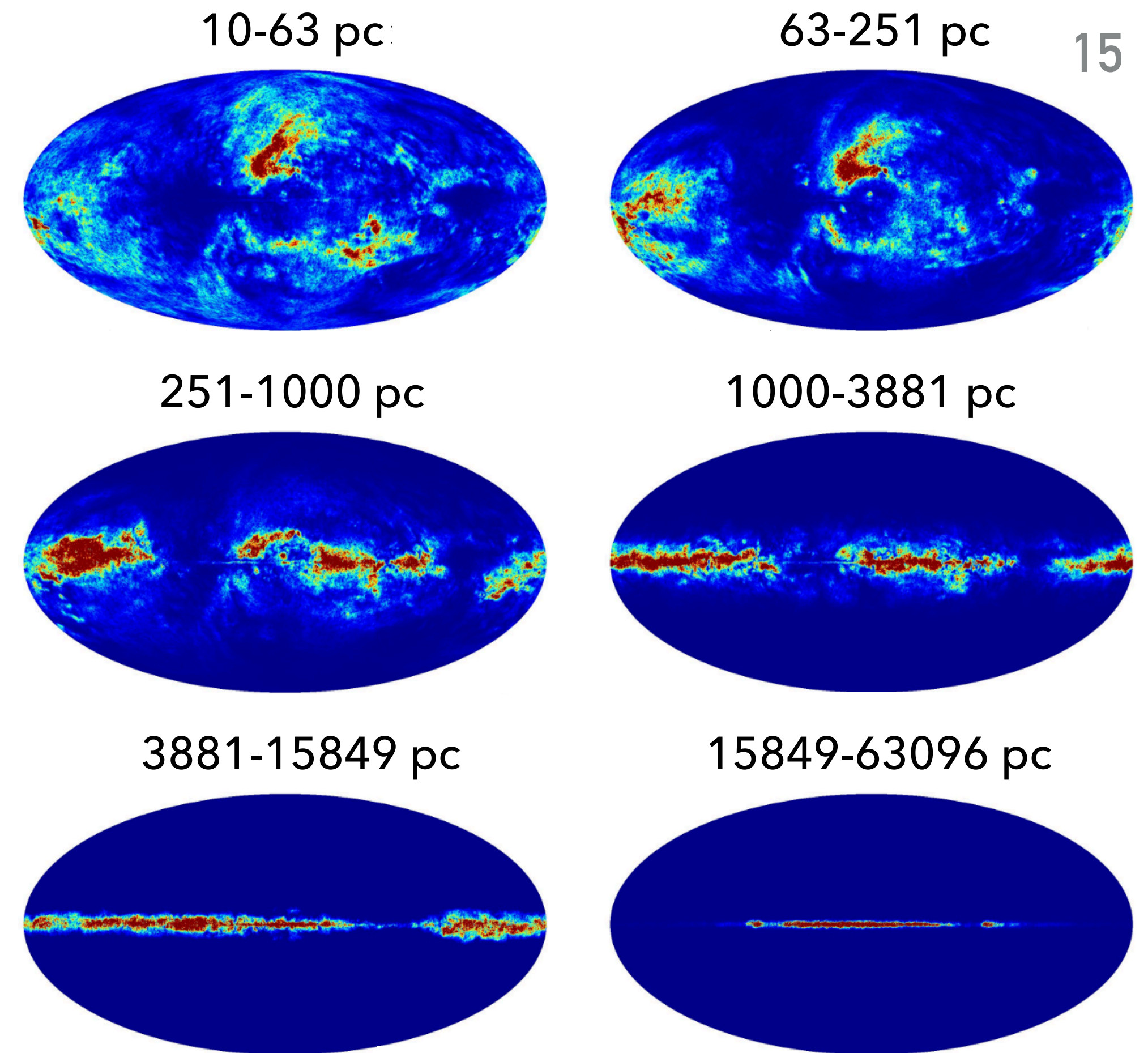
# TESTS ON (OUT-OF-DISTRIBUTION) PYSM MODELS

15

- ▶ *Medium complexity*
  - ▶ **d1** (spatially varying  $\beta_d$ )
  - ▶ **d10** (variation of d1, stronger  $\beta_d$  variations)
  - ▶ **s5** (spatially varying  $\beta_s$ )

# TESTS ON (OUT-OF-DISTRIBUTION) PYSM MODELS

- ▶ *Medium complexity*
  - ▶ **d1** (spatially varying  $\beta_d$ )
  - ▶ **d10** (variation of d1, stronger  $\beta_d$  variations)
  - ▶ **s5** (spatially varying  $\beta_s$ )
- ▶ *High complexity*
  - ▶ **d12** (spat. varying  $\beta_d$ , line-of-sight SED variations)
  - ▶ **s7** (Non-power-law SED: curved index)



