A Solution to the Hierarchy Problem with Non-Linear Quantum Mechanics

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(With David E Kaplan)

The Hierarchy Problem

$$m_H^2 \qquad\qquad m_H^2$$

Observed: $(125\ \mathrm{GeV})^2$

Theory? $(10^{19}\ \mathrm{GeV})^2$

Why?

Standard Model?

1947

Symmetry?

2010s

Gravity?

2010s

Relaxation?

2016s

Correct Quantum Hamiltonian?

Outline

1. Non-Linear Quantum Mechanics

2. Equations of Motion

3. Hierarchy Problem

4. Conclusions

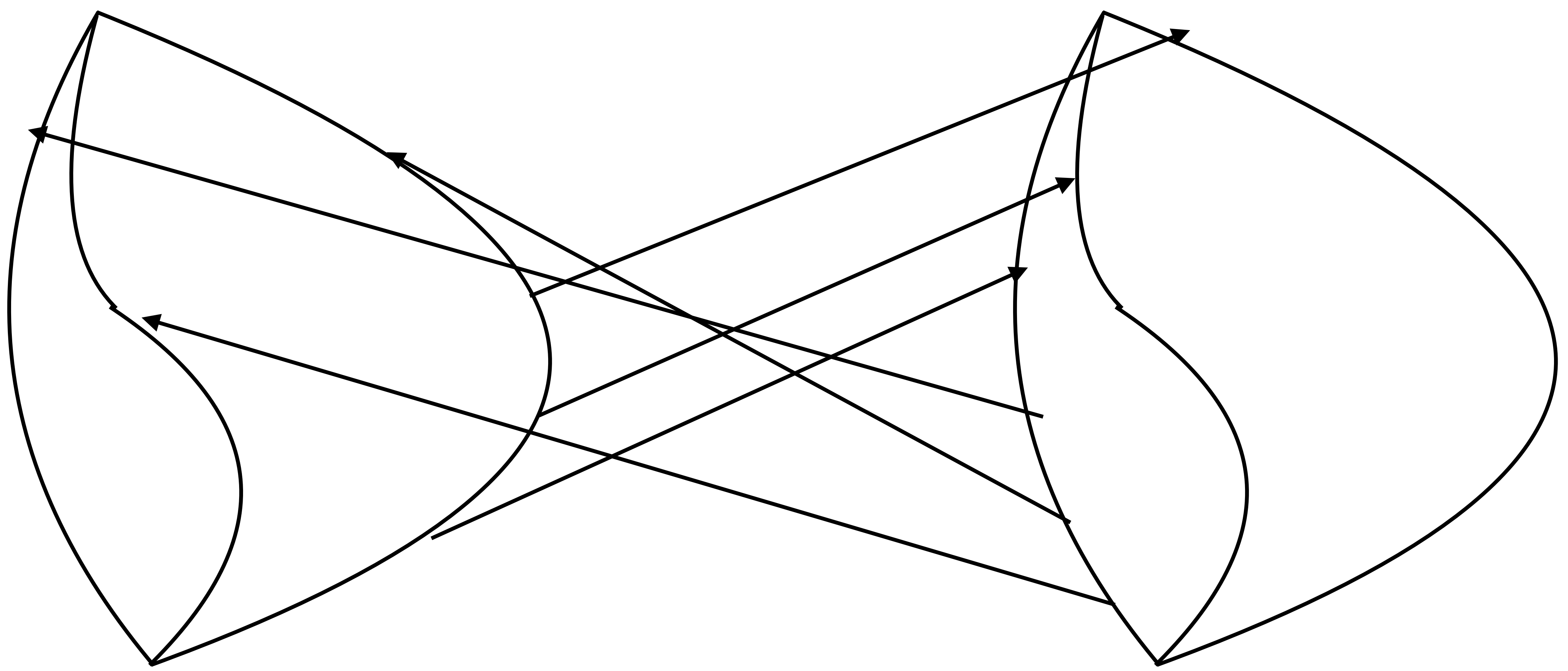
Nonlinear Quantum Mechanics

Quantum Mechanics

No parameterized deviation from quantum algebra (i.e. quantized states)

Why should time evolution be linear?

Electron Coupled to Electromagnetism



Electron paths do not interact via electromagnetism

Paths of two electrons interact causally (QFT)

Why can't path talk to itself?

Right Language: QFT

Quantum Mechanics

States: Vectors in Hilbert space: $|\chi(t)\rangle$

Operators: Matrix on Hilbert space: H

Linear Quantum Mechanics

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

Matrix on Vector - gives another vector

Non-Linearity

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle + \langle\chi(t)|O|\chi(t)\rangle H_2|\chi(t)\rangle$$

Causality - bake in using QFT

Linear Quantum Field Theory

Schrodinger Picture of Quantum Field Theory

$|\chi(t)\rangle$ Quantum State of Fields (e.g. in Fock states)

$\phi(x)$ Time Independent Operators

$$H = \int d^3x\, \mathcal{H}\left(\phi(x), \pi(x)\right)$$

Time Evolution

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = \int d^3x \left(\Pi_\phi(x)^2 + \nabla_i\phi\nabla^i\phi + \cdots + \lambda\phi(x)\bar{\Psi}(x)\Psi(x)\right)|\chi(t)\rangle$$

Locality Baked In - all operators at same spatial point x

Non-Linear Quantum Field Theory

Schrodinger Picture of Quantum Field Theory

$|\chi(t)\rangle$ Quantum State of Fields (e.g. in Fock states)

$\phi(x)$ Time Independent Operators

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle + \langle\chi(t)|O|\chi(t)\rangle H_2|\chi(t)\rangle$$

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = \left(H_L + \epsilon\int d^3x\,\left(\langle\chi(t)|\phi(x)|\chi(t)\rangle\bar{\Psi}(x)\,\Psi(x)\right)\right)|\chi(t)\rangle$$

Locality Baked In - all operators at same spatial point x

100 years of quantum mechanics: how big is ε?

