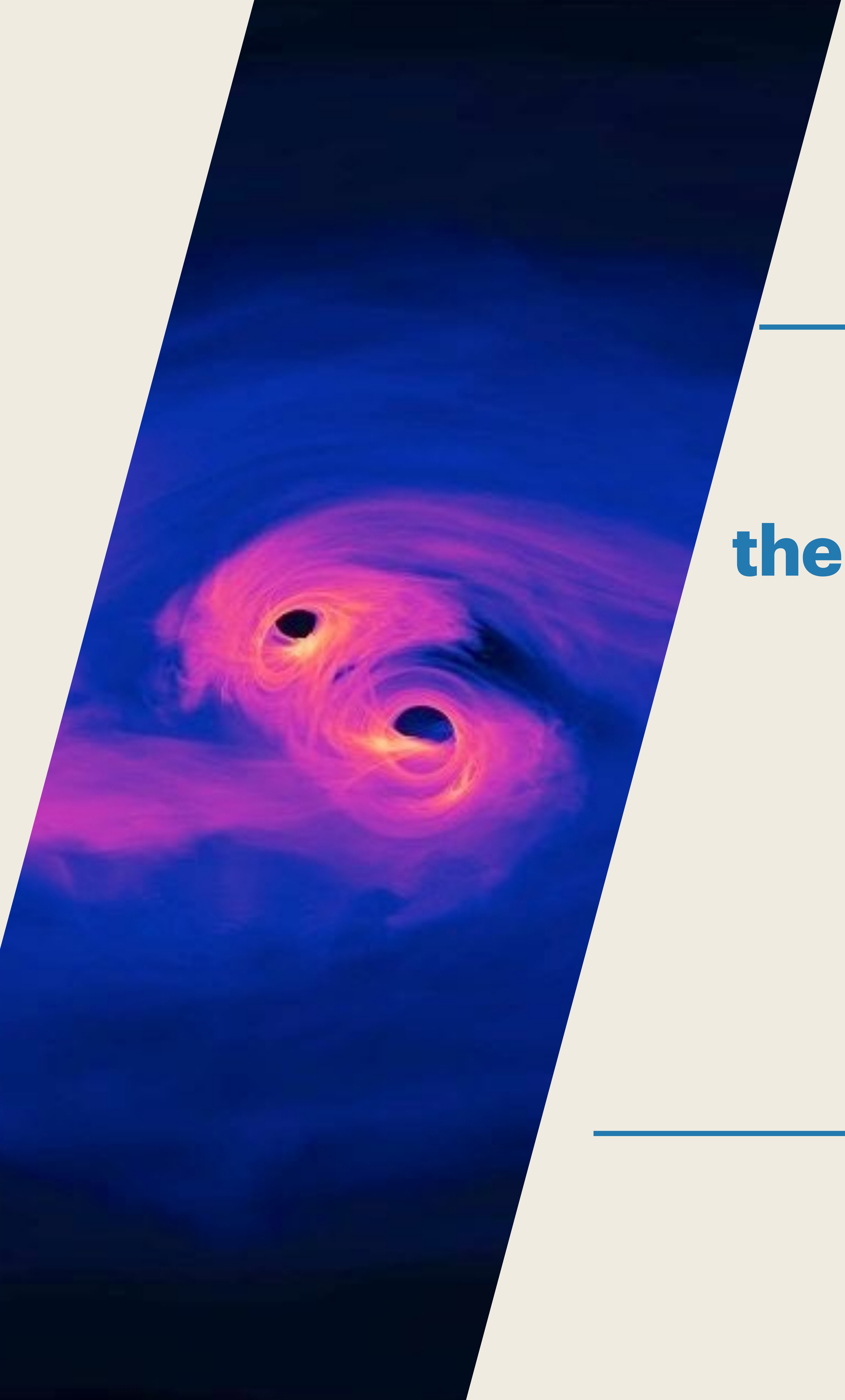

FC & M.Oguri

Phys. Rev. D**112**(2025)083502

Constraining the lensing dispersion from the angular clustering of binary black hole mergers

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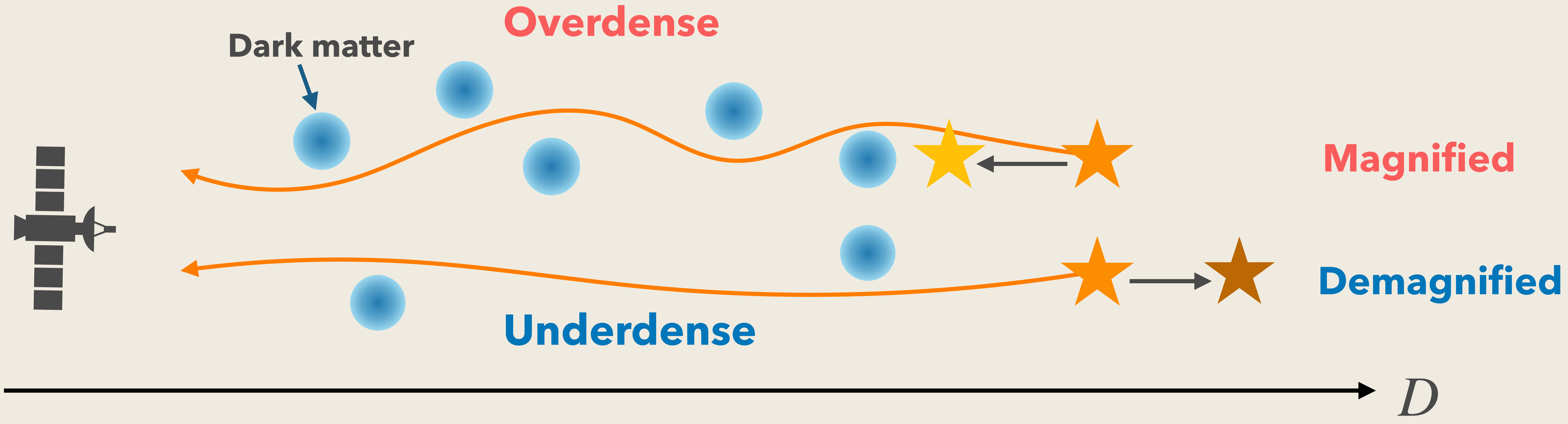
@ COSMO-26, Leiden

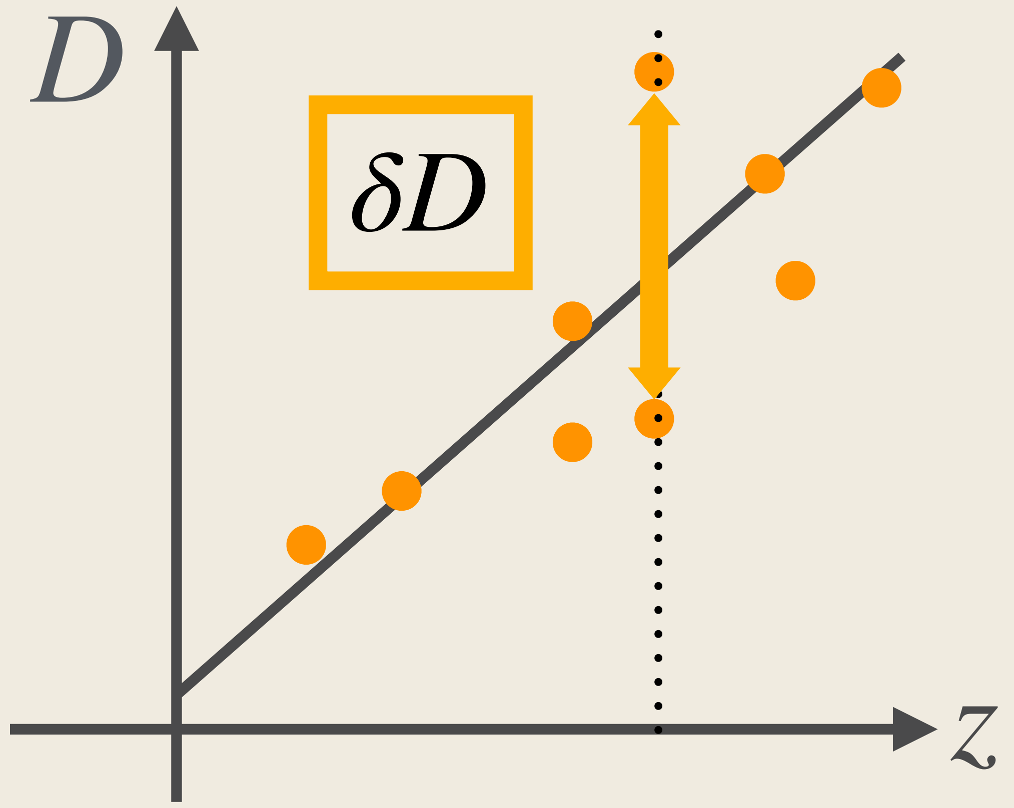
29th International Conference on Particle Physics & Cosmology

24-28 August 2026

What is the lensing dispersion?



The distance–redshift relation



The lensing dispersion

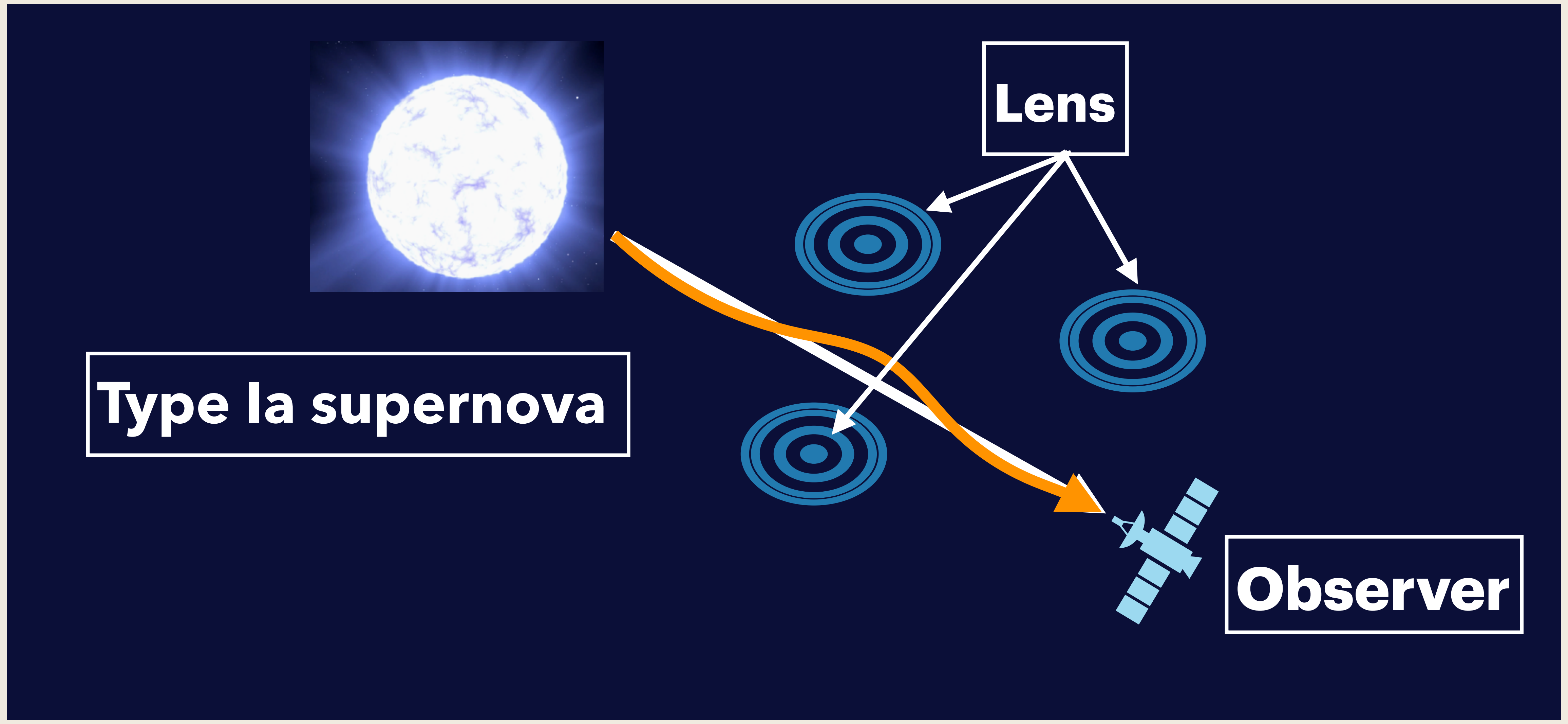
$$\langle \kappa^2 \rangle \sim \langle \kappa(\boldsymbol{\theta}, z)\kappa(\boldsymbol{\theta}', z') \rangle$$

