Spectral decomposition in stochastic inflation

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BASED ON ARXIV:2606.09690

Stochastic inflation

Starobinsky 1986

Inflaton coarse-grained over a Hubble patch:
classical motion + quantum jumps

Access to cosmological super-Hubble perturbations
beyond linear perturbation theory

Field motion

Slow-roll

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2(\phi)}$$

Field motion

Slow-roll

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2(\phi)} + \frac{H(\phi)}{2\pi}\xi(N)$$

$$\langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

Field motion

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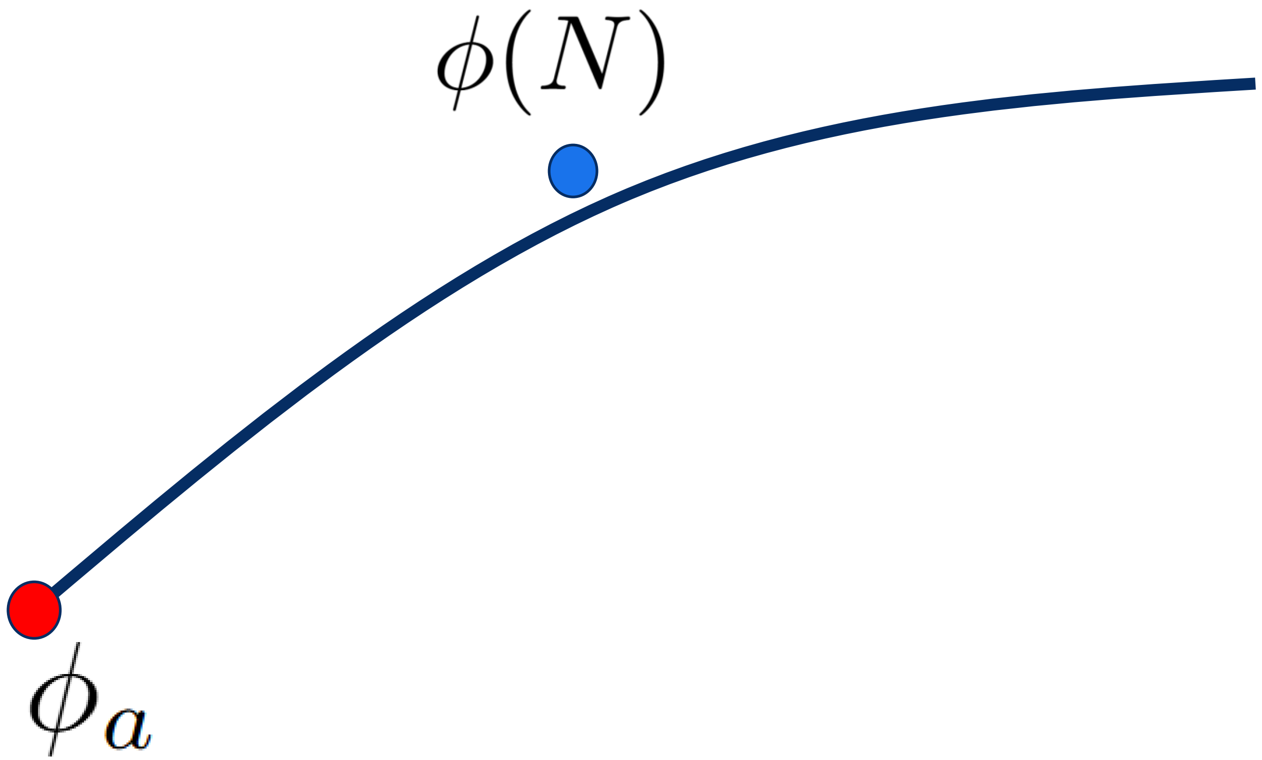
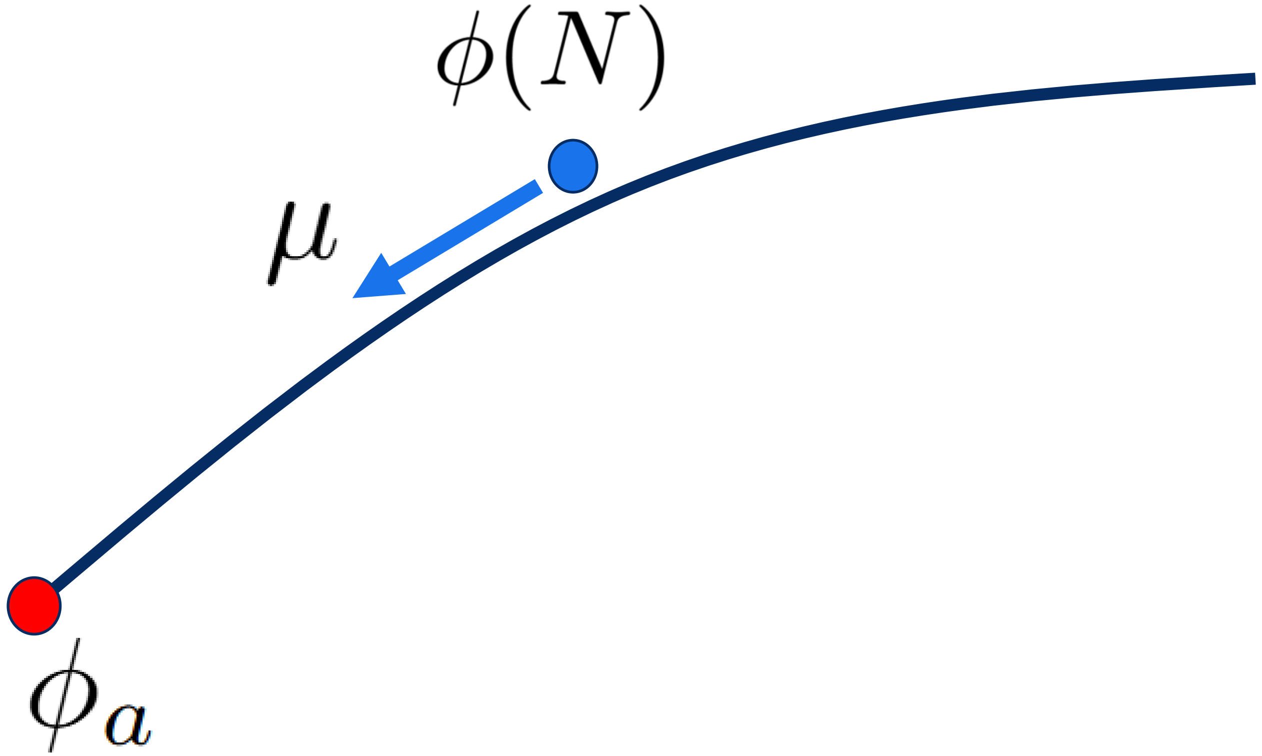
General attractor

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = \mu(\phi) + \sigma(\phi)\xi(N)$$

