Probing the 3+1 model in the SHiP experiment

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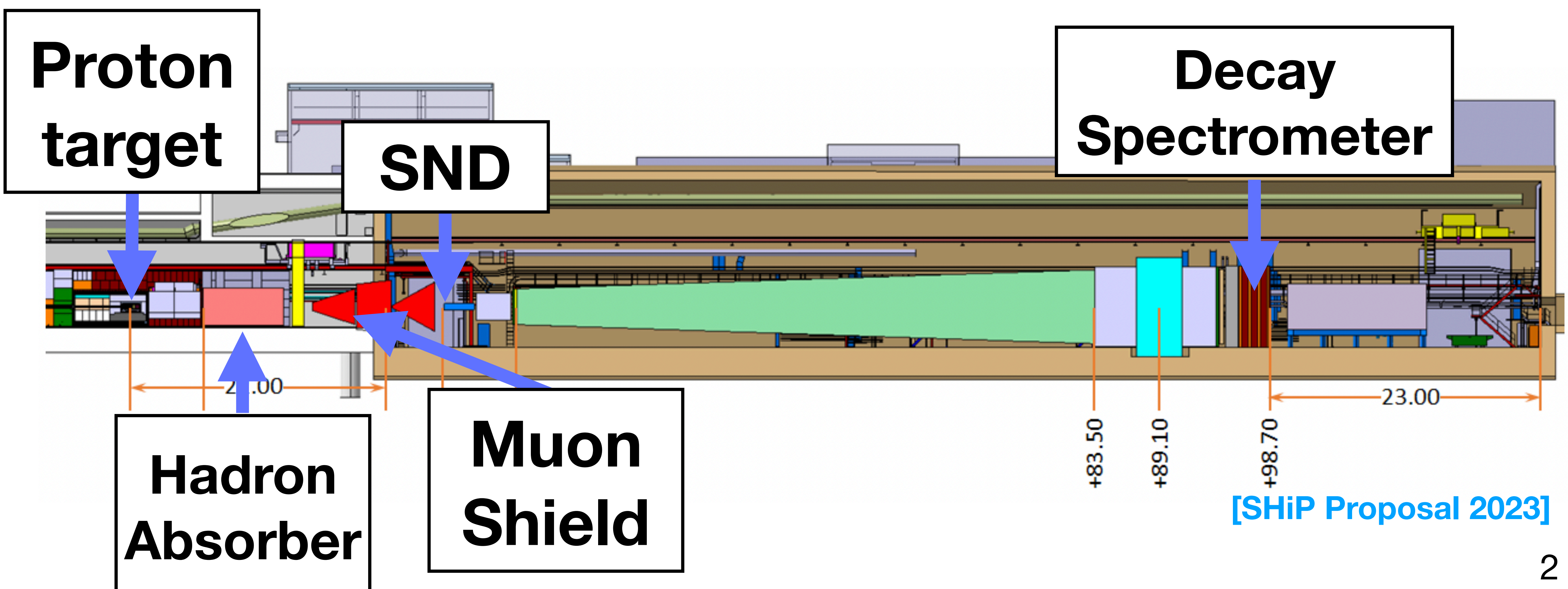
SHiP experiment

- SHiP(**S**earch for **Hi**dden **P**article) is constructed for feebly interacting particles (long range particles, such as hidden gauge, high M sterile neutrinos, etc)
- SND: Scattering and Neutrino Detector (for neutrino physics)



[SHiP Proposal 2023]

