Fine-tuning in inflation:
effects, constraints, and case studies

Wenqi Ke

University of Minnesota

Based on:
2504.12283,
2510.15137,
2603.02389

SI 2026

Two related topics:

I – Inflation is fine-tuned

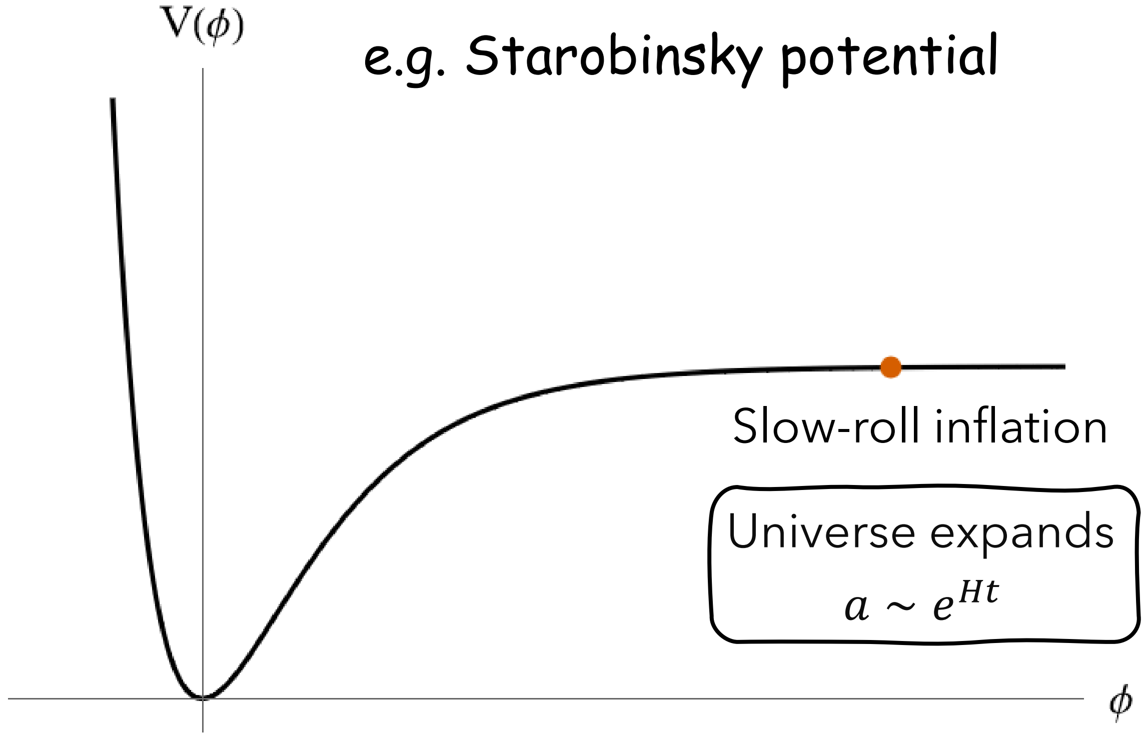
How much fine-tuning is allowed?

II – Inflation gets radiative corrections

What's the constraint on the couplings?

Single-field inflation

e.g. Starobinsky potential



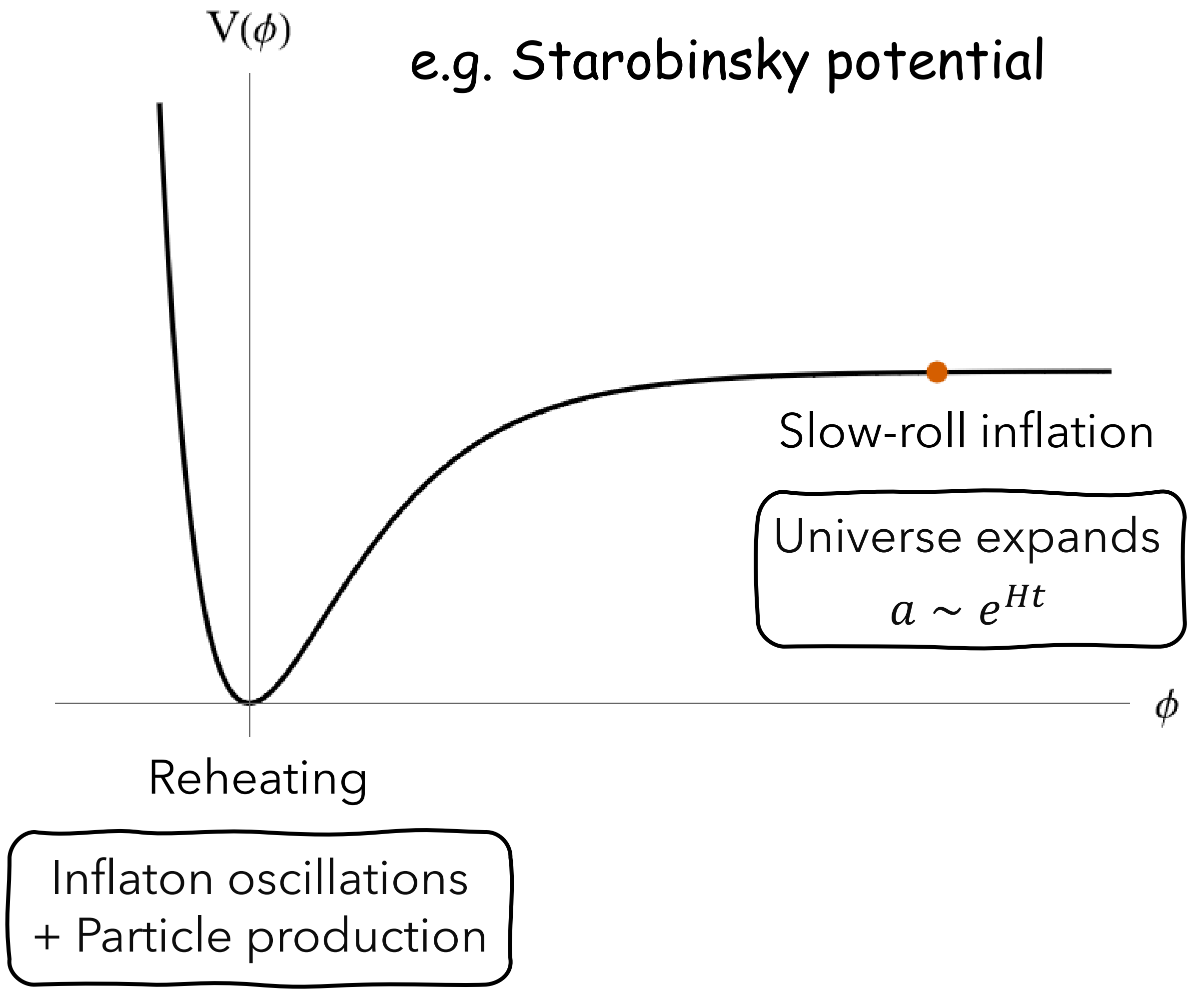
Slow-roll inflation

Universe expands
$a \sim e^{Ht}$

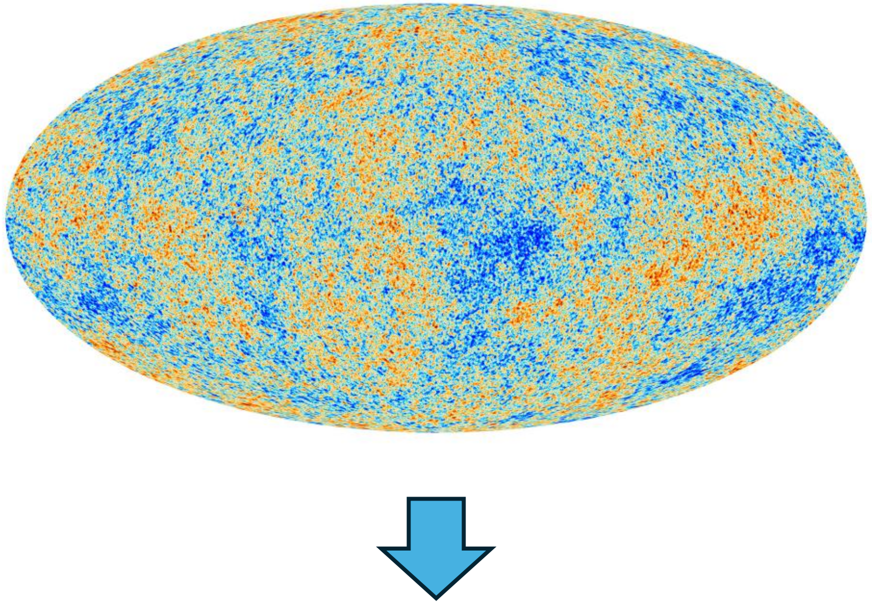
Reheating

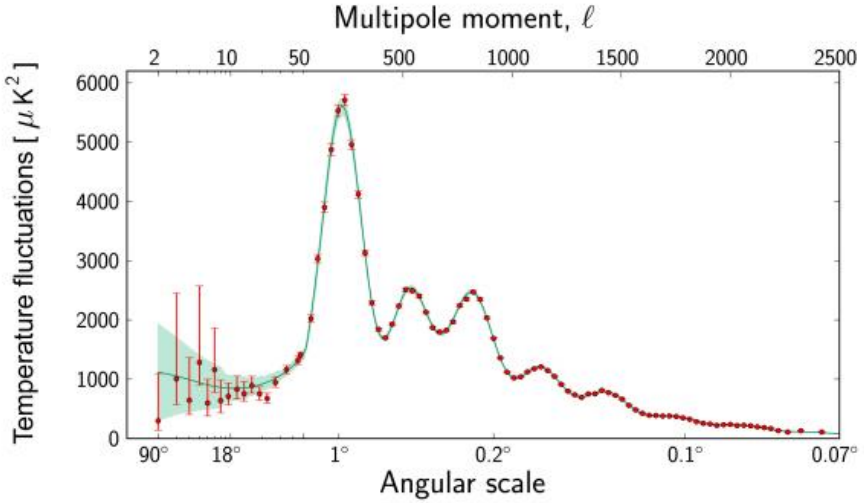
Inflaton oscillations
+ Particle production

Single-field inflation



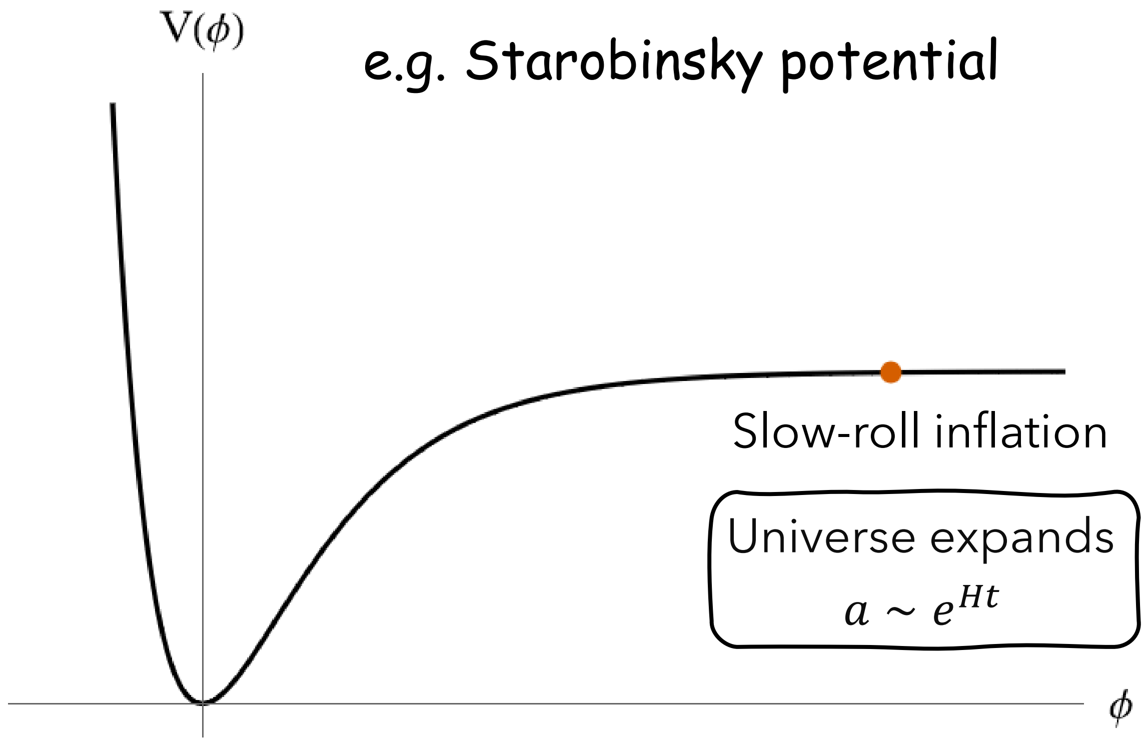
Inflation is testable





Single-field inflation

e.g. Starobinsky potential



Slow-roll inflation

Universe expands
$a \sim e^{Ht}$

Reheating

Inflaton oscillations
+ Particle production

CMB observables:

Spectral index: $n_s = n_* \simeq 1 - 6\epsilon_* + 2\eta_*$

Tensor-to-scalar ratio: $r = r_* \simeq 16\epsilon_*$

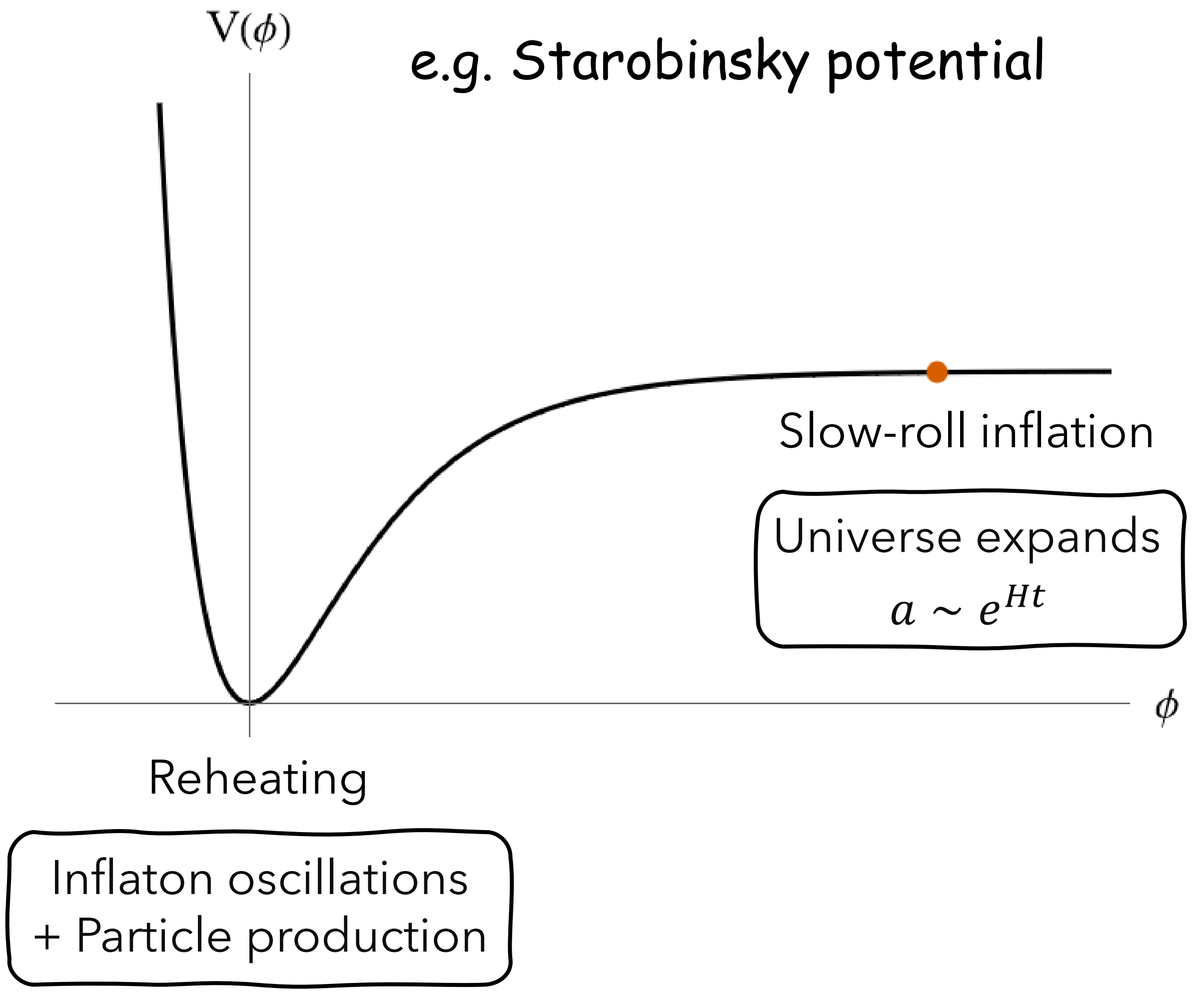
at the pivot scale

Important quantities:

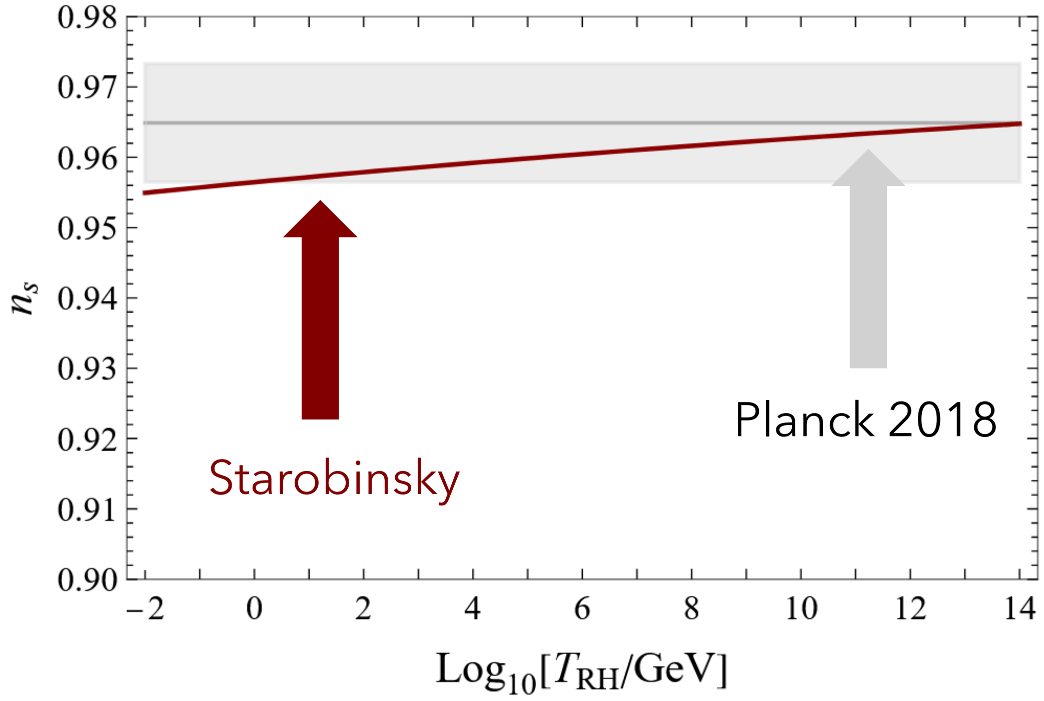
Reheating temperature
$$\rho_\phi(T_{RH}) = \rho_{rad}(T_{RH})$$

Number of e-folds $N(a) \equiv \ln(a_{\text{end}}/a)$

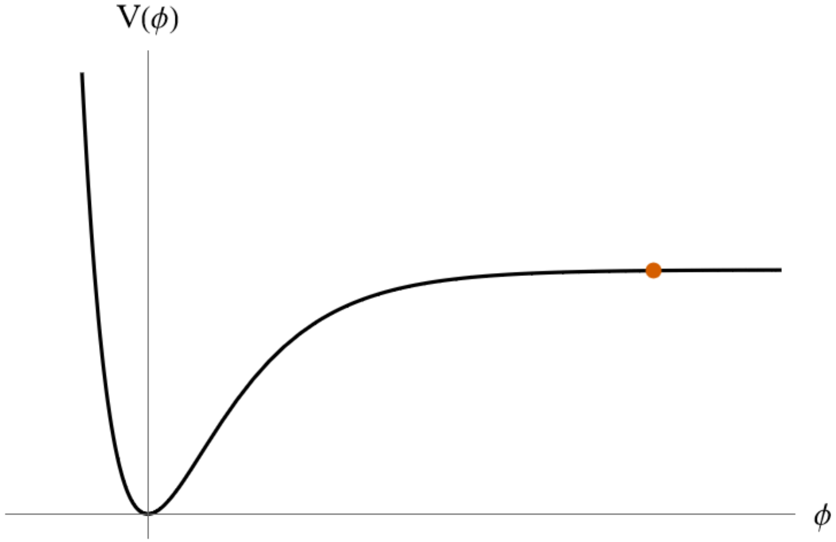
Single-field inflation



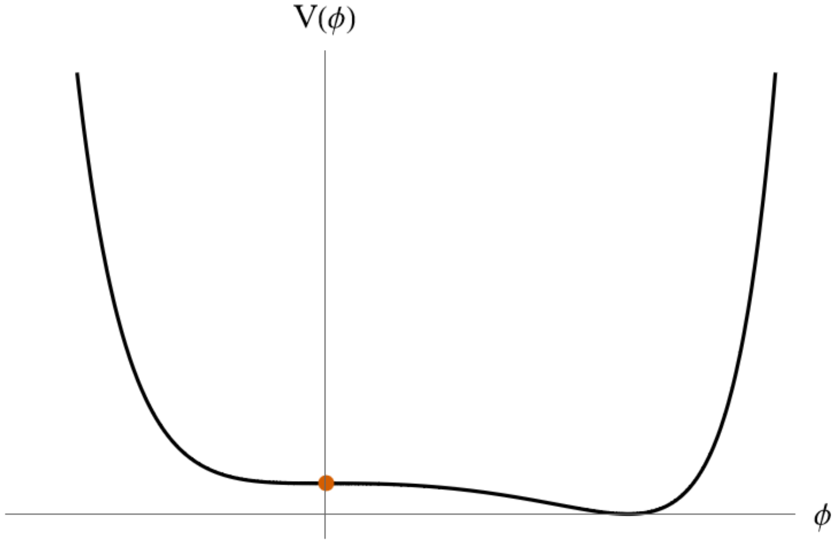
So far very successful



Inflation is fine-tuned, for example:



The potential has to be **flat enough**



The inflaton should sit at the right place **initially**

Linde, Westphal [0712.1610]

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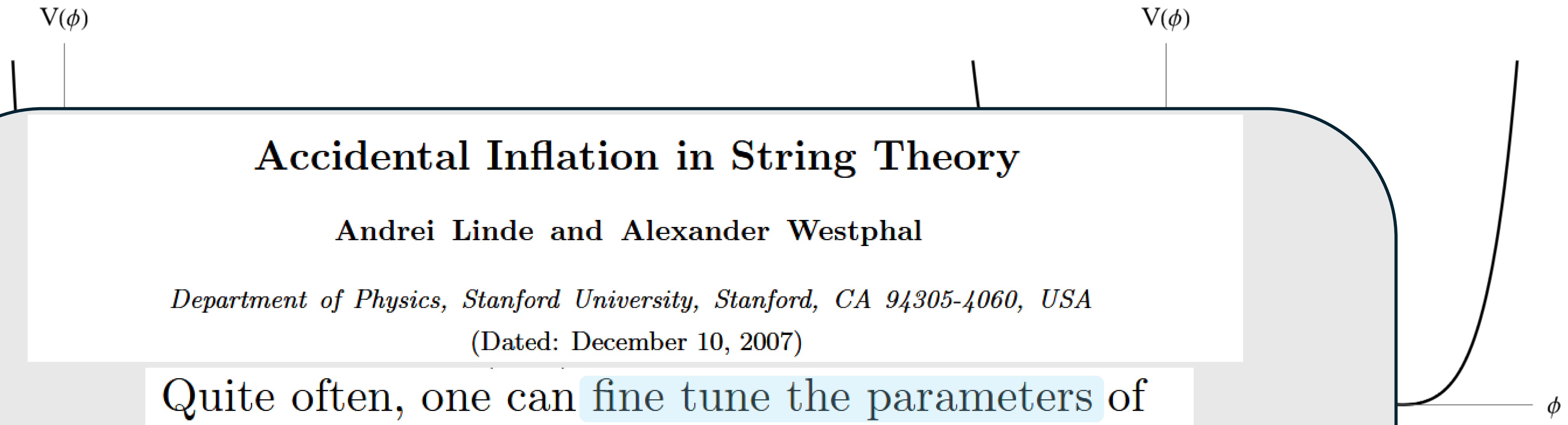


Accidental Inflation in String Theory

Andrei Linde and Alexander Westphal

Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

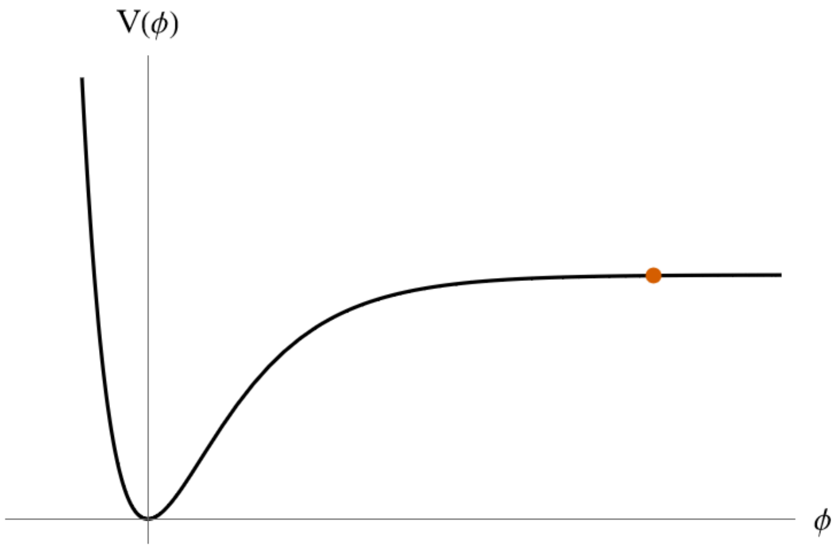
(Dated: December 10, 2007)

Quite often, one can fine tune the parameters of the theory and achieve desirable flatness of the potential for some small range of the values of the scalar fields. In such models, inflation in a certain sense happens by accident: One must have the parameters of the models fixed in a very narrow range, and in addition, one should hope that the scalar field from the very beginning by some happy accident was in the required range of its values where inflation is possible.

at the

Westphal [0712.1610]

Inflation is fine-tuned, for example:



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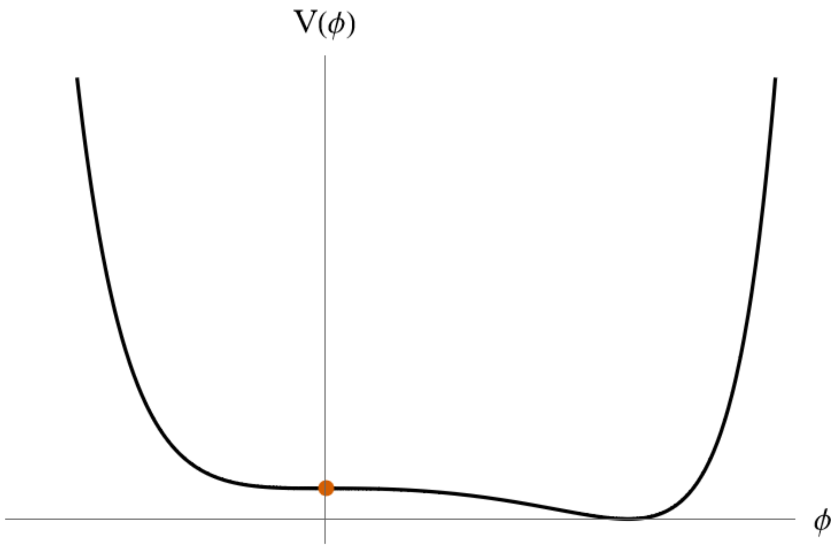


The inflaton should sit at the right place **initially**

Linde, Westphal [0712.1610]

Many ways to *deform* the potential:

Higher curvature terms (R^n), radiative corrections (2nd topic), higher dimensional operators, or just a tiny change of the coupling constants (1st topic)…

How much can the parameters vary?

