

Quantum Field Theory Of Cosmological Perturbations Induced by Ultralight Dark Matter

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Based on: T. Prokopec & M. Vecchioni, *arXiv:2607.27133*

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Cosmology

Why model dark-matter backreaction?

$$\delta T_{\mu\nu}^{\text{DM}} \overset{\text{backreact}}{\rightleftharpoons} \delta g_{\mu\nu} \begin{cases} \phi + \psi & \rightarrow \text{Lensing probes} \\ h_{ij}^{\text{TT}} & \rightarrow \text{SGWB probes} \end{cases}$$

Constraining DM physics

Signatures in $\Omega_{\text{GW}}(f)$, $\phi + \psi$



DM constraints

Inferring primordial spectra

$$P_{\zeta,h}^{\text{obs}}(k) = |T_{\zeta,h}^{\text{DM}}(k; \eta_0)|^2 P_{\zeta,h}^{\text{prim}}(k)$$



Requires accurate T^{DM}

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Requires accurate T^{DM}

What is the correct description of the dark matter sector?

Previous studies: backreaction of classical ULDM

Gravitational Potentials

Khmelnitsky & Rubakov (2014)

- Classical ULDM oscillations
→ oscillating ϕ, ψ
→ characteristic PTA timing signal.

Hwang & Noh (2022)

- Relativistic perturbative treatment
→ additional oscillatory component.

Gravitational Waves

Delgado (2023)

- Classical ULDM oscillations
→ oscillating ϕ, ψ
→ parametric enhancement of GW.

Jenks & Kamionkowski (2026)

- Classical ULDM with Gauss-Bonnet coupling
→ modified GW amplitude, phase and propagation speed.

From Quantum to Classical ULDM

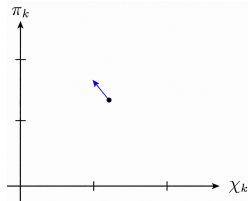
Is a classical field description enough?

ULDM $\left\{ \begin{array}{l} \text{Misalignment production} \\ \text{High-occupancy } (N_k \gg 1) \end{array} \right\}$ approximately classical state $|C\rangle$

$$\langle C | \hat{\chi} | C \rangle = \bar{\chi}$$

$$\langle C | \hat{\chi} \hat{\chi} | C \rangle_c \approx 0$$

Classical phase-space



From Quantum to Classical ULDM

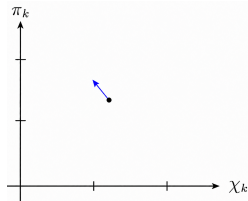
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Classical phase-space



BUT

$N_k \gg 1 \not\Rightarrow$ classical state

Inflation squeezes distribution

To what extent two-point correlations
influence **graviton dynamics**?

Semiclassical QFT description

1-loop 2PI action $\Gamma_{2\text{PI}}$ + in-in formalism

Quantum statistics $\rightleftharpoons$ Classical gravity

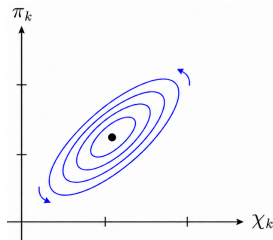
Gaussian state $|G\rangle$

$$\overbrace{\langle G|\hat{\chi}|G\rangle = \bar{\chi} \quad \langle G|\hat{\chi}\hat{\chi}|G\rangle_c = i\Delta}^{\text{condensate} \quad \text{two-point statistics}}$$

$\phi_0, \theta_0 \quad \quad N_k, S_k, \varphi_k$

$$i\Delta = \underbrace{i\Delta_V}_{\text{vacuum}} + \underbrace{i\Delta_N}_{\text{occupancy}} + \underbrace{i\Delta_S}_{\text{squeezing}}$$

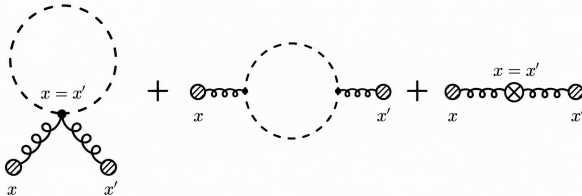
Phase-space distribution



1-Loop backreaction

Graviton self-energy

$$[\mu\nu\mathcal{L}^{\rho\sigma}](x;x') * h_{\rho\sigma}(x') = [\mu\nu\Sigma_R^{\rho\sigma}](x;x') * h_{\rho\sigma}(x')$$



Adiabatic retarded graviton self-energy $[\mu\nu i\Sigma_R^{\rho\sigma}]$:

- 21 tensorial structures;
- Vacuum, occupancy, squeezing and condensate contributions;
- Vacuum sector regularized through dimensional renormalization;
- Satisfies Noether-Ward identities (T. Prokopec, 2025).

