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Non-Linear Dynamics and Primordial Black Hole Formation During Kination

based on JCAP07(2026)048, ArXiv: 2507.19166
with P. Giannadakis, L. Heurtier, E. A. Lim

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Overview

- Why Primordial Black Holes

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- Why Kination

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- Why Primordial Black Holes
- Why Kination
    - Primordial Black Hole Formation

FLRW Cosmology

- FLRW Metric $ds^2 = a^2(\eta)(-d\eta^2 + \gamma_{ij}dx^i dx^j),\ \mathcal{H} \equiv \frac{a'(\eta)}{a(\eta)}$
- Equation of State $p = w\rho, \qquad \rho \propto a^{-3(1+w)}$
- The Standard Cosmological Evolution

Inflation	$w \approx -1, \qquad \rho \approx \text{const}$
$\downarrow$	
Radiation Epoch	$w = -1/3, \qquad \rho \propto a^{-4}$
$\downarrow$	
Matter Epoch	$w = 0, \qquad \rho \propto a^{-3}$
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Late Time Cosmological Constant	$w \approx -1$

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? $\longrightarrow$	No Strong Constraints
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Non Standard Epoch

GRChombo

- GR + a minimally coupled Scalar Field

$$S = \int d^4x \sqrt{-g} \left(\frac{m_{\mathrm{pl}}^2}{16\pi} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

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  - An epoch where the dynamics are dominated by the kinetic energy of a rolling scalar field

$$\rho_K \equiv \frac{1}{2a^2} \phi'^2 \gg V(\phi)$$

$$\rho_K \gg \rho_\nabla \equiv \frac{1}{2a^2} (\nabla \phi)^2$$

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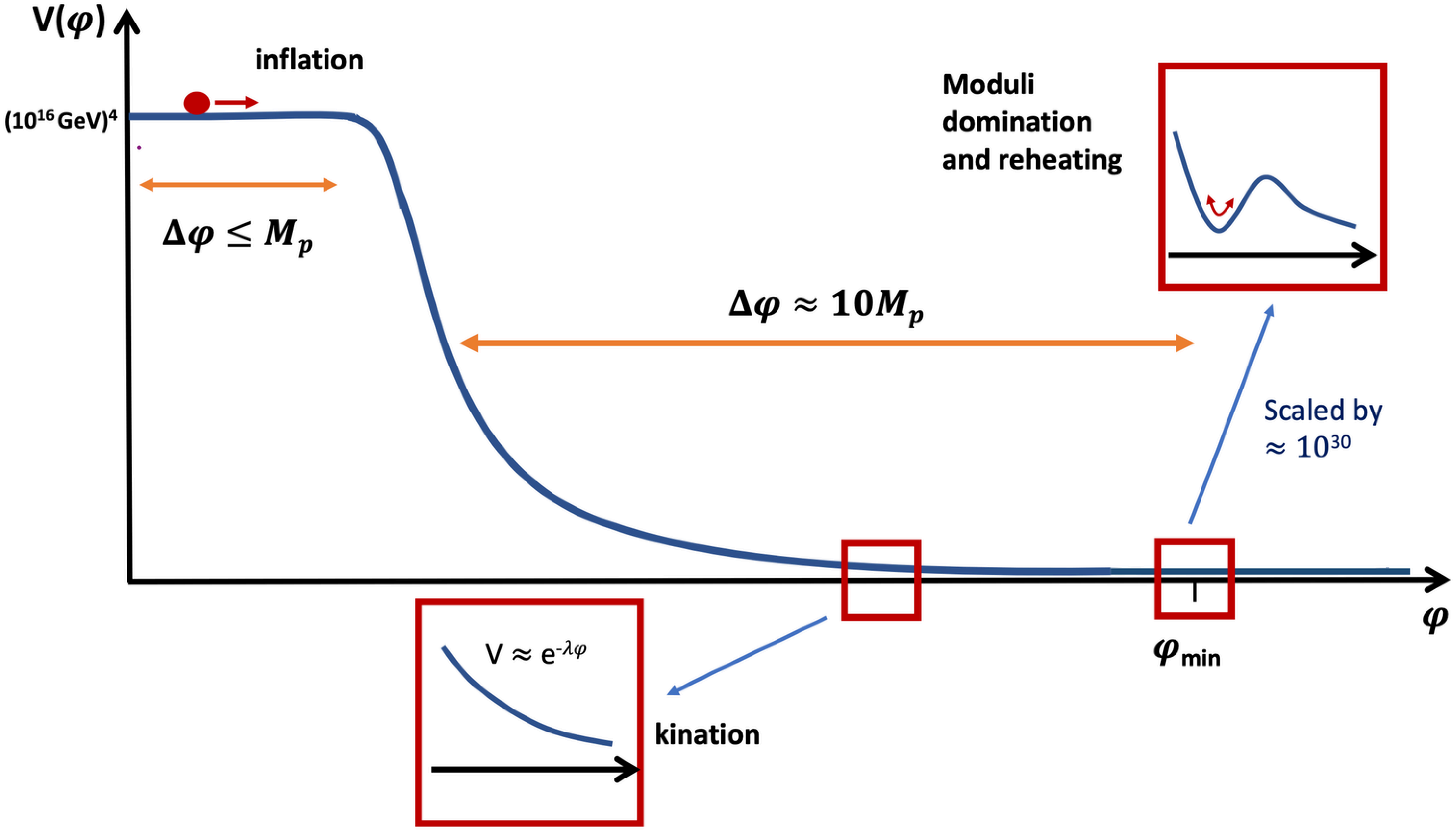
$$\rho \propto a^{-6}$$

Inflationary Models



Large Volume Scenario (LVS)

arXiv: 2303.04819
arXiv: 2401.04064

Linear Perturbation Theory

- Linear Scalar Self-Perturbation in FLRW background

$$\phi(\eta,\vec{x}) = \bar{\phi}(\eta) + \delta\phi(\eta,\vec{x}) = \bar{\phi}(\eta) + \frac{f(\eta,\vec{x})}{a}, \qquad \delta\phi(x) = \int \frac{d^3k}{(2\pi)^3}\delta\phi_k e^{ikx}$$