Example: Starobinsky inflation

Two ways to get the potential:

1 Modified gravity

$$A = \int d^4x \sqrt{-g}\left(-\frac{1}{2}R + \frac{\alpha}{2}R^2\right)$$

Starobinsky potential

$$A = -\frac{1}{2}\int d^4x \sqrt{-\tilde{g}}\left[\tilde{R} - \partial^\mu\phi\partial_\mu\phi + \frac{1}{4\alpha}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2\right]$$

How much can the parameters vary?

Example: Starobinsky inflation

Two ways to get the potential:

2 Supergravity

Kähler potential

Superpotential

$$K = -3\ln(T + T^* - |\chi|^2/3)$$

$$W = M\left(\frac{1}{2}\chi^2 - \frac{\lambda}{3\sqrt{3}}\chi^3\right)$$

λ is a free parameter

Scalar potential: $V = e^K[g^{m\bar{n}} D_m W D_{\bar{n}} W^* - 3|W|^2]$

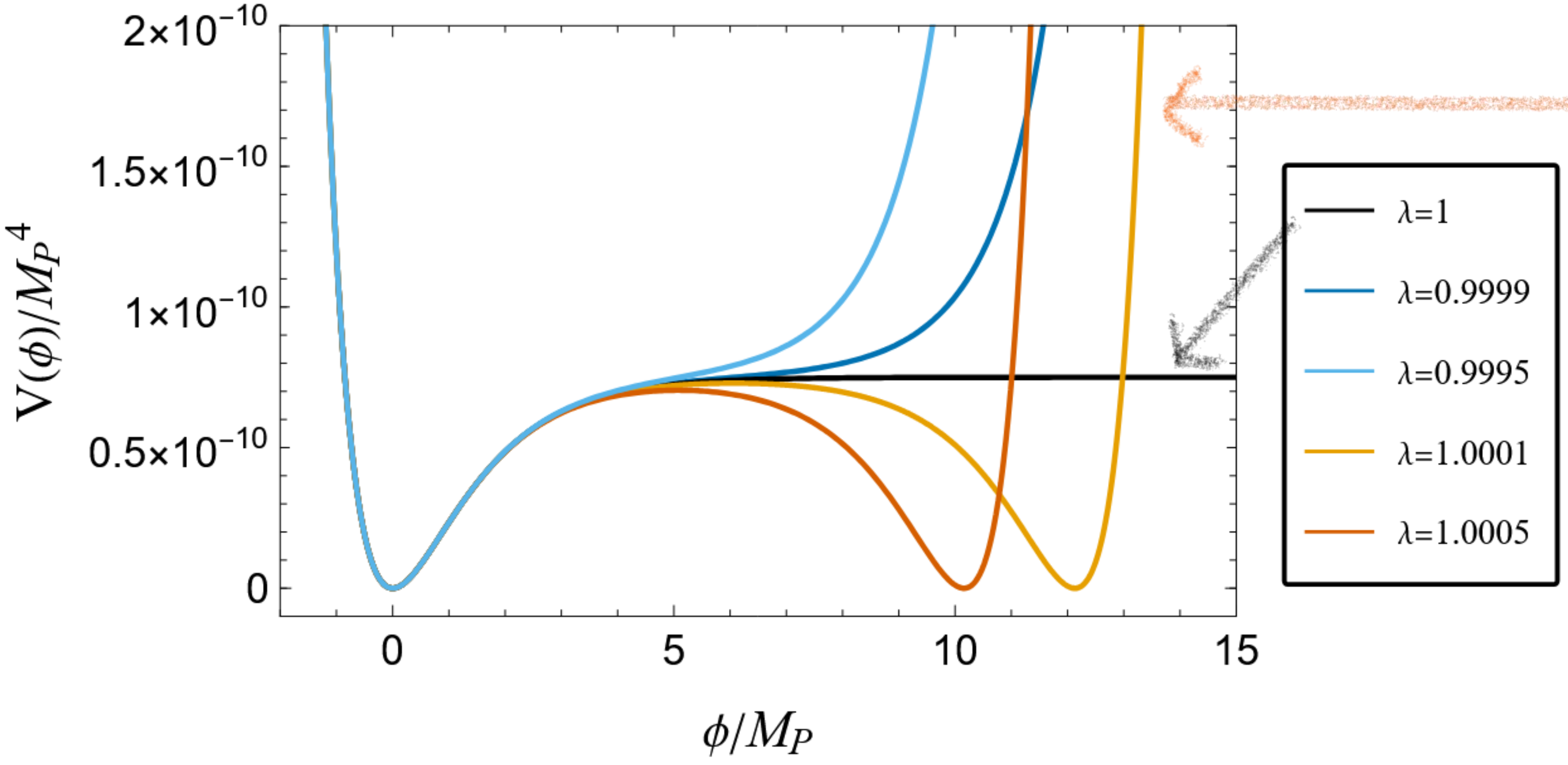
$\Rightarrow$ **Starobinsky potential if $\lambda = 1$**

How much can the parameters vary?

$\lambda = 1$ gives Starobinsky $\longrightarrow$ What about $\lambda \neq 1$?

I. Antoniadis, J. Ellis, WK, D. Nanopoulos, K. Olive [2504.12283]

General scalar potential: $$V(\phi) = 3M^2 \sinh^2(\phi/\sqrt{6}) \left(\cosh\left(\phi/\sqrt{6}\right) - \lambda \sinh\left(\phi/\sqrt{6}\right)\right)^2$$



0.0001 change in λ makes a big difference!

$\lambda \neq 1$:

- Smaller flat portion

Inflation may not be successful

- A barrier appears!

Affects the initial condition

How much can the parameters vary?

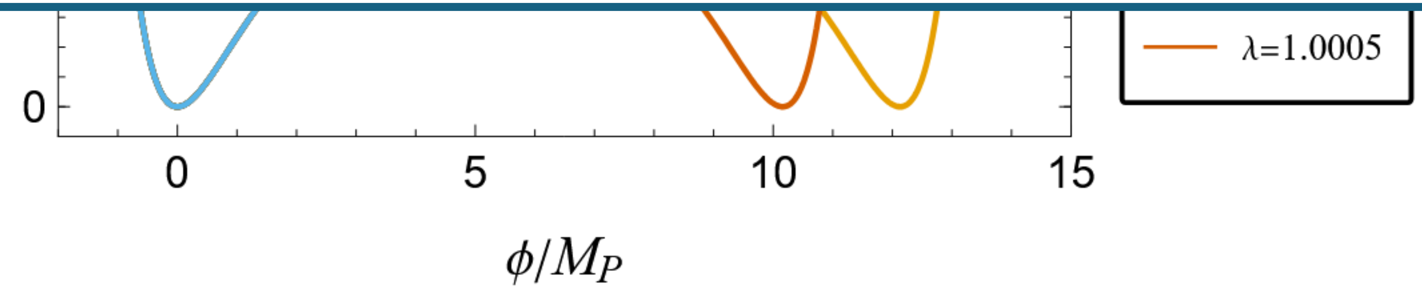
Compare with the experiments!

2018 Planck+BICEP/Keck: $n_s = 0.9649 \pm 0.0084$ (2σ)

$r < 0.036$

Last year

ACT DR6 (DESI+Planck): $n_s = 0.9743 \pm 0.0068$ (2σ)



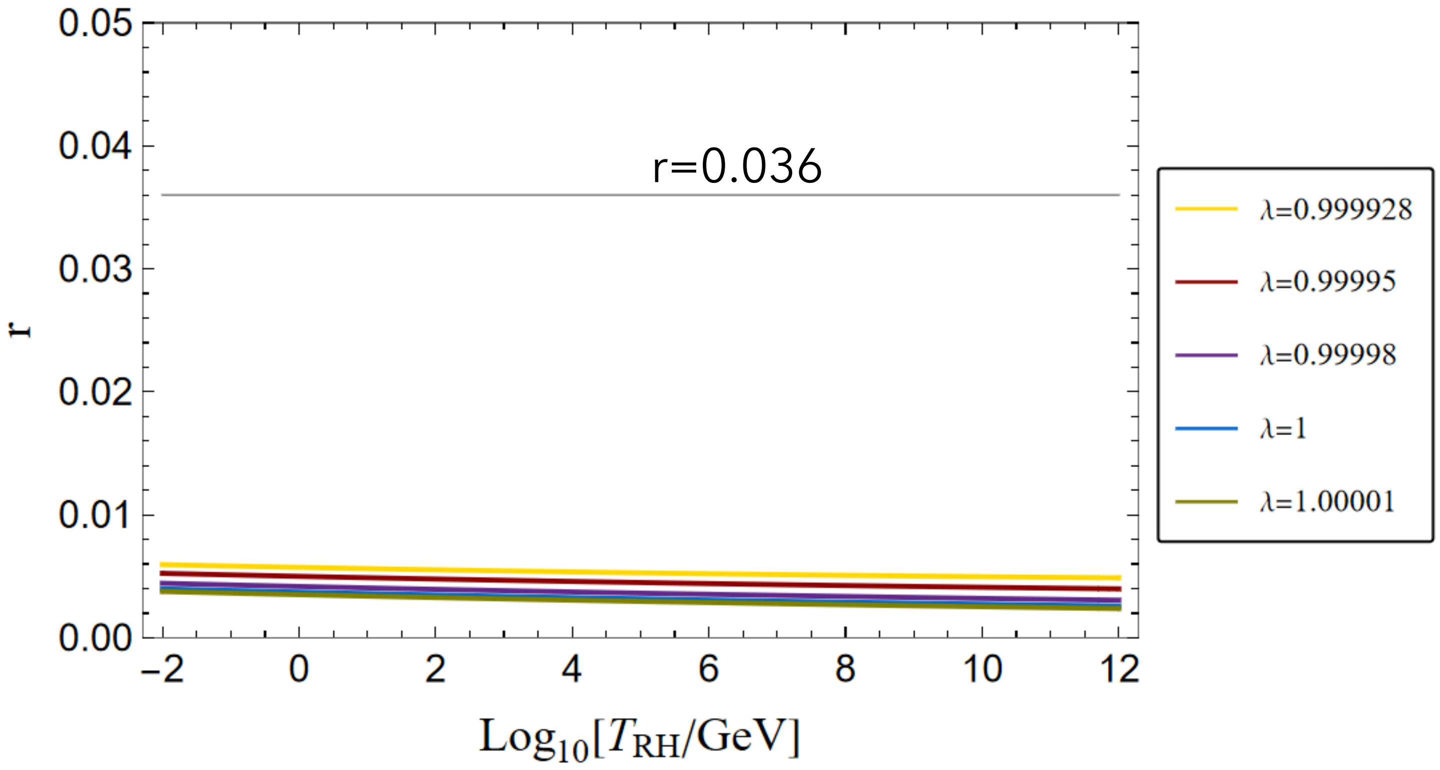
essful

- A barrier appears!