$$(\because\ \boldsymbol{\theta} \sim \boldsymbol{\theta}', z \sim z')$$

Small-scale density fluctuations

The lensing dispersion $\langle \kappa^2 \rangle$ > useful cosmological probe

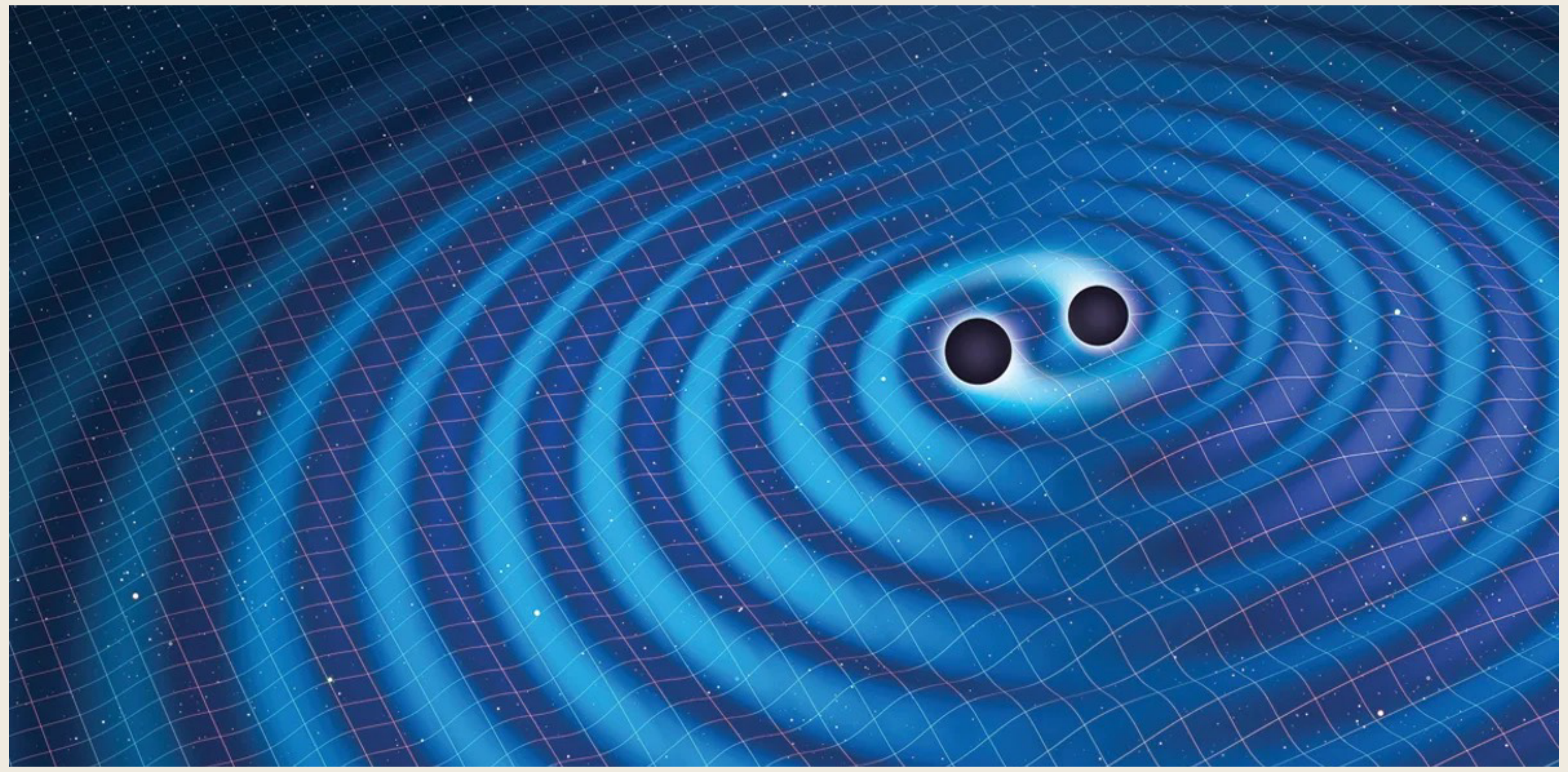
Observation of $\langle \kappa^2 \rangle$ by Type Ia supernova



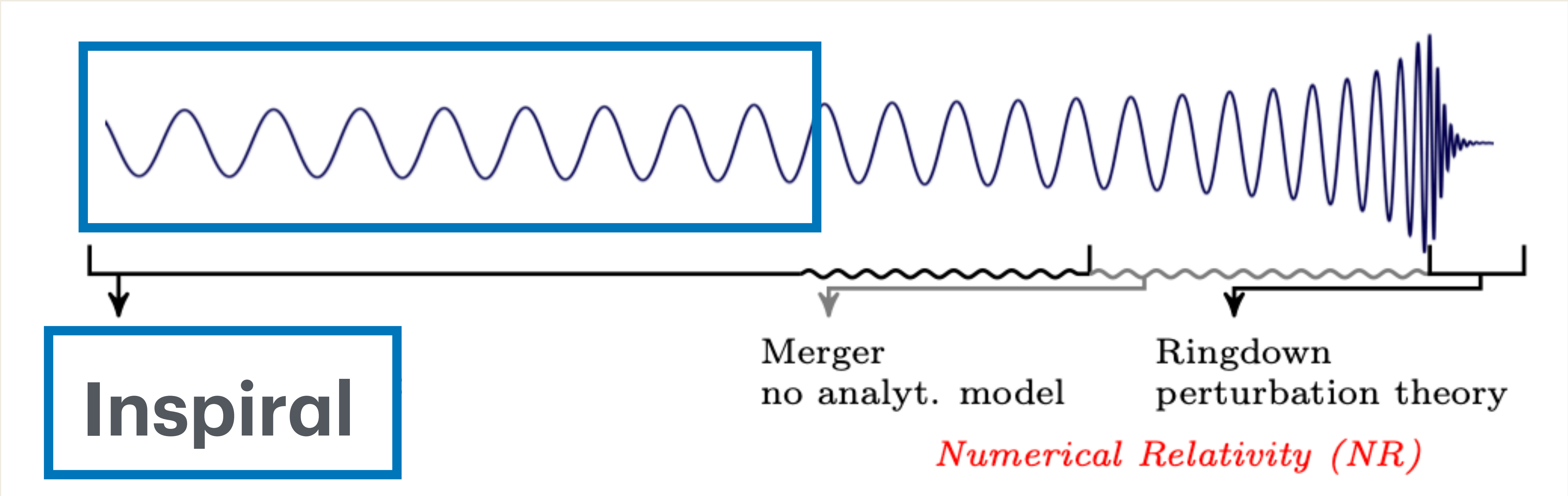
For instance……

- Abundance of primordial black holes (PBH) (Zumalacárregui & Seljak 2018)
- Upper limit on neutrino masses (Hada and Futamase, 2016; Agrawal et al., 2019)
- Small-scale cosmological density power spectrum (Ben-Dayan & Takahashi 2016)

Measuring luminosity distance D_L from GW



MARK GARLICK/SPL/Getty



Ohme 2012

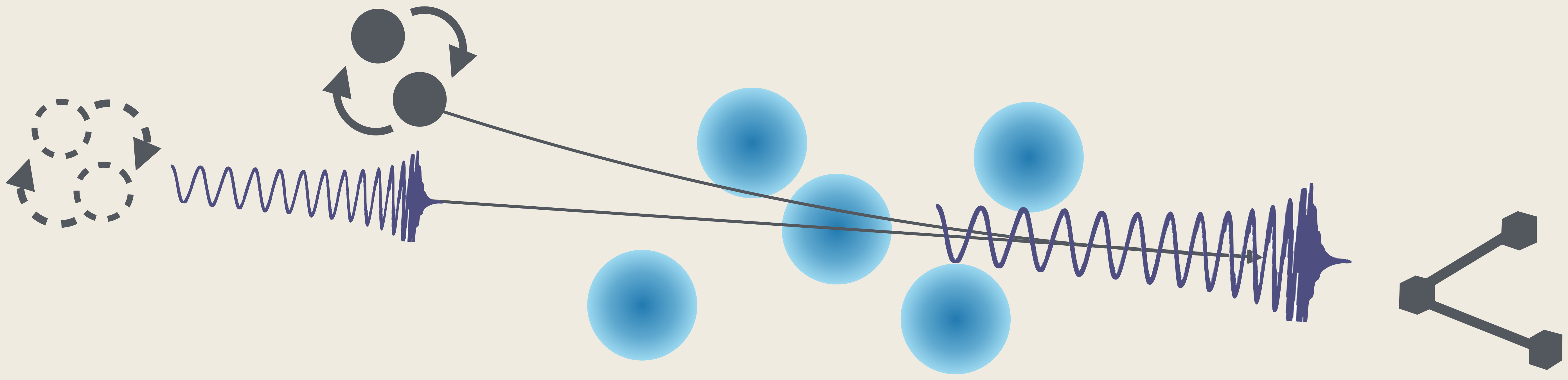
$$\dot{f}(t) \propto M_z^{5/3} f^{11/3} \qquad h_{+\times}(t) \propto \frac{M_z^{5/3} f^{2/3}}{\boxed{D_L}} \qquad \left(M_z \equiv (1+z)M_c\right)$$

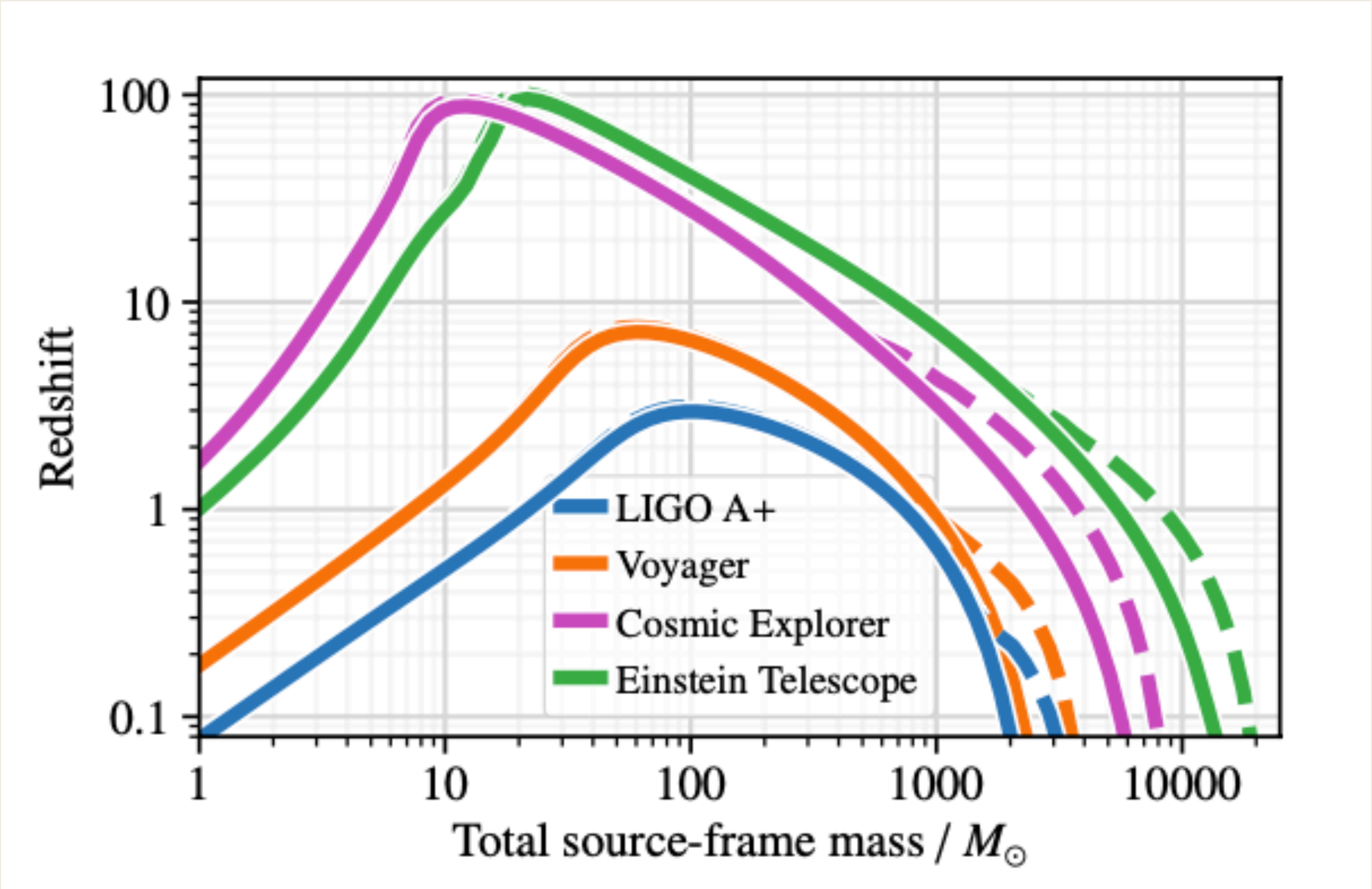
$f, \dot{f}$: **frequency, frequency evolution** $h_+, h_\times$: **strain of plus mode, cross mode** M_c : **chirp mass**

Gravitational waves provide direct distance measurements!