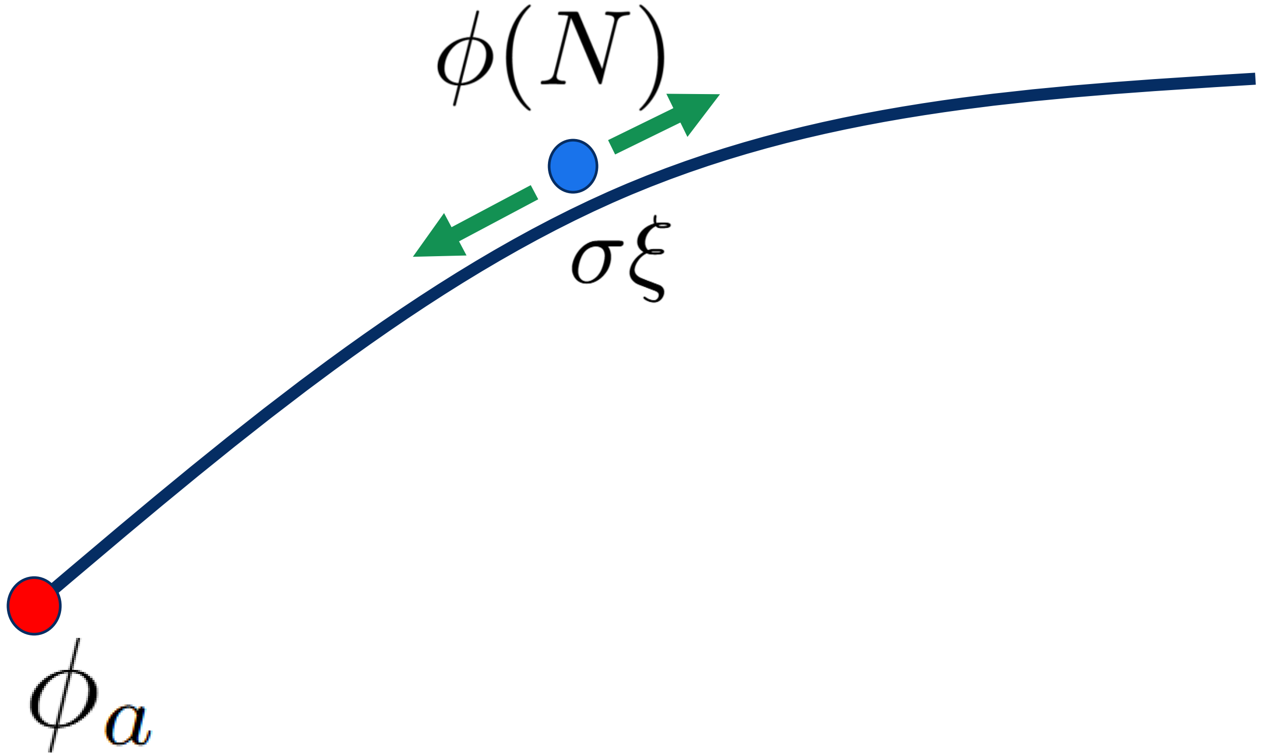
Field motion



Field motion



Field motion



Field distribution

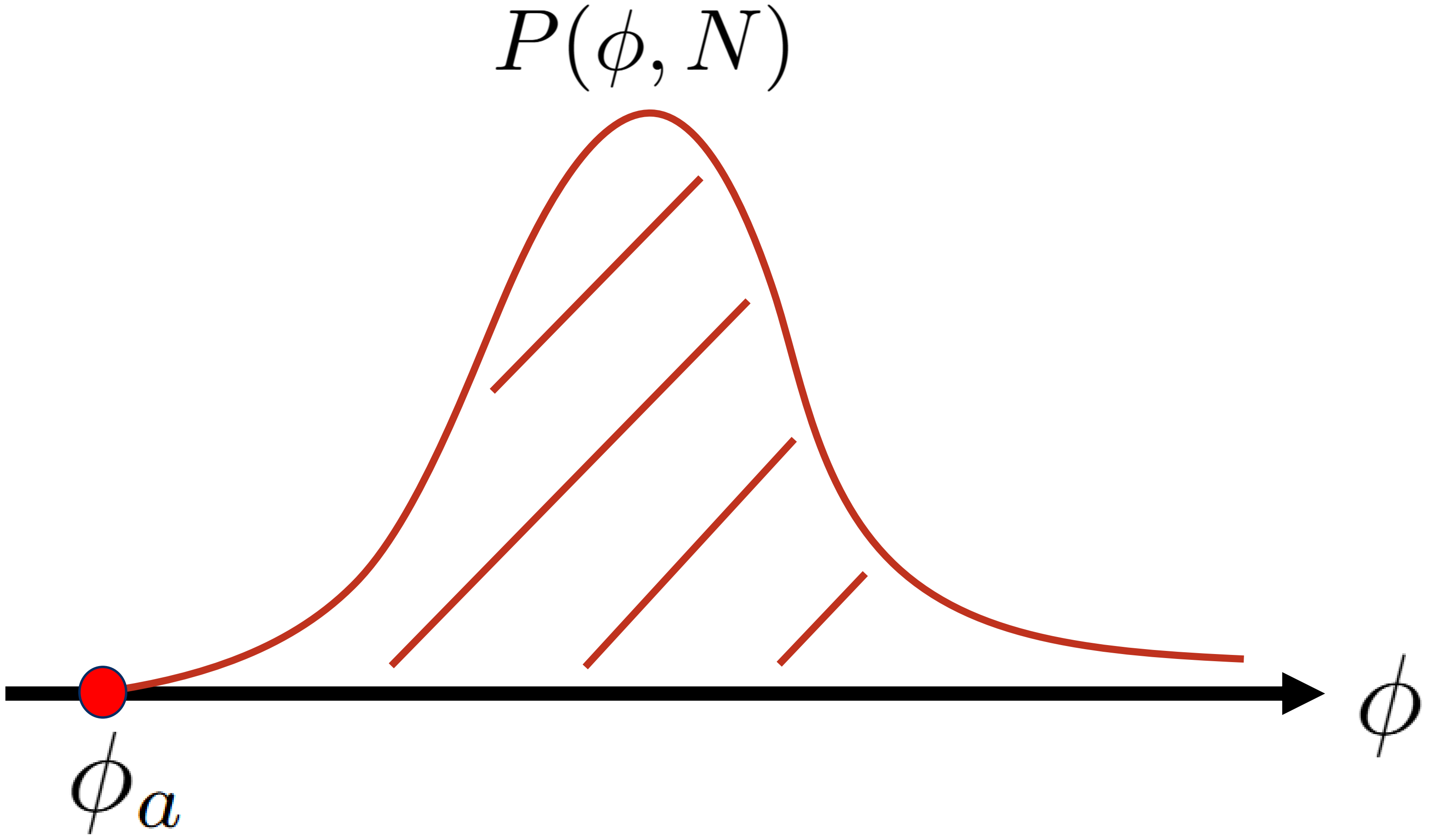
Fokker-Planck

$$\partial_N P(\phi, N) = \underbrace{\partial_\phi \left[\partial_\phi \left(\frac{1}{2}\sigma^2(\phi) P(\phi, N)\right) - \mu(\phi) P(\phi, N)\right]}_{\equiv \mathcal{L}_{\mathrm{FP},\phi} P(\phi, N)}$$

