

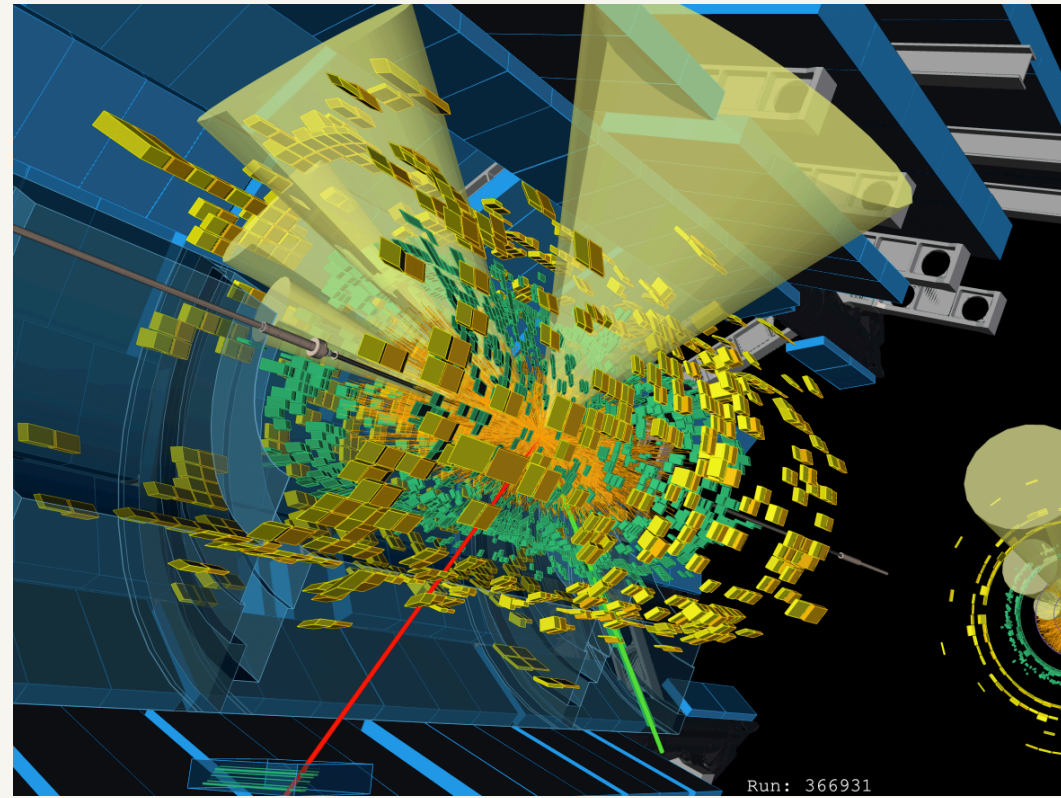
Quantum Information Science for Particle & Collider Physics

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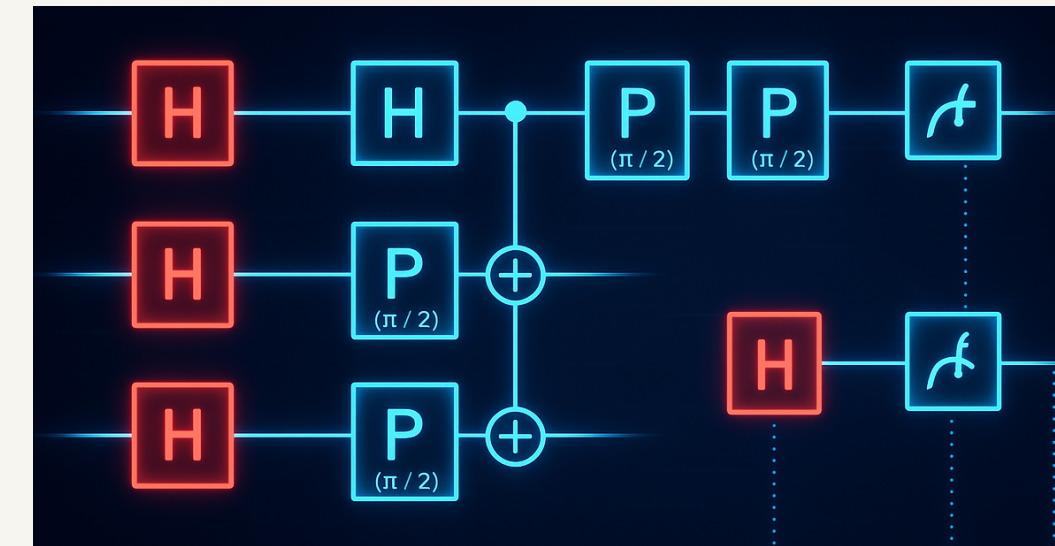
30th International Summer Institute (SI2026), 8 August 2026

Particle Collider



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Quantum Information Processing Device