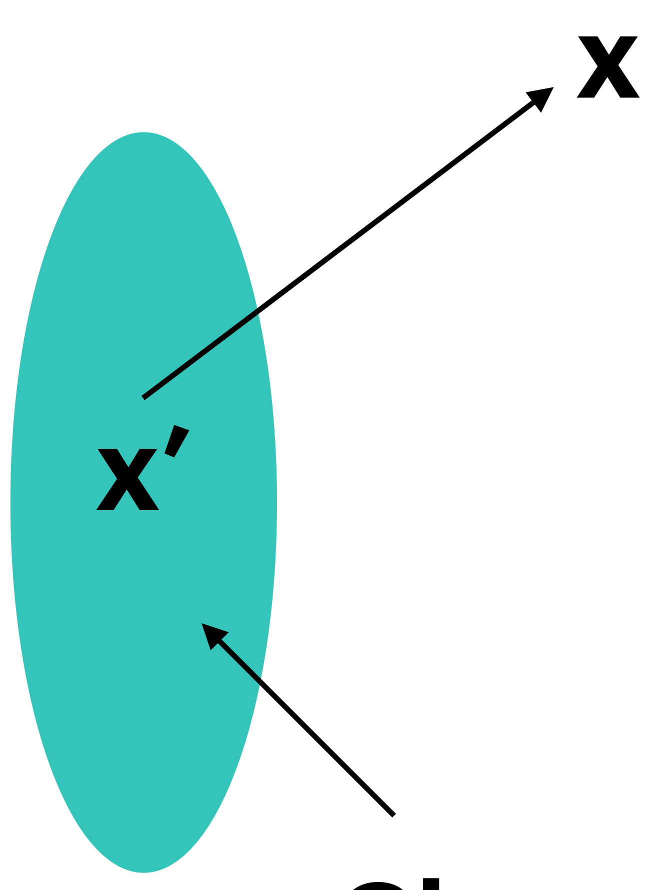
Single Particle

$$\mathcal{H} \supset \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Suppose we have a ψ particle - how does its wave-function evolve?

To zeroth order, ψ just sources the Φ field

Straightforward Computation of Expectation Value



$$\langle\chi|\phi\left(x\right)|\chi\rangle = \int d^4x'\psi^*\left(x'\right)\psi\left(x'\right)G_R\left(x-x'\right)$$