SHiP experiment

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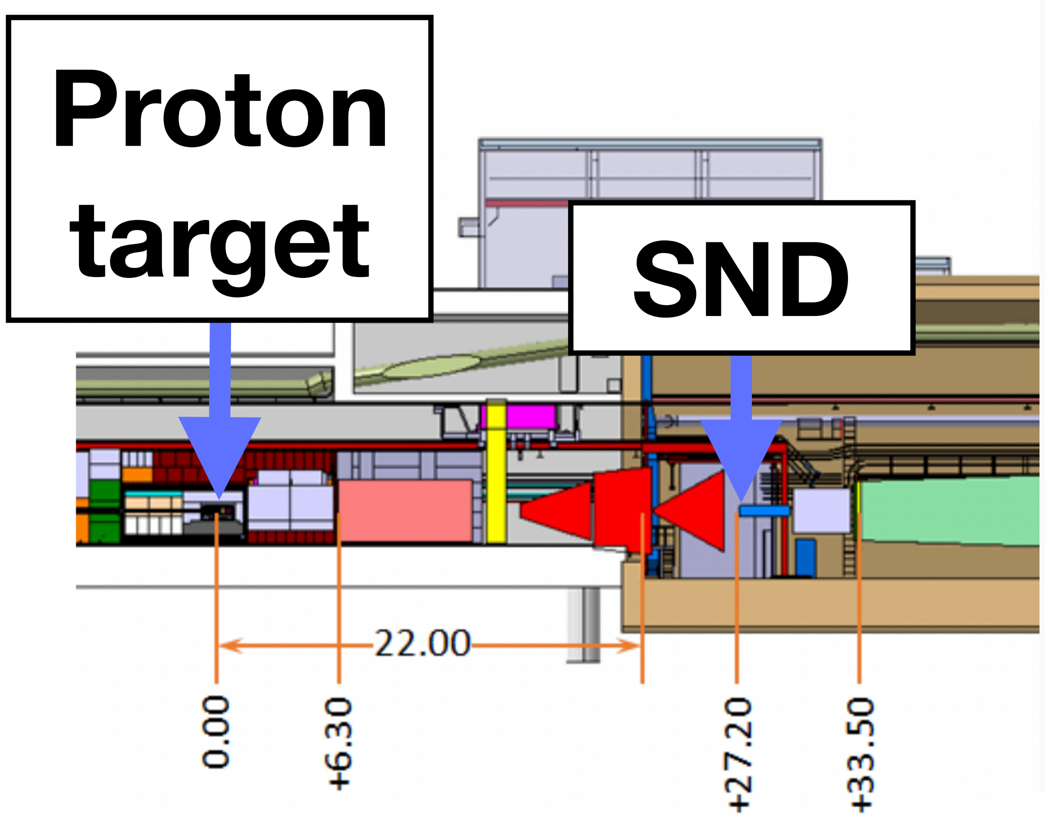
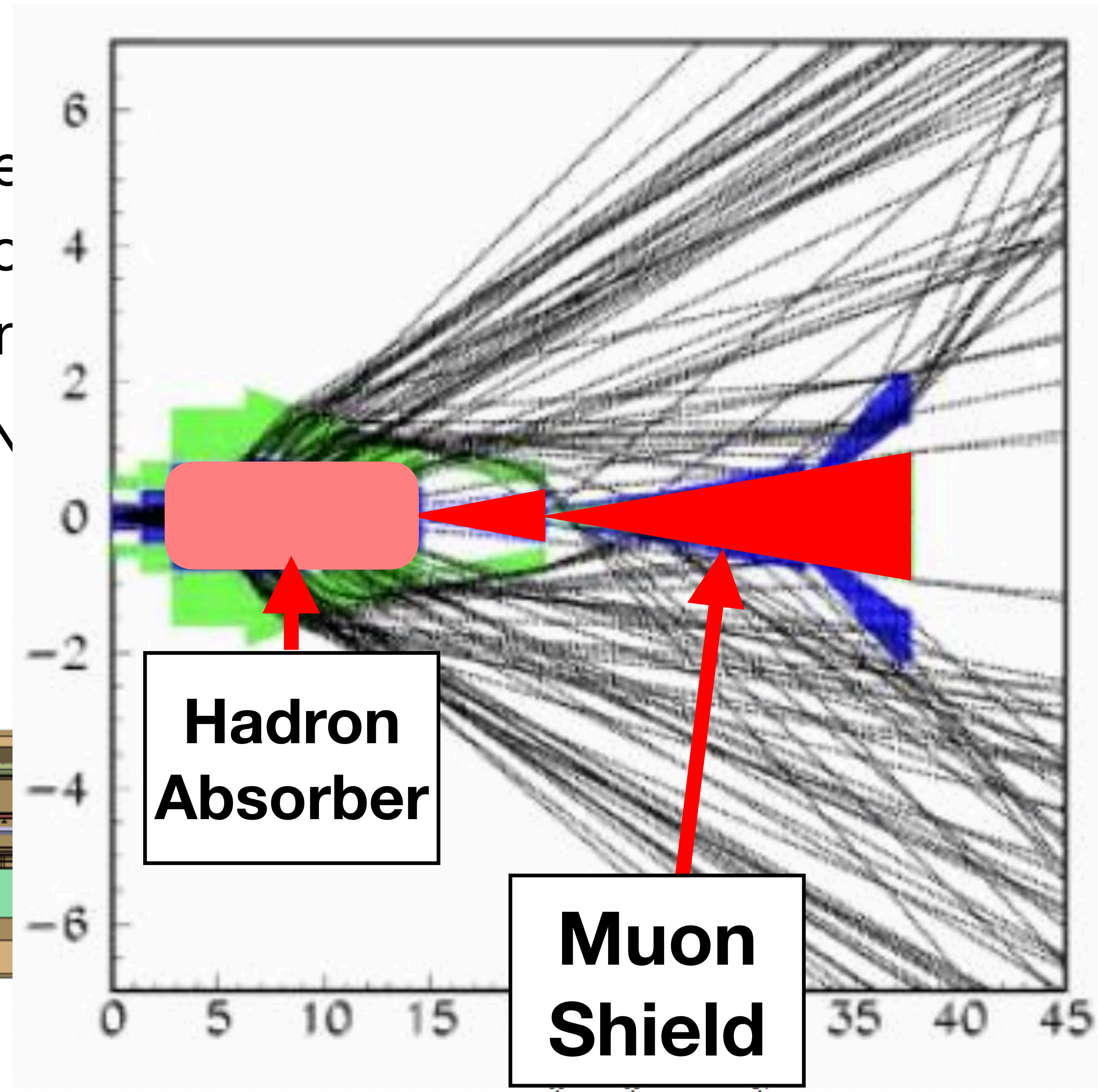
  interacting particles (lo
  gauge, high M sterile r
- SND: Scattering and N





[SHiP Proposal 2023]

The 3+1 model

The 3+1 model: add a "**single**" neutrino mass eigenstate into the original 3-flavor neutrino model

Disappearance: $P_{\alpha\alpha} = 1 - 4|U_{\alpha 4}|^2(1 - |U_{\alpha 4}|^2)\sin^2\left(\frac{\Delta m_{41}^2 L}{4E_\nu}\right)$

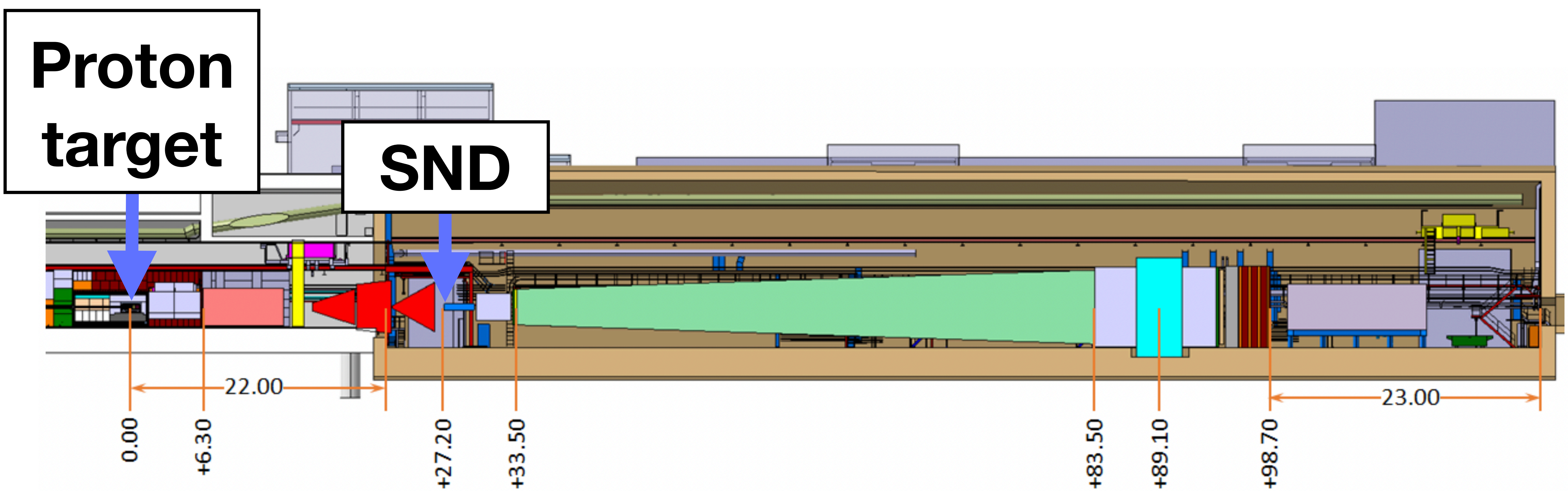
Appearance: $P_{\alpha\beta} = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2\sin^2\left(\frac{\Delta m_{41}^2 L}{4E_\nu}\right)$

