An On-Shell Bootstrap Program of Cosmological Correlators

Zhong-Zhi Xianyu (鲜于中之)
Department of physics, Tsinghua University

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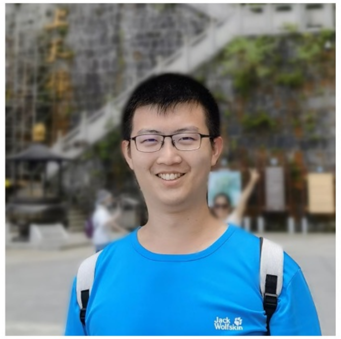
刘皓源
Haoyuan Liu

秦哲涵
Zhehan Qin



吴家毅
Jiayi Wu

张洪语
Hongyu Zhang

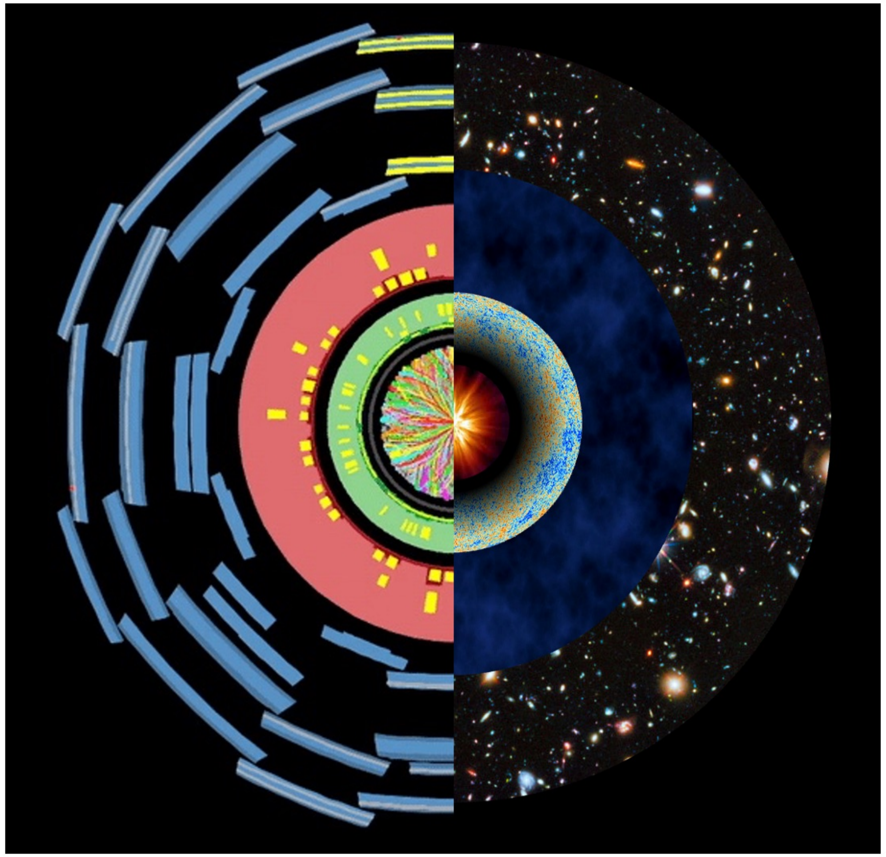
Based on

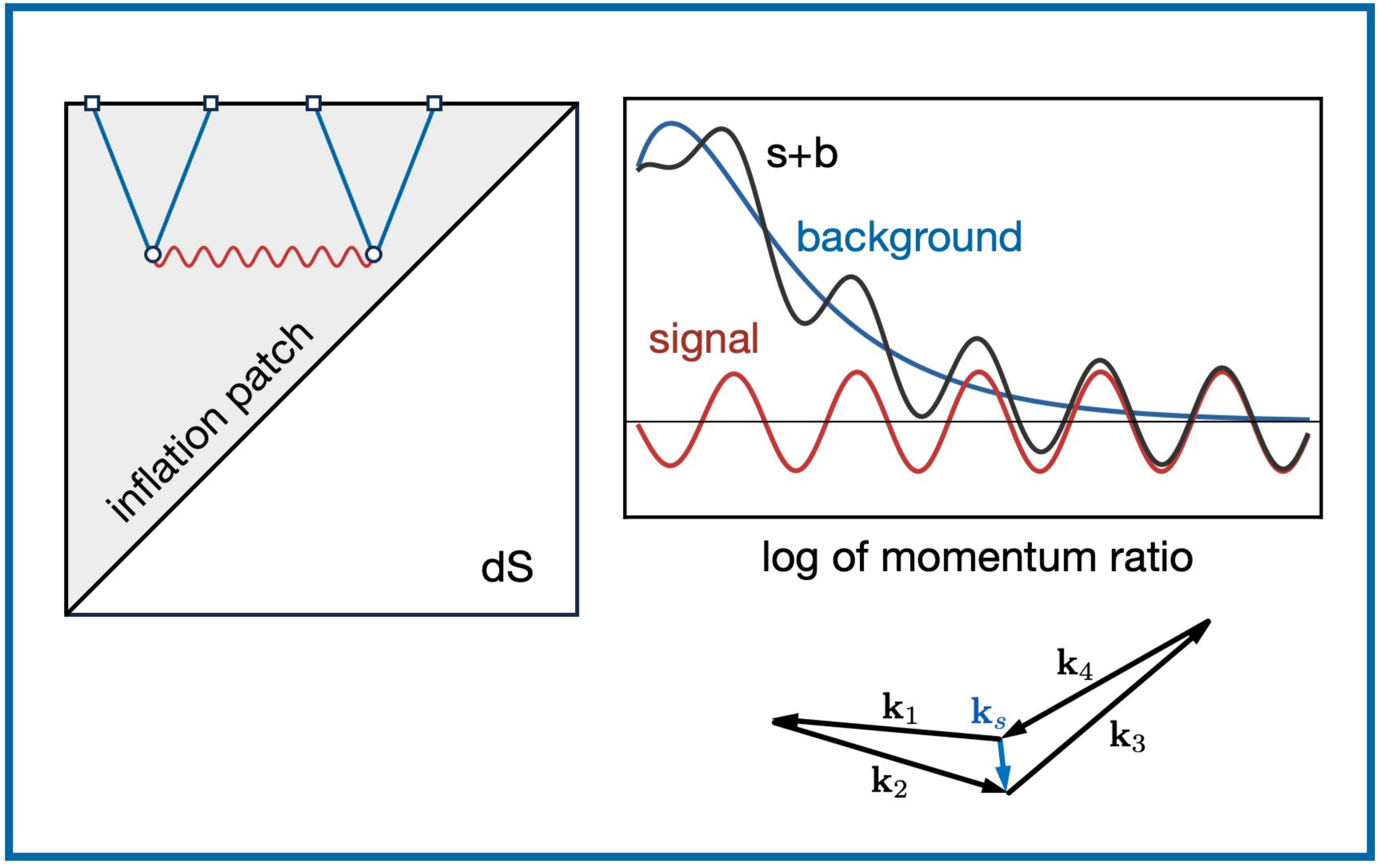
Haoyuan Liu, Zhehan Qin, Jiayi Wu, ZX, Hongyu Zhang, 2606.02686

and also

ZX, Hongyu Zhang, 2211.03810
Qin, ZX, 2304.13295, 2308.14802
Haoyuan Liu, Zhehan Qin, ZX, 2407.12299

