Gauge Symmetries and Gauge-Invariant Approaches

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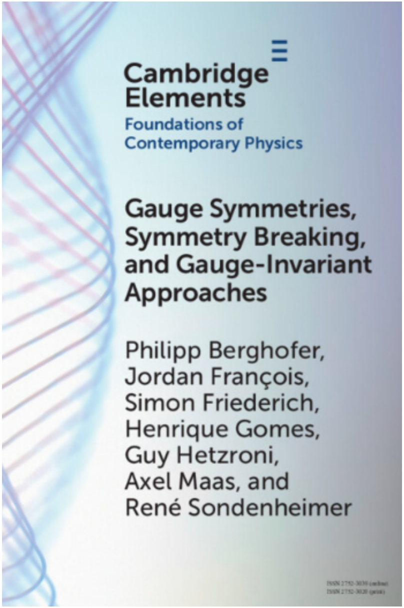
Contents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Gauge Symmetry, the Dressing Field Method, and the Uniqueness of Dressings

Recently, the dressing field method (DFM) has emerged as an effective mathematical tool for reducing gauge symmetries by constructing gauge-invariant variables from the field space of a theory. Assuming that DFM provides a suitable way to extract the physical content of a theory, we investigate its ontological implications. Prima facie, DFM suggests that extended dressed quantities constitute the fundamental building blocks of reality. A serious challenge to this interpretation, however, is that it remains unclear whether—and under which conditions—*unique* dressings are possible. This paper seeks to clarify that issue.

1. Gauge symmetries

- We conceptualize symmetry as invariance under transformation.
- Accordingly, types of symmetries can be distinguished according to types of transformations.

- global transformations: transformations that are performed identically at each point in spacetime.
- local transformations: transformations that are performed arbitrarily at each point in spacetime.

- A physical theory possesses a gauge symmetry if the theory remains invariant under a local transformation.
- A physical object is gauge-invariant if it remains the same with respect to a certain local transformation.

The significance of gauge symmetries

- Modern physics is written in the language of gauge field theories.
- The Standard Model is a gauge theory in the sense that is rests on *internal* local symmetries.
- General relativity is a gauge theory in the sense that it rests on an *external* local symmetry.

Interpreting gauge symmetries: A very broad distinction

- Realist interpretations: Interpretations of gauge theories that take gauge symmetries to be physically real and allow realist commitments to gauge variant quantities.

- Redundancy interpretations: Interpretations that consider gauge symmetries to be surplus mathematical structure and restrict realist commitments to gauge-invariant quantities.

The problem

The problem for realist interpretations is that for quantities on the same gauge orbit we can neither by theory nor by experiment determine which quantity is realized.

Gauge freedom

Gauge freedom implies that our gauge theory is mathematically underdetermined. This underdetermination corresponds to an indeterminism.

Desiderata

Any interpretation of gauge symmetries should aim at fulfilling the following desiderata:

- D1: To avoid ontological indeterminism.
- D2: To avoid ontological commitments to quantities that are not measurable even in principle.
- D3: To avoid surplus mathematical structure that has no direct ontological correspondence.

Remark

Interpretations that consider gauge variant quantities to be real struggle with D1 and D2.

Interpretations that restrict ontological commitments to gauge-invariant quantities typically struggle with D3.

Reducing gauge symmetries

- Gauge fixing and the mechanism of spotaneous symmetry breaking are established ways to reduce gauge symmetries.
- Both methods, however, face a number of problems such as the Gribov ambiguity or Elitzur's theorem.
- Alternative: Pursuing manifestly gauge-invariant approaches.

2. The received view

It has become the received view to say that while symmetries typically relate different physical states (such as in Galileo's famous ship example), *gauge* symmetries relate *mathematically distinct* descriptions to the *same* physical state (see Earman 2004, 1233; Norton 2003, 114; Maudlin 2002, 3)

The received view

There is some agreement among philosophers and physicists that

1. gauge symmetries do not constitute symmetries of nature,
2. are not physically real but rather are mathematical redundancy that can be used to describe reality but does not represent structures of reality,
3. only gauge-invariant quantities can be physically real quantities.

Physicist quotes

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- And Anthony Zee states that "gauge invariance is strictly speaking not a 'real' symmetry but merely a reflection of the fact that we used a redundant description," declaring that "gauge theories are also deeply disturbing and unsatisfying in some sense: They are built on a redundancy of description" (Zee 2010, 187).