Advantages of using gravitational waves from binary black holes

Binary black hole (BBH)





Hall 2022

1. Obtain luminosity distances with small systematics
2. Probe higher redshifts $z > 2$ than supernova surveys

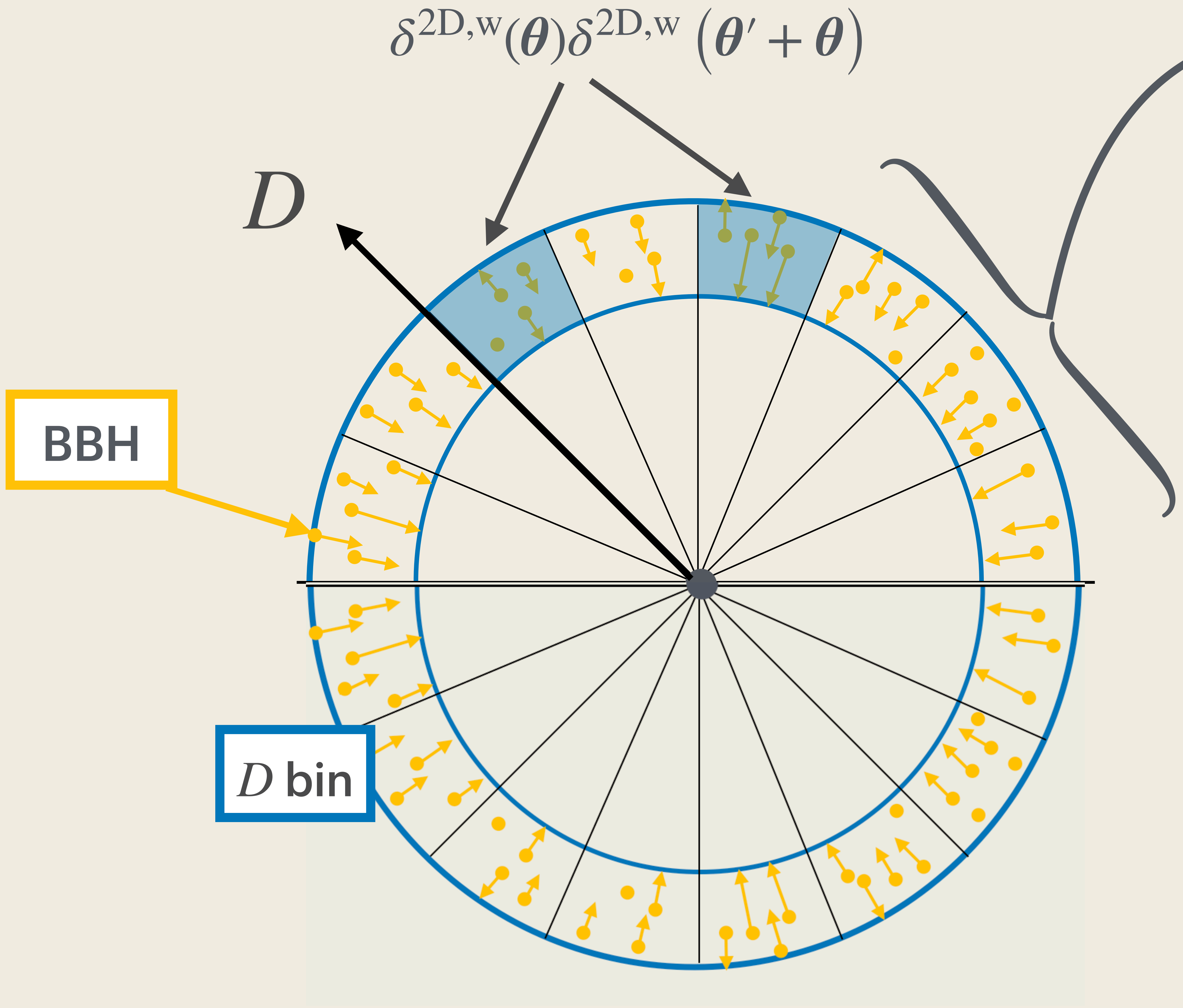
Identifying the BBH’s redshift is challenging

1. Dark Sirens: Mergers without an electromagnetic counterpart.
2. Poor Localization of the gravitational wave source on the sky.

➡ We cannot measure BBH’s redshift directly

Aim: Measure lensing dispersion using gravitational waves, even when redshifts are unknown.

Method：Using angular clustering of GWs on the sky



Large scale : $\ell \sim 100 - 300$

➡ **GW's density distribution = linear bias**

$$\delta_{\rm GW} = b_{\rm GW}\delta_{\rm m} \quad b_{\rm GW} = 1.5$$

$\delta_{\rm m}$: matter density distribution

Angular clustering signal

Auto-correlation angular power spectrum

$C^{\rm ww}(\ell)$ + **lensing effect**

➡ **Lensing-induced $C^{\rm ww}(\ell)$ deviation**

➡ **Lensing dispersion $\langle \kappa^2 \rangle$**

Method: Lensing effect changes D of the GW source

Weak lensing effect on luminosity distance

$$D = \frac{\bar{D}}{\sqrt{\mu}} \simeq \frac{\bar{D}}{\sqrt{1 + 2\kappa + 3\kappa^2 + \gamma^2}} \qquad \mu \simeq 1 + 2\kappa + 3\kappa^2 + \gamma^2$$

$\bar{D}$: Standard luminosity distance in a uniform isotropic universe

μ : magnification , κ : lensing convergence, γ : shear field

We compute the angular clustering signal, including lensing effects up to second order, and capture $\langle \kappa^2 \rangle$ contribution.

Method: New power spectrum formula of GW sources with $\langle\kappa^2\rangle$

NEW

$$C^{\mathrm{ww}}(\ell) = C^{\mathrm{ss}}(\ell) + C^{\mathrm{st}}(\ell) + C^{\mathrm{su}}(\ell) + C^{\mathrm{tt}}(\ell)$$

intrinsic clustering

$$C^{\mathrm{ss}}(\ell) = \int_0^\infty dz\ W^{\mathrm{s}}(z) W^{\mathrm{s}}(z) \frac{H(z)}{\chi^2} b_{\mathrm{GW}}^2 P_{\mathrm{m}}\left(\frac{\ell+1/2}{\chi}; z\right)$$

intrinsic clustering ✕ lensing effect

$$C^{\mathrm{st}}(\ell) = -2\int_0^\infty dz W^{\mathrm{s}}(z) W^{\mathrm{t}}(z) \frac{H(z)}{\chi^2} b_{\mathrm{GW}}^2 P_m\left(\frac{\ell+1/2}{\chi}; z\right) \boxed{\langle\kappa^2\rangle}$$

$$C^{\mathrm{su}}(\ell) = 2\int_0^\infty dz W^{\mathrm{s}}(z) W^{\mathrm{u}}(z) \frac{H(z)}{\chi^2} b_{\mathrm{GW}}^2 P_m\left(\frac{\ell+1/2}{\chi}; z\right) \boxed{\langle\kappa^2\rangle}$$

lensing effect

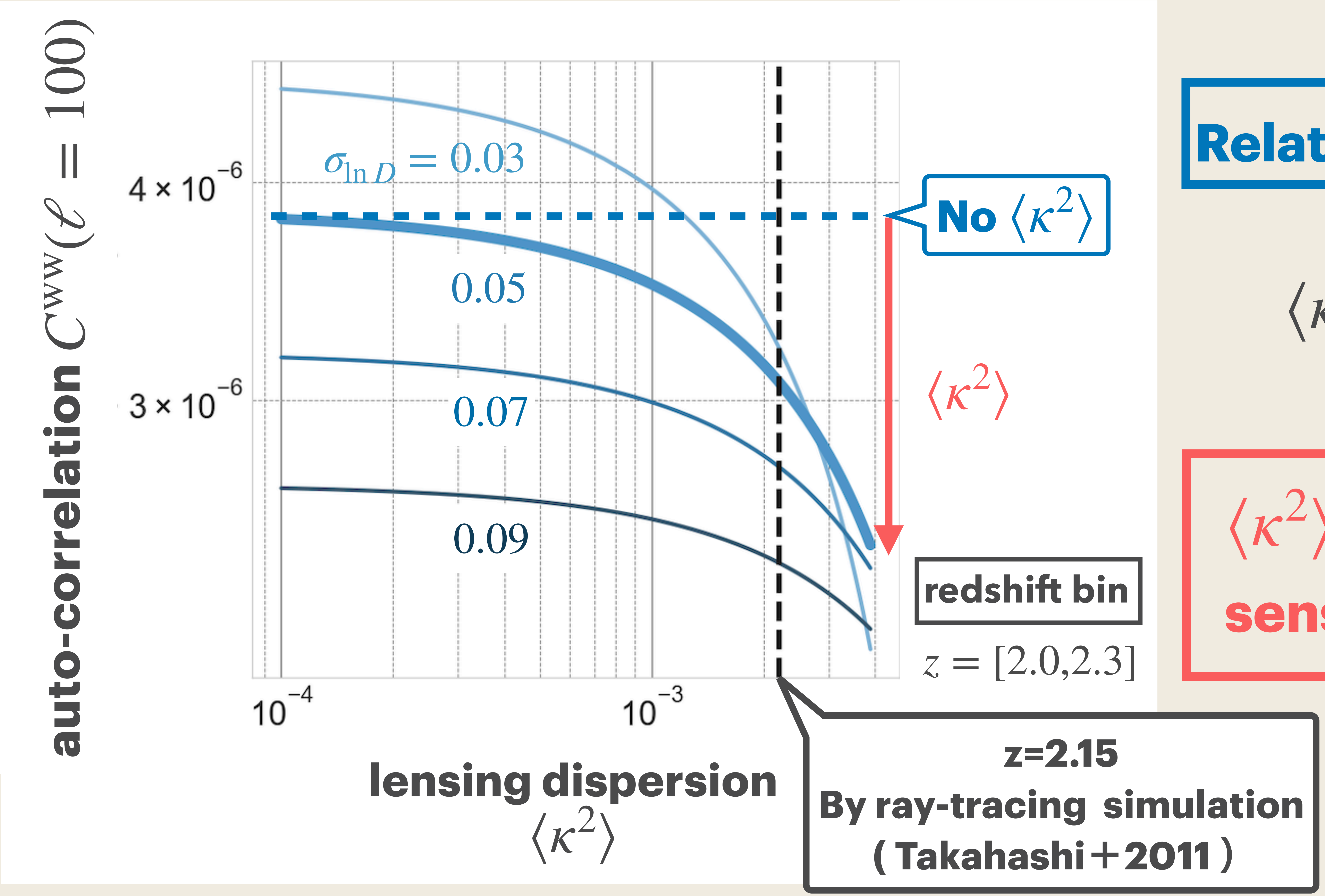
$$C^{\mathrm{tt}}(\ell) = \int_0^\infty dz \int_0^\infty dz' W^{\mathrm{t}}(z) W^{\mathrm{t}}(z') \int_0^{\min(z,z')} dz'' W^\kappa(z''; z)\, W^\kappa(z''; z') \frac{H(z'')}{\chi''^2} P_{\mathrm{m}}\left(k = \frac{\ell+1/2}{\chi''}, z''\right)$$

b_{GW} : bias of GW sources

χ : comoving distance

P_m: linear matter power spectrum

Result: Relation between $\langle\kappa^2\rangle$& $C^{\mathrm{WW}}(\ell)$



Relation between $\langle\kappa^2\rangle$& $C^{\mathrm{WW}}(\ell)$

$\langle\kappa^2\rangle \uparrow \Rightarrow C^{\mathrm{WW}}(\ell) \downarrow$

$\langle\kappa^2\rangle$ induced ΔC_ℓ indicates sensitivity to the dispersion.