- EoM, Mukhanov-Sasaki Equation

$$\bar{\phi}'' + 2\mathcal{H}\bar{\phi}' = 0,\ V \to 0$$

$$f_k'' + (k^2 - \frac{a''}{a})f_k = 0$$

$$\longrightarrow$$

$$\bar{\phi} \sim \ln \eta$$

$$f_k(\eta) = A\sqrt{\eta}J_0(k\eta) + B\sqrt{\eta}Y_0(k\eta)$$

Linear Perturbation Theory

- Subhorizon Perturbation $k\eta \gg 1$

- Superhorizon Perturbation $k\eta \ll 1$

Linear Perturbation Theory

- Subhorizon Perturbation $k\eta \gg 1$

$$f_k = A\cos(k\eta) + B\sin(k\eta),$$

$$\delta\phi_k \sim \frac{\cos(k\eta)}{a} \propto a^{-1}$$

$$\langle \rho_\nabla \rangle \sim \left\langle \frac{\delta\phi_k^2}{a^2} \right\rangle \propto a^{-4}$$

$$\langle \rho_{\delta K} \rangle = \left\langle \left(\frac{f_k'}{a^3} - \mathcal{H}\frac{f_k}{a^2} \right) \bar{\phi}' \right\rangle + \left\langle \left(\frac{f_k'^2}{2a^4} + \mathcal{H}^2 \frac{f_k^2}{2a^2} - \mathcal{H}\frac{f_k' f_k}{a^3} \right) \right\rangle \propto a^{-4}$$

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$$\rho_{\delta\phi} = \langle \rho_\nabla + \rho_{\delta K} \rangle \propto a^{-4} \Rightarrow \delta = \frac{\rho_{\delta\phi}}{\bar{\rho}} \propto a^2$$

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$$f_k = c_1 a + c_2 a \ln a,$$

$$\delta\phi_k \sim A + B\ln a$$

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$$\langle \rho_{\delta K} \rangle \propto a^{-6}$$

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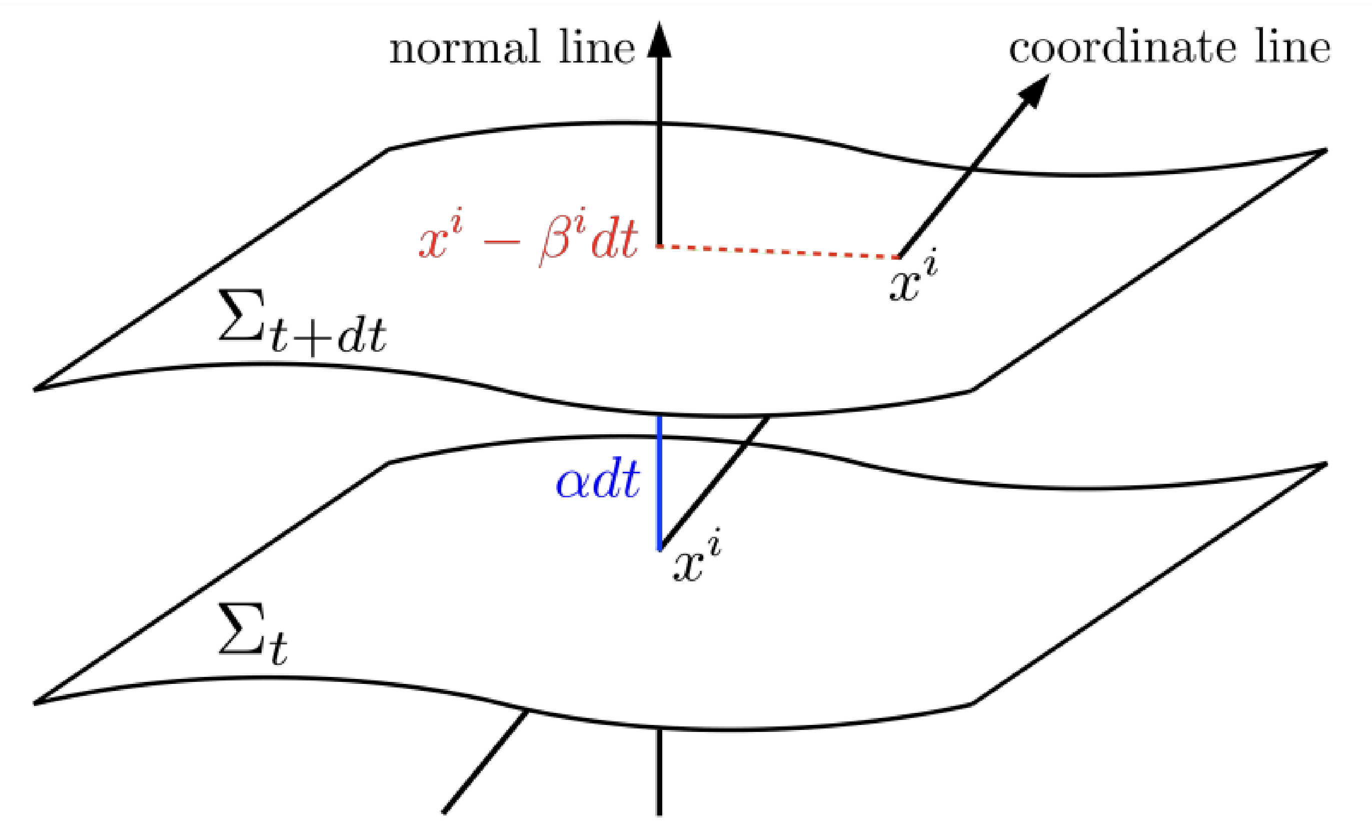
$$\rho_{\delta\phi} \propto a^{-2} \Rightarrow \delta = \frac{\rho_{\delta\phi}}{\bar{\rho}} \propto a^4 \longrightarrow$$

