Global Monopole Networks Beyond Constant Scaling

Logarithmic evolution, primordial black holes, and gravitational waves

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MAIN RESULT

The monopole abundance keeps growing slowly with scale hierarchy—enhancing late-time PBHs and tilting the associated GW spectrum.

01 Why scaling monopoles matter

$O(3) \to O(2)$ with $\pi_2(S^2) = \mathbb{Z}$

$\rho_{grad}(r) \sim v^2/r^2$ $E(R) \sim 4\pi v^2 R$

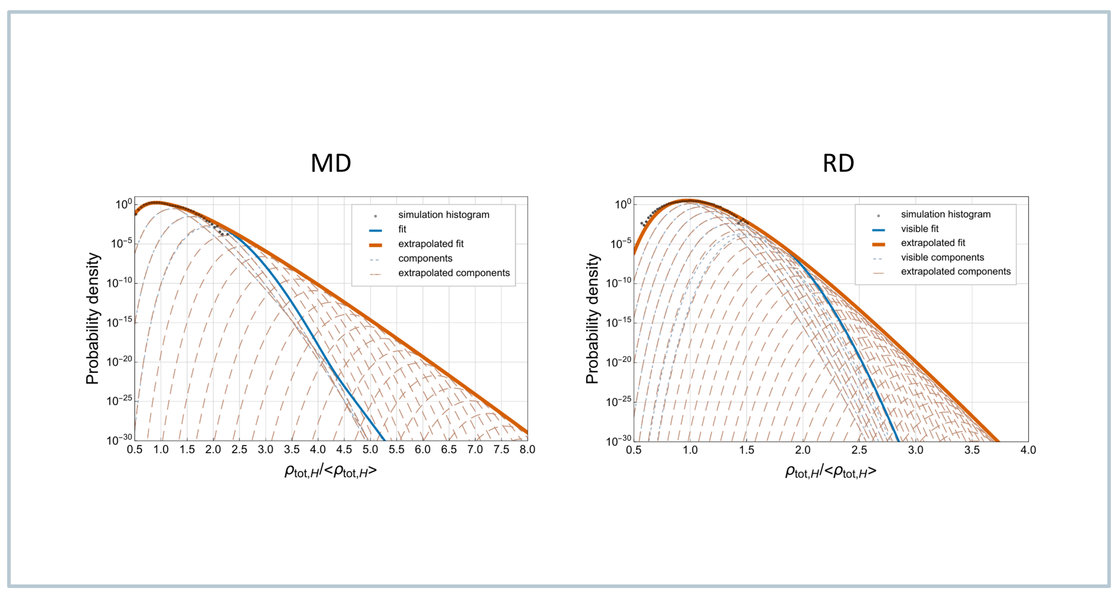
$N_{M,H} = \xi H^3$ $\rho_M/\rho_{bg} \sim (4\pi/3)\, \xi\, v^2/M_{pl}^2$

- Global—or very weakly gauged—monopoles retain horizon-scale gradient fields.
- Long-range attraction drives annihilation and a scaling network.
- The usual ansatz assumes ξ, the number per Hubble volume, is constant.

3D classical lattice simulations | matter & radiation eras | p = −3…1 | fat-core + fixed-core check

the initial power spectrum of the scalar field $\mathcal{P}_{a,b} = \frac{1}{4}\left(\frac{k}{k_{\rm UV}}\right)^p k_{\rm UV}^{-1}\delta_{a,b}$

03 Rare Hubble patches seed PBHs



High-$N_{M,H}$ sectors generate an extended energy tail far beyond the directly sampled histogram.

