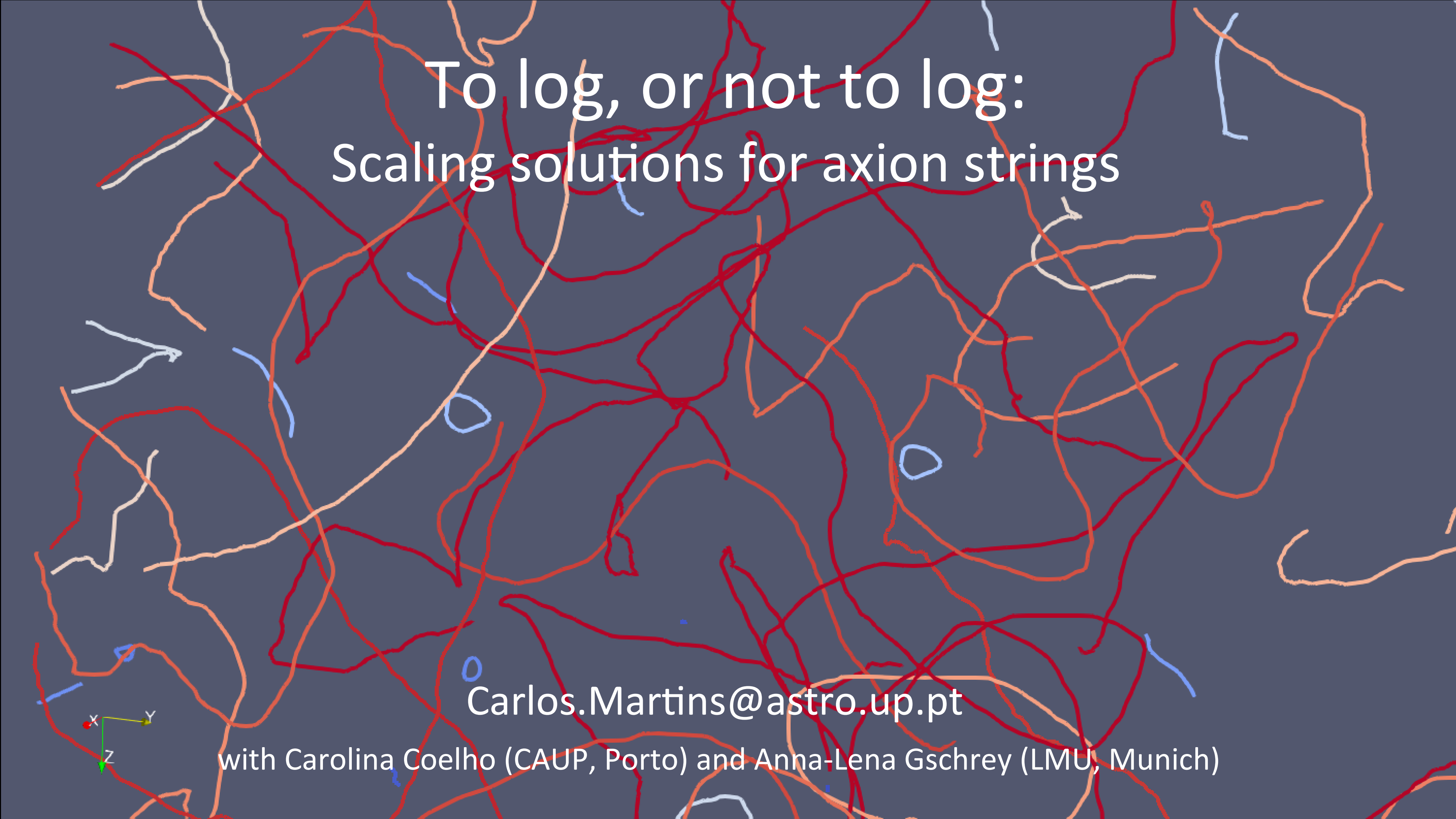


To log, or not to log:

Scaling solutions for axion strings

Carlos.Martins@astro.up.pt

with Carolina Coelho (CAUP, Porto) and Anna-Lena Gschrey (LMU, Munich)

Topological Defects

Defects form at cosmological phase transitions *[Kibble 1976]*; if they are stable, they can survive up to the present:

- Discrete symmetry breaking produces domain walls (pathological)
- Axial symmetry breaking produces cosmic strings (mostly benign)
- Spherical symmetry breaking produces monopoles (fine if global)

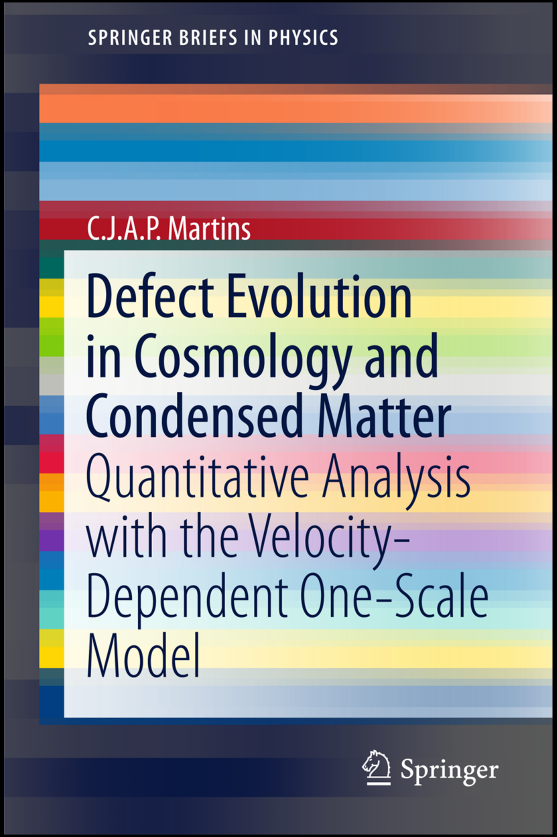
They are fossil relics constraining the underlying models; their study is a crucial part of any credible attempt to describe the very early universe

No 'ivory tower' physics: experimentally studied in liquid crystals, superconductors, superfluids, … and also biochemistry (cf. protein folding), geology (cf. tafoni), etc.

Interesting for analytic modeling since they span a wide range of scales, from cosmological to microphysics, while subject to the same physical processes

Quantitative Defect Evolution

Cosmic defects are highly non-linear objects, two inter-related approaches needed

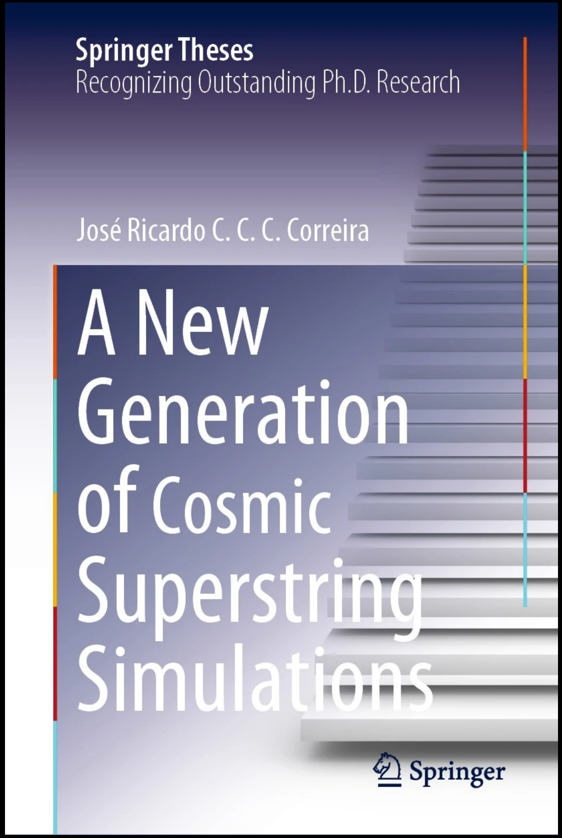


Analytic (thermodynamical) modeling

- VOS model is precise and accurate, but must be calibrated by matching numerical simulations