Affects the initial condition

CMB observables for different λ

$$V(\phi) = 3M^2 \sinh^2(\phi/\sqrt{6}) \left(\cosh\left(\phi/\sqrt{6}\right) - \lambda \sinh\left(\phi/\sqrt{6}\right)\right)^2$$

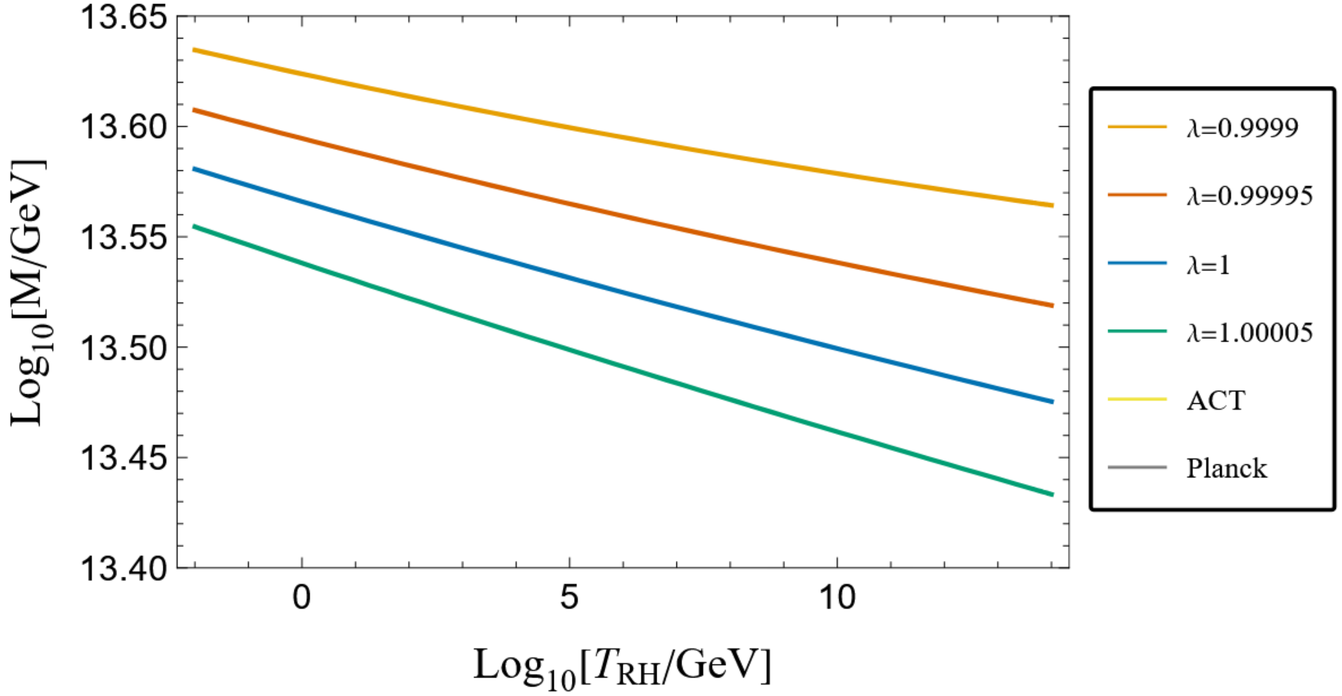


r doesn't vary so much compared to 0.036

We can't see 'fine-tuning' here

CMB observables for different λ

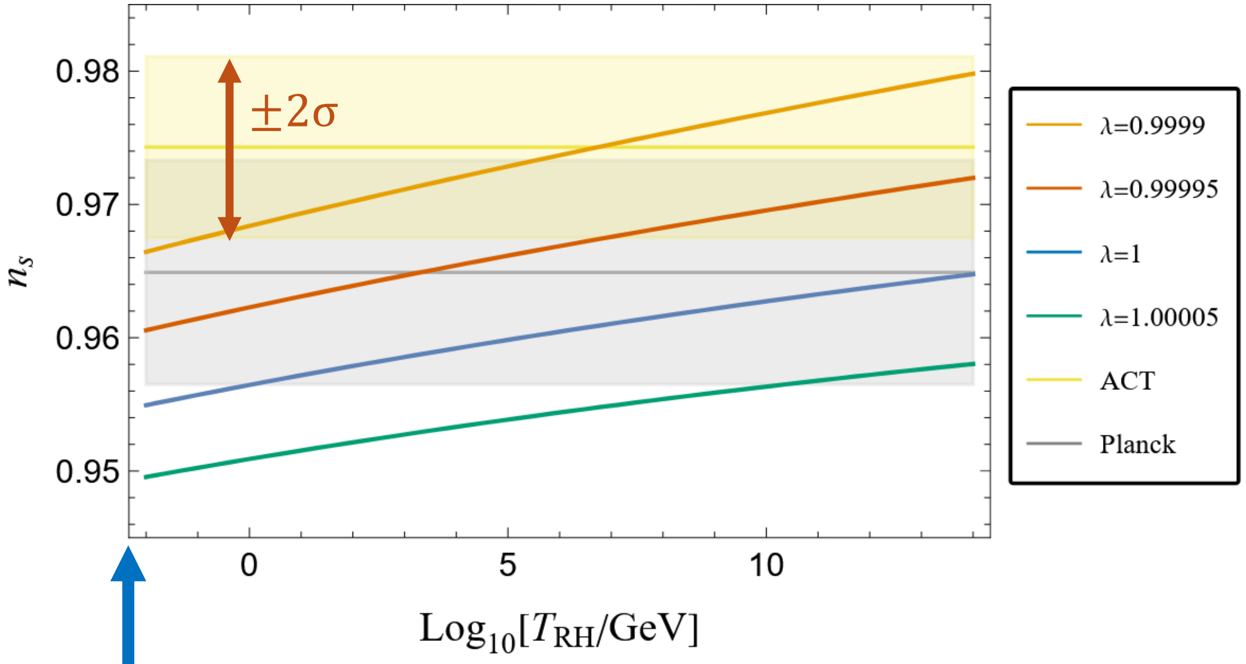
$$V(\phi) = 3M^2 \sinh^2(\phi/\sqrt{6}) \left(\cosh\left(\phi/\sqrt{6}\right) - \lambda \sinh\left(\phi/\sqrt{6}\right)\right)^2$$



M (related to the scale of inflation) isn't so sensitive either

CMB observables for different λ

$$V(\phi) = 3M^2 \sinh^2(\phi/\sqrt{6}) \left(\cosh\left(\phi/\sqrt{6}\right) - \lambda \sinh\left(\phi/\sqrt{6}\right)\right)^2$$



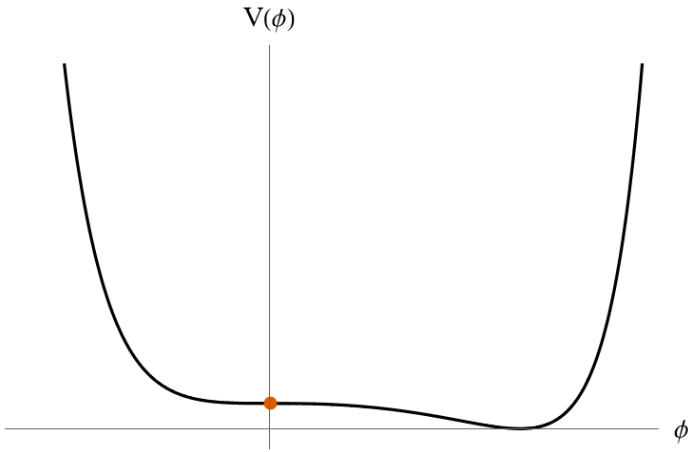
But n_s does vary a lot!

For *ACT* bound, $\lambda = 1$ (Starobinsky) is disfavored, and $\lambda \simeq 0.9999$ is prefered

Lower bound on T_{RH} from BBN → Lower bound on $\lambda > 0.99984$ (combined with Planck bound)

Not just one model!

$$V(\phi) \simeq M^2\left(1 - 4\phi^3 + \frac{13}{2}\phi^4 + \cdots\right)$$



❖ Example of « new inflation »

Holman, Ramond, Ross (1984)

❖ ***Ruled out*** because $n_s \simeq 0.91$

↓

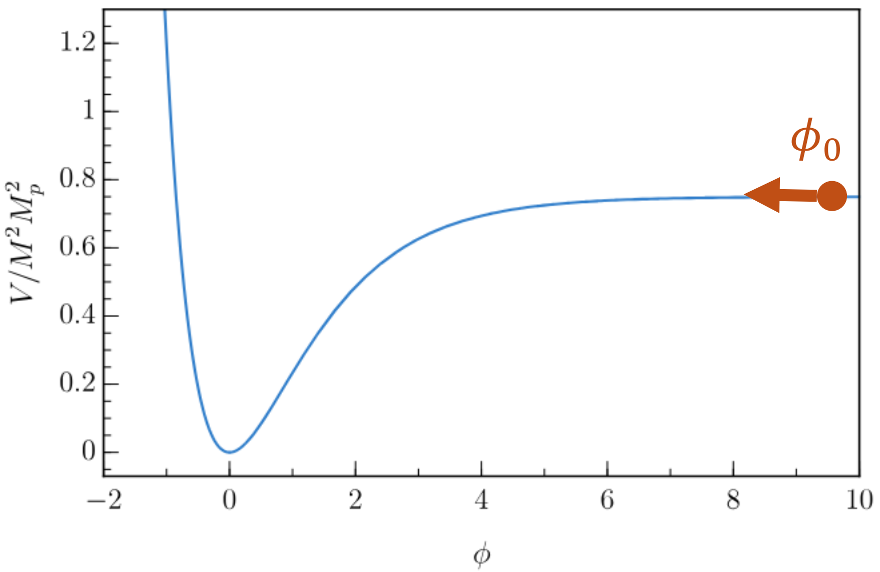
Fine-tuning can **save** this model

and many other examples…

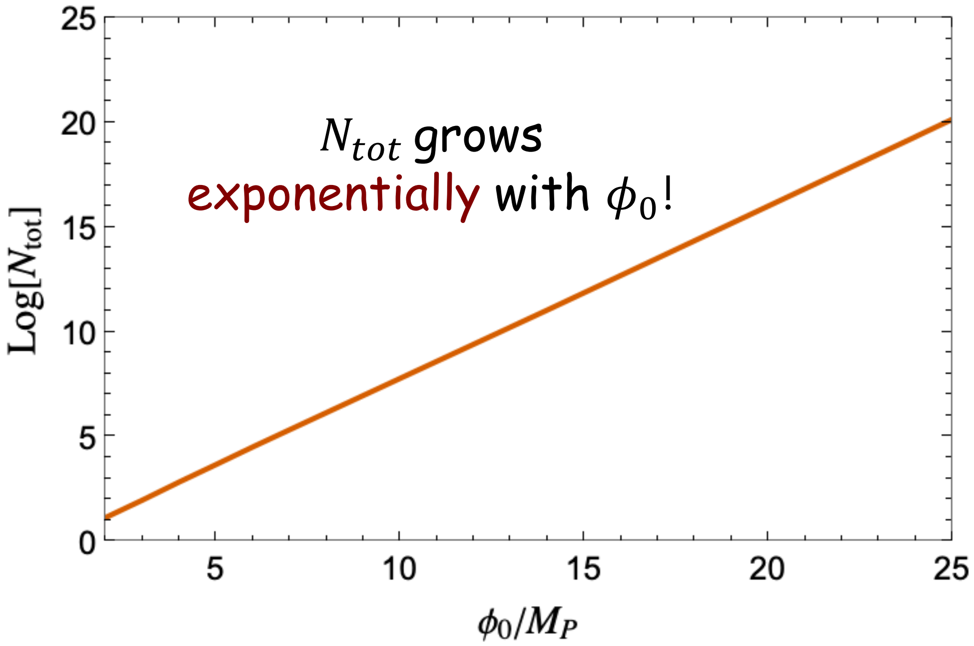
An issue with the initial value

Back to Starobinsky… What's the initial field value?

Starobinsky $\lambda = 1$



Larger ϕ_0, larger N_{tot}



The inflaton can start with $\phi_0 \gg M_P$

An issue with the initial value

Total number of efolds: $N_{tot} \equiv \ln(a_{\text{end}}/a_0)$

Total growth of volume: $\boxed{V \sim e^{3N_{tot}}}$

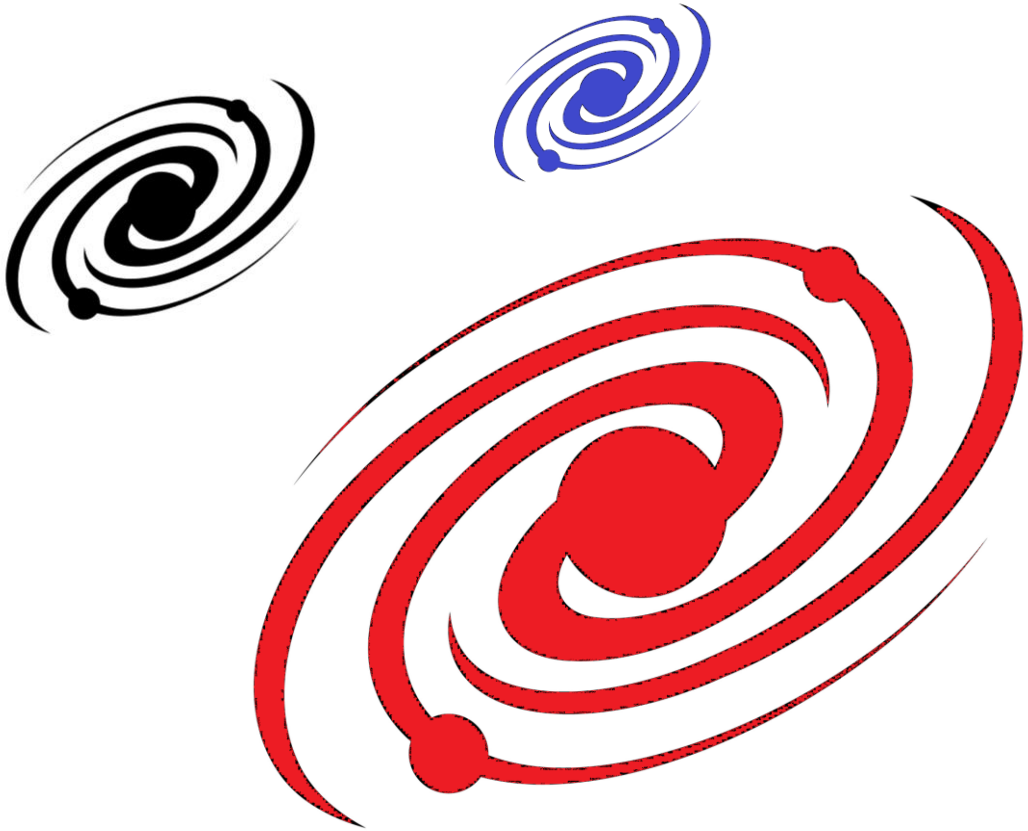
ϕ_0

$\phi_1 > \phi_0$

$\phi_2 < \phi_0$

More probable that
we come from a patch
with a larger initial value!