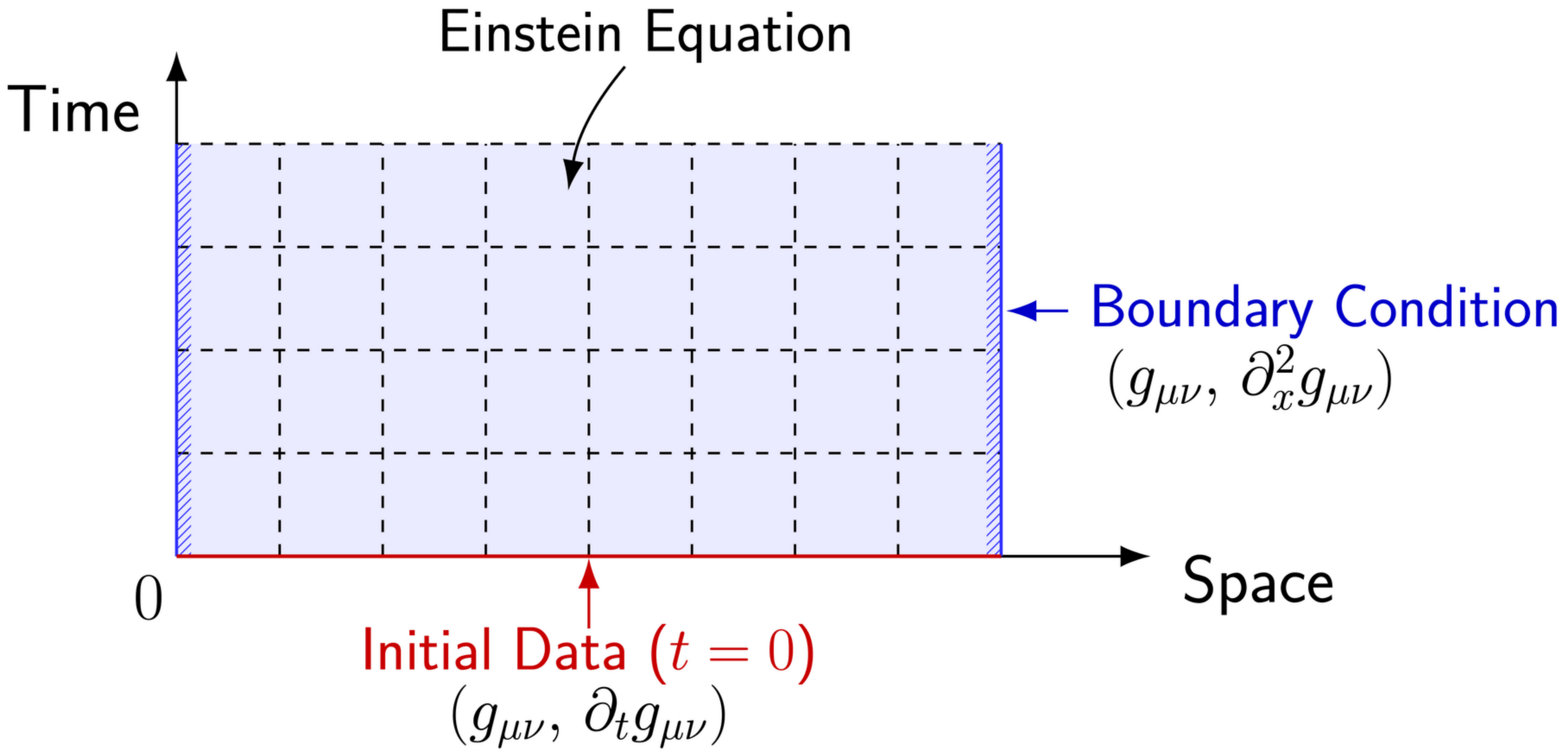
Strong gravitational backreaction?
Black hole formation?

NR for Cosmologists

- Numerical Relativity = Turning GR into Initial Value Problems

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$





NR for NRists

- CTTK Initial Condition Solver

$$\alpha = \chi = 1 \qquad \beta^i = B^i = 0 \qquad \tilde{\gamma}_{ij} = \delta_{ij}$$

- CCZ4 Evolution

$$8\pi(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) = R_{\mu\nu} + 2\nabla_{(\mu}Z_{\nu)} - 2\kappa_1 n_{(\mu}Z_{\nu)} + \kappa_1(1+\kappa_2)g_{\mu\nu}n_\alpha Z^\alpha, \; Z^\mu \to 0$$

- Gauge Conditions

$$\partial_t\alpha = -2\alpha(K - \langle K\rangle - 2\Theta) + \beta^i\partial_i\alpha, \qquad \partial_t\beta^i = \frac{3}{4}B^i, \qquad \partial_t B^i = \partial_t\tilde{\Gamma}^i - B^i$$

- Matter

$$\phi, \quad \Pi \equiv \frac{1}{\alpha}(\dot{\phi} - \beta^i\partial_i\phi)$$

Simulation Setup

- Assume Pure Kination $V(\phi) = 0$

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Simulation Setup

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- Pseudo-isotropic scalar field setup

$$\phi_0 = \cancel{\bar{\phi}_0} + \frac{\Delta\phi}{3}\sum_{i=1}^{3}\cos(\frac{2\pi N_{\phi,0}x_i}{L})$$

$$\dot{\phi}_0 = \sqrt{\frac{3}{4\pi}}m_{\rm pl}H_0 + \frac{\Delta\dot{\phi}}{3}\sum_{i=1}^{3}\cos(\frac{2\pi N_{\dot{\phi},0}x_i}{L} + \theta)$$

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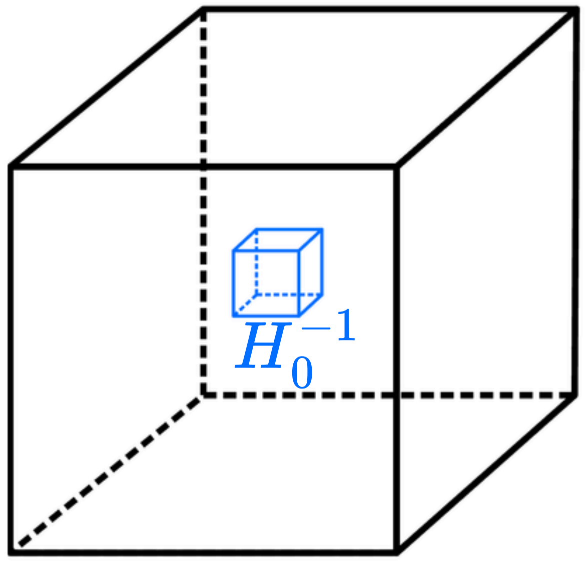
- Simulation Domain

$$L \equiv (N_{\phi,0} H_0)^{-1} = (N_{\phi,0}\dot{\phi}_0)^{-1}\sqrt{\frac{3}{4\pi}} m_{\rm pl}$$