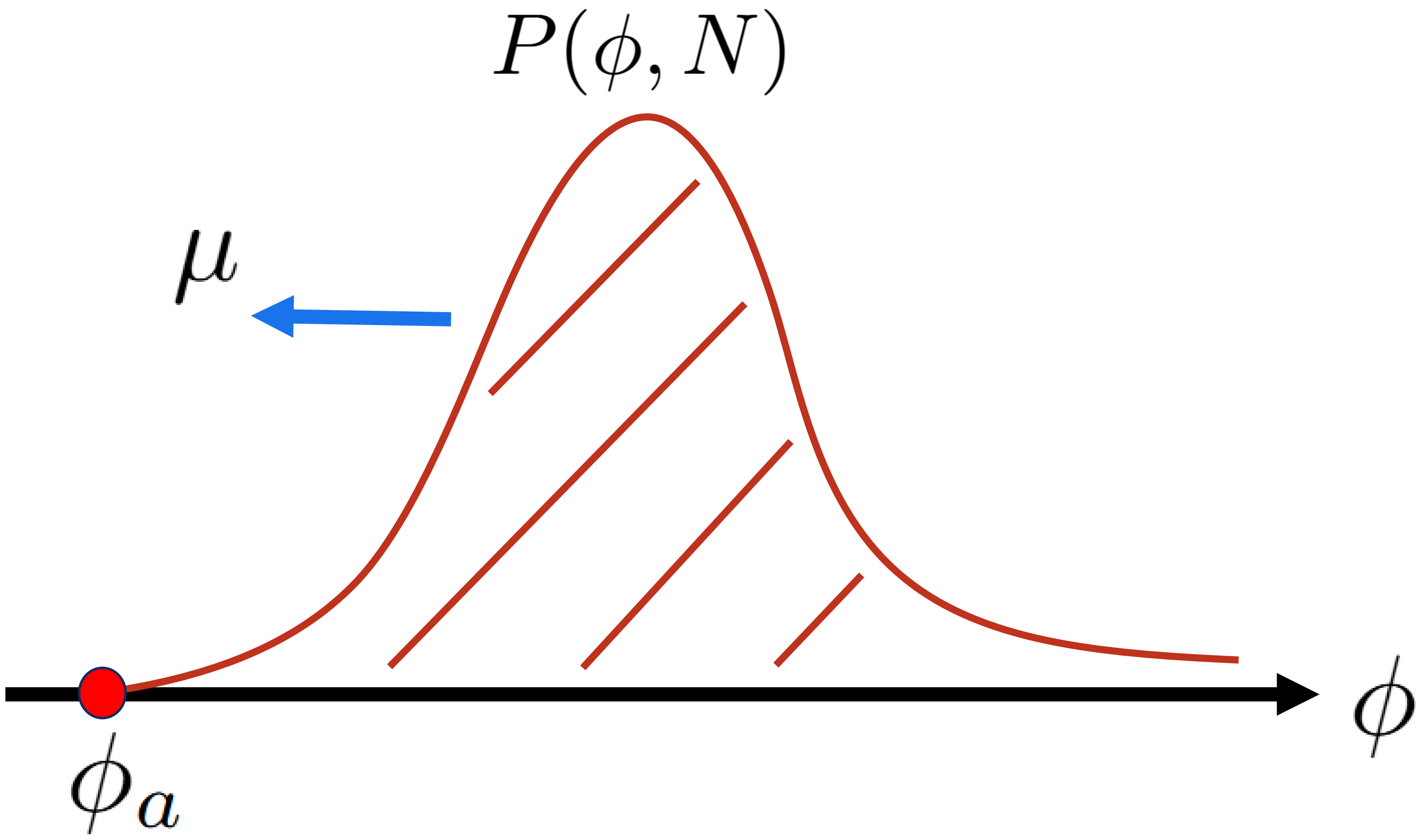
Adjoint Fokker-Planck

$$\partial_N P_{\mathrm{FPT}}(N, \phi_0) = \underbrace{\frac{1}{2}\sigma^2(\phi_0)\partial^2_{\phi_0} P_{\mathrm{FPT}}(N, \phi_0) + \mu(\phi_0)\partial_{\phi_0} P_{\mathrm{FPT}}(N, \phi_0)}_{\equiv \mathcal{L}^\dagger_{\mathrm{FP},\phi_0} P_{\mathrm{FPT}}(N, \phi_0)}$$

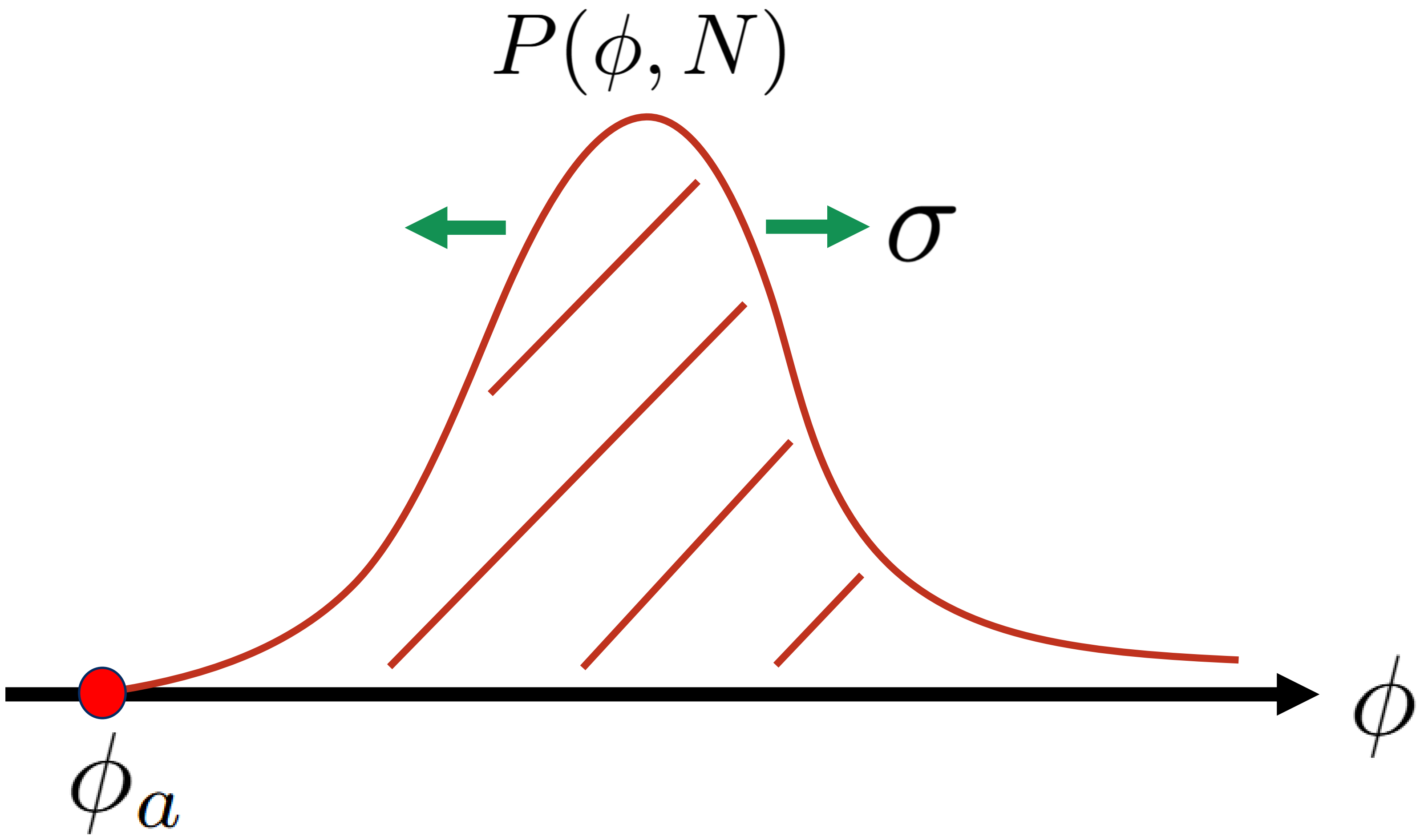
Field distribution



Field distribution



Field distribution



Spectral decomposition

Mishra et al, 2026 [2606.00176]
Tomberg 2026 [2606.09690]

$$\mathcal{L}_{\mathrm{FP},\phi} u_n(\phi) = -\lambda_n u_n(\phi)$$

$$\mathcal{L}^\dagger_{\mathrm{FP},\phi} \bar{u}_n(\phi) = -\lambda_n \bar{u}_n(\phi)$$

$$\bar{u}_n(\phi) = w(\phi) u_n(\phi)$$

$$w(\phi) \equiv \sigma^2(\phi) \exp\left(-\int^\phi \frac{2\mu(\tilde{\phi})}{\sigma^2(\tilde{\phi})} \mathrm{d}\tilde{\phi} \right)$$

$$\int_{\phi_{\mathrm{low}}}^{\phi_{\mathrm{high}}} \bar{u}_i(\phi) u_j(\phi) \mathrm{d}\phi = \delta_{ij}$$

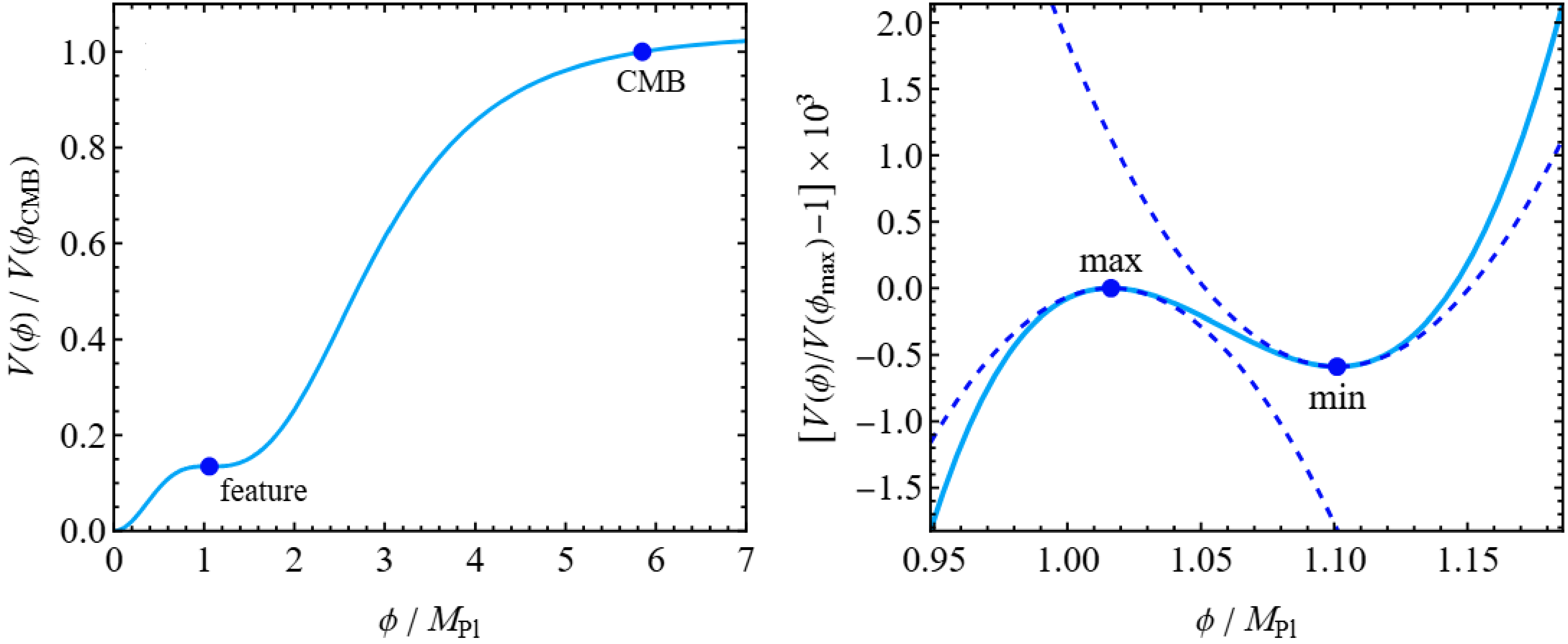
$$\sum_{n=1}^{\infty} \bar{u}_n(\tilde{\phi}) u_n(\phi) = \delta(\tilde{\phi} - \phi)$$

