

A Parameter Scan of Current-Carrying Cosmic Strings

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Superconducting Strings

- in reality, fields do not exist in isolation
 - the responsible field for string formation will couple to other existing fields
 - extra degrees of freedom (e.g. conserved current) (E. Witten 1985)
- if symmetry of current-carrying field is a gauged $U(1)$: strings become superconducting
- for global $U(1)$: strings are current-carrying, but not superconducting
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Two different approaches:

- 3D field theory numerical simulations limited by resolution, box sizes and dynamical range
- analytic models like the VOS model

The Velocity-dependent One-Scale (VOS) model

- basic idea: start from statistical physics and turn it into thermodynamics
- more precise: take microphysics given by Nambu-Goto action and then average over whole network obtaining evolution equations for macroscopic quantities
- possible extension: take into account charges and currents arriving at Charge-Velocity-dependent One-Scale (CVOS) model
- problem: unknowns remain while obtaining equations of motion
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$U(1) \times U(1)$ strings

- $\mathcal{L} = |D_\mu \phi|^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |\mathcal{D}_\mu \sigma|^2 - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} - V(\phi, \sigma)$
- With fields (ϕ, A_μ) and (σ, B_μ) and
$$D_\mu = \partial_\mu - ie_\phi A_\mu, \quad \mathcal{D}_\mu = \partial_\mu - ie_\sigma B_\mu$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$
- Take the potential of Saffin, 2005:
$$V(\phi, \sigma) = \frac{\lambda_\phi}{4} (|\phi|^2 - \phi_0^2)^2 + \frac{\lambda_\sigma}{4} (|\sigma|^2 - \sigma_0^2)^2 - \kappa (|\phi|^2 - \phi_0^2)(|\sigma|^2 - \sigma_0^2)$$

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ϕ_0, σ_0 : VEVs of both fields

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- $\lambda_\phi, \lambda_\sigma$: scalar couplings

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κ : coupling constant

Initial Conditions

Critical points (ϕ^2, σ^2) at:

$$(0, 0), \quad (\phi_0^2, \sigma_0^2), \quad \left(\phi_0^2 - \frac{2\kappa\sigma_0^2}{\lambda_\phi}, 0\right), \quad \left(0, \sigma_0^2 - \frac{2\kappa\phi_0^2}{\lambda_\sigma}\right)$$

if $\kappa < -\frac{1}{2}\sqrt{\lambda_\phi\lambda_\sigma}$ then red and orange points are minima
→ range for superconducting strings

Ensure stability of vacuum: $\lambda_\phi\phi_0^4 > \lambda_\sigma\sigma_0^4$

Extra condition from condensate mass: $m_{\sigma,+}^2 > m_{\sigma,-}^2$

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Resulting in equations of motion for the scalar fields:

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} = D^j D_j \phi - a^2 \phi \left[\frac{\lambda_\phi}{2} (|\phi|^2 - \phi_0^2) - \kappa (|\sigma|^2 - \sigma_0^2) \right] \quad (1)$$

$$\ddot{\sigma} + 2\frac{\dot{a}}{a}\dot{\sigma} = \partial^j \partial_j \psi - a^2 \sigma \left[\frac{\lambda_\sigma}{2} (|\sigma|^2 - \sigma_0^2) - \kappa (|\phi|^2 - \phi_0^2) \right] \quad (2)$$

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And for the gauge fields (we set $e_\sigma = 0 \rightarrow U(1)_\phi$ local and $U(1)_\sigma$ global):

$$\dot{F}_{0j} = \partial_j F_{ij} - 2a^2 e_\phi^2 \text{Im}[\phi^* D_j \phi] \quad (3)$$

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With Gauss' law for the first sector:

$$\partial_i F_{0i} - 2a^2 e_\phi^2 \mathcal{J} [\phi^* \dot{\phi}] \phi = 0 \quad (4)$$

and a charge conservation equation for the second sector:

$$\partial_\mu J = \sigma \partial_\mu \sigma = 0 \quad (5)$$

The Simulation

We visualize the evolution of the string network by showing:

- Isosurfaces of $|\phi| = 0.7$
- Isosurfaces of $|\sigma| = 0.4$
- Colors: Phase of $\phi \in [-\pi, \pi]$

Full Parameter Space

$$\kappa < -\frac{1}{2}\sqrt{\lambda_\phi\lambda_\sigma}$$

$$\lambda_\phi\phi_0^4 > \lambda_\sigma\sigma_0^4$$

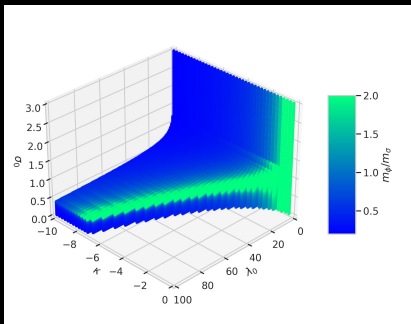
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We fix values for the Higgs mass:

$$\lambda_\phi = 2, \quad e_\phi = 1, \quad \phi_0 = 1 \\ \rightarrow m_\phi^2 = 2$$