Philosopher quotes

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- “formal redundancy” (Martin 2003)
- “descriptive fluff” (Earman 2004, 1239)

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Thus: Reformulate (particle) physics in a manifestly gauge-invariant fashion!

The gauge transformations of classical electrodynamics

The fields $\vec{E}$ and $\vec{B}$ can be represented by a pair of potentials $\vec{A}$ and V such that

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t},$$

$$\vec{B} = \nabla \times \vec{A}.$$

Here A is called the vector potential and V the scalar potential.
Due to the fact that the curl of a gradient is zero, there is a kind of underdetermination between A and B:

$$\vec{B} = \nabla \times \vec{A} = \nabla \times (\vec{A} + \nabla f)$$

for arbitrary smooth functions f.
This means that the same magnetic field can be represented by many mathematically distinct vector potentials.

The gauge transformations of classical electrodynamics

The following transformations are the gauge transformations of the theory:

$$\begin{aligned} \vec{A} &\to \vec{A} + \nabla f, \\ V &\to V - \frac{\partial f}{\partial t}. \end{aligned} \tag{1}$$

The fields $\vec{E}$ and $\vec{B}$ remain invariant under these transformations and Maxwell's equations remain satisfied: Maxwell's equations possess a gauge symmetry; $\vec{E}$ and $\vec{B}$ are gauge-invariant physical quantities.

QED

The Lagrangian for QED reads:

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu A_\mu\psi - m\bar{\psi}\psi. \tag{2}$$

- ψ: spinor field (electron–positron field)
- A_μ: electromagnetic (photon) field
- γ^μ: Dirac matrices
- m: electron/positron mass

Importantly, $\mathcal{L}_{\text{QED}}$ is invariant under the gauge transformations:

$$A_\mu(x) \to A_\mu(x) + \partial_\mu f(x), \tag{3}$$

$$\psi(x) \to e^{-ief(x)}\psi(x). \tag{4}$$

A concrete example

In QED, the gauge-variant electron field ψ can be turned into a gauge-invariant quantity via:

$$\psi^P(x) = D(x)\,\psi(x), \tag{5}$$

where $D(x)$ is the Dirac phase factor:

$$D(x) = \exp\left(-ie\int_x^\infty dy_\mu\, A_\mu(y)\right). \tag{6}$$

The resulting field $\psi^P(x)$ is gauge-invariant.

"Our first motivation for dressing the Lagrangian quarks is to note that they are not gauge invariant. The gauge symmetry shows that not all fields are physically significant. To produce a gauge invariant quark field it is necessary to dress the fermions with coloured gluons. In QED electrons must also be dressed with photons, as will be discussed below, but a major difference is that photons are not electrically charged, in contrast to the colour charged gluons." (Lavelle and McMullan 1997)

- QED: Dressing electrons with photons
- QCD: Dressing quarks with gluons.

"[T]he matter fields of QED are not gauge invariant and cannot be identified with physical particles. In previous papers we have shown that by a process of dressing the matter fields with a cloud made out of the vector potentials we can construct fields which are locally gauge invariant. Many such invariant fields can be manufactured. The next task is thus twofold: one must identify which of the gauge invariant fields correspond to physically significant variables and carry out the construction of these fields." (Bagan et al 2000)

"The idea is that in order to describe physical particles, the gauge noninvariant Lagrangian fields must be combined into gauge invariant composites, or dressed, fields." (Ilderton et al 2010)

4. The Dressing Field Method

Goal: Reduce or eliminate gauge symmetry by constructing gauge-invariant, composite fields.

Setup:

- Gauge theory with structure group H
- Fields (e.g. connection A, matter fields φ)
- Gauge transformations: $\gamma(x) \in \mathcal{H}$

Key properties:

- $u : M \to H$ s.t. $u^\gamma = \gamma^{-1} u$
- A^u, φ^u are **gauge invariant** (or transform under a reduced group)
- Gauge redundancy is **removed or reduced**
- The construction is a **field redefinition**, not a gauge fixing

Conceptual Insight

DFM replaces gauge-variant variables by composite fields that parametrize the **physical (reduced) configuration space**.

Gauge transformations

Let $P(M, H)$ be a principal bundle with structure group H.