Spectral decomposition

$$P(\phi, N|\phi_0) = \sum_{n=1}^{\infty} \bar{u}_n(\phi_0) u_n(\phi) e^{-\lambda_n N}$$

$$P_{\mathrm{FPT}}(N, \phi_0) = \sum_{n=1}^{\infty} \bar{u}_n(\phi_0) \frac{1}{2} \sigma^2(\phi_a) \partial_{\phi_a} u_n(\phi_a) e^{-\lambda_n N}$$

PBH models



1.0
0.8
0.6
0.4
0.2
0.0
$V(\phi)\ /\ V(\phi_{\mathrm{CMB}})$
CMB
feature
0
1
2
3
4
5
6
7
$\phi\ /\ M_{\mathrm{Pl}}$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
$[V(\phi)/V(\phi_{\mathrm{max}})-1] \times 10^3$
max
min
0.95
1.00
1.05
1.10
1.15
$\phi\ /\ M_{\mathrm{Pl}}$

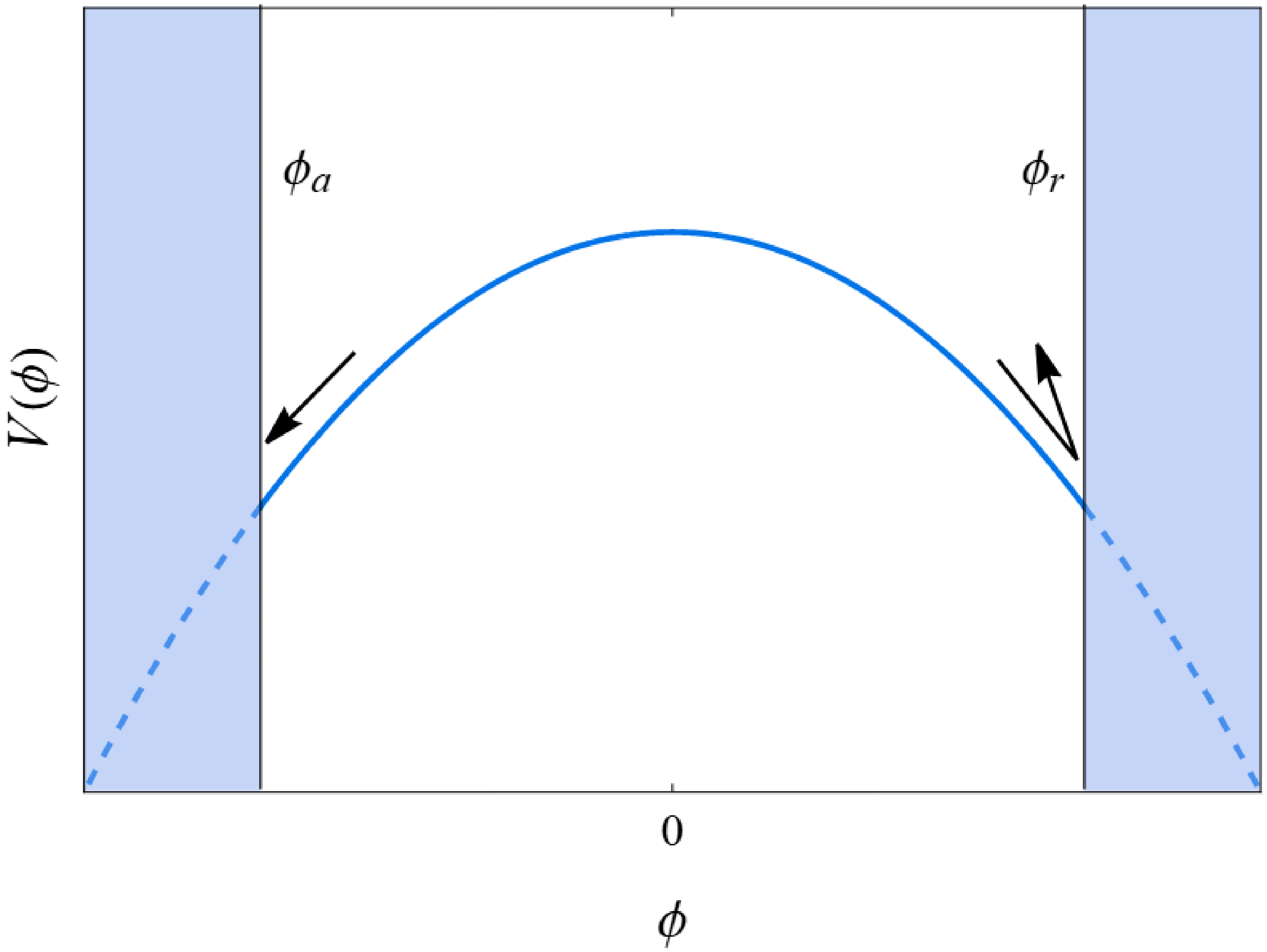
Hilltop inflation

Classical trajectory:

$$\phi_{\mathrm{cl}} = \phi_0 e^{\frac{\epsilon_2}{2}N}$$

$$\mu(\phi) = \frac{\epsilon_2}{2}\phi$$

$$\sigma = \frac{H}{2\pi}\frac{\sigma_c^{3/2}\sqrt{\pi}}{2}|H_\nu(\sigma_c)|$$



Hilltop inflation

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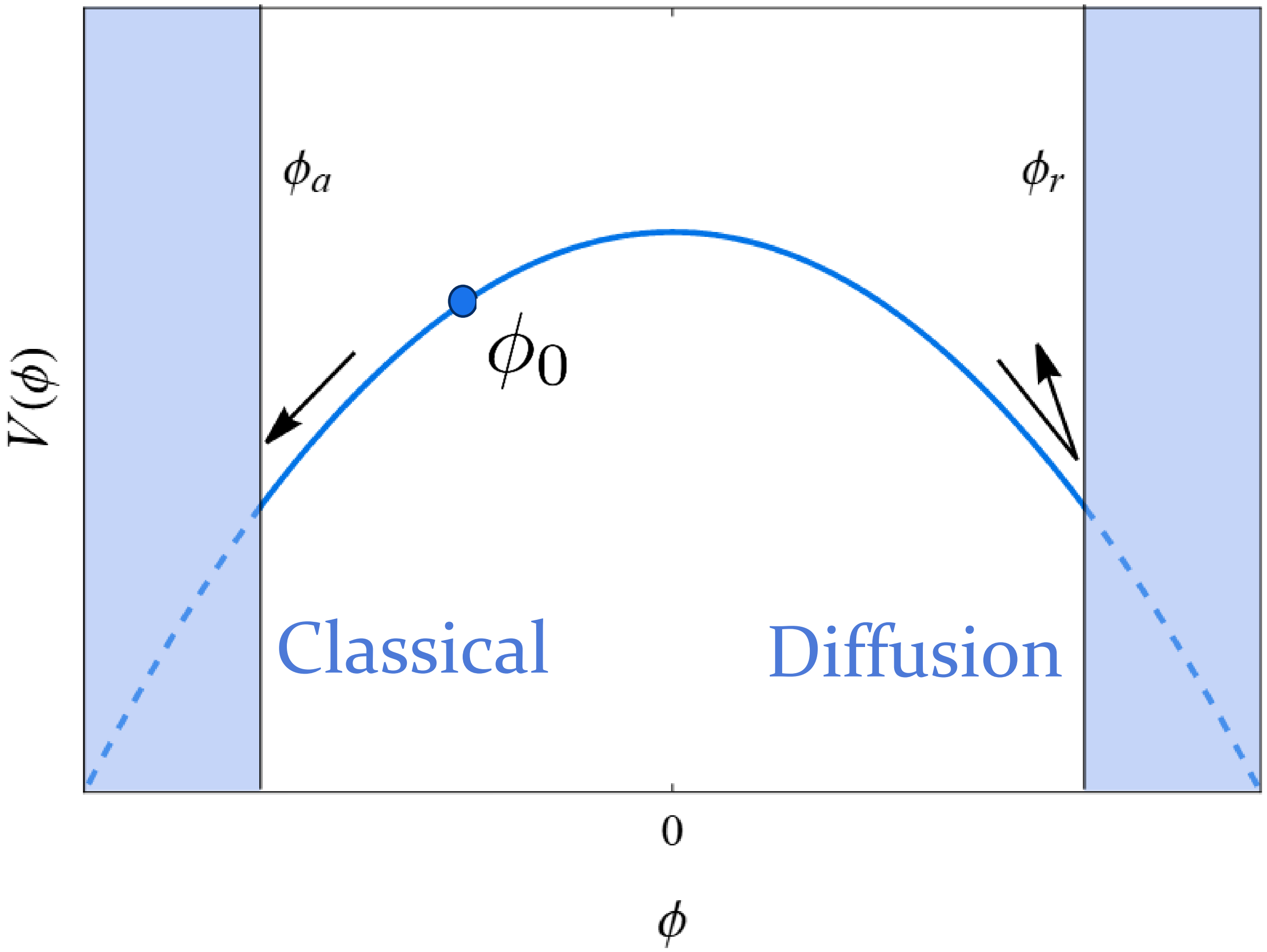


Hilltop inflation

Scaling: $\varphi \equiv \sqrt{\frac{\epsilon_2}{2}}\frac{\phi}{\sigma}$ $\quad \tilde{N} \equiv \frac{\epsilon_2}{2}N$ $\quad \tilde{\lambda}_n \equiv \frac{2}{\epsilon_2}N$

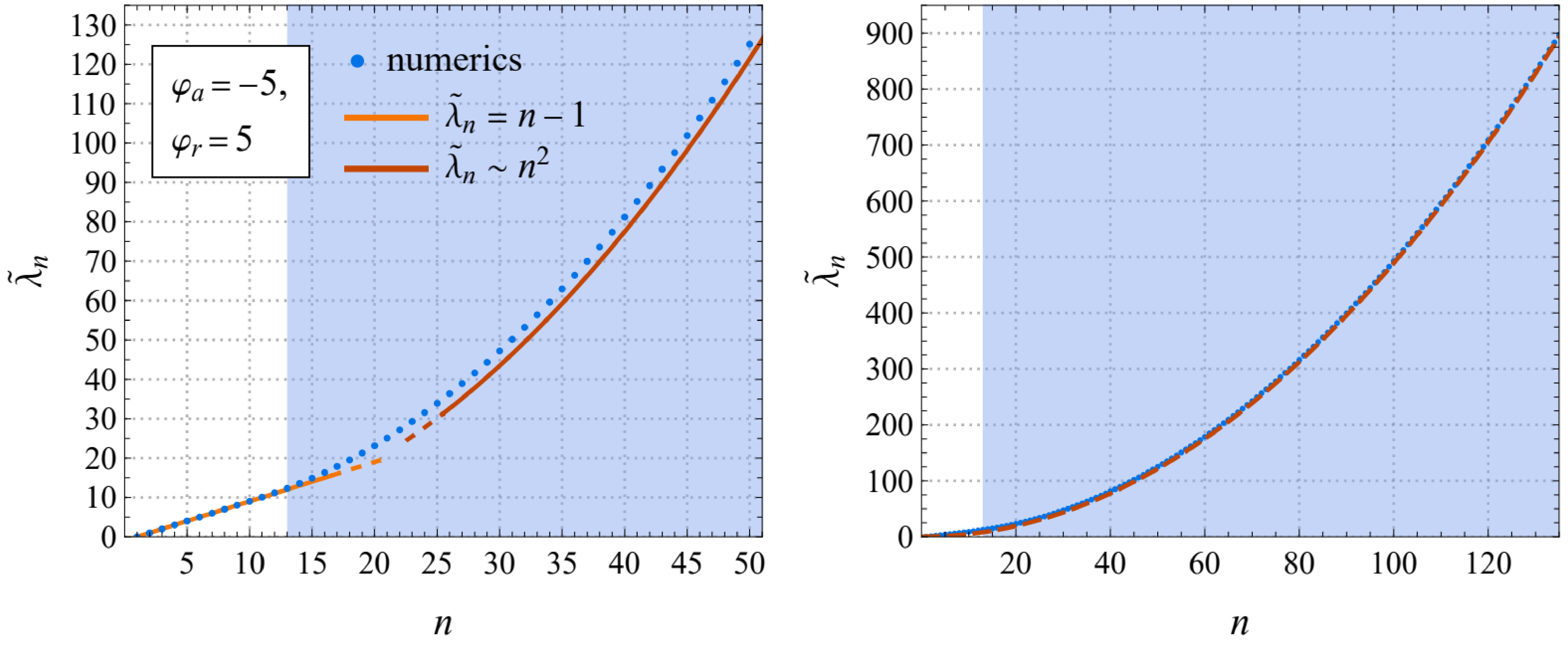
Eigenfunctions: $\bar{u}_n(\varphi) = A_n \bar{u}_{A,\tilde{\lambda}_n}(\varphi) + B_n \bar{u}_{B,\tilde{\lambda}_n}(\varphi)$

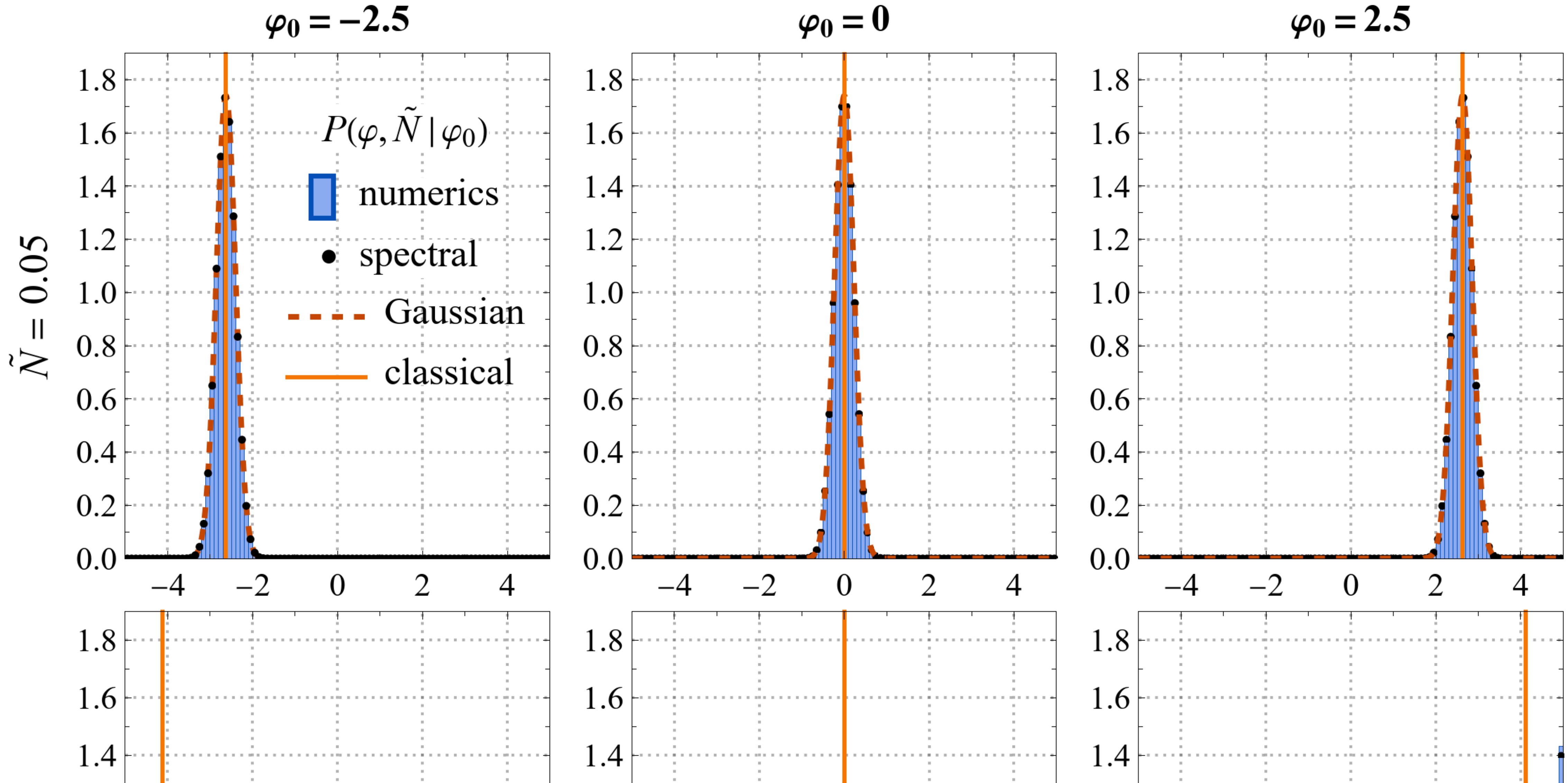
$$\bar{u}_{B,\tilde{\lambda}}(\varphi) \equiv \varphi \cdot {}_1F_1\left(\frac{1+\tilde{\lambda}}{2}; \frac{3}{2}; -\varphi^2\right)$$