(May be problematic)



An issue with the initial value

Transplanckian field value $\longrightarrow$ Quantum gravity regime

Swampland Distance Conjecture

H. Ooguri, C. Vafa (hep-th/0605264)

Transplanckian field excursion

⬇

Tower of light states ➡ Including massive spin 2

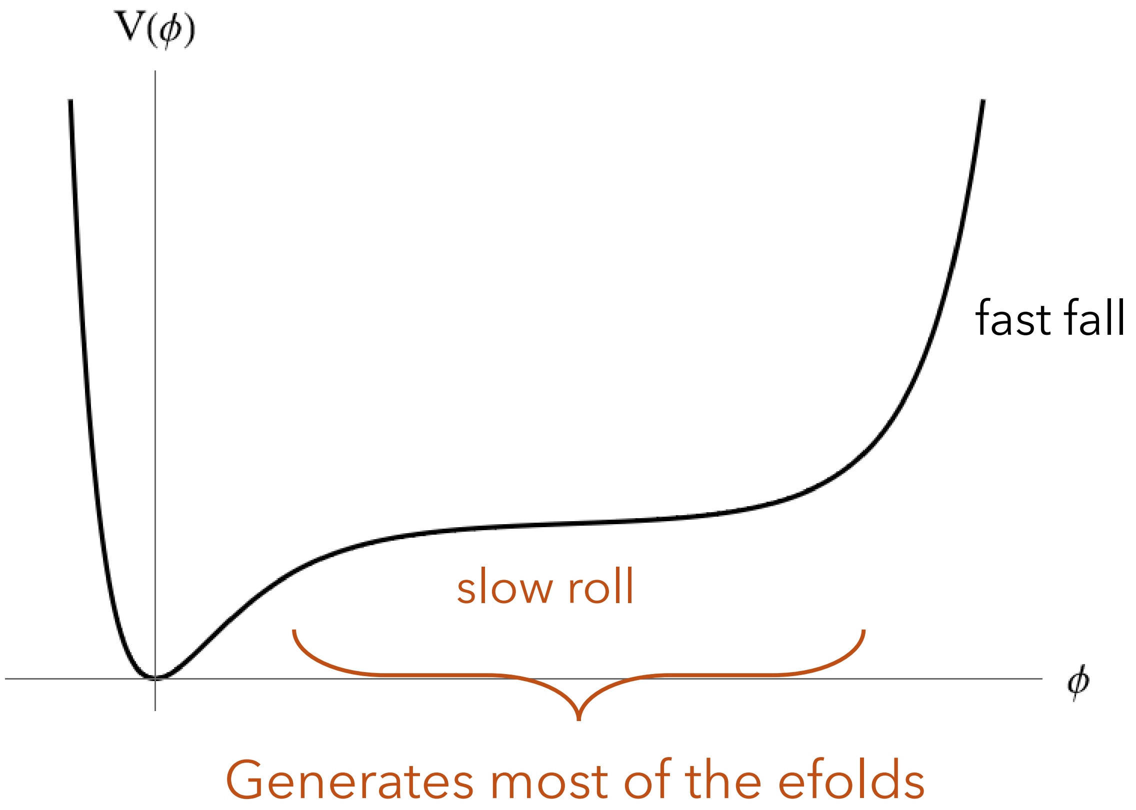
⬇ ⬇

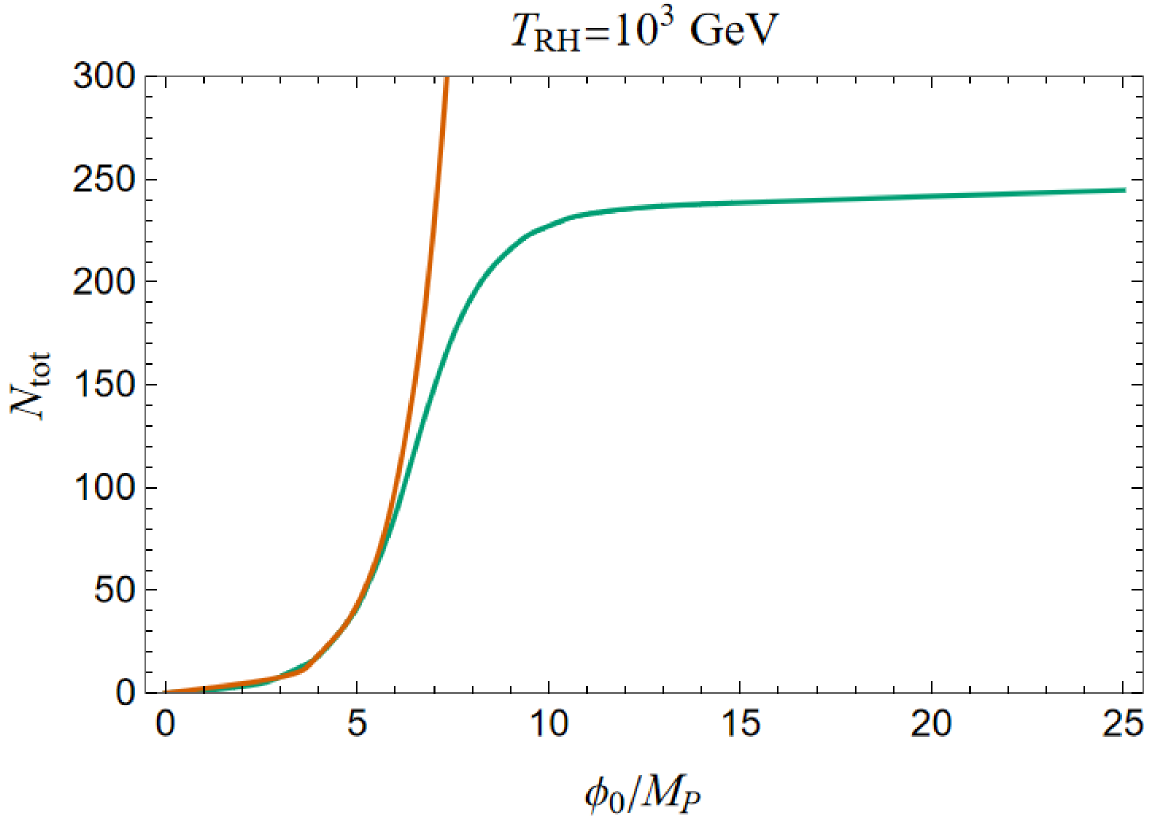
Breakdown of the EFT **Spin 2 ligher than Hubble violates unitarity**

Limit on the field excursion: $\frac{\delta\phi}{M_P} < c \ln \frac{M_P}{H} \simeq O(10)$

Effect of the fine-tuning: a barrier appears!

$\lambda = 0.99995$ Deformation of Starobinsky potential





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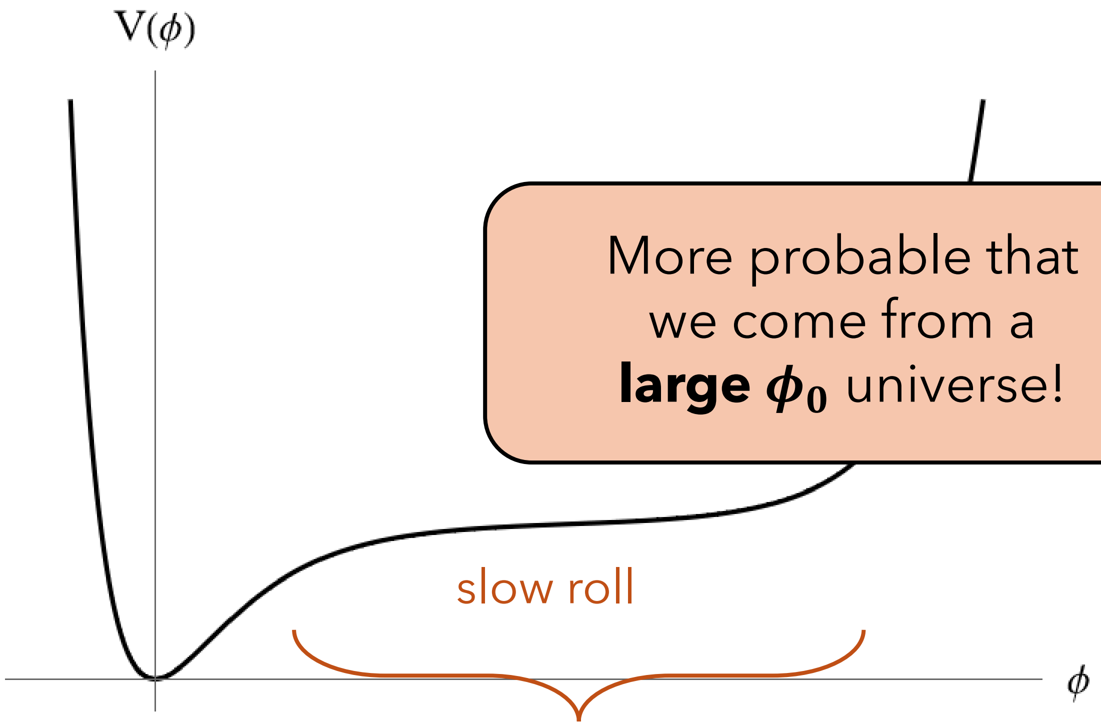
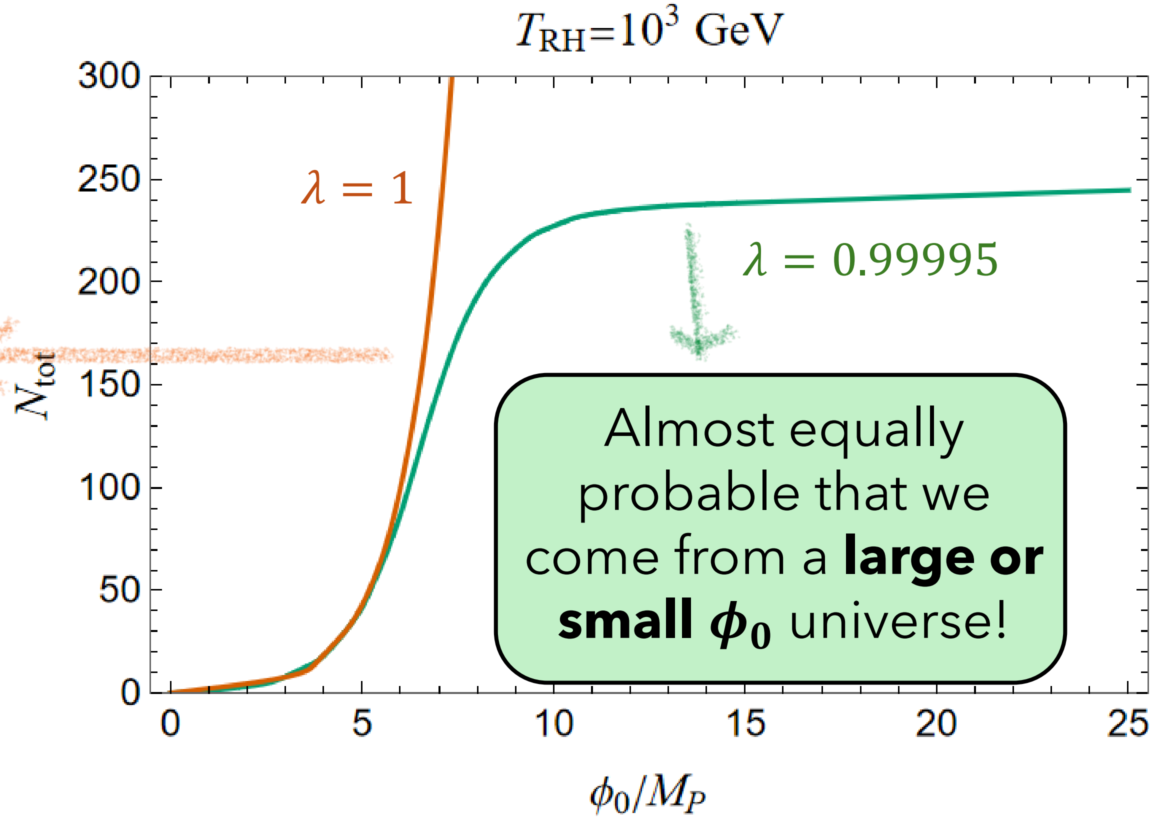


More probable that we come from a **large ϕ_0** universe!



Almost equally probable that we come from a **large or small ϕ_0** universe!

Main messages (about fine-tuning)

(1) $\lambda = 1$ or $\lambda = 0.9999(\dots)$ may just be different *accidents*, but some accidents can save the model experimentally and theoretically

(2) Which accident do we have? Is λ calculable?