$\delta_H \simeq f_\phi\,(R_H - 1)$

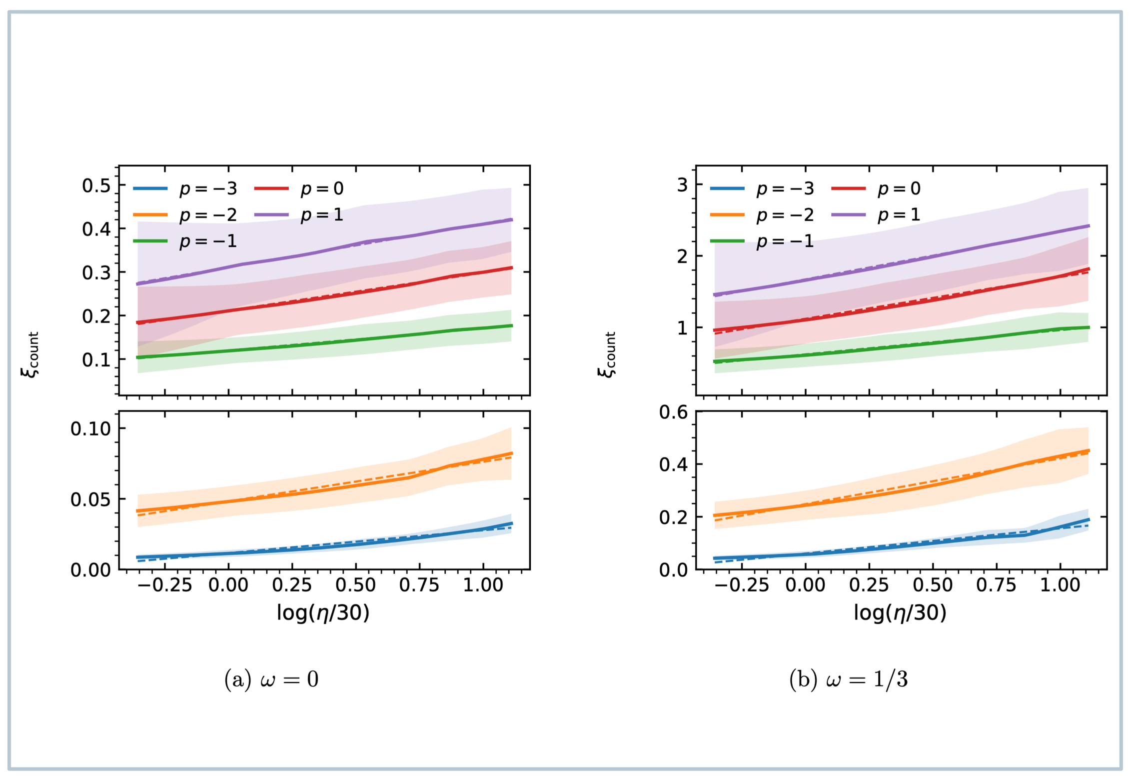
$v \gtrsim 0.1\, M_{pl} \Rightarrow$ non-negligible collapse

- Stochastic defect number → order-one Hubble-patch overdensity.
- Formation throughout scaling produces a broad mass spectrum.
- $M_{PBH}(H) \simeq 4\pi\, \gamma_{col}\, M_{pl}^2/H$.

$$f_\phi \equiv \frac{\langle \rho_{\rm tot,H}\rangle}{\rho_{\rm bg}}$$

$$R_H = \frac{\rho_{{\rm tot},H}}{\langle \rho_{{\rm tot},H}\rangle}$$

02 Constant scaling fails



For every tested initial spectrum, ξ_{count} retains a positive late-time drift (solid curves; logarithmic fits are dashed).
ω is the background equation-of-state parameter.