Boundaries:

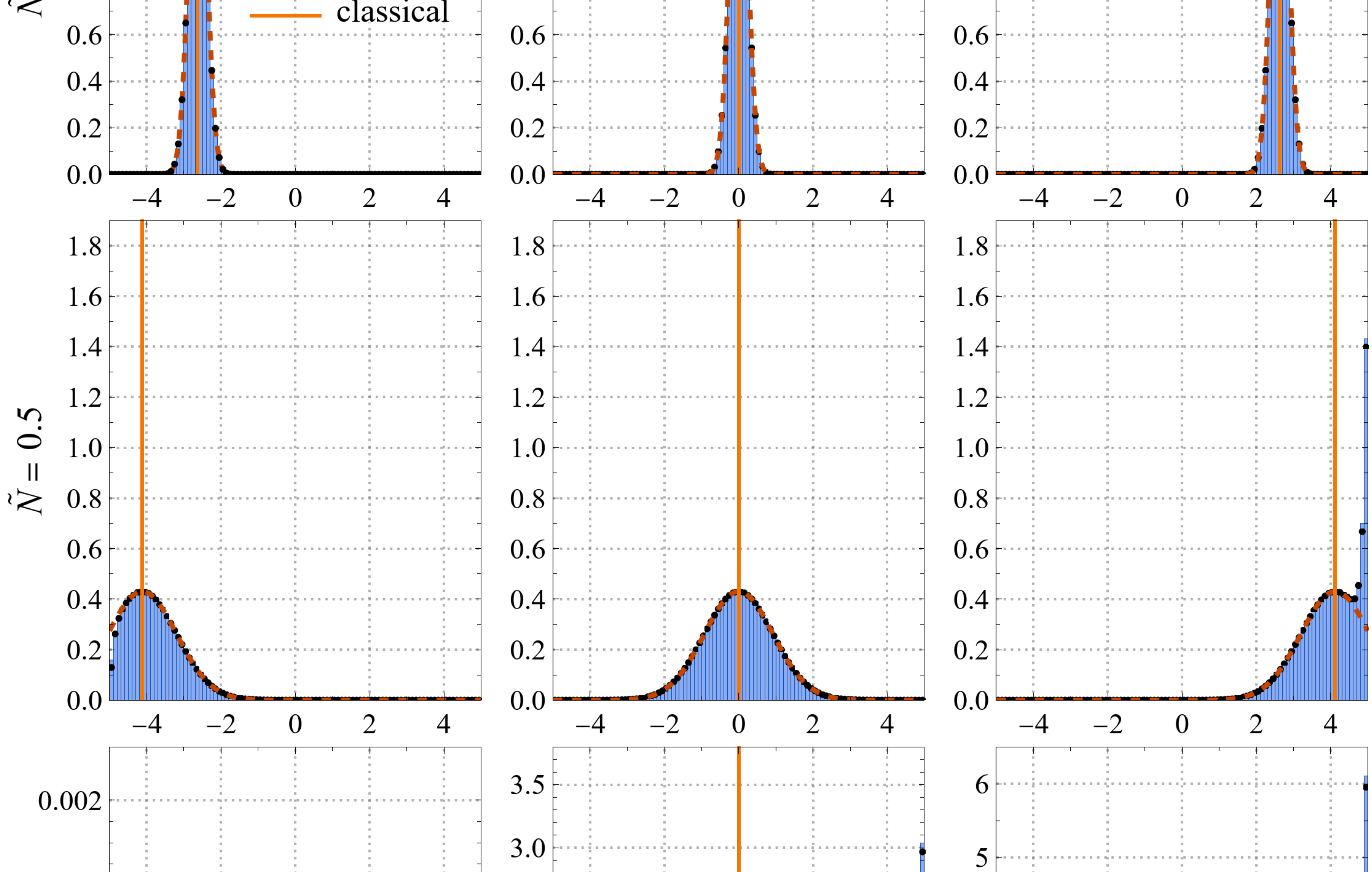
$\bar{u}_n(\varphi_a) = 0 \qquad \bar{u}'_n(\varphi_r) = 0$

$$\bar{u}_{A,\tilde{\lambda}}(\varphi) \equiv {}_1F_1\left(\frac{\tilde{\lambda}}{2}; \frac{1}{2}; -\varphi^2\right)$$

