Forbidden Dark Matter with Sommerfeld Enhancement

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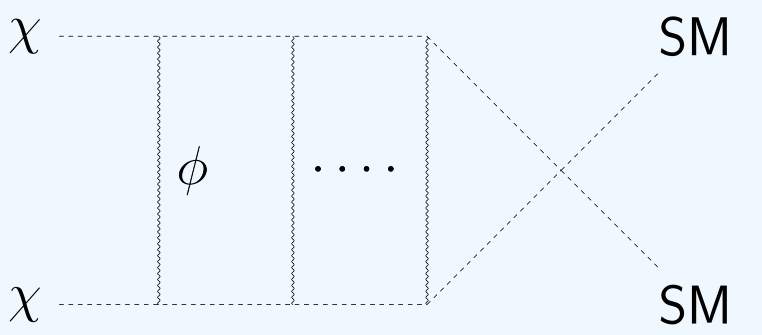
Sommerfeld Enhancement

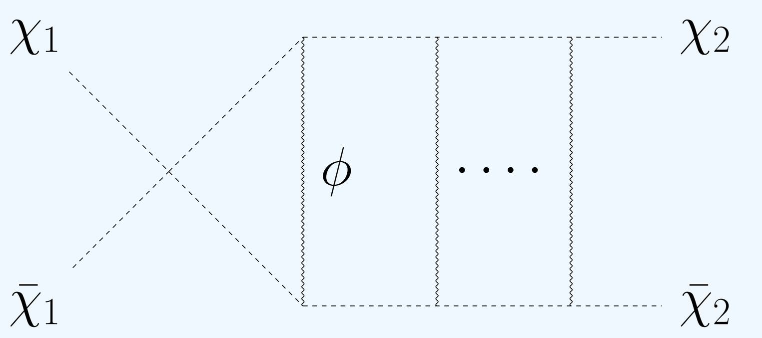
- The relic abundance of WIMP depends on $\langle \sigma v_{\rm rel}\rangle$.
- When calculating $\langle \sigma v_{\rm rel}\rangle$, we need to care about the **Sommerfeld enhancement** (SE).



- The enhanced cross section: $\sigma v_{\rm rel} = S(v_{\rm rel}) \times \sigma_{\rm tree} v_{\rm rel}$
- We need to solve Schrödinger eq. to obtain $S(v_{\rm rel})$.

SE in Forbidden Channels

- The long-range force between annihilation products can modify $\sigma v_{\rm rel}$.
- χ_1: DM with mass m_1
- χ_2 Annihilation product with mass $m_2 > m_1$
  $\to$ **Forbidden channel** [Griest, Seckel (1991), D'Agnolo, Ruderman (2016)]



- χ_2 and $\bar{\chi}_2$ must decay into thermal plasma.
  $\to$ Final-state SE depends on the lifetime

Previous Work

Cutoff method [Cui, Luo (2020)]
Timescale of long-range interaction $\sim (m_2 v_2^2)^{-1}$ vs. lifetime Γ^{-1}