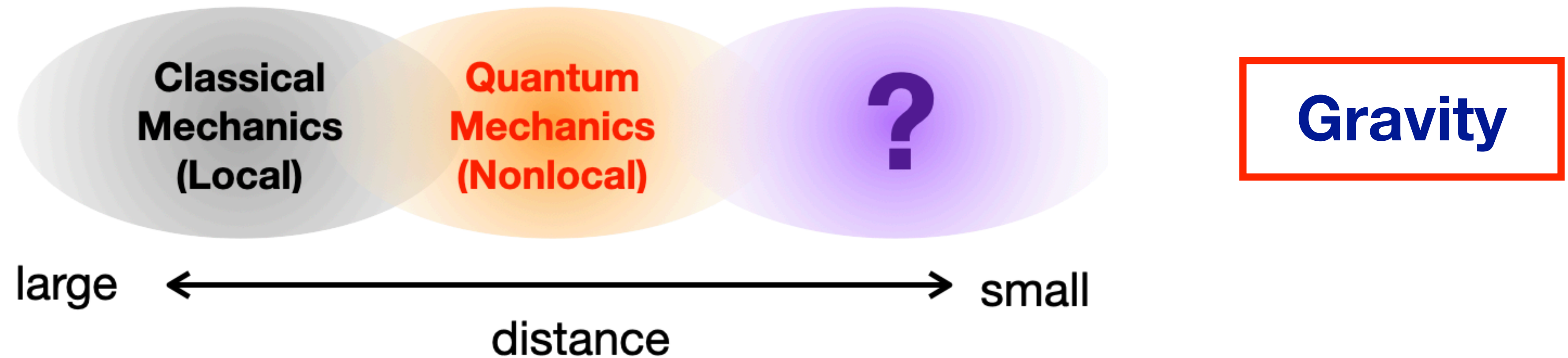
Motivation: Gives us a new perspective! Fun to explore!

Precision test of SM: Enhancing new physics searches

A fresh look at QFT from QIT: Can we predict SM parameters?
Area law: cross-section = entanglement entropy

High Energy Test of Quantum Mechanics Beyond QFT detection

High Energy Test of Quantum Mechanics



- QM might be a low energy effective theory of more fundamental short-distance theory.
- Currently, **no LHC analysis** can distinguish between anomalies from **QFT-based BSM** and those from **beyond-QM**

★ Non-linear extensions of QM:

Weinberg (1989), Polchinski (1991),
D.E.Kaplan, S.Rajendran, (2021)

$$i\partial_t|\chi\rangle = \int d^3x \left[\hat{\mathcal{H}}(x) + \langle\chi|\hat{\mathcal{O}}_1(x)|\chi\rangle \hat{\mathcal{O}}_2(x) \right] |\chi\rangle$$

non-linear state-dependent term

Plan

Lecture 1:

- **QI Tools:** Density Matrix, Entanglement, Bell nonlocality
- **QI Measurements @ Collider:** $t\bar{t}$, $\tau\text{-}\tau$, ZZ

Lecture 2:

- **Open Quantum Mechanics**
 - ◇ Reconstruction of Quantum Channel (probing NLQM)
 - ◇ Decoherence @ Colliders

Quantum Mechanics

Three line summary of Quantum Mechanics:

Linear time-evolution: $|\psi(t)\rangle = \hat{U}(t) |\psi(0)\rangle$ $\hat{U}(t) = e^{-i\hat{H}t}$

Born rule: $\text{Prob}(a_k|\psi) = |\langle a_k|\psi\rangle|^2$ $\hat{A}|a_k\rangle = a_k|a_k\rangle$

Mean value: $\langle \hat{A} \rangle_\psi = \sum_k a_k \text{Prob}(a_k|\psi) = \langle \psi|\hat{A}|\psi\rangle$

When we don't know the pure state exactly,

Ensemble: $\{ q_1; |\psi_1\rangle, q_2; |\psi_2\rangle, \dots \}$ $q_i \geq 0, \sum_i q_i = 1$

All physical info is encoded in the **density operator**:

$$\hat{\rho} = \sum_i q_i |\psi_i\rangle \langle \psi_i|$$

Time Evolution: $\hat{\rho}(t) = \hat{U}(t) \hat{\rho}(0) \hat{U}(t)^\dagger$ $\hat{U}(t) = e^{-i\hat{H}t}$

Probability: $\text{Prob}(a_k|\rho) = \sum_i q_i \text{Prob}(a_k|\psi_i) = \text{Tr} (\hat{\Pi}_k \hat{\rho})$ $\hat{\Pi}_k = |a_k\rangle \langle a_k|$

Mean value: $\text{Tr} (\hat{A}\hat{\rho}) = \sum_i q_i \text{Tr} (\hat{A} |\psi_i\rangle \langle \psi_i|) = \sum_i q_i \langle \psi_i | \hat{A} | \psi_i \rangle$

