Angular momentum of vacuum bubbles in a first order phase transition

Jan Tristram Acuña

National Institute of Physics, University of the Philippines Diliman,
Quezon City, Philippines

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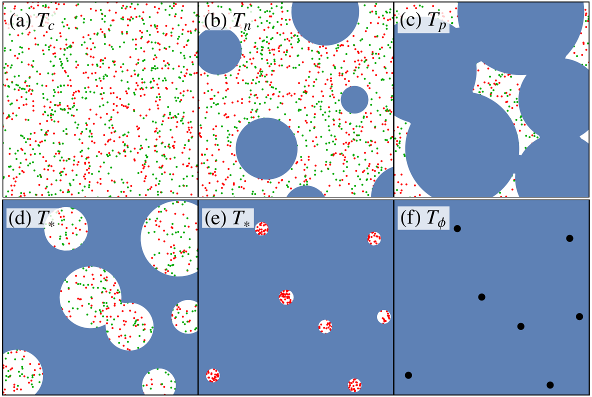
Outline

talk based on JTA, D. Marfatia, and P. Y. Tseng; JCAP 06 (2026) 042 (2505.09202)

Compact objects: PBH and Fermi balls

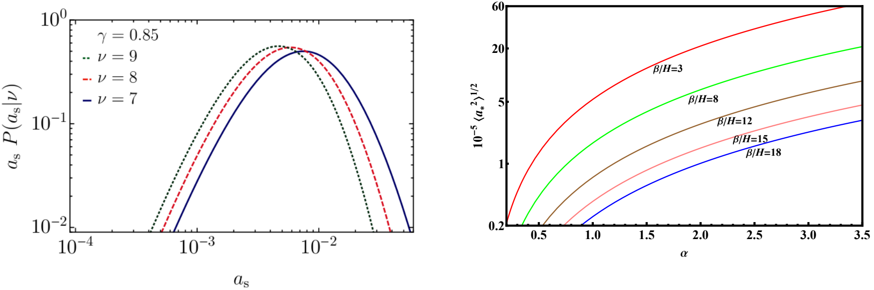


$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_{\rm eff}(\phi, T) + \bar{\chi}\left(i\gamma^\mu\partial_\mu - m\right)\chi - g_\chi\phi\bar{\chi}\chi.$$

$$M_{\rm FB} = Q_{\rm FB}\left[12\pi^2\Delta V_{\rm eff}(T_*)\right]^{1/4} \simeq M_{\rm PBH}, \quad s_{\rm FB}, s_{\rm PBH} = ???$$

K. Kawana and K.P. Xie (2022)

Spin of PBHs



- Left panel: assumed modifications to primordial curvature power spectrum (*i.e.* change γ), then derived spin distribution
- Right panel: assumed curvature power spectrum modified at the FOPT[1], then used formalism in the left panel

[1] Liu et. al. PRL **130**, 051001 (2208.14086)

Left panel: De Luca et. al. (2020); right panel: Banerjee and Dey (2024)

Spin of vacuum bubbles: our assumptions

- decoupled dark and visible sectors with respective temperatures
- singlet scalar, generic quartic potential, $T_0^2 > 0$

$$V_{eff} = \frac{\lambda}{4}\phi^4 - (AT + C)\phi^3 + D(T^2 - T_0^2)\phi^2 - \frac{\pi^2}{90} g_\rho T^4$$

