

Towards a Detection of Post-Born Lensing

Ewan Chamberlain, Mathew Robertson, Antony Lewis

University of Sussex

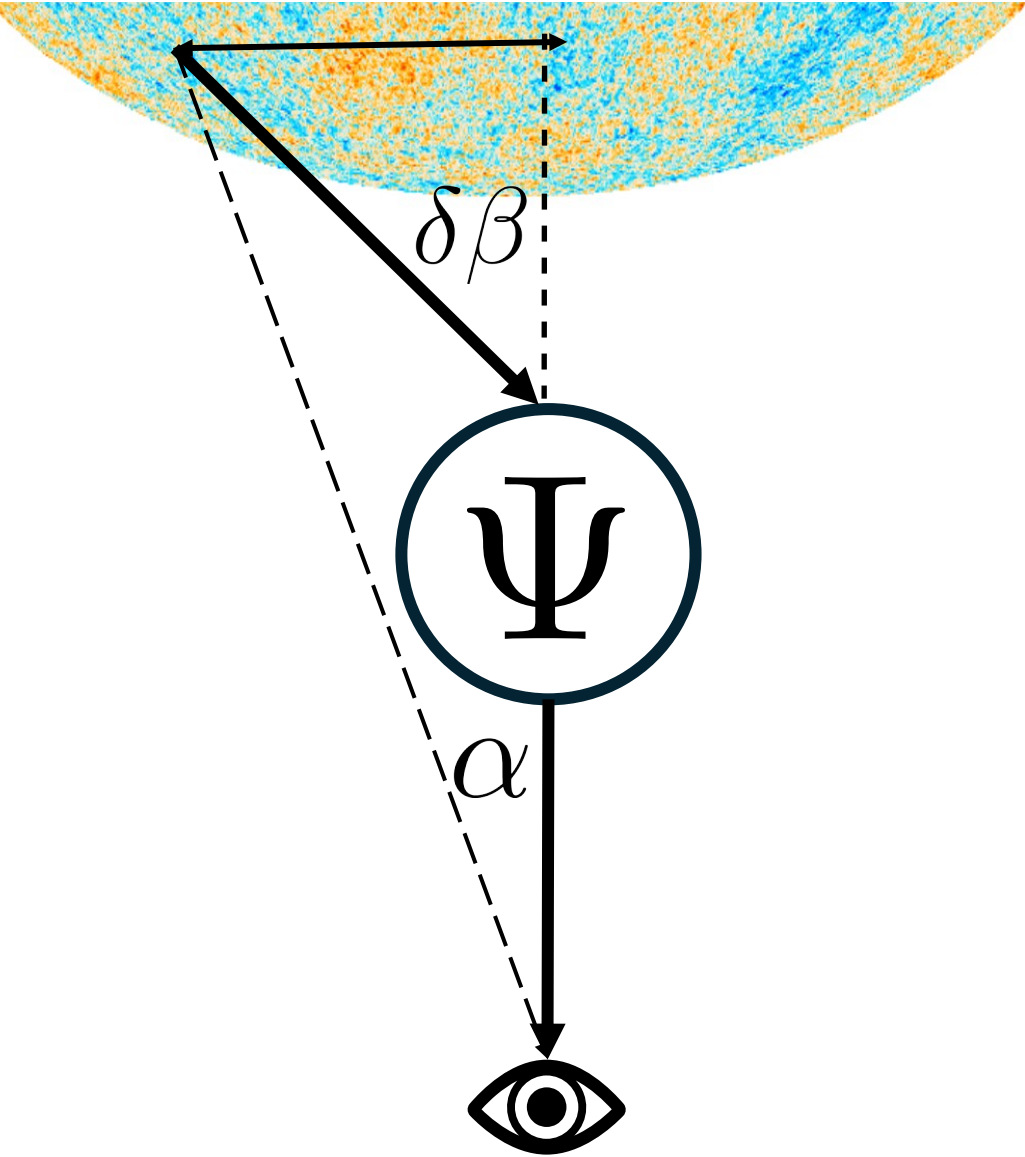


Science and
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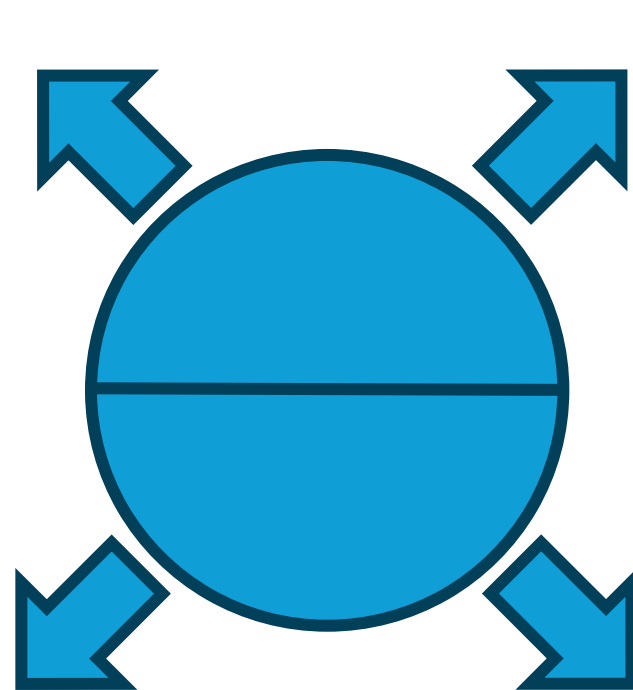
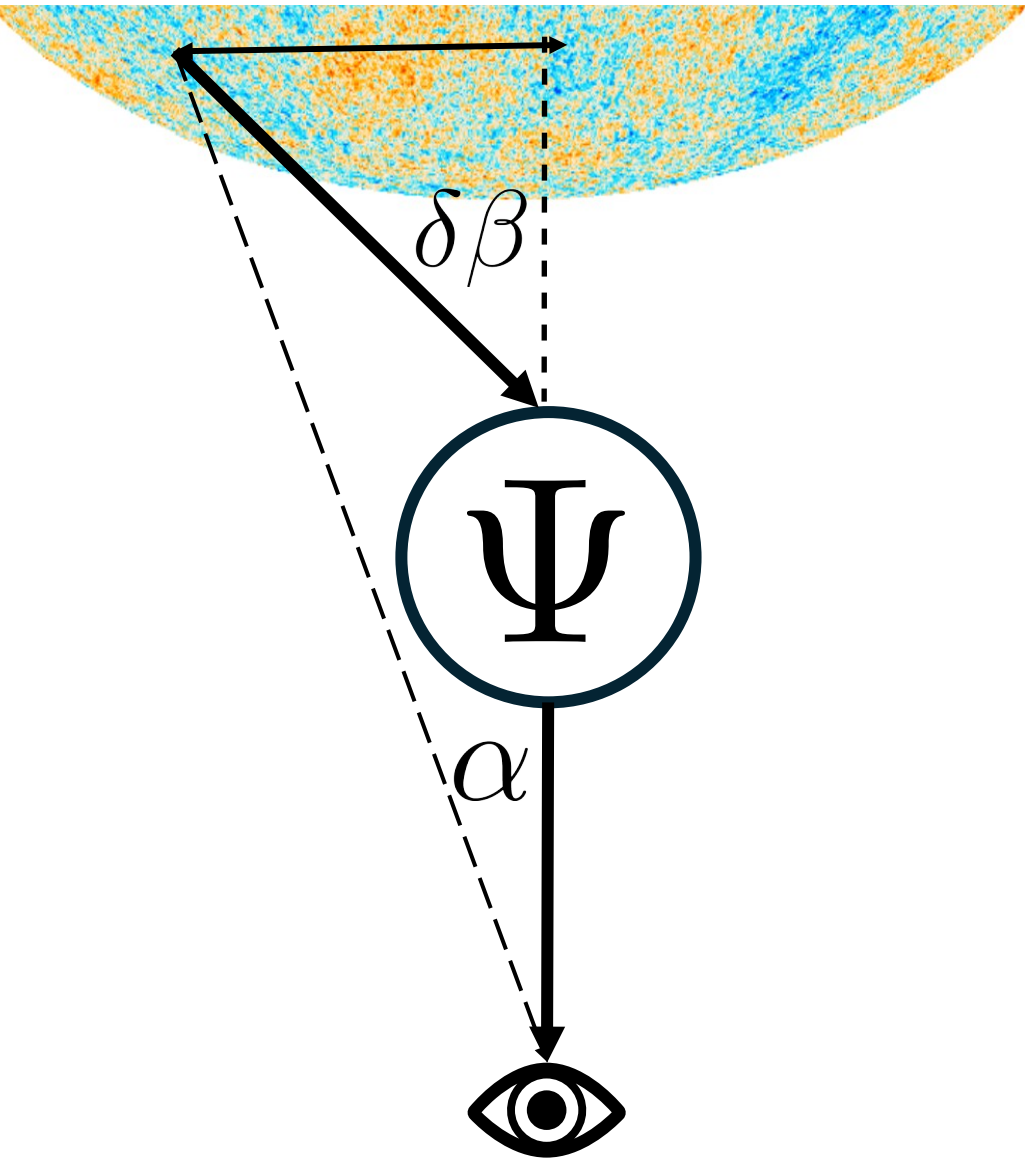