Equations for gravitational potentials (ϕ, ψ)

Sub-Hubble modes

$$\begin{aligned}
 a^2 \nabla^2 \psi = & \kappa^2 \phi \left[\frac{3}{8} I_2^N + \frac{3a^2 m^2}{8} I_1^N + \frac{1}{8} I_2^S + \frac{a^2 m^2}{8} I_1^S \right] \\
 & + \kappa^2 3\psi \left[\frac{1}{8} I_2^N - \frac{1}{8} I_2^S + \frac{a^2 m^2}{4} I_1^S + \frac{\phi_0}{16a} a^2 m^2 (1 - 3 \cos[2(mt - \theta_0)]) \right] \\
 & + \text{NON-LOCAL TERMS.}
 \end{aligned}$$

$$\begin{aligned}
 a^2 (\nabla^2 \phi + 3(\partial_\eta + 2\mathcal{H})\partial_\eta \psi) \\
 = & \kappa^2 \phi \left[\frac{3}{4} I_2^N + \frac{3a^2 m^2}{8} I_1^N - \frac{1}{4} I_2^S + \frac{a^2 m^2}{8} I_1^S + \frac{\phi_0}{16a} a^2 m^2 (7 - 9 \cos[2(mt - \theta_0)]) \right] \\
 & + \kappa^2 3\psi \left[\frac{5}{12} I_2^N + \frac{3a^2 m^2}{8} I_1^N - \frac{7}{12} I_2^S + \frac{5a^2 m^2}{8} I_1^S + \frac{\phi_0}{16a} a^2 m^2 (7 - 9 \cos[2(mt - \theta_0)]) \right] \\
 & + \text{NON-LOCAL TERMS.}
 \end{aligned}$$

 Occupancy (adaibatic)

 Squeezing ($\omega \approx 2m$)

 Condensate ($\omega = 2m$)

Gravitational waves - Sub-Hubble modes

State-dependent effects

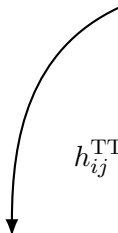
$$\left[\partial_\eta^2 + k^2 - \frac{2}{\eta^2} + \frac{\kappa^2}{3a^2} (I_2^N + I_2^S(\eta)) \right] a h_{ij}^{\text{TT}} = 0$$



Coherent state

(no coupling $\Rightarrow$ free propagation)

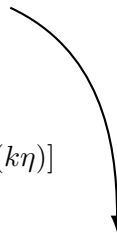
$$h_{ij}^{\text{TT}} = \frac{1}{k\eta} [A(k) j_1(k\eta) + B(k) y_1(k\eta)]$$



Mixed state

(graviton mass correction)

- amplitude correction δh
- phase correction $\delta \Phi$



Squeezed state

(oscillating graviton mass)

- parametric resonance

Gravitational waves - Pure squeezed ULDM state

Parametric resonance

$$S_k \approx N_k$$

$$\left\{ \partial_\eta^2 + k^2 - \frac{2}{\eta^2} + \frac{\kappa^2 \bar{P}_{\text{DM},0}}{a^3} [1 + \cos(2\Omega_s \eta)] \right\} a h_{ij}^{\text{TT}} = 0$$

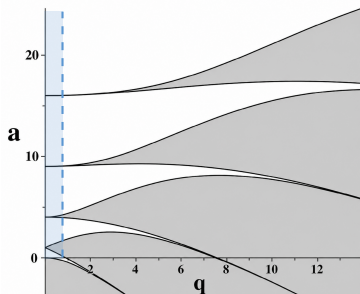
Mathieu equation

$$\left. \begin{aligned} \omega_{\text{res}} &= \Omega_s = \frac{am}{3} + \text{rel. corrections} \sim a(\eta) \\ \omega_h &= k + \text{sub-Hubble corrections} \sim \text{const} \end{aligned} \right\} \text{GW modes pass through the resonance band}$$

$$\text{Band width: } \Delta\omega \sim \frac{\kappa^2 \bar{P}_{\text{DM},0}}{2a^3} \implies \text{**narrow-band regime**}$$

Parametric resonance: enhancement results

Band structure¹



 Narrow-band regime: $q \ll 1$, $a > 0$

Mathieu coefficients

$$a \approx 4 \Omega_S^{-2} k^2$$

$$q \approx 2 \Omega_S^{-2} \kappa^2 P_{\text{DM},0} / a^3$$

Resulting enhancement²

- Modes undergoing enhancement:

$$\lambda_{\text{com}} \lesssim 10^3 - 10^6 \text{ pc.}$$

- **Max enhancement:**

$$\exp\left(\frac{\pi}{32} \frac{\kappa^4 P_{\text{DM},0}^2}{\Omega_S^3 \mathcal{H} a^6}\right) < 1 + 10^{-12}$$

¹M. Geeraerts and T. Van Doorsselaere (2021).

²Y. Shtanov, J.H. Traschen and R.H. Brandenberger (1995)

Conclusions

Take-away message:

- QFT 2PI framework provides a systematic way of estimating 2-point backreaction effects on gravitational perturbations.
- Squeezed ULDM induces parametric resonance on primordial gravitational waves.
- For a minimally coupled model in the sub-Hubble regime the relative amplification is below observable levels.

Phenomenology beyond this regime:

- Backreaction on super-Hubble GW modes.
- Quantum backreaction on scalar gravitational potentials.
- Non-minimal couplings and non-Gaussian states.

Thank you
for your attention