Δm_{41}^2 with maximum sensitivity

$$\frac{\Delta m_{41}^2 L}{4\langle E_\nu \rangle} \rightarrow \frac{\boxed{(10^3\,\mathrm{eV}^2)} \cdot (27\,\mathrm{m})}{4 \times (30\,\mathrm{GeV})} \approx 1$$

SHiP as the 3+1 model prober

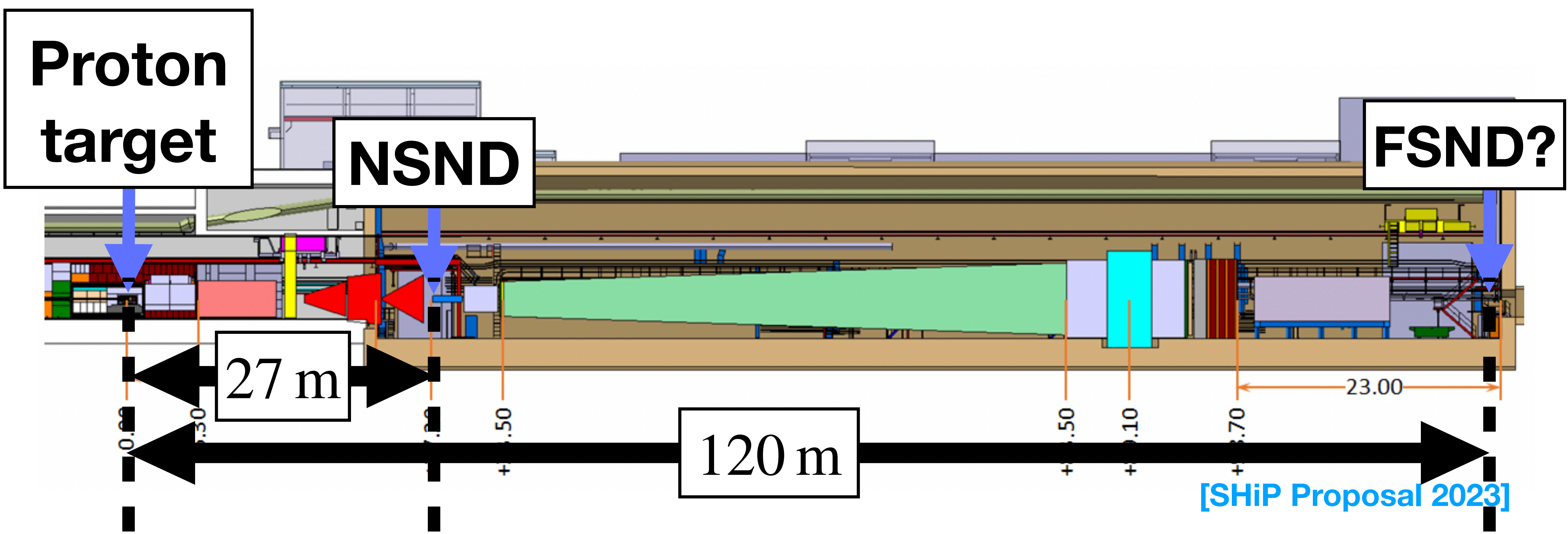
- Main objective: to utilize SHiP as a short baseline experiment
- The distance between Proton target and SND ⇒ baseline
- SND could probe CC events on every three flavor



[SHiP Proposal 2023]

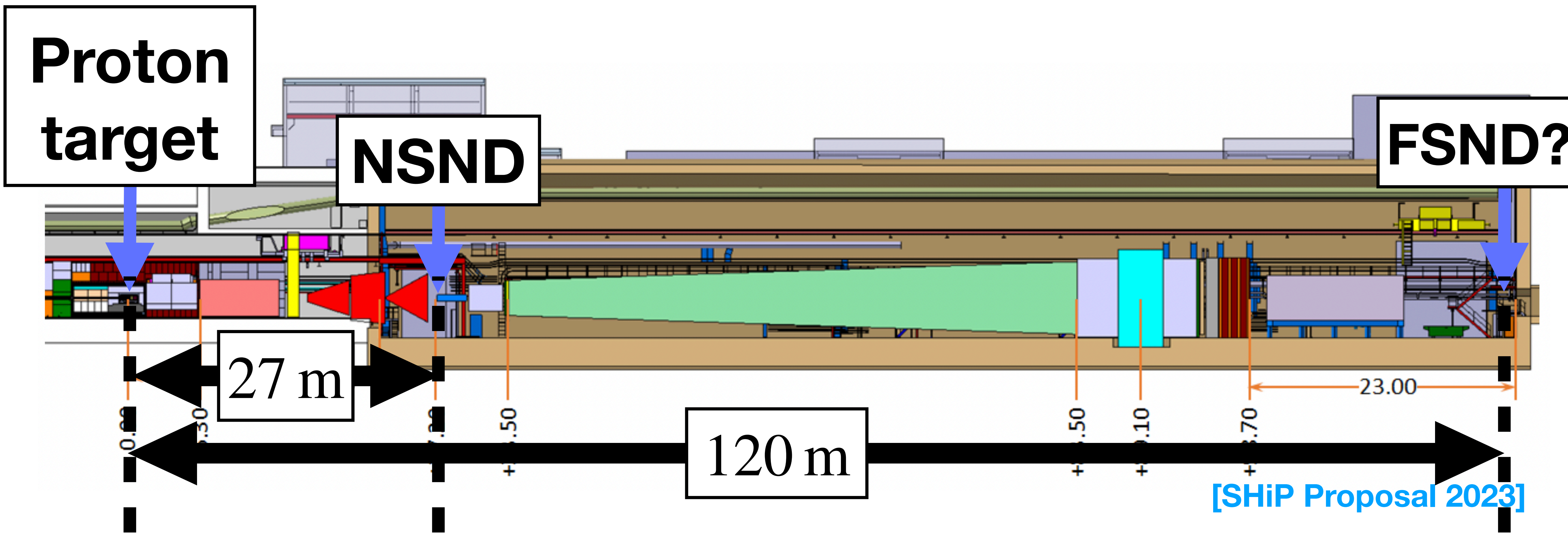
SHiP as the 3+1 model prober

- Far SND (FSND) was proposed in our previous study to enhance sensitivity. 2403.04191
- For comparison, original SND is labeled as Near SND (NSND).
- Length of SNDs: $2.6\,\mathrm{m}$



SHiP as the 3+1 model prober

- # of events at NSND: given by the proposal [SHiP Proposal 2023]
- We assumed that the energy spectrum is the same for both NSND and FSND, with a certain # of event ratio $R_{\mathrm{F/N}}$.