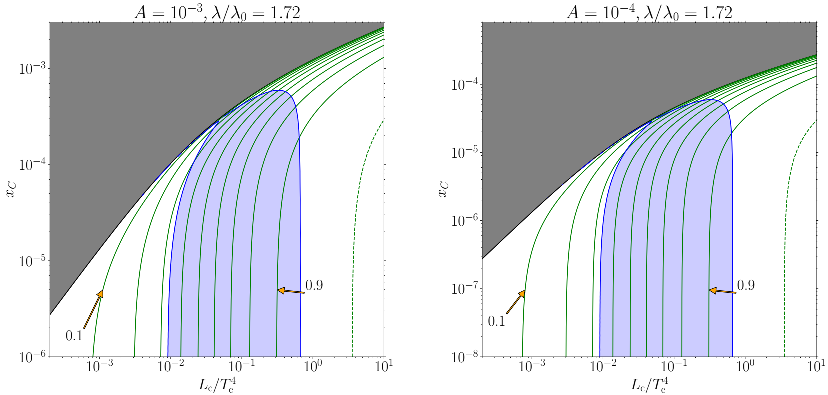
- nearly scale-invariant primordial curvature power spectrum

$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3 P_{\mathcal{R}}(k)}{2\pi^2}, \quad P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3} A_{\rm s} \left(\frac{k}{k_{\rm s}}\right)^{n_{\rm s}-1}$$

- spherical false vacuum bubbles
- angular momentum due to fluid density and velocity perturbations
- long wavelength limit
- single fluid regime

$$p_{\rm D} = F p_{\rm FV} + (1-F) p_{\rm TV}\,, \quad c^2_{\rm s,D} = \frac{\dot{p}_{\rm D}}{\dot{\rho}_{\rm D}} = \frac{s_{\rm D} - \left(\frac{\dot{F}}{H}\right)\frac{\Delta p}{T}}{T\,\frac{\partial s_{\rm D}}{\partial T} - \left(\frac{\dot{F}}{H}\right)\frac{\Delta \rho}{T}}$$

Procedure: Choose FOPT parameters

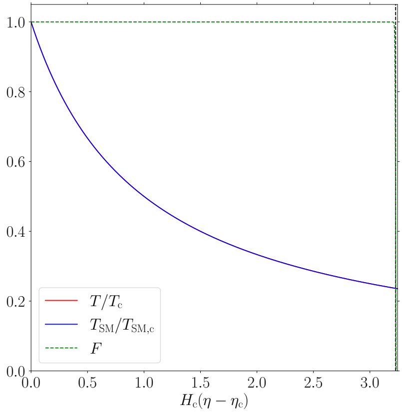


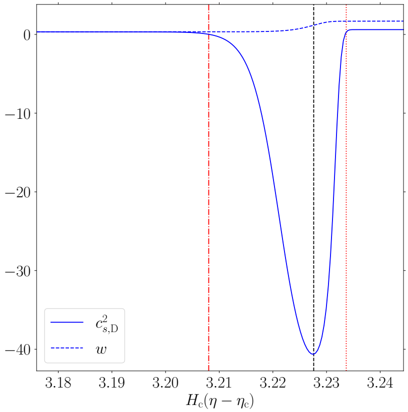
- Choice of FOPT parameters:

$$\{A, C, D, T_0, \lambda\} \to \{A, x_C, T_c, L_c/T_c^4, \lambda\}$$

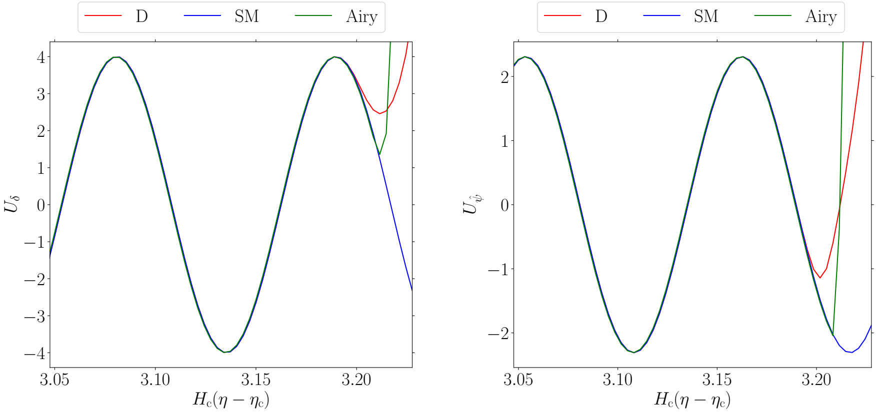
- Reference $\lambda_0 \simeq 0.129(A/0.1)^{4/3}$