Result: Ratio of signal to noise S/N

Ratio of signal to noise

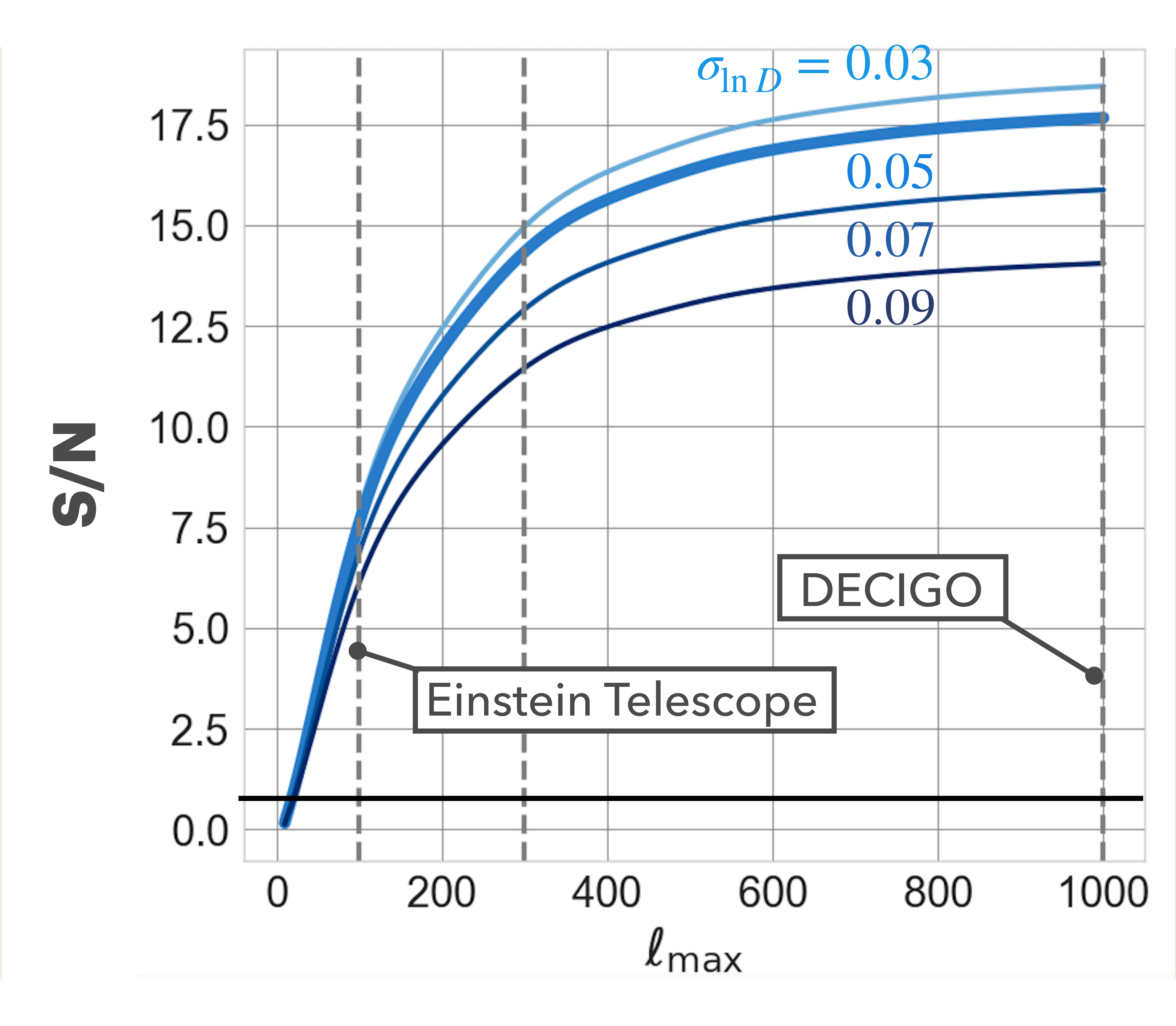
$$\sigma^2 \sim \mathrm{Cov} = \delta_{\ell\ell'} \frac{2}{2\ell+1}\left(C_\ell^{\mathrm{ww}} + \frac{1}{\bar{n}_{\mathrm{GW}}}\right)^2$$

$\bar{n}_{\mathrm{GW}}$: average number density of GWs

$$\mathrm{S/N} \equiv \sqrt{\sum_{\ell=0}^{\ell_{\max}} \frac{C_\ell^{\mathrm{ww}2}}{\mathrm{Cov}}}$$

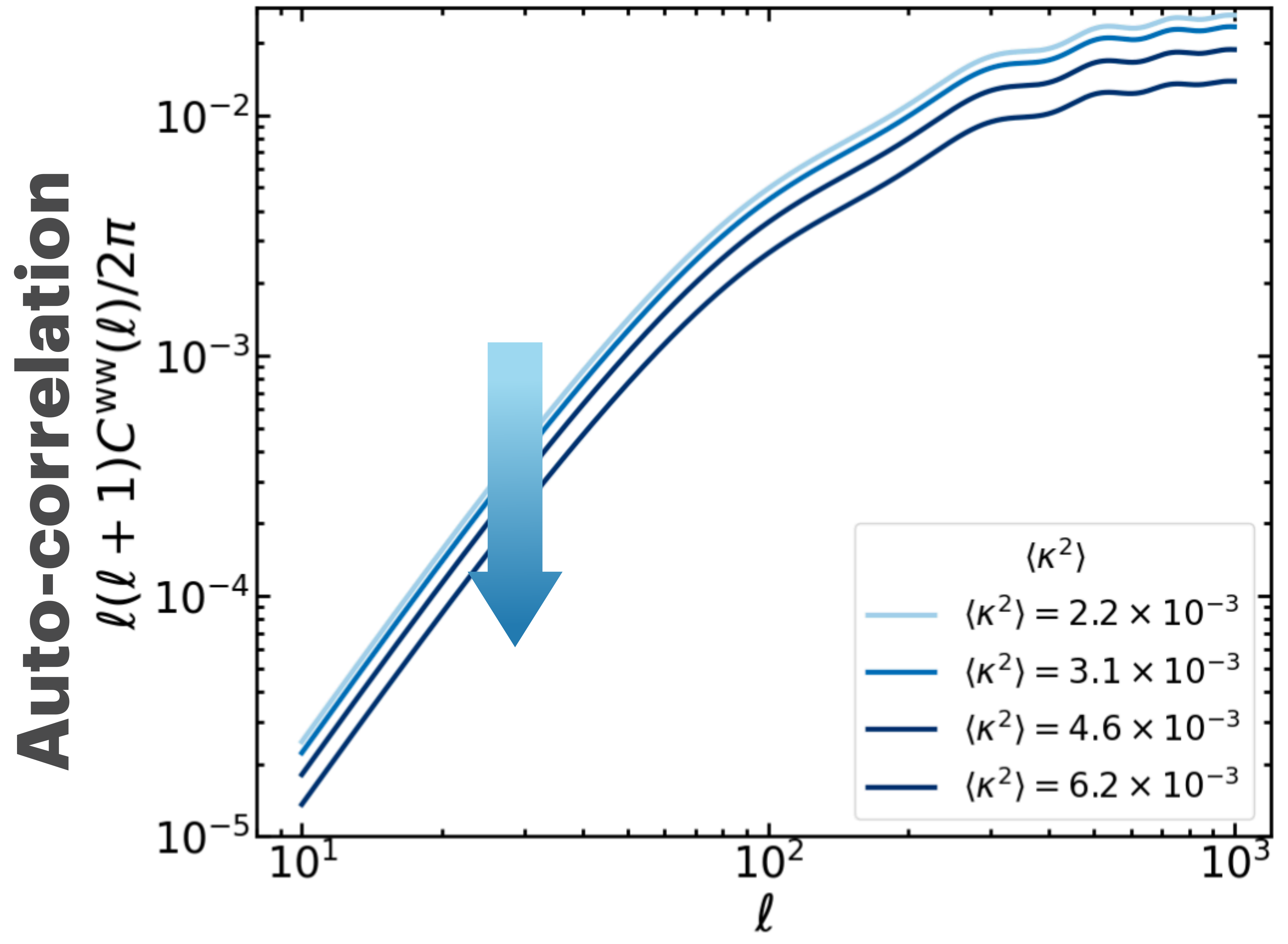
Angular resolution $\theta \sim \dfrac{\pi}{\ell_{\max}}$

The correlation signal is observable with next-generation observatories!