Simulations: canonical & new approches

- Field theory, Nambu-Goto, AMR, …
- Bottlenecked by hardware resources and computation time
- Key development CPU → GPU based *[Correia & Martins 2017]*
- Results not fully self-consistent



The VOS Model

Roadmap in *[Kibble 1985]*: do thermodynamics rather than statistical physics; first full implementation in *[Martins & Shellard 1996ab]*, adding velocity dynamics

Use the microphysics (e.g., the Goto-Nambu action for strings) to obtain macroscopic dynamical equations for macroscopic quantities, by taking suitable averages

Simplest model has one lengthscale and RMS velocity; for the particular case of cosmic strings

$$
\begin{aligned}
2\frac{dL}{dt} &= \frac{L}{\ell_s} + \frac{L}{\ell_d}v^2 + \tilde{c}v \\
\frac{dv}{dt} &= (1-v^2)\left[\frac{k(v)}{L} - \frac{v}{\ell_d}\right]
\end{aligned}
$$

Network evolution due to energy flow energy from large to small scales (other mechanisms also play a role): note the similarities with Richardson cascade in continuum mechanics

Driving forces, damping, energy losses, radiation channels, etc require accurate modeling, and quantitative calibration in matching simulations: done for simplest walls and strings

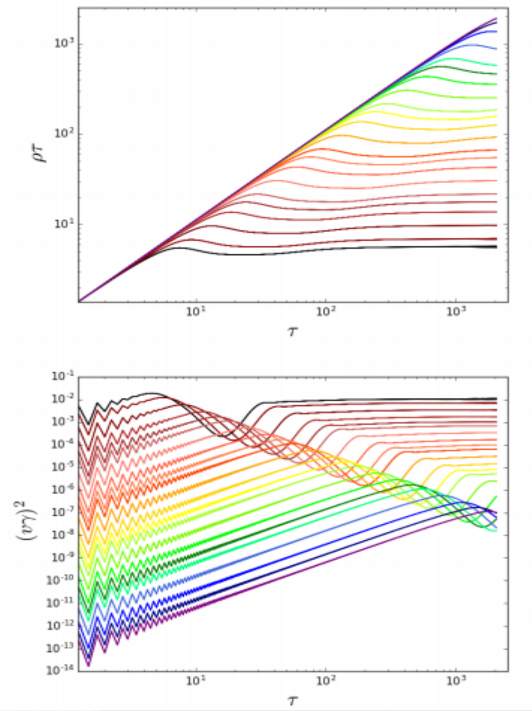
The Scaling Landscape

Attractor large-scale linear scaling solutions for plain cosmic strings and walls are well established and have been confirmed by everybody who has simulated them

Large-scale scaling does not guarantee scaling of the network's small scale features (energy could still flow between components and/or scales)

Sufficiently large Hubble damping leads to a linear scaling regime with non-relativistic network: linear regime is in fact a Kibble regime

	Friction Damping	*Hubble Damping*
Energy losses negligible (transient)	**Conformal stretching** $L \propto a\,, \quad v \propto \frac{\ell_f}{a} \propto a^n$	**Thawing regime** $L \propto a\,, \quad v \propto \frac{1}{n\lambda}\frac{t}{a} \propto \frac{1-\lambda}{n\lambda}\tau\,.$
Energy losses important (attractor)	**Kibble regime** $L \propto (\ell_f t)^{1/2}\,, \quad v \propto \left(\frac{\ell_f}{t}\right)^{1/2}$	**Linear regime** $L \propto t\,,\ v \propto \mathrm{const}\,.$



Wiggly Strings

Cosmologically realistic networks have short wavelength propagation modes (wiggles), optimally described through their (multi)fractal properties *[Martins & Shellard 2006]*

Strings are not Brownian on correlation length scales, only on horizon scales; at the scale of the correlation length, they are closer to a self-avoiding random walk