Charge Density of ψ

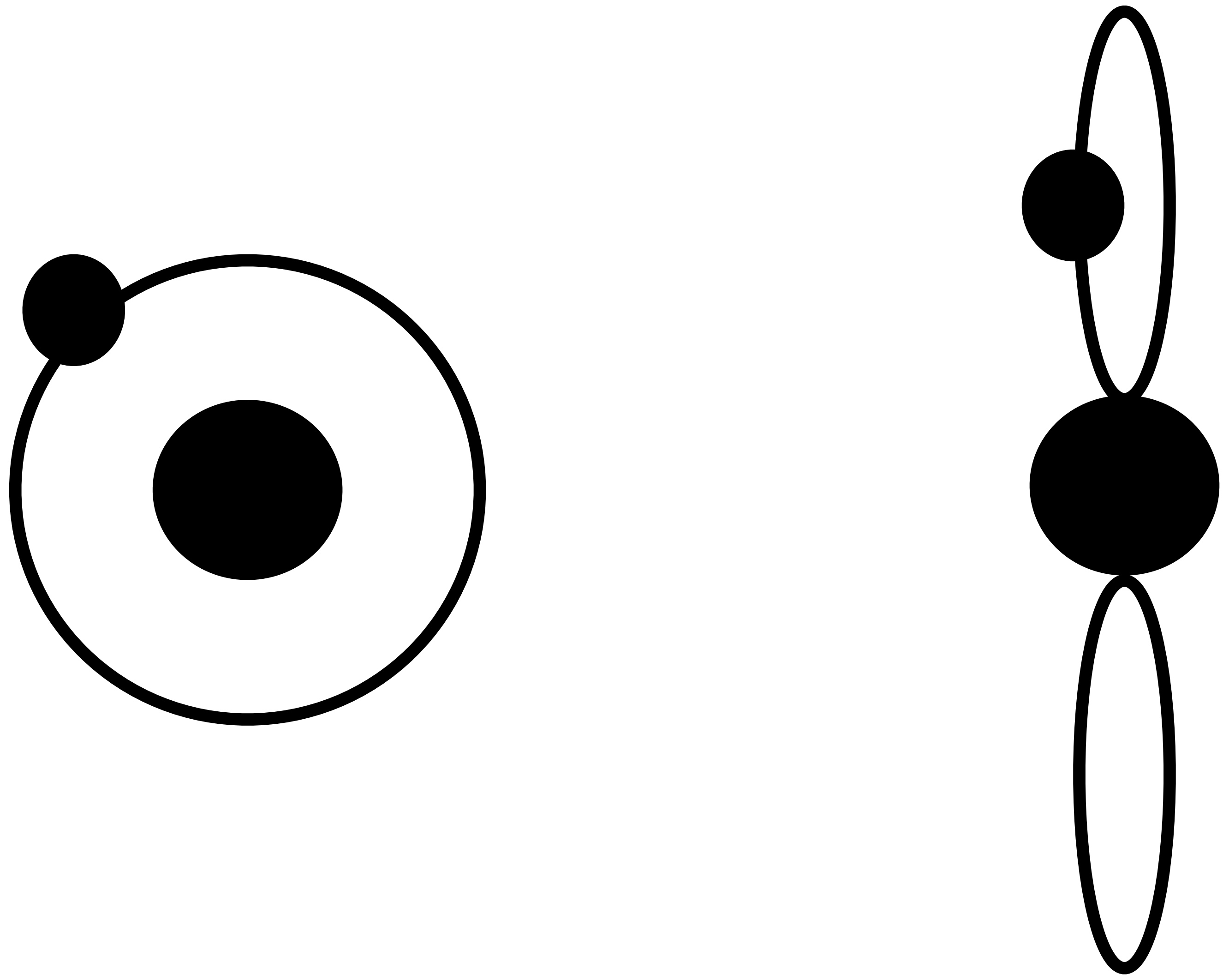
Causal Green’s Function

Schrodinger Equation

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Single particle equation derived from field theory
Equation depends upon theory (Yukawa, Φ^4 etc)

$$i\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = \left(H + \epsilon\int d^4x'\,\Psi^*\left(x\right)\Psi\left(x'\right)G_R\left(x;x'\right)\right)\Psi\left(t,\mathbf{x}\right)$$

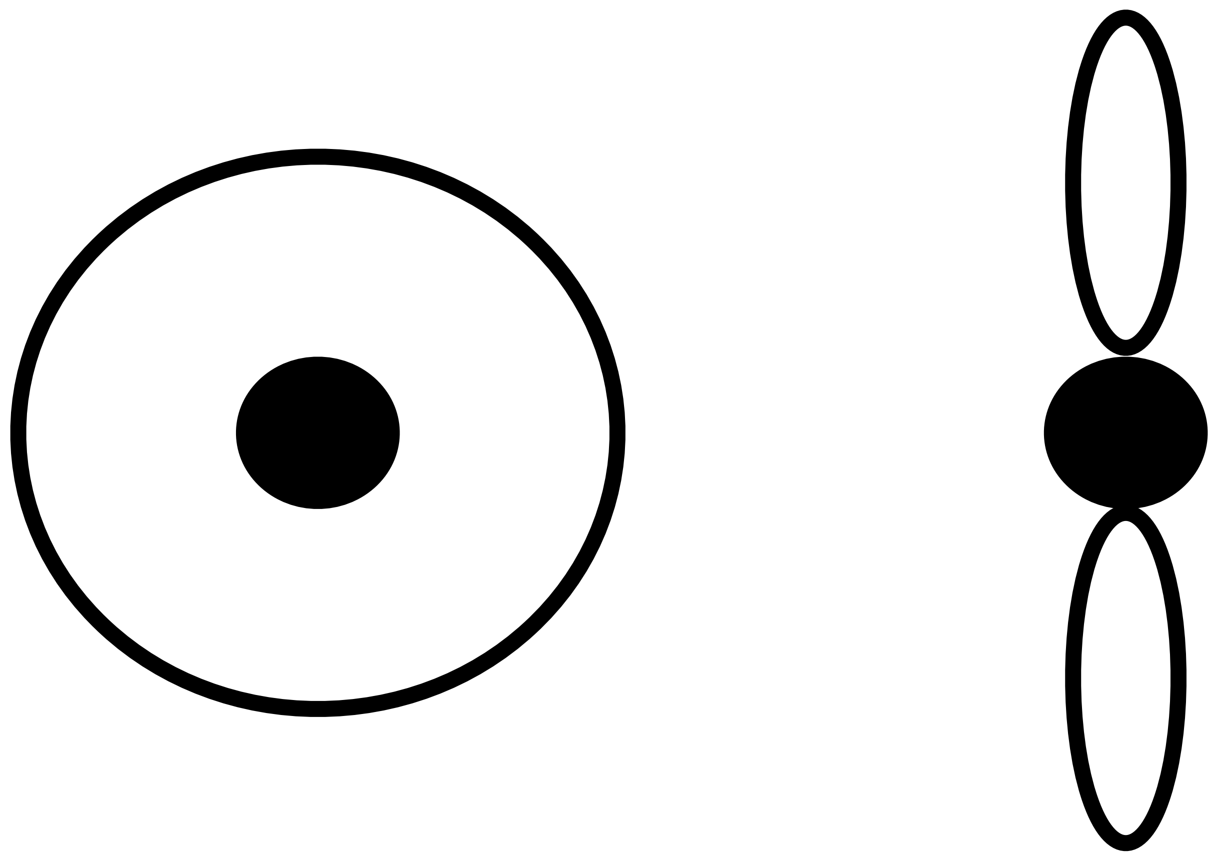


Fixed Central particle

Self interaction of wave-function breaks degeneracy of levels

Constraints

What does this do to the Lamb Shift?