General 3+N oscillation

General parametrization on U(N) mixing: $(N-1)^2 + N - 1$ DoF with:

$\dfrac{N(N-1)}{2}$ # of angles & $\dfrac{(N-1)(N-2)}{2}$ CP violating phases,
$N-1$ # of mass eigenvalues.

Then what if:
1. masses of 3 active neutrino mass eigenstates are much smaller than other masses,
2. and if neutrino traveling length is short enough that we could ignore the oscillation between active neutrinos (i.e., short baseline):

$4(N-3)$ # of angles + masses, while phases are ignored.

Why the 3+1 model still needs for experiments?

Goodness of fit: how much “fit” do we expect?

$$\chi^2 = \text{(Goodness of fit)} = \min_{\theta} \sum_{i}^{N_{\text{bins}}} \frac{[N_{\text{observed},i} - N_{\text{expected},i}(\theta)]^2}{N_{\text{expected},i}(\theta)}$$

Approximately, the value χ^2 follows chi square distribution with the mean value $N_{\text{bins}} - (\text{DoF})$ & std of $\sqrt{2[N_{\text{bins}} - (\text{DoF})]}$.

In my paper, in total 21 (42 for NSND+FSND) # of bins are used.
(And it wouldn’t be much larger for the actual experimental side of analysis)

Then… for 21 # of bins:

3+1 model $(\theta_{14}, \theta_{24}, \theta_{34}, \Delta m_{41}^2) \rightarrow$ mean of 17, std of 5.8
3+2 model (8 DoF) $\rightarrow$ mean of 13, std of 5.1

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Then… for 21 # of bins:

3+1 model $(\theta_{14}, \theta_{24}, \theta_{34}, \Delta m^2_{41}) \rightarrow$ mean of 17, std of 5.8 ~3 sigma
3+2 model (8 DoF) $\rightarrow$ mean of 13, std of 5.1 ~2.5 sigma

Why the 3+1 model still needs for experiments?

$$-\mathcal{L}_{M_\nu} = M_{Dij}\bar{\nu}_{si}\nu_{Lj} + \frac{1}{2}M_{Nij}\bar{\nu}_{si}\nu^c_{sj} + \text{h.c.}$$

ν_L = Left handed neutrinos that interacts with standard particles.

ν_s = unknown right handed(or whatever chiral spinor) that interacts with ν_L.

We can make any conspiracy that induces any oscillation. Instead, why don't we use the minimal model as a barometer for new physics? → 3+1 model

The FC method with nuisance parameters

PhysRevD.57.3873

Sensitivity: confidence region of the most probable data set

The confidence region with given observations $\boldsymbol{D}$ & $\boldsymbol{\varphi}$:

$$\mathrm{CR}(\boldsymbol{D}, \boldsymbol{\varphi}; \alpha) = \{\boldsymbol{\theta} \,|\, \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) > \alpha\},$$

$\alpha \equiv$ Significance Level
=1-(Confidence Level)

where $\mathfrak{p}$ is the p-value is defined as

$$1 - \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) \equiv \int_0^{\lambda(\boldsymbol{D},\boldsymbol{\varphi},\boldsymbol{\theta})} \rho(\lambda' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi}) d\lambda',$$

where $\rho(\lambda' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})$ is a probability density function of the test statistic λ'.

Probability distribution of samples

$$P(\boldsymbol{O}, \boldsymbol{\varphi}' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi}) = \prod_{\alpha,i} \mathrm{Pois}(O_{\alpha,i} \,|\, \mu_{\alpha,i}) \times \rho(\boldsymbol{\varphi}' \,|\, \boldsymbol{\phi}),$$

- $\boldsymbol{\phi}$ should be chosen in order to perform repeated sampling
- Profiled FC method: choose $\boldsymbol{\phi}$ that maximize $P(\boldsymbol{D}, \boldsymbol{\varphi} \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})$.
  1503.07622, 2207.14353
  - $\mu_{\alpha,i} \equiv s_{\alpha,i} + b_{\alpha,i}$
  - The test statistic:

$$\lambda(\boldsymbol{O}, \boldsymbol{\varphi}', \boldsymbol{\theta}) \equiv -2 \log \frac{\max_{\boldsymbol{\phi}} P(\boldsymbol{O}, \boldsymbol{\varphi}' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})}{\max_{\boldsymbol{\theta},\boldsymbol{\phi}} P(\boldsymbol{O}, \boldsymbol{\varphi}' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})}$$

Parametric bootstrap

https://hep-physics.rockefeller.edu/luc/talks/ParametricBootstrap.pdf

The point estimate (or mean) of nuisance parameters

$$\rho(\boldsymbol{\varphi} \,|\, \boldsymbol{\phi}) = \prod_j \mathcal{N}_{(-0.5,\infty)}(\varphi_j \,|\, \phi_j, \sigma_{\text{norm}})$$

- The uncertainty of nuisance parameters is determined by the data of auxiliary measurements $\boldsymbol{D}_{\text{aux}}$.
- Ideally, auxiliary measurements should be sampled as well as the experimental data.
- Instead, a *parametric bootstrap* samples the point estimate $\boldsymbol{\varphi}$ of nuisance parameters $\boldsymbol{\phi}$.
- Basically, $\boldsymbol{D}_{\text{aux}}$ is replaced by $\boldsymbol{\varphi}$.

Signal estimate

Contribution of flavor β on the 3+1 model

$$s_{\alpha,i} = \sum_{\beta}^{e,\mu,\tau} (1 + \phi_\beta + \phi_{\beta,i}) \boxed{\int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_\mathrm{d}} dL \int_0^\infty dE_\mathrm{rec} \varepsilon_\alpha P_{\beta\alpha} \frac{d^2 N_\beta}{dE_\nu dL} \frac{\sigma_{\nu_\alpha A}}{\sigma_{\nu_\beta A}} \rho(E_\mathrm{rec} \,|\, E_\nu)}$$

- Energy reconstruction uncertainty: $20\ \%$ CERN-THESIS-2017-064
- Detection efficiency:

$$\varepsilon_e = 30\ \%$$ 1904.05686

$$\varepsilon_\mu = 40\ \%$$ 1904.05686

$$\varepsilon_\tau = 10\ \%$$ CERN-THESIS-2021-091 $\phi_{\tau-h} \geq 2.8\ \mathrm{rad}$

- Nuisance parameters are the same for both NSND and FSND.