$L = \lambda_0$

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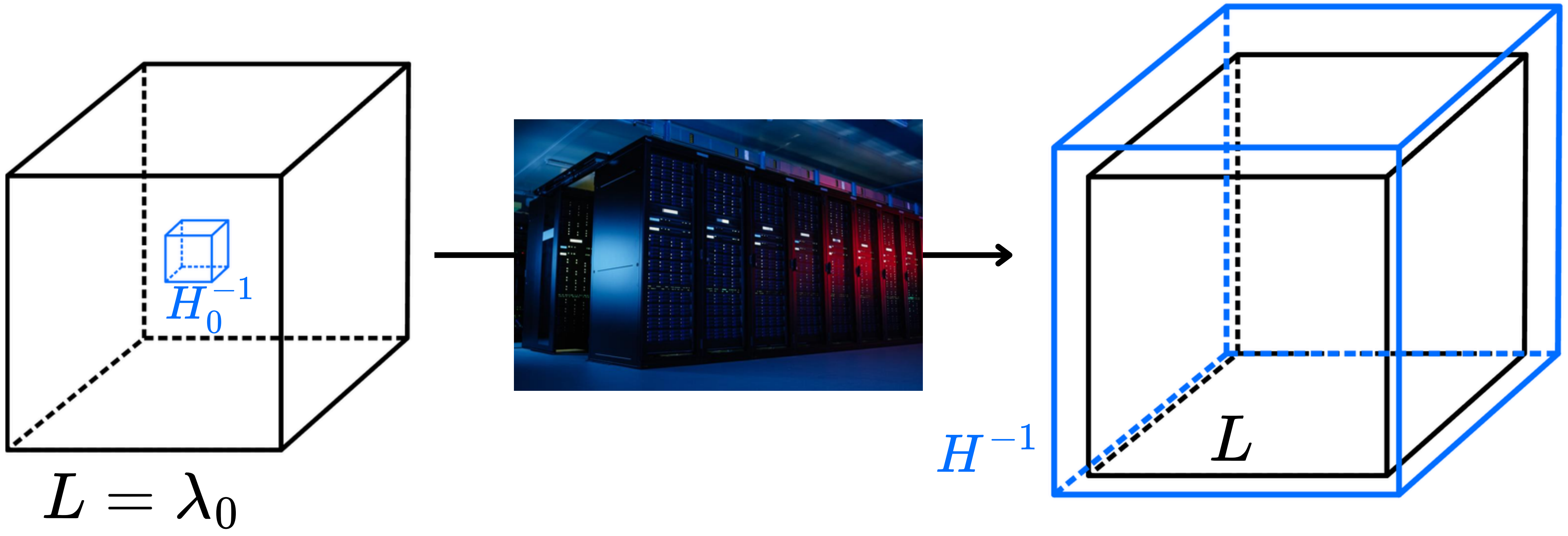

Diagnostics

- Superhorizon modes re-enter at $a_{\mathrm{re-entry}} \approx \sqrt{\lambda_0 H_0}$

- Energy Density

$$\bar{\rho} \equiv \frac{1}{2}\langle \Pi \rangle^2$$

$$\rho_\nabla \equiv \frac{1}{2}\gamma^{ij}\partial_i\phi\partial_j\phi$$

$$\langle \rho_{\delta K} \rangle \equiv \frac{1}{2}\langle \Pi^2 \rangle - \frac{1}{2}\langle \Pi \rangle^2$$

$$\langle \rho_{\delta\phi} \rangle = \langle \rho_{\delta K} \rangle + \langle \rho_\nabla \rangle$$

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- What did we learn?

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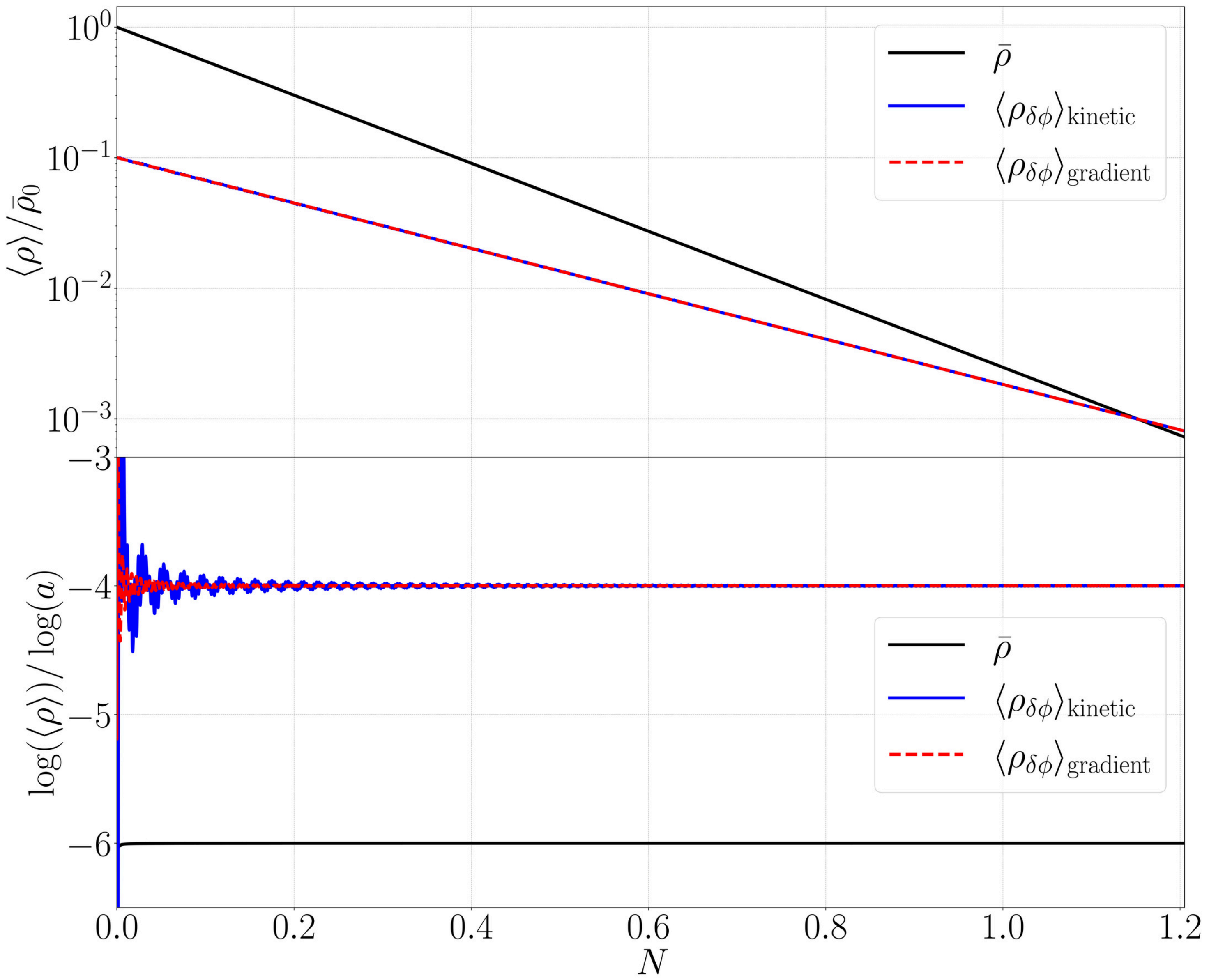
Results

- Subhorizon Perturbation



$$\lambda_0 = 0.01 H_0^{-1}, \quad \delta_0 = \langle\rho_{\delta\phi,0}\rangle/\bar{\rho}_0 = 0.1$$

$$\bar{\rho} \propto a^{-6}$$

$$\langle\rho_{\delta\phi}\rangle \propto a^{-4}$$

Subhorizon perturbation takes over and become the new radiation background with no strong gravitational backreaction. In order to see gravitaional collapse for such mode, we require $\delta_0 \gg 1$.

