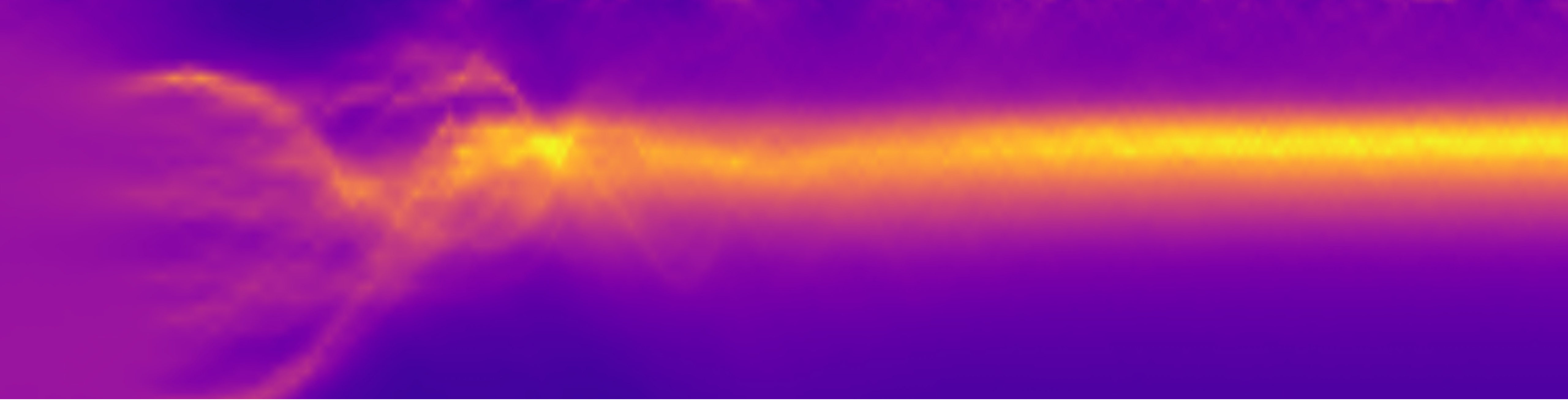

Coherent and Incoherent Dark Matter

A unified two-fluid approach

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COSMO 26 - Leiden

INSTYTUT FIZYKI POLSKIEJ AKADEMII NAUK

Funded by the European Union

Motivation: Bosonic Dark Matter

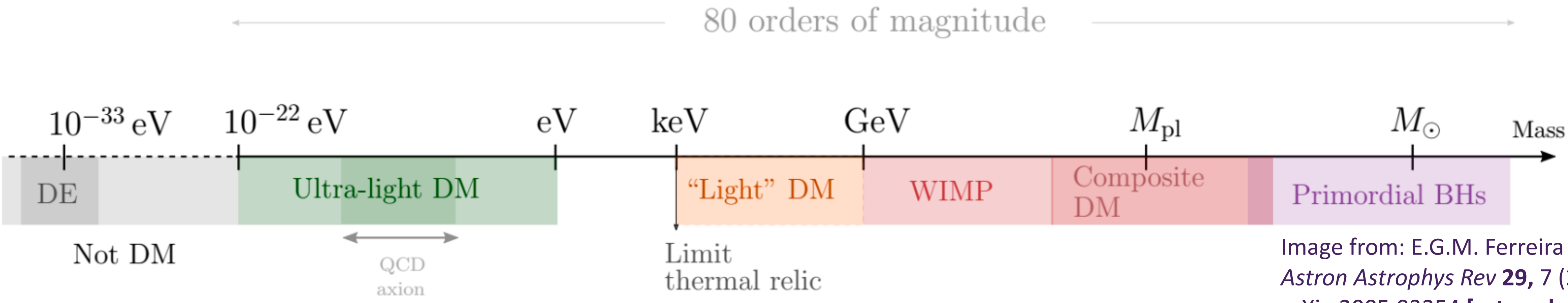

Image from: E.G.M. Ferreira *Astron Astrophys Rev* **29,** 7 (2021), arXiv:2005.03254 **[astro-ph.CO]**.