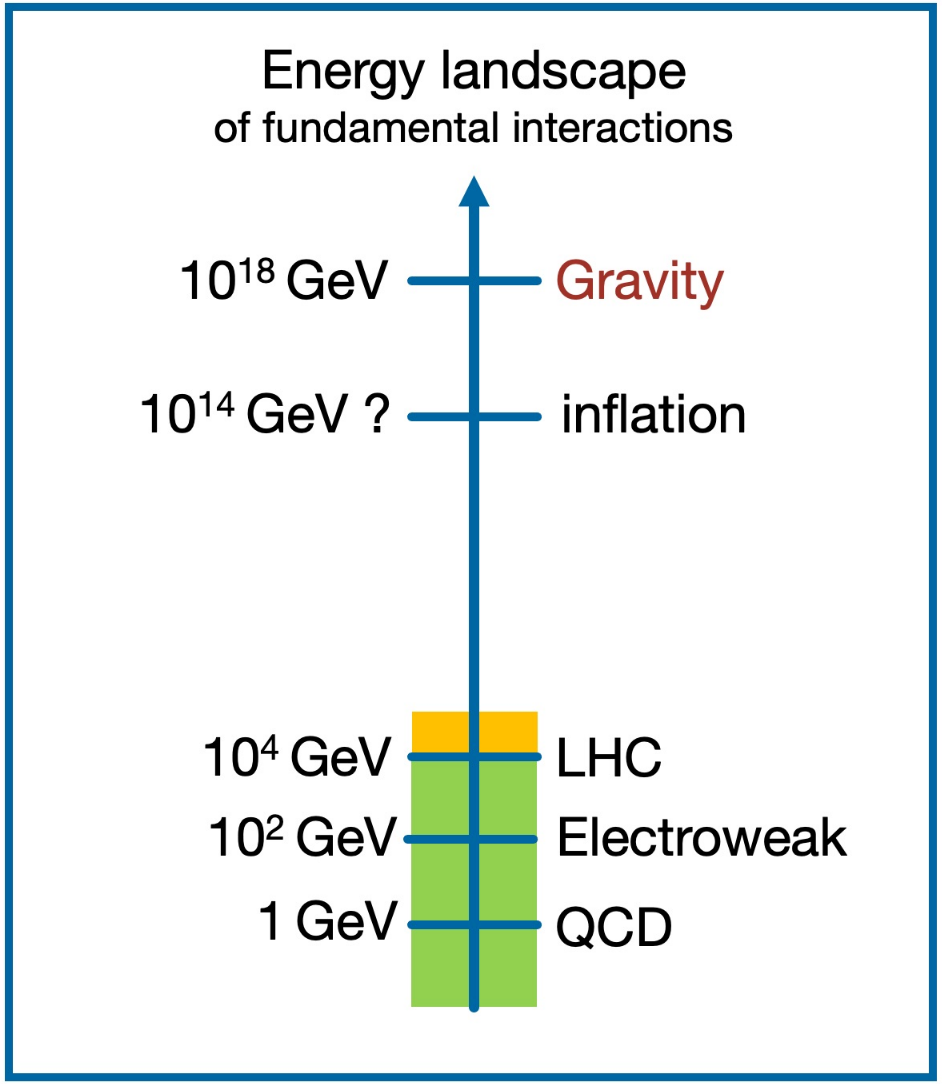
Cosmological collider physics

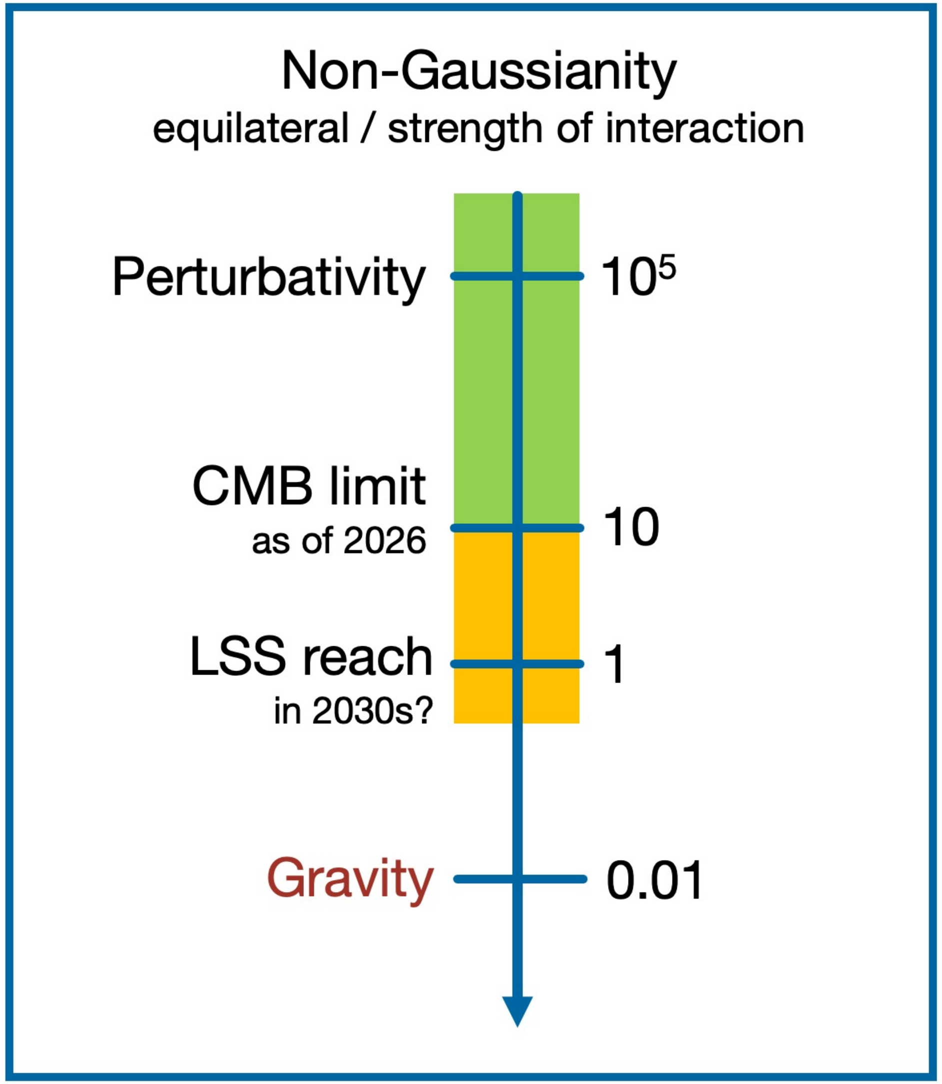




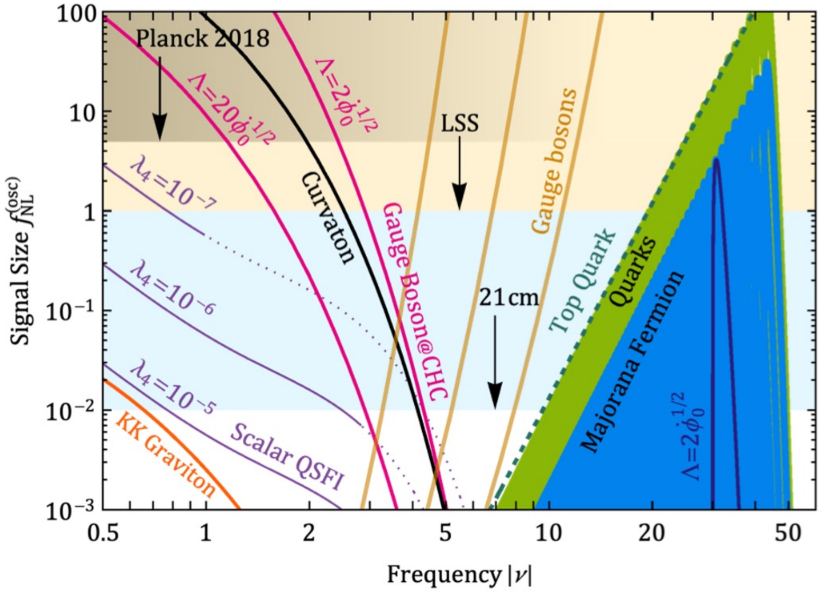
[Chen, Wang, 0911.3380; Arkani-Hamed, Maldacena, 1503.08043 and many more]

Cosmological collider physics





Cosmological collider physics | Particle Phenomenology



[Lian-Tao Wang, ZX, 1910.12876]

Over the years, many particle models identified in SM/BSM, with naturally large signals

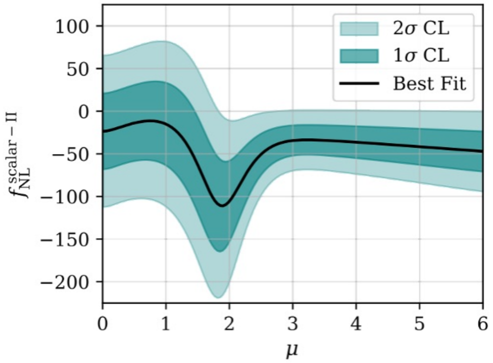
Many fascinating stories which are still ongoing

The CC signals can be there, and deserve to be treated seriously

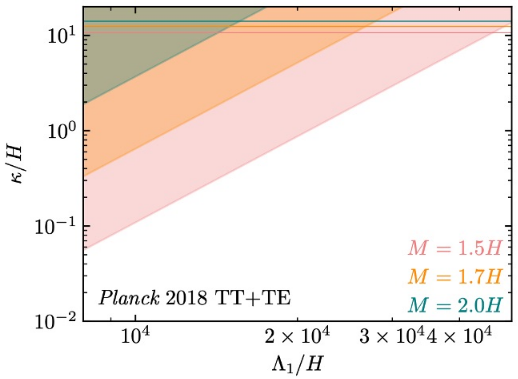
Cosmological collider physics | data analysis

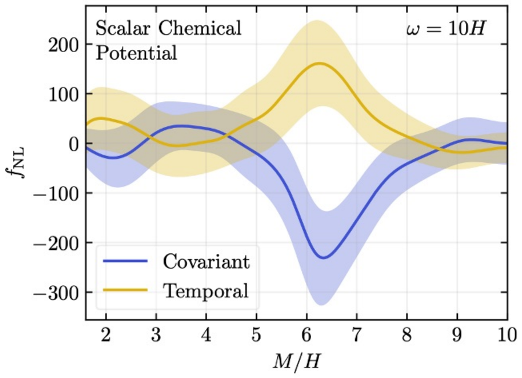
CMB-based (Planck)

The first and a comprehensive search of single-exchange signals
[Sohn, Wang, Fergusson, Shellard, 2404.07203, (w/ Suman), 2512.22085]