Trace formula:

$$\langle e_n | e_m \rangle = \delta_{nm} \quad \sum_n |e_n\rangle \langle e_n| = 1$$

$$\text{Tr} (\hat{X} |\alpha\rangle \langle \beta|) \equiv \sum_n \langle e_n | (\hat{X} |\alpha\rangle \langle \beta|) | e_n \rangle = \sum_n \langle \beta | e_n \rangle \langle e_n | \hat{X} | \alpha \rangle = \langle \beta | \hat{X} | \alpha \rangle$$

$$\implies \text{Tr} (\hat{\Pi}_k \hat{\rho}) = \sum_i q_i \text{Tr} (|a_k\rangle \langle a_k| \psi_i\rangle \langle \psi_i|) = \sum_i q_i |\langle a_k | \psi_i \rangle|^2 = \text{Prob}(a_k | \rho)$$

$$\implies \text{Tr} (\hat{A} \hat{\rho}) = \sum_i q_i \text{Tr} (\hat{A} |\psi_i\rangle \langle \psi_i|) = \sum_i q_i \langle \psi_i | \hat{A} | \psi_i \rangle = \langle \hat{A} \rangle_\rho$$

$$\implies \text{Tr} (\hat{\rho}) = \sum_i q_i \text{Tr} (|\psi_i\rangle \langle \psi_i|) = \sum_i q_i \langle \psi_i | \psi_i \rangle = \sum_i q_i = 1$$

$$\hat{\rho} = \sum_i q_i |\psi_i\rangle \langle \psi_i|$$

$$\hat{\Pi}_k = |a_k\rangle \langle a_k|$$

Property of the density operator:

PSD = Positive Semi-Definite

$$\hat{\rho} = \sum_i q_i |\psi_i\rangle \langle \psi_i|$$

$$\Rightarrow \left\{ \begin{array}{l} \text{Hermitian: } \hat{\rho}^\dagger = \hat{\rho} \\ \text{PSD: } \hat{\rho} \geq 0 \quad \left(\langle \varphi | \hat{\rho} | \varphi \rangle \geq 0, \forall |\varphi\rangle \right) \\ \text{Unit Trace: } \text{Tr } \hat{\rho} = 1 \end{array} \right.$$

Converse is true: Let $\mathcal{S}(\mathcal{H})$ be the set of all Hermitian, unit-trace and PSD operators. Every element of $\mathcal{S}(\mathcal{H})$ corresponds to a physical state.

$$\rho \xrightarrow{\text{Hermitian}} \rho^{\text{diag}} = \text{diag}(p_1, p_2, \dots) \quad \left\{ \begin{array}{ll} p_i \geq 0 & \leftarrow \text{PSD} \\ \sum_i p_i = 1 & \leftarrow \text{Unit Trace} \end{array} \right.$$

Density matrix $\rightarrow$ pure-state ensemble is *not unique*

- Consider the following state

$$\rho = p |0\rangle\langle 0| + (1 - p) |1\rangle\langle 1|$$

- Define $|\psi_{\pm}\rangle = c_{\theta} |0\rangle \pm s_{\theta} |1\rangle$

$$\langle \psi_+ | \psi_- \rangle = c_{\theta}^2 - s_{\theta}^2$$

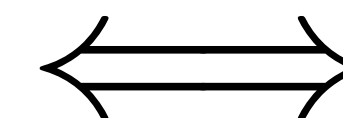
$$|\psi_{\pm}\rangle\langle \psi_{\pm}| = c_{\theta}^2 |0\rangle\langle 0| \pm c_{\theta}s_{\theta} [|0\rangle\langle 1| + |1\rangle\langle 0|] + s_{\theta}^2 |1\rangle\langle 1|$$

- Define $\rho_{\theta} = \frac{1}{2} |\psi_+\rangle\langle \psi_+| + \frac{1}{2} |\psi_-\rangle\langle \psi_-| = \underline{c_{\theta}^2 |0\rangle\langle 0| + s_{\theta}^2 |1\rangle\langle 1|}$

$\rightarrow \rho = \rho_{\theta} |_{c_{\theta}^2=p}$

$$\begin{cases} |0\rangle & \cdots & p \\ |1\rangle & \cdots & 1 - p \end{cases}$$

physically
indistinguishable



$$\begin{cases} |\psi_+\rangle & \cdots & \frac{1}{2} \\ |\psi_-\rangle & \cdots & \frac{1}{2} \end{cases}$$

How much is the state pure ?

$$\hat{\rho}_{\text{mix}}^{\text{max}} = \sum_{i=1}^d \frac{1}{d} |\psi_i\rangle\langle\psi_i| \quad (d \equiv \dim\mathcal{H})$$

$$\hat{\rho}_{\text{pure}} = |\psi\rangle\langle\psi| \quad \{q_i\} = \{1, 0, 0, \dots\}$$

$$\hat{\rho}_{\text{mix}} = \sum_i q_i |\psi_i\rangle\langle\psi_i| \quad \{q_i\} = \{q_1, q_2, \dots\}$$