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CMB lensing

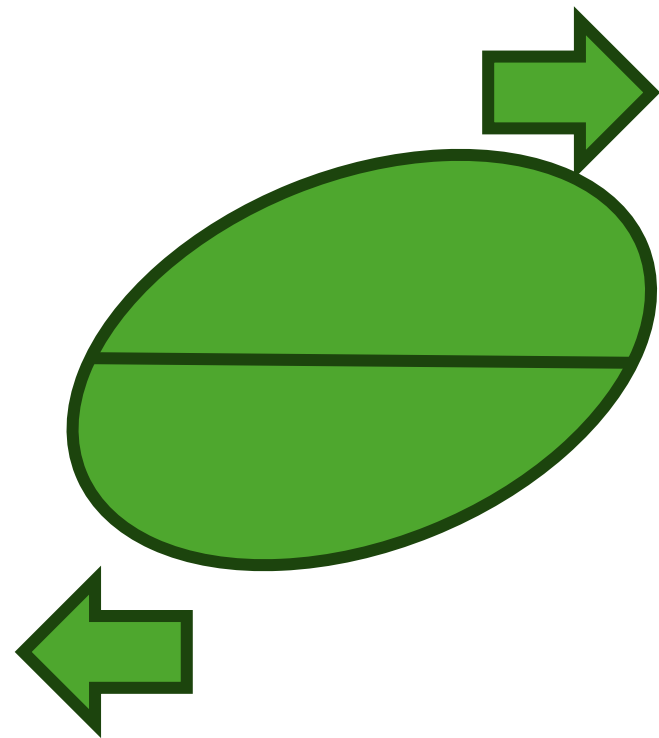


CMB lensing



Convergence

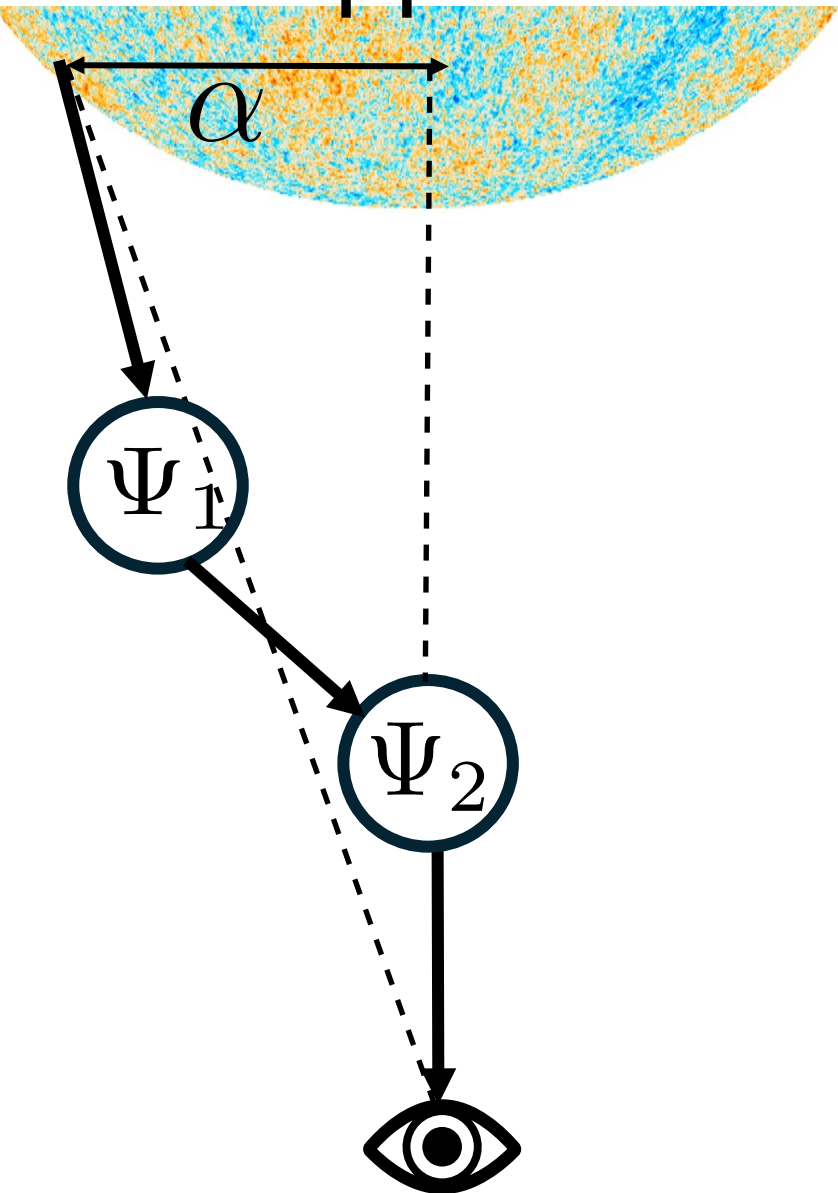
$$\kappa = -\frac{1}{2} \nabla \cdot \alpha$$



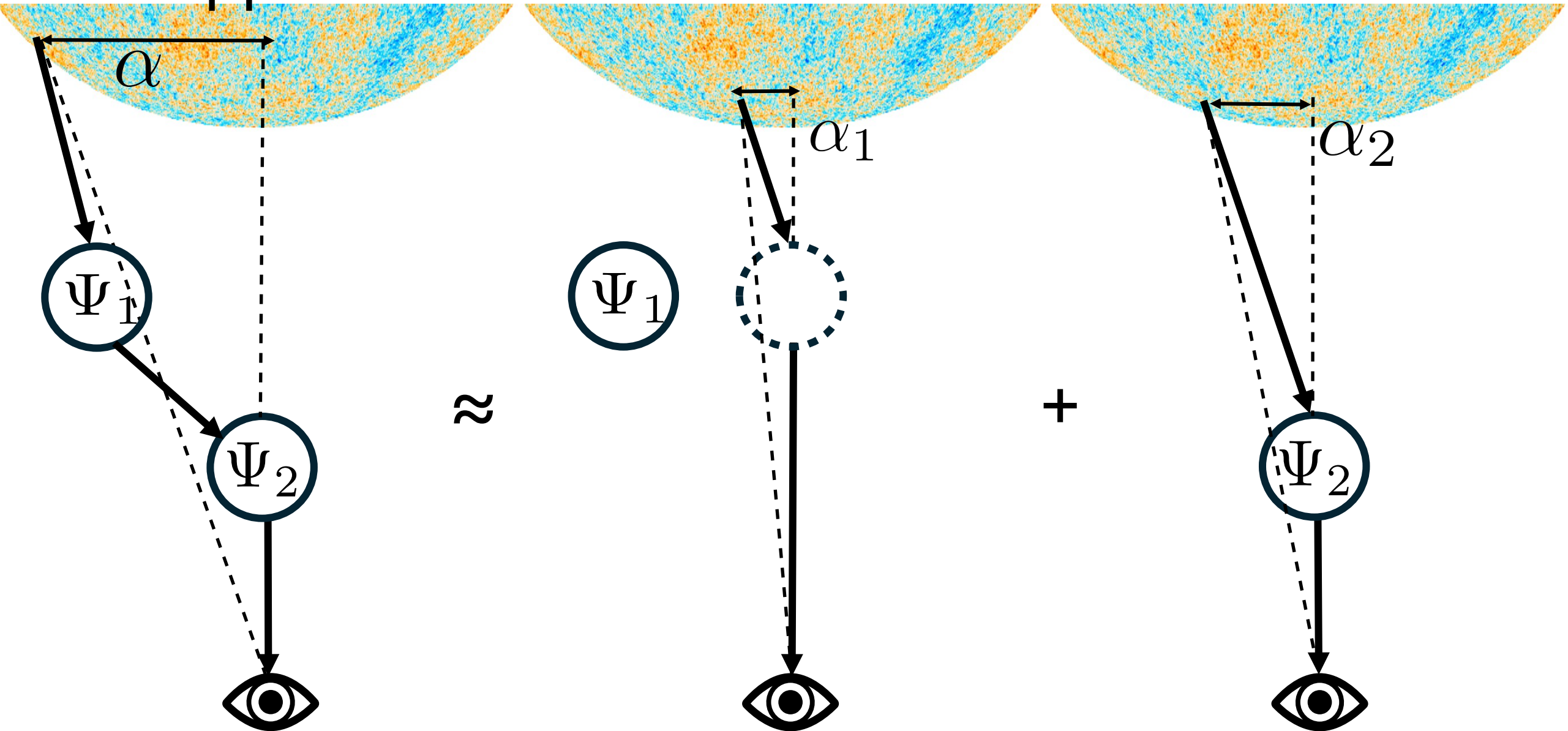
Shear

$$\gamma = \gamma_1 + i\gamma_2$$

Born Approximation



Born Approximation



Post-Born Corrections

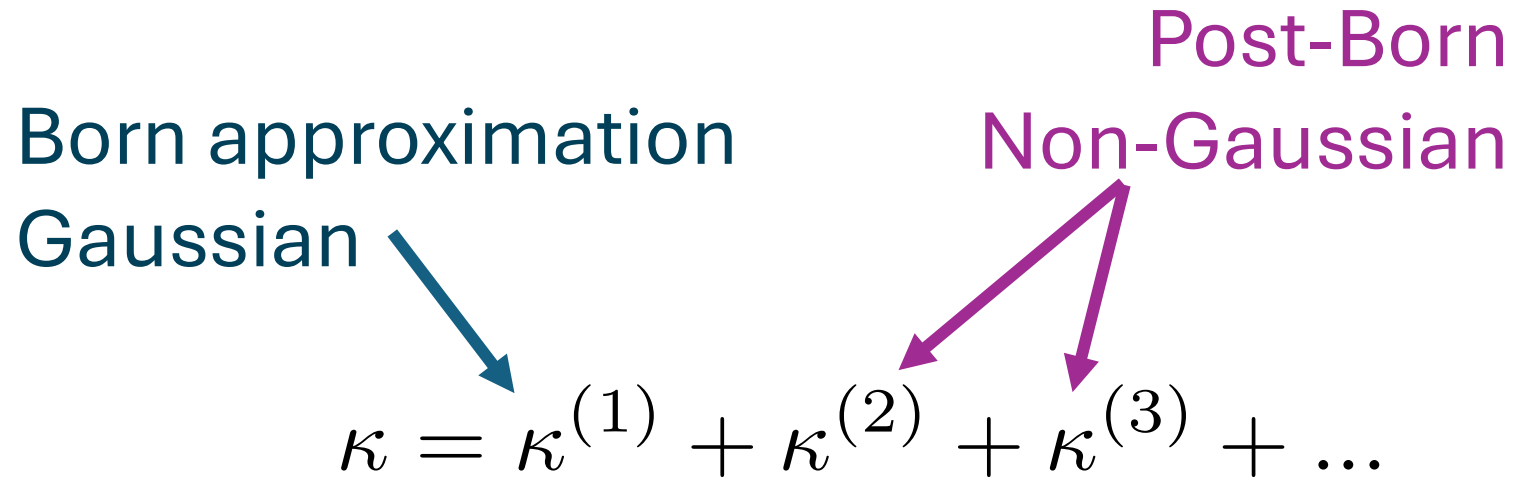
Born approximation
Gaussian


$$\kappa = \kappa^{(1)} + \kappa^{(2)} + \kappa^{(3)} + \dots$$

Post-Born Corrections

Born approximation
Gaussian

Post-Born
Non-Gaussian



The diagram illustrates the expansion of the convergence κ into a series of terms. A blue arrow points from the text 'Born approximation Gaussian' to the first term $\kappa^{(1)}$ in the equation. Two purple arrows point from the text 'Post-Born Non-Gaussian' to the subsequent terms $\kappa^{(2)}$ and $\kappa^{(3)}$ in the equation.