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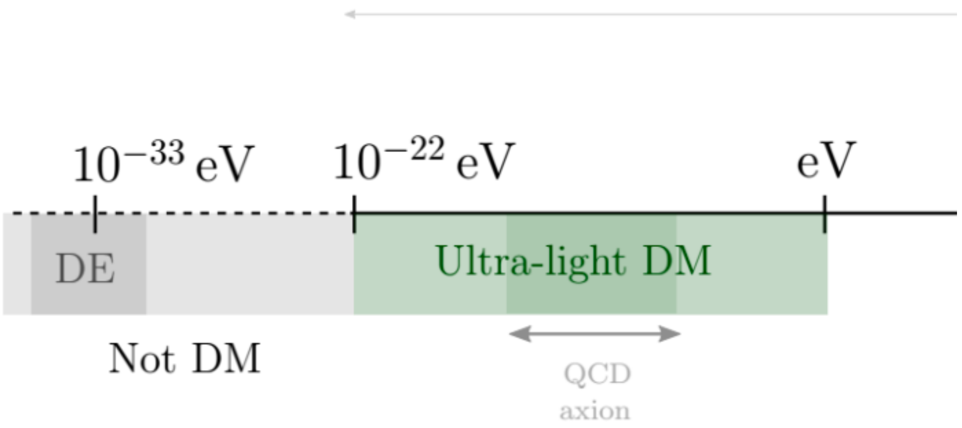

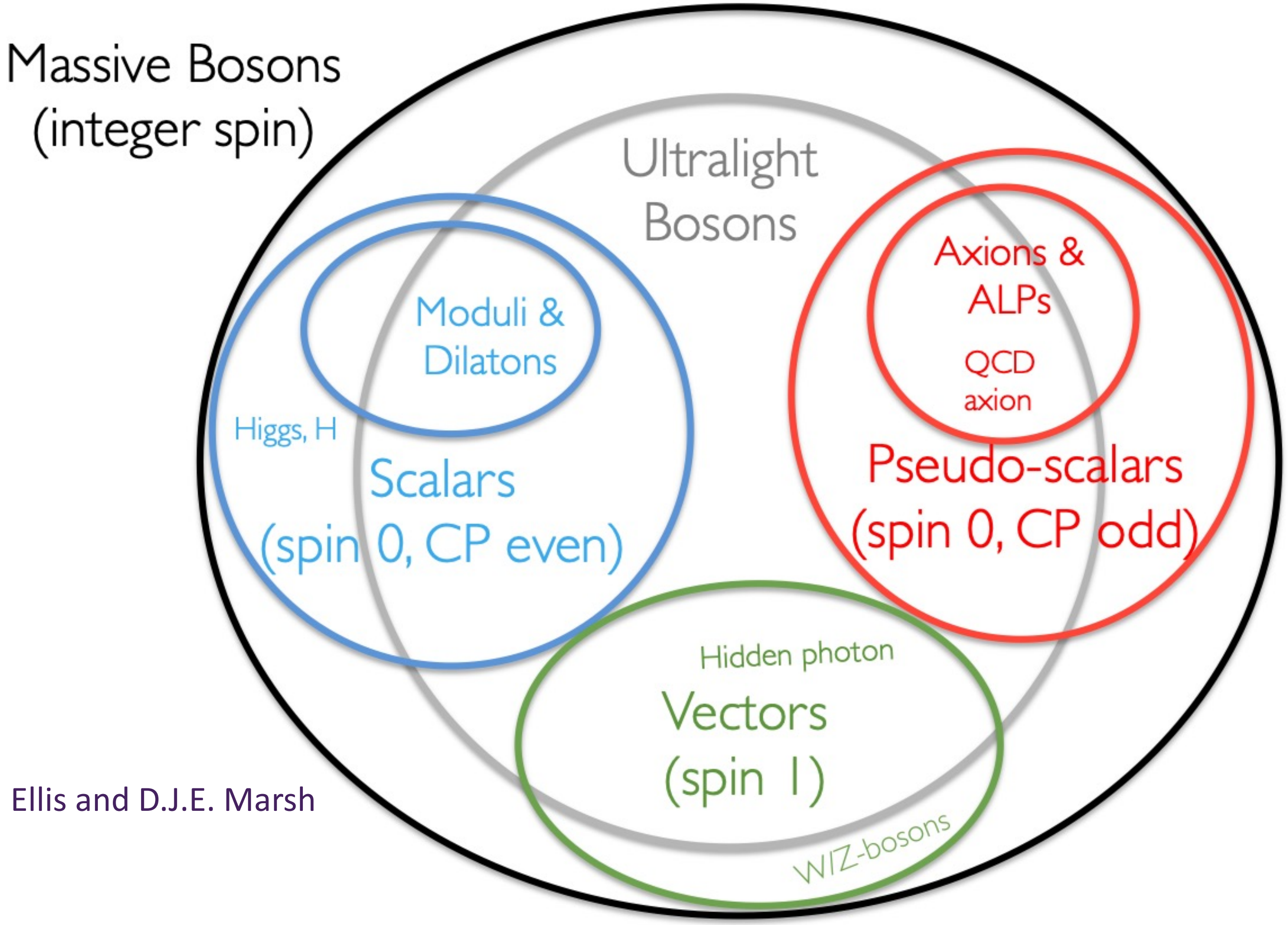

Image from: F. Chadha-Day, J. Ellis and D.J.E. Marsh
Sci.Adv. 8 (2022) 8, abj3618
arXiv:2105.01406 **[hep-ph]**.