Procedure: Track background evolution

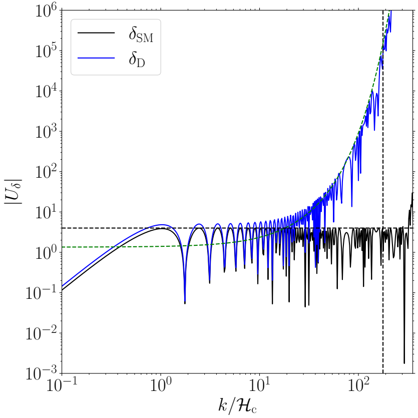


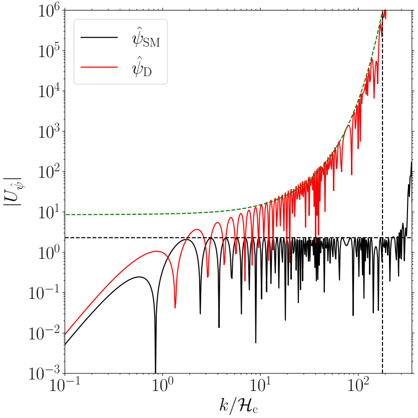


Procedure: Evolve perturbations



Procedure: Evaluate transfer functions





Procedure: Calculate angular momentum integral

$$
\begin{aligned}
J^2_{\mathrm{CM,rms}} &= \left(\frac{3}{4} A_{\mathrm{s}} M R\right)^2 \tilde{x}_0^2 \mathcal{C}(\tilde{x}_0, x_0) \\
\mathcal{C}(\tilde{x}_0, x_0) &\equiv A_{\mathrm{s}}^{-2} \int_0^\infty dz\ z \int_0^\infty dz'\ z' \\
&\quad \times \int_{-1}^{1} dx\ (1 - x^2)\ W^2(z, z'; \tilde{x}_0) \Delta^2_{\mathcal{R}}(k) \Delta^2_{\mathcal{R}}(k')\,, \\
W(z, z'; \tilde{x}_0) &\equiv \frac{U_{\mathrm{eff}}(k) U_\psi(k') \left(\mathcal{F}_{k+k'} - 3\mathcal{F}_k \mathcal{G}_{k'}\right)}{z'} - (z \leftrightarrow z') \\
z \equiv k/(\sqrt{3}\mathcal{H}), &\quad \tilde{x}_0 \equiv \sqrt{3} H R, \quad x_0(\eta) = R/a(\eta)
\end{aligned}
$$

- $\mathcal{F}$ and $\mathcal{G}$ are form factors associated with spherical volumes.
- $U_I \equiv Q_I/\mathcal{R}_k$: transfer functions

Main result: spin estimates

- Basic definitions

$$s_{\rm rms} = \frac{R(\eta)}{G_{\rm N} M(\eta)} v_{\rm eff} \, ,$$

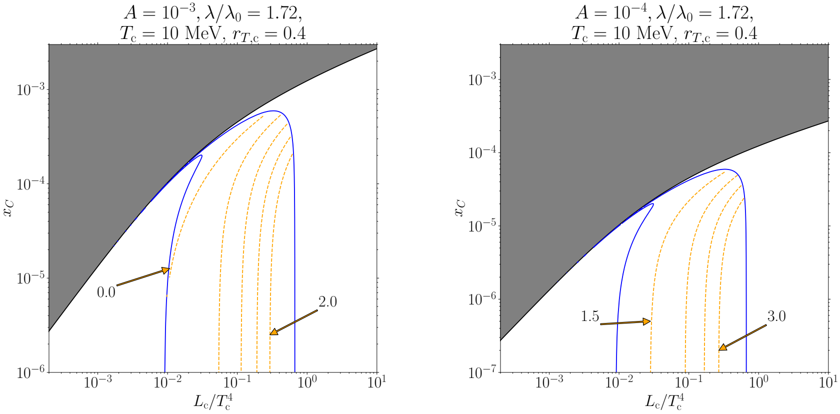
$$v_{\rm eff} = \frac{3}{4} \, A_{\rm s} \tilde{x}_0(\eta) \mathcal{C}^{1/2}(\tilde{x}_0, x_0)$$