Proton at Fixed Location

2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function spread over some region (e.g. trap size ~ 100 nm)
Expectation value of electromagnetic field diluted
In neutral atom - heavily suppressed, except at edges!

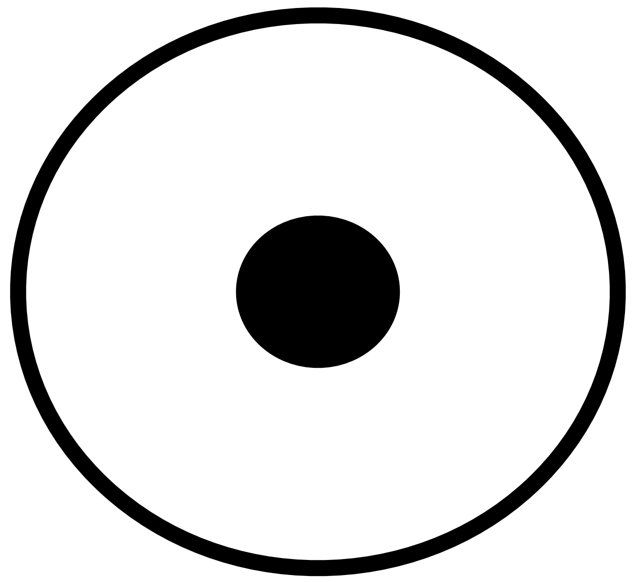
$\varepsilon < 10^{-2}$

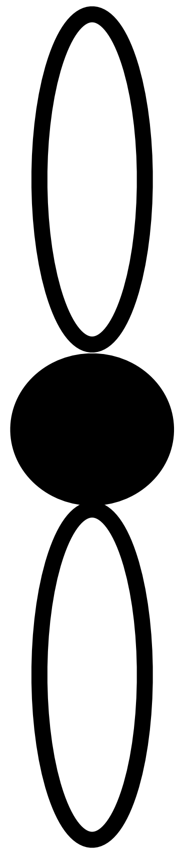
Key Point

Energy Depends on Entire Wave-Function

Similarly, kills possible bounds on QCD and gravity

Emergent "Linearity"





$$H \supset \lambda\phi\bar{\Psi}\Psi + \epsilon\langle\chi|\phi|\chi\rangle\bar{\Psi}\Psi$$

Want to measure ε

Go create some ϕ and look for effect

$$|\chi\rangle = \alpha|U\rangle + \beta|M\rangle \implies \langle\chi|\phi|\chi\rangle = |\alpha|^2\langle U|\phi|U\rangle + |\beta|^2\langle M|\phi|M\rangle$$

<U|ϕ|U> is non zero, but <M|ϕ|M> = 0

Cannot undo small α - hide large ε

For large β entirely dominated by |M>

Local Exploitability completely determined by unchangeable initial conditions

Stark Difference from Linear Quantum Mechanics

“EFT” with Operators and States

$$\boxed{H_L \supset \left(\int d^3x\,\lambda\,\boxed{\phi\left(x\right)\bar{\Psi}\left(x\right)\Psi\left(x\right)} + \frac{\phi^2}{\Lambda}\bar{\Psi}\Psi\right)}$$

$$H_{NL} \supset \epsilon\left(\int d^3x\,\boxed{\langle\chi\left(t\right)|\phi\left(x\right)|\chi\left(t\right)\rangle}\bar{\Psi}\left(x\right)\Psi\left(x\right)\right)$$

Free QFT does not generate interactions, but…

Linear theory does not generate non-linearity, but…

Low Energy Physics?

Low “Overlap” Physics?

Emergent "Linearity"

$$H \supset \lambda\phi\bar{\Psi}\Psi + \epsilon\langle\chi|\phi|\chi\rangle\bar{\Psi}\Psi$$

$$|\chi\rangle = \alpha|U\rangle + \beta|M\rangle \implies \langle\chi|\phi|\chi\rangle = |\alpha|^2\langle U|\phi|U\rangle + |\beta|^2\langle M|\phi|M\rangle$$

Do all effects of non-linear quantum mechanics vanish if α << 1?

Not if <M|ϕ|M> is non-zero

Two fields for which the expectation value is non-zero
Gravity(<M|g|M> != 0) and the Higgs(<M|h|M> !=0)

Both are sources of huge hierarchy problems

Both Hamiltonians could be corrected by non-linear terms

The Classical Equations of Motion

The Classical Equations of Motion

$$H = \frac{p^2}{2m} + V\left(x\right)$$

$$\dot{x} = \frac{\partial H}{\partial p} \implies \dot{x} = \frac{p}{m} \qquad \dot{p} = -\frac{\partial H}{\partial x} \implies \dot{p} = -\frac{\partial V}{\partial x}$$

Where do these come from?

$$i\frac{\partial|\chi\left(t\right)\rangle}{\partial t} = H|\chi\left(t\right)\rangle$$

Ehrenfest's Theorem

$$\frac{d\langle\chi\left(t\right)|O|\chi\left(t\right)\rangle}{dt} = -i\langle\chi\left(t\right)|[O, H]|\chi\left(t\right)\rangle$$