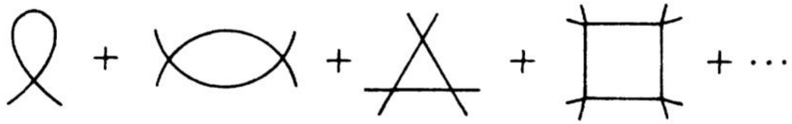
(3) There are other sources that deform the potential, such as

radiative corrections

Radiative corrections

The effective scalar potential

e.g. Tree-level:  $V_0 = \frac{1}{4}\lambda\phi^4$

One-loop: 

Summing up one-particle-irreducible diagrams

More generally, if the scalar couples to **other fields**. We have:

$$V_1(\phi) = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i}(2s_i+1)M_i^4(\phi^2)\left[\log\frac{M_i^2(\phi^2)}{\mu^2} - c_i\right]$$

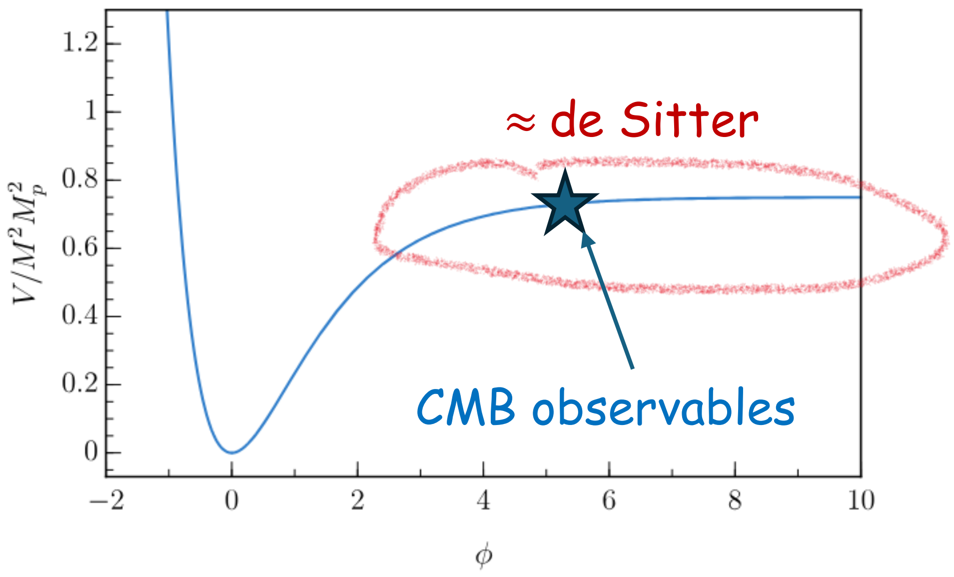
$c_i = 3/2$ for scalars & fermions

$c_i = 5/6$ for vector bosons

$M_i^2(\phi^2)$: the field-dependent mass of the particle i

Radiative corrections are always there: from the self coupling, or coupling to other fields

Two subtleties in inflation (1)



We need the one-loop potential in ***de Sitter***

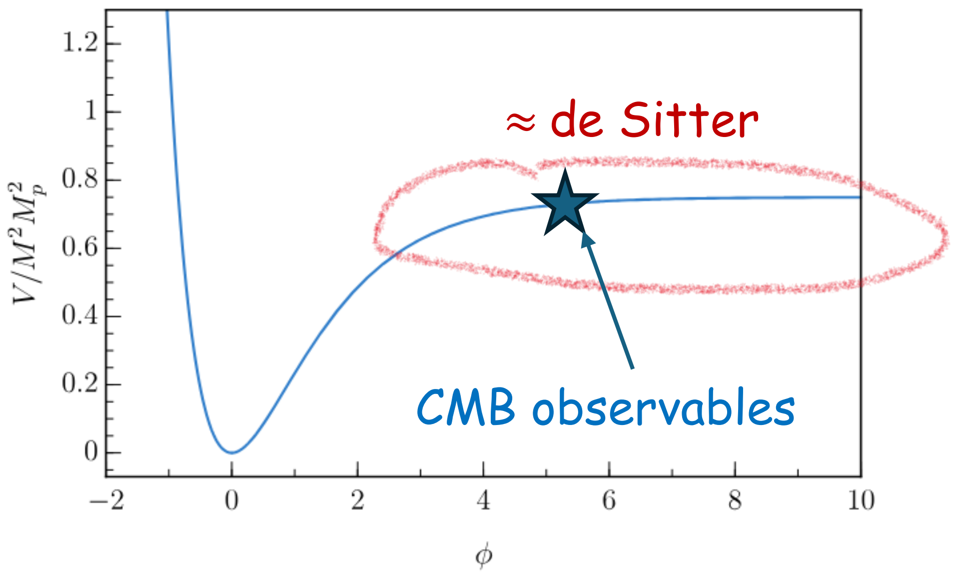
Markkanen, Nurmi, Rajantie, Stopyra [1804.02020]

What changes in curved spacetime?

#1: Curvature contributes to the *effective mass*.

#2: Curvature adds *extra pieces* in the potential.

Two subtleties in inflation (1)



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What changes in curved spacetime?

#1: Curvature contributes to the *effective mass*.

#2: Curvature adds *extra pieces* in the potential.

For inflaton potential, the dS correction is actually **negligible**

$(V_1 \sim H^4) \ll (V_0 \sim M_P^2 H^2)$

Two subtleties in inflation (2)

The effective mass evolves with time

e.g. $y\phi(t)\bar{f}f$

can vary from $y\,M_P$ to 0

$$V_1(\phi) = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i}(2s_i+1)M_i^4(\phi^2)\left[\log\frac{M_i^2(\phi^2)}{\mu^2} - c_i\right]$$

Avoid large log's!

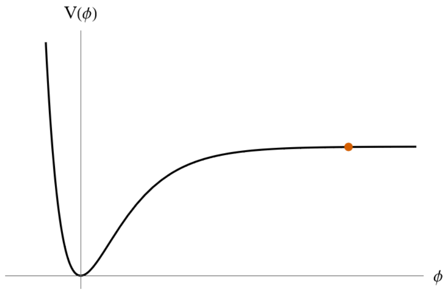
RG-improve the scalar potential: $\frac{d}{d\log\mu}V_{eff} = 0$

Beta functions
Running couplings e.g. $y(\mu)$

Radiative corrections in Starobinsky model

J. Ellis, T. Gherghetta, K. Kaneta, WK, K. Olive [2510.15137]

Tree-level potential: $V_0(\phi) = \frac{3}{4} m^2 M_P^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right)\right]^2$



- Inflaton self-coupling -

Effective mass: $M^2(\phi) \simeq V''(\phi) - 2H^2$

One-loop potential: $V_1(\phi) \simeq \frac{1}{64\pi^2}\left[M^4(\phi)\left(\log\frac{|M^2(\phi)|}{\mu^2} - \frac{3}{2}\right) - \frac{2H^4}{15}\log\frac{|M^2(\phi)|}{\mu^2}\right]$

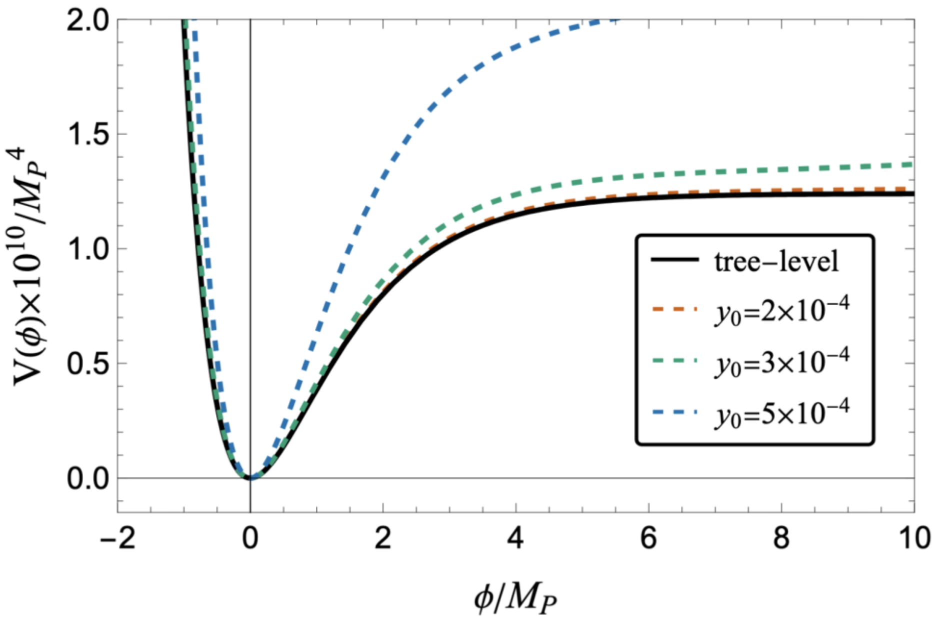
$$V_0 \simeq 3M_P^2 H^2 \gg V_1$$

tiny

- Coupling to a fermion -

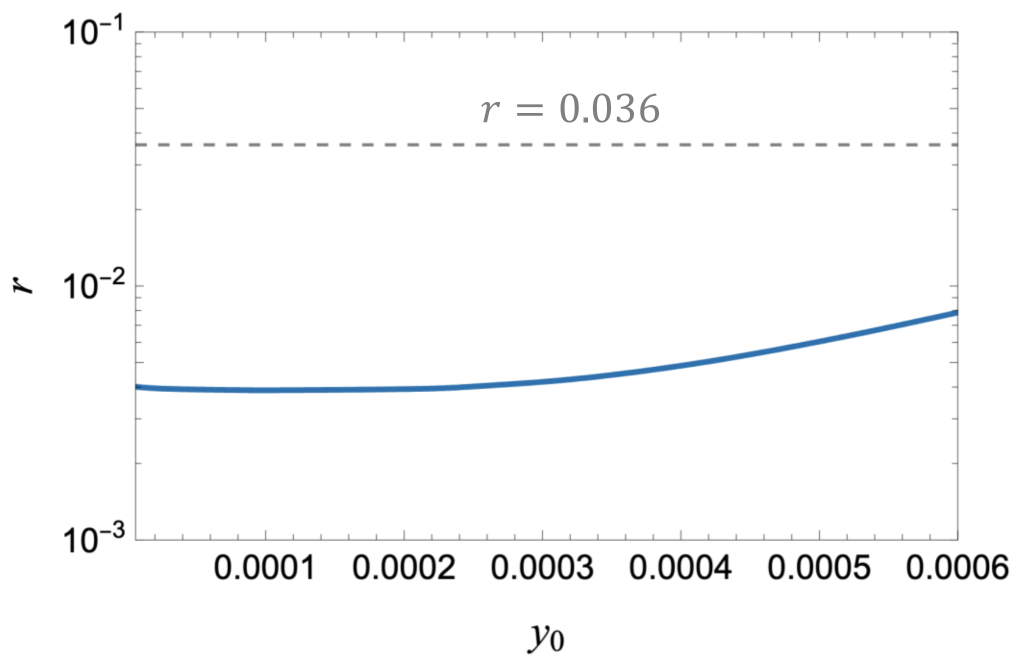
$y\phi\bar{f}^c f/2$ (*f is massless*)

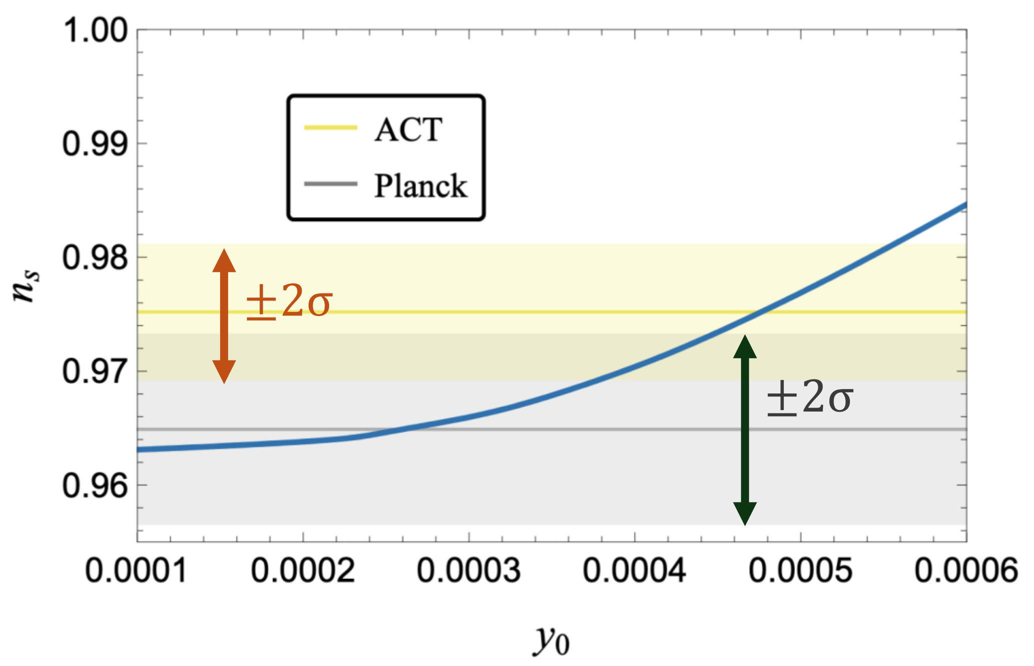
Tree-level
vs.
One-loop corrected potential



CMB observables change ⟶ Bound on Yukawa coupling

- Coupling to a fermion -





Stronger constraint from n_s!