$$\kappa = \kappa^{(1)} + \kappa^{(2)} + \kappa^{(3)} + \dots$$

Post-Born Corrections

Born approximation
Gaussian

Post-Born
Non-Gaussian

$$\kappa = \kappa^{(1)} + \kappa^{(2)} + \kappa^{(3)} + \dots$$

$$\kappa^{(2)} = \kappa^{(2L)} + \kappa^{(2D)}$$

Lens-lens
coupling

Post-Born Corrections

Born approximation
Gaussian

Post-Born
Non-Gaussian

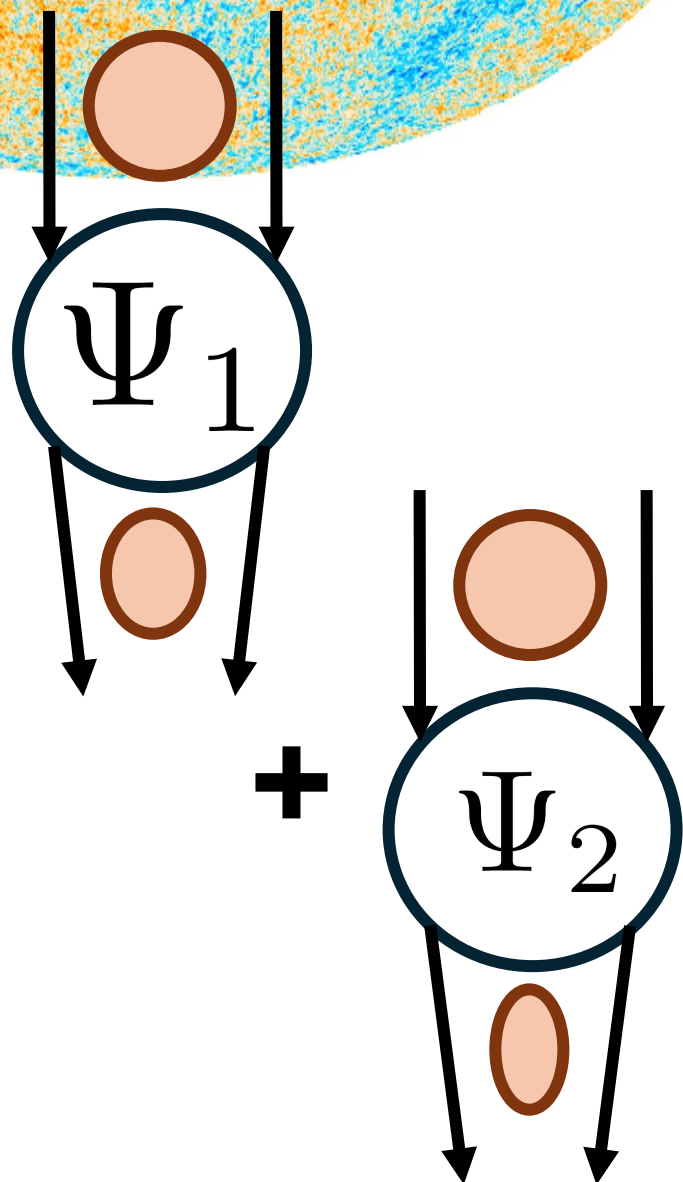
$$\kappa = \kappa^{(1)} + \kappa^{(2)} + \kappa^{(3)} + \dots$$

$$\kappa^{(2)} = \kappa^{(2L)} + \kappa^{(2D)}$$

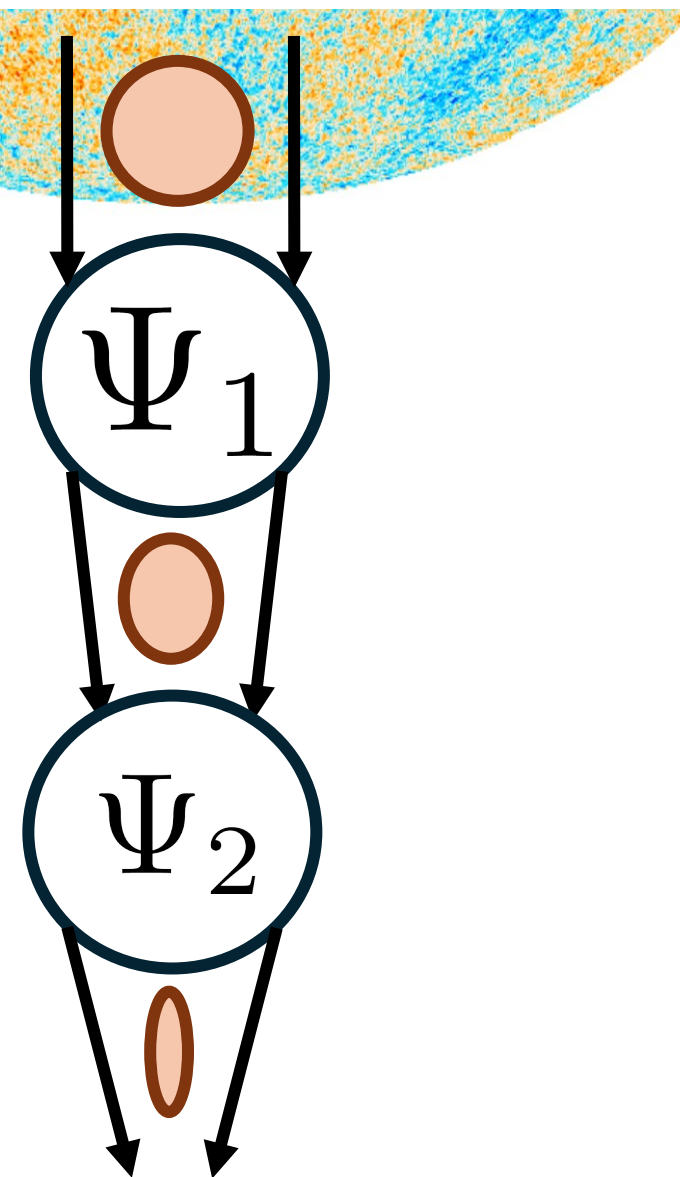
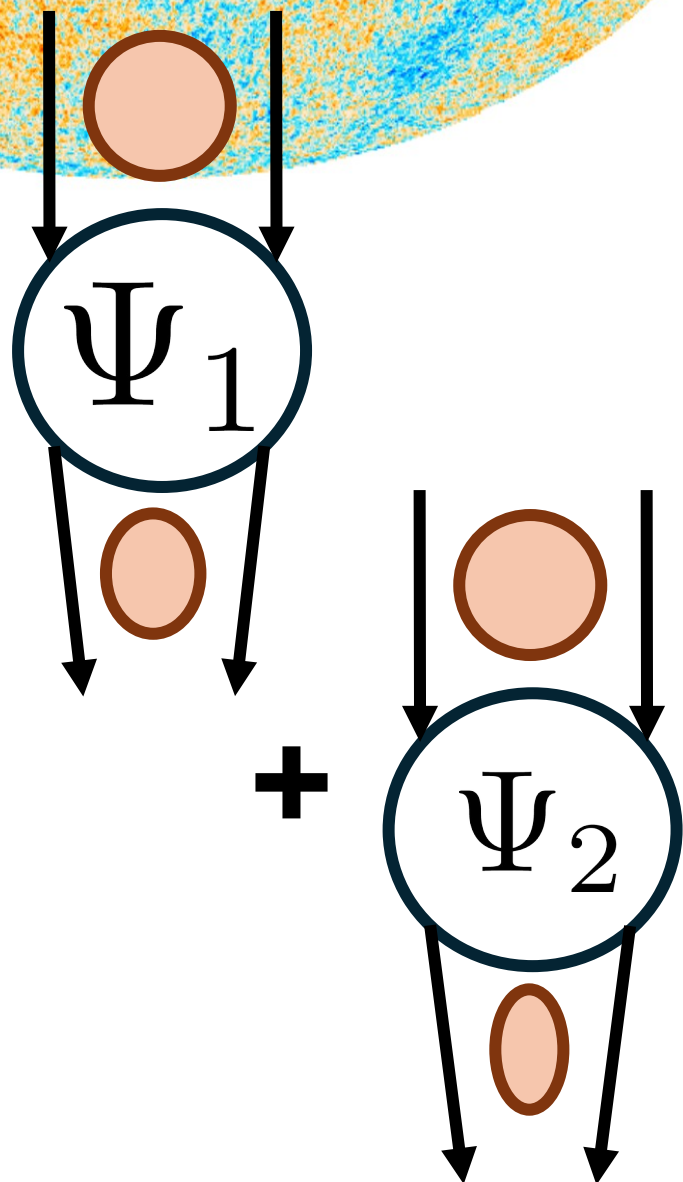
Lens-lens
coupling