$$r_T = \frac{T}{T_{\rm SM}}$$

- Spin estimate

$$s_{\rm rms,*} = \left(\frac{2}{3}\right)^{1/2} \left(\frac{\beta_*/H_*}{v_{\rm w,*}}\right)^2 \left\{ 1 + \frac{g_{\rho,\rm SM}(T_*)}{g_\rho} r_{T,*}^{-4} \left[1 - (1 - F_*) \frac{30}{\pi^2 g_\rho} \frac{\Delta\rho(T_*)}{T_{\rm c}^4} \frac{T_{\rm c}^4}{T_*^4}\right]^{-1} \right\} v_{\rm eff,*}$$

Main result: spin estimates



- Orange dashed curves: contours of constant $\log_{10}[s_{\rm rms,*}/(v_{\rm eff,*}/A_{\rm s})]$
- $v_{\rm eff,*}/A_{\rm s}$: $10^{-5} \sim 10^{-3} \Rightarrow s_{\rm rms,*} \sim \mathcal{O}(10^{-5}) - \mathcal{O}(10)$
- Can obtain spins above unity (cf. Earth: around 889)

Summary and takeaways

- Compact objects can be generated during FOPTs
- We provided an estimate of the spins of the progenitor vacuum bubbles
- Spin is an integral over Fourier modes of the product of the density, velocity perturbations, weighted over a form factor kernel
- We self-consistently tracked the background evolution and perturbations, given an FOPT potential
- We obtained a scaling relation between the RMS spin and FOPT parameters

Backup 1: Background evolution equations

$$\frac{dx}{da} = \frac{1}{ah}\,, \frac{dy}{da} = -\frac{y}{a}\,,$$

$$\frac{dy_{\rm SM}}{da} = -\frac{y_{\rm SM}}{a}\frac{1}{1+\frac{1}{3}\frac{d\ln g_{s,\rm SM}}{d\ln T_{\rm SM}}}\,, \frac{d(\ln F)}{da} = \frac{g_1}{a}v_{\rm w}\ \frac{1}{ah}\,,$$

$$\frac{dg_{i+1}}{da} = \frac{g_i}{a}v_{\rm w}\ \frac{1}{ah}\,, \quad 1 \le i \le 2, \quad \frac{dg_3}{da} = -8\pi a^3\frac{\Gamma(T_{\rm c}y)}{H_{\rm c}^4}\ \frac{1}{ah}\,,$$

$$\frac{d\tilde{\eta}}{da} = \frac{1}{a^2h}$$

$$g_n(t) \equiv -\frac{4\pi}{3}\ \frac{3!}{(3-n)!}H_{\rm c}^{4-n}\int_{t_{\rm c}}^{t} dt'\ \frac{\Gamma(T(t'))}{H_{\rm c}^4}a^3(t')\ r^{3-n}(t,t')\,.$$

Backup 2: Continuity and Euler equations in the uniform Hubble gauge

$$\frac{\delta'_{\rm D}}{\mathcal{H}} - \frac{k}{\mathcal{H}}\hat{\psi}_{\rm D} + 3(1+w_{\rm D})\Psi_k + 3(c^2_{s,{\rm D}} - w_{\rm D})\delta_{\rm D} = 0\,,$$

$$\frac{\hat{\psi}'_{\rm D}}{\mathcal{H}} + (1-3w_{\rm D})\hat{\psi}_{\rm D} + c^2_{s,{\rm D}}\frac{k}{\mathcal{H}}\delta_{\rm D} + (1+w_{\rm D})\frac{k}{\mathcal{H}}\Psi_k = 0\,,$$