$$v_{\rm cut} = \sqrt{\Gamma/m_2}$$

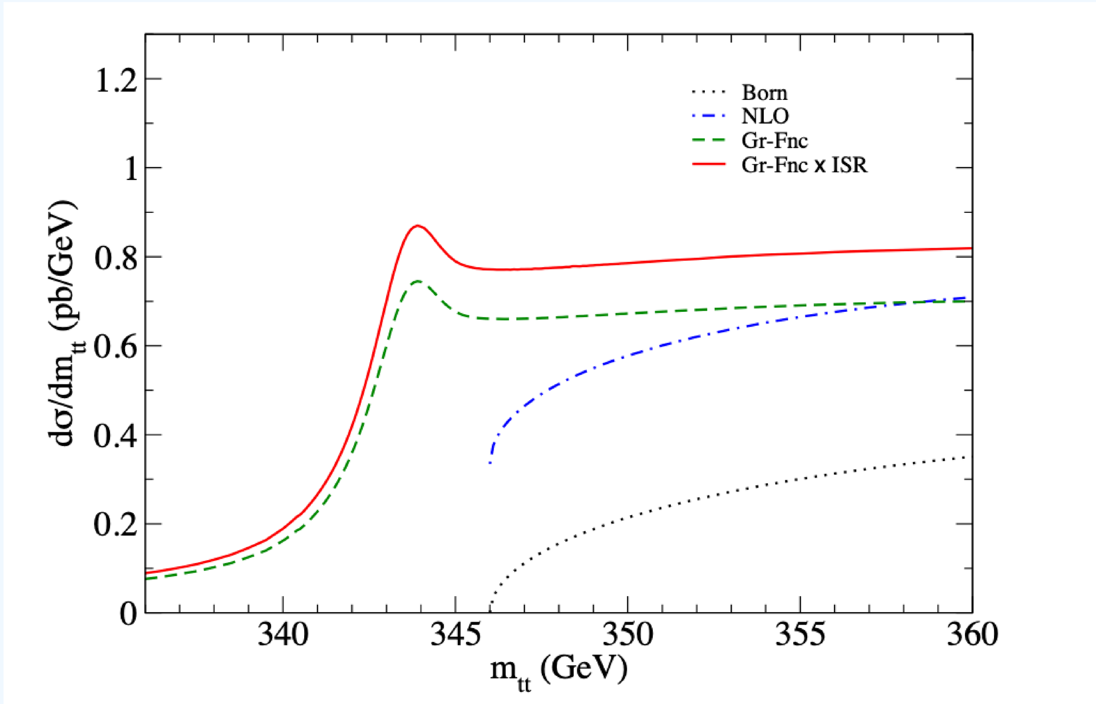
$$S_f^{(\rm cut)}(v_2) = \begin{cases} 1 & (0 < v_2 < v_{\rm cut}) \\ S_f(v_2) & (v_{\rm cut} \le v_2) \end{cases}$$

SE is assumed to be ineffective near the threshold

$t\bar{t}$ production [Fujii, Hagiwara, Murayama, Sumino (1993) etc.]

$$\left[-\frac{1}{m_t}\nabla^2 + V_{\rm QCD}(r) - (E + i\Gamma_t)\right] G_t(\boldsymbol{r}; E + i\Gamma_t) = \delta^3(\boldsymbol{r})$$

$$\sigma \propto {\rm Im}\, G_t(\boldsymbol{0}; E + i\Gamma_t)$$



[Figure from Hagiwara, Sumino, Yokoya (2008)]
Bound states enhance the cross section.

Formulation

We need to solve the Schrödinger eq.

$$\left[-\frac{1}{2\mu_1}\nabla^2 - E\right]\psi_1(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_2(\boldsymbol{r})$$

$$\left[-\frac{1}{2\mu_2}\nabla^2 + V(r) - (E_2 + i\Gamma)\right]\psi_2(\boldsymbol{r}) = u\delta^3(\boldsymbol{r})\psi_1(\boldsymbol{r})$$

$$E_2 = E - 2(m_2 - m_1), \quad p_1 = \sqrt{2\mu_1 E}, \quad p_2 = \sqrt{2\mu_2 E_2 + i\Gamma}$$

Boundary conditions at $r \to \infty$

$$\psi_1(\boldsymbol{r}) \to e^{ip_1 z} + f_1(\theta)\frac{e^{ip_1 r}}{r}$$

$$\psi_2(\boldsymbol{r}) \to f_2(\theta)\frac{e^{ip_2 r}}{r}$$

Annihilation cross section

$$\sigma v_{\rm rel} \propto {\rm Im}\, G_2(\boldsymbol{0}; E_2 + i\Gamma) \quad \text{where} \quad \left[-\frac{1}{2\mu_2}\nabla^2 - \frac{\alpha}{r} - (E_2 + i\Gamma)\right] G_2(\boldsymbol{r}; E_2 + i\Gamma) = \frac{1}{2\mu_2}\delta^3(\boldsymbol{r}).$$