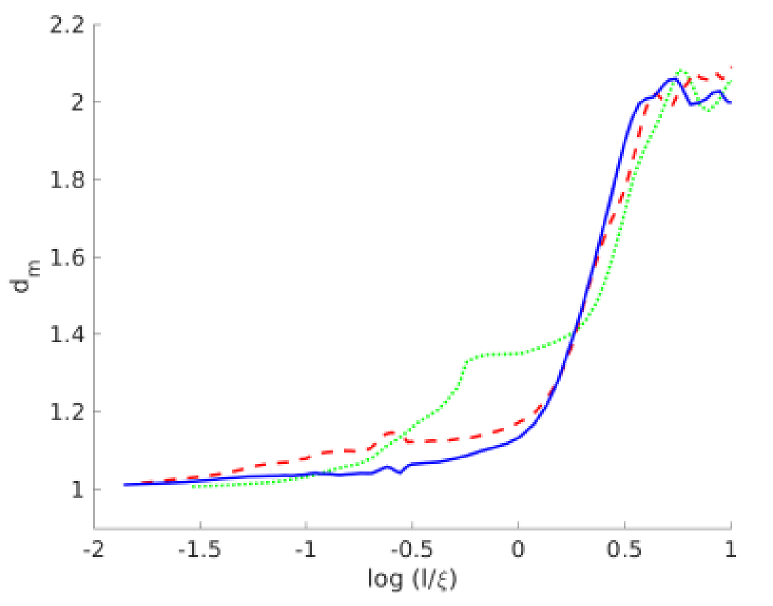
Fractal structure evolves such that any physical scale always loses power; wiggles may scale, disappear or grow, also depending on coarse-graining scale *[Almeida & Martins 2021]*

$$2\frac{dL}{dt} = HL\left[3 + v^2 - \frac{(1-v^2)}{\mu^2}\right] + \frac{cfv}{\sqrt{\mu}}$$

$$2\frac{d\xi}{dt} = H\xi\left[2 + \left(1+\frac{1}{\mu^2}\right)v^2\right] + v\left[k\left(1-\frac{1}{\mu^2}\right) + c\left(f_0+s\right)\right] + \left[d_m\left(\ell\right)-1\right]\frac{\xi}{\ell}\frac{d\ell}{dt}$$

$$\frac{dv}{dt} = \left(1-v^2\right)\left[\frac{k(v)}{\xi\mu^2} - Hv\left(1+\frac{1}{\mu^2}\right) - \frac{1}{1+\mu^2}\frac{\left[d_m\left(\ell\right)-1\right]}{v\ell}\frac{d\ell}{dt}\right]$$

$$\frac{1}{\mu}\frac{d\mu}{dt} = \frac{v}{\xi}\left[k\left(1-\frac{1}{\mu^2}\right) - c\left(f - f_0 - s\right)\right] - H\left(1-\frac{1}{\mu^2}\right) + \frac{\left[d_m\left(\ell\right)-1\right]}{\ell}\frac{d\ell}{dt}$$



Varying Tension Strings

Two phenomenological approaches to introducing time-dependent tension
- Macroscopic, at the level of the VOS model itself *[Yamaguchi 2005]*
- Microscopic (and propagated to the VOS model) *[Conlon et al. 2024]*

Cosmologically, the two approaches correspond to opposite assumptions on the physical impact of the varying tension *[Coelho et al. 2026]*

$$\frac{1}{\ell_s} = 2H$$
$$\frac{1}{\ell_d} = 2H + \frac{1}{\ell_f}$$

$$\frac{1}{\ell_s} = 2H + \frac{1}{\mu}\frac{d\mu}{dt}$$

Macroscopic: stretching length changed

$$\frac{1}{\ell_d} = 2H + \frac{1}{\ell_f} + \frac{1}{\mu}\frac{d\mu}{dt}$$

Microscopic: damping length changed

A meaningful comparison is the wiggly model, for which both characteristic lengths are changed, but in different ways:

$$\frac{1}{\ell_{s,wig}} = 2H + \left(1 - \frac{1}{\mu_{wig}^2}\right) H$$

$$\frac{1}{\ell_{d,wig}} = 2H - \left(1 - \frac{1}{\mu_{wig}^2}\right) H + \frac{1}{\ell_f}$$