Purity:

$$\gamma(\rho) \equiv \text{Tr}\rho^2 = \sum_i q_i^2$$

$$\text{max mixed} \quad \frac{1}{d} \leq \gamma(\rho) \leq 1 \quad \text{pure}$$

von Neumann entropy:

$$S_V(\rho) \equiv -\text{Tr}[\rho \log_2 \rho] = -\sum_i q_i \log_2 q_i$$

$$\text{pure} \quad 0 \leq S_V(\rho) \leq \log_2 d \quad \text{max mixed}$$

Qubit:

$$\{ |\uparrow_z\rangle, |\downarrow_z\rangle \} = \{ |+_z\rangle, |-_z\rangle \} = \{ |0_z\rangle, |1_z\rangle \}$$

$$\sigma_z |\pm_z\rangle = \pm |\pm_z\rangle$$

Compare the following two states:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|+_z\rangle + |-_z\rangle) = |+_x\rangle \quad \text{Prob}(\pm_z|\psi) = \frac{1}{2},$$

$$\text{Prob}(+_x|\psi) = 1,$$

$$\text{Prob}(-_x|\psi) = 0$$

$$\rho_{\text{mix}}^{\text{max}} = \frac{1}{2} |+_z\rangle\langle+_z| + \frac{1}{2} |-_z\rangle\langle-_z| \quad \text{Prob}(\pm_z|\rho_{\text{mix}}^{\text{max}}) = \frac{1}{2} \quad \text{Prob}(\pm_x|\rho) = \frac{1}{2}$$

Matrix representation:

$$\langle i|j\rangle = \delta_{ij} \quad \rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}, \quad \rho_{ij} = \langle i|\hat{\rho}|j\rangle$$

$$\langle \tilde{\alpha}|\tilde{\beta}\rangle = \delta_{\alpha\beta} \quad \tilde{\rho} = \begin{pmatrix} \tilde{\rho}_{00} & \tilde{\rho}_{01} \\ \tilde{\rho}_{10} & \tilde{\rho}_{11} \end{pmatrix}, \quad \tilde{\rho}_{\alpha\beta} = \langle \tilde{\alpha}|\hat{\rho}|\tilde{\beta}\rangle$$

Unitary relation:

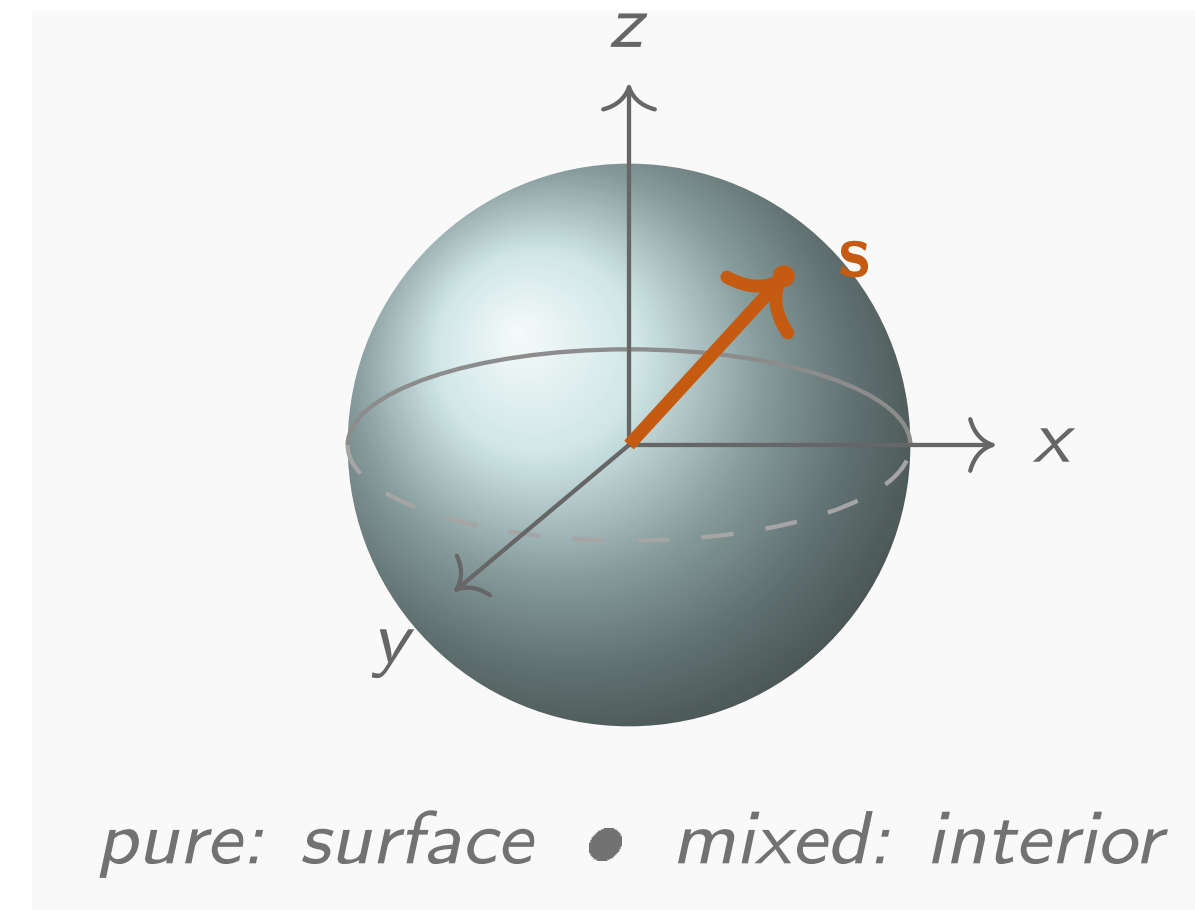
$$\begin{aligned} |\tilde{\alpha}\rangle &= U_{\alpha i} |i\rangle \\ U^\dagger U &= 1 \end{aligned} \quad \Longrightarrow \quad \tilde{\rho}_{\alpha\beta} = \langle \tilde{\alpha}|\hat{\rho}|\tilde{\beta}\rangle = U_{\alpha i}^* \underbrace{\langle i|\hat{\rho}|j\rangle}_{\rho_{ij}} U_{j\beta}^T$$