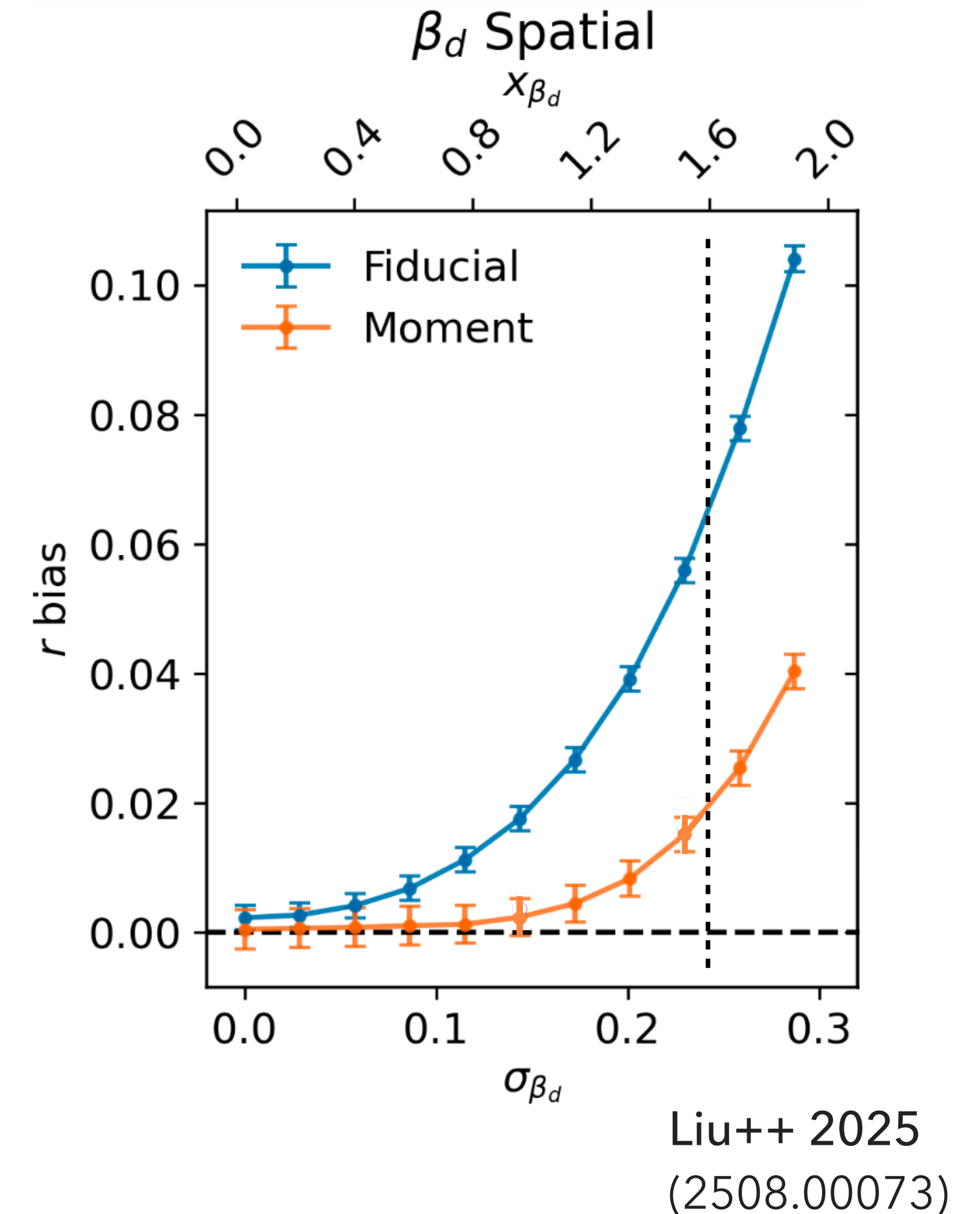
Martínez-Solaache, Karakci, Delabrouille 2018  
(1706.04162) 3D dust model based on  
starlight reddening data

The Pan-Experiment Galactic  
Science Group, 2025 (2502.20452)