Ray deflection

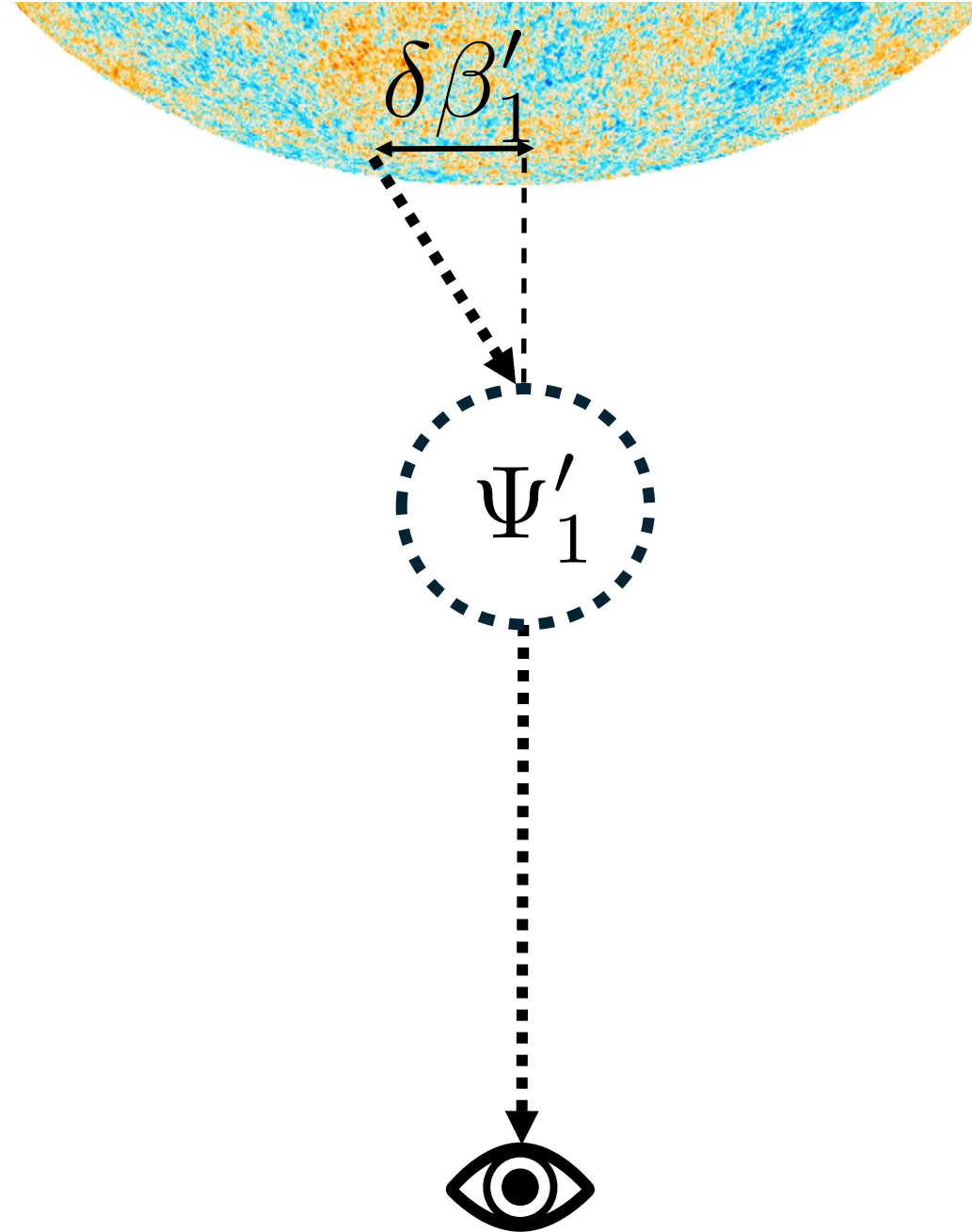
Lens-Lens Coupling



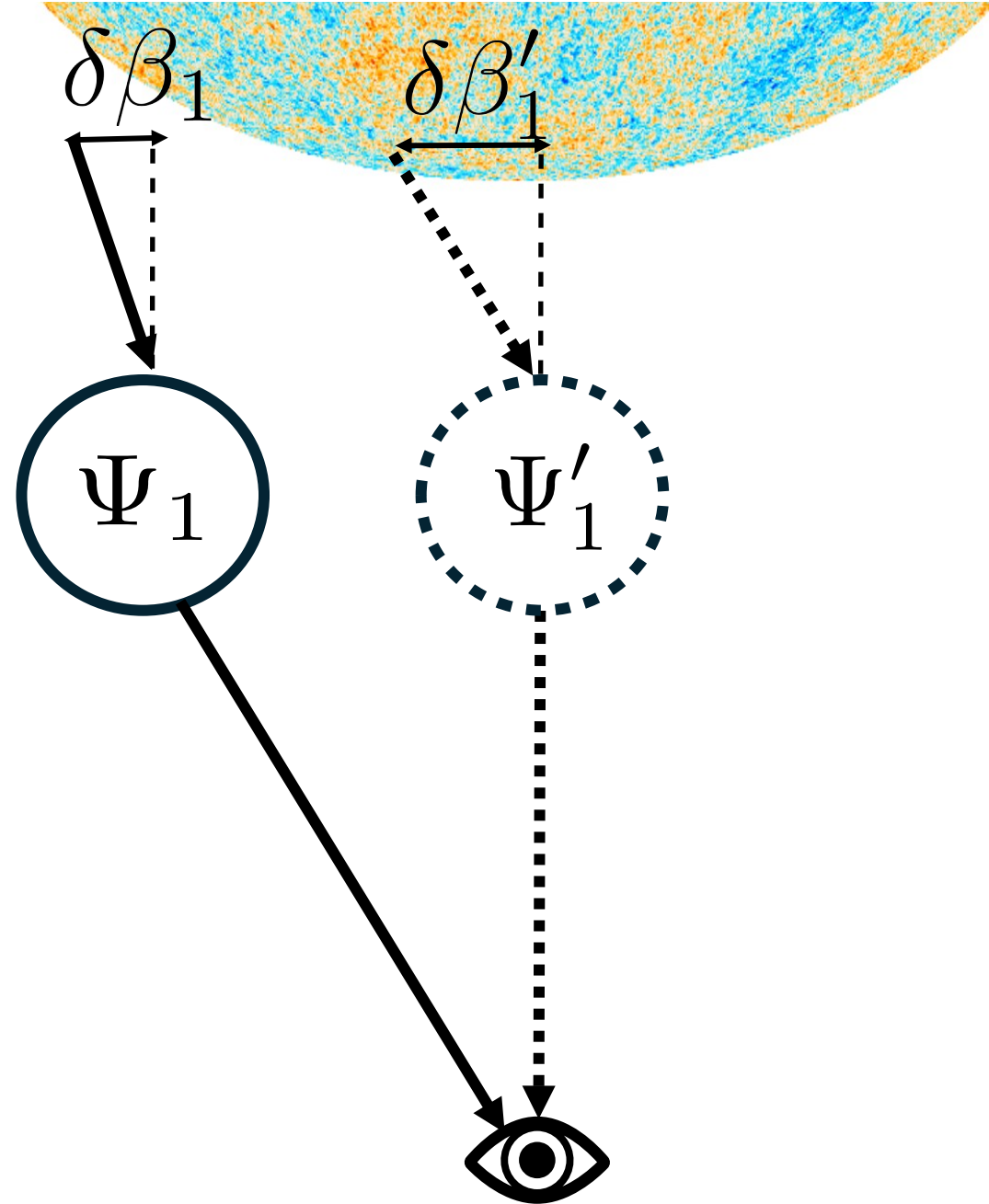
Lens-Lens Coupling



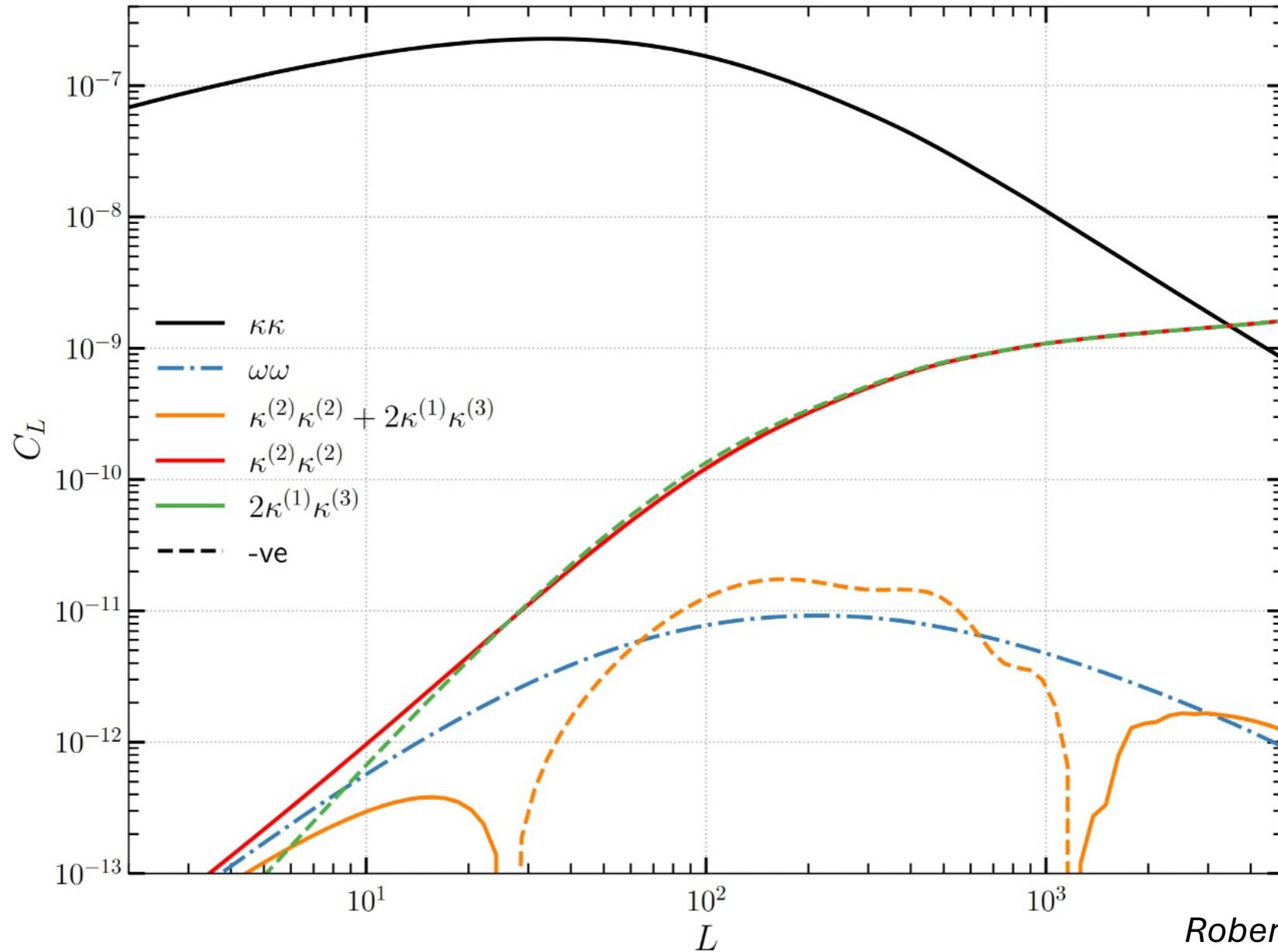
Ray Deflection