Varying Tension String Scaling Solutions

Full analysis of power-law cases reported in *[Coelho et al. 2026]*

$$a(t) \propto t^{\lambda}$$
$$\mu(t) \propto t^{\delta},$$

Scaling solution	Macroscopic	Microscopic
$\ell_f = const$	Normalization	Unchanged
$T = const$	Scaling	Scaling
Minkowski	Normalization	Scaling
Stretching	Scaling	Scaling
Kibble	Scaling	Scaling
Linear	Normalization	Normalization
de Sitter	Scaling	Normalization
Contracting	Scaling	Scaling

Impact depends on the modeling approach and also on the specific scenario under consideration: in most cases the scaling solution itself changes, but in the linear (i.e. scale-invariant) scaling regime only its normalization is impacted

Specifically, one can write the linear scaling solution as (where δ_1 and δ_2 correspond to the macroscopic and microscopic power-law time dependencies respectively)

$$L = \sqrt{\frac{k(k+\tilde{c})}{(2\lambda+\delta_2)(2-2\lambda-\delta_1)}}\,t$$
$$v = \sqrt{\frac{2-2\lambda-\delta_1}{2\lambda+\delta_2}\frac{k}{k+\tilde{c}}},$$

Axion Strings

The axion is the leading solution to the strong CP problem, and a dark matter candidate

In these models a symmetry breaking phase transition at the Peccei-Quinn scale forms global strings, whose tension has logarithmic time dependence; at QCD phase transition, ca. 70 e-folds later, the string network decays and the axion acquires a mass

First simulations *[Ryden et al. 1990]* report v~0.4, and a number of defects per horizon (ζ, which primarily impacts the axion dark matter abundance) is constant to within a logarithmic factor

Recent simulations disagree on whether ζ is constant or log-dependent; note that in these the maximum value of the log is <10, while the physically relevant cosmological one is around 70.

Initial conditions are in a single corner of parameter space, with low densities and high velocities: this is unrepresentative and likely unphysical

Interestingly, within numerical uncertainties all groups agree on the value of ζ at the end of their simulations; the disagreement is only on whether or not ζ is a constant

Reference	ζ	v	Log?
Hindmarsh et al. [18]	1.2 ± 0.2	Not reported	No
Hindmarsh et al. [19]	1.2 ± 0.1	0.61 ± 0.01	No
Correia et al. [20]	1.2 ± 0.1	0.57 ± 0.01	No
Saikawa et al. [21]	1.2 ± 0.3	Not reported	Yes
Kim et al. [22]	0.9 ± 0.2	Not reported	Yes
Buschmann et al. [23]	1.2 ± 0.2	Not reported	Yes
Klaer & Moore [24]	1.0 ± 0.1	0.61 ± 0.05	Yes
Kawasaki et al. [25]	1.1 ± 0.3	0.52 ± 0.05	Yes
Gorghetto et al. [26]	1.0 ± 0.3	0.66 ± 0.05	Yes

Large Log Expectations

Networks have finite lifetime: asymptotic solutions provide physical insight but their relation to simulations is subtle, considering the factor O(10) difference between the simulated and physical logs

For ~constant log tension expect number of strings per Hubble volume decreases, and so does their velocity, but network density increases

$$\frac{(L/t)^2_{ax}}{(L/t)^2_{std}} = \frac{N}{N-1}$$
$$\frac{\zeta_{ax}}{\zeta_{std}} = \frac{v^2_{ax}}{v^2_{std}} = 1 - \frac{1}{N}$$

$$\frac{\rho_{ax}}{\rho_{std}} = N - 1$$

Goldstone boson mode is further radiation channel and mediates inter-string forces (proportional to 1/R~1/L), enhancing intercommutings and loop production: expect larger values of (c,k)