# TESTS ON (OUT-OF-DISTRIBUTION) PYSM MODELS

15

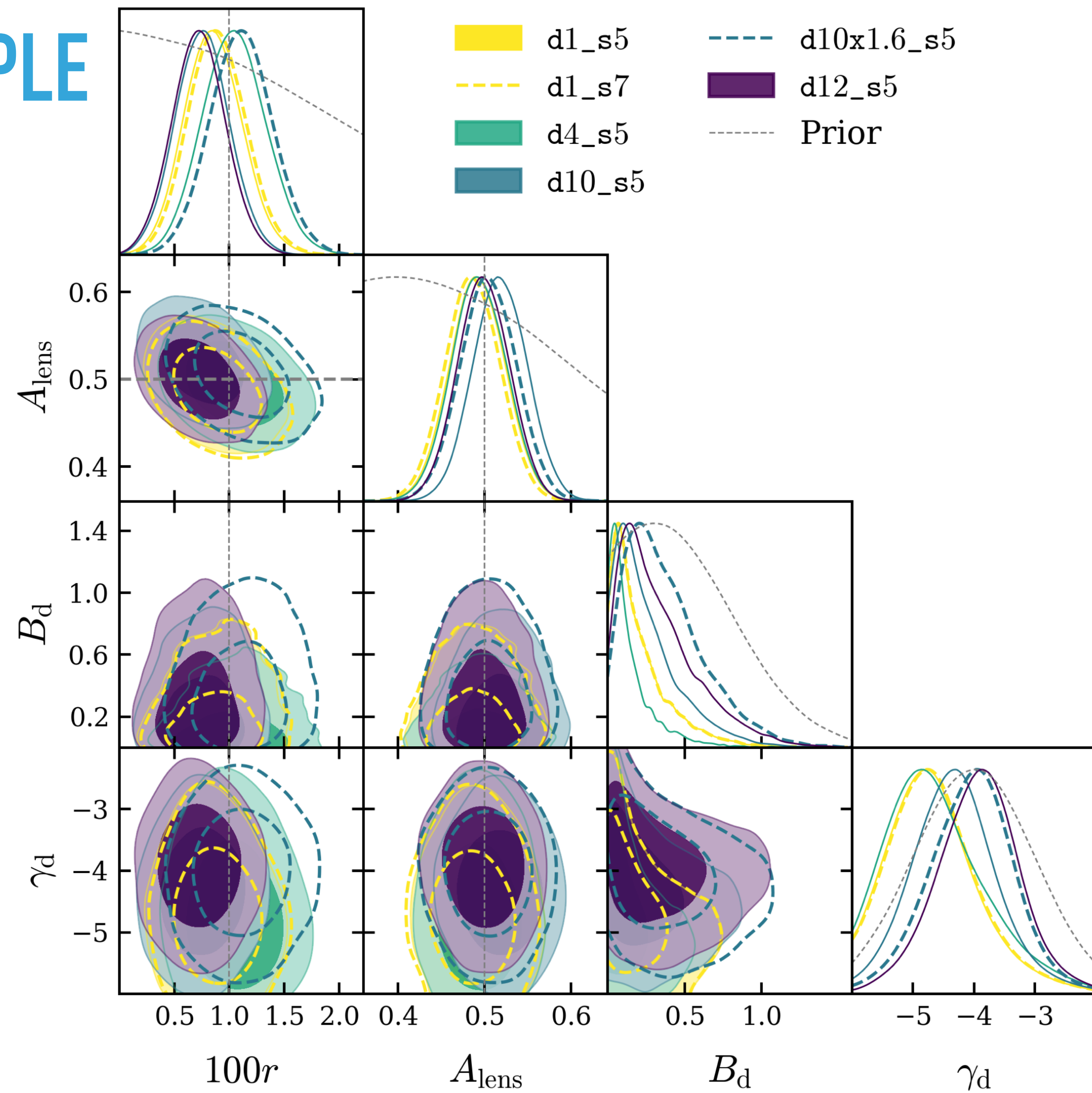
- ▶ *Medium complexity*
  - ▶ **d1** (spatially varying  $\beta_d$ )
  - ▶ **d10** (variation of d1, stronger  $\beta_d$  variations)
  - ▶ **s5** (spatially varying  $\beta_s$ )
- ▶ *High complexity*
  - ▶ **d12** (spat. varying  $\beta_d$ , line-of-sight SED variations)
  - ▶ **s7** (Non-power-law SED: curved index)
- ▶ *Extra-high complexity*
  - ▶ **d10x1.6** (d10 model with 1.6 more spatial variations in  $\beta_d$ )



The Pan-Experiment Galactic  
Science Group, 2025 (2502.20452)