Results

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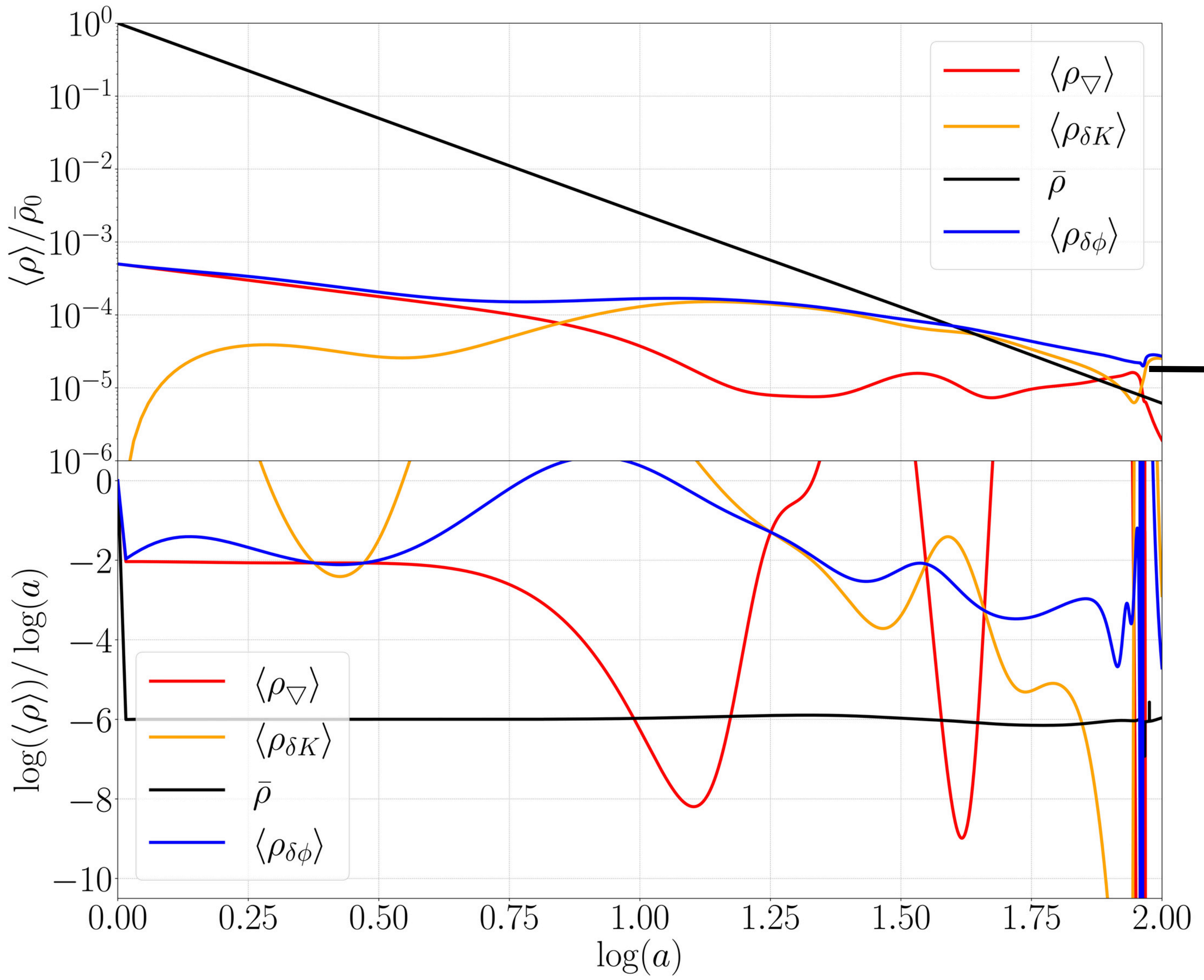

Black Hole Formed

$$\lambda_0 = 20H_0^{-1}, \quad \delta_0 = \langle\rho_{\delta\phi,0}\rangle/\bar{\rho}_0 = 5\times10^{-4}$$