Maximally mixed state:

$$\rho_{\text{mix}}^{\text{max}} = \frac{1}{2} |+_z\rangle\langle+_z| + \frac{1}{2} |-_z\rangle\langle-_z| \quad \Longrightarrow \quad \frac{1}{2} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \quad \textit{basis independent}$$

Bloch representation:

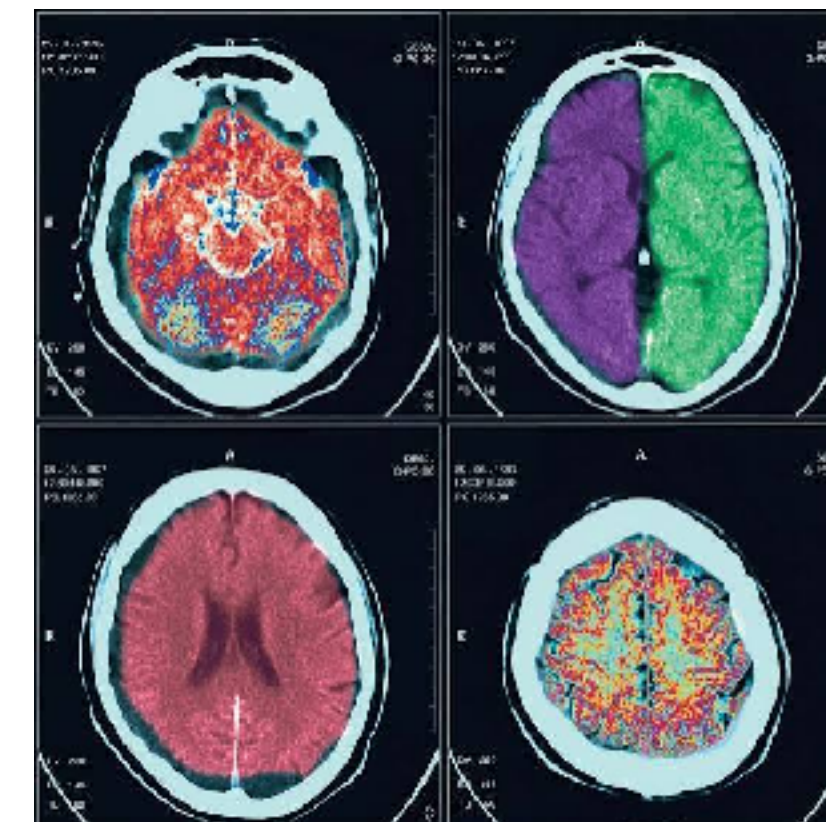
$$\left. \begin{array}{l} \text{Tr } \rho = 1 \\ \rho^\dagger = \rho \\ \rho \geq 0 \end{array} \right\} \implies \rho = \frac{1}{2} \left(\mathbb{1} + \sum_{i=1}^3 B_i \sigma_i \right) \quad B_i \in \mathbb{R}, \quad \sum_i B_i^2 \leq 1$$



Experimental reconstruction of the state (**Quantum State Tomography**):

$$B_i = \text{Tr} [\sigma_i \rho] = \langle \sigma_i \rangle_\rho$$

$$\text{Tr} (\sigma_i \sigma_j) = 2 \delta_{ij}$$



$$\mathcal{H}_A \ni \{|+\rangle_A, |-\rangle_A\}$$



$$\mathcal{H}_B \ni \{|+\rangle_B, |-\rangle_B\}$$

2-qubit: $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B \ni \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$

General state:

$$\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i (\sigma_i \otimes \mathbb{1}_2) + \bar{B}_j (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right)$$

Fano coeffs.:

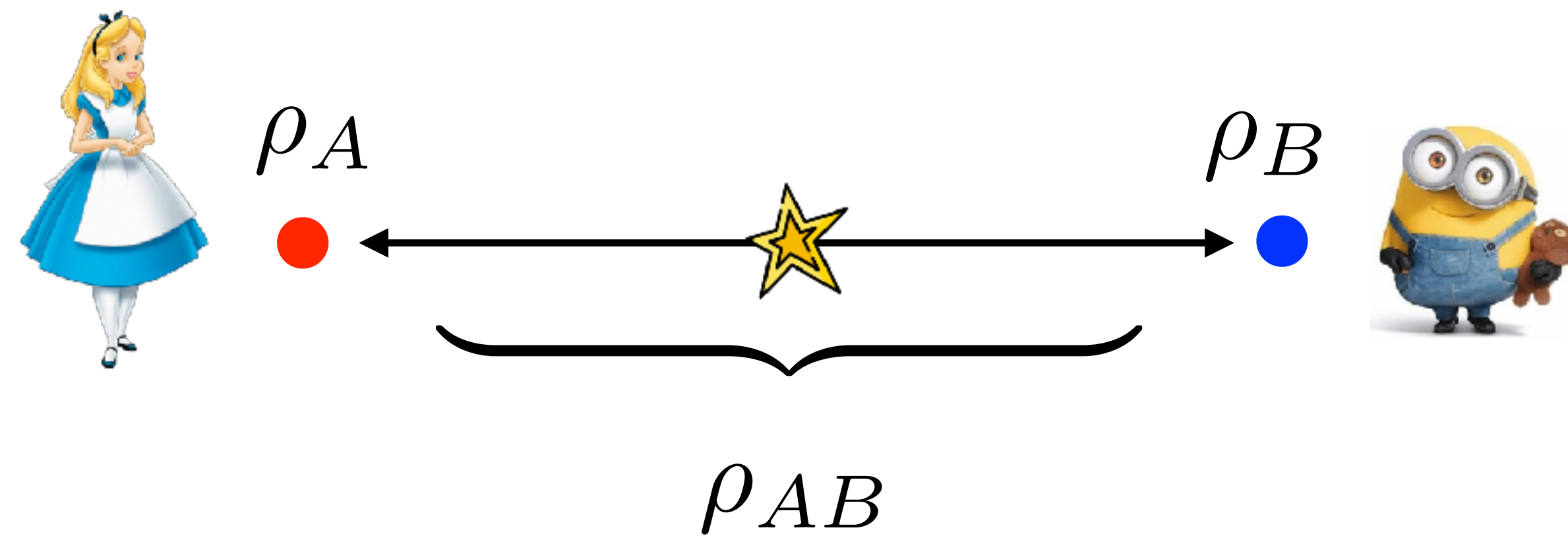
$$B_i = \text{Tr} [(\sigma_i \otimes \mathbb{1}) \hat{\rho}] = \langle \sigma_i^{(A)} \rangle_\rho \quad \longleftarrow \text{A's spin average}$$

$$\bar{B}_i = \text{Tr} [(\mathbb{1} \otimes \sigma_i) \hat{\rho}] = \langle \sigma_i^{(B)} \rangle_\rho \quad \longleftarrow \text{B's spin average}$$

$$C_{ij} = \text{Tr} [(\sigma_i \otimes \sigma_j) \hat{\rho}] = \langle \sigma_i^{(A)} \sigma_j^{(B)} \rangle_\rho \quad \longleftarrow \text{spin-spin correlation}$$

$$\hat{\rho} \geq 0 \Rightarrow B_i^2, \bar{B}_j^2, \|C\|_{\text{eig}}^{\max} \leq 1$$

$$\text{Tr} (\sigma_i \sigma_j) = 2 \delta_{ij} \quad 18$$



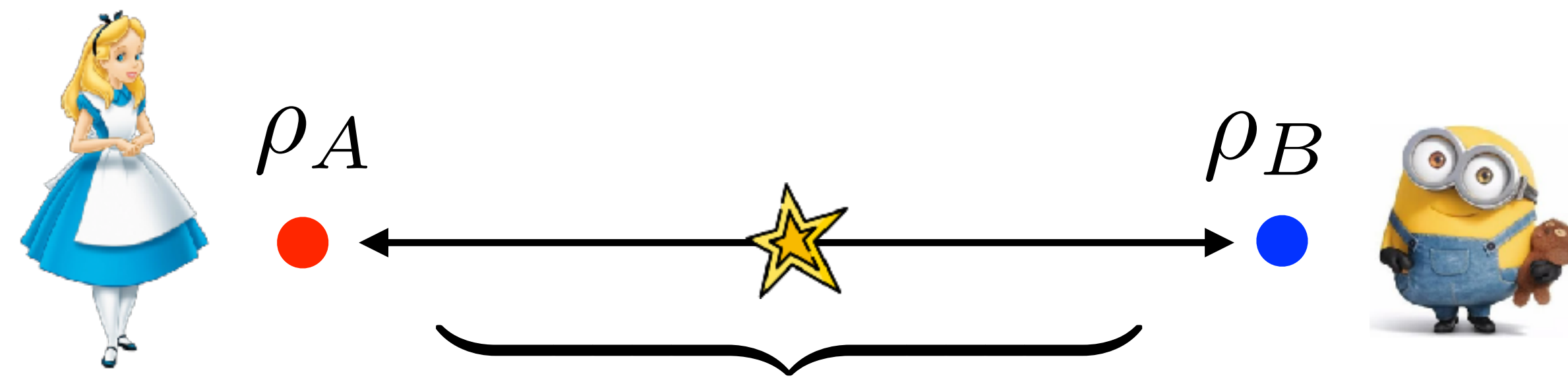
Alice's local (reduced) state ρ_A is defined such that

$$\langle O_A \rangle = \text{Tr} \left[(O_A \otimes \mathbb{1}) \cdot \rho_{AB} \right] = \text{Tr}_A \left[O_A \cdot \rho_A \right] \quad \forall O_A$$

$$= \sum_{k,\ell} \left[{}_A \langle \ell | \otimes {}_B \langle k | \right) \left[(O_A \otimes \mathbb{1}_B) \rho_{AB} \right] (| \ell \rangle_A \otimes | k \rangle_B \right]$$