# PYSM POSTERIOR EXAMPLE

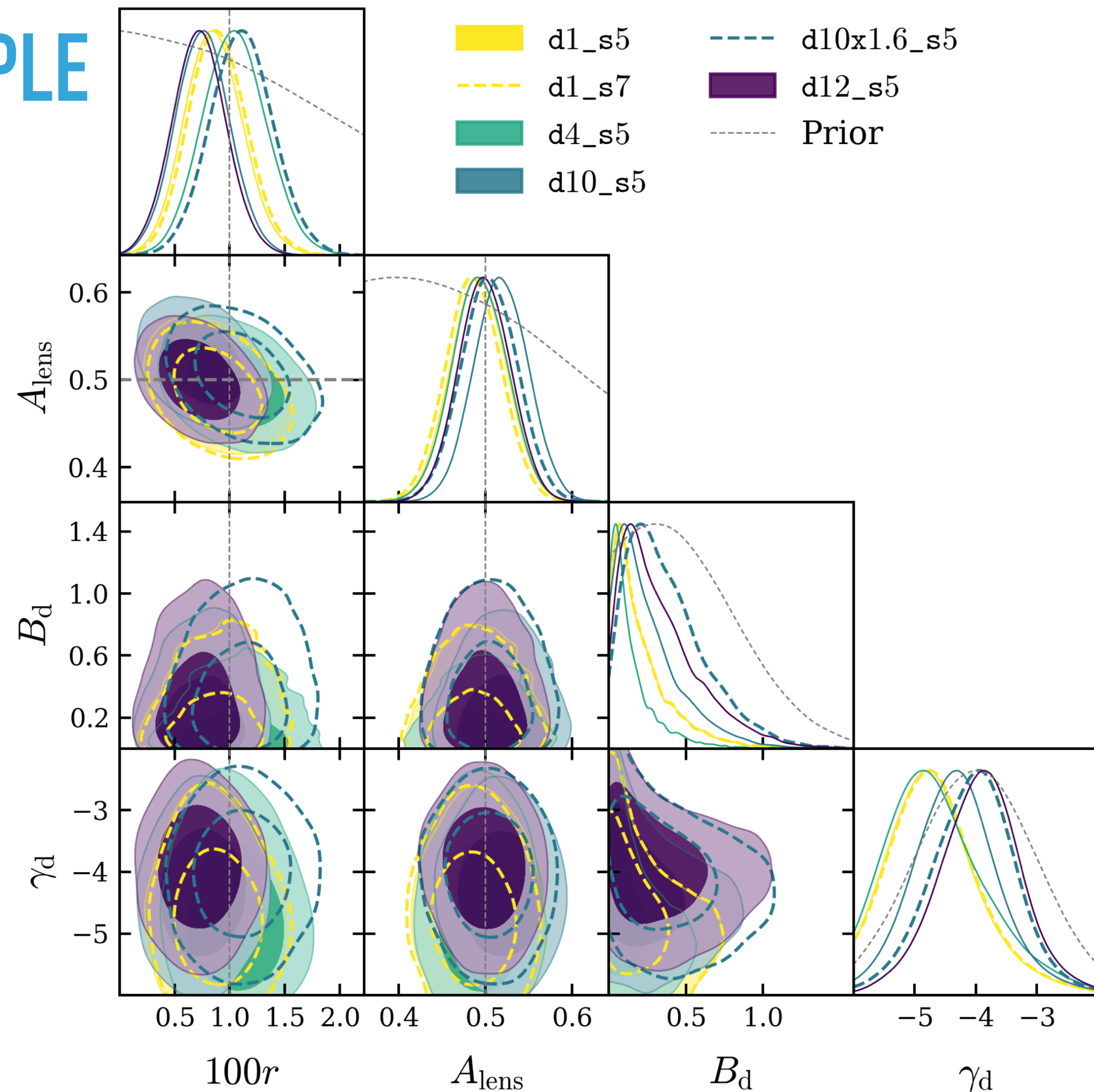
16



# PYSM POSTERIOR EXAMPLE

16

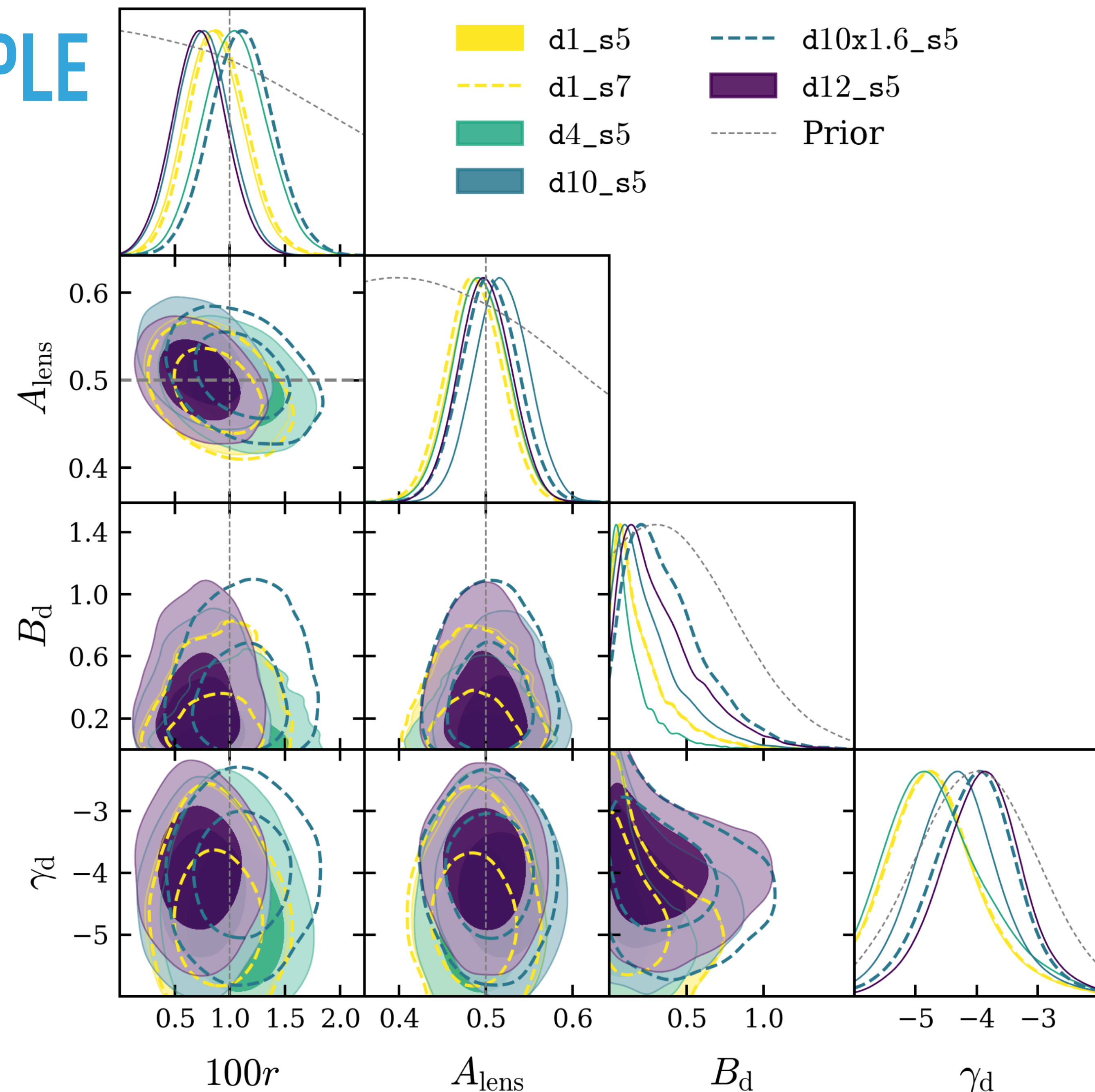
- $r$  inference appears unbiased for all PySM models