Results

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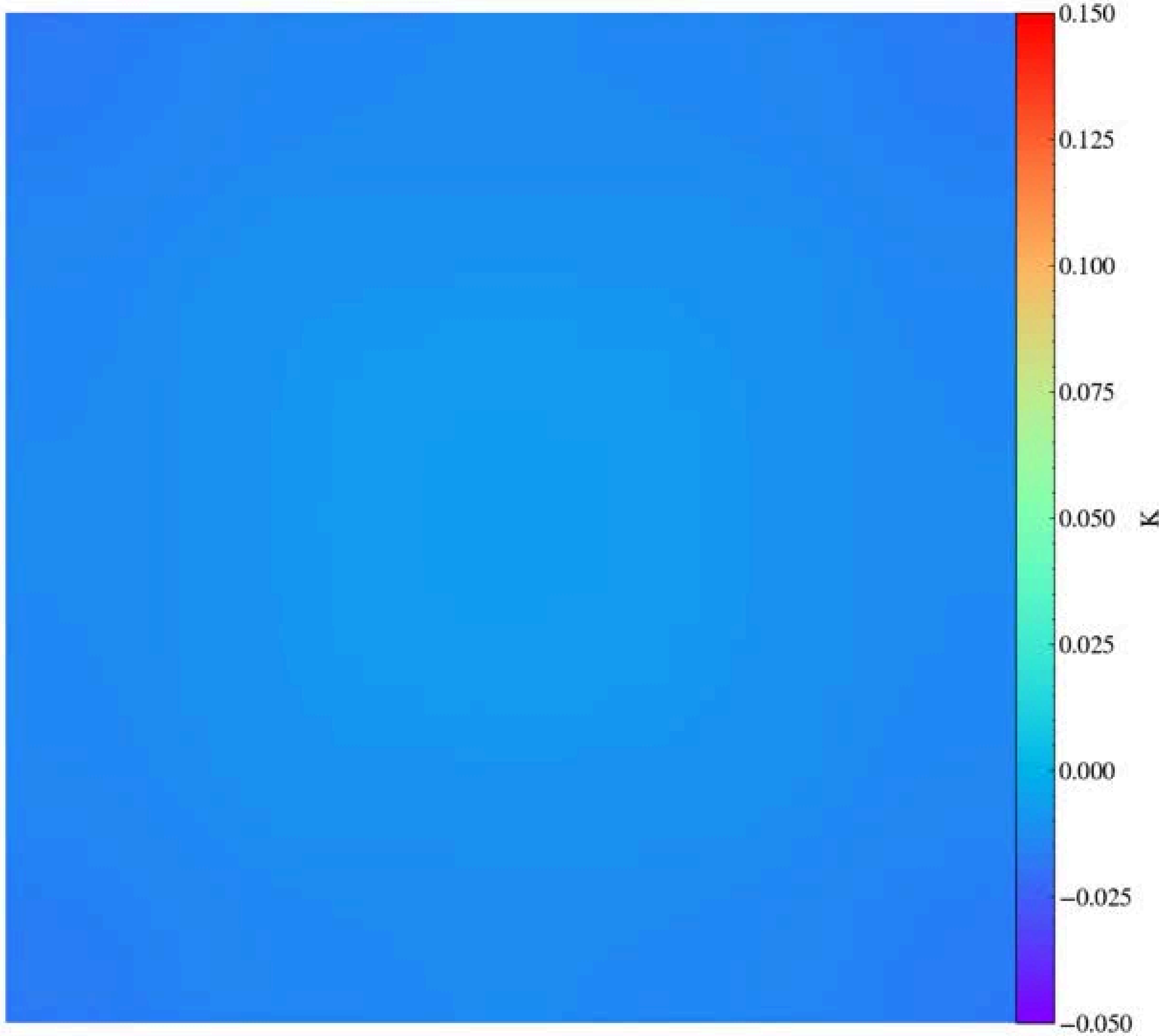

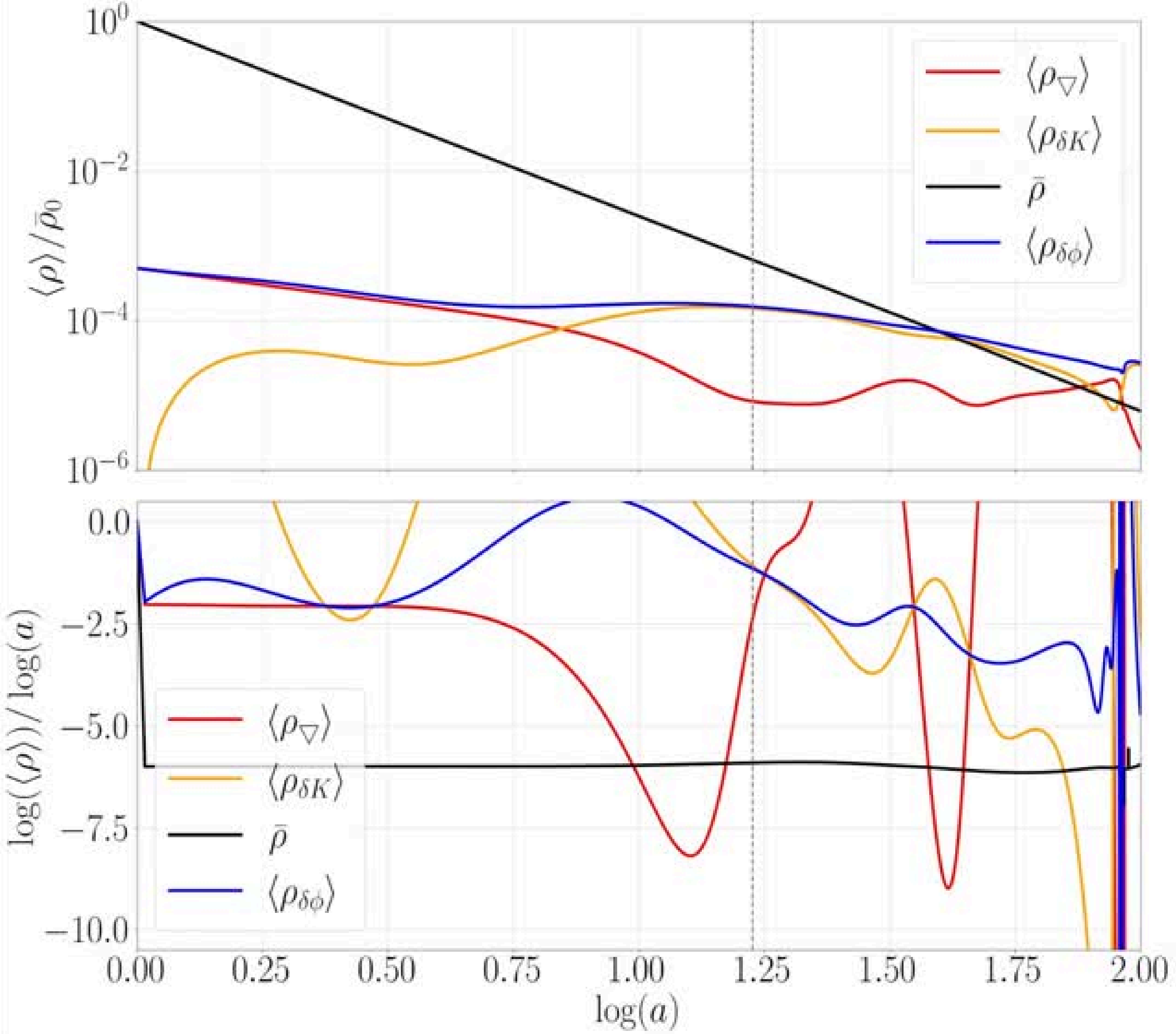

https://www.youtube.com/watch?v=0msEzENelt8

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Critical Density Contrast

- Perturbative analysis[1] estimated the critical density contrast for spherical collapse based on the Press-Schechter Theory is

