Foundations of General-Relativistic Gauge Field Theory

Torino, March 2026

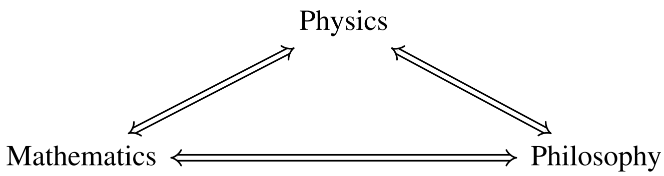
Opening remarks

(By Jordan François – Edited and slightly expanded)

We are pleased to welcome you all for this event which, judging by the turnout, is already a small success. Any additional measure of it will be due to our lecturers and participants of course, but overall success of this initiative should be credited first to my co-organisers: Above all to Dr. Lucrezia Ravera, from DISAT here at Polito, not only for being at the origin of idea of this school, but for bearing most of the responsibilities and cognitive load required for the organization, down to the last detail: most, if not all, of you will indeed have interacted primarily with her. Then, warm thanks go to Dr. Vincenzo Antonelli, from DISMA at Polito, who when rooms became hard to find stepped-in to offer us the hospitality of the mathematics department; which happens to be in line with the spirit of this school.

Speaking of, a word about the idea behind this school: We aim to reawaken, or revitalize, among the younger generation of aspiring academics, awareness of a tradition grounded in the idea that fundamental (theoretical and mathematical) Physics is best appreciated, understood, and ultimately *practiced*, when paying close attention to both the technical and conceptual foundations of our *best*, i.e. *empirically established*, *theoretical frameworks*. That is, the best practice for those willing to study and contribute to fundamental physics, is to aim for as strong as possible a command of; first, A) the mathematical language & structures in which it is nested; and second, B) the philosophical network of questions and insights that innervates/permeates it, or often **emerges** from it.

The fruitfulness of this tradition is abundantly attested by history of Physics and Mathematics, as well as of Philosophy thereof, as it documents the synergy and rich organic relations between the three disciplines.[1]



Neglecting the conceptual depth and rigor of A) and/or B) is thus to miss the full beauty of the complete picture.

One may indeed profitably remember the extent of the cross-fertilization between Physics and Mathematics, and how much the former is indebted to the numerous contributions of mathematicians! Many will effortlessly come to mind (the portraits of some being in this very room): e.g. Newton, Euler, Laplace, Lagrange, Gauss, Fourier, Hamilton, Grassmann, Riemann, Maxwell, Heaviside, Poincaré, Minkowski, Hilbert, Noether, Levi-Civita, Weyl, É. Cartan, Wigner [Physics Nobel 1963], Von Neumann, etc. But also in closer times Penrose [Physics Nobel 2020], Atiyah, Singer, Berezin, etc. This illustrates the obvious; mastery of mathematics – the language of Nature, per Galileo's historic words – makes it easier to do good (even great) physics.

No less important is the inclination to conceptual thinking stemming from philosophical literacy, which played a key role in the establishment of new paradigmatic frameworks – a.k.a."revolutionary periods" of Science, per the well known terminology of T. Kuhn. Many regarded as "founding fathers" of XXth century physics were indeed highly educated in, and sensitive to, philosophical thinking, which decisively influenced their physics and breakthroughs. This shows most clearly with Einstein, for both relativity theories (1905, 1915) and his contributions to quantum theory; e.g. the EPR paper of 1935 sprang from his longstanding technico-conceptual debate with Bohr in the 20s and early 30s (including the famous episode of the 1927 Solvay conference). Einstein and Bohr were not outliers in their inclinations; of the same cloth from their great generation were notably Schrödinger (1926, 1935) and Weyl (1918, 1929), and in the younger generation Heisenberg (1925, QM) and F. London. Later still, consider e.g. Bohm, Aharonov (AB effect 1959), Everett (1957), and of course Bell (1964).

[1] It is may be useful to recall that, while the terms "mathematician" and "philosopher" date back to greek antiquity, the terms "physicist" and "scientist" were coined in 1833-34 by the English polymath William Whewell and systematically used in his 1840 book "*The Philosophy of the Inductive Sciences*" – in particular "Scientist" as a practitioner of Sciences was devised in analogy to "artist" as a practitioner of Arts. Before then, those studying the natural world were referred to as "natural philosophers", and Physics called "*natural philosophy*"; Newton's *magnus opus* is entitled "*Philosophiæ naturalis principia mathematica*", i.e. "*Mathematical principles of natural philosophy*".

These examples – and similar others revealing deep relations between Mathematics and Philosophy[2] – shows that, though pragmatic given human limitations, strict (academic) compartmentalization of sciences is ultimately artificial, especially so near the boundaries, and should be ignored when knowledge, in its unity, is pursued *from first principles* and with rigor and clear thinking.

Our topic here is *general-relativistic gauge field theory* (gRGFT) which is, besides quantum theory, the most fundamental playground for modern physics. This is because it ties together some of the most basic notions with deep ideas: the structure of spacetime and matter/fields/energy with *local* symmetries (diffeomorphisms & gauge symmetries). Dealing with such fundamental matters requires sophisticated tools, both technical/mathematical and conceptual/philosophical. The payoff are an unprecedented unity of physics and unexpected new insights.

Our main lecturer will each try to give you a taste of a different aspects of gRGFT:

Dr. Marc Geiller, from ENS Lyon, FR, will talk about the fascinating topic of Noether theorems, conserved charges, and asymptotic symmetries, which has elicited a renewed interests in the past few year, in part due to its clear connection/applications to gravitational waves astrophysics which started in earnest in September 2015.

Dr. Philipp Berghofer, from Graz University, AT, will expose you to the fundamentals of philosophy of physics, especially on the nature of space or spacetime, as well as to the subtlety symmetries – especially gauge/local symmetries – introduce in our understanding of the picture of the world depicted by our best theories.

Dr. Ravera will tackle the issue of systematically extracting the physical content of a theory, i.e. that which is invariant under local symmetries. In particular, she will introduce you to an a new approach that, though you are unlikely to have seen elsewhere, constitute a broad framework with many applications, both retrospective and new.

I will start by laying the basic technical ground of the topic at hand; the differential geometry of connections on principal fiber bundles. Most of it is textbook material, only here articulated in a more systematic and logically progressive way, some of it you will probably have not yet seen elsewhere.

These lectures, alongside the several contributions by guests and participants, is our program for this three days event. In addition to this food for thought, we also propose to feed the body via the excellent catering offered during coffee breaks and at lunch. We hope you enjoy all of it.

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[2]History of which shows, one the one hand, that philosophers made fundamental contributions to Mathematics; like Descartes in 1637 with the algebraicization of geometry (which Newton despised!), or Leibniz in the late 1670 with the integro-differential calculus and our modern notations (which Newton disliked!), or yet Russell in the early XXth (set theory, *Principia*), etc. And, on the other hand, it reminds us that the very basis of Mathematics have been revamped, from the late XIXth century to almost the mid XXth century, in light of deep foundational philosophical questions: This is the so-called "foundational crisis of mathematics", leading to the development of Set theory (Cantor, Dedekind, Zermelo, Frankel), formal Logic (Frege, Gödel, Cohen), Computation theory (Church, Turing), and onward to Category theory (H. Cartan, Eilenberg, Grothendieck, Lawvere) and modern Homotopy Type Theory (originating, again, with Russell).