The Classical Equations of Motion

$$H = \frac{p^2}{2m} + V(x) \qquad \frac{d\langle \chi(t)|O|\chi(t)\rangle}{dt} = -i\langle \chi(t)|[O,H]|\chi(t)\rangle$$

$$[x,p] = i$$

$$\frac{d\langle x\rangle}{dt} = -i\langle [x, \frac{p^2}{2m}]\rangle = \frac{\langle p\rangle}{m} \qquad \frac{d\langle p\rangle}{dt} = -i\langle [p, V(x)]\rangle = -\langle \frac{\partial V}{\partial x}\rangle$$

These identities are true for all quantum states

For very narrow (i.e. coherent) states, replace <V(x)> by V(<x>)

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} \qquad \frac{d\langle p\rangle}{dt} = -\frac{\partial V(\langle x\rangle)}{\partial\langle x\rangle}$$

Classical Field Theory

$$H = \int d^3x \frac{1}{2} \left(\pi^2 + \nabla_i \phi \nabla^i \phi\right) + V\left(\phi\right)$$

$$\dot{\phi} = \frac{\partial H}{\partial \pi} = \pi \qquad \dot{\pi} = -\frac{\partial H}{\partial \phi} = \nabla^2 \phi - \frac{\partial V}{\partial \phi}$$

Where do these come from?

$$i \frac{\partial |\chi\left(t\right)\rangle}{\partial t} = H |\chi\left(t\right)\rangle$$

$$\frac{d\langle \chi\left(t\right) | O | \chi\left(t\right)\rangle}{dt} = -i \langle \chi\left(t\right) | [O, H] | \chi\left(t\right)\rangle$$

Classical Field Theory

$$H = \int d^3x \frac{1}{2}\left(\pi^2 + \nabla_i\phi\nabla^i\phi\right) + V\left(\phi\right)$$

$$\left[\phi\left(x\right), \pi\left(x'\right)\right] = i\delta\left(x - x'\right)$$

$$\frac{\partial\langle\pi\rangle}{\partial t} = -i\langle[\pi, H]\rangle = \langle\nabla^2\phi - \frac{\partial V}{\partial\phi}\rangle$$

These identities are true for all quantum states

For coherent states

$$\frac{\partial\langle\pi\rangle}{dt} = \nabla^2\langle\phi\rangle - \frac{\partial V\left(\langle\phi\rangle\right)}{\partial\phi}$$

Non-Linear Quantum Mechanics

$$H = H_L + \int d^3x g \boxed{\langle \chi(t) | \phi^2 | \chi(t) \rangle} \phi^2$$

$$\frac{\partial \langle \pi \rangle}{\partial t} = -\langle [\pi, H] \rangle = \cdots + \boxed{2} g \langle \phi^2 \rangle \langle \phi^2 \rangle$$

Expectation value is a c-number. Commutes with π

2 instead of 4.
This will solve the hierarchy problem

The Hierarchy Problem

Why is the Hierarchy Problem hard to solve?

A Version of Weinberg's No Go

$$V\left(h\right) = -\mu^2 h^2 + \lambda h^4 + \dots$$

Need: $$\frac{\partial V}{\partial h} = 0 \implies -2\mu^2 + 4\lambda h^2 = 0$$

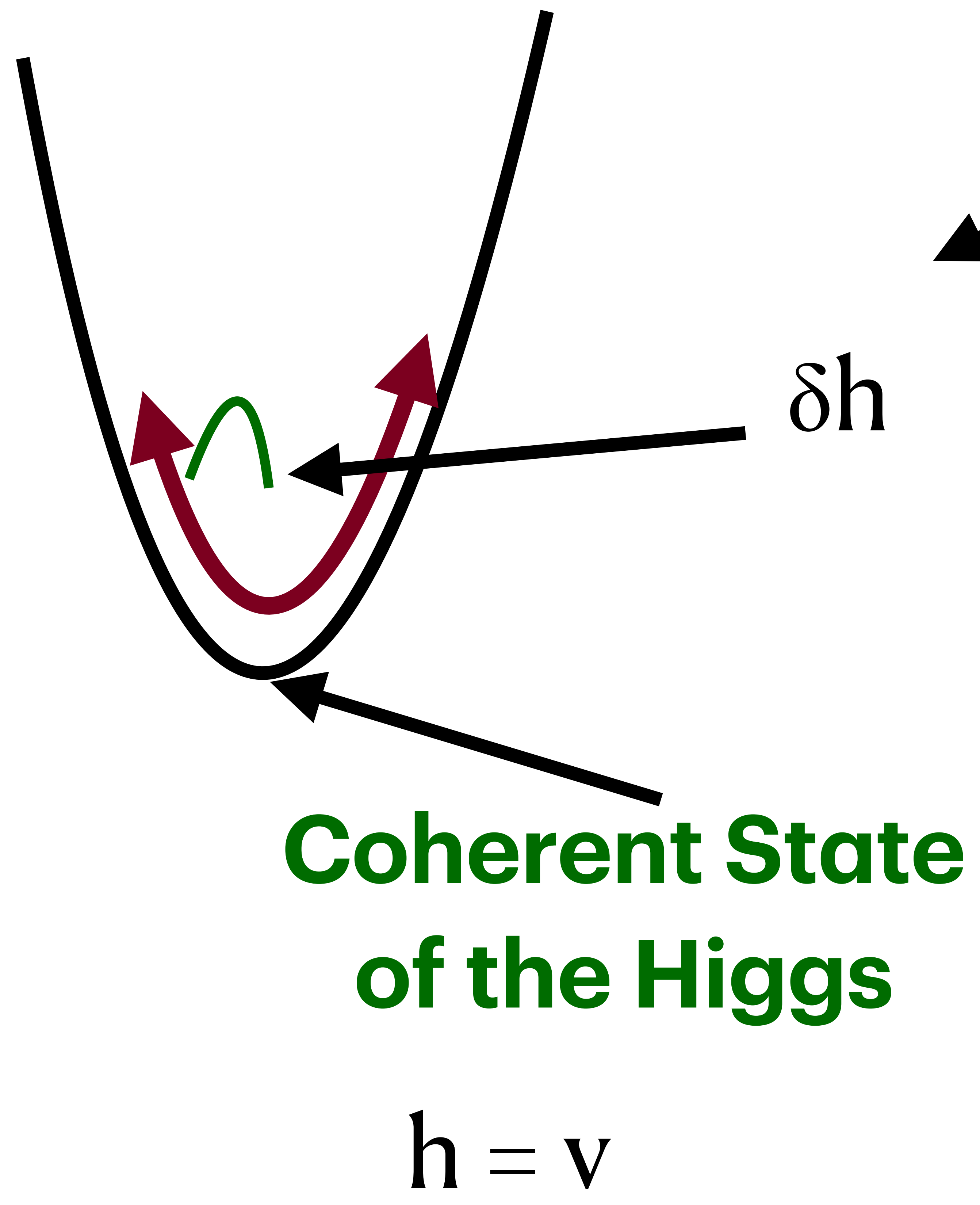
At that minimum: $$\frac{\partial^2 V}{\partial h^2} \ll \mu^2 \implies -2\mu^2 + 12\lambda h^2 \approxeq= 0$$