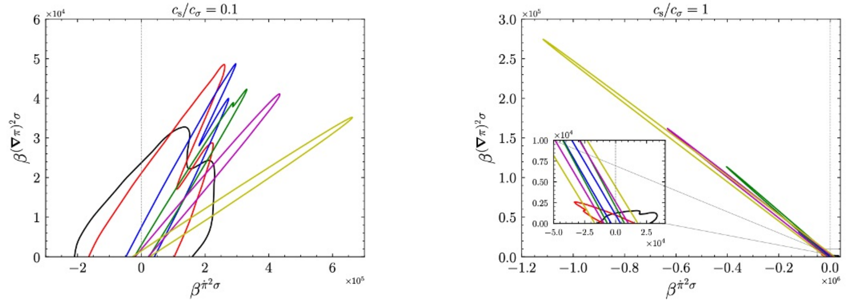
The first constraint on the classic triple exchange in quasi-single-field inflation and scalar chemical potential [Kumar, Lu, ZX, Zhang, 2603.15728, 2604.07434]



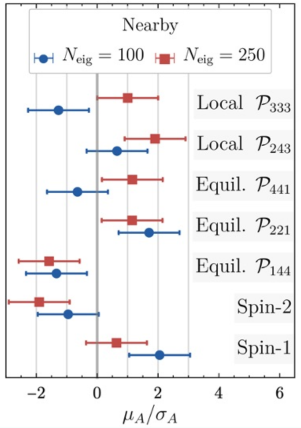


LSS-based (BOSS)

Constraints on inflaton-massive state couplings in EFT of inflation
[Cabass et al., 2404.01894]



Parity-violating cosomlogical collider signal search from LSS trispectra
[Bao, Wang, ZX, Zhong, 2504.02931]



Cosmological collider physics

Phenomenology

BSM particle physics

primordial cosmology

Theory

quantum field theory in dS

cosmological correlators

Data analysis

template bank

data processing

Cosmological correlators | the problem

- Weakly coupled QFTs in the bulk (loop expansion works)
- dS isometries often broken (especially dS boosts, sometimes dilitation as well)
- They suggest us to develop analytical methods that exploit bulk perturbativity in a model independent way but do not heavily rely on full dS isometries
- The goals:
  1. Explicit results of correlators in terms of well-defined functions
  2. Understanding the analytic structures: complete characterization of their singularities
  3. Analytical continuation and efficient numerical strategies
- A comment on tensor structure:

$$\langle \varphi_{\boldsymbol{k}_1} \cdots \varphi_{\boldsymbol{k}_n} \rangle' \sim \sum_{\text{tensor structure}} \text{tensorial factor} \times \text{scalar integral}$$

Cosmological correlators | challenges and progress

- Perturbative in the bulk | Typically boost/dilatation breaking | tensor structure × scalar integral

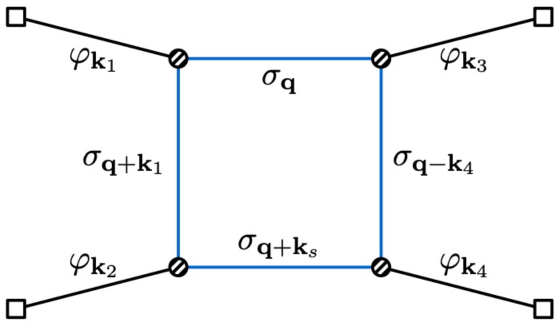
$$\mathcal{T}(\{\boldsymbol{k}\}) \sim \int \mathrm{d}\tau \int \mathrm{d}^d\boldsymbol{q} \times (-\tau)^p \times e^{\mathrm{i}E\tau} \times \mathrm{H}_{\mathrm{i}\tilde{\nu}}\Big[-K(\boldsymbol{q},\boldsymbol{k})\tau\Big] \times \theta(\tau_i - \tau_j)$$

vertex int | loop int | ext line | bulk line

[See Chen, Wang, ZX, 1703.10166 for a review]

- Challenges: Special functions & nested time integrals
  Complexity increases with # of loops AND vertices

- Many exciting new results over the years
  differential equations [Arkani-Hamed, Baumann, Lee, Pimentel 1811.00024, Aoki, Pinol, Sano, Yamaguchi, Zhu, 2404.09547, and many more]
  partial MB rep [Qin, ZX 2205.01692 etc]
  family-tree decomposition [ZX, Zang, 2309.10849 etc]
  complete tree solutions [Liu, ZX, 2412.07843 etc]
  chemical potential, sound speed, features, loops…

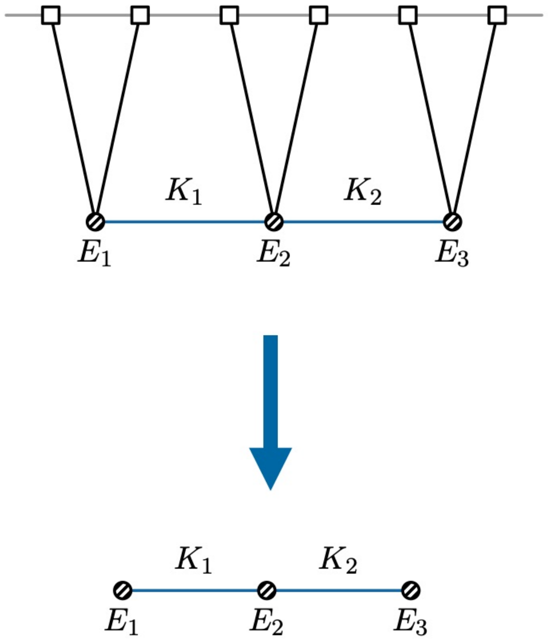


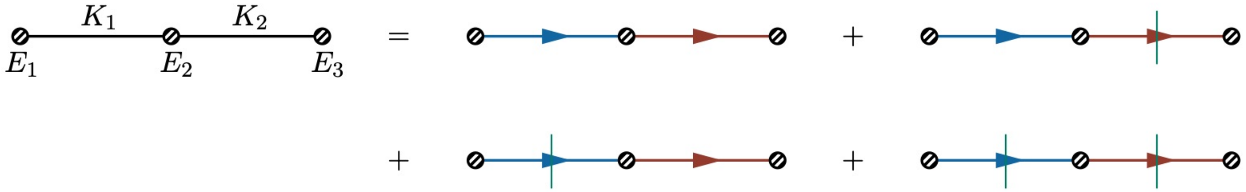
Massive family trees: WYSIWYG rules for all trees

[Liu, ZX, 2412.07843]

$$\boxed{\mathcal{G} = \sum_{i\in 2^{\{K\}}} \underset{i}{\mathrm{Cut}}\,[\mathcal{G}]}$$

- The complete solution to arbitrary massive tree is the sum of the CIS (completely inhom sol) and all of its cuts.
- CIS => massive family tree
- Cuts => "tuned" (♯ or ♭) massive family trees





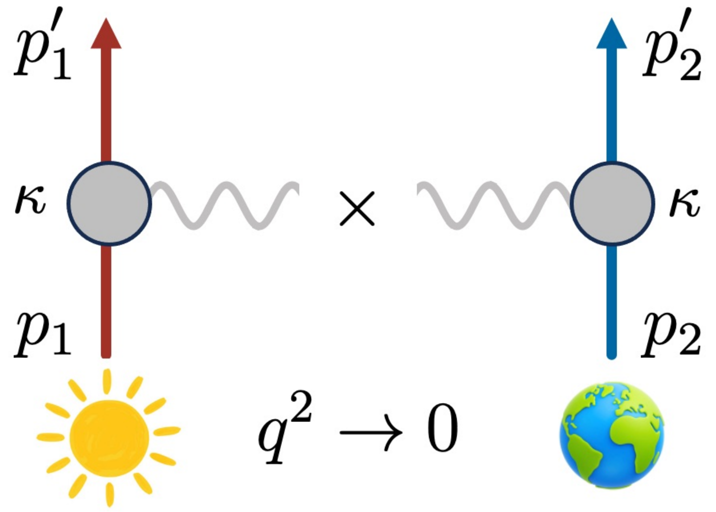
$$\mathcal{G}_3 = \llbracket 123 \rrbracket + \llbracket 1^{\sharp_1} \rrbracket \Big(\llbracket 2^{\sharp_1} 3 \rrbracket + \llbracket 2^{\flat_1} 3 \rrbracket \Big) + \llbracket 12^{\sharp_2} \rrbracket \Big(\llbracket 3^{\sharp_2} \rrbracket + \llbracket 3^{\flat_2} \rrbracket \Big)$$
$$+ \llbracket 1^{\sharp_1} \rrbracket \Big(\llbracket 2^{\sharp_1 \sharp_2} \rrbracket + \llbracket 2^{\flat_1 \sharp_2} \rrbracket \Big) \Big(\llbracket 3^{\sharp_2} \rrbracket + \llbracket 3^{\flat_2} \rrbracket \Big) + \text{shadows}$$

This talk:

Bootstrapping inflationary correlators (tree & loop) from their on-shell data

On-shell bootstrap | lessons from scattering amplitudes

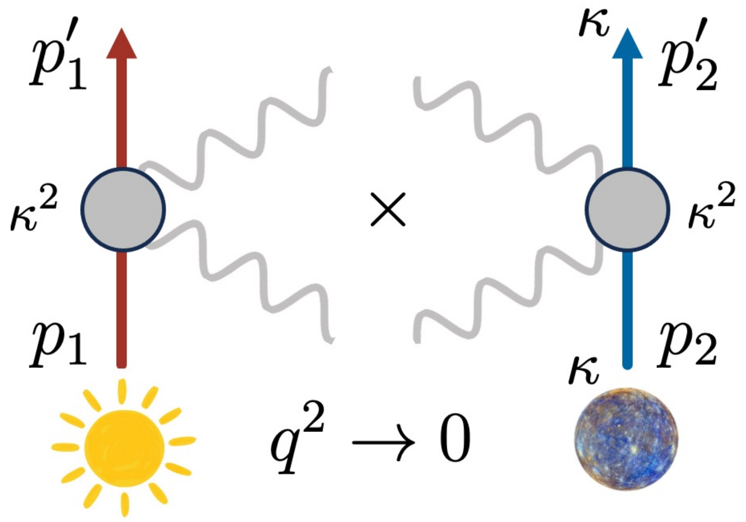
Tree level: on-shell poles



$$\mathcal{A}(q^2) = \kappa m_1 \times \frac{1}{q^2} \times \kappa m_2 + \text{regular}$$

That’s why the Earth orbits

Loop level: unitarity cuts



$$\mathcal{A}(q^2) = \int \frac{\mathrm{d}^4\ell}{(2\pi)^4} \frac{C_1 \times C_2}{\ell^2 (q-\ell)^2} + \text{regular}$$

That’s why Mercury precesses

On-shell bootstrap | cosmological counterparts

- Lesson: The on-shell data from an amplitude are often simpler and physically more transparent than the whole thing => Bootstrapping the amplitude from the on-shell data

Flat-space scattering amplitudes	Cosmological correlators
Analytic structure poles & branch points from on-shell limit	**Analytic structure** physical and unphysical branch cuts
On-shell data on-shell poles (tree) & cuts (loop)	**On-shell data** CC signal branch cuts **(tree & loop)**
Bootstrap strategy BCFW (tree) & unitarity cuts (loop)	**Bootstrap strategy** Dispersion relations

Analytic structures | tree-level example

- All singularities of a correlator come in two types:
- 1. Physical singularities

 1a) Nonlocal-signal branch point $K \to 0$

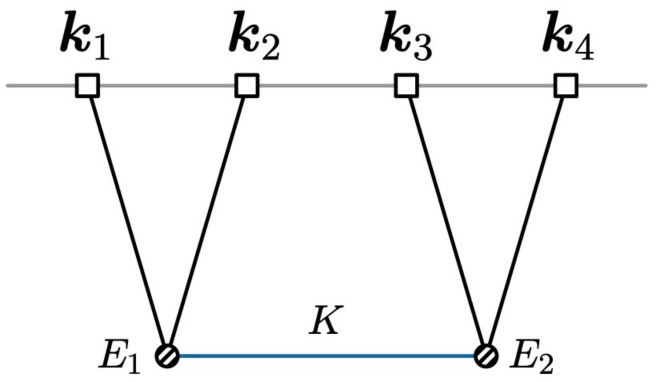
 1b) Local-signal branch point E_1 or $E_2 \to \infty$

- They probe low-energy on-shell physics
- 2. Unphysical singularities

 2a) Partial-energy "pole" $E_1 + K$ or $E_2 + K \to 0$

 2b) Total-energy "pole" $E_1 + E_2 \to 0$

- They probe high-energy physics



$$E_1 = |\boldsymbol{k}_1| + |\boldsymbol{k}_2|$$

$$E_2 = |\boldsymbol{k}_3| + |\boldsymbol{k}_4|$$

$$K = |\boldsymbol{k}_1 + \boldsymbol{k}_2|$$

Analytic structures | tree-level example

- All singularities of a correlator come in two types:
- 1. Physical singularities

1a) Nonlocal-signal branch point $K \to 0$

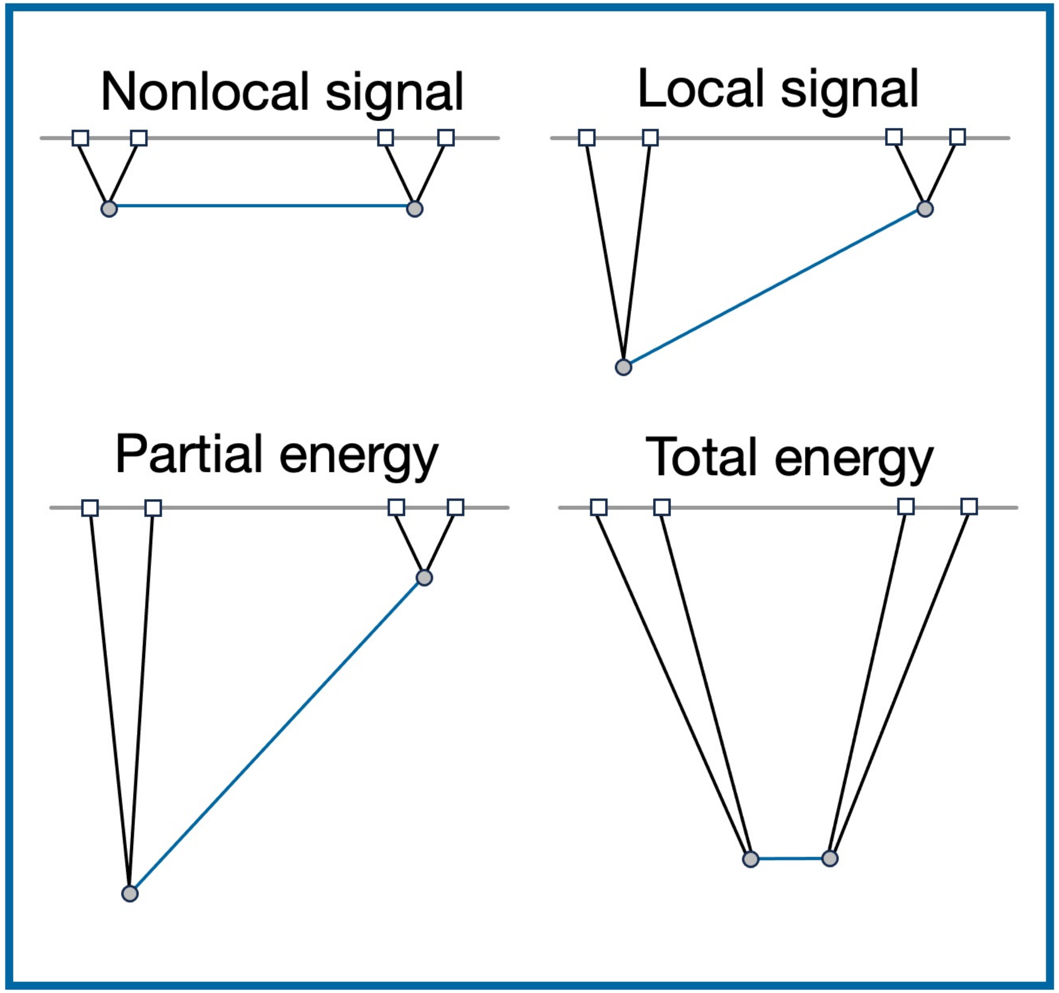
1b) Local-signal branch point E_1 or $E_2 \to \infty$

- They probe low-energy on-shell physics
- 2. Unphysical singularities

2a) Partial-energy “pole” $E_1 + K$ or $E_2 + K \to 0$

2b) Total-energy “pole” $E_1 + E_2 \to 0$

- They probe high-energy physics

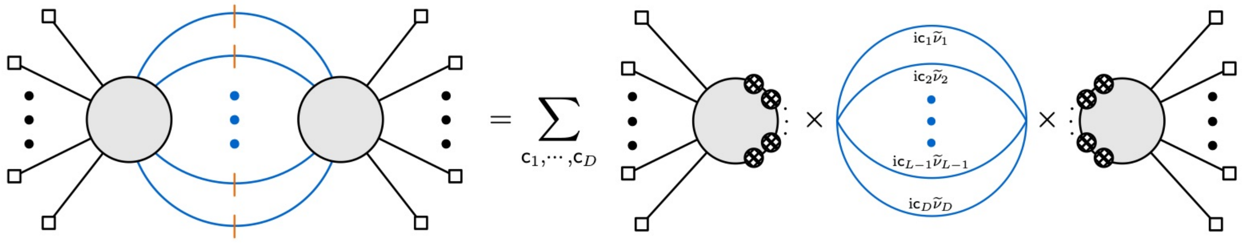


On-shell data | nonlocal signals (NS)

[Qin, ZX, 2304.13295, 2308.14802]

- NSs are from on-shell produced heavy states stretched over superhorizon distances

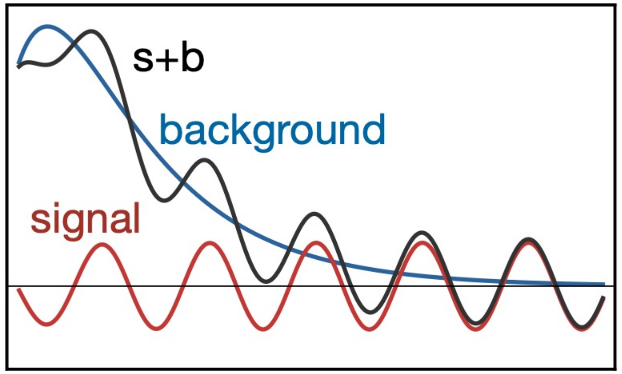
- Correlators factorize at NS branch points, similar to an amplitude at on-shell poles, but true to all loops!