$$\frac{\delta'_{\rm SM}}{\mathcal{H}} - \frac{k}{\mathcal{H}}\hat{\psi}_{\rm SM} + 4\Psi_k = 0, \quad \frac{\hat{\psi}'_{\rm SM}}{\mathcal{H}} + \frac{1}{3}\frac{k}{\mathcal{H}}\delta_{\rm SM} + \frac{4}{3}\frac{k}{\mathcal{H}}\Psi_k = 0$$

$$\left[\left(\frac{k}{\mathcal{H}}\right)^2 + \frac{9}{2}(1+w)\right]\Psi_k + \frac{3}{2}(1+3c_s^2)\delta = 0$$

$$\delta \equiv \frac{\rho_{\rm D}\delta_{\rm D} + \rho_{\rm SM}\delta_{\rm SM}}{\rho_{\rm D}+\rho_{\rm SM}},$$

$$w \equiv \frac{\rho_{\rm D}w_{\rm D} + \rho_{\rm SM}w_{\rm SM}}{\rho_{\rm D}+\rho_{\rm SM}}, \quad c_s^2 \equiv \frac{\rho'_{\rm D}c^2_{s,{\rm D}} + \rho'_{\rm SM}c^2_{s,{\rm SM}}}{\rho'_{\rm D}+\rho'_{\rm SM}}$$

Backup 3: Relevant FOPT quantities

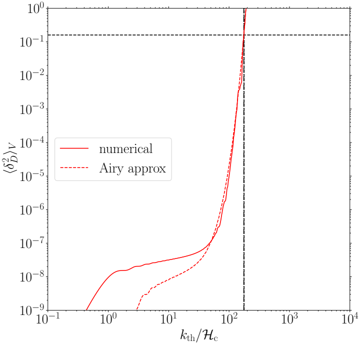
$$-\ln F(t) = \frac{4\pi}{3} \int_{t_{\rm c}}^{t} dt' \; \Gamma(T(t')) \; a^3(t') r^3(t,t')$$

$$r(t,t') \equiv \int_{t'}^{t} dt'' \; \frac{v_{\rm w}(t'')}{a(t'')}$$

$$\Gamma(T) = T^4 \left[\frac{S_3(T)}{2\pi T}\right]^{3/2} \exp\left[-\frac{S_3(T)}{T}\right]$$

$$v_{\rm w} = 1 - \left(\frac{T}{T_{\rm c}}\right)^{n_{\rm w}} , \quad n_{\rm w} = 8$$

Backup 4: $k_{\rm BH}$ and $k_{\rm cut}$



$$\langle\delta^2(\vec{r})\rangle_V \equiv \frac{1}{V_R}\int_R d^3\vec{r}\,\langle\delta^2(\vec{r})\rangle$$

$$= \lim_{k_{\rm th}\to\infty}\int_0^{k_{\rm th}}\frac{dk}{k}\,\frac{k^3 P_{\mathcal{R}}(k)}{2\pi^2}\,|U_\delta(k)|^2$$

$$k_{\rm cut} = \frac{\pi}{R_*},\,\langle\delta^2(\vec{r})\rangle_V\Big|_{k_{\rm th}=k_{\rm BH}} = 0.16$$

Backup 4: Center-of-mass angular momentum and distribution

$$\vec{J}_{\rm CM}(\eta) = 4\pi\bar{\rho}R^5 \int d\Pi_k d\Pi_{k'}\ (\vec{k}\times\vec{k}')\ \delta_{\rm eff}(\vec{k},\eta)$$
$$\times \left[\mathcal{F}(|\vec{k}+\vec{k}'|x_0) - 3\mathcal{F}(kx_0)\mathcal{G}(k'x_0)\right] \frac{\hat{\psi}(\vec{k}',\eta)}{k'}$$

$$P\left(\vec{J}\,\middle|\,x_0,\eta\right) = \frac{3}{J^2_{\rm CM,rms}(x_0,\eta)\sqrt{2\pi}} \exp\left[-\frac{3\vec{J}^2}{2J^2_{\rm CM,rms}(x_0,\eta)}\right]$$