※ Any independency between NSND and FSND could lower the sensitivity.

Background estimate

$$b_{\tau,i} = (1 + \phi_\mu + \phi_{\mu,i}) \int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_\mathrm{d}} dL \int_0^\infty dE_\mathrm{rec} R_{\mathrm{s/b}}^{-1} \varepsilon_\tau P_{\mu\mu} \frac{d^2 N_\tau}{dE_\nu dL} \rho(E_\mathrm{rec} \,|\, E_\nu)$$

- Background: tau only, signal-to-bkg ratio $R_{\mathrm{s/b}} = 2$ in the three-flavor model, assuming $\phi_{\tau-h} \geq 2.6\,\mathrm{rad}$. CERN-THESIS-2021-091

Purity of ν_e : $96\,\%$, ν_μ : $99.5\,\%$ at OPERA experiment

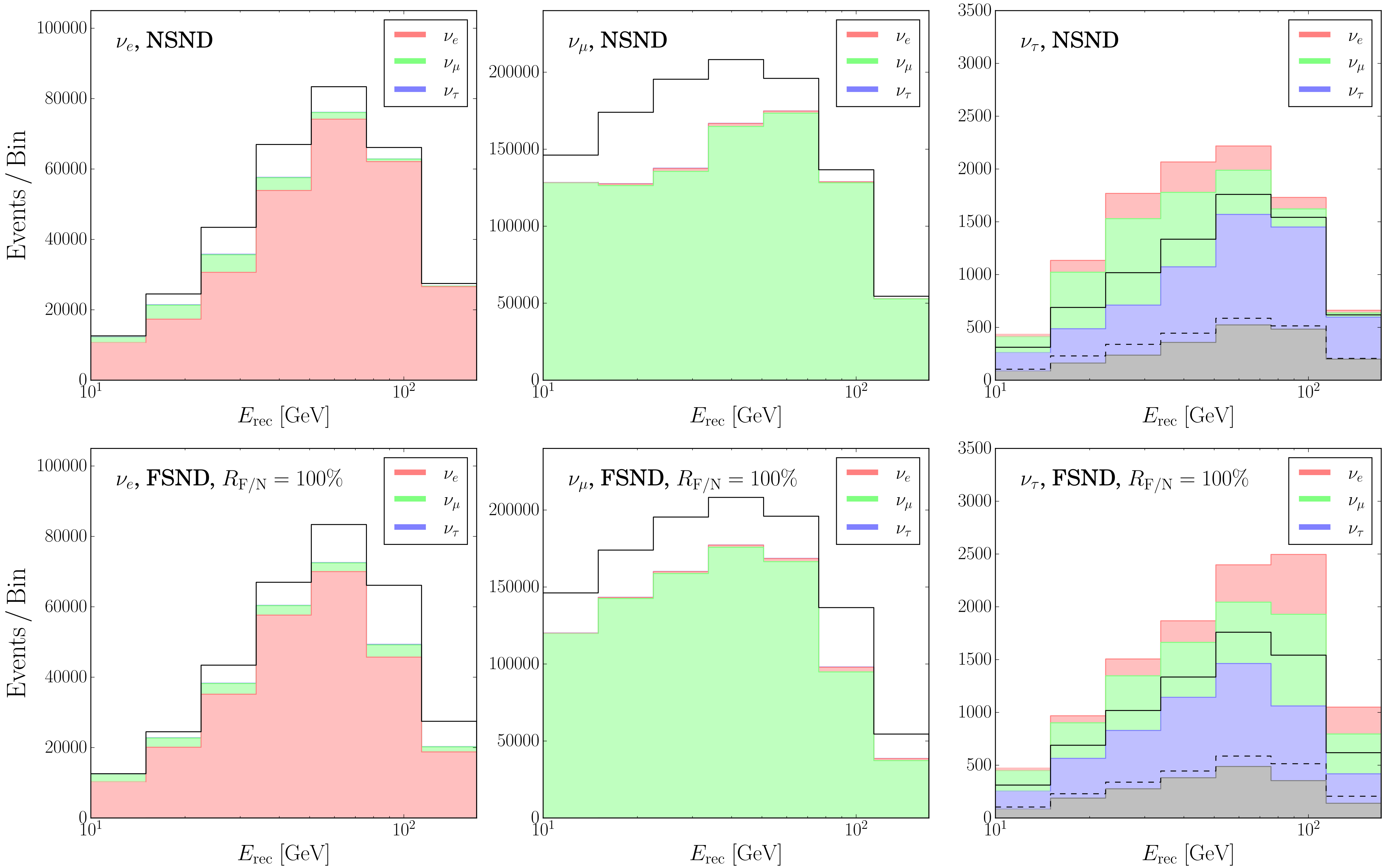
1904.05686

- The majority of the tau neutrino background originates from muon neutrinos.
- Therefore, the background is affected by muon neutrino disappearance.

Choice of energy bin

The energy bin range: $(E_i^{\mathrm{start}}, E_i^{\mathrm{end}})$, $E_0^{\mathrm{start}} = 10\,\mathrm{GeV}$, $E_i^{\mathrm{end}} = 1.5 \times E_i^{\mathrm{start}}$

$\Delta m_{41}^2 = 1000\ \mathrm{eV}^2,\ |U_{e4}|^2 = |U_{\mu 4}|^2 = |U_{\tau 4}|^2 = 0.1$



CC DIS events at SND w/o mixing

Within 5 years of operation with a total of 2×10^{20} POT:

	Electron	Muon	Tau
Signal	3.2×10^5	1.1×10^6	7.2×10^4
Bkg	0	0	3.6×10^4

We draw the sensitivity for cases when

- $R_{\mathrm{F/N}} = 0 \Leftrightarrow$ NSND only (Blue)
- $R_{\mathrm{F/N}} = 10\,\%$ (Purple)
- $R_{\mathrm{F/N}} = 100\,\%$ (Red)