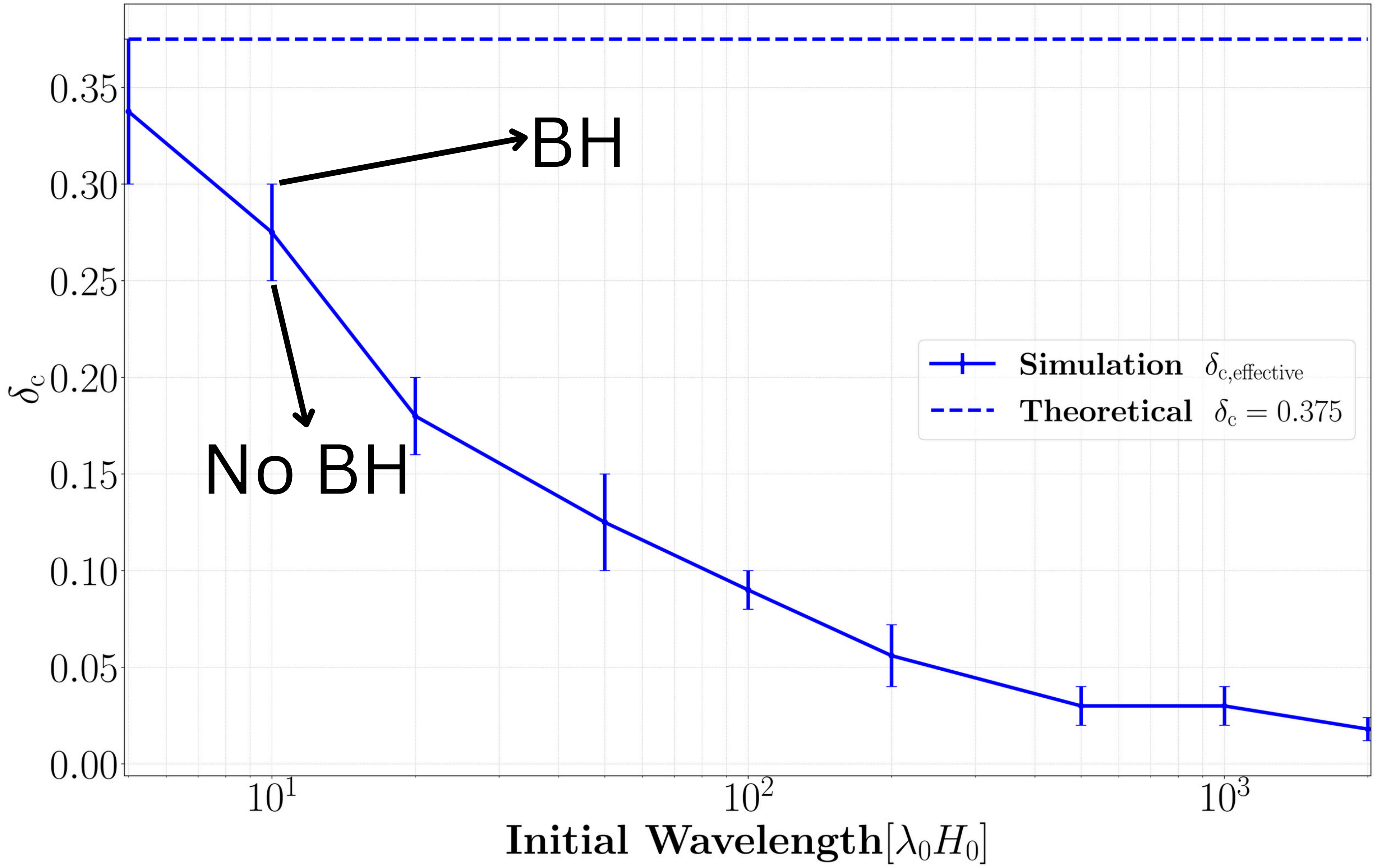
$$\delta_c \approx \frac{3(1+w)}{5+3w}\sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right) = 0.375$$

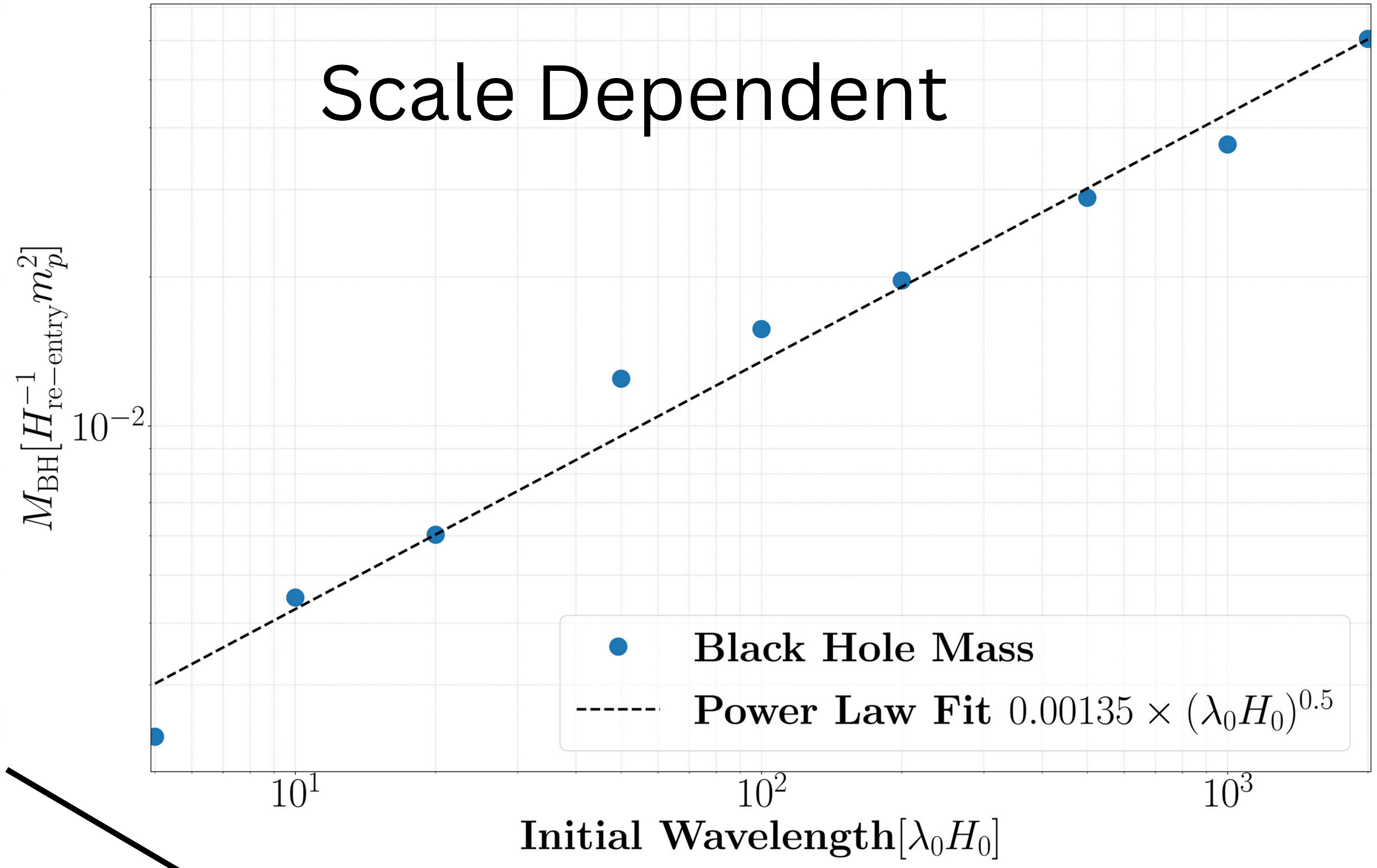
- Recent numerical work[2] has shown that the critical density contrast is actually closer to $0.5 \lesssim \delta_c \lesssim 0.8$ for spherical collapse. And spherically symmetric initial condition favours gravitational collapse[3].
- Extrapolate an effective density contrast for simulations with black hole formation assuming an FLRW kination background up to mode horizon re-entry.