- $\mathcal{F}$ and $\mathcal{G}$ are form factors associated with spherical volumes.

cf. P. J. E. Peebles (1969)

Backup 5: Benchmark points

	BP-1	BP-2	BP-3	BP-4	BP-5	BP-6
A	10^{-3}	10^{-3}	10^{-3}	10^{-4}	10^{-4}	10^{-4}
x_C	1.1063×10^{-4}	2.4432×10^{-4}	2.4131×10^{-4}	1.712×10^{-5}	3.97×10^{-7}	8.19×10^{-6}
λ	3.6917×10^{-4}	3.6917×10^{-4}	4.8063×10^{-4}	1.7136×10^{-5}	1.7136×10^{-5}	2.2309×10^{-5}
$L_\mathrm{c}/T_\mathrm{c}^4$	5.0456×10^{-2}	9.0072×10^{-2}	3.998×10^{-2}	7.2479×10^{-2}	2.8264×10^{-2}	1.9733×10^{-2}
T_c (MeV)	10^3	10^{-2}	10	10^3	10	10^5
$r_{T,\mathrm{c}}$	0.4	0.4	0.4	0.4	0.1	0.4
T_* (MeV)	3.4306×10^2	2.3654×10^{-3}	2.4216	3.0246×10^2	3.8815	3.3936×10^4
$T_{\mathrm{SM},*}$ (MeV)	8.9392×10^2	5.9135×10^{-3}	6.2448	7.9351×10^2	4.1986×10^1	8.5999×10^4
$\sqrt{3}H_* R_*$	1.1716×10^{-3}	1.7169×10^{-3}	2.3446×10^{-3}	2.9002×10^{-4}	2.2558×10^{-4}	3.4178×10^{-4}
$M_*/M_\odot$	2.88×10^{-14}	0.2359	8.73×10^{-8}	5.44×10^{-16}	3.8×10^{-15}	5.3×10^{-20}
α_*	1.2×10^{-5}	8.1792×10^{-4}	3.19×10^{-5}	5.1×10^{-5}	2.71×10^{-7}	4.99×10^{-6}
β_*/H_*	2.2962×10^3	1.5653×10^3	1.1432×10^3	9.309×10^3	1.3535×10^4	8.1307×10^3
$v_{\mathrm{w},*}$	0.9998	1	1	0.9999	0.9995	0.9998
$s_{\mathrm{rms},*}/(v_{\mathrm{eff},*}/A_\mathrm{s})$	7.3349	0.1379	0.2581	1.2304×10^2	1.1132×10^4	1.0651×10^2
$\tilde{\eta}_* - \tilde{\eta}_i$	1.2187×10^{-2}	2.5631×10^{-2}	3.2774×10^{-2}	3.8714×10^{-3}	2.08×10^{-3}	3.9741×10^{-3}
$\tilde{\eta}_*$	1.8899	3.2276	3.1085	2.2708	1.5419	1.9378
$c_{s,\mathrm{D}*}^2$	-1.664×10^1	-4.0616×10^1	-4.0832×10^1	-1.3081×10^1	-6.9385	-1.4892×10^1
$v_{\mathrm{eff},*}/A_\mathrm{s}$	8.2735×10^{-4}	2.4278×10^{-4}	7.6×10^{-5}	1.2037×10^{-3}	1.2595×10^{-3}	3.8337×10^{-4}
$k_\mathrm{BH}/k_\mathrm{cut}$	1.3085	0.9904	1.3485	0.9900	1.5711	1.3911
s_rms	6.0685×10^{-3}	3.3471×10^{-5}	1.96×10^{-5}	0.1481	1.4021×10^1	4.0831×10^{-2}