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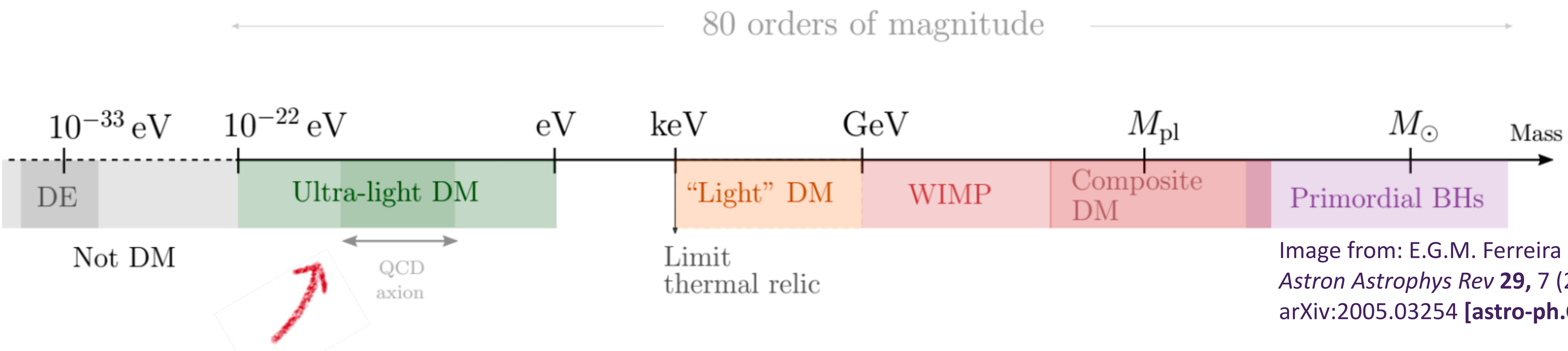


Image from: E.G.M. Ferreira *Astron Astrophys Rev* **29,** 7 (2021), arXiv:2005.03254 **[astro-ph.CO]**.

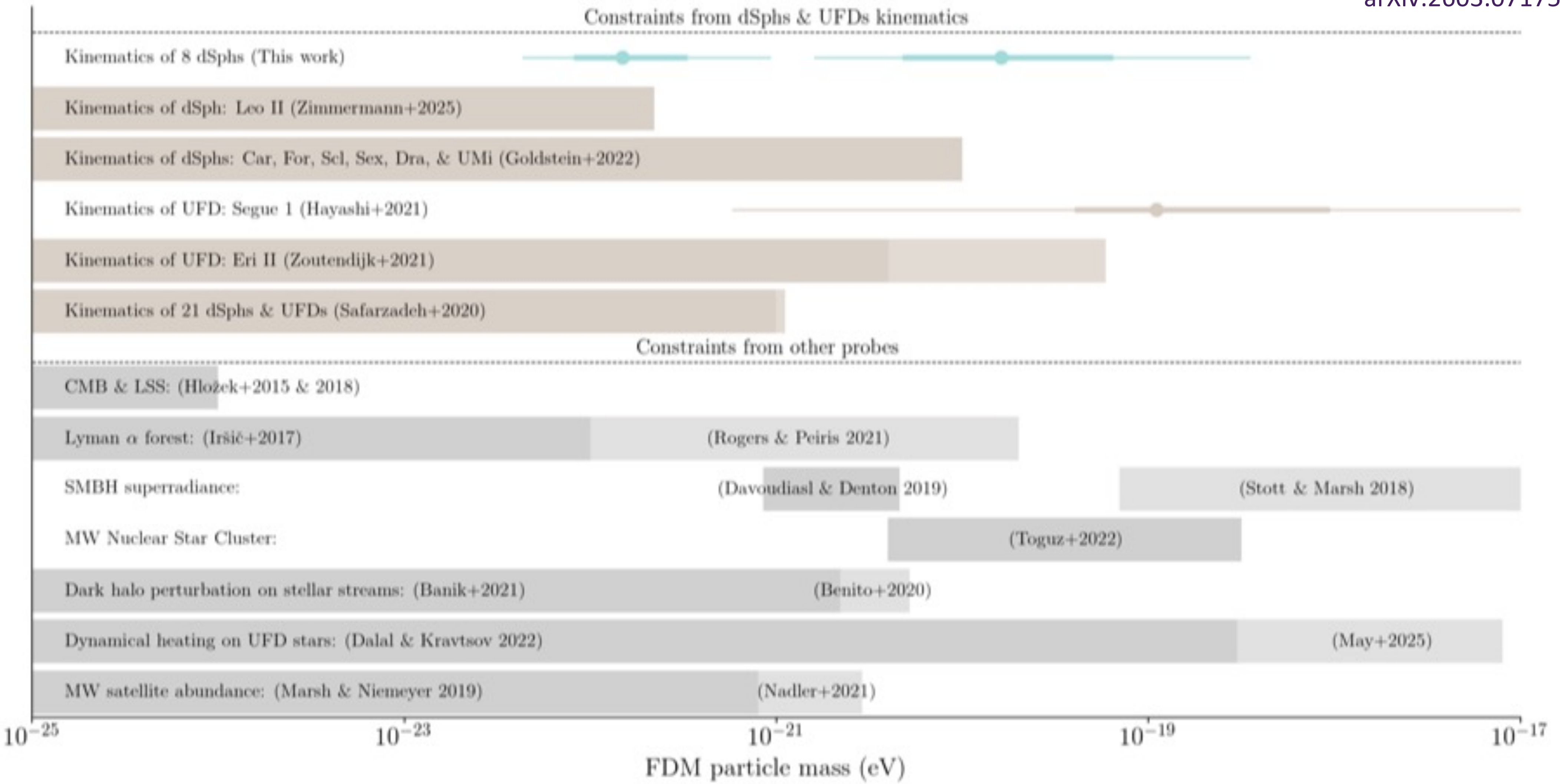
This sector only contains bosons. It has attracted attention solving CDM problems as:

- Missing satellites
- Cusp-Core problem
- Etc.

De Broglie wavelength of order of galactic length scales $\lambda_{\rm dB} \equiv \dfrac{2\pi}{mv} = 0.48\,{\rm kpc}\left(\dfrac{10^{-22}\,{\rm eV}}{m}\right)\left(\dfrac{250\,{\rm km/s}}{v}\right)$

Constraints on Ultra-light dark matter

Image from: Wardana D. et al, arXiv:2603.07175 **[astro-ph.GA]**.



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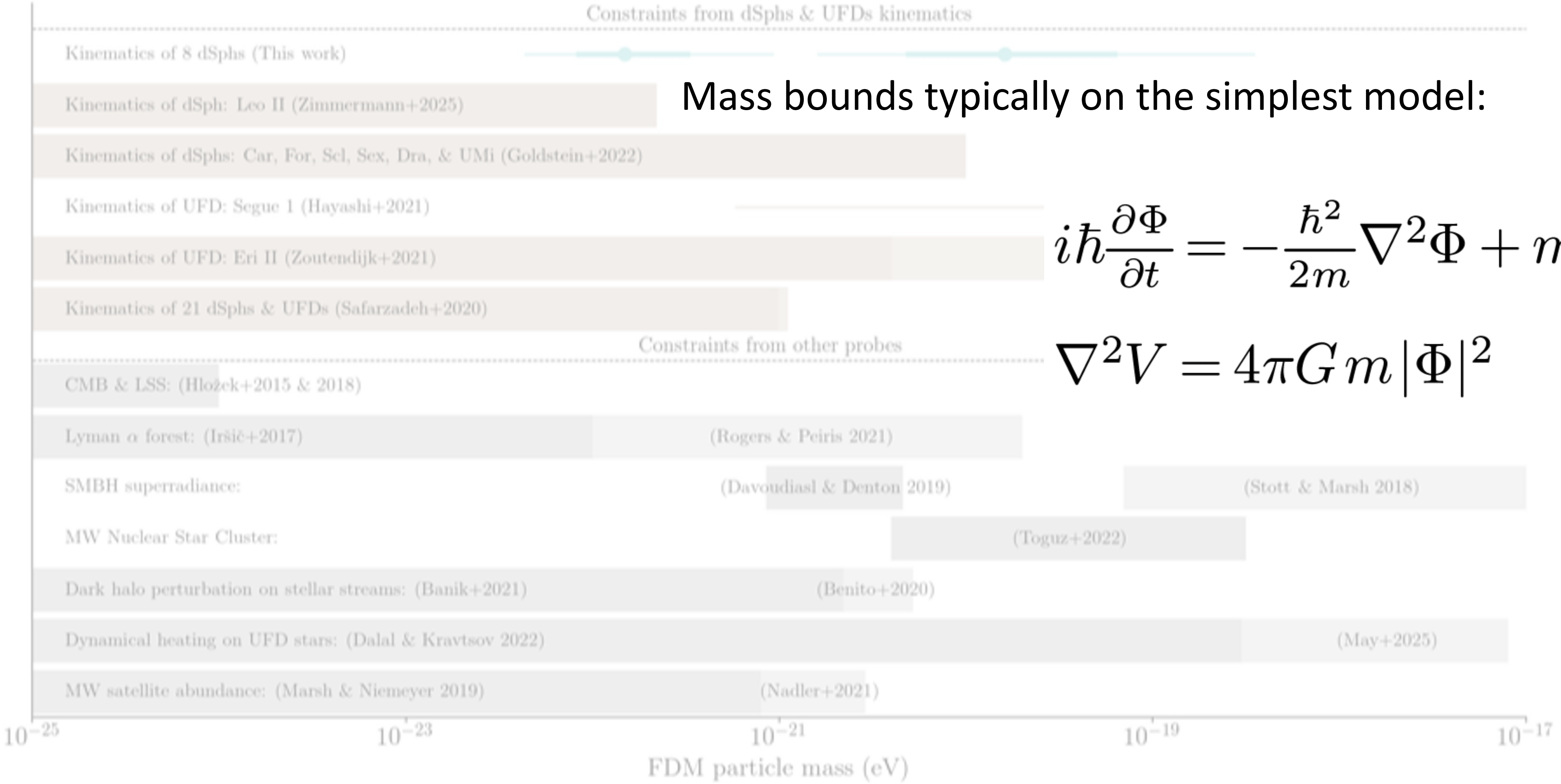