$$= \sum_{\ell} {}_A \langle \ell | \left(O_A \sum_k {}_B \langle k | \rho_{AB} | k \rangle_B \right) | \ell \rangle_A = \text{Tr}_A \left[O_A \cdot \text{Tr}_B (\rho_{AB}) \right]$$

$$\rho_A = \text{Tr}_B [\rho_{AB}]$$



Pure singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B \right]$$

Alice's reduced state:

$$\begin{aligned} \rho_A &= \text{Tr}_B |\psi\rangle\langle\psi| = \frac{1}{2} \text{Tr}_B \left[\left(|01\rangle - |10\rangle \right) \left(\langle 01| - \langle 10| \right) \right] \\ &= \frac{1}{2} \text{Tr}_B \left[\underbrace{|01\rangle\langle 01|}_{\langle 1|1\rangle_B} - \underbrace{|01\rangle\langle 10|}_{\cancel{\langle 1|0\rangle_B}} - \underbrace{|10\rangle\langle 01|}_{\cancel{\langle 0|1\rangle_B}} + \underbrace{|10\rangle\langle 10|}_{\langle 0|0\rangle_B} \right] = \frac{1}{2} \left[|0\rangle\langle 0|_A + |1\rangle\langle 1|_A \right] = \rho_{\text{mix}}^{\text{max}} \end{aligned}$$

“The best possible knowledge of a whole does not include best possible knowledge of its parts”

Erwin Schrödinger

Entanglement

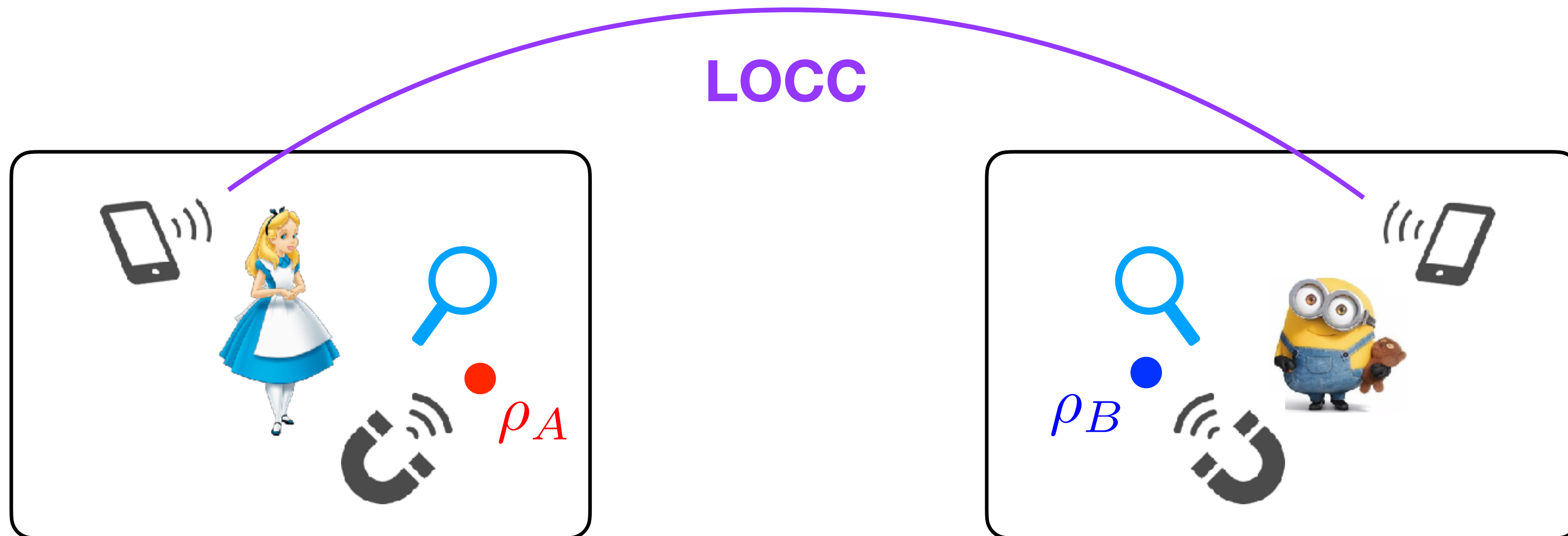
⌘ The state is called **separable** if it admits the expression

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

$$p_i \geq 0, \sum_i p_i = 1$$

otherwise it is called **entangled**.