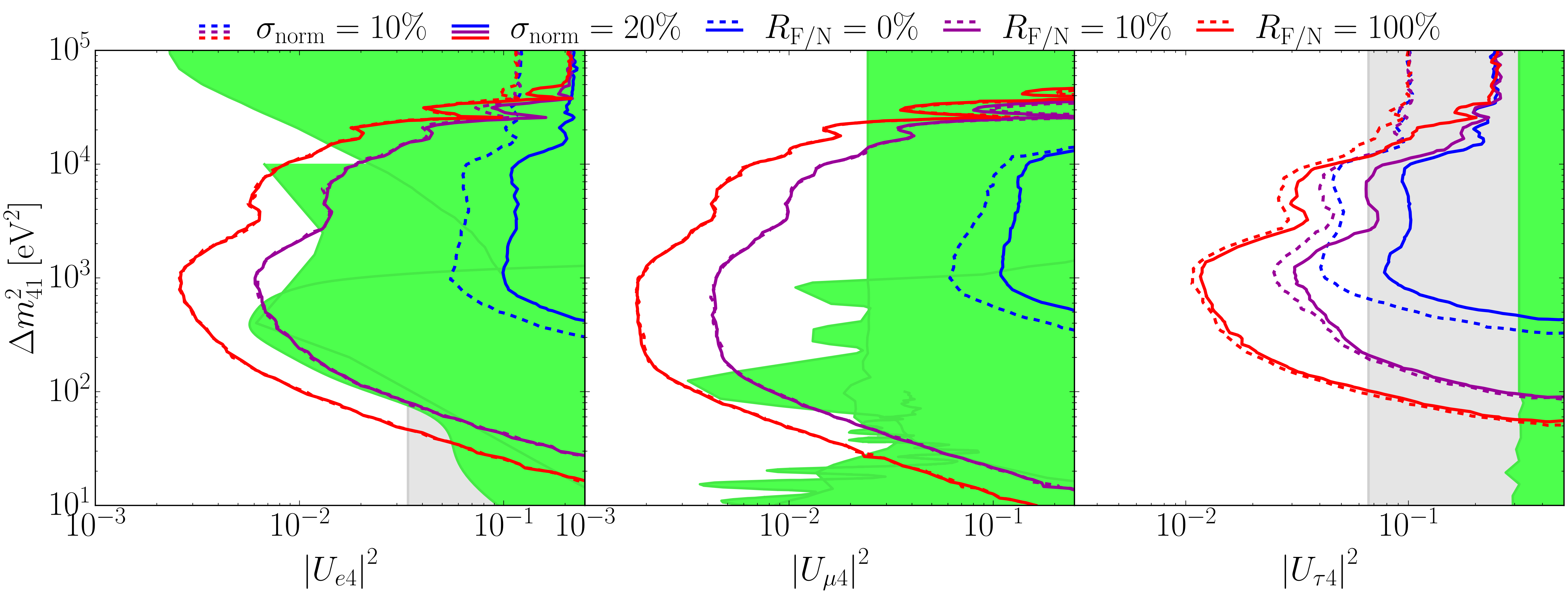
&

- $\sigma_{\mathrm{norm}} = 10\,\%$ (Dashed)
- $\sigma_{\mathrm{norm}} = 20\,\%$ (Solid).

Choice of parameter space

1. Sensitivities on the mixing with single flavor
   - Only disappearance happens.
   - The FC method on the plane $(\Delta m_{41}^2, |U_{\alpha 4}|^2)$.
2. Sensitivities on the mixing with two flavors
   - The FC method on the plane $(|U_{\alpha 4}|^2, |U_{\beta 4}|^2)$, for given $\Delta m_{41}^2 = (50\,\mathrm{eV}^2,\ 250\,\mathrm{eV}^2,\ 1000\,\mathrm{eV}^2,\ 5000\,\mathrm{eV}^2)$.