# PYSM POSTERIOR EXAMPLE

16

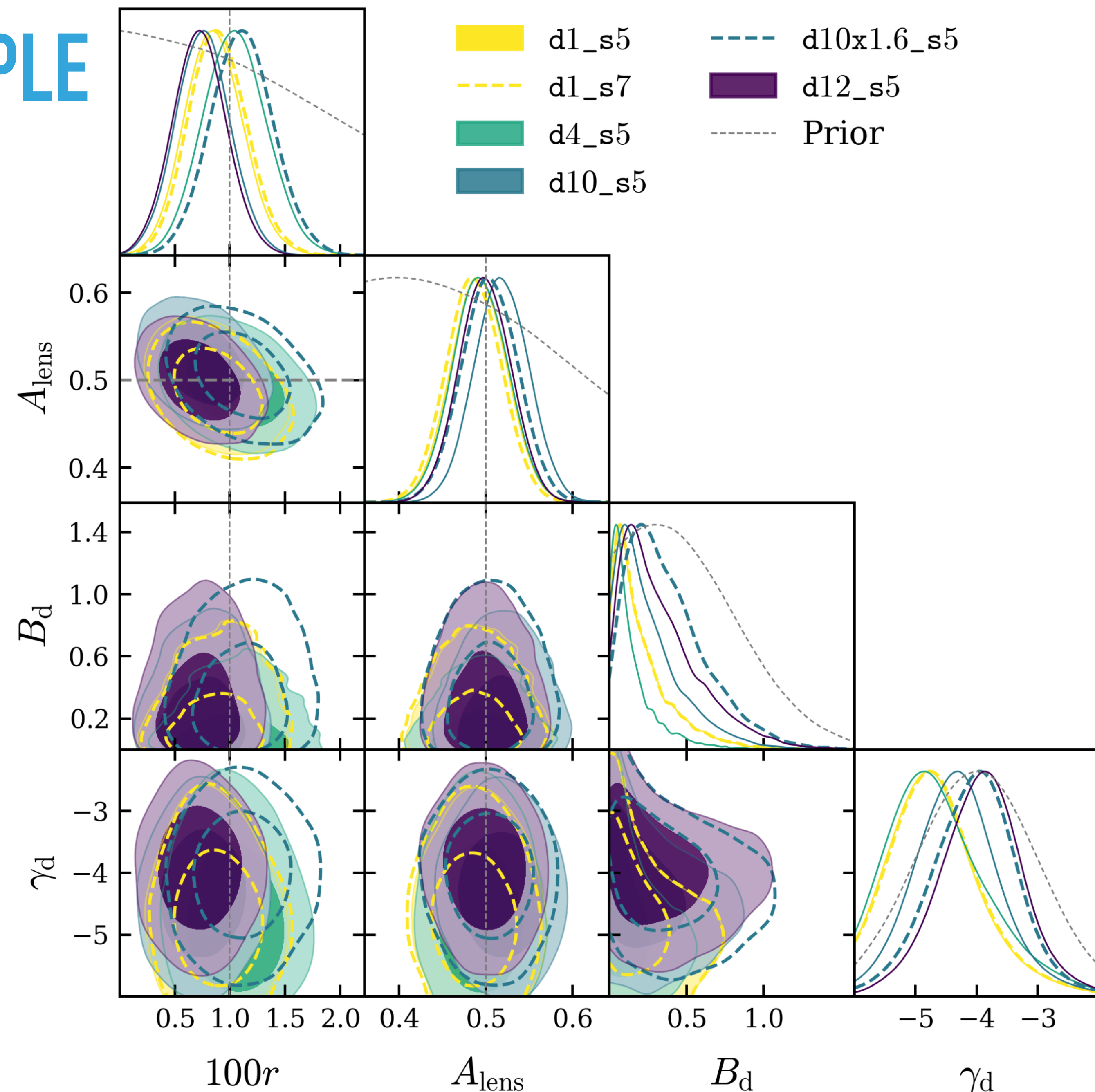
- ▶  $r$  inference appears unbiased for all PySM models
- ▶ SBI trained on simple isotropic simulations