Planck: $y \leq 4.5 \times 10^{-4}$

ACT: $3.8 \times 10^{-4} \leq y \leq 5.6 \times 10^{-4}$

Reheating temperature

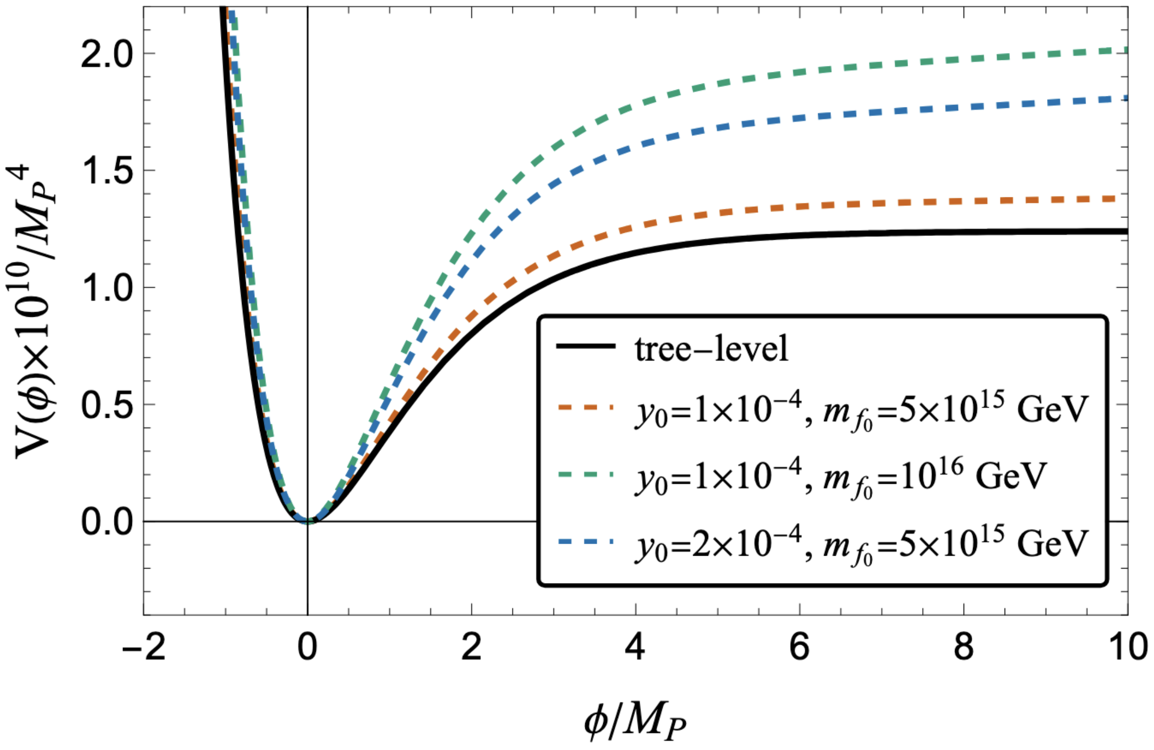
$$\frac{\pi^2}{30} g_{RH} T_{RH}^4 = \frac{12}{25} \left(\Gamma_\phi M_P\right)^2$$

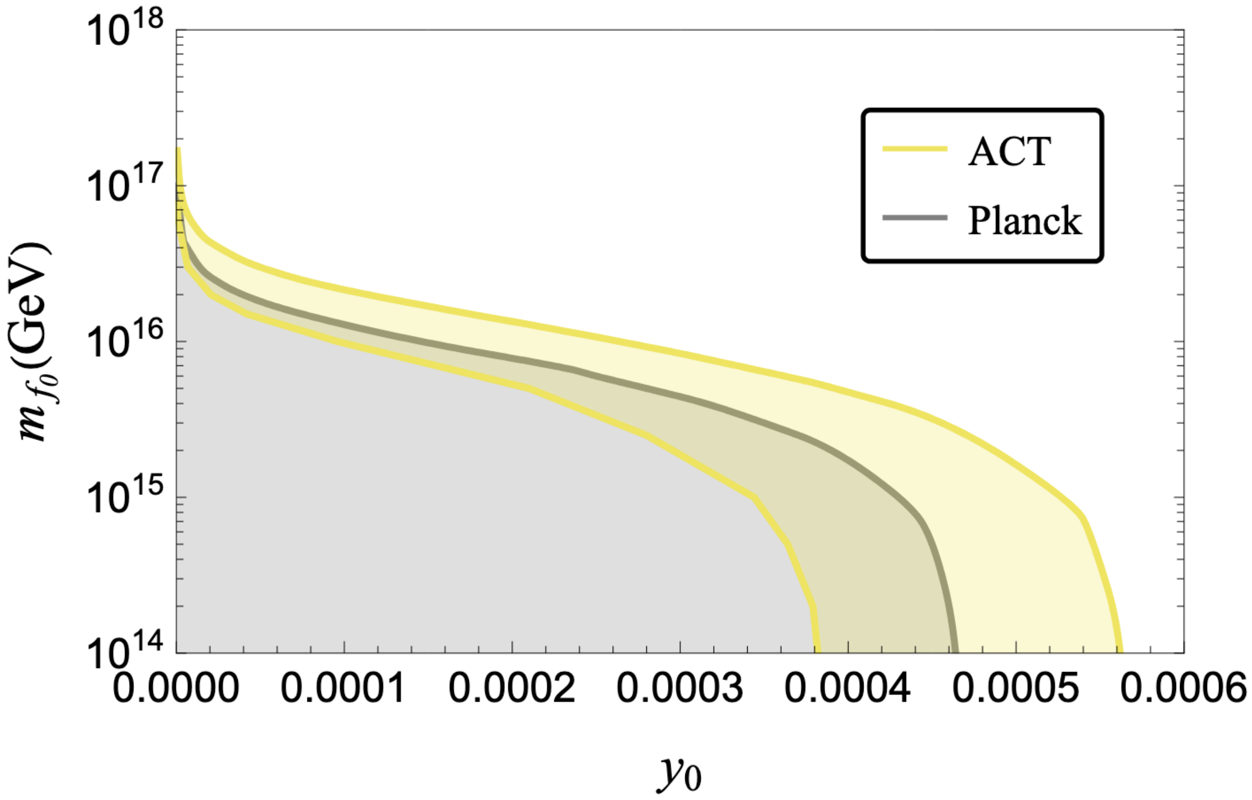
Planck: $T_{RH} \leq 2 \times 10^{11} \mathrm{GeV}$

ACT: $1.7 \times 10^{11} \leq T_{RH} \leq 2.8 \times 10^{11} \mathrm{GeV}$

- Massive fermion -

$y\phi\bar{f}^c f/2 + m_f \bar{f}^c f/2$





n_s constrains the Yukawa coupling AND the bare mass

e.g. For $y \simeq 0.0002$:

Planck implies $m_f \lesssim O(10^{16})\text{GeV}$

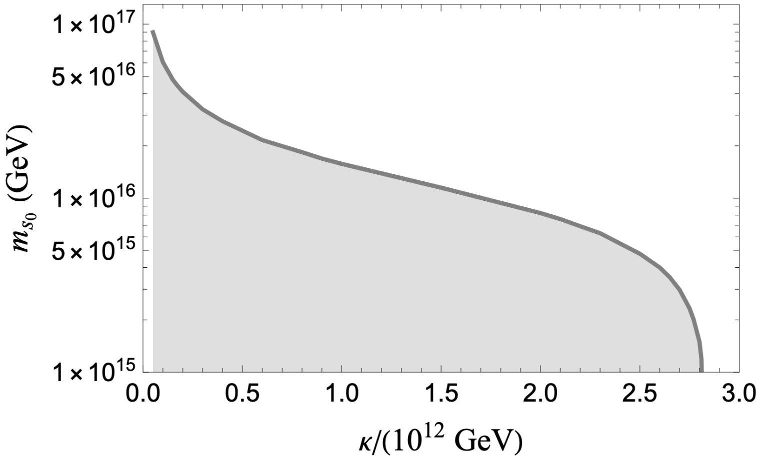
ACT implies $m_f \simeq O(10^{16})\text{GeV}$

Similarly for other couplings

- Coupling to a scalar - $\kappa\phi s^2/2$

- Massive scalar - $\kappa\phi s^2/2 + m_s^2 s^2/2$

Planck: $\kappa \leq 4 \times 10^{12}\,\mathrm{GeV}$



etc...

Radiative corrections must be kept under control

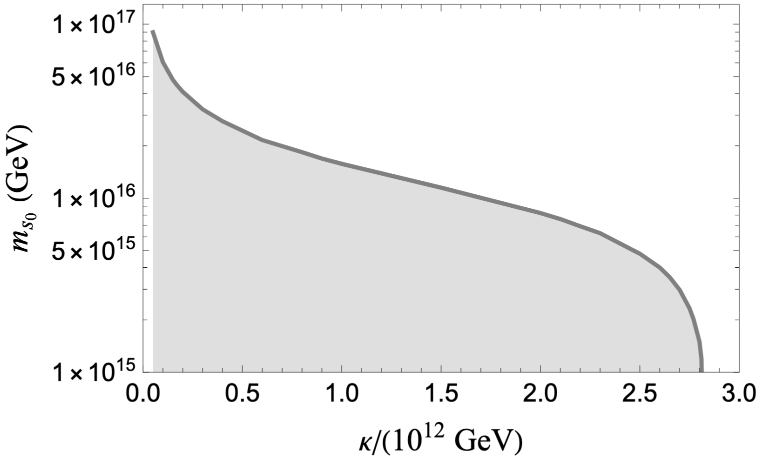
Similarly for other couplings

- Coupling to a scalar - $\kappa\phi s^2/2$

- Massive scalar - $\kappa\phi s^2/2 + m_s^2 s^2/2$

etc...

Planck: $\kappa \lesssim 4 \times 10^{12}\,\mathrm{GeV}$



Radiative corrections must be <u>kept under control</u>

*Can any **symmetry** do that?*

J. Ellis, T. Gherghetta, K. Kaneta, WK, K. Olive [2603.02389]

Radiative corrections in supergravity

= Supersymmetric gravity

Inflation from supergravity

Kähler potential

$$K = -3\ln(T + T^* - |\chi|^2/3)$$

Superpotential

$$W = M\left(\frac{1}{2}\chi^2 - \frac{\lambda}{3\sqrt{3}}\chi^3\right)$$

Scalar potential: $V = e^K[g^{m\bar{n}} D_m W D_{\bar{n}} W^* - 3|W|^2]$

Starobinsky potential if $\lambda = 1$

J. Ellis, T. Gherghetta, K. Kaneta, WK, K. Olive [2603.02389]

Radiative corrections in supergravity

= Supersymmetric gravity

Inflation from supergravity

Kähler potential

Determines the scalar kinetic term

Gets *loop corrections*

Superpotential

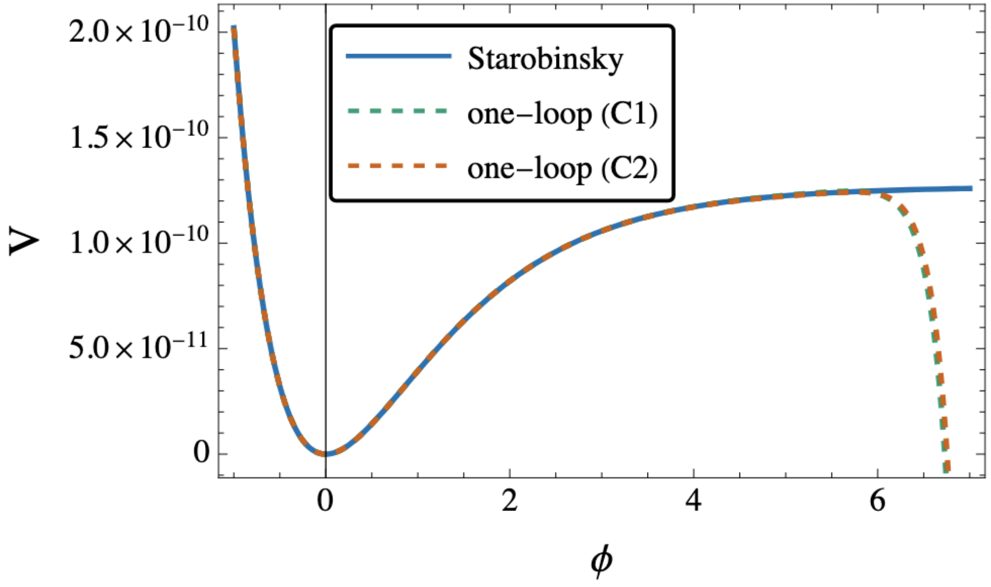
<u>*Protected*</u> by SUSY

"Non-renormalization theorem"

Scalar potential: $V = e^K[g^{m\bar{n}} D_m W D_{\bar{n}} W^* - 3|W|^2]$

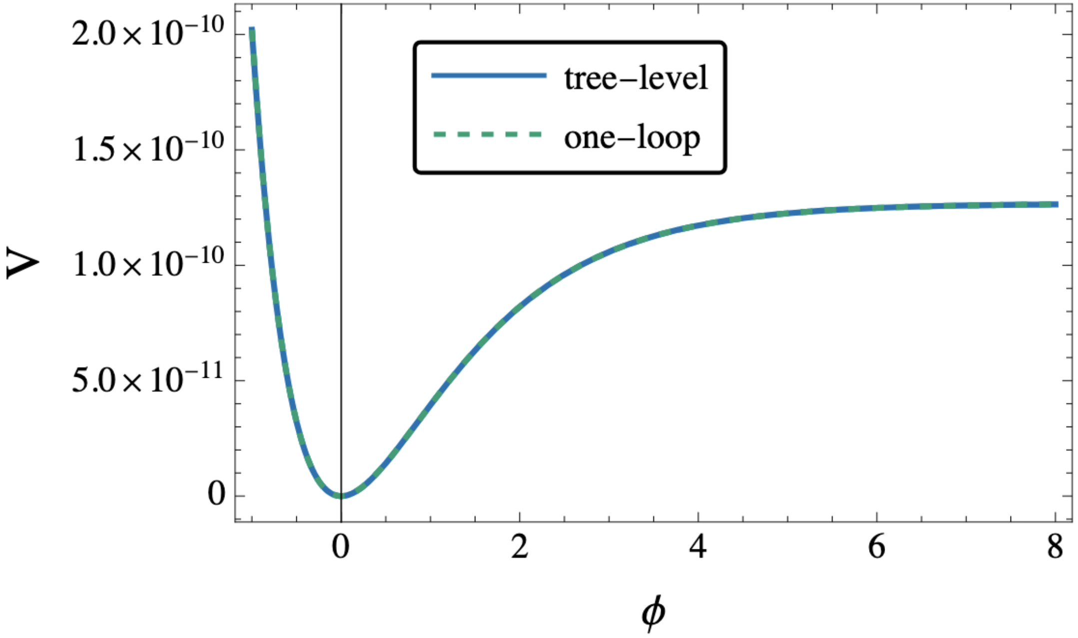
Some models have better controlled corrections

Symmetry badly broken



e.g. "Wess-Zumino model"

Symmetry breaking kept small



e.g. "Cecotti model"

Radiative corrections are tiny!