Ray Deflection

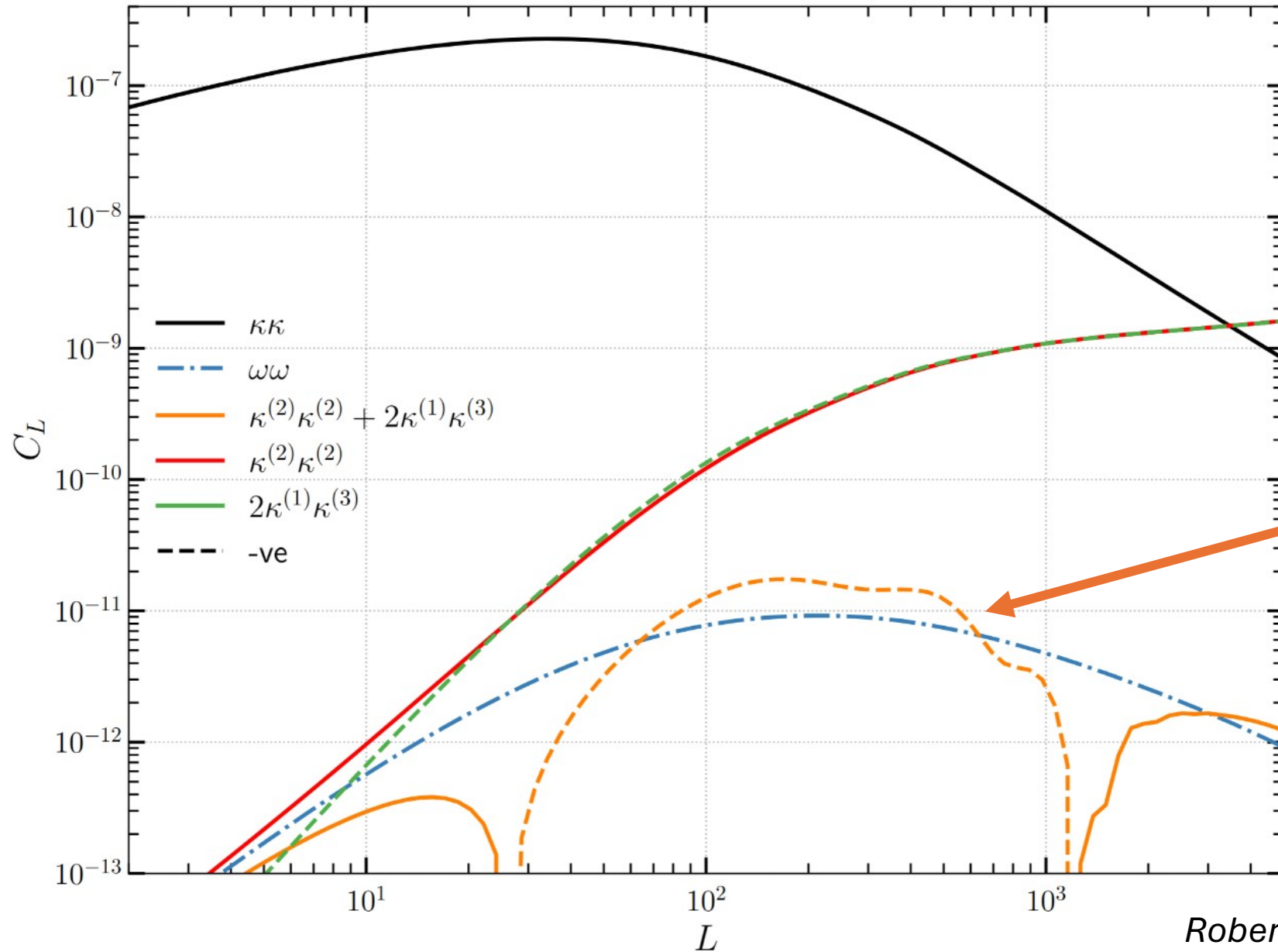


Power Spectra



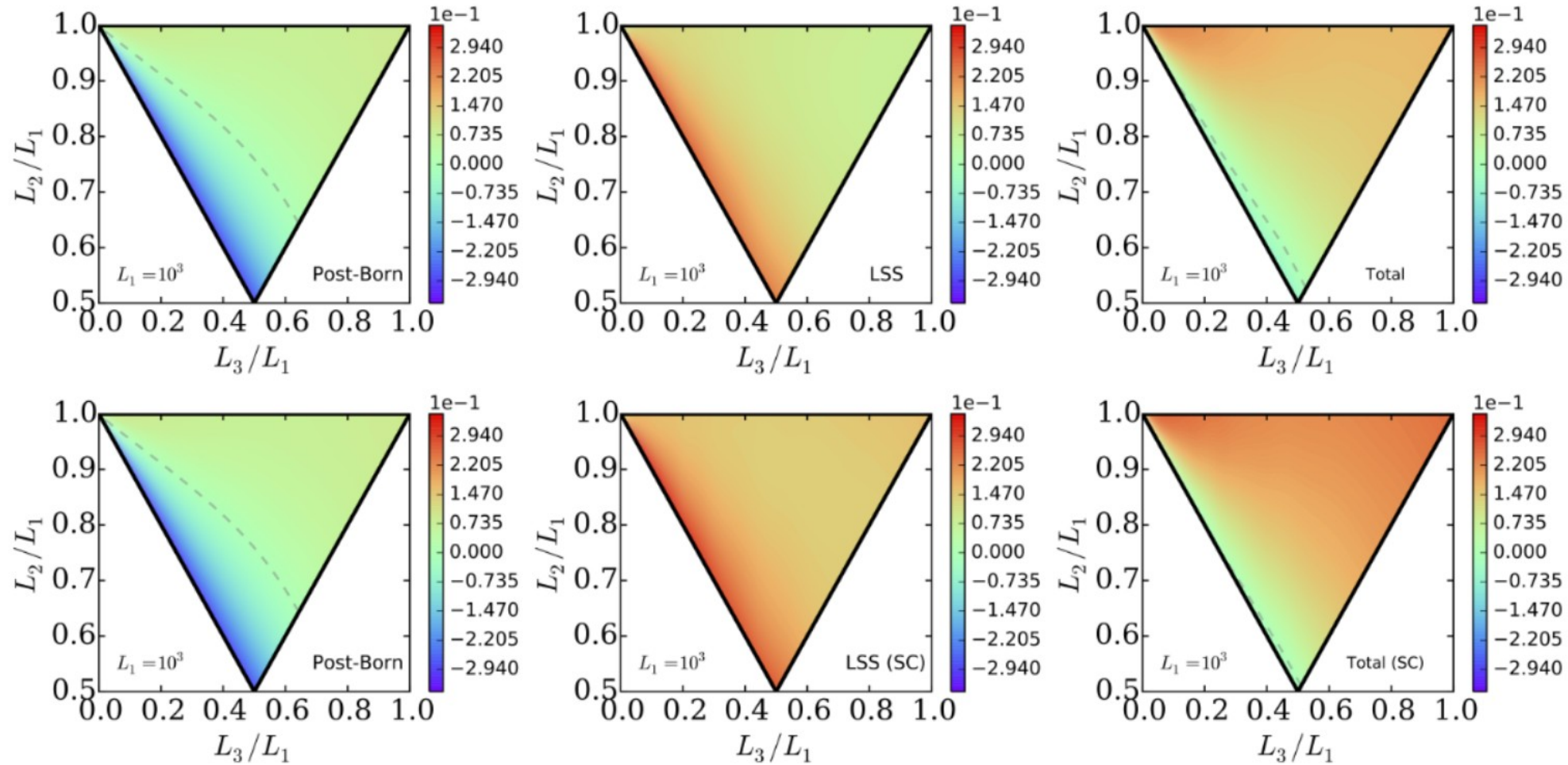
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Power Spectra