Mass bounds typically on the simplest model:

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + mV\Phi$$

$$\nabla^2 V = 4\pi G m|\Phi|^2$$

More physics in the bosonic picture

The simple way to extend the model is adding a self-interaction:

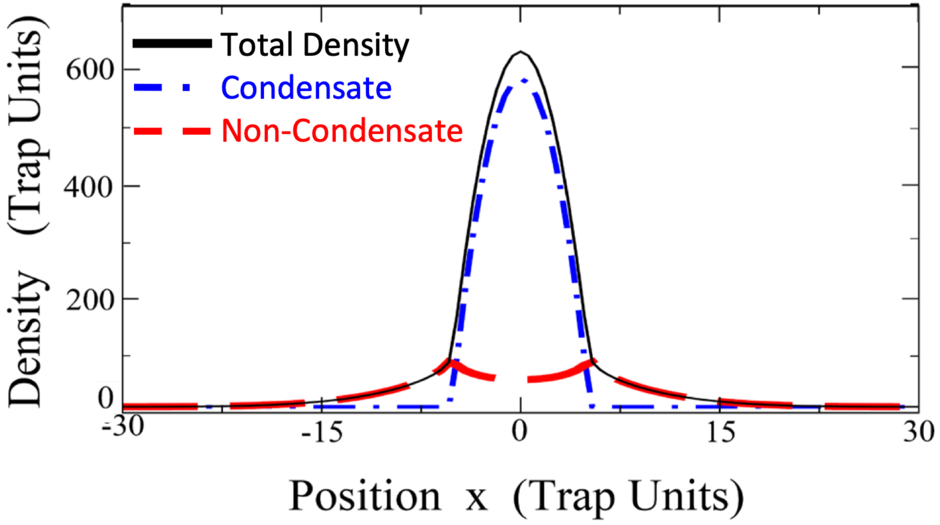
$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + mV\Phi + g|\Phi|^2\Phi$$

$$\nabla^2 V = 4\pi G m|\Phi|^2$$

Some studies has been conducted considering this.