$$\delta_{c,\text{effective}} \approx a^4_{\text{re}-\text{entry}}\delta_0 = (\lambda_0 H_0)^2 \langle \rho_{\delta\phi,0}\rangle / \bar{\rho}_0$$

1.arXiv:1309.4201
2.arXiv:2007.05564
3.arXiv: 2410.03452

Critical Density Contrast





3.8 efolds of kination

out of the max allowed ~10

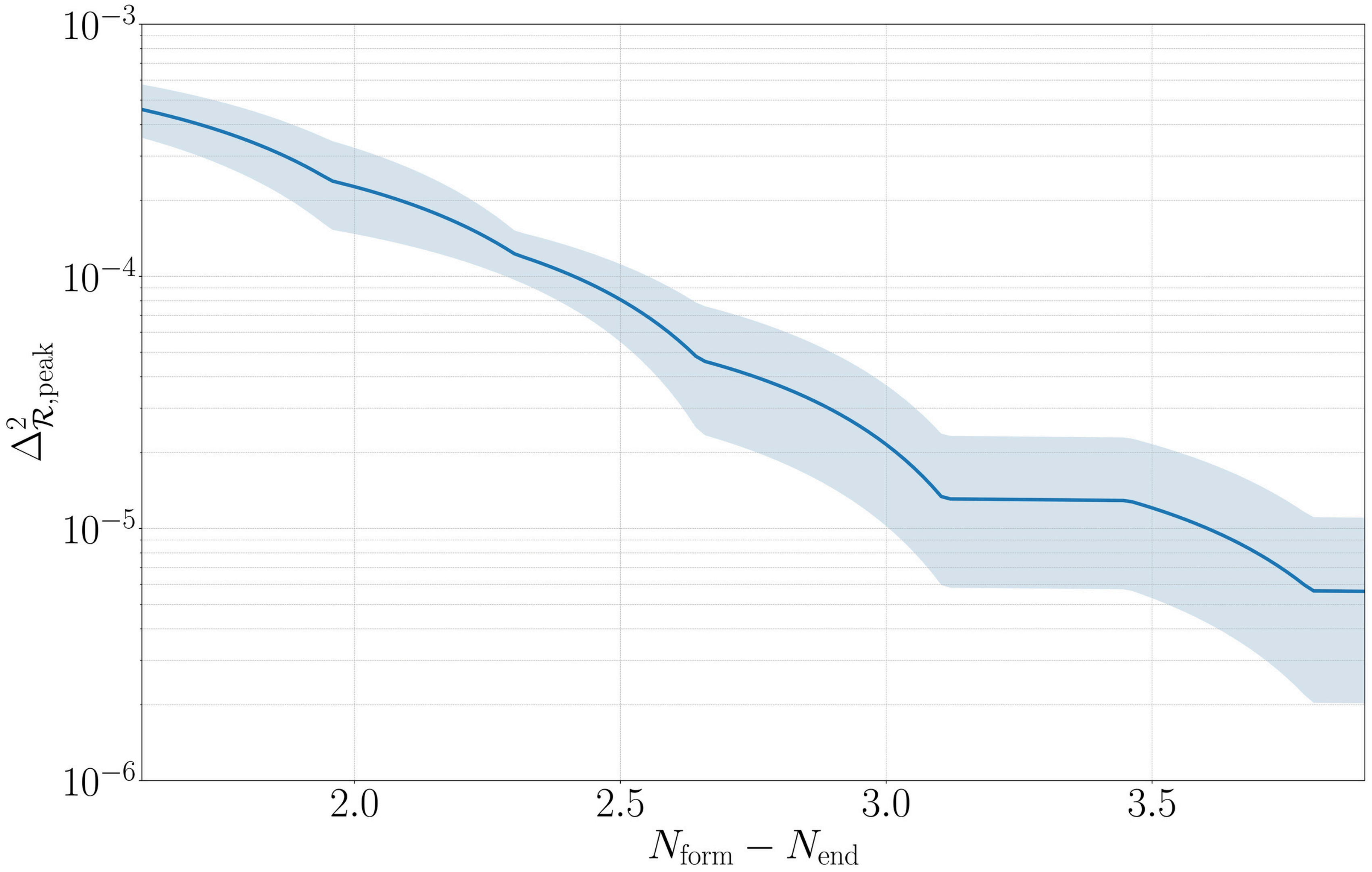
Critical Density Contrast

- To form PBHs, we typically require a quasi-monochromatic enhancement in the primordial curvature power spectrum

$$\Delta^2_{\mathcal{R},\mathrm{peak}} \sim 10^{-2} \gg \Delta^2_{\mathcal{R},CMB} \sim 10^{-9}$$

- If there is a period of prolonged kination, we would only require a moderate enhancement of

$$\Delta^2_{\mathcal{R},\mathrm{peak}} \sim \mathcal{O}(10^{-5} \sim 10^{-6})$$



Summary and Outlook

- Linear perturbations during kination would eventually take over as radiation background;
- Superhorizon perturbations are easier to trigger collapses to PBHs;
- PBH formation during kination requires a smaller ($O(10^{-5}$~$10^{-6}))$ enhancement in the curvature power spectrum
- Future directions/work in progress: extend the numerical study to the maximum allowed ~10 efolds (current bound), include potentials and assume inhomogeneities following a gaussian power spectrum.

Thank you!

arXiv:2507.19166, **C. Cheng**, P. Giannadakis, L. Heurtier, E. A. Lim

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