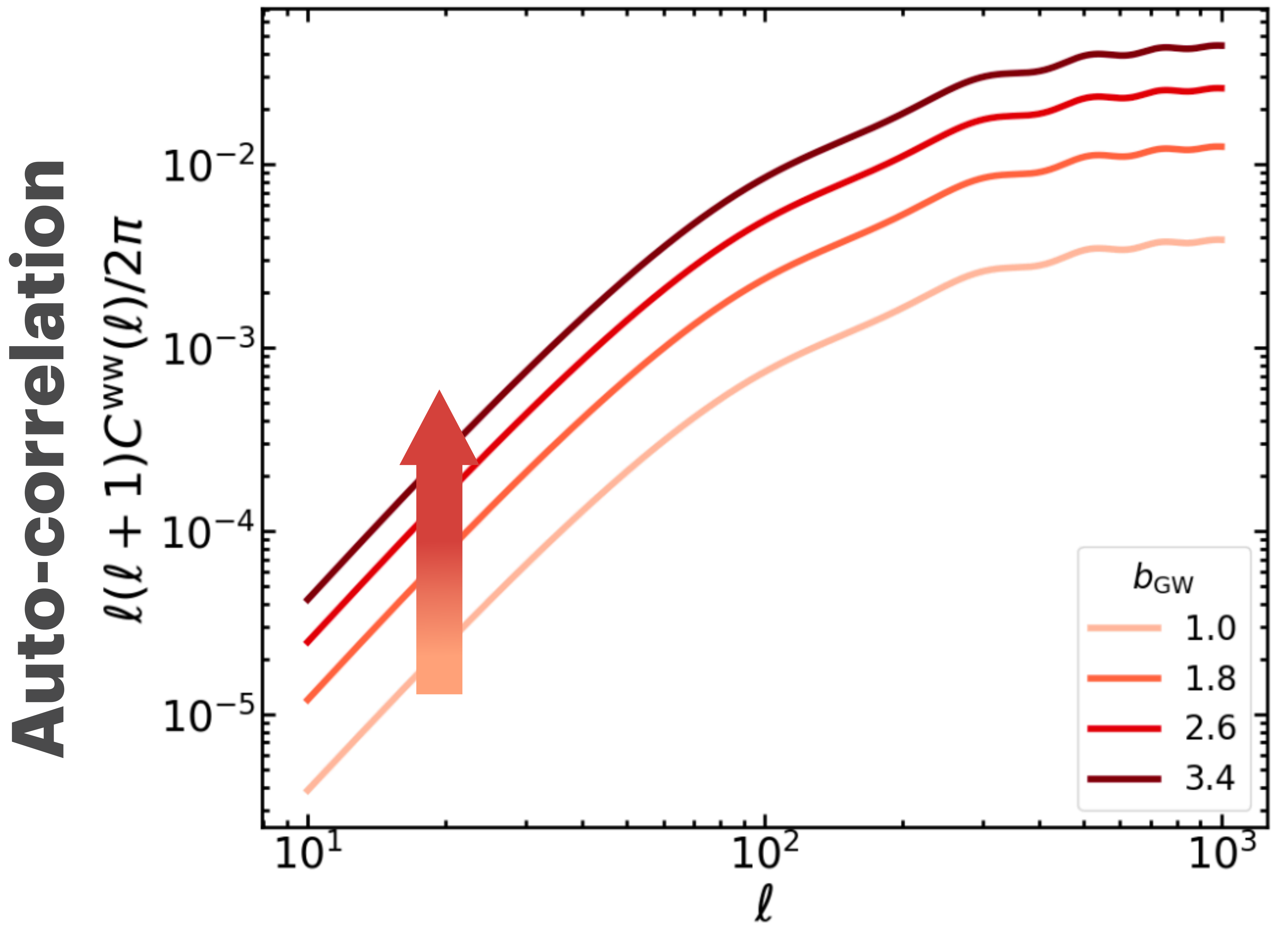


Result：Strong degeneracy between $\langle\kappa^2\rangle$ vs $b_{\rm GW}$

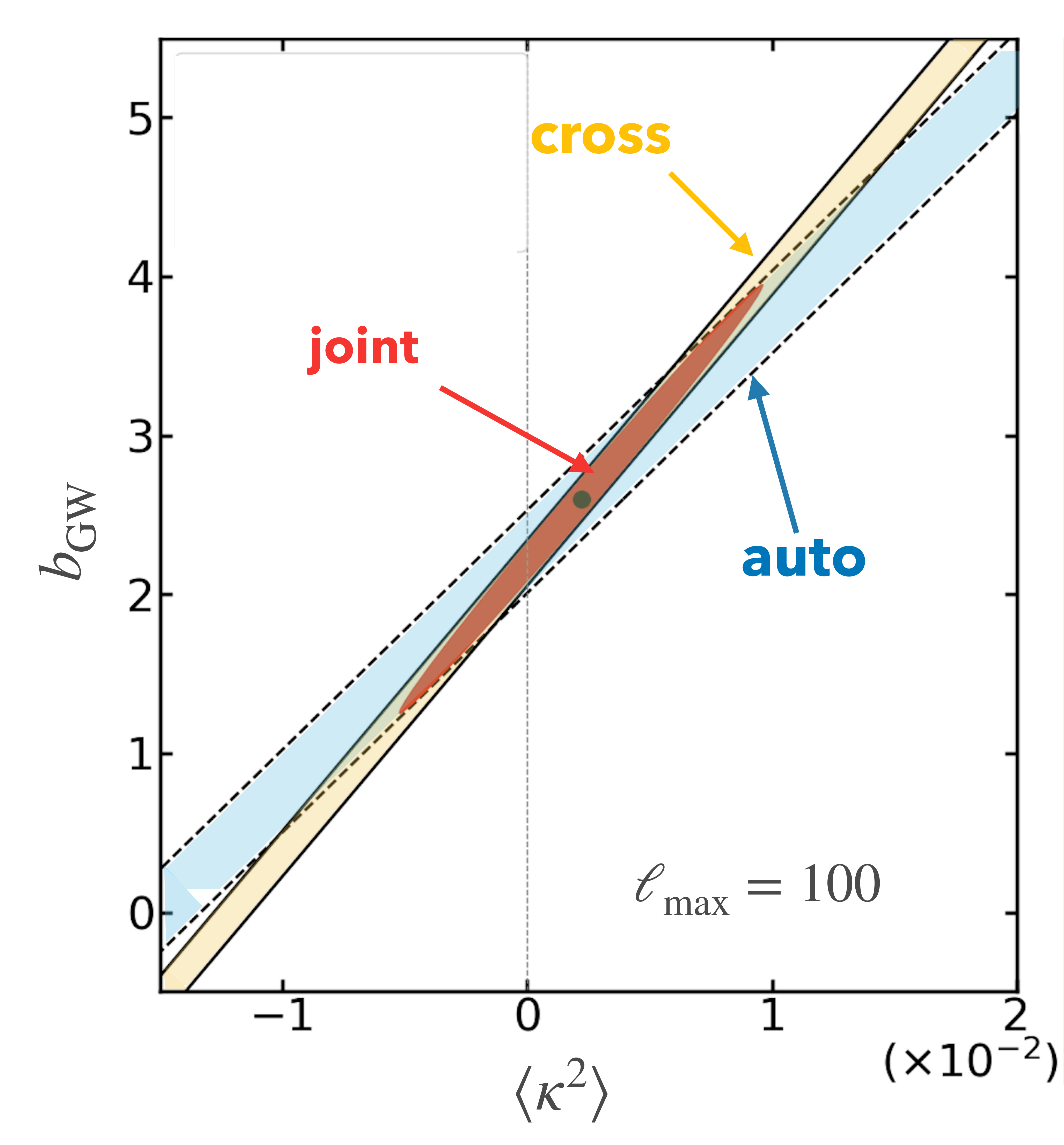
$\langle\kappa^2\rangle$ variation ($b_{\rm GW}$ fixed)



$b_{\rm GW}$ variation ($\langle\kappa^2\rangle$ fixed)



Result: 1σ confidence region in the $\langle\kappa^2\rangle$-b_{GW} plane



Joint analysis

- Auto-correlation of BBHs
- Cross-correlation between BBHs and spec-z galaxy

➡ **The degeneracy is partially broken**

Opening a path to constrain $\langle\kappa^2\rangle$!

Fiducial value

$\langle\kappa^2\rangle = 2.2 \times 10^{-3} \quad \sigma_{\ln D} = 0.05 \quad b_{\mathrm{GW}} = 2.6$

Future Work : Toward Better Constraints on $\langle \kappa^2 \rangle$

Approach 1 : Utilizing the scale dependence of the bias parameter

Effect of $\langle \kappa^2 \rangle \sim$ scale-independent **+** $\delta_{\mathrm{GW}} = b_{\mathrm{GW}}(\boxed{k})\delta_{\mathrm{m}}$

Approach 2 : Constraining formation channels at low-z and predicting b_{GW} at high-z

At low-z
- More precise luminosity distances
- Better sky localization
- Weaker gravitational lensing impact

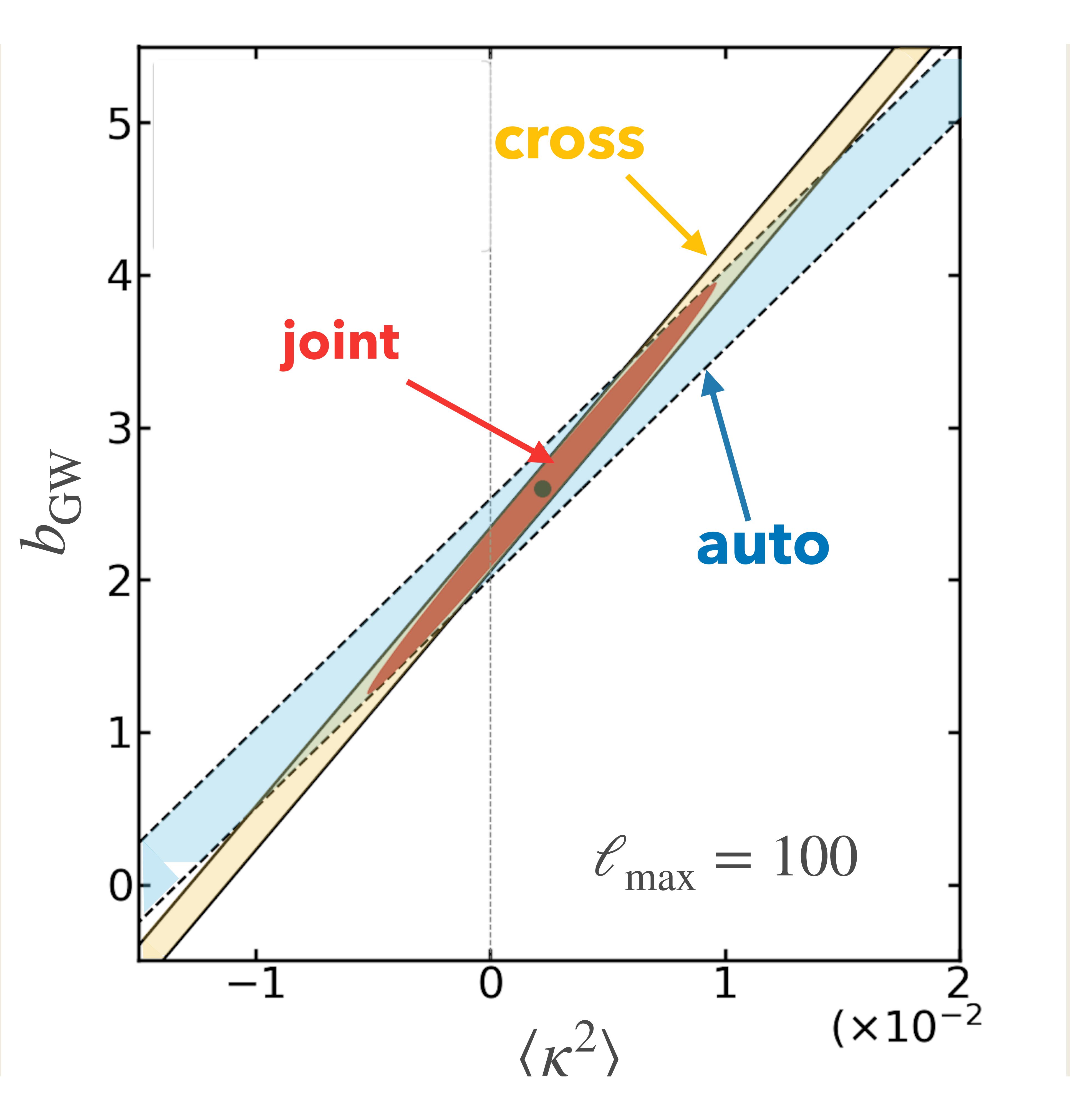
➡ **BBH formation channel** ➡ **predicting b_{GW} to high-z**

Better knowledge of b_{GW} enables significantly tighter constraints on $\langle \kappa^2 \rangle$ with this method