In the macroscopic and microscopic approaches, asymptotic solutions are unchanged (no log); however, in the wiggly case there would be negative asymptotic logs…

… also in the network's RMS velocity

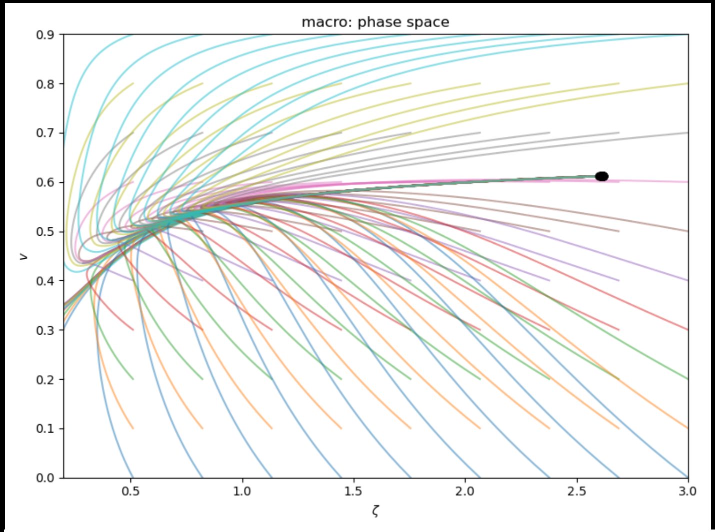
… implying a positive log for ζ

… and different behaviours for ξ and L

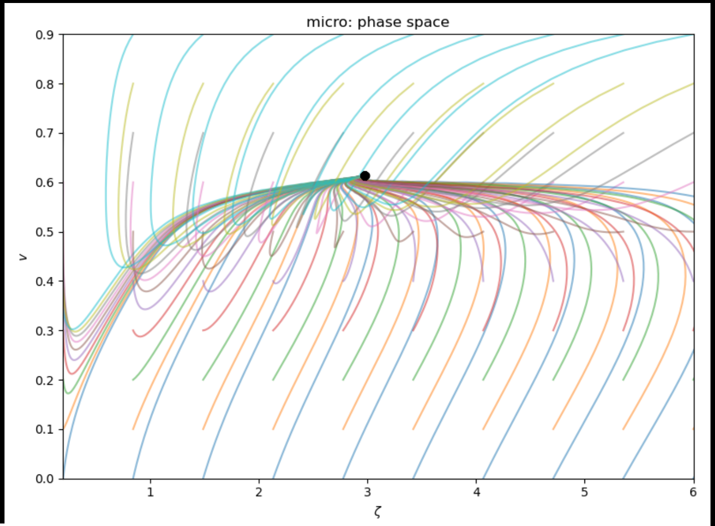
(recall that $\xi^2 = \mu L^2$)

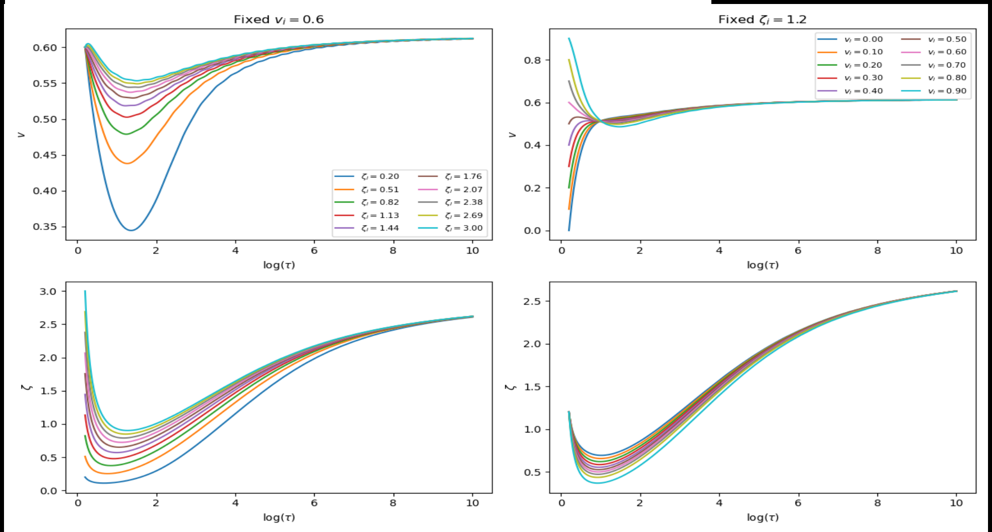
$$v = \frac{1}{\mu} \propto \frac{1}{\log t}$$
$$\xi = \frac{2k}{\mu} t \propto \frac{t}{\log t}$$
$$L = \frac{2k}{\mu^{3/2}} t \propto \frac{t}{\log^{3/2} t}$$