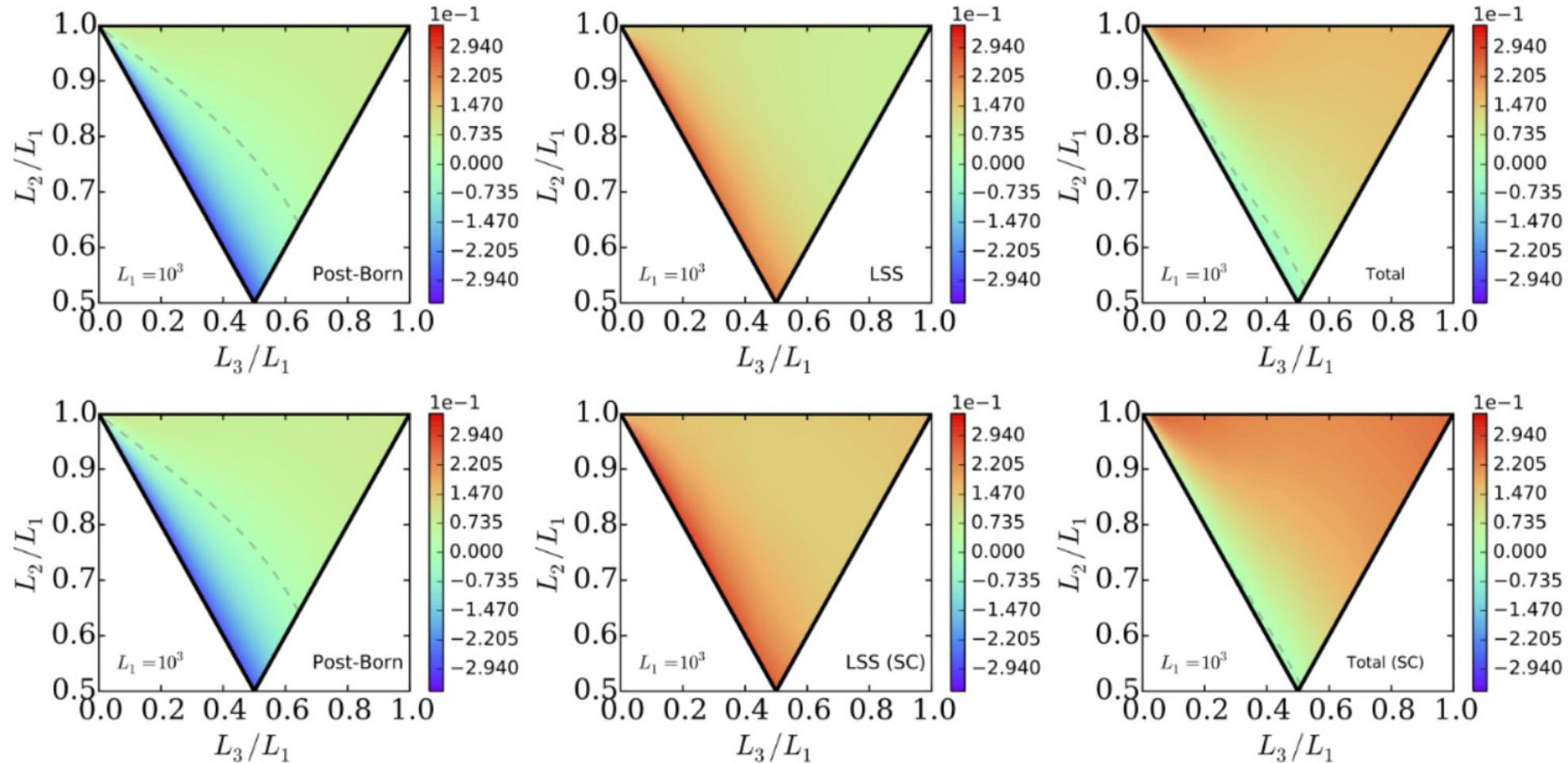
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Bispectra



Bispectra

Post-Born and LSS
bispectra similar size!

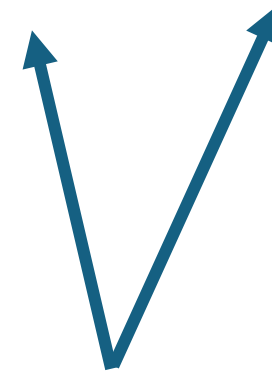


Quadratic Estimator

$$\hat{\phi}(\mathbf{L}) = N(\mathbf{L}) \int \frac{d^2\mathbf{L}'}{(2\pi)^2} G^{XY}(\mathbf{L}', \mathbf{L} - \mathbf{L}') \tilde{X}(\mathbf{L}') \tilde{Y}^*(\mathbf{L} - \mathbf{L}')$$

Quadratic Estimator

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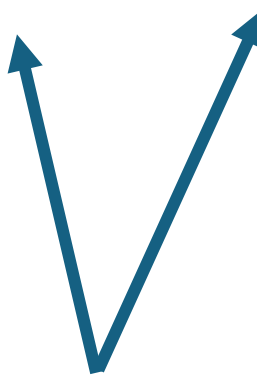
Tracer fields, off
diagonal
elements

Quadratic Estimator

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Weight function



Tracer fields, off
diagonal
elements

Quadratic Estimator

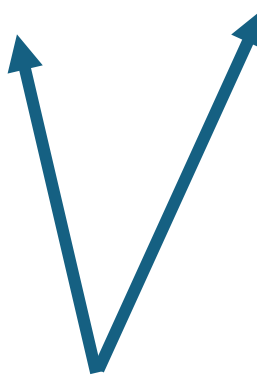
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Normalization



Weight function



Tracer fields, off
diagonal
elements

$$\hat{\kappa}^{\text{tem}}(\mathbf{L}) = \frac{F_L^{-1}}{2} \int \frac{d^2 \mathbf{L}_1}{(2\pi)^2} b_{LL_1 L_2}^{\kappa^{(2)} ij, \text{pB}} \bar{a}^i(\mathbf{L}_1) \bar{a}^j(\mathbf{L}_2)$$

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Inverse covariance
weighted tracers

Kappa (2) Template

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$$\hat{\kappa}^{\text{tem}}(\mathbf{L}) = \frac{F_L^{-1}}{2} \int \frac{d^2 \mathbf{L}_1}{(2\pi)^2} b_{LL_1 L_2}^{\kappa^{(2)ij, \text{pB}}} \bar{a}^i(\mathbf{L}_1) \bar{a}^j(\mathbf{L}_2)$$

Post-Born
bispectrum



Inverse covariance
weighted tracers



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Fisher matrix

Post-Born
bispectrum

Inverse covariance
weighted tracers

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Inverse covariance
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Bispectrum!

$$\langle C_L^{\hat{\kappa}^{\text{tem}} \kappa^{(2)}} \rangle = C_L^{\kappa^{(2)} \kappa^{(2)}}$$

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$$\langle C_L^{\hat{\kappa}^{\text{tem}} \kappa^{(2)}} \rangle = C_L^{\kappa^{(2)} \kappa^{(2)}}$$

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Bispectrum!

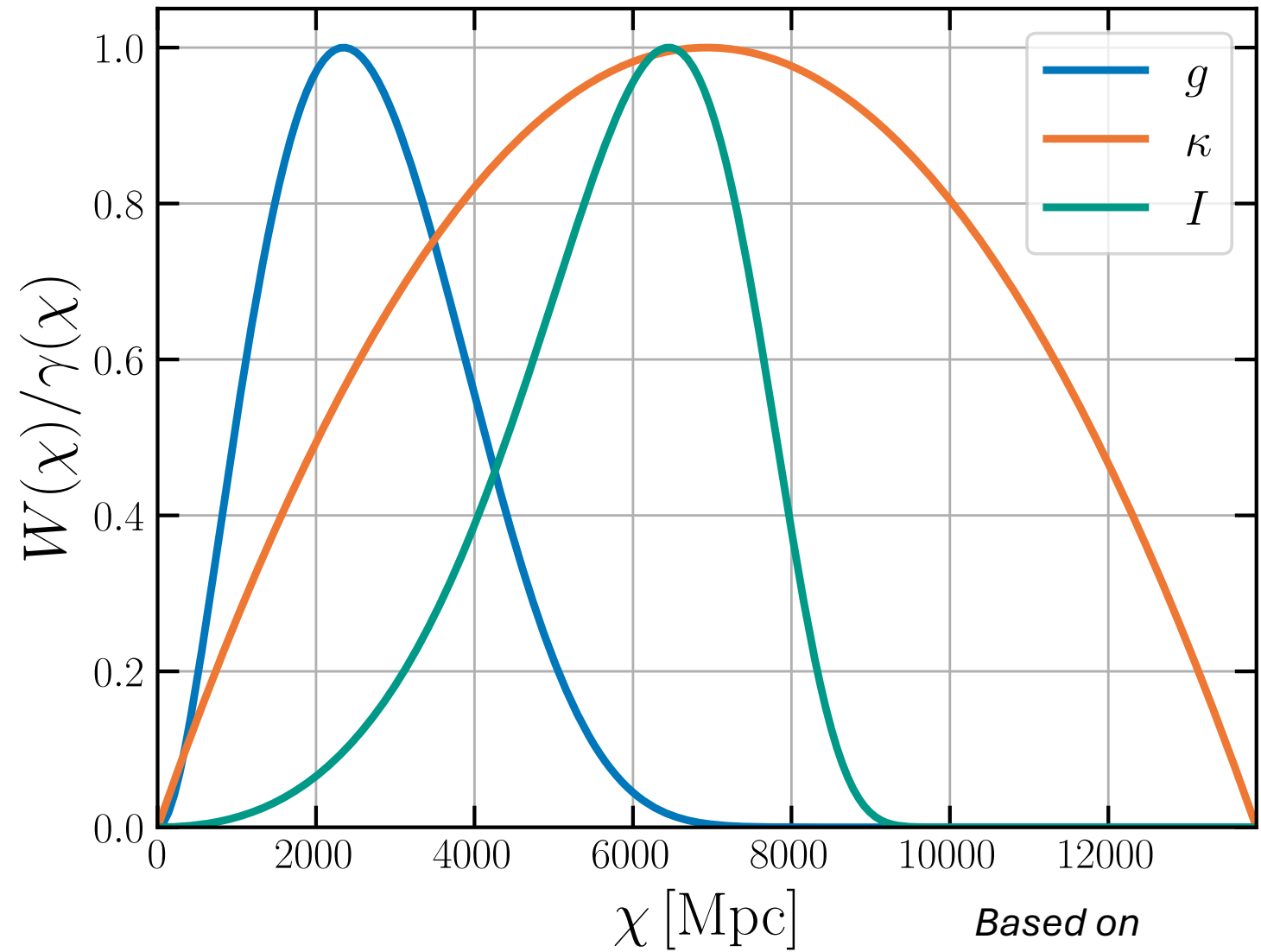
$$\langle C_L^{\hat{\kappa}^{\text{tem}} \kappa^{(2)}} \rangle = C_L^{\kappa^{(2)} \kappa^{(2)}}$$

$$\langle C_L^{\hat{\kappa}^{\text{tem}} \hat{\kappa}^{\text{tem}}} \rangle = F_L^{-1} C_L^{\kappa^{(2)} \kappa^{(2)}}$$

$$\langle C_L^{\hat{\kappa}^{\text{tem}} \kappa} \rangle = C_L^{\kappa^{(2)} \kappa^{(2)}} + N_L^{\text{pB}} + N_L^{\tilde{\delta}} + N_L^{\text{LSS}}$$