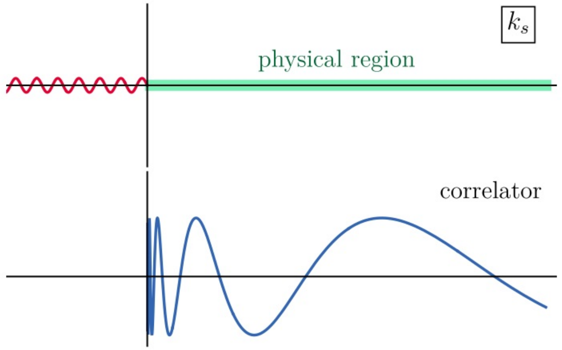
$$\lim_{P\to 0} \mathcal{T}(P) = \mathcal{L} \times \mathcal{S}(P) \times \mathcal{R} + \text{analytic}$$

- If the NS is all you want, the life is much easier

- Recover the whole thing from NS alone?



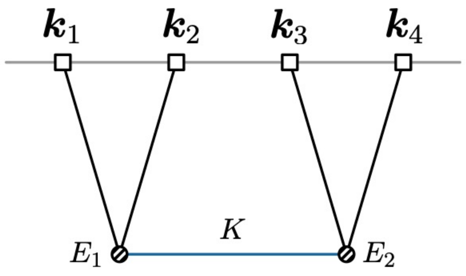
Nonlocal signal



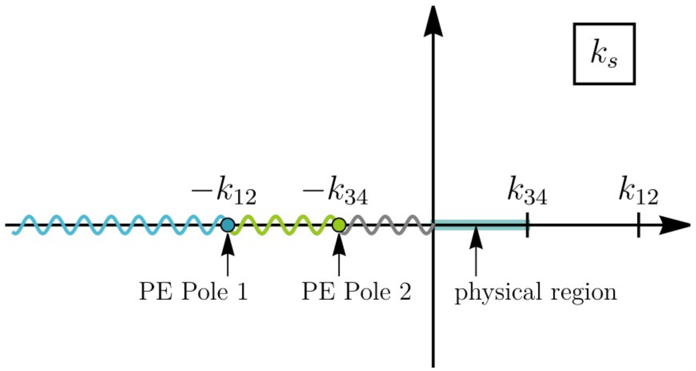
On-shell bootstrap | line dispersion for tree graphs

[Liu, Qin, ZX, 2407.12299]

- Identify the correct complex plane where all discontinuities expressible with on-shell data

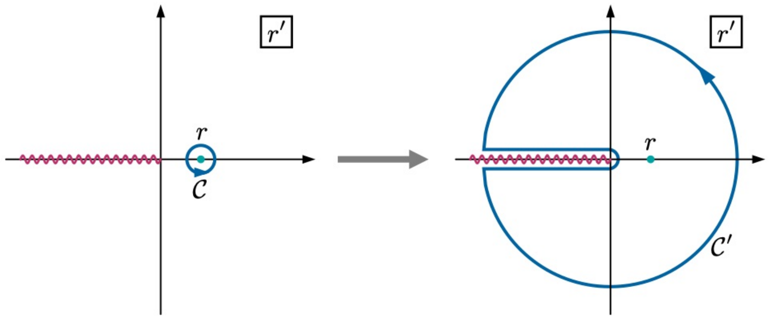


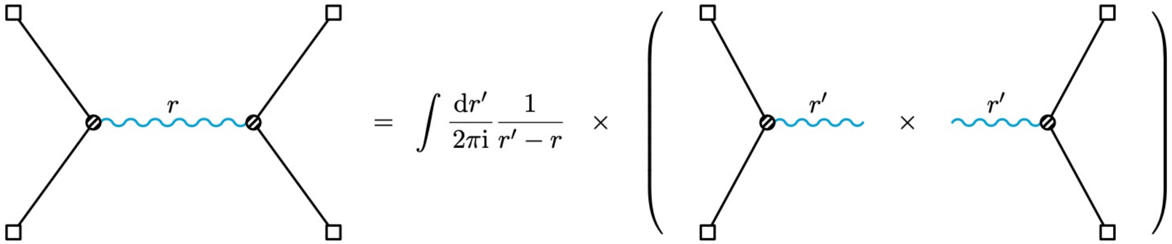
$$r_1 = K/E_1 \qquad x = E_2/E_1$$



$$\underset{k_s}{\mathrm{Disc}}\, \mathcal{I}(k_s) = \underset{k_s}{\mathrm{Disc}} \Big[\mathcal{I}_{\mathrm{NS}}(k_s) - \mathcal{I}_{\mathrm{NS}}(-k_s)\Big]\theta(-k_s)$$

$$\mathcal{I}(r_1, x) = \frac{r_1}{2\pi \mathrm{i}} \int_{-\infty}^{0} \frac{\mathrm{d}r}{r(r-r_1)} \underset{r_1}{\mathrm{Disc}} \Big[\mathcal{I}_{\mathrm{NS}}(r_1, x) - \mathcal{I}_{\mathrm{NS}}(-r_1, x)\Big]$$

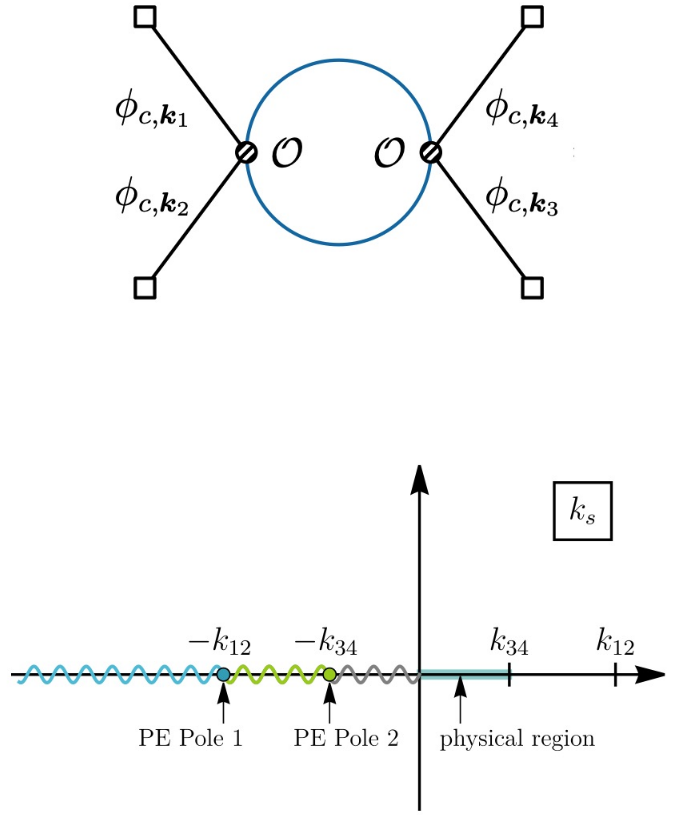




[Liu, Qin, ZX, 2407.12299]

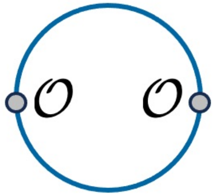
Loop correlators | analytic structure

- Extending to pheno-relevant loops; bubbles for this work
- Crucially, (renormalized) bubble graphs share identical analytic structure with single-exchange trees
  -- a conceptual foundation of our program
- Nonlocal signal, local signal, partial energy, total energy
  [a few ongoing works]
- On-shell data (nonlocal signal) got in several ways:
  PMB [Qin, ZX, 2205.01692] or spectral [ZX, Zhang, 2211.03810]
- In principle, there should exist a line dispersion relation for bubble graphs reducing the whole graph to a dispersive integral over its nonlocal signal
- Better shortcuts exist with spectral input…

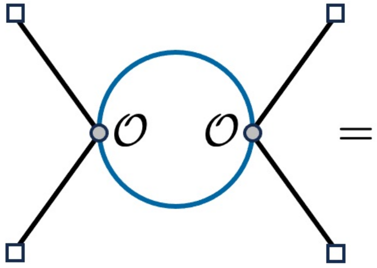


Loop correlators | spectral decomposition

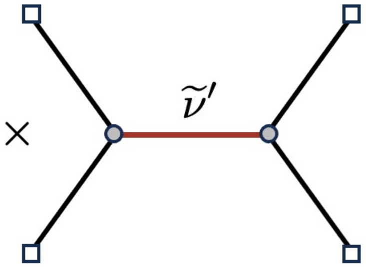
- A dS covariant bubble, as 2pt function of a composite operator $\mathcal{O}$, decomposable to a linear superposition of massive propagators with masses ν'
- Insert it into the correlator and switch the order of time and spectral integral, and we get a spectral rep of bubble graphs
- The spectral density $\rho^{\mathcal{O}}(\widetilde{\nu}')$ computable in a few ways (EdS or EAdS)
  [Marolf, Morrison, 1006.0035, Loparco et al., 2306.00090]
- Essentially 3j symbols of SO(5) when analytically continued to a sphere (EdS)



$$= \int d\widetilde{\nu}'\, \rho^{\mathcal{O}}(\widetilde{\nu}') \times$$



$$= \int d\widetilde{\nu}'\, \rho^{\mathcal{O}}(\widetilde{\nu}') \times$$



$$\rho^{\sigma^2}(\widetilde{\nu}') = \frac{\widetilde{\nu}' \sinh(\pi\widetilde{\nu}')}{16\pi^5\Gamma\left(\frac{3}{2}+\mathrm{i}\widetilde{\nu}'\right)\Gamma\left(\frac{3}{2}-\mathrm{i}\widetilde{\nu}'\right)} \times \prod_{\mathsf{a},\mathsf{b},\mathsf{c}=\pm} \Gamma\left[\frac{3}{4}+\frac{1}{2}\mathrm{i}(\mathsf{a}+\mathsf{b})\widetilde{\nu}+\frac{1}{2}\mathrm{i}\mathsf{c}\widetilde{\nu}'\right]$$

Loop correlators | on-shell data are discrete!