Two different conditions - cannot simultaneously satisfy

Generic Potential: Minimum is not a place where second derivative needs to be small

The Higgs in Linear Quantum Mechanics

The Higgs Particle: Fluctuation around the minimum



The Higgs Mass: Energy Needed to create zero (or really, low) momentum fluctuation

$$|\chi\left(t\right)\rangle = \alpha|v+\delta h\rangle + \beta|M\rangle$$

$$i\frac{d|\chi(t)\rangle}{dt} = H|\chi\left(t\right)\rangle \implies i\frac{d|\delta h\rangle}{dt} = H|\delta h\rangle$$

Mass independent of precise Higgs wave-function or the wave-function of the entire quantum state

Will not be true in non-linear quantum mechanics

Higgs Production at the LHC

$$|\chi(t)\rangle = |U\rangle$$

What is the Higgs Field that is created?

$$|\chi(t)\rangle = |0\rangle + \sqrt{\frac{1}{L^3 m_h^3}\frac{\sigma_{\mathrm{pp}\to h}}{L^2}}\,|h\rangle$$

$$\langle\chi(t)|h^2|\chi(t)\rangle = v^2 + \boxed{\frac{1}{L^3 m_h^3}\frac{\sigma_{\mathrm{pp}\to h}}{L^2}}\,m_h^2$$

Highly Suppressed

Higgs Production at the LHC

$$|\chi(t)\rangle = \alpha|U\rangle + \beta|M\rangle$$

What is the Higgs Field that is created?

We exist in a small part of the wave-function!

$$\langle\chi(t)|h^2|\chi(t)\rangle = v^2 + |\alpha|^2 \frac{1}{L^3 m_h^3} \frac{\sigma_{\mathrm{pp}\to h}}{L^2} m_h^2$$

Bottom-line: $\langle\chi(t)|h^2|\chi(t)\rangle = v^2$

The Higgs Mass in Non-Linear Quantum Mechanics

$$V\left(h\right) = -\mu^2 h^2 + g\langle\chi|h^2|\chi\rangle h^2$$

Stationary Points $\frac{d\langle\pi\rangle}{dt} = -i\langle[\pi, H]\rangle = \boxed{\left(-2\mu^2 + 2g\langle h^2\rangle\right)}\langle h\rangle = 0$

What is the energy needed to create δh?

State dependent - but at the LHC: $\langle\chi\left(t\right)|h^2|\chi\left(t\right)\rangle = v^2$

$$i\frac{d|\chi(t)\rangle}{dt} = H|\chi\left(t\right)\rangle \implies i\frac{d|\delta h\rangle}{dt} = \int d^3x\left(-\mu^2 h^2 + g\langle\chi|h^2|\chi\rangle\right)h^2 + \ldots|\delta h\rangle$$

Higgs Mass: $\boxed{-2\mu^2 + 2g\langle h^2\rangle}$ **Same Condition**

The Hierarchy Problem

Simplicity: $$|\chi(t)\rangle = |U\rangle$$

$$V(h) = -\mu^2 h^2 + \frac{g}{2}\langle\chi|h^2|\chi\rangle h^2 + \frac{\lambda}{4}h^4$$

Stationary Points: $$\langle h\rangle = 0, \langle h\rangle = \frac{\mu^2}{\frac{g}{2} + \lambda/4}$$