# PYSM POSTERIOR EXAMPLE

16

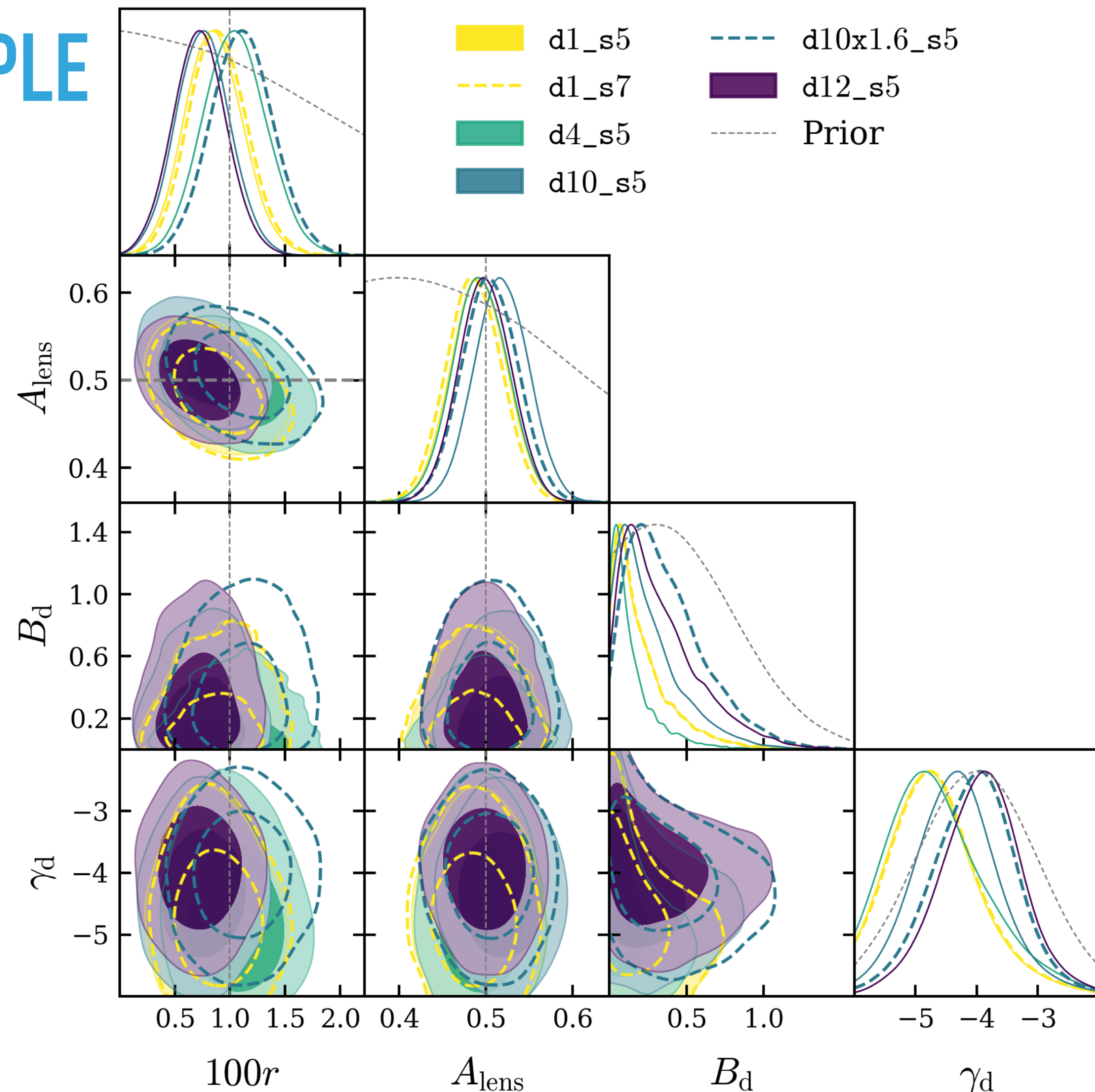
- ▶  $r$  inference appears unbiased for all PySM models
- ▶ SBI trained on simple isotropic simulations
- ▶ Same SBI model works for all PySM simulations without any model-specific tuning



# PYSM POSTERIOR EXAMPLE

16

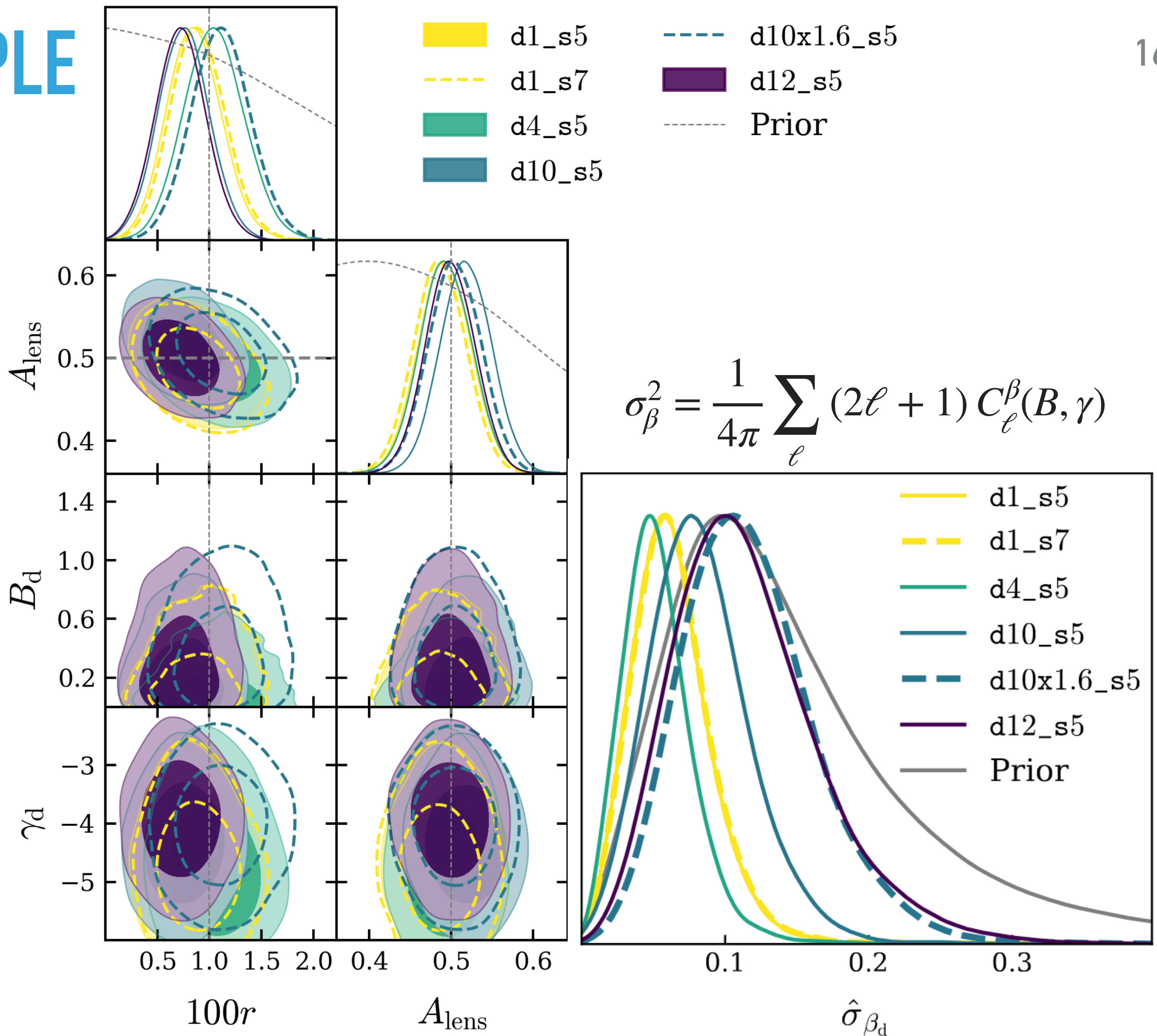
- ▶  $r$  inference appears unbiased for all PySM models
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- ▶ Inferred  $\beta_d$  power spectrum ( $C_{\ell}^{\beta_d} = B_d \ell^{\gamma_d}$ ) correctly traces complexity of models