[ZX, Zhang, 2211.03810, Zhang, 2507.19318]

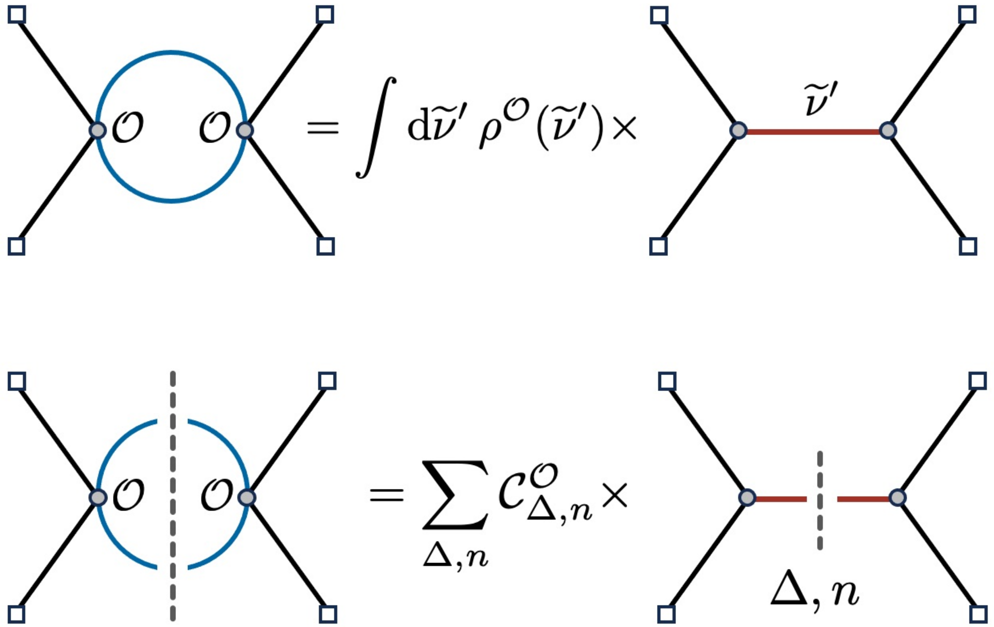
- The spectral integral is a mess but doable thanks to the meromorphicity of $\rho^{\mathcal{O}}(\widetilde{\nu}')$

$$\mathcal{J}^{\mathcal{O}} = \mathcal{J}^{\mathcal{O}}_{\rm NS} + \mathcal{J}^{\mathcal{O}}_{\rm LS} + \mathcal{J}^{\mathcal{O}}_{\rm BG}$$

- The answer is complicated, but the on-shell part (NS) has an inspiring form

$$\mathcal{J}^{\mathcal{O}}_{\rm NS} = \sum_{\Delta=\Delta_\pm} \sum_{n=0}^{\infty} \mathcal{C}^{\mathcal{O}}_{\Delta,n}\, \mathbf{I}^{2\Delta+2n}_{\rm NS}$$

- It's a discrete sum over nonlocal signals of quasinormal modes (QNM) with definite scaling dimension (rather than fields of definite mass)



$$\Delta = \frac{3}{2} \pm \mathrm{i}\widetilde{\nu}, \quad n \in \mathbb{N}$$

$$\mathcal{C}^{\sigma^2}_{\Delta,n} = \frac{(4\Delta+4n-3)(\tan^2(\pi\Delta)-1)\,(n+1)_{1/2}\,(2\Delta+n-2)_{1/2}}{32\pi^2(\Delta+n)_{1/2}\,(\Delta+n-1)_{1/2}}$$

Bootstrapping loops | conjecture of a shortcut

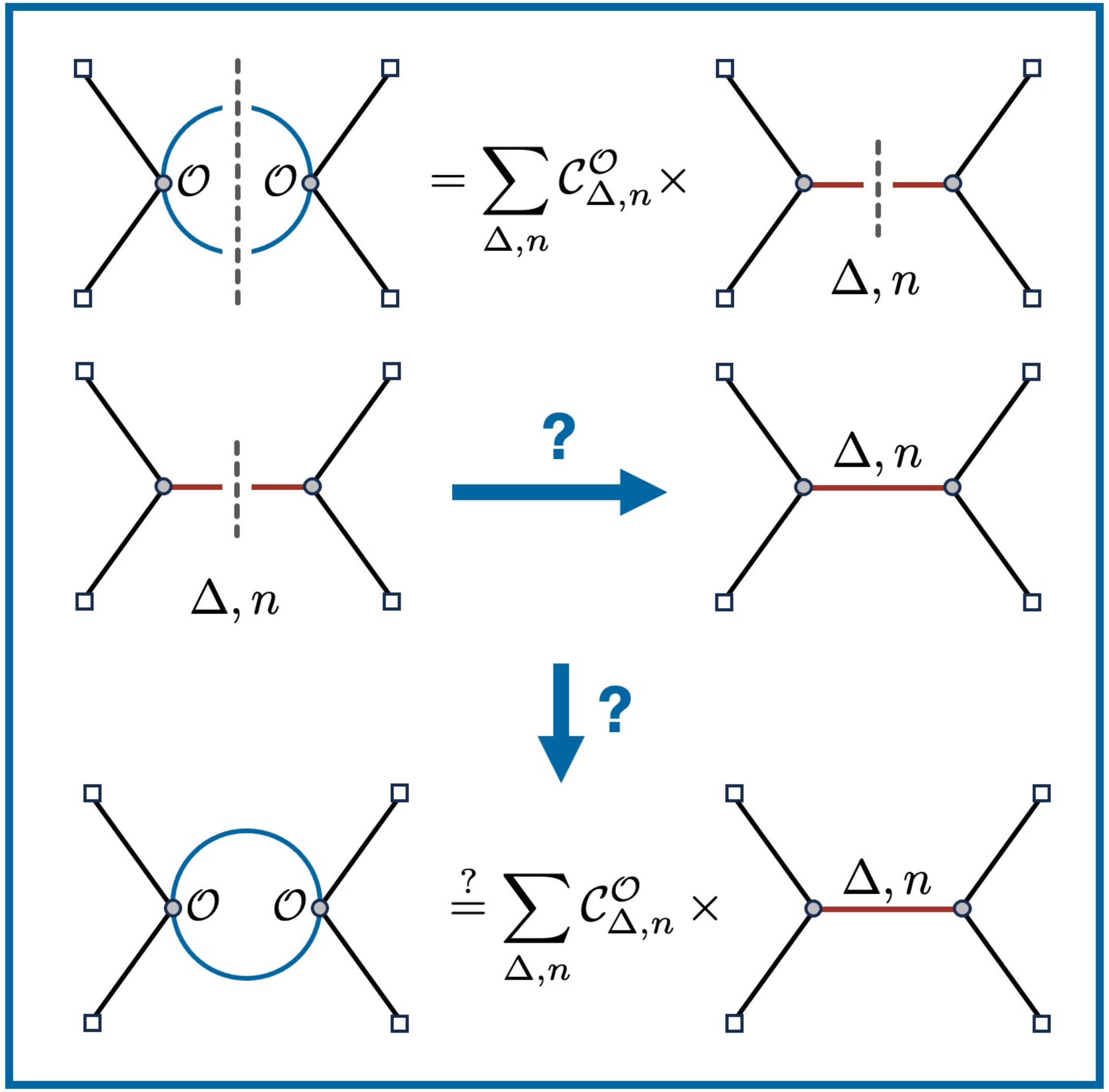
- A spectral computation reducing bubble NS to QNM NS

$$\mathcal{J}^{\mathcal{O}}_{\mathrm{NS}} = \sum_{\Delta=\Delta_\pm} \sum_{n=0}^{\infty} \mathcal{C}^{\mathcal{O}}_{\Delta,n}\, \mathbf{I}^{2\Delta+2n}_{\mathrm{NS}}$$

- We may imagine a tree graph $\mathbf{I}^{2\Delta+2n}$ mediated by a QNM of definite dimension (rather than a field of definite mass) and further conjecture that it has the same analytic structure as a normal tree graph