Remember: $e_\sigma = 0$

$\rightarrow U(1)_\phi \times U(1)_\sigma$ corresponds to local Higgs and global condensate fields

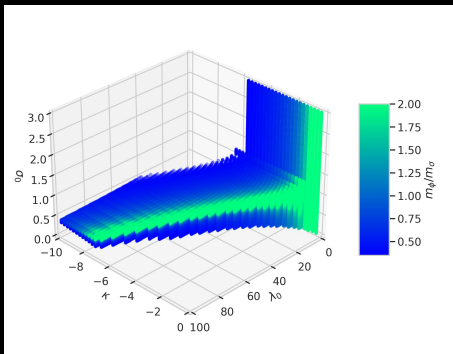
Free parameters:

λ_σ, σ_0 and κ

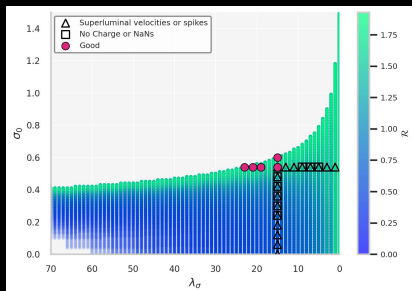
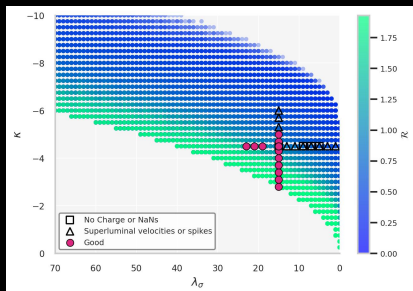
Further Constraints

$$1/(m_{\sigma,+}^2 - m_{\sigma,-}^2) \cdot 1/dx > 0.7$$

For $dx = 0.25$:

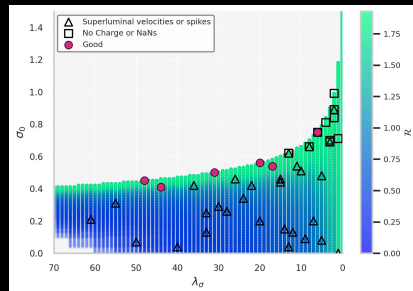
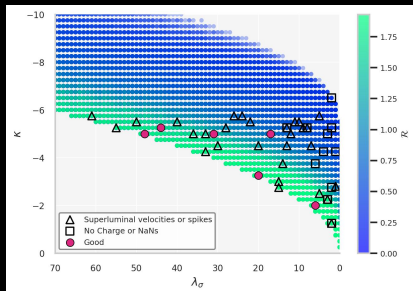


First Steps



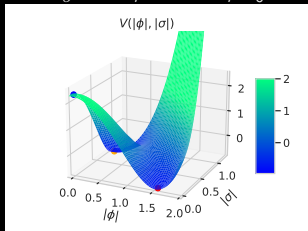
Runs performed on 4096^3 grid on the Leonardo Cluster

Wider Variety of Input Parameters

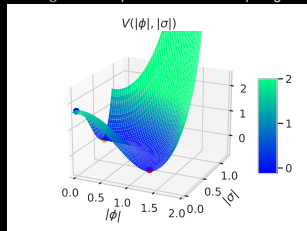


The potentials

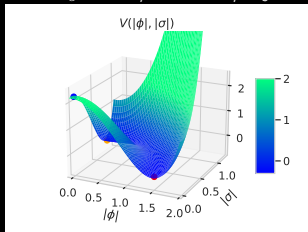
Set A: $\lambda_\sigma = 17$, $\kappa = -5$, $\sigma_0 = 0.54$



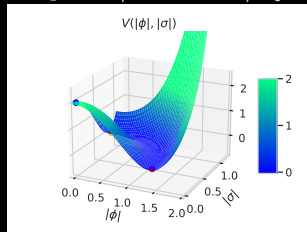
Set B: $\lambda_\sigma = 44$, $\kappa = -5.25$, $\sigma_0 = 0.41$



Set C: $\lambda_\sigma = 31$, $\kappa = -5$, $\sigma_0 = 0.5$



Set D: $\lambda_\sigma = 20$, $\kappa = -3.25$, $\sigma_0 = 0.56$



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- RMS velocity of string-forming field:

$$\bar{v}_{\phi}^2 = \frac{2R}{1+R}, \quad R = \frac{\sum_x |\Pi|^2 f(\phi, \sigma)}{\sum_{x,i} |D_{x,i}^+ \phi|^2 f(\phi, \sigma)}$$