$\xi_{count}(\eta) = A_{30} + B \log(\eta/30)$

$\gamma \equiv d \log \xi_{count} / d \log(m_r/H)$

$\gamma \approx 0.5$

Why γ is the useful diagnostic

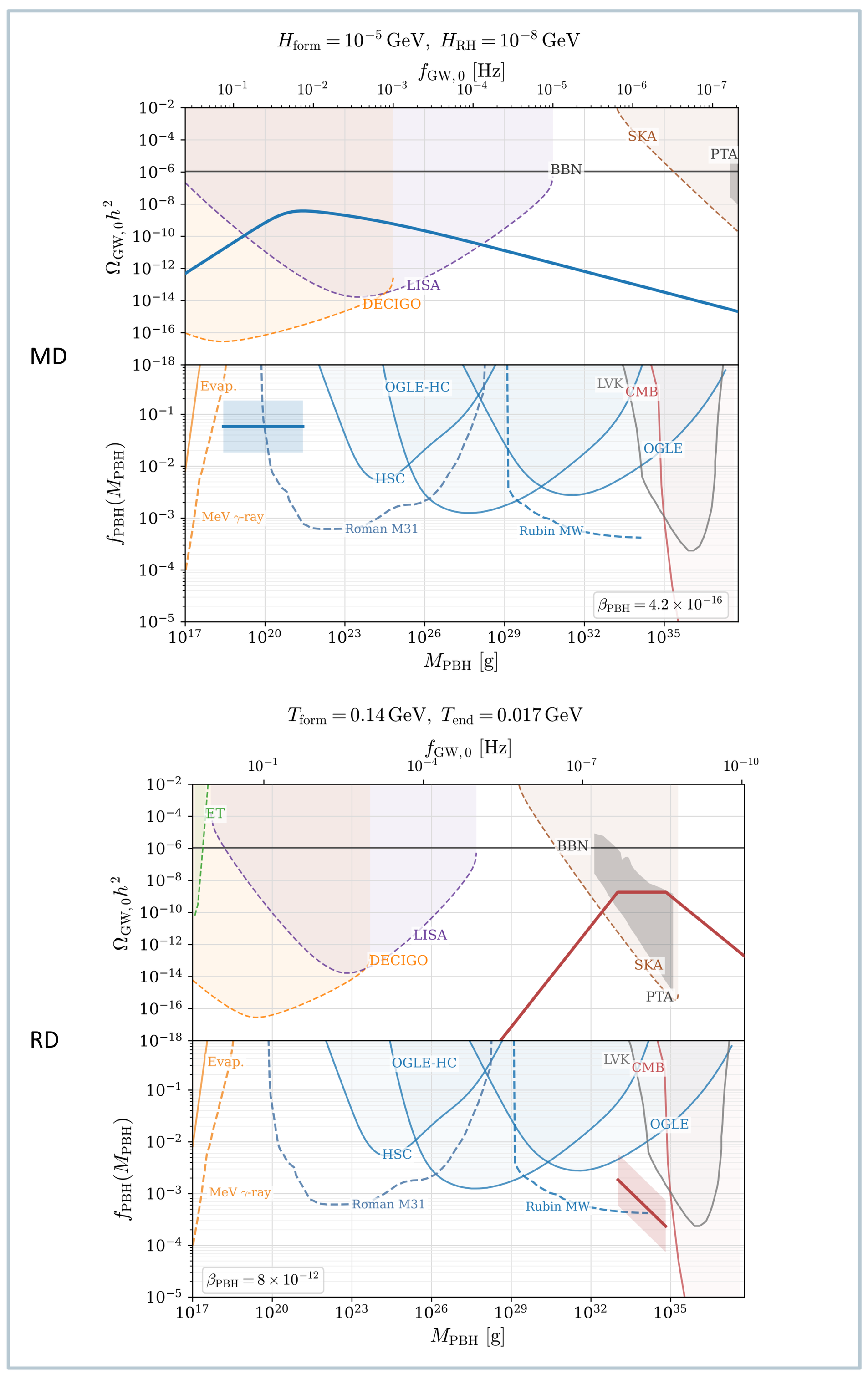
- The additive slope B differs strongly between matter and radiation domination.
- Normalizing to the physical hierarchy m_r/H removes most background dependence.
- For blue initial spectra, γ is comparatively stable against the initial conditions.
- A fixed-core run independently gives $\gamma \simeq 0.52$–0.53.

Possible dynamical origin

$\tau_{ann} \simeq D\,[c_0 + c_1 \log(D/r_{core}) + \dots]$

Nonzero impact parameters or angular momentum can delay core collisions. Hubble friction and Goldstone radiation must first dissipate angular momentum, leaving a logarithmic dependence on m_r/H.

04 One network, correlated observables



Example early matter-dominated history: the same formation epoch fixes both GW frequency and PBH mass; projected reaches include SKA, LISA, ET, and DECIGO.

SAME HUBBLE SCALE

$H_{formation} \to \{M_{PBH}, f_{GW,0}\}$

The duration of scaling sets the spectral width; the network energy sets the amplitude.

A magnetic-charge smoking gun

O(10)% of Hubble patches carry nonzero winding

If gauge screening terminates scaling, PBHs can inherit hidden magnetic charge. For $v = O(0.1)M_{pl}$, the magnetic Coulomb force can be comparable to gravity on unscreened scales—modifying binaries, mergers, and GW signals.

What changes when ξ is not constant?

PBH spectra

Late-forming and magnetically charged PBHs are enhanced.

GW spectra

A radiation-era plateau can acquire a blue tilt.

Dark sectors

Axion, dark-photon, and monopole relic estimates shift.

Next test

Extend dynamic range and model annihilation with angular momentum.

TAKEAWAY

Global-monopole scaling is not exactly stationary: a slow growth in ξ links defect dynamics directly to stronger late-time PBHs and a modified stochastic GW background.

[1] W. Nakano & W. Yin, Constant Scaling Fails for Global Monopole Networks, arXiv:2607.05517. [2] D. Aburatani, W. Nakano & W. Yin, PBHs and GWs from Scaling Monopoles, arXiv:2606.31937.