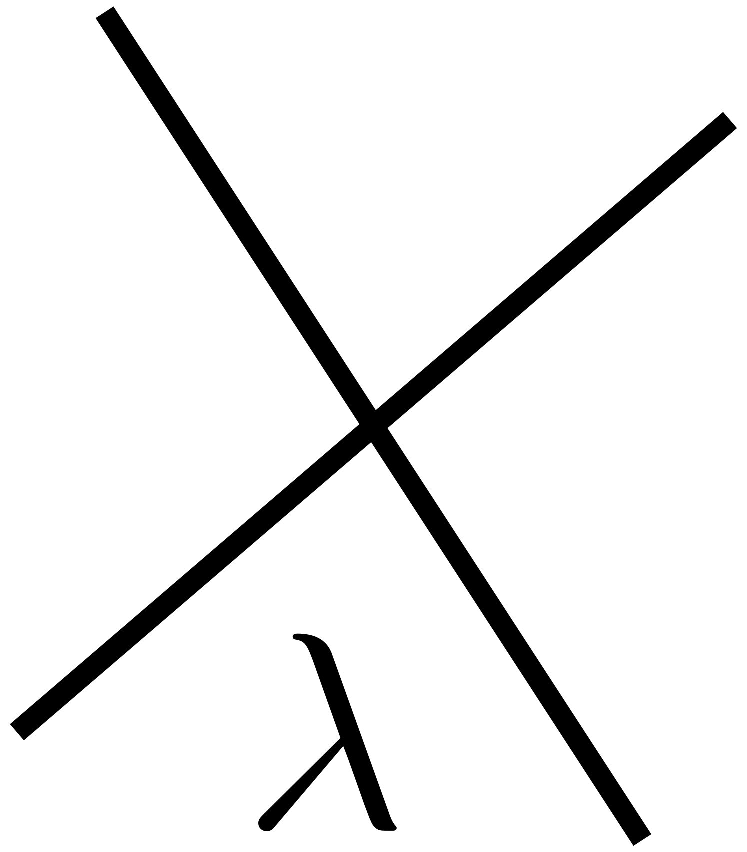
Fluctuations δh: **Unstable** **Mass**

$$m_h = \sqrt{\frac{4\mu^2}{(2g+\lambda)}} \qquad g \sim \frac{\mu^2}{m_w^2} \gg 1$$

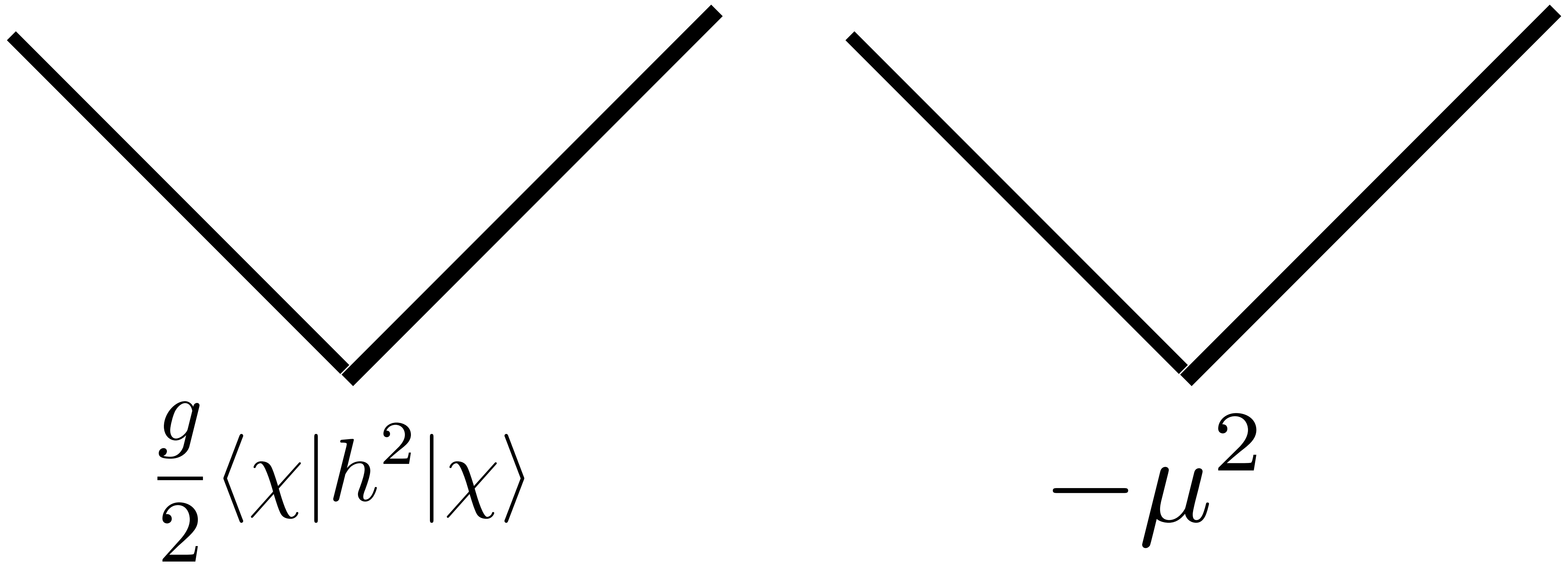
Perturbativity?

$$V(h) = -\mu^2 h^2 + \frac{g}{2}\langle\chi|h^2|\chi\rangle h^2 + \frac{\lambda}{4}h^4$$

$$g \sim \frac{\mu^2}{m_w^2} \gg 1$$



Usual quartic leads to loops



Just a mass correction

Perturbativity?