Main messages (about radiative corrections)

- Inflation generically gets radiative corrections!
- CMB bounds constrain the inflaton couplings & reheating temperature
- Generally, inflation is sensitive to many sources of deformation

  e.g. R^n higher curvature terms, higher dim operaters

How is inflation protected from these deformations?

Bottom-up: constrain these operators using experiments

Top-down: any *symmetries* constraining these operators?

Thank you!

Backup slides

$$k_* = a_* H_*$$

$$\sqrt{V_*/3M_P^2}$$

$$\ln \frac{k_*}{a_0 H_0} = \ln \left(\frac{a_*}{a_e} \frac{a_e}{a_{rad}} \frac{a_{rad}}{a_0} \frac{H_*}{H_0} \right)$$

N_*

$$\left(\frac{g_0}{g_{reh}}\right)^{\frac{1}{3}} T_0/T_{rad}$$

Assuming entropy conservation after reheating

$$\ln \frac{a_e}{a_{rad}} = \frac{1}{3(1+w_{int})} \ln \frac{\rho_{rad}}{\rho_{end}}$$

$$N_* = \ln\left[\frac{1}{\sqrt{3}}\left(\frac{\pi^2}{30}\right)^{1/4}\left(\frac{43}{11}\right)^{1/3}\frac{T_0}{H_0}\right] - \ln\left(\frac{k_*}{a_0 H_0}\right) - \frac{1}{12}\ln g_{\rm reh}$$
$$+ \frac{1}{4}\ln\left(\frac{V_*^2}{M_P^4 \rho_{\rm end}}\right) + \frac{1 - 3w_{\rm int}}{12\,(1+w_{\rm int})}\ln\left(\frac{\rho_{\rm R}}{\rho_{\rm end}}\right) \longrightarrow \text{Depend on } T_{RH}$$

$\Rightarrow$ N_* and T_{RH} are **related**

Observables:

1) Scalar power spectrum amplitude: $A_s \simeq \frac{V_*}{24\pi^2 \epsilon_* M_P^4}$

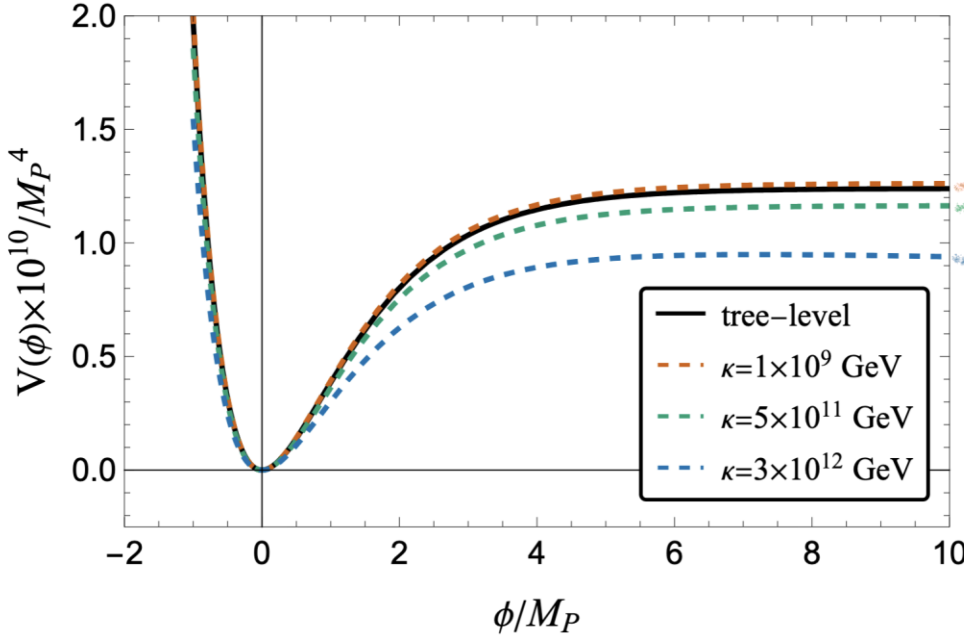
2) Scalar spectral index: $n_s = n_* \simeq 1 - 6\epsilon_* + 2\eta_*$

3) Tensor-to-scalar ratio: $r = r_* \simeq 16\epsilon_*$

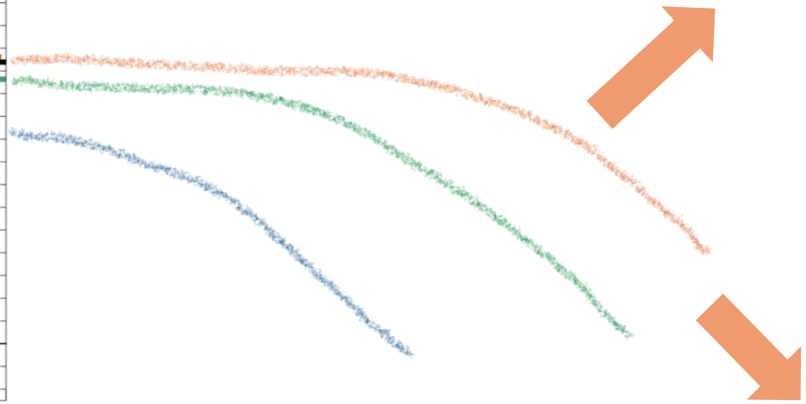
They also depend on T_{RH}

Scalar coupling $\kappa\phi s^2/2$ *(s is massless & real)*

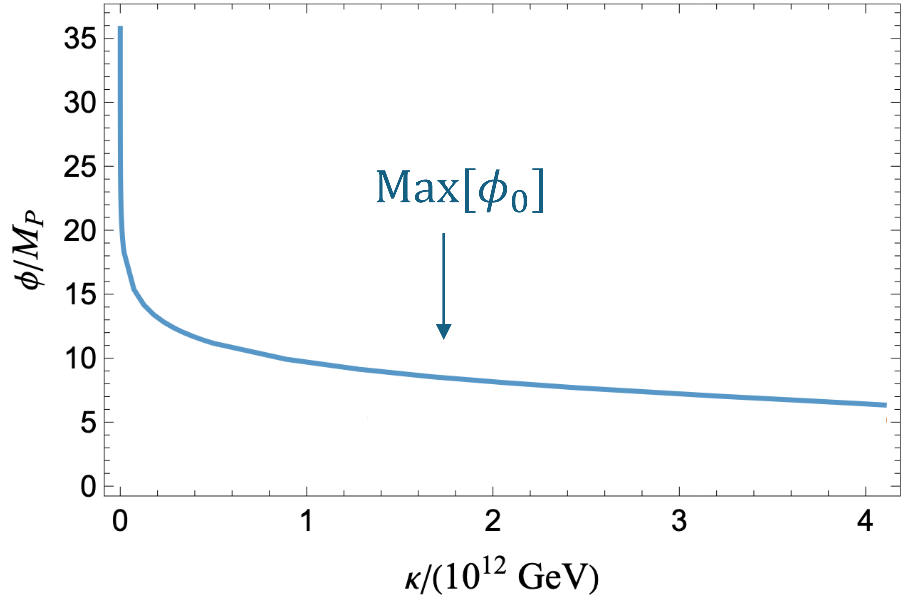
$M^2(\phi) \gg H^2$ requires $\kappa \gg 10^7\,\mathrm{GeV}$



The scalar coupling **bends down** the inflaton potential at large field value

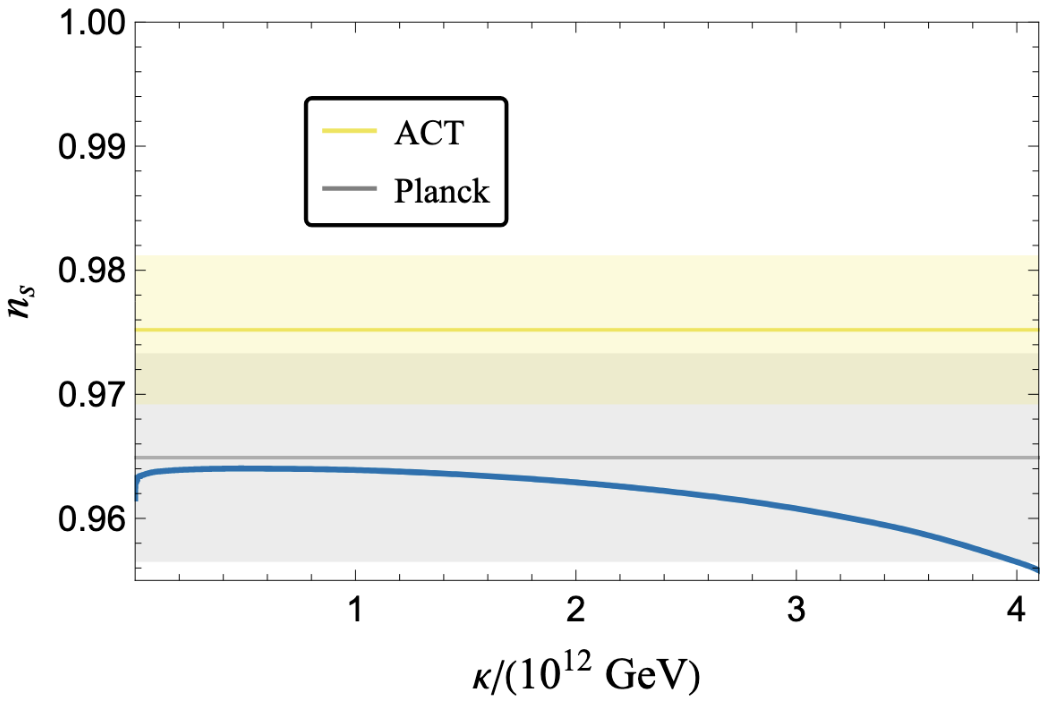


The initial inflaton value has an **upper bound!**



Scalar coupling $\kappa\phi s^2/2$ *(s is massless & real)*

$M^2(\phi) \gg H^2$ requires $\kappa \gg 10^7\,\text{GeV}$



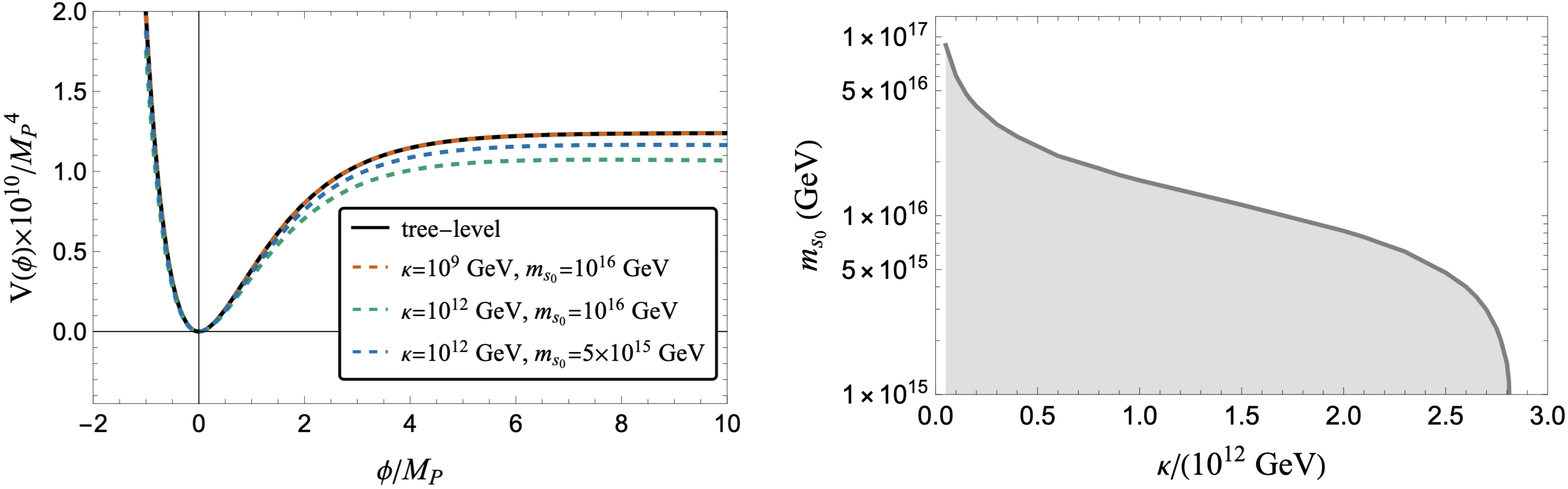
- Different from the fermion case, the scalar coupling **reduces** n_s for large κ
- The **ACT** bound is **never satisfied**

Reheating temperature

$$\frac{\pi^2}{30} g_{RH} T_{RH}^4 = \frac{12}{25}\left(\Gamma_\phi M_P\right)^2$$

Planck: $\kappa \le 4 \times 10^{12}\,\text{GeV}$ $\longrightarrow$ **Planck:** $T_{RH} \le 4.2 \times 10^{13}\,\text{GeV}$

Coupling with massive scalar $\kappa\phi s^2/2 + m_s^2 s^2/2$



❖ Again, the one-loop correction from the scalar coupling **reduces** n_s

❖ Again, the **ACT** bound is **never satisfied**

❖ **Planck** bound on n_s gives an upper limit on m_s for each κ