- Charge and current estimators:

$$\bar{Q}_f^2, \bar{J}_f^2 = \frac{\sum_x \mathcal{J}(\sigma^* \partial_{0,i} \sigma)^2 f(\phi, \sigma)}{\sum_x \rho f(\phi, \sigma)}$$

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- Derived diagnostics:

$$K = \bar{Q}_f^2 - \bar{J}_f^2, \quad Y = \frac{\bar{Q}_f^2 + \bar{J}_f^2}{2}$$

$$\omega_\sigma = \frac{\bar{Q}_f^2 - \bar{J}_f^2/3}{\bar{Q}_f^2 + \bar{J}_f^2}$$

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Chirality: $K = 0$ (light-like),
 electric (time-like): $\bar{Q}_f^2 > \bar{J}_f^2$, magnetic (space-like): $\bar{Q}_f^2 < \bar{J}_f^2$.

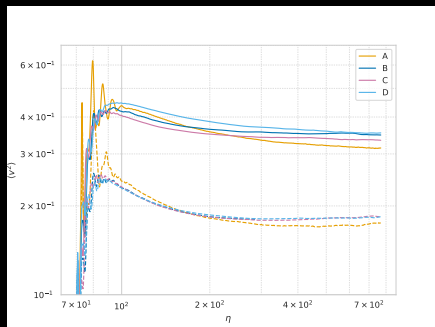
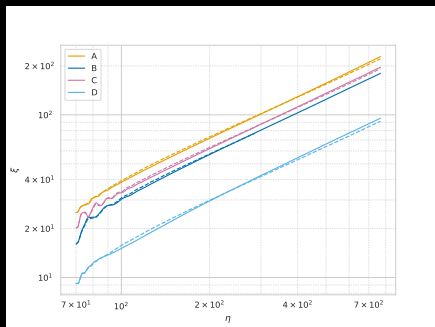
Current-Carrying Strings

Linear scaling defined by **mean string separation** and **velocity** behaving as $\xi \propto (\eta - \eta_0)^\mu$ and $\bar{v}^2 \propto \eta^\nu$ with $\mu = 1$ and $\nu = 0$.

Initializing ϕ as gaussian random field close to vacuum expectation value and σ as small-amplitude fluctuations around zero. After diffusion time, scaling is reached.

- solid lines: radiation era
- dashed lines: fast expansion

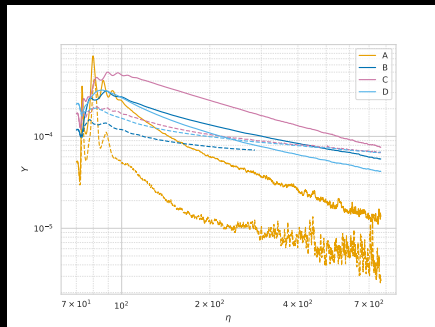
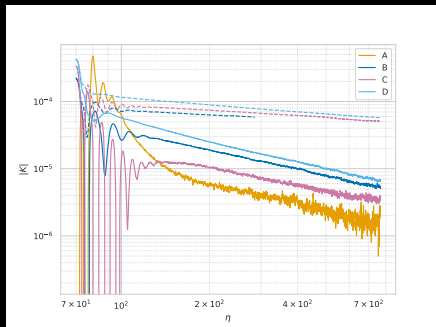
Runs performed on a 6144^3 grid on the Leonardo Cluster.



Charge and Current

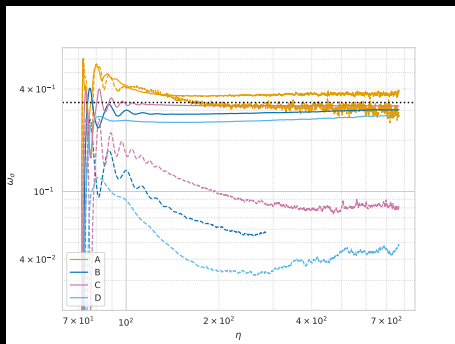
We define **charge** and **current** relative to total local energy density:

$$K = Q^2 - J^2, \quad Y = 0.5(Q^2 + J^2)$$



Equation of State

Equation of state: $\omega_\sigma = \frac{Q^2 - J^2/3}{Q^2 + J^2}$ for identifying if strings are electric, chiral or magnetic



Summary

- Performed a scan of the parameter space for current-carrying cosmic strings.
- Identified a restricted region where the evolution is numerically stable and well-behaved.
- Used initial conditions from this region for longer simulations and observable analysis.

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Next Steps

- Further explore the boundary of the stable parameter region.
- Investigate scaling behaviour dependence on initial conditions in more detail.