- Then, we would conjecture that

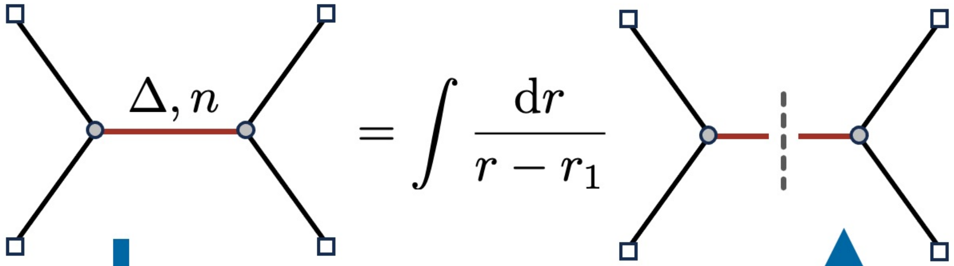
$$\mathcal{J} \stackrel{?}{=} \sum_{\Delta=\Delta_\pm} \sum_{n=0}^{\infty} \mathcal{C}^{\mathcal{O}}_{\Delta,n}\, \mathbf{I}^{2\Delta+2n}$$



Bootstrapping loops | justifying the shortcut

- The single-QNM-exchange does have the same analytic structure as a normal tree
- The two cases differed mildly by a boundary condition, too mild to alter the singularity structure
- As a result, we can prove a line dispersion relation for QNM-mode-exchange graph
- Then, we can insert the on-shell data, as a discrete sum, into the dispersive integral
- Switching the order of integral and the sum, we arrive at the desired result

$$\mathbf{I}^{\Delta}(r_1, x) = \frac{r_1}{2\pi \mathrm{i}} \int_{-\infty}^{0} \frac{\mathrm{d}r}{r(r - r_1)} \times \operatorname*{Disc}_{r} \left[\mathbf{I}^{\Delta}_{\mathrm{NS}}(r, x) - \mathbf{I}^{\Delta}_{\mathrm{NS}}(-r, x) \right]$$

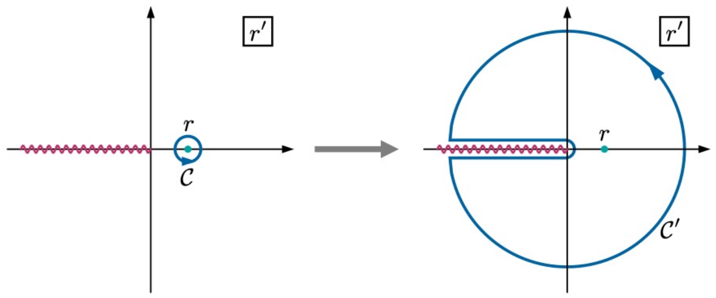


$$\mathcal{J}^{\mathcal{O}}_{\mathrm{NS}} = \sum_{\Delta,n} \mathcal{C}^{\mathcal{O}}_{\Delta,n} \, \mathbf{I}^{2\Delta+2n}_{\mathrm{NS}}$$

$$\mathcal{J}^{\mathcal{O}} = \sum_{\Delta,n} \mathcal{C}^{\mathcal{O}}_{\Delta,n} \, \mathbf{I}^{2\Delta+2n}$$

Bootstrapping loops | An ultraviolet patch

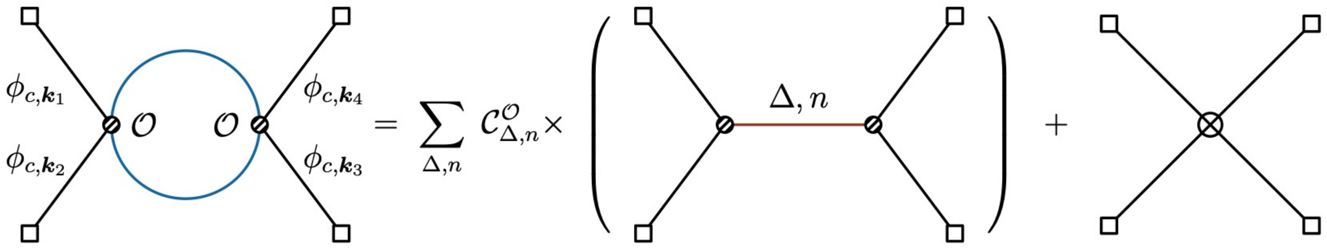
- The previous result not quite right: UV problem
- The series is divergent. However, the divergences are local and removed exactly by counterterms



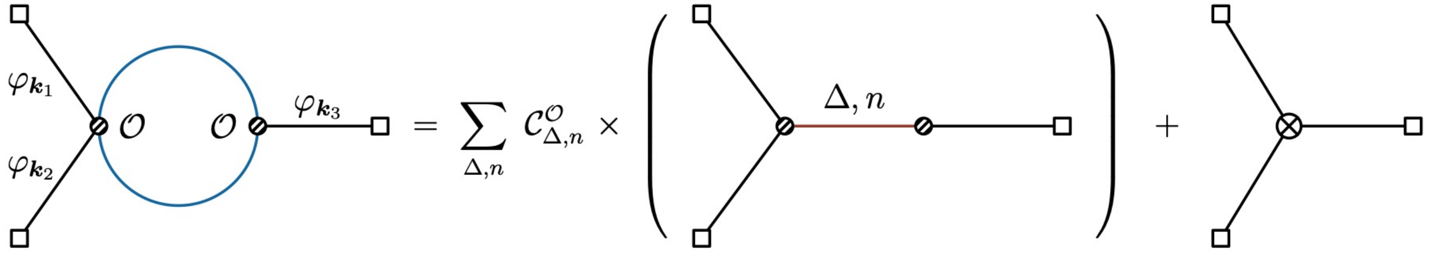
- The lesson: There are always a finite number of local terms undetermined by a loop computation alone; They should be determined by renormalization conditions

The bubble loop is a discrete sum over QNM trees up to local counterterms

$$\mathcal{J}^{\mathcal{O}} = \sum_{\Delta,n} \mathcal{C}^{\mathcal{O}}_{\Delta,n}\, \mathbf{I}^{2\Delta+2n} + \text{local terms}$$



Example | inflaton 3-point function with a massive bubble



$$u = 2k_3/(k_1+k_2+k_3)$$

$$\mathcal{K}(u) = \mathcal{K}_{\rm S}(u) + \mathcal{K}_{\rm BG}(u)$$

on-shell data

- Step 1: A spectral computation to determine the on-shell data, and, in particular, the spectral coefficients

$$\mathcal{K}_{\rm S} = \sum_{\Delta,n} \mathcal{C}^{\sigma^2}_{\Delta,n}\, \mathbf{I}^{2\Delta+2n}_{\text{3-pt, S}}$$

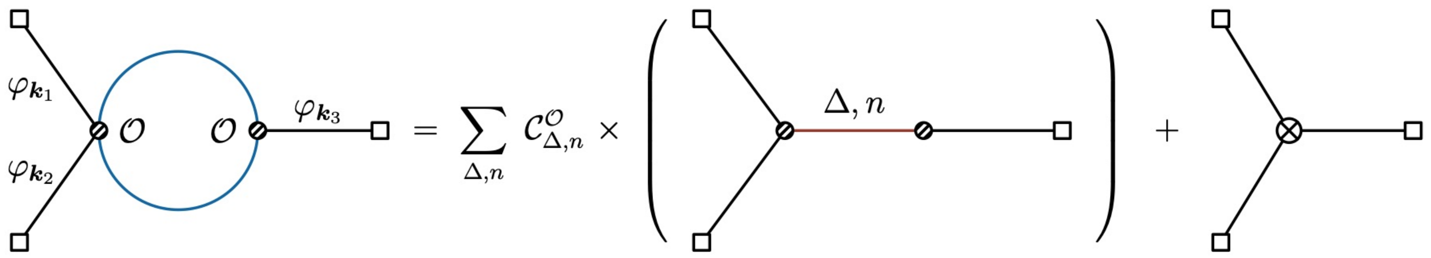
$$\mathcal{C}^{\sigma^2}_{\Delta,n} = \frac{(4\Delta+4n-3)(\tan(\pi\Delta)^2-1)\,(n+1)_{1/2}\,(2\Delta+n-2)_{1/2}}{32\pi^2(\Delta+n)_{1/2}\,(\Delta+n-1)_{1/2}}$$

$$\mathbf{I}^{\Delta}_{\text{3-pt, S}}(u) = -\frac{\pi}{4}\,\frac{\cos(\pi\Delta)+1}{\sin(2\pi\Delta)}\,u^{\Delta+1}\,{}_2\mathcal{F}_1\left[\begin{matrix}\Delta+1,\Delta-1\\ 2\Delta-2\end{matrix}\middle|\,u\right]$$

- Here $\mathbf{I}^{\Delta}_{\text{3-pt,S}}$ is the on-shell signal of a 3-point function with a QNM exchange:

$$\mathbf{I}^{\Delta}_{\text{3-pt}} = \mathbf{I}^{\Delta}_{\text{3-pt,S}} + \mathbf{I}^{\Delta}_{\text{3-pt,BG}}$$

Example | inflaton 3-point function with a massive bubble



$$u = 2k_3/(k_1+k_2+k_3)$$

$$\mathcal{K}(u) = \mathcal{K}_{\rm S}(u) + \mathcal{K}_{\rm BG}(u)$$

on-shell data

- Step 2: Take the naïve sum $\mathcal{K} = \sum_{\Delta,n} \mathcal{C}^{\sigma^2}_{\Delta,n}\, \mathbf{I}^{2\Delta+2n}_{\text{3-pt}}$, but the resulting background is divergent

$$\mathcal{K}_{\rm BG} = \sum_{\Delta,n} \frac{\pi\cot(\pi\Delta)}{8\cos(2\pi\Delta)}\, \mathcal{C}^{\sigma^2}_{\Delta,n}\, u^3 \times {}_3\mathcal{F}_2\left[\begin{matrix} 1,1,3 \\ 2\Delta+2n, 3-2\Delta-2n \end{matrix}\middle| u\right]$$