SE factor

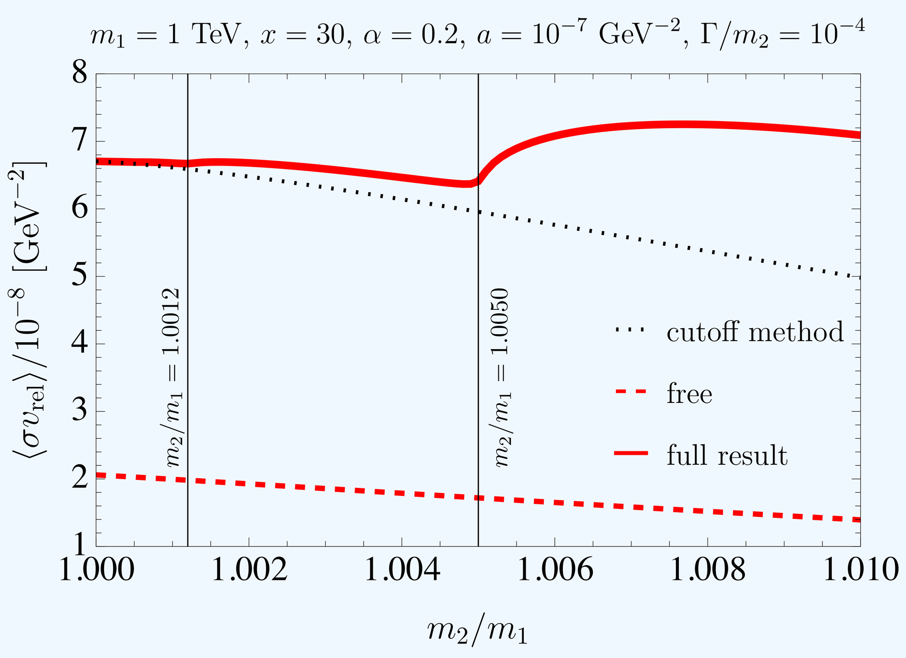
$$S_f(E_2, \Gamma) = \frac{{\rm Im}\, G_2(\boldsymbol{0}; E_2 + i\Gamma)}{{\rm Im}\, G_2^{\rm free}(\boldsymbol{0}; E_2 + i\Gamma)}$$

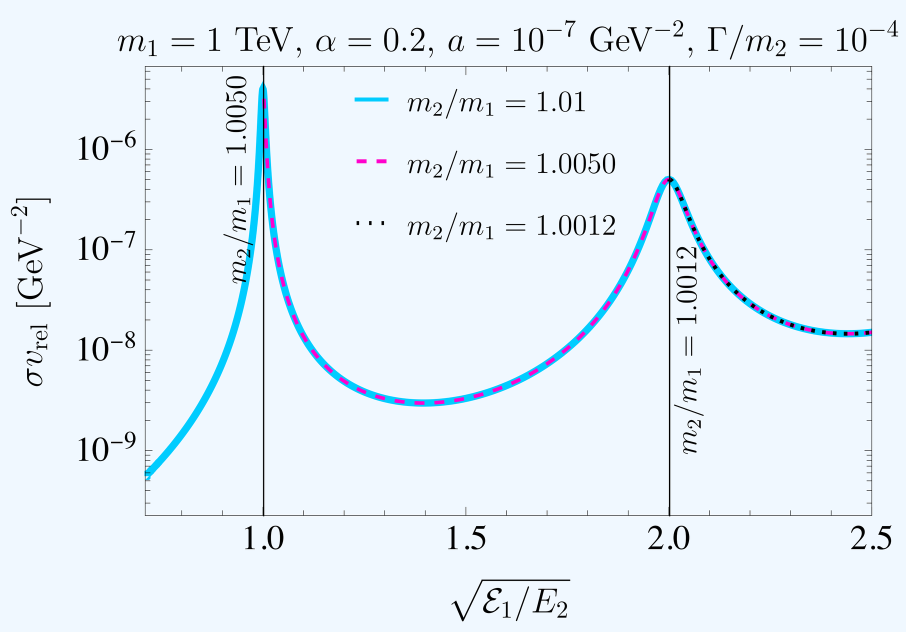
Thermally Averaged Cross Section

We calculate $\langle \sigma v_{\rm rel}\rangle$ for $V(r) = -\alpha/r$. (s-wave)

Our method: $\langle \sigma v_{\rm rel}\rangle = \dfrac{x^{3/2}}{2\sqrt{\pi}}\displaystyle\int_0^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} S_f(v_{\rm rel}) \sigma_{\rm tree} v_{\rm rel}$