Higgs Energy Correction?

$$|\chi(t)\rangle = |U\rangle$$

$$\langle\chi(t)|h^2|\chi(t)\rangle = v^2 + \frac{1}{L^3 m_h}\frac{\sigma_{\text{pp}\to h}}{L^2} m_h^2$$

$$\delta m_h^2 \sim g\frac{1}{L^3 m_h}\frac{\sigma_{\text{pp}\to h}}{L^2} \sim \left(\frac{M_{pl}}{m_w}\right)^2 \frac{1}{L^3 m_w}\frac{\sigma_{\text{pp}\to h}}{L^2} \ll m_w^2$$

Even more suppressed if we are a tiny part of the wave-function

Early Universe Coherent Field Value

$$g\langle h^2\rangle \lesssim \mu^2 \implies h \lesssim m_w$$

Can produce particles at cut-off

Many Worlds?

$$|\chi(t)\rangle = \alpha|U\rangle + \sum_i \beta_i |M_i\rangle$$

N worlds with N higgs - coupled together by expectation value

$$V_1(h_1) = -\mu^2 h_1^2 + \frac{g}{2}\langle\chi|h^2|\chi\rangle h_1^2 + \frac{\lambda}{4} h_1^4$$

$$V_2(h_1) = -\mu^2 h_2^2 + \frac{g}{2}\langle\chi|h^2|\chi\rangle h_2^2 + \frac{\lambda}{4} h_2^4$$

Do they all end up in the right minimum?

$$\langle h_1\rangle = \langle h_2\rangle = \sqrt{\frac{4\mu^2}{2g+\lambda}}$$

Only Stable Minimum

Signals

$$V\left(h\right) = -\mu^2 h^2 + \frac{g}{2}\langle\chi|h^2|\chi\rangle h^2 + \frac{\lambda}{4}h^4$$

Identical Collider Phenomenology as the Standard Model

$$g\langle h^2\rangle \lessapprox \mu^2 \implies h \lessapprox m_w$$

Early universe signals could be different (e.g. phase transitions)

$$V\left(h\right) = -\mu^2 h^2 + \frac{g}{2}\langle\chi|h^2|\chi\rangle h^2 + \frac{\lambda}{4}h^4 + \kappa\langle h\rangle h^3 + \dots$$

Deviations from Standard Model Higgs Potential

Conclusions

The Cosmological Constant Problem

Λ Λ

Observed: 10^{-12} eV^4

Theory: 10^{48} eV^4

Why?

Standard Model?	**Symmetry?**	**Relaxation?**	**Gravity?**
1947	4000 BC	1980s, 2016s	2000s

The correct quantum Hamiltonian?

Cosmological Constant Problem

$$H_{GR} = \int d^3x \sqrt{-g} \left(\mathcal{H}_G \left(\pi_{ij}, g_{ij} \right) + \mathcal{H}_M \left(g_{ij}, \pi_\phi, \phi \right) \right)$$

$$\mathcal{H}_M = \left(\pi_\phi^2 + g_{ij} \nabla^i \phi \nabla^j \phi + V\left(\phi \right) \right)$$

$$\text{Vacuum: } |V\rangle, \ \mathcal{H}_M |V\rangle = \Lambda^4 |V\rangle$$

$$i \frac{d|V\rangle}{dt} = H_{GR}|V\rangle = \int d^3x \sqrt{-g} \left(H_G \left(\pi_{ij}, g_{ij} \right) + \Lambda^4 \right) |V\rangle$$

$$\frac{d}{dt} \langle V | \pi_{ij} | V \rangle = \Lambda^4 \langle V | [\pi_{ij}, \sqrt{-g}] | V \rangle + \cdots = \Lambda^4 \langle V | \frac{\partial \sqrt{-g}}{\partial g_{ij}} | V \rangle + \dots$$

π and g do not commute - vacuum energy gravitates

A Classical Volume Element

$$H = \int d^3x \boxed{\langle\Psi|\sqrt{-g}|\Psi\rangle} \left(\mathcal{H}_G\left(\pi_{ij}, g_{ij}\right) + \mathcal{H}_M\left(g_{ij}, \pi_\phi, \phi\right)\right)$$

$$\langle\Psi|\sqrt{-g}|\Psi\rangle = \sqrt{-\mathrm{Det}\left(\langle\Psi|g_{\mu\nu}|\Psi\rangle\right)}$$

$$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$$

Transforms as $\sqrt{-g}$

Vacuum: $|V\rangle$, $\mathcal{H}_M|V\rangle = \Lambda^4|V\rangle$

$$i\frac{d|V\rangle}{dt} = H_{GR}|V\rangle = \int d^3x \, \langle V|\sqrt{-g}|V\rangle \left(H_G\left(\pi_{ij}, g_{ij}\right) + \Lambda^4\right)|V\rangle$$

A Classical Volume Element

$$i\frac{d|V\rangle}{dt} = H_{GR}|V\rangle = \int d^3x \, \langle V|\sqrt{-g}|V\rangle \left(H_G\left(\pi_{ij}, g_{ij}\right) + \Lambda^4\right)|V\rangle$$

$$\frac{d}{dt}\langle V|\pi_{ij}|V\rangle = \Lambda^4 \langle V|[\pi_{ij}, \boxed{\langle V|\sqrt{-g}|V\rangle}]|V\rangle + \ldots$$

Pure number - commutator vanishes!

Vacuum energy does not gravitate!

Paper to appear....