But this is not the only possible extra physics in bosons…

Cold Atom Physics: Coherent and Incoherent modes

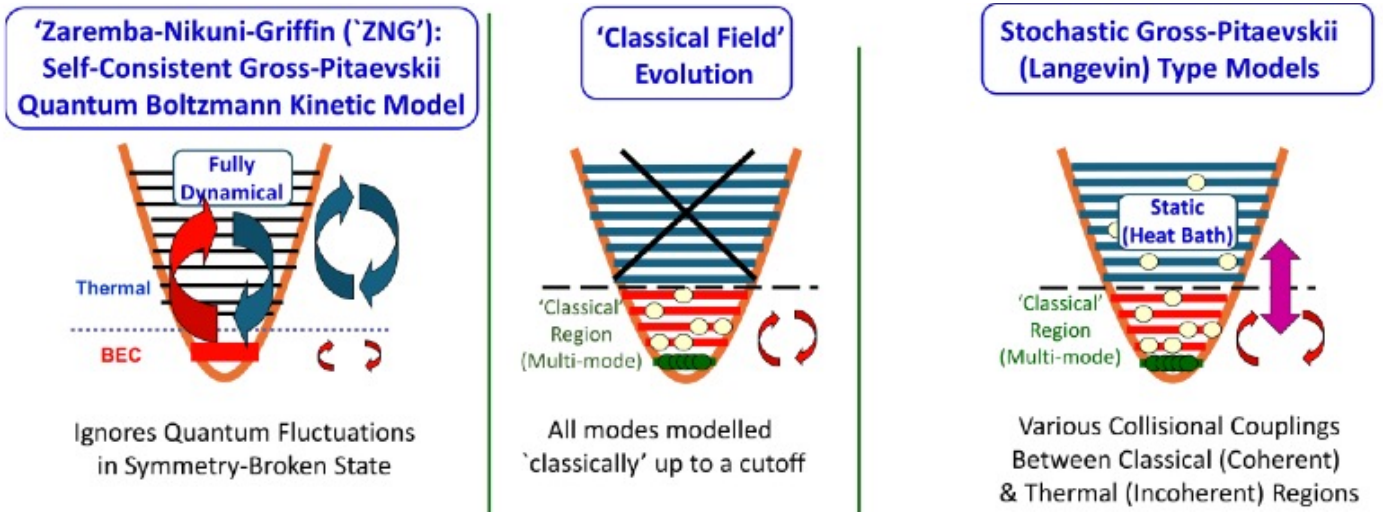


Coexistence between a coherent and incoherent modes:

- Zaremba-Nikuni-Griffin (ZNG) Model

  E. Zaremba, T. Nikuni and A. Griffin
  J. Low Temp. Phys. **116**, 277 (1999)
  arXiv:cond-mat/9903029

- Stochastic Gross-Pitaevskii Model



C. W. Gardiner and M. J. Davis
J. Phys. B: At. Mol. Opt. Phys. **36**, 4731 (2003)
arXiv:cond-mat/0308044

Cold Atom Physics: Coherent and Incoherent modes

This picture seems to be present in ultra-light bosonic models

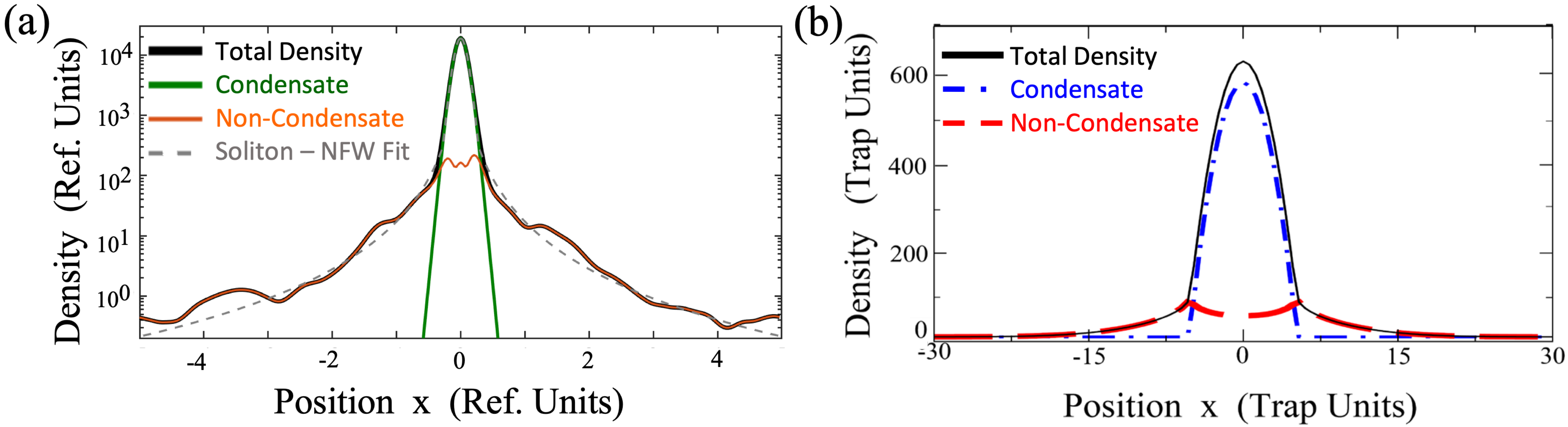


Image from: N. Proukakis, G. Rigopoulos and A.S.,
Phys. Rev. D 108 (8 2023), p. 083513
arXiv:2303.02049 **[astro-ph.CO]**.

Cold Atom Physics: Quantum corrections - LHY

In Cold-Atoms is known that quantum fluctuations modify the energy beyond mean-field approximation: the Lee-Huang-Yang correction.