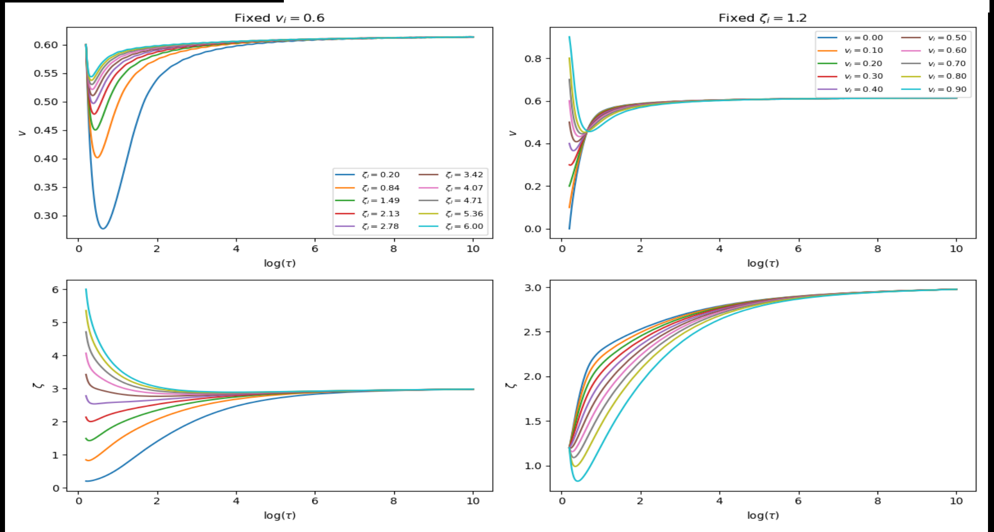
Small Log Expectations



NB: Plots in this slide assume a loop chopping efficiency c=0.57 as in *[Moore et al. 2002]*