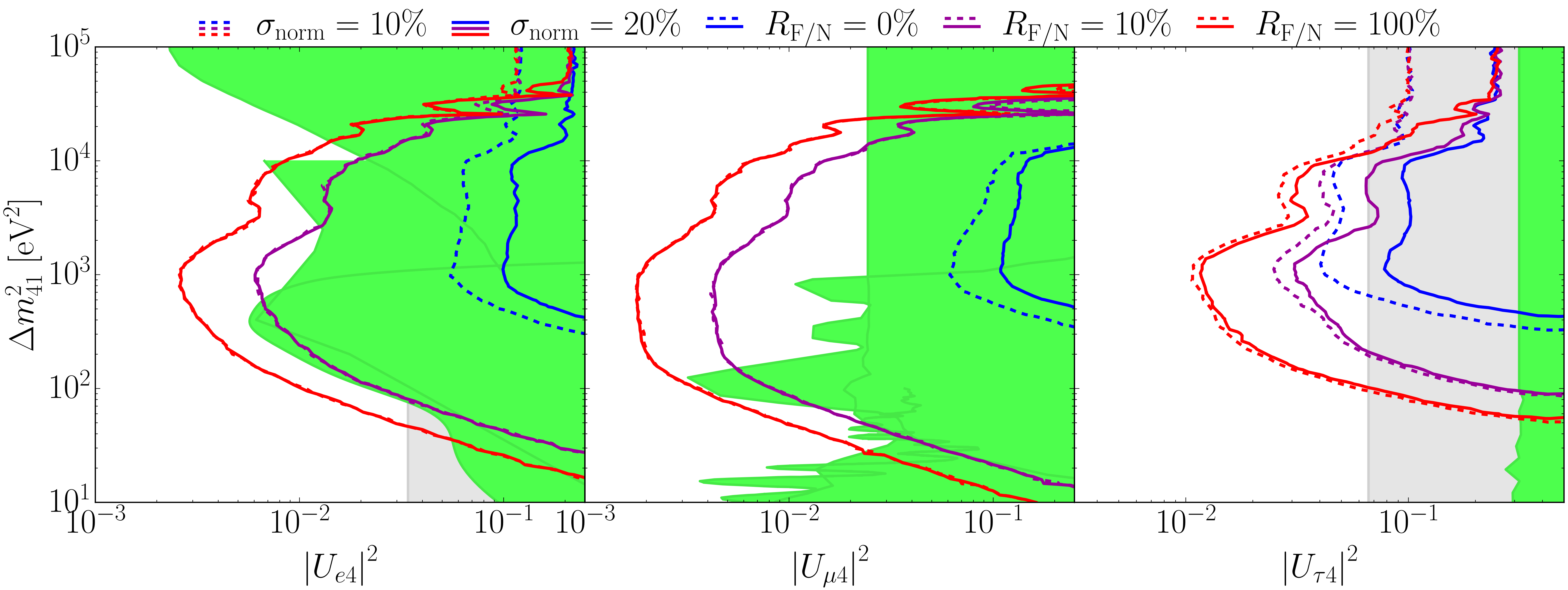
Sensitivities on single flavor mixing



Green: constraints excluded by other experiments

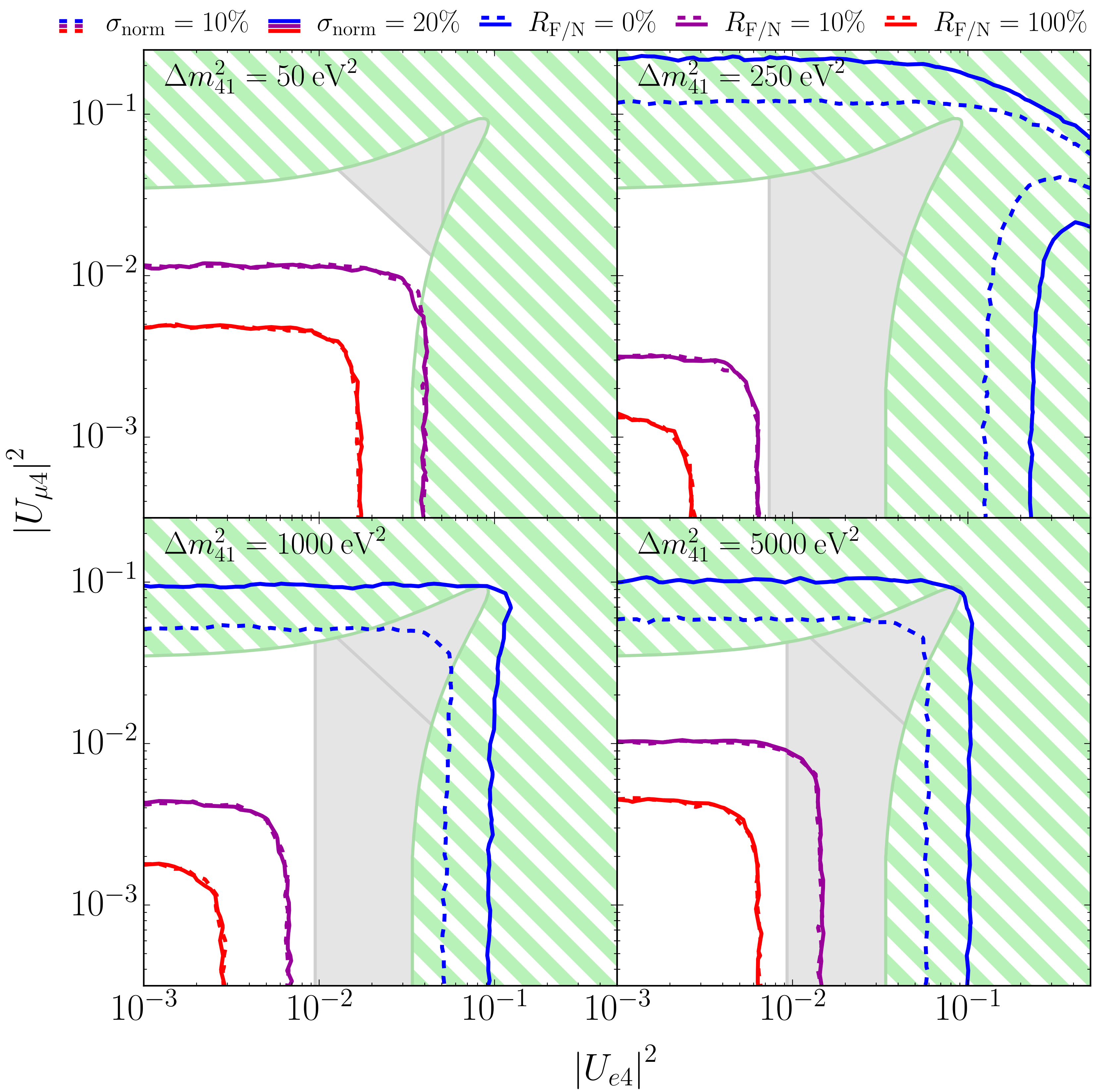
Grey: limit on mixing angle from different parameter space

Sensitivities on single flavor mixing



- Electron flavor: Tritium beta decay experiments
  1307.5687, 2201.11593, 2207.06337
- Muon flavor: CCFR, MiniBooNE+SciBooNE, MINOS
  PRL 52 (1984) 1384 1208.0322 1710.06488
- Tau flavor: NOvA, IceCube-DeepCore (Grey)
  2409.04553 2407.01314

Sensitivities on two flavor mixing ($e - \mu$)



Dashed light green:

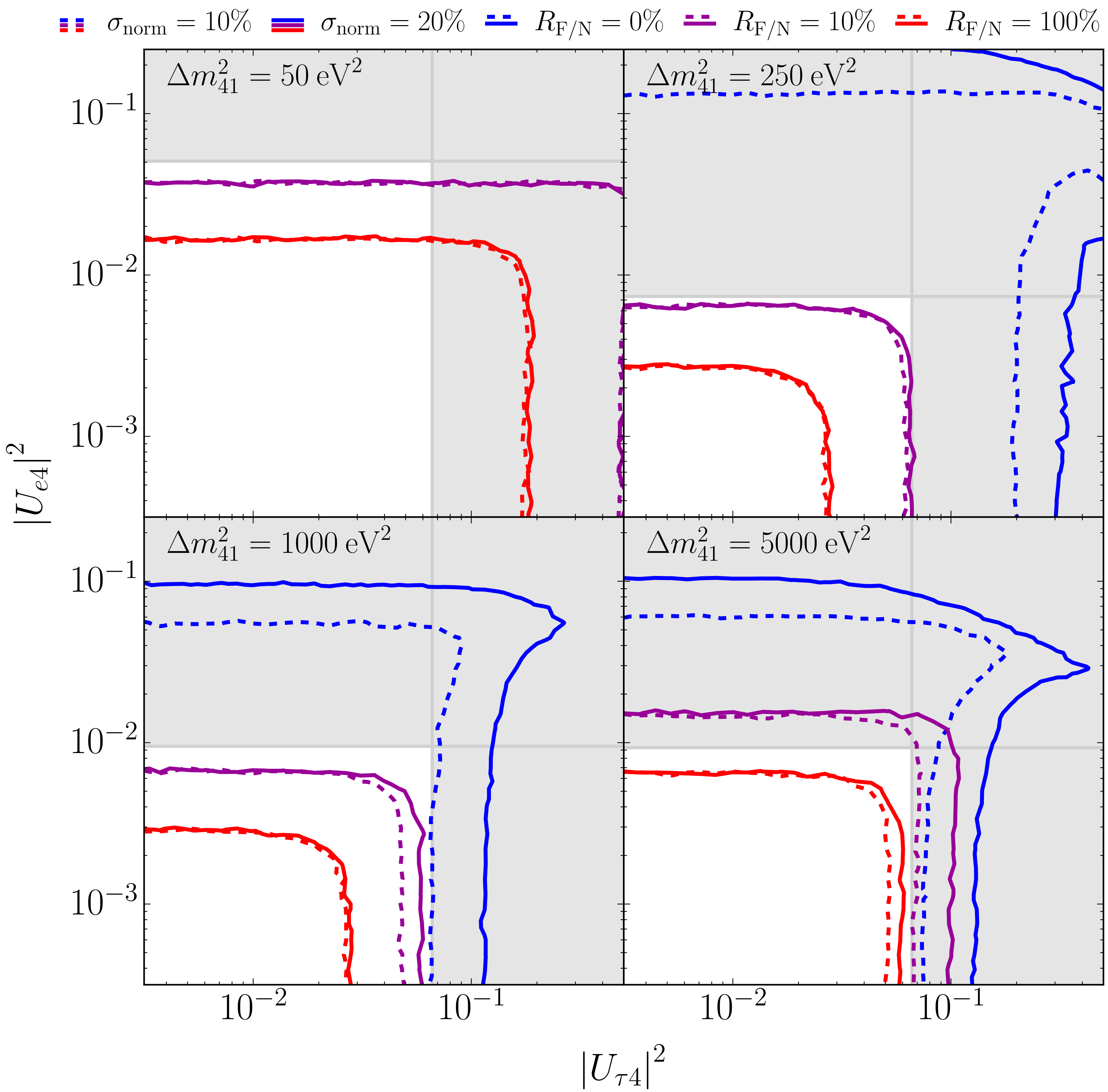
Super-Kamiokande
1410.2008

Grey:

Tritium beta decay exp.,
1307.5687, 2201.11593, 2207.06337

MicroBooNE (diagonal)
2210.10216

Sensitivities on two flavor mixing ($\tau - e$)



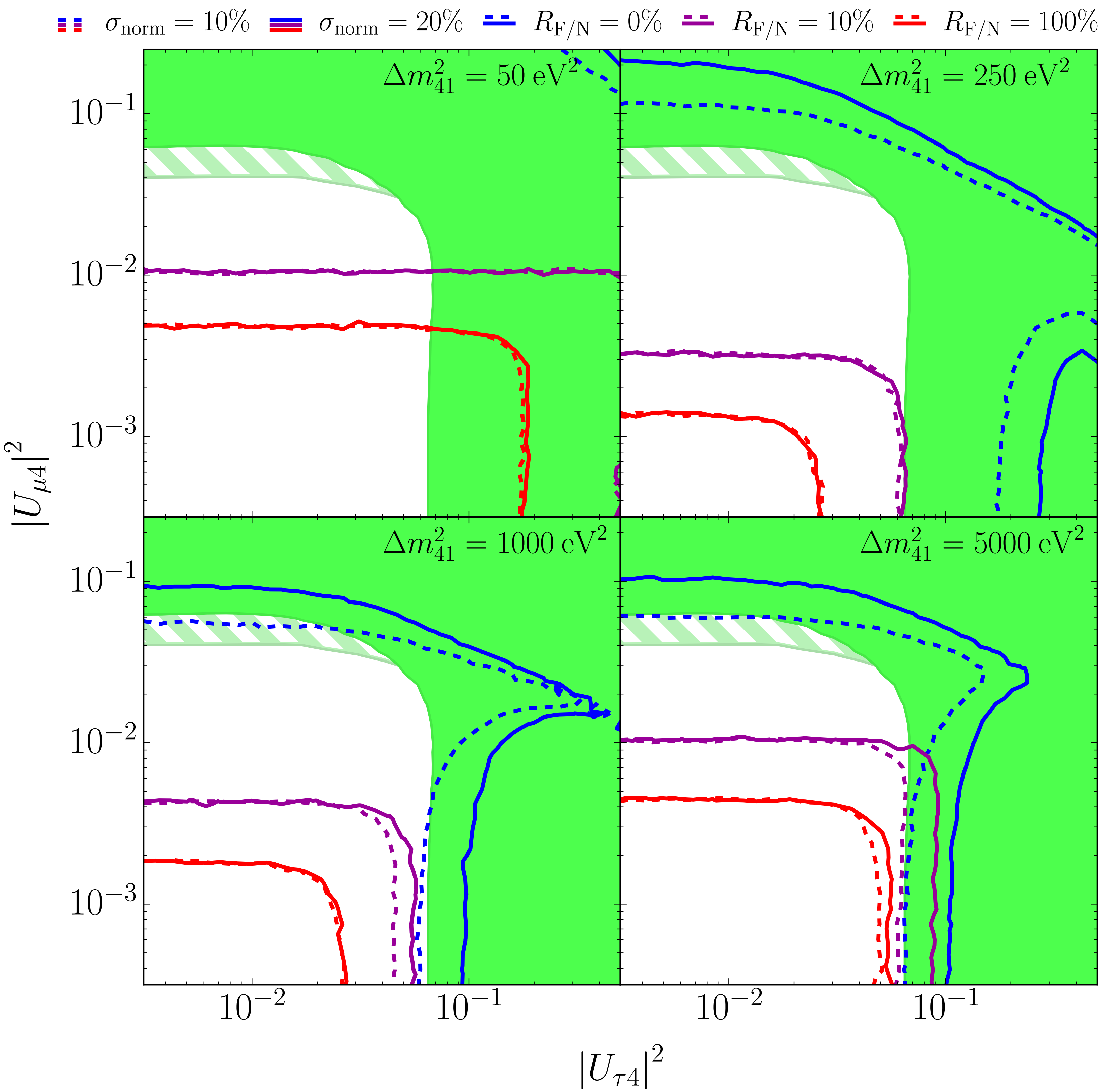
Horizontal grey:

Tritium beta decay exp.,
1307.5687, 2201.11593, 2207.06337

Vertical grey:

IceCube-DeepCore
2407.01314

Sensitivities on two flavor mixing ($\tau - \mu$)



Green:

IceCube-DeepCore

2407.01314

Light Green:

Super-Kamiokande

1410.2008

Conclusion

- Using the FC method with a parametric bootstrap, we estimate sensitivity of SHiP to the $3+1$ model across six $(\sigma_{\mathrm{norm}}, R_{\mathrm{F/N}})$ configurations.

- Reducing σ_{norm} from 20% to 10% roughly doubles sensitivity in NSND, while dependency on σ_{norm} is disappeared with FSND.

- FSND with $R_{\mathrm{F/N}} = 100\,\%$ enhances sensitivities on muon, tau, and electron mixing by factors of 10, 7, and 3 near $\Delta m^2_{41} \sim 10^3\,\mathrm{eV}^2$.

- The cancellation between appearance–disappearance generate kinks only on NSND only cases.