# PYSM POSTERIOR EXAMPLE

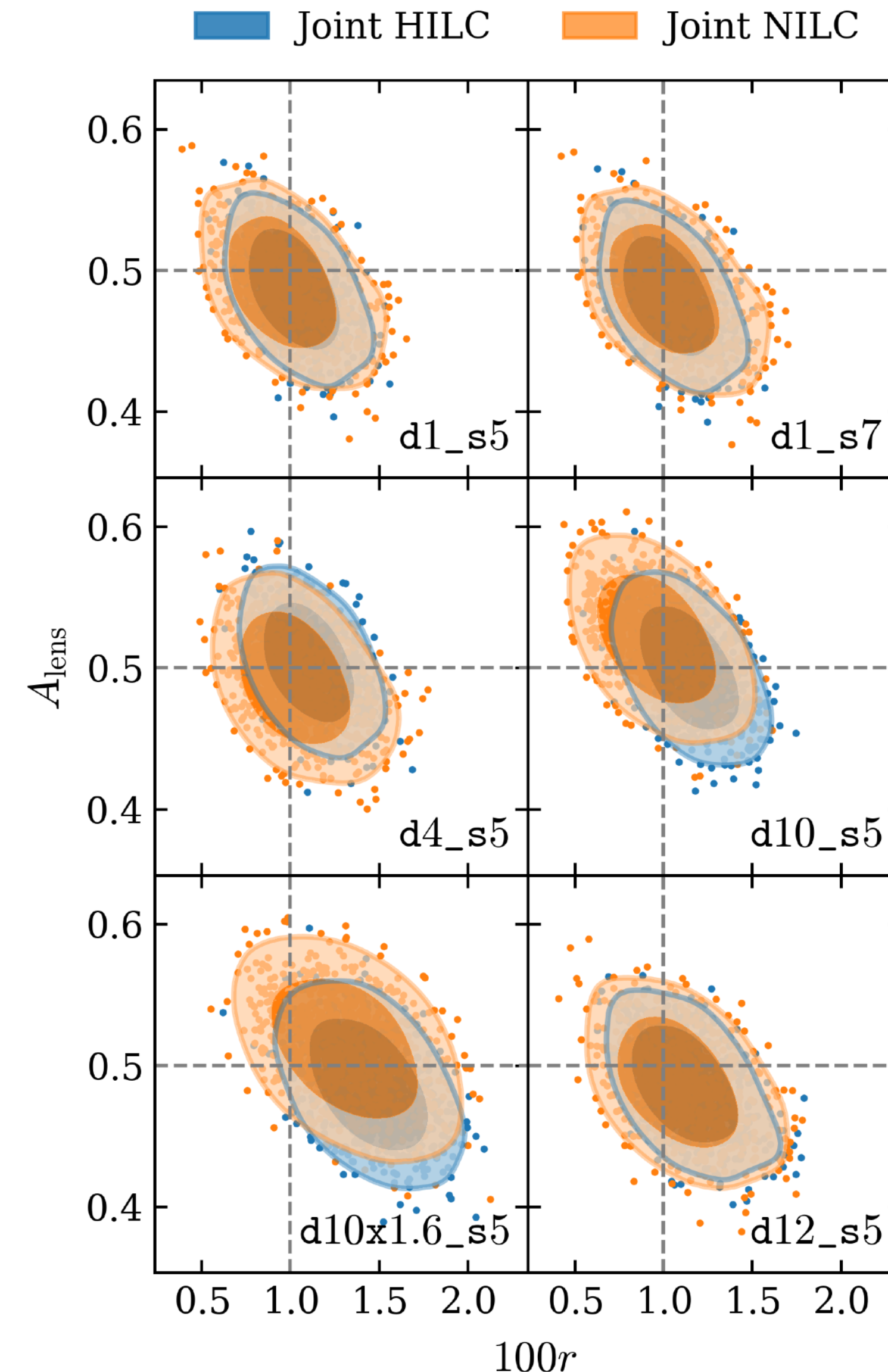
16

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- ▶ Harmonic ILC becomes biased for most complex models:
  - ▶ NILC anisotropic cleaning helps with robustness
- ▶ Small increase in variance for NILC vs HILC is consequence of simplistic (isotropic) simulations
- ▶ NILC variance expected to decrease with more realistic training set