Tracer Fields

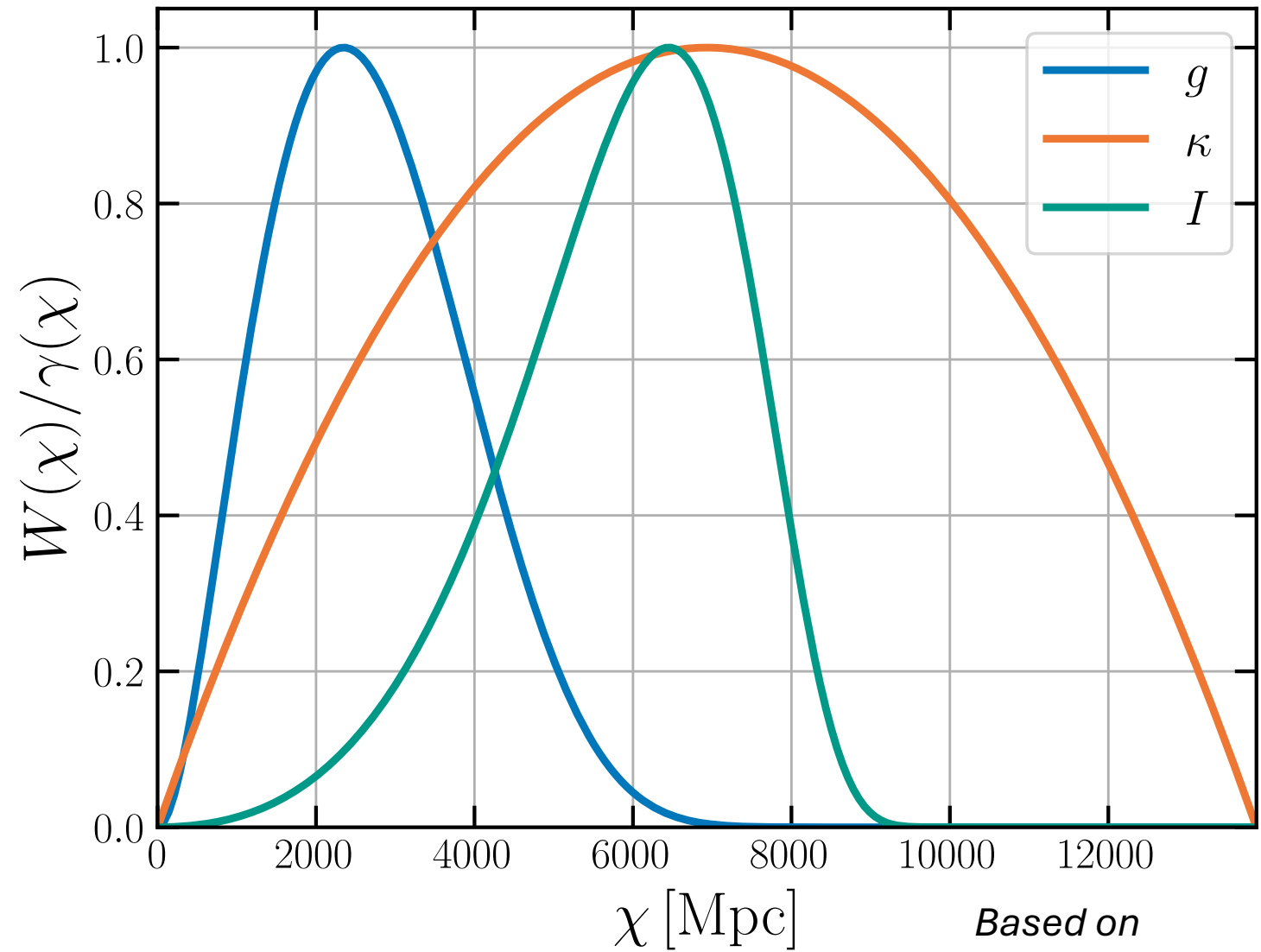
- CMB lensing convergence κ
 - ACT DR6
 - $40 \leq L \leq 2\,000$



*Based on
Robertson 2023*

Tracer Fields

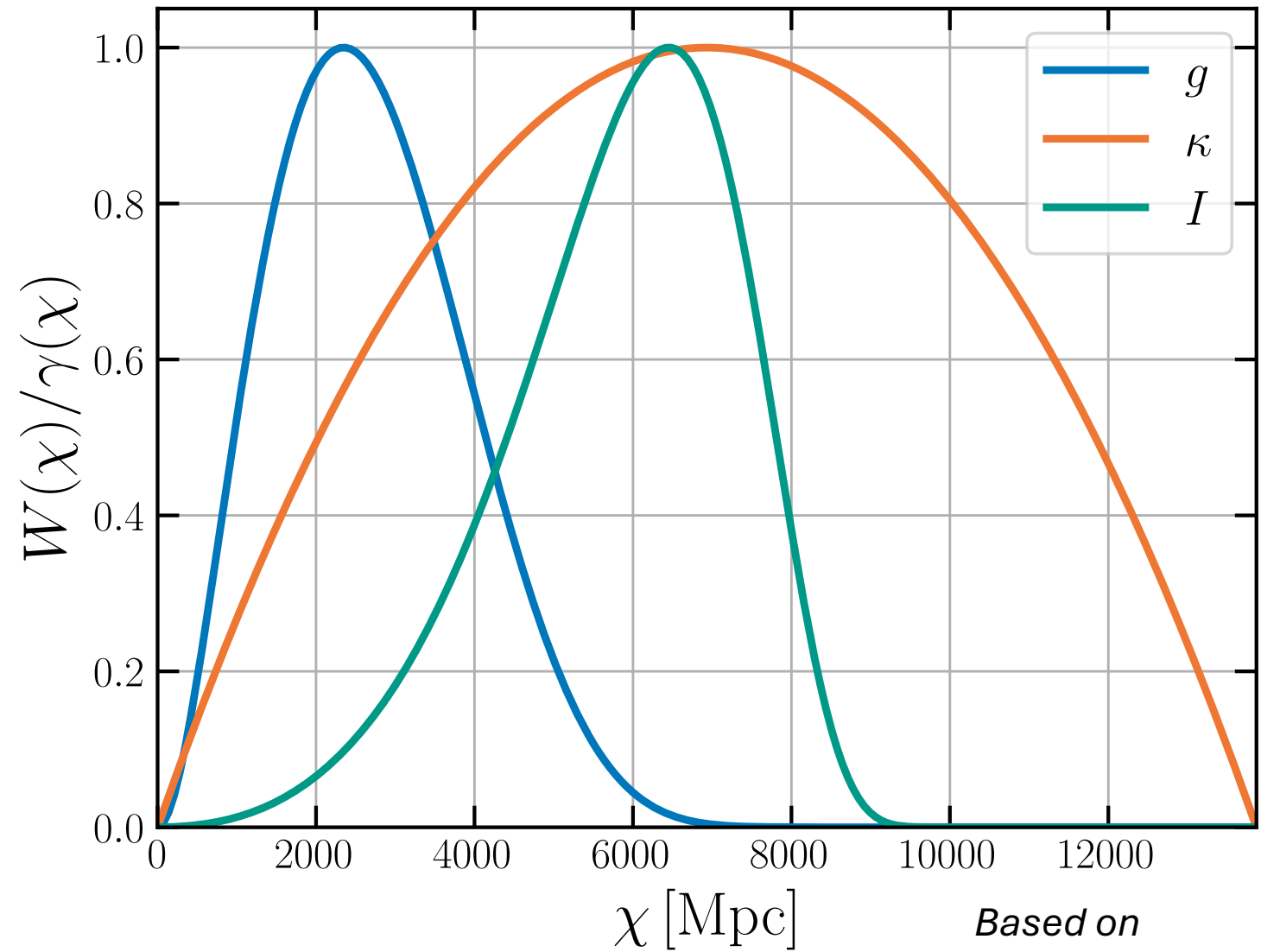
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 - $40 \leq L \leq 2\,000$
- Galaxy count g
 - unWISE
 - $100 \leq L \leq 2\,000$



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Tracer Fields

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- Cosmic infrared background I
 - *Planck* 353 GHz
 - $110 \leq L \leq 2\,000$