This correction has been extensively studied in Dipolar Quantum Droplets:

$$i\hbar\dot{\psi} = \left[-\frac{\hbar^2\nabla^2}{2m} + g|\psi|^2 + \Phi(\mathbf{r}) + \gamma_{\mathrm{QF}}|\psi|^3\right]\psi(\mathbf{r})$$

LHY term

$$\gamma_{\mathrm{QF}} = \frac{32}{3}g\sqrt{\frac{a_s^3}{\pi}}\left(1 + \frac{3}{2}\epsilon_{\mathrm{dd}}^2\right)$$

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LHY term

$$\gamma_{\mathrm{QF}} = \frac{32}{3}g\sqrt{\frac{a_s^3}{\pi}}\left(1+\frac{3}{2}\epsilon_{\mathrm{dd}}^2\right)$$

Important: The power of this correction depends on the kind of interaction
(we will see later that for gravity it is NOT the same as contact interaction)

The model

- We use Schwinger-Keldysh formalism for a single spin-0 bosonic field ϕ that we split

$$\phi = \Phi + \varphi$$

Low energy-momentum
(Coherent)

High energy-momentum
(Non-Coherent)

- The Nambu space allows the existence of LHY-like terms

$$\begin{aligned} S_{eff} \;=\; & \frac{1}{2}\int d^4x \,(\, F^{cl*}(x)\,,\;\; F^{q*}(x)\,)\begin{pmatrix} 0 & \mathcal{L}_0 \\ \mathcal{L}_0 & 0 \end{pmatrix}\begin{pmatrix} F^{cl}(x) \\ F^{q}(x) \end{pmatrix} \\ & -\frac{1}{2}\int d^4x \int d^4x' \,(\, F^{cl*}(x)\,,\;\; F^{q*}(x)\,)\,\hat{\mathbb{S}}_{(c)}(x,x')\begin{pmatrix} F^{cl}(x') \\ F^{q}(x') \end{pmatrix} \end{aligned}$$

$$F^{cl} = \begin{pmatrix} \Phi^{cl} \\ \Phi^{cl*} \end{pmatrix}, \quad F^{q} = \begin{pmatrix} \Phi^{q} \\ \Phi^{q*} \end{pmatrix} \qquad \hat{\mathbb{S}}_{(c)} = \begin{pmatrix} 0 & \mathbb{S}^{A}_{(c)} \\ \mathbb{S}^{R}_{(c)} & \mathbb{S}^{K}_{(c)} \end{pmatrix} \qquad \mathbb{S}^{R}(x,x') = \begin{pmatrix} \Sigma^{R}(x,x') & \Sigma_{\rm an}(x,x') \\ \bar{\Sigma}_{\rm an}(x,x') & \Sigma^{R*}(x,x') \end{pmatrix}$$

The model – Without LHY corrections

- Ignoring anomalous correlators, the equations for both components are:

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + mV\Phi + g(|\Phi|^2 + 2\tilde{n})\Phi$$

$$\frac{\partial f}{\partial t} + \frac{k}{m}\nabla f - \nabla(mV + 2g(|\Phi|^2 + \tilde{n}))\nabla_k f = 0$$

$$\nabla^2 V = 4\pi G m(|\Phi|^2 + \tilde{n})$$

$$\tilde{n} = \int \frac{d^3k}{(2\pi\hbar)^3} f$$

Higher order in the couplings adds extra phenomenology

N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 108 (8 2023), p. 083513 arXiv:2303.02049 **[astro-ph.CO]**
And arXiv:2407.08860 **[astro-ph.CO]**

The model – Without LHY corrections

- Passing to hydrodynamical formulation with universe expansion:

$$\ddot{\delta}_c + 2H\dot{\delta}_c + \left(\frac{\hbar^2 k^4}{4m^2a^4} - 4\pi G\bar{\rho}f + \frac{g\bar{\rho}fk^2}{m^2a^2}\right)\delta_c - (1-f)\left(4\pi G\bar{\rho} - \frac{2g\bar{\rho}\,k^2}{m^2a^2}\right)\delta_{\rm nc} = 0$$

$$\ddot{\delta}_{\rm nc} + 2H\dot{\delta}_{\rm nc} - \left(4\pi G\bar{\rho}(1-f) - \frac{1}{a^2}\left(\frac{2g\bar{\rho}(1-f)}{m^2} + \frac{5\kappa\bar{\rho}^{2/3}(1-f)^{2/3}}{3}\right)k^2\right)\delta_{\rm nc} - f\left(4\pi G\bar{\rho} - \frac{2g\bar{\rho}\,k^2}{m^2a^2}\right)\delta_c = 0$$