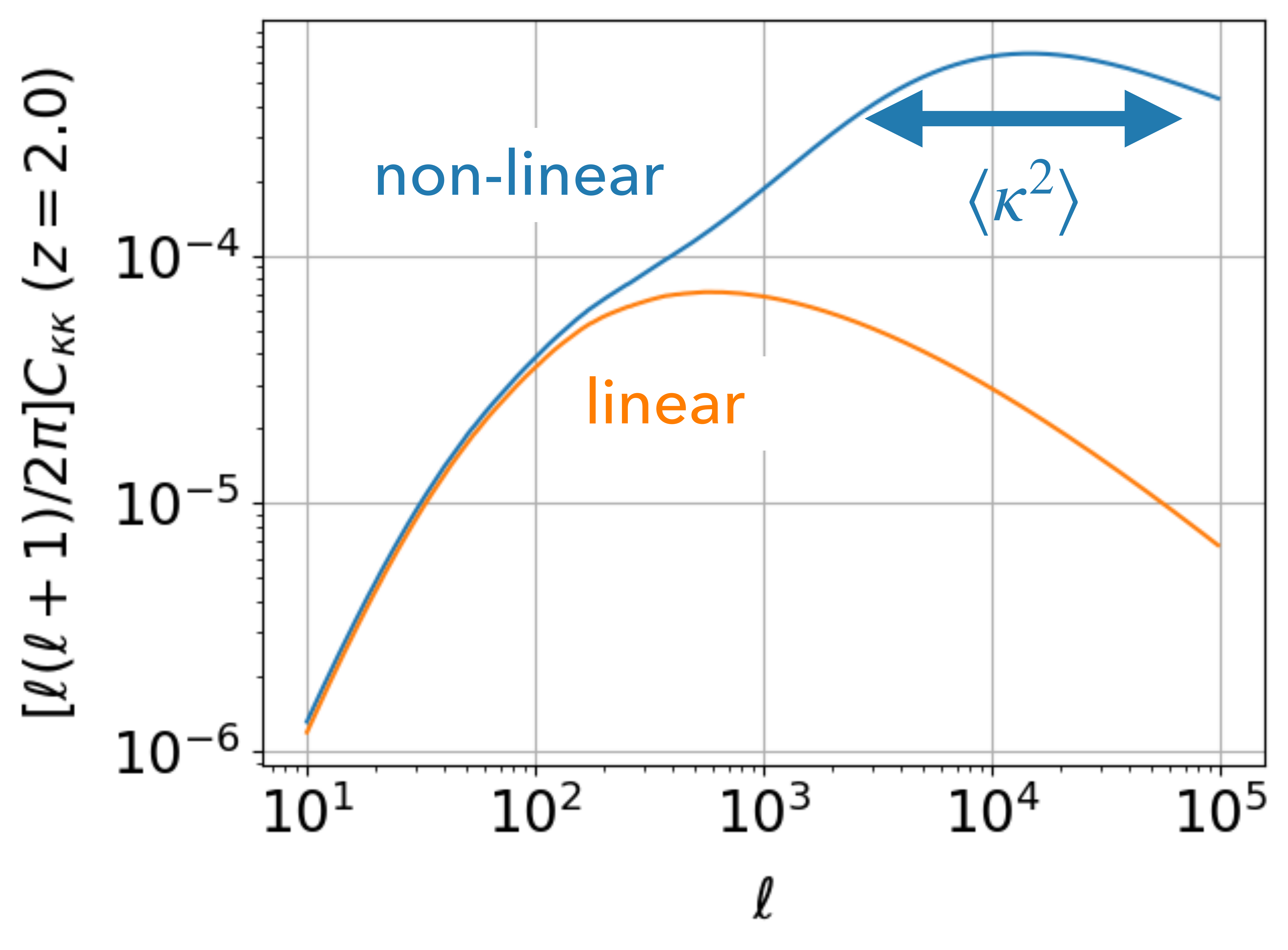
Summary

FC & Oguri
Phys. Rev. D**112**(2025)083502

- GW can measure luminosity distance directly + probe lensing dispersion to higher redshifts.
- Since many events are dark sirens, we propose a method utilizing angular clustering in luminosity-distance bins, excluding redshifts.
- The amplitudes of the angular power spectra of gravitational wave sources: decreasing function of the lensing dispersion.
- The signal should be observable with next-generation detectors.
- Joint analysis partially breaks degeneracies, opening a path to constrain lensing dispersion.



The correlation of lensing effect



Takahashi et al., 2012

Preparation for auto-correlation angular power spectrum

Observational error：log-normal distribution

$$p\left(D_{\mathrm{obs}} \mid D\right)=\frac{1}{\sqrt{2\pi}\sigma_{\ln D}} \exp\left[-x^2\left(D_{\mathrm{obs}}\right)\right] \frac{1}{D_{\mathrm{obs}}} \qquad x\left(D_{\mathrm{obs}}\right) \equiv \frac{\ln D_{\mathrm{obs}}-\ln D}{\sqrt{2}\sigma_{\ln D}}$$

The average projected number density of GW sources $\bar{n}_i^{\mathrm{w}}$：

$$\bar{n}_i^{\mathrm{w}}=\int_0^{\infty} dz \frac{\chi^2}{H(z)} S_i(z) T_{\mathrm{obs}} \frac{\dot{n}_{\mathrm{GW}}(z)}{1+z}$$

The duration of the observation : $T_{\mathrm{obs}}=5\ (\mathrm{yr})$

BH merger rate : $\dot{n}_{\mathrm{GW}}=10^{-5}\ \left(h^3 \mathrm{Mpc}^{-3} \mathrm{yr}^{-1}\right)$

Liner bias： $\delta_{\mathrm{GW}}=b_{\mathrm{GW}} \delta_m \quad b_{\mathrm{GW}}=2.6$

Limber approximation:

When θ ~ 1/l ≪ 1, only the power spectrum of sources at the same radial distance from the observation point contributes to the calculation.

SELECTION FUNCTION

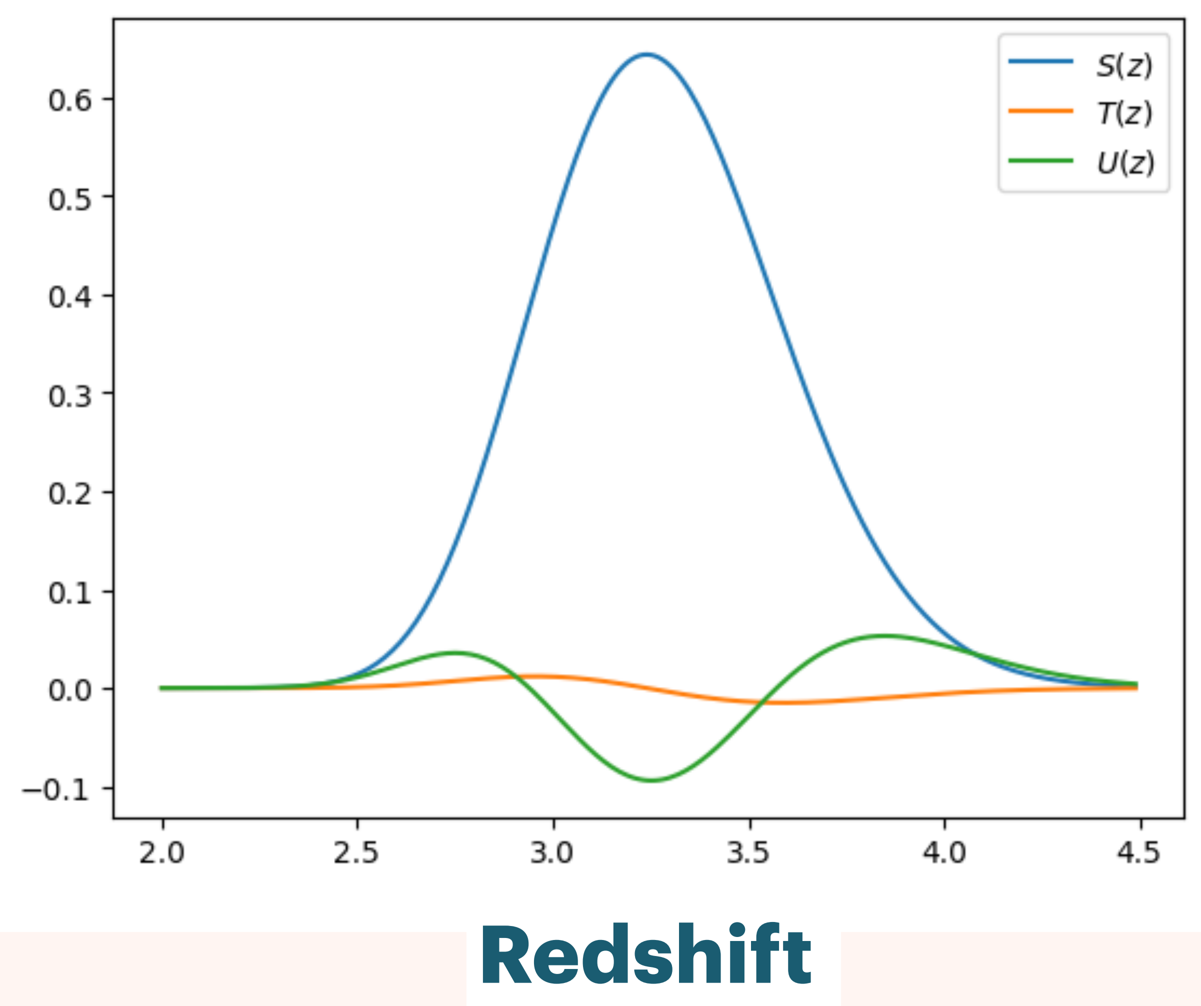
The zero order

$$S(z) \equiv \frac{1}{2}\left[\operatorname{erfc}\left\{x\left(D_{\min}\right)\right\} - \operatorname{erfc}\left\{x\left(D_{\max}\right)\right\}\right]$$

Expand to the second order with a small amount of Δx

$$x\left(D_{\mathrm{obs}}\right) \simeq \bar{x} + \boxed{\frac{1}{\sqrt{2}\sigma_{\ln D}}\left(\kappa + \frac{1}{2}\kappa^2 + \frac{1}{2}\gamma^2\right)}$$

$$= \Delta x$$



$$S(z) = \bar{S}(z) + T(z)\left(\kappa + \frac{\kappa^2}{2} + \frac{\gamma^2}{2}\right) + U(z)\kappa^2$$

The first order

$$T(z) = \frac{1}{\sqrt{2\pi}\sigma_{\ln D}}\left[-\exp\left\{-x^2\left(D_{\min}\right)\right\} + \exp\left\{-x^2\left(D_{\max}\right)\right\}\right]$$

The second order

$$U(z) = \frac{1}{2\sqrt{\pi}\sigma^2_{\ln D}}\left[\bar{x}\left(D_{min}\right)\exp\left\{-\bar{x}^2\left(D_{\min}\right)\right\} - \bar{x}\left(D_{max}\right)\exp\left\{-\bar{x}^2\left(D_{max}\right)\right\}\right]$$

$$W^{\mathrm{s}} = \frac{1}{\bar{n}^{\mathrm{w}}}\frac{\chi^2}{H(z)}\bar{n}_{\mathrm{GW}}(z)S(z) \quad W^{\mathrm{t}}(z) = \frac{1}{\bar{n}^{\mathrm{w}}}\frac{\chi^2}{H(z)}\bar{n}_{\mathrm{GW}}(z)T(z) \quad W^{\mathrm{u}}(z) = \frac{1}{\bar{n}^{\mathrm{w}}}\frac{\chi^2}{H(z)}\bar{n}_{\mathrm{GW(z)}}U(z)$$

cross-correlation of GWs and Galaxy

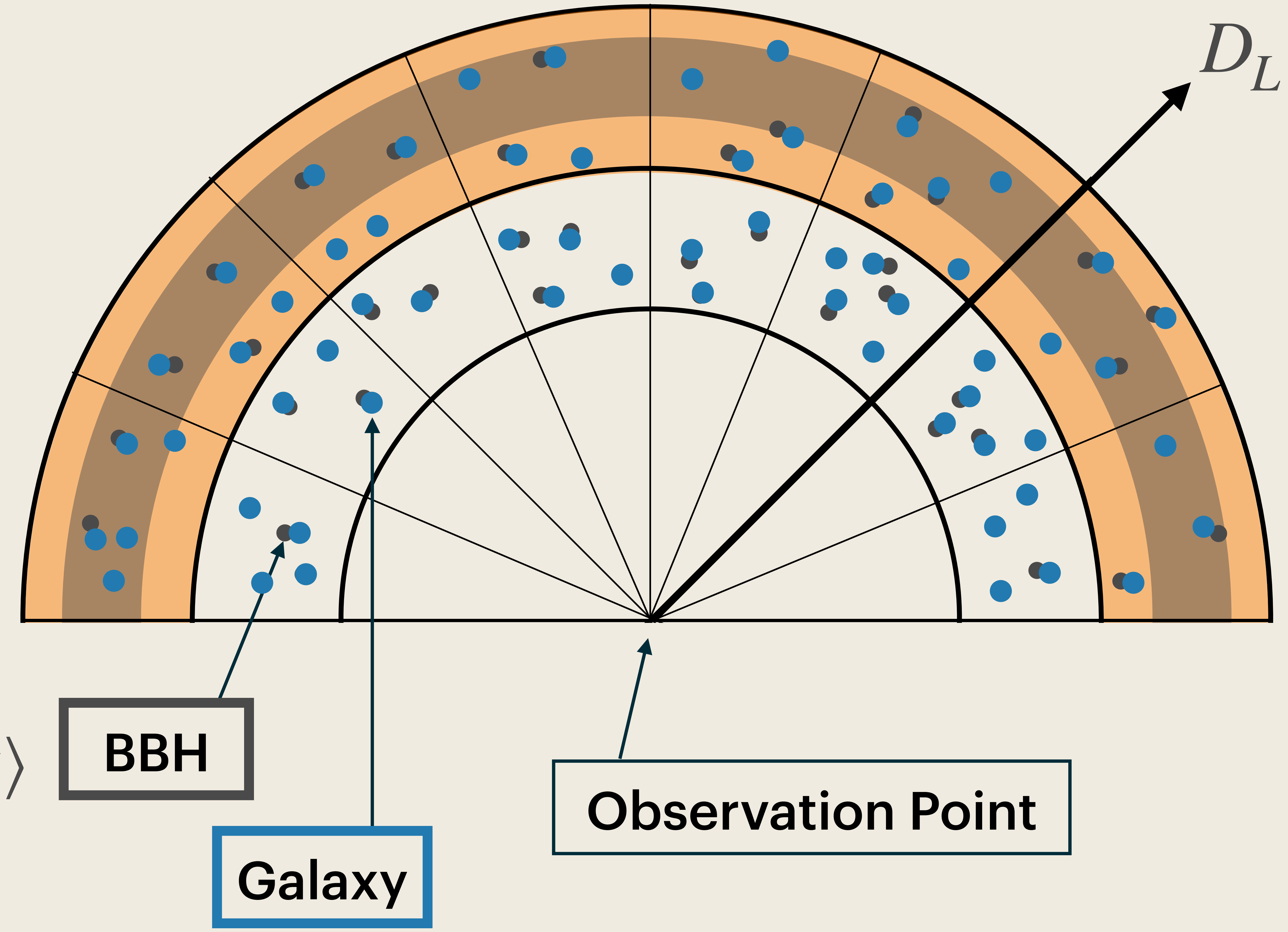
cross-correlation power spectrum

$$C^{\mathrm{wg}}(\ell) = C^{\mathrm{sg}}(\ell) + C^{\mathrm{tg}}(\ell) + C^{\mathrm{ug}}(\ell)$$