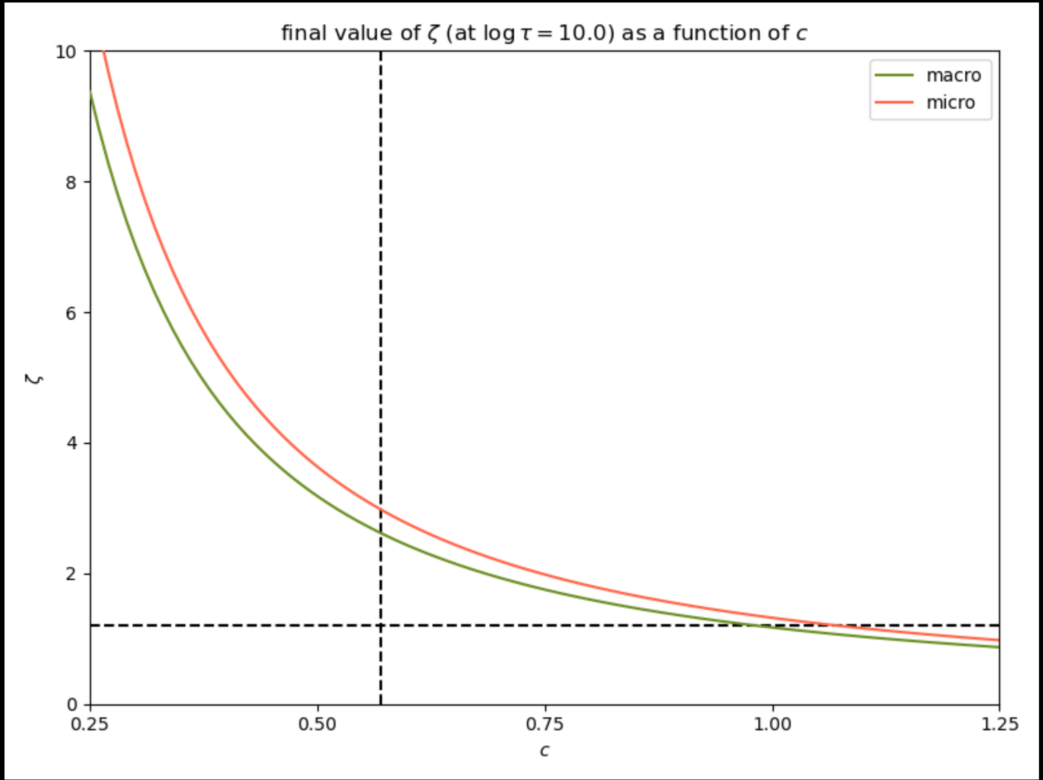


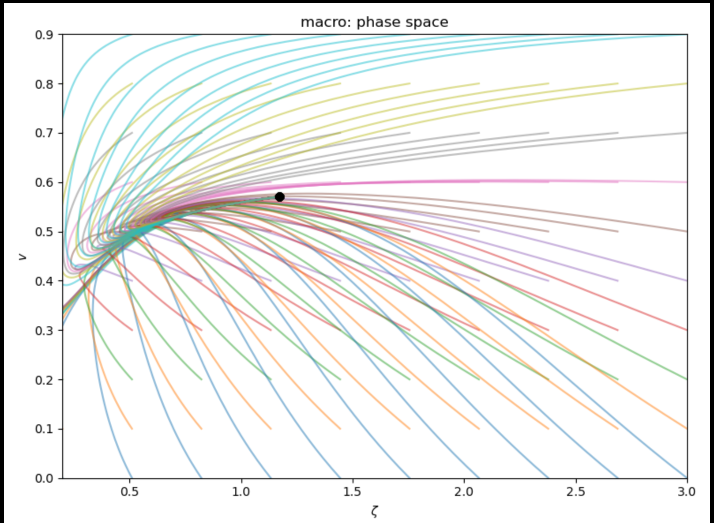


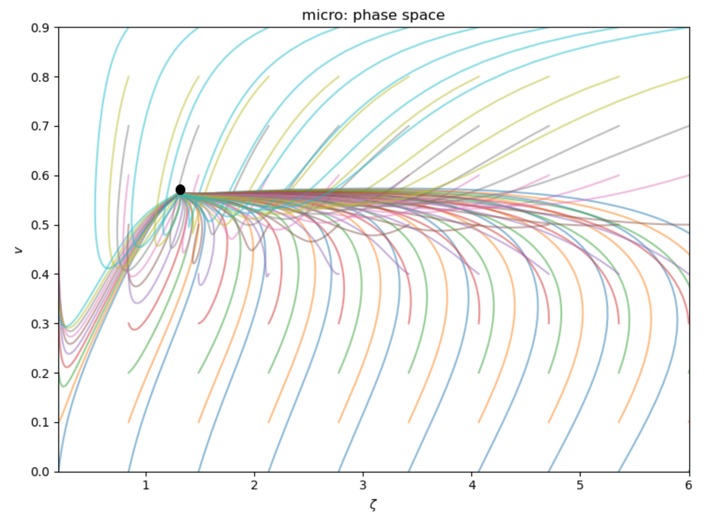
To c, or not to c

Energy loss mechanisms critically impact ζ, and thereby the axion dark matter aboundance

Beware your initial conditions, and please report velocities!








To be continued...
Centro de Astrofísica
da Universidade do Porto
IN THE
SKY