Parameters →

m: boson mass

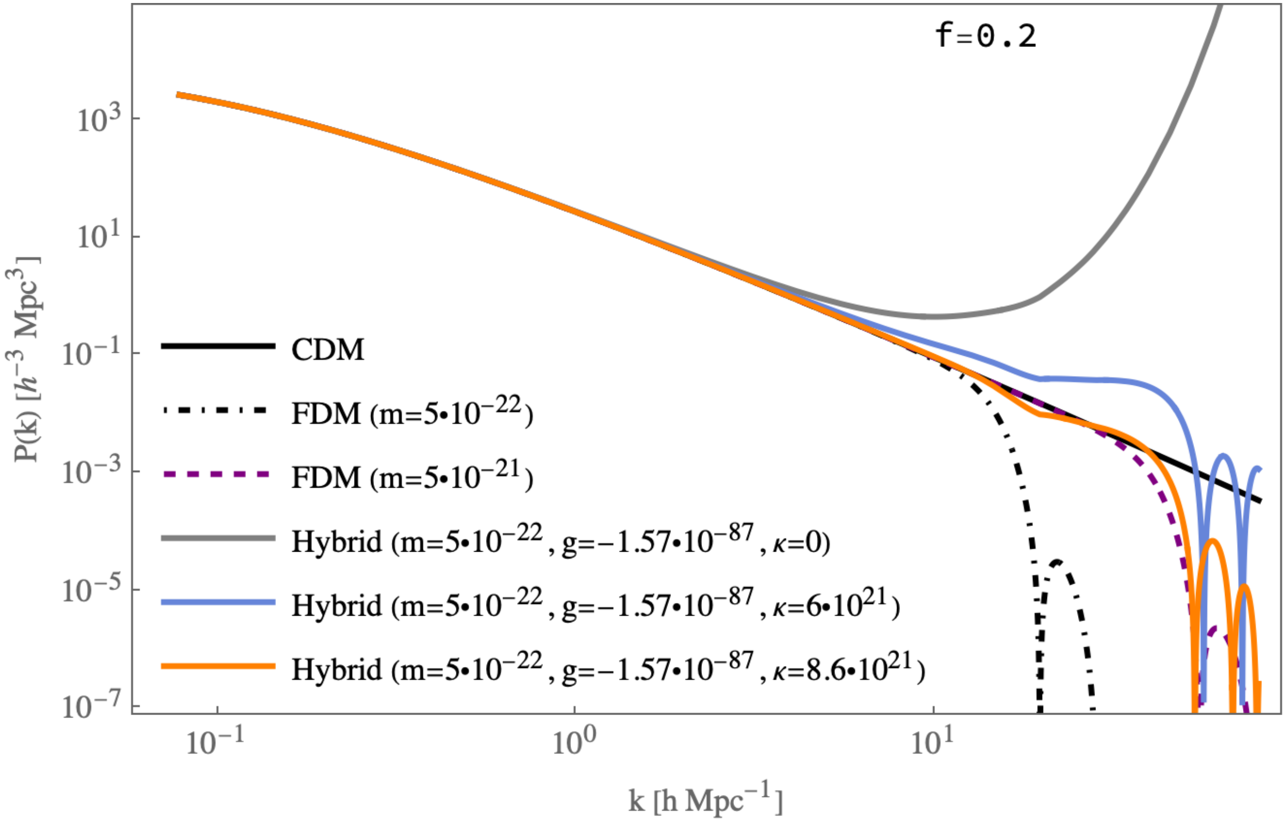
f : condensate fraction

g : self-interaction

κ : particle pressure ($P = \kappa\tilde{\rho}^{5/3}$)

Linear Regime

Hybrid model looks like FDM of higher mass, and the model can have impact on the existence of early structures depending on the choice of parameters



N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 110 (2 2024), p. 023504 ArXiv:2311.05280 **[astro-ph.CO]**

The model – LHY corrections

- Let us to consider only coherent part with the correction only for gravity:

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + mV\Phi - \frac{2}{\pi^{3/4}}\left(1 - \frac{\sqrt{\pi}\,\Gamma\left(\frac{7}{4}\right)}{3\Gamma\left(\frac{5}{4}\right)}\right)\frac{G^{5/4}m^{11/4}}{\hbar^{1/2}}|\Phi|^{1/2}\Phi$$

$$\nabla^2 V = 4\pi G m\, n_{\rm tot}$$

with

$$n_{\rm tot} = |\Phi|^2 + \frac{2}{3\pi^{5/4}}\left(1 + \frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{4}\right)}{2\Gamma\left(-\frac{1}{4}\right)}\right)\left(\frac{Gm^3}{\hbar^2}\right)^{3/4}|\Phi|^{3/2}$$

The power in the bosonic field is different than in pure contact interaction.
The question is how relevant could be this term.

The model – LHY corrections

- If we neglect the gravitational LHY term and we consider only the contact interaction LHY term:

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + mV\Phi + gn_{\rm tot}\Phi + \frac{4}{3}\frac{g^{5/2}m^{3/2}}{\pi^2}|\Phi|^3\Phi$$

$$\nabla^2 V = 4\pi G m n_{\rm tot}$$

If only g is relevant for the LHY term, keeping it as a free parameter (not as in the axion cases) could be of impact.

Summary and Comments

- The consideration of extra physics in bosonic dark matter modifies the parameter space and we need to be careful with the bounds on ultra-light models.

- LHY term could be relevant only for the self-interaction coupling case but more work is needed to determine the real impact.

Thanks

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