$$C^{\mathrm{sg}}(\ell) = \int_0^\infty dz W^{\mathrm{s}}(z) W^{\mathrm{g}}(z) \frac{H(z)}{\chi^2} b_{\mathrm{GW}} b_{\mathrm{g}} P_{\mathrm{m}} \left(\frac{\ell + 1/2}{\chi}; z \right)$$

$$C^{\mathrm{tg}}(\ell) = -\int_0^\infty dz W^{\mathrm{t}}(z) W^{\mathrm{g}}(z) \frac{H(z')}{\chi'^2} b_{\mathrm{GW}} b_{\mathrm{g}} P_{\mathrm{m}} \left(\frac{\ell + 1/2}{\chi'}; z' \right) \langle \kappa^2 \rangle$$

$$C^{\mathrm{ug}}(\ell) = \int_0^\infty dz W^{\mathrm{u}}(z) W^{\mathrm{g}}(z) \frac{H(z')}{\chi'^2} b_{\mathrm{GW}} b_{\mathrm{g}} P_{\mathrm{m}} \left(\frac{\ell + 1/2}{\chi'}; z' \right) \langle \kappa^2 \rangle$$



χ : **comoving distance**

b_{GW} : **bias of GW sources**

b_{g} : **bias of galaxies**

P_m: **linear matter power spectrum**

Why is Log-normal distribution chosen?

GW observation

		Error
GWXXXXX1 : SN=13	○ →	$\pm 13\,\%$
GWXXXXX2 : SN=1	×	
GWXXXXX3 : SN=20	○ →	$\pm 20\,\%$
GWXXXXX4 : SN=5	×	

error propagation

$$h_{+\times}(t) \propto \frac{1}{D_L} \longrightarrow \sigma_{h_\pm} = \sigma_D$$

→**Luminosity distance error is a relative error**

relative error → **log-normal distribution**

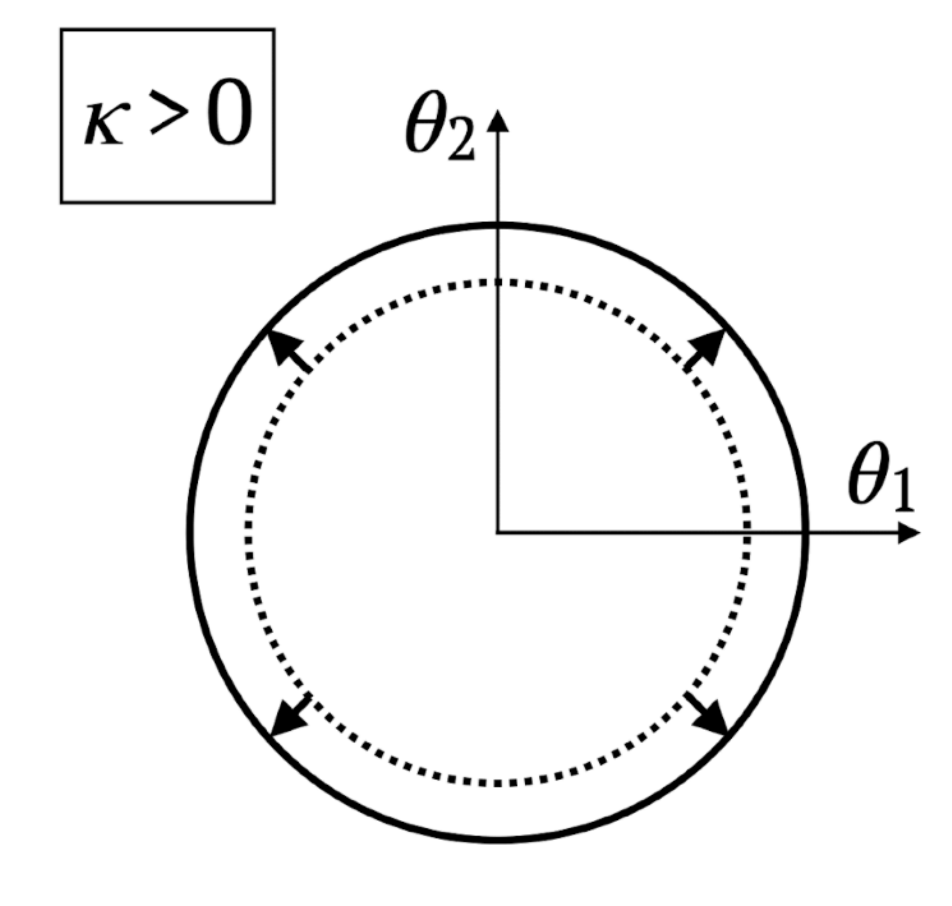
absolute error → **normal distribution**

$$x\left(D_{\mathrm{obs}}\right) \equiv \frac{\ln D_{\mathrm{obs}} - \ln D}{\sqrt{2}\sigma_{\ln D}}$$

Lensing convergence and its dispersion

The lensing convergence field $\kappa(\boldsymbol{\theta}, z)$ is the projection of the matter density field $\bar{\rho}_m(z)$ and magnify the images.

$$\kappa(\boldsymbol{\theta}, z) = \int_0^z dz' \frac{\bar{\rho}_{\mathrm{m}}(z')}{H(z')(1+z')\Sigma\mathrm{crit}(z'; z)} \delta_{\mathrm{m}}(\boldsymbol{\theta}, z') \equiv \int_0^z dz' W^{\kappa}(z'; z)\, \delta_{\mathrm{m}}\left(\boldsymbol{\theta}, z'\right)$$



$$\langle \kappa(\boldsymbol{\theta}, z) \kappa(\boldsymbol{\theta}', z') \rangle = \int_0^z dz'' W^{\kappa}(z''; z)\, \delta_{\mathrm{m}}\left(\boldsymbol{\theta}, z''\right) \int_0^{z'} dz''' W^{\kappa}(z'''; z')\, \delta_{\mathrm{m}}\left(\boldsymbol{\theta}', z'''\right)$$

From the ray tracing simulation

$\langle \kappa \rangle = -2 \langle \kappa^2 \rangle, \langle \gamma^2 \rangle = \langle \kappa^2 \rangle$ (Takahashi+2011)

LENSING CONVERGENCE DISPERSION $\langle \kappa^2 \rangle$

Magnification is approximate as follows

$$\mu = \frac{1}{(1-\kappa)^2 - \gamma^2} \simeq 1 + 2\kappa + 3\kappa^2 + \gamma^2$$

Average expansion rate

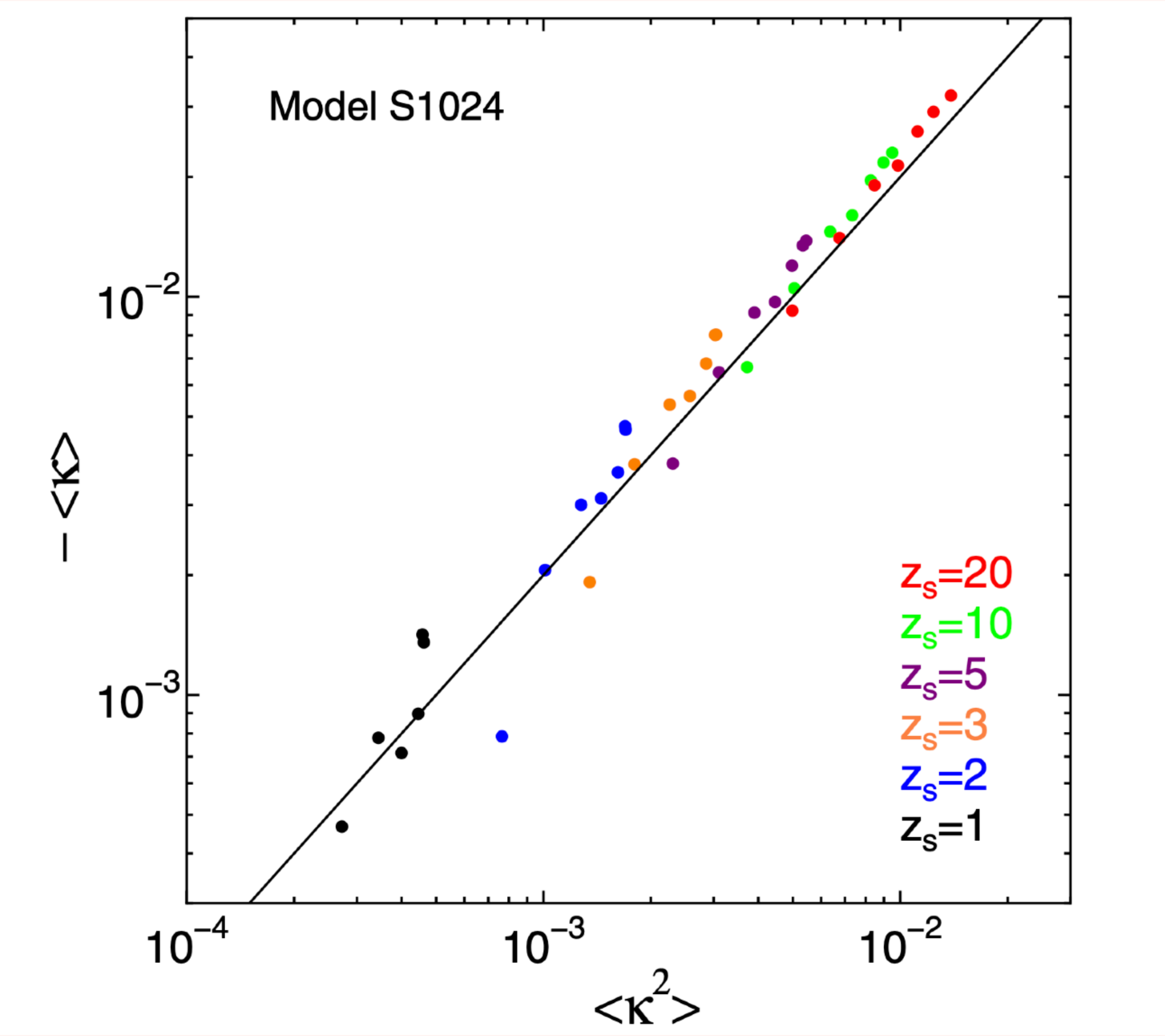
in terms of source $\langle \mu \rangle = 1$

$$\langle \mu \rangle = 1 \simeq 1 + 2\langle \kappa \rangle + 3\left\langle \kappa^2 \right\rangle + \left\langle \gamma^2 \right\rangle$$

From the approximation of

the weak gravity lens effect $\left\langle \kappa^2 \right\rangle = \left\langle \gamma^2 \right\rangle$