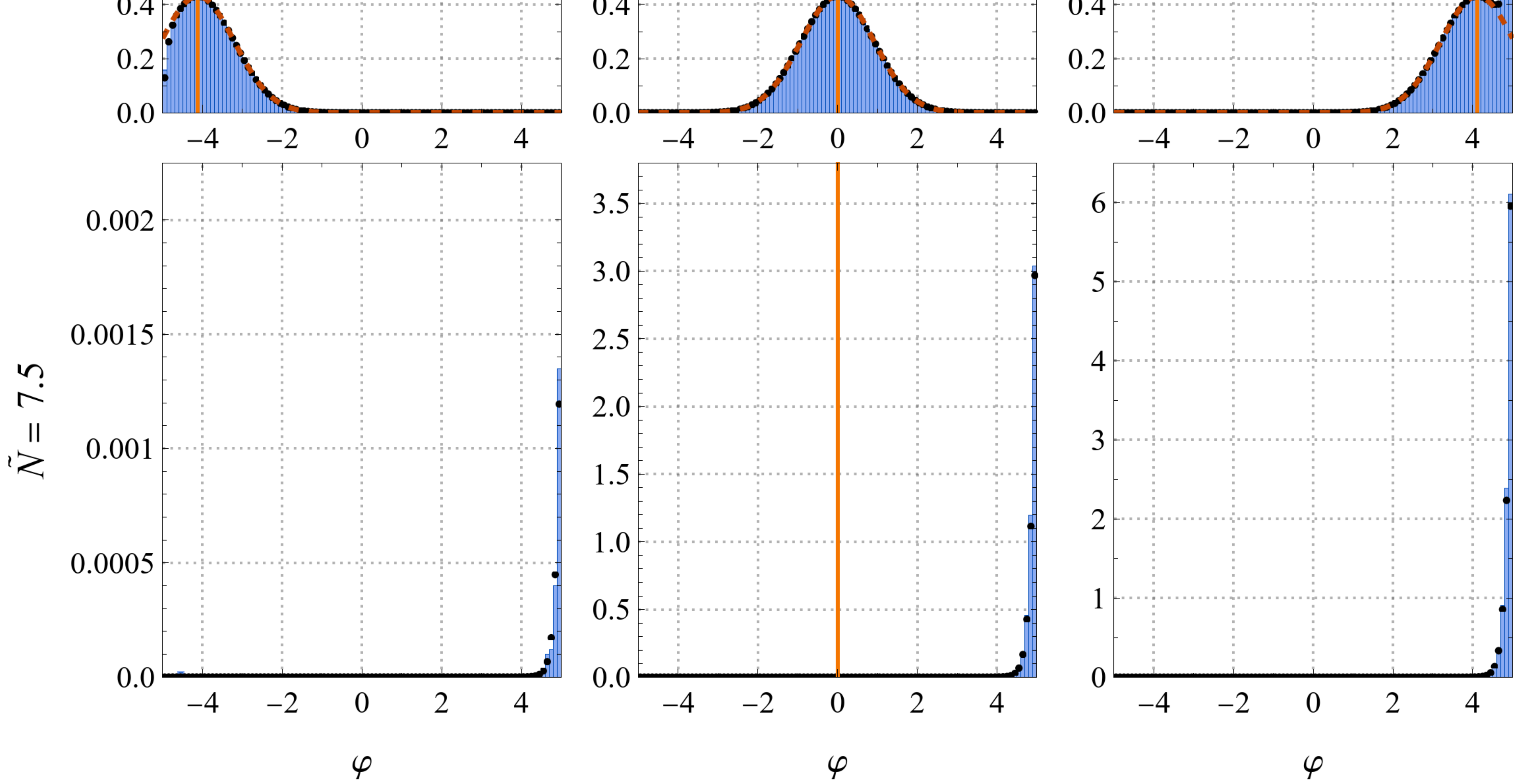

$\varphi_a = -5,$
$\varphi_r = 5$
numerics
$\tilde{\lambda}_n = n - 1$
$\tilde{\lambda}_n \sim n^2$
$\tilde{\lambda}_n$
n
$\tilde{\lambda}_n$
n



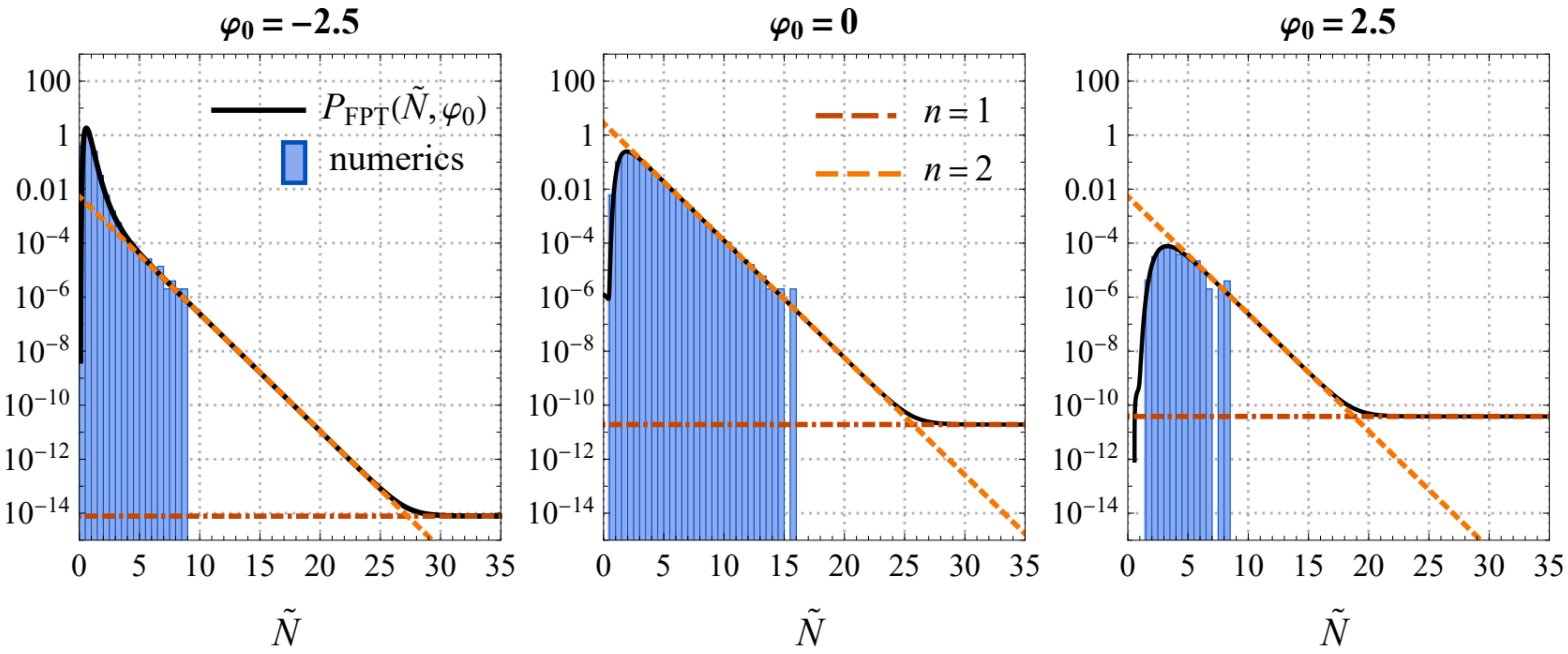
$\varphi_0 = -2.5$
$\varphi_0 = 0$
$\varphi_0 = 2.5$
$\tilde{N} = 0.05$
$P(\varphi, \tilde{N} \,|\, \varphi_0)$
numerics
spectral
Gaussian
classical



classical
$\tilde{N} = 0.5$



$\tilde{N} = 7.5$
φ
φ
φ



$\varphi_0 = -2.5$
$\varphi_0 = 0$
$\varphi_0 = 2.5$
$P_{\rm FPT}(\tilde{N}, \varphi_0)$
numerics
$n = 1$
$n = 2$
100
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
0 5 10 15 20 25 30 35
$\tilde{N}$

Hilltop inflation: ΔN

Coarse-grained curvature perturbation:

$$\Delta N_c = N_c + \bar{N}_c(\phi_c) - \bar{N}$$

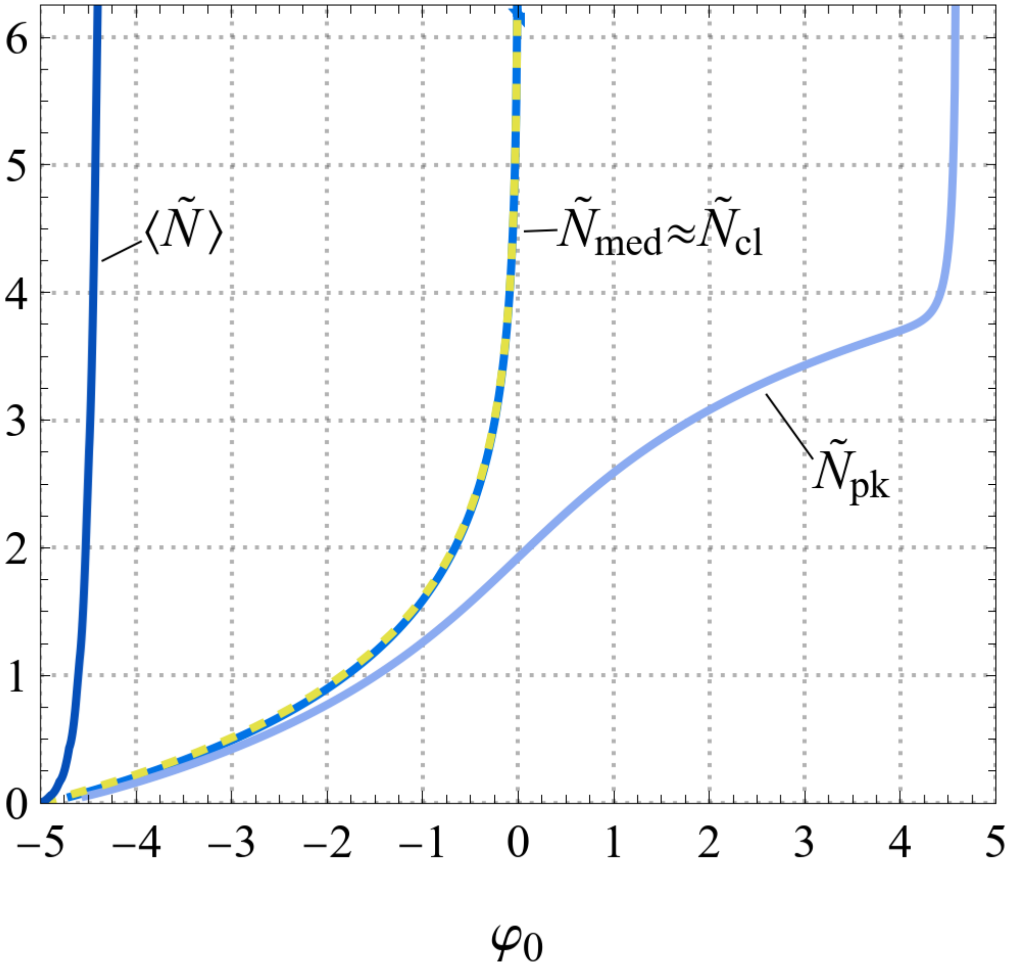
Coarse-graining scale

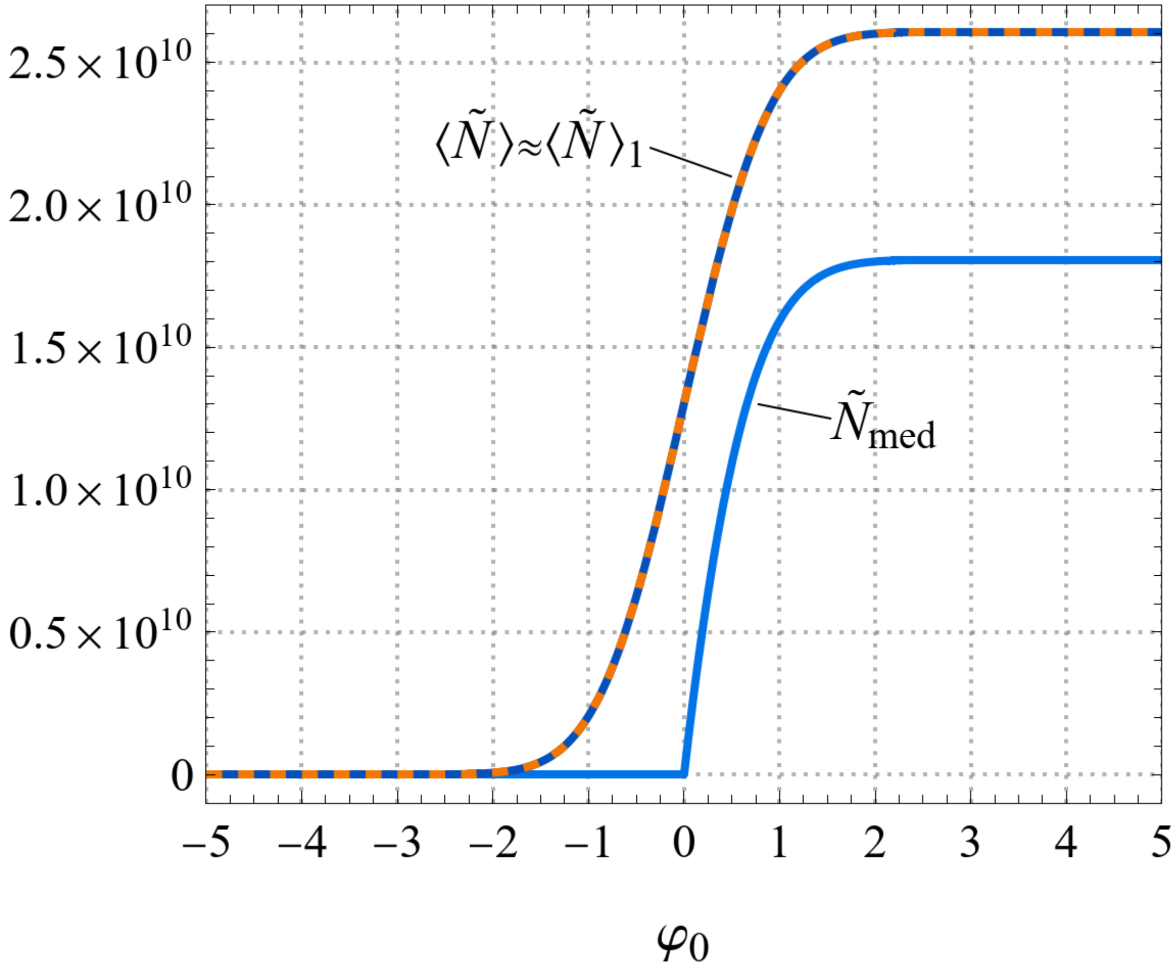
Stochastic

$$P(\Delta N_c) = P(\phi_c, N_c) \left| \frac{\mathrm{d}\phi_c}{\mathrm{d}\bar{N}_c} \right|$$

Background?

Use median!



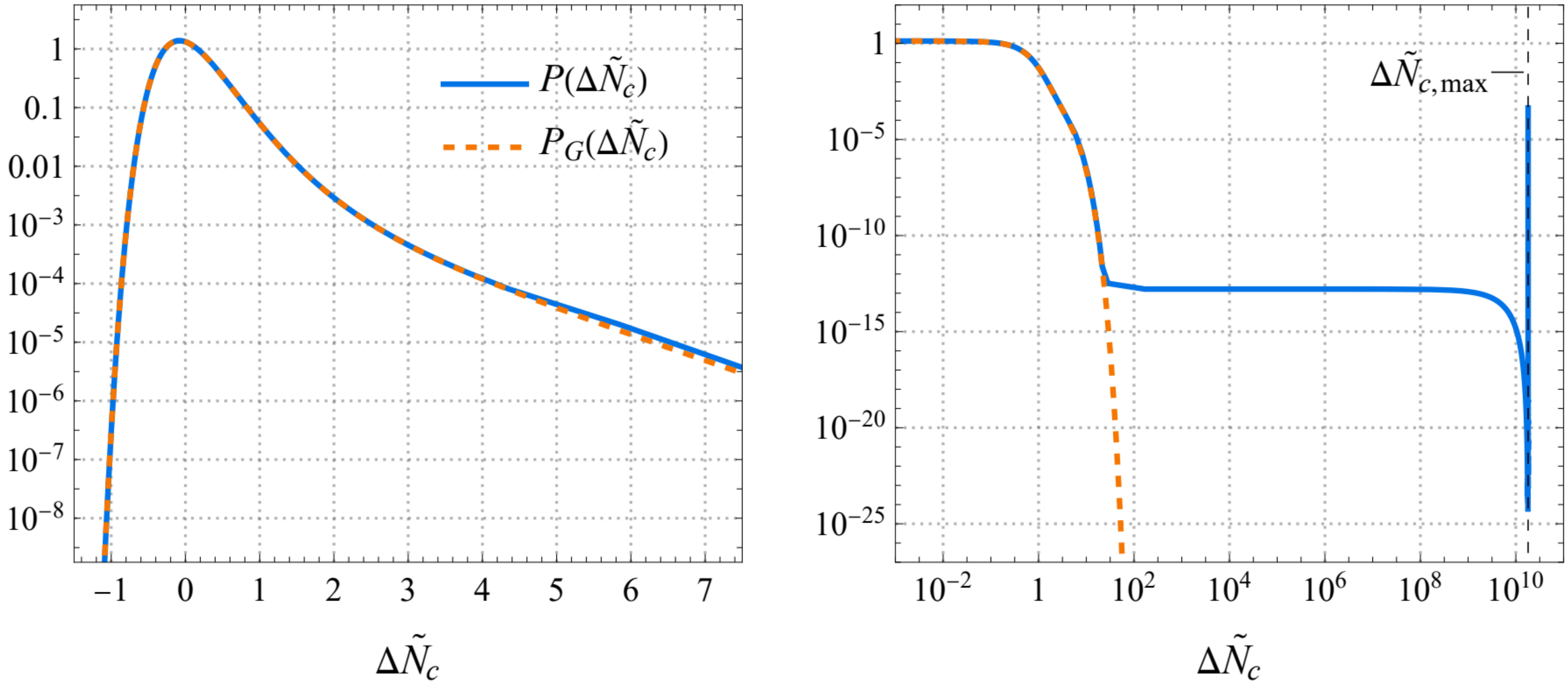


Mean fails as background!

Use median?

Beyond hilltop?

Hilltop inflation: ΔN



Summary

Spectral method is a powerful tool for studying large perturbations in stochastic inflation

Mean e-folds bad description for the background: use median instead?