Under $\gamma : M \to H$:

$$A^{\gamma} = \gamma^{-1} A \gamma + \gamma^{-1} d\gamma, \tag{7}$$

$$\phi^{\gamma} = \gamma^{-1} \phi. \tag{8}$$

- Lagrangian is invariant
- Fields transform nontrivially
- Physical observables should be gauge invariant

Motivation for the Dressing Field Method

- Bare fields are not gauge invariant
- Only gauge-invariant quantities are physically meaningful

DFM idea:

- Construct gauge-invariant variables directly from fields
- Extract the physical content of the theory

Dressing field and dressed fields

A dressing field satisfies:

$$u^{\gamma} = \gamma^{-1} u. \tag{9}$$

Define dressed fields:

$$A^{u} := u^{-1} A u + u^{-1} du, \tag{10}$$

$$\phi^{u} := u^{-1} \phi. \tag{11}$$

By construction:

$$(A^{u})^{\gamma} = A^{u}, \qquad (\phi^{u})^{\gamma} = \phi^{u}.$$

- Gauge symmetry is removed
- Physical (gauge-invariant) variables obtained

Dressings vs gauge transformations

Despite the formal similarity

$$A^\gamma = \gamma^{-1} A \gamma + \gamma^{-1} d\gamma,$$
$$\phi^\gamma = \gamma^{-1} \phi.$$

$$A^u := u^{-1} A u + u^{-1} du,$$
$$\phi^u := u^{-1} \phi.$$

- $u \notin \mathcal{H}$
- Not a gauge transformation
- Not a gauge fixing

Dressings construct new variables rather than selecting a gauge.

5. Philosophical implications

- Not the (excitations of) the gauge variant "elementary" fields we know from the SM constitute what is physically fundamental but the built-up gauge-invariant fields that result form the elementary fields being dressed up by the dressing fields.
- For instance, electrons would no longer be conceived as excitations of the elementary (non-gauge-invariant) electron field but of the built-up gauge-invariant field consisting of the elementary electron field "dressed up" by a photon cloud.
- Reality ultimately consists of extended objects, not point-like particles.
- Eliminating gauge-dependent objects as physical objects might well be the most consequential shift in the way in which we portray nature since the advent of quantum field theory.

Two possible interpretations

Interpretation 1

Only the composite (dressed) field is physically real. The gauge-variant elementary field and the dressing field are merely *mathematical tools*.

Interpretation 2

The gauge-variant elementary field and the dressing field possess physical reality. Through their interaction, something *observable* and mathematically determinate emerges.

Open Question

If unique dressings are possible (for our best theories), does this have an impact on which interpretation to prefer?

Residual transformations of the first kind

Suppose the dressing removes only $K \subset H$.

Remaining symmetry:

$$J := H/K. \tag{12}$$

- Genuine gauge symmetry
- Acts nontrivially on dressed fields

Interpretation: First kind

- Residual symmetry of the first kind = leftover gauge symmetry
- Indicates incomplete dressing

Conceptually unproblematic:

- Just means not all gauge freedom has been removed

Transformations of the second kind

Even after removing all gauge symmetry, there may remain an ambiguity in the choice of dressing field.

If u is a dressing field, for any other possible dressing field u' we have

$$u' = u\xi, \tag{13}$$

with $\xi^\gamma = \xi$.

Define:

- $\mathcal{G}$: group of such ξ

Action

On dressing:

$$u^{\xi} := u\xi. \tag{14}$$

On dressed fields:

$$(\phi^{u})^{\xi} := \phi^{u\xi}, \tag{15}$$

$$(\phi^{u})^{\xi} := (\phi^{\xi})^{u\xi}. \tag{16}$$

Interpretation: Second kind

- Parametrize ambiguity in dressing choice
- Not inherited gauge symmetry
- New type of symmetry-like structure

Interpretation: Second kind

- Parametrize ambiguity in dressing choice
- Not inherited gauge symmetry
- New type of symmetry-like structure

- $\mathcal{G}$ trivial $\Rightarrow$ unique dressing
- $\mathcal{G}$ discrete $\Rightarrow$ finite choices
- $\mathcal{G}$ continuous $\Rightarrow$ infinitely many dressings

QED: No unique dressing!

"The gauge-variant operator $\phi(x)$, which can be thought of as creating a charged particle at x, can be dressed to give an operator

$$\Phi(x) = \phi(x)e^{iqV(x)}$$

where the electromagnetic dressing $V(x)$ is a functional of A_μ. [...] The dressed operator can be thought of as creating a particle, together with its EM field, in some configuration determined by the particular choice of dressing $V(x)$. There are infinitely many such possible dressings." (Giddings and Weinberg 2020, 7)

Dressing ambiguity $\neq$ Gauge indeterminacy

Although there may not be unique dressings, dressing ambiguities differ fundamentally from gauge redundancy!