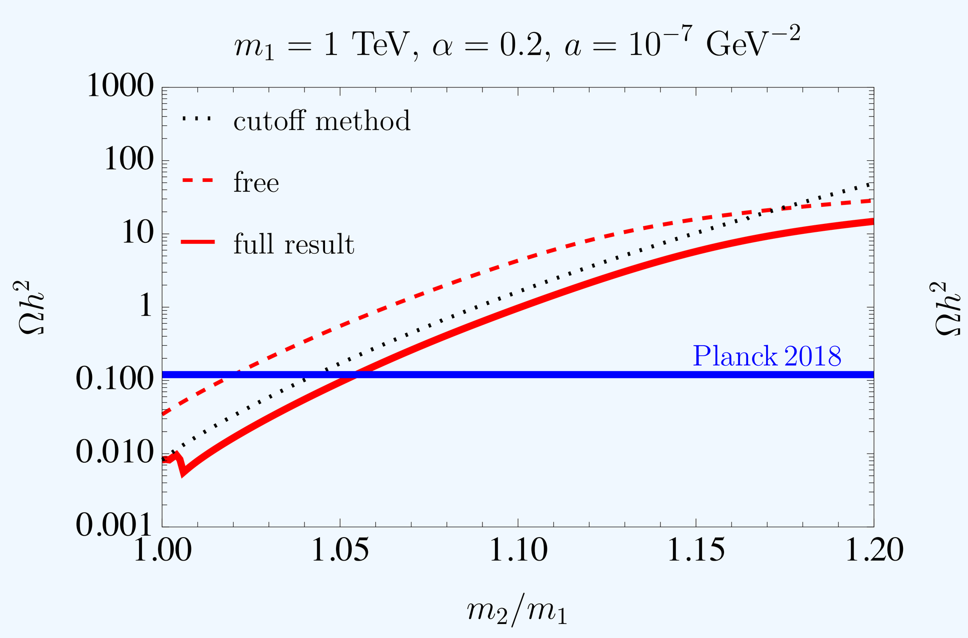
Cutoff method: $\langle \sigma v_{\rm rel}\rangle^{(\rm cut)} = \dfrac{x^{3/2}}{2\sqrt{\pi}}\displaystyle\int_{v_{\rm th}}^\infty dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} S_f^{(\rm cut)}(v_{\rm rel}) \sigma_{\rm tree} v_{\rm rel}$

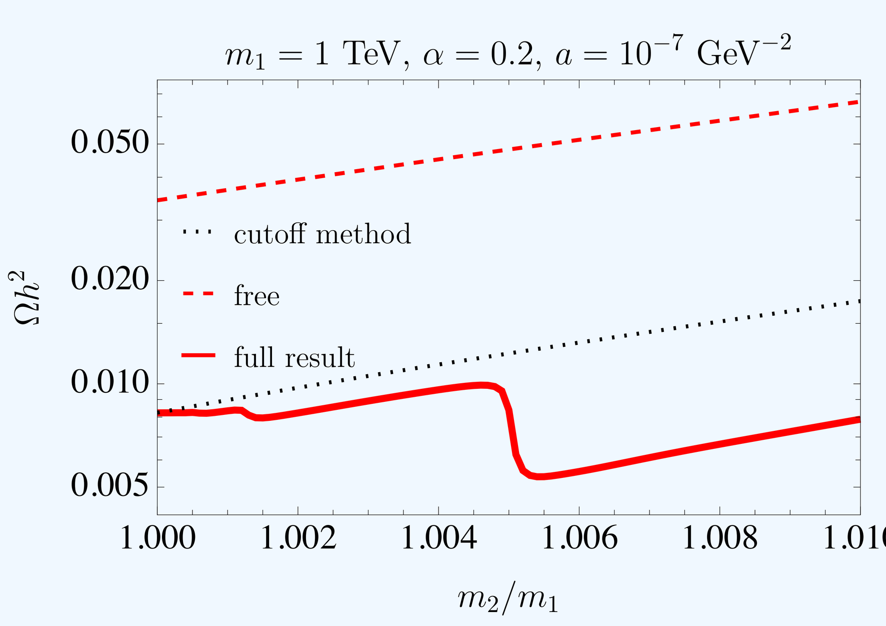




- Binding Energy: $\mathcal{E}_n = -\mu_2\alpha^2/(2n^2)$.
- Minimum of E_2: $E_2^{\rm min} = -2m_1(m_2/m_1 - 1)$.
  Resonance of bound states contribute if $E_2^{\rm min} \le \mathcal{E}_n$

DM Relic Abundance





- Small m_2/m_1: Resonances of bound states
- Large m_2/m_1: Annihilation processes e.g. $\chi_1\bar{\chi}_1 \to \chi_2\bar{\chi}_2^*$
  Prediction is modified by bound states!

Summary

- Final-state SE is significant in forbidden channels.
- Annihilation products must decay $\to$ Lifetime should be considered.
- We solve the Schrödinger eq. with the decay width of annihilation product.
- **Resonances of bound states enhances $\langle \sigma v_{\rm rel}\rangle$**