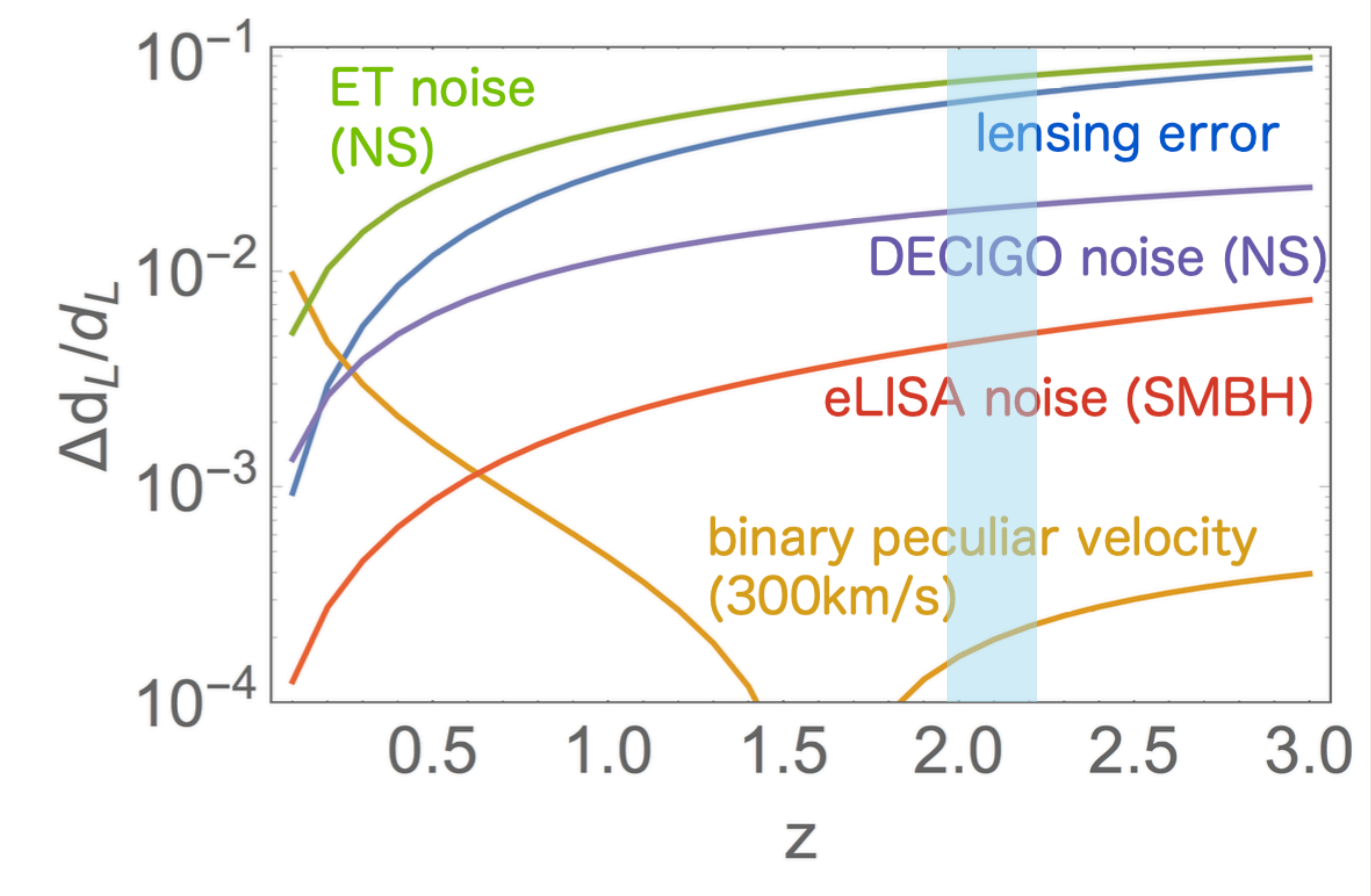
$$\langle \kappa \rangle \simeq -2\left\langle \kappa^2 \right\rangle$$



Ray-tracing Simulationsによる結果

(Takahashi＋2011)

The error of luminosity distance



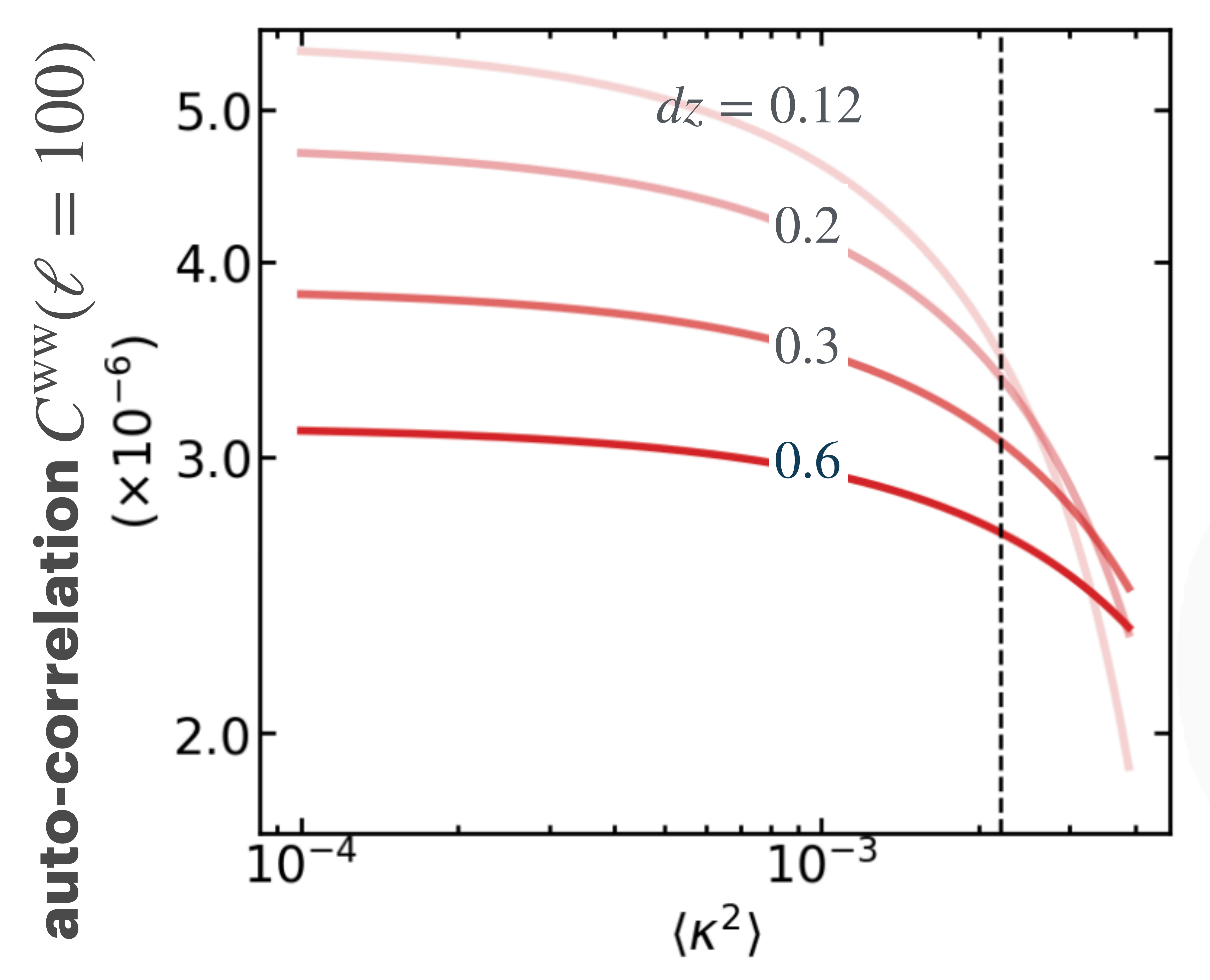
$$\frac{\Delta d_L(z)}{d_L(z)} = \sqrt{\sigma_{inst}^2(z) + \sigma_{lens}^2(z) + \sigma_{pv}^2(z)}$$

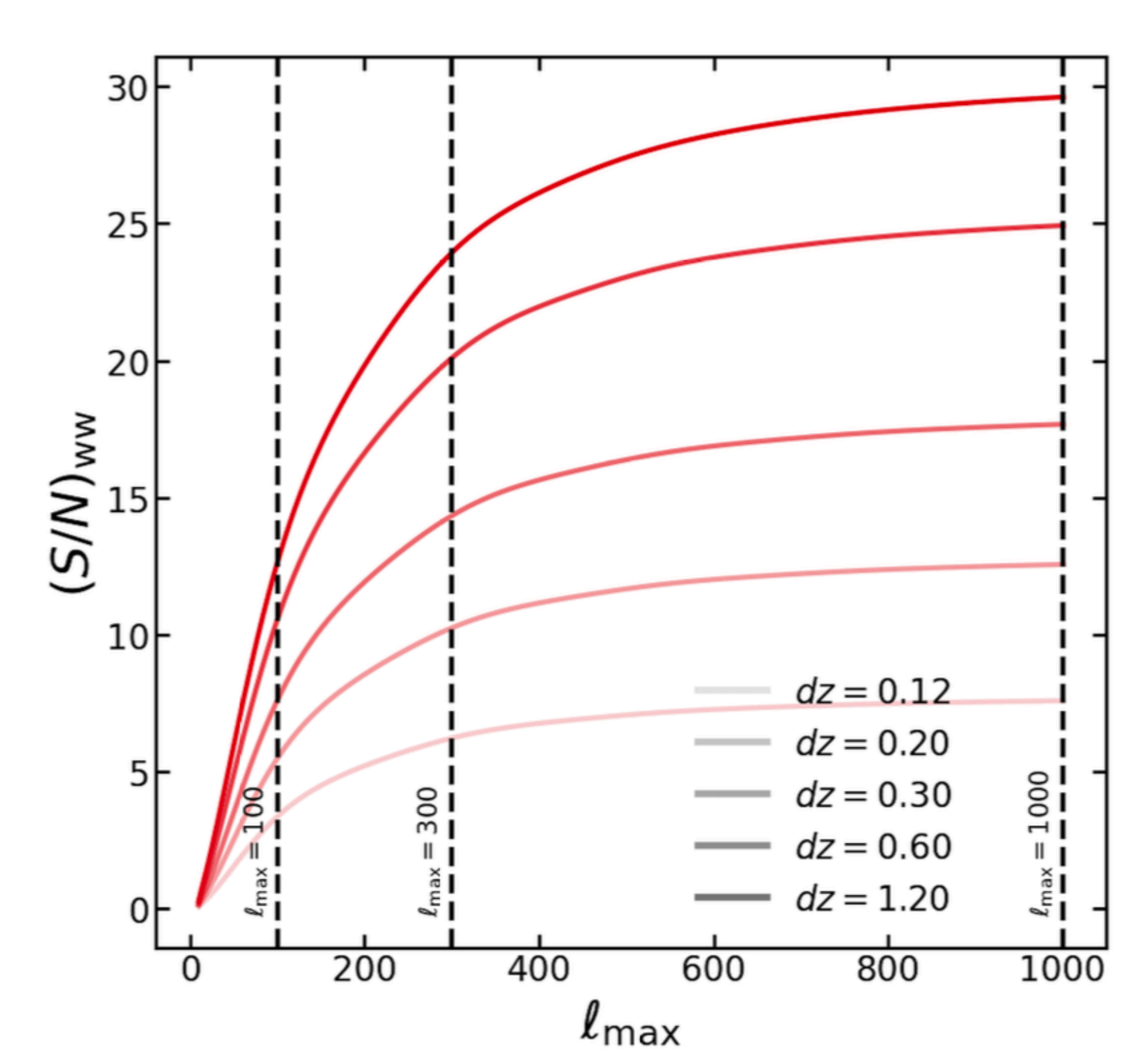
The relative magnitude of errors

$$\sigma_{inst}^2(z) > \sigma_{lens}^2(z) \gg \sigma_{pv}^2(z)$$

Nishizawa, Taruya, Saito, PRD (2011)

Bin size dependent of the results





Comparison with another way of incorporating lensing effects

the gravitational lensing effect is expressed by increasing the probability distribution dispersion.

$$\sigma_{lnD} \longrightarrow \sigma = \sqrt{\sigma_{lnD}^2 + \sigma_\kappa^2}$$

Differences can be seen where the observation error is small.

The distribution is uniformly expanded, so the distribution's distortion cannot be fully expressed in this way?