Gauge transformations

- Relate different descriptions of the **same physical state**.
- Lead to indeterminism if gauge-variant quantities are taken as real.

Dressing transformations

- Change the **definition of the physical observable**.
- Correspond to different constructions of gauge-invariant operators.

Once a dressing is chosen:

- The dressed state evolves uniquely under the dynamics.
- No indeterminacy in time evolution arises.

Conceptual Distinction

	Gauge Redundancy	Dressing Ambiguity
Origin	Gauge symmetry	Non-unique dressing
Meaning	Redundant description	Different observables
Physical states	Identical	Possibly distinct constructions
Effect on dynamics	Apparent indeterminism	Deterministic evolution
Role in theory	Must be removed	Reflects freedom in representation

Toward a third relational interpretation?

Interpretation 1

Only the composite (dressed) field is physically real. The gauge-variant elementary field and the dressing field are merely *mathematical tools*.

Interpretation 2

The gauge-variant elementary field and the dressing field possess physical reality. Through their interaction, something *observable* and mathematically determinate emerges.

Interpretation 3

None of the fields is ontologically fundamental.
What is fundamental are the gauge-invariant relations encoded in the dressed fields.

Conceptual Payoff

- Gauge invariance is achieved relationally
- Degrees of freedom *co-define* each other
- No need for:
  - eliminativism (Interpretation 1)
  - reification of auxiliary structure (Interpretation 2)

Conclusion

The DFM naturally supports a form of relational ontology:

only invariant relations among degrees of freedom are physically meaningful.

Thank You!

DFM and Relational Interpretation (Internal Symmetries)

With ϕ-dependent dressing fields, the DFM admits a natural relational interpretation.

The dressed fields

$$\phi^{u[\phi]} = \{A^{u[A,\varphi]}, \varphi^{u[A,\varphi]}\} \tag{17}$$

result from a *reshuffling of degrees of freedom* of the bare fields ϕ.

- Physical and gauge (purely redundant) d.o.f. are mixed
- Pure gauge modes are eliminated (partially or fully)

Relational Meaning (Internal Case)

Crucially, some internal d.o.f. of ϕ are used to define the dressing field $u[\phi]$.

These are then used to coordinatize the remaining d.o.f. of ϕ:

$$\phi \quad \longrightarrow \quad \phi^{u[\phi]} \tag{18}$$

Key idea

The physical content is encoded in relations among field degrees of freedom, not in the individual fields themselves.

Thus, $\phi^{u[\phi]}$ are:

gauge-invariant relational variables

Internal Point-Coincidence Argument

Dressed fields achieve gauge invariance relationally:

- Physical d.o.f. *coordinatize* and *co-define* each other

Interpretation

This implements the internal point-coincidence argument:

- Only coincidences between field values are physical
- Gauge-dependent structure is eliminated

$\Rightarrow$ Immunity to the internal hole argument

All of this is achieved already at the *kinematical level*.

Point-Coincidence Argument (GR Case)

Dressed fields achieve Diff(M)-invariance relationally:

- Spatiotemporal d.o.f. co-define each other
- Physical content = coincidences of field values

Interpretation

This formally implements the point-coincidence argument:

- Only coincidences of field values are observable
- Manifold points have no independent physical identity

$\Rightarrow$ Immunity to the hole argument

Again, this is achieved at the *kinematical level*.

Summary: Why DFM is Relational

- Dressing fields are constructed from the fields themselves
- They use part of the system to define the rest
- Gauge invariance is achieved relationally, not by reduction alone

Conceptual Payoff

- Physical observables = relations among degrees of freedom
- Direct implementation of (generalized) point-coincidence arguments
- Immunity to (generalized) hole arguments

DFM = a constructive realization of relationalism

OSR?

The ontological picture presented here does not follow the traditional substantivalist versus relationalist dichotomy.

Rather, it suggests a relationalist realist view of a principal bundle as representing the manifold of physical spatio-temporal and internal events, which supervene on the fundamental physical fields of the theory.

The d.o.f. of these fields are coextensive with the mutually co-defining network of their (invariant) relations.

This may nicely align with a “moderate” form of ontic structural realism regarding the fundamental fields of gRGFT, regarding relata and relations as inseparable and on equal-footing ontologically.

Stachel (2014): “dynamic structural realism”