Distribution of posterior means of  $r$  and  $A_{\text{lens}}$  for different CMB and noise realizations 17

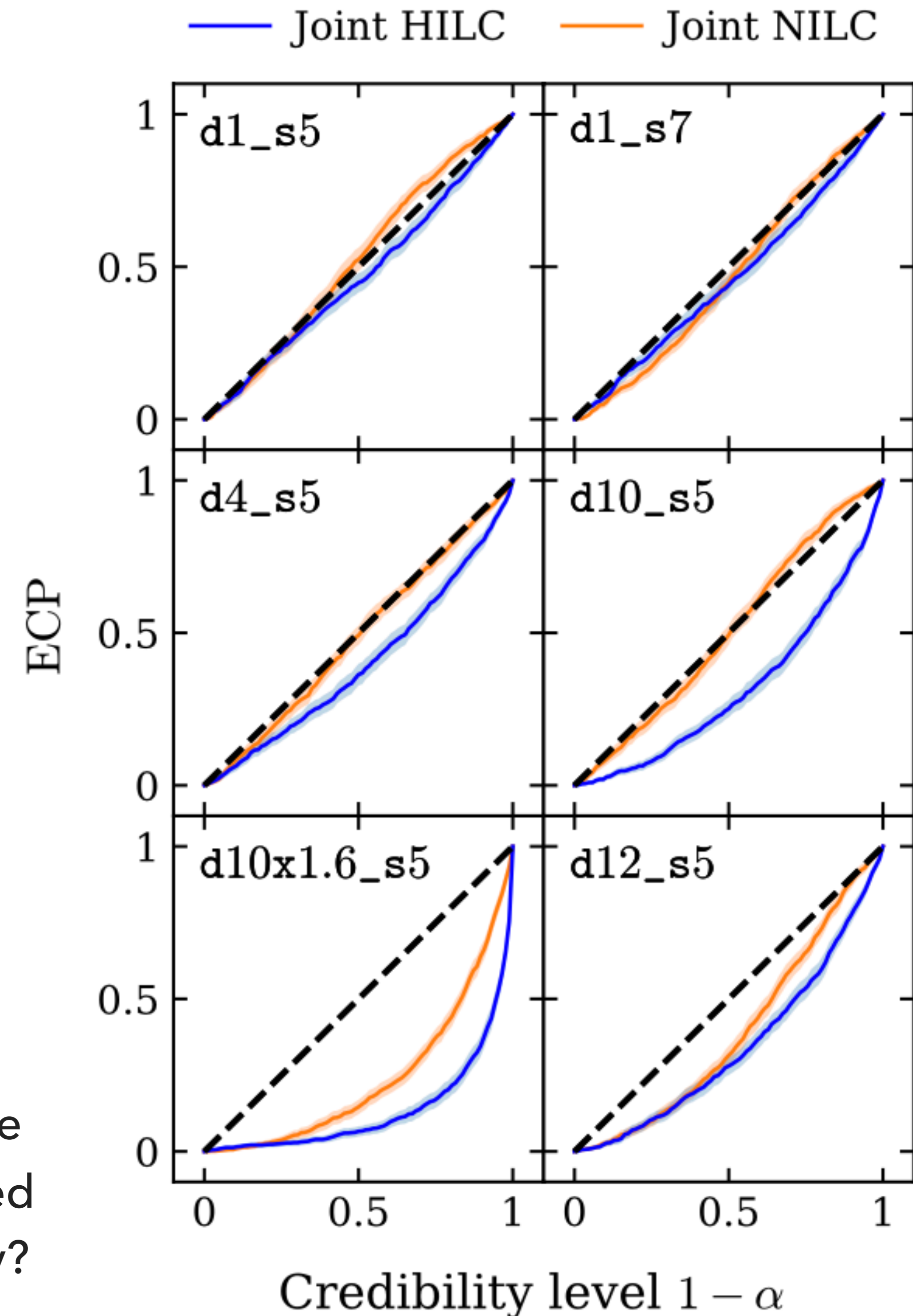


# PYSM BIAS

18

- ▶ Harmonic ILC (HILC) becomes biased for most complex models:
  - ▶ NILC anisotropic cleaning helps with robustness
- ▶ Coverage tests also reveal biases for NILC-based compression, but actual biases are small ( $<1\sigma$ ) and always below HILC bias

ECP = expected coverage probability: i.e. does the true value ( $r$  and  $A_{\text{lens}}$ ) lie inside a credibility interval (defined from the posterior estimate) with the correct frequency?



We obtain unbiased and low-variance results for the most complex (dust + sync) PySM foreground models on 20% sky fraction, with a setup with lower noise than the optimistic SO noise forecasts

- ▶ Importance of using foreground maps together with CMB map (20% to 400% reduction in  $\sigma(r)$  compared to CMB-only NILC/constrained-NILC)
- ▶ NILC-based data compression leads to unbiased results on realistic foreground simulations (*i.e. able to do SBI without accurate simulations*)
- ▶ Promising direction that combines best parts of existing methods → appears to be already good enough for Simons Observatory
- ▶ Future: adding instrumental realism in simulations, lensing/de-lensing, exploring different foreground models, reducing simulation budget with high/low-fidelity split