- **L**ocal **O**peration and **C**lassical **C**ommunication does **not** generate entanglement



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- Entanglement implies a *past interaction* of the two systems:



$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B \right]$$

How can one check if the state is entangled?

$$\rho \neq \sum_i p_i \rho_i^A \otimes \rho_i^B$$

⌘ Peres-Horodecki criterion:

- Let $\{|i\rangle\}$ ($i = 1, \dots, d_A$) be a basis of $\mathcal{H}_A$ and $\{|\alpha\rangle\}$ ($\alpha = 1, \dots, d_B$) be a basis of $\mathcal{H}_B$,

$$[\rho]_{(i,\alpha),(j,\beta)} = \langle i | \langle \alpha | \hat{\rho} | j \rangle | \beta \rangle$$

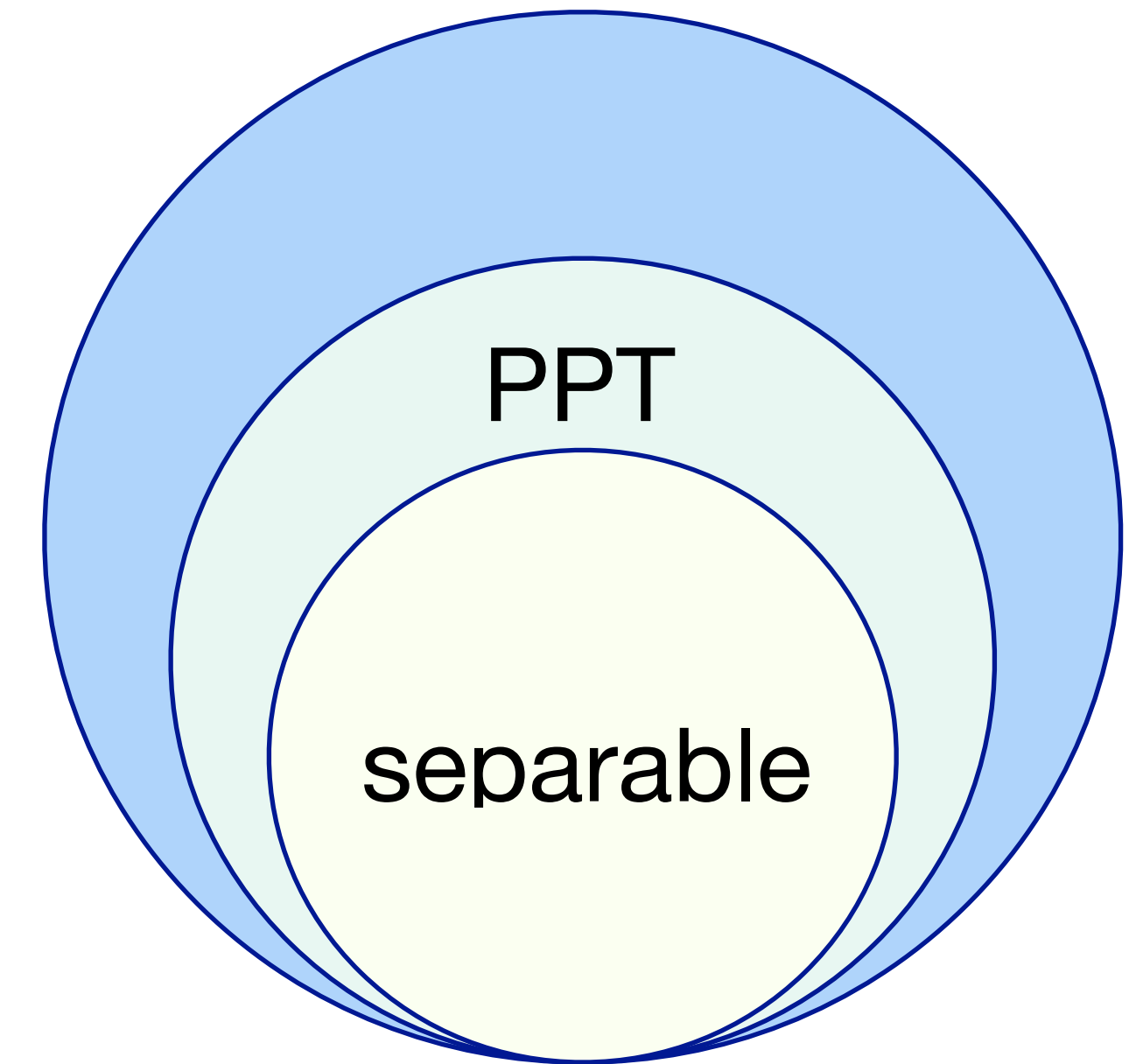
- Define the partial transpose of B as $\rho^{T_B} = [\rho]_{(i,\beta),(j,\alpha)}$
- If state is separable, $\rho^{T_B} = \sum_i p_i \rho_i^A \otimes (\rho_i^B)^T$ is Hermitian, Unit-Trace, PSD

If one finds a **negative eigenvalue** in ρ^{T_B} , the state is guaranteed to be **entangled**.

$\Rightarrow$ **necessary** condition, it is also **sufficient** for $(d_A, d_B) = (2,2), (2,3), (3,2)$.

⌘ If $\rho^{T_B} \geq 0$, the state ρ is called
“**Positive-Partial-Transpose (PPT)**”.

⌘ All separable states are PPT.



• If state is separable, $\rho^{T_B} = \sum_i p_i \rho_i^A \otimes (\rho_i^B)^T$ is Hermitian, Unit-Trace, PSD

If one finds a **negative eigenvalue** in ρ^{T_B} , the state is guaranteed to be **entangled**.

⇒ **necessary** condition, it is also **sufficient** for $(d_A, d_B) = (2,2), (2,3), (3,2)$.