- An analysis of large-*n* limit shows that the divergence comes totally from the u^3 term, which is removed by a counterterm $\Delta\mathcal{L} \propto a\varphi'^3$, leaving a finite BG with a renormalization constant:

$$\mathcal{K}_{\rm BG} = Cu^3 + \sum_{\Delta,n} \frac{\pi\cot(\pi\Delta)}{8\cos(2\pi\Delta)} \mathcal{C}^{\sigma^2}_{\Delta,n} u^4 \times {}_3\mathcal{F}_2\left[\begin{matrix} 1,2,4 \\ 1+2\Delta+2n, 4-2\Delta-2n \end{matrix}\middle| u\right]$$

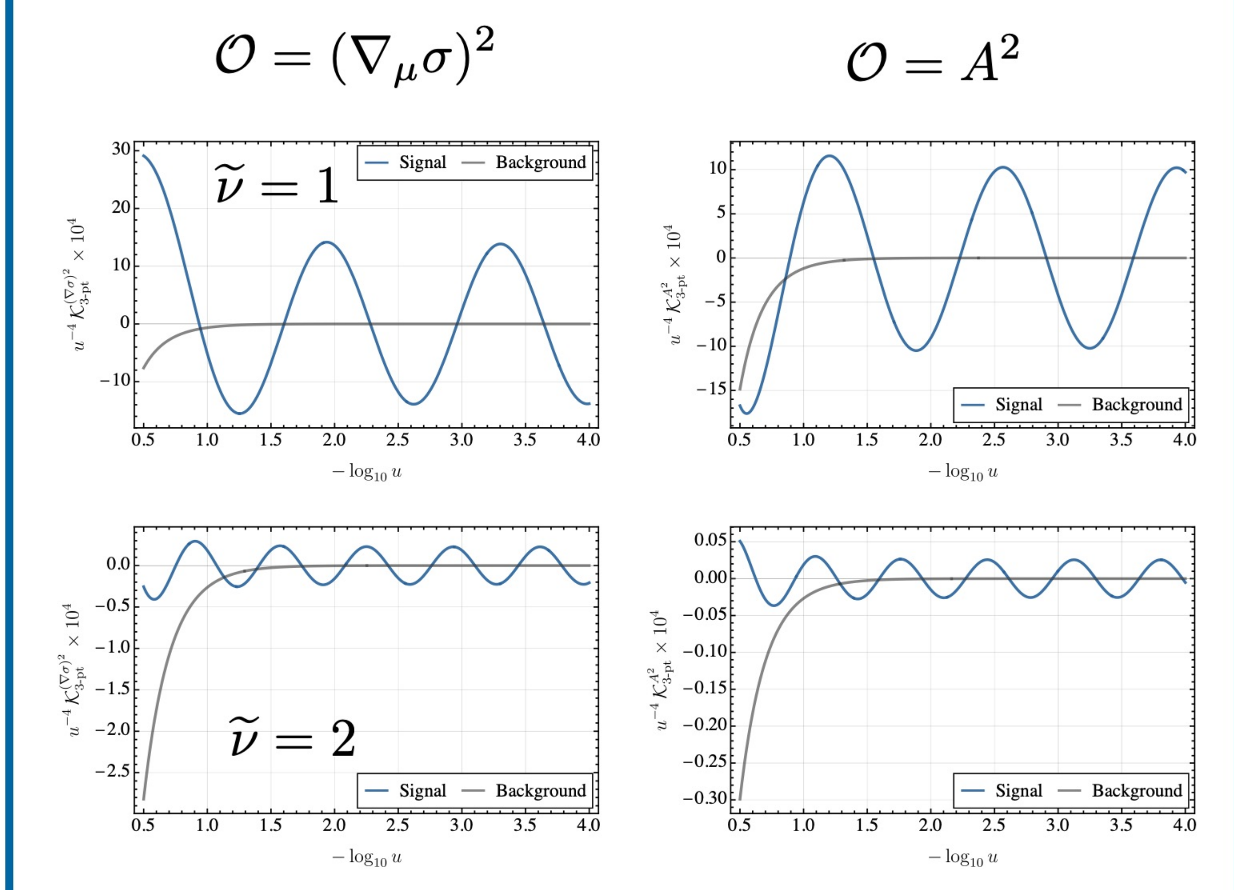
renormalization-independent background

Example | spinning and derivatively-coupled bubbles | pheno implication

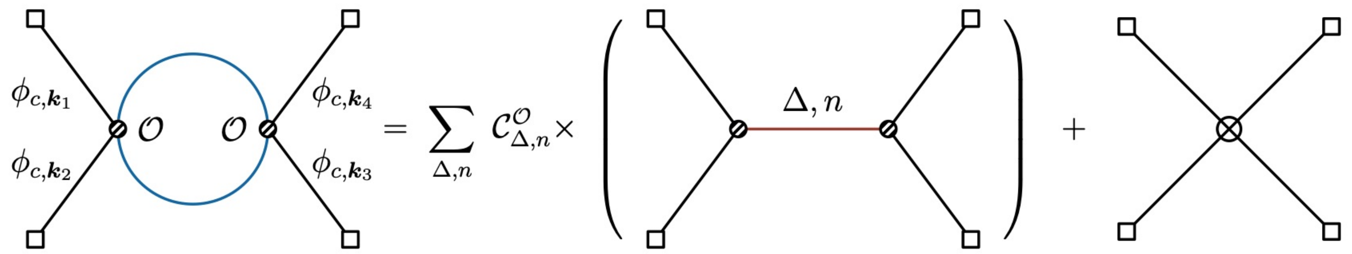
- We have on-shell bootstrapped a few 3pt and 4pt graphs with external conformal/massless scalars and massive spin-0 or spin-1 bubble loops
- Parametric dependences on mass are transparent in the squeezed and large mass limits
- Examples of inflaton bispectra:

$$K^{(\nabla\phi)^2}_{\text{3pt,S}} \sim \frac{1}{2}\,\text{Re}\left[\widetilde{\nu}^6 e^{-2\pi\widetilde{\nu}+\text{i}\pi/2}\left(\frac{u}{4}\right)^{4-2\text{i}\widetilde{\nu}}\right]$$

$$K^{A^2}_{\text{3pt,S}} \sim 24\,\text{Re}\left[\widetilde{\nu}_A^2 e^{-2\pi\widetilde{\nu}_A+\text{i}\pi/2}\left(\frac{u}{4}\right)^{4-2\text{i}\widetilde{\nu}_A}\right]$$

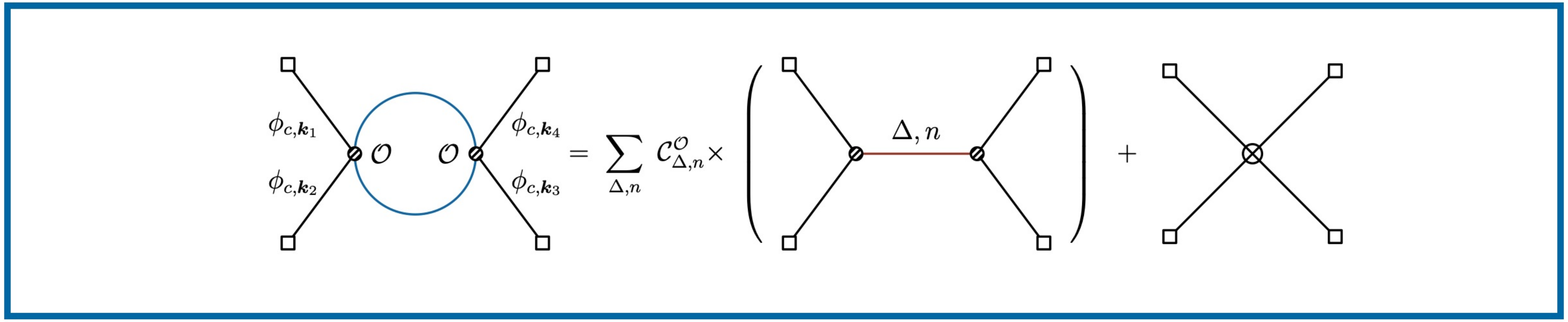


Discussions | lessons



- On-shell data determine everything up to local counterterms [dispersion]
- On-shell data are discrete: meromorphicity of overtones [spectrum]
- Spectral dispersion: an on-shell bootstrap program, potentially powerful for loops
- On-shell advantages: free of off-shell complications: gauge dep / FP ghost / etc

Discussions | outlook



- Easy targets: bubbles & bananas of covariant dispersions (arbitrary mass, spin, and total angular momentum)
- Noncovariant couplings also doable! $\mathcal{O} = (\sigma')^2 \Rightarrow \mathcal{O}_{\mu\nu} = \nabla_\mu \sigma \nabla_\nu \sigma$ with $\mu = \nu = 0$
- Noncovariant dispersions (chemical potential) more challenging, but meromorphicity holds!
- Other topologies (triangle, box) possible in principle; complications mainly from kinematics

Thank you!