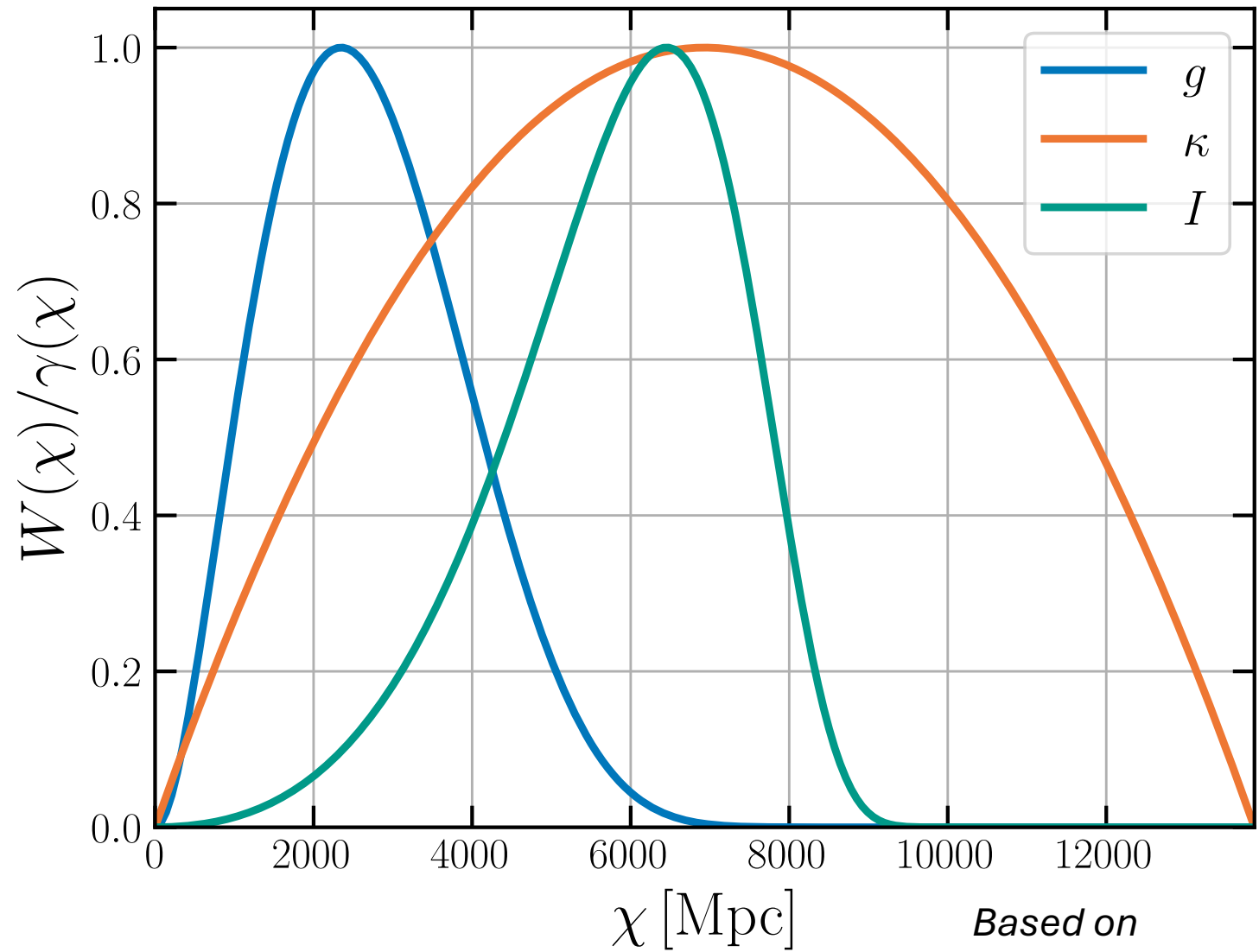


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Tracer Fields

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$$a(\mathbf{L}) = \int d\chi W_a(\chi) \delta(\mathbf{L}, \chi)$$



*Based on
Robertson 2023*

Is this detectable?

S/N	$\{\kappa\} [\sigma]$	$\{g\} [\sigma]$	$\{I\} [\sigma]$	$\{\kappa, g\} [\sigma]$	$\{\kappa, I\} [\sigma]$	$\{g, I\} [\sigma]$	$\{\kappa, g, I\} [\sigma]$
$\kappa^{\text{tem}} \kappa^{(2)}$	<u>6.3</u>	8.6	4.5	12	18	28	29
$\kappa^{\text{tem}} \kappa$	0.6	8.4	4.5	6.3	15	27	25
$\kappa^{\text{tem}} \kappa$ (inc. $\tilde{\delta}$)	0.6	0.4	0.8	1.2	0.7	0.7	1.0
$\kappa^{\text{tem}} \Delta\kappa$ (inc. $\tilde{\delta}$)	2.2	2.5	1.7	4.1	5.0	7.5	8.0

Robertson 2025

Is this detectable?

S/N	$\{\kappa\} [\sigma]$	$\{g\} [\sigma]$	$\{I\} [\sigma]$	$\{\kappa, g\} [\sigma]$	$\{\kappa, I\} [\sigma]$	$\{g, I\} [\sigma]$	$\{\kappa, g, I\} [\sigma]$
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Robertson 2025

Yes!

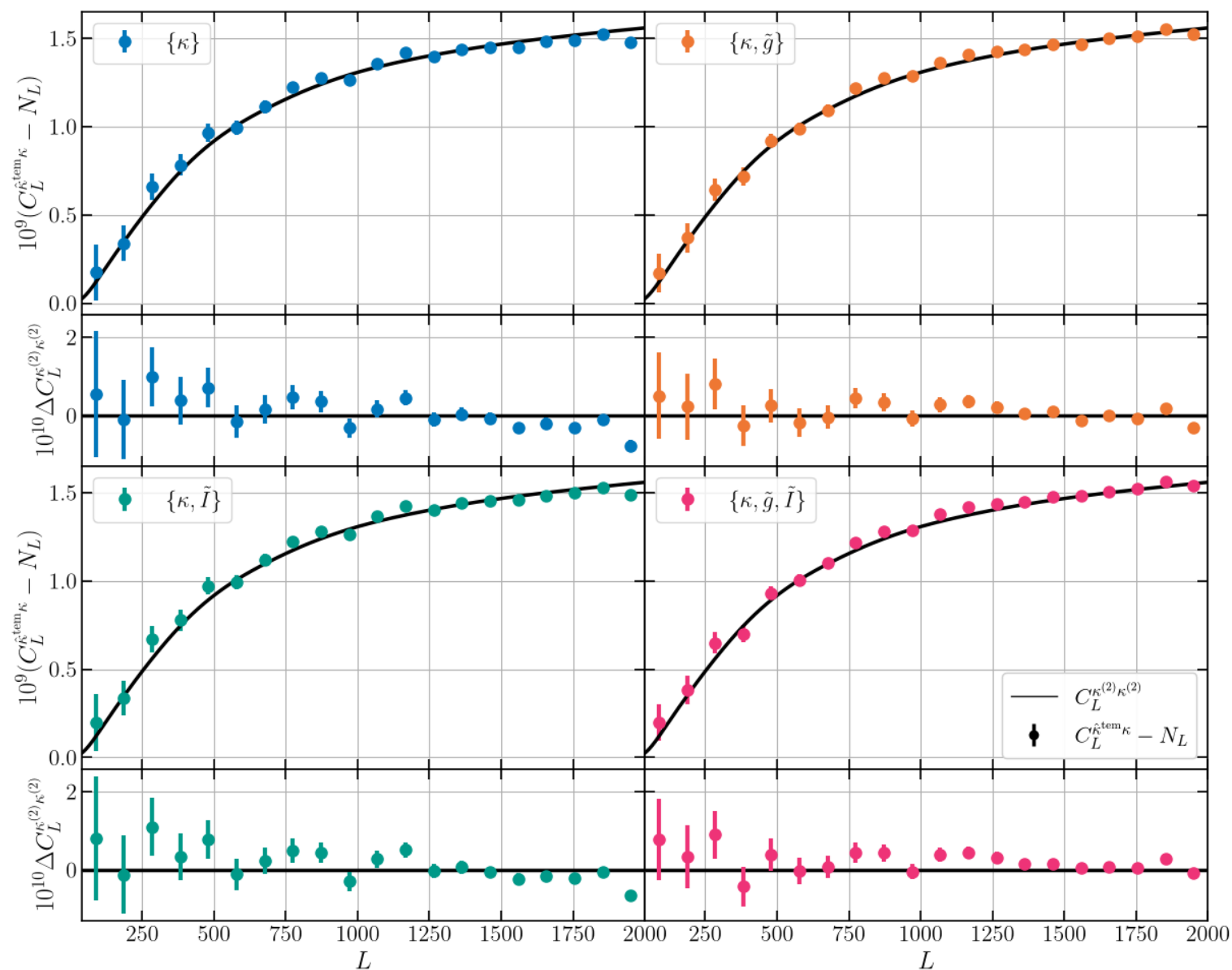
DEMNUni (*Carbone+ 2016*)

- **Dark Energy and Massive Neutrino Universe**
 - N -body simulation suite.
 - Initial redshift $z_{\text{in}} = 99$.
 - Comoving volume $(2 h^{-1} \text{ Gpc})^3$.
 - 2048^3 dark matter particles.

DEMNUi (*Carbone+ 2016*)

- **Dark Energy and Massive Neutrino Universe**
 - N -body simulation suite.
 - Initial redshift $z_{\text{in}} = 99$.
 - Comoving volume $(2 h^{-1} \text{ Gpc})^3$.
 - 2048^3 dark matter particles.
- Ray-traced κ map
- g and l maps created from δ shells.
- g and l maps lensed with κ shells $\rightarrow \tilde{g}, \tilde{l}$

Results from N -body sims



- Results from simulations match theory well
- General increase in power on large scales, reduction on small scales

Next steps

- Apply simulated noise to signal maps.
- Construct template with a reconstructed $\hat{k}$ from QE.
 - New bias bispectra:

$$N_{\hat{k}\hat{k}}^{(0)}, N_{\hat{k}}^{(1)}, N_{A_1}^{(2)}, N_{C_1}^{(2)}$$

- Account for other foregrounds (e.g.) tSZ.
- Construct template with real data and get a detection.

- Weak lensing is approximated along the LoS using the Born approximation.
- To account for coupled lensing events we apply post-Born corrections.
- We outline a method of detecting these post-Born corrections and detect the post-Born signal in an N -body simulation .
- After applying the method to reconstructed lensing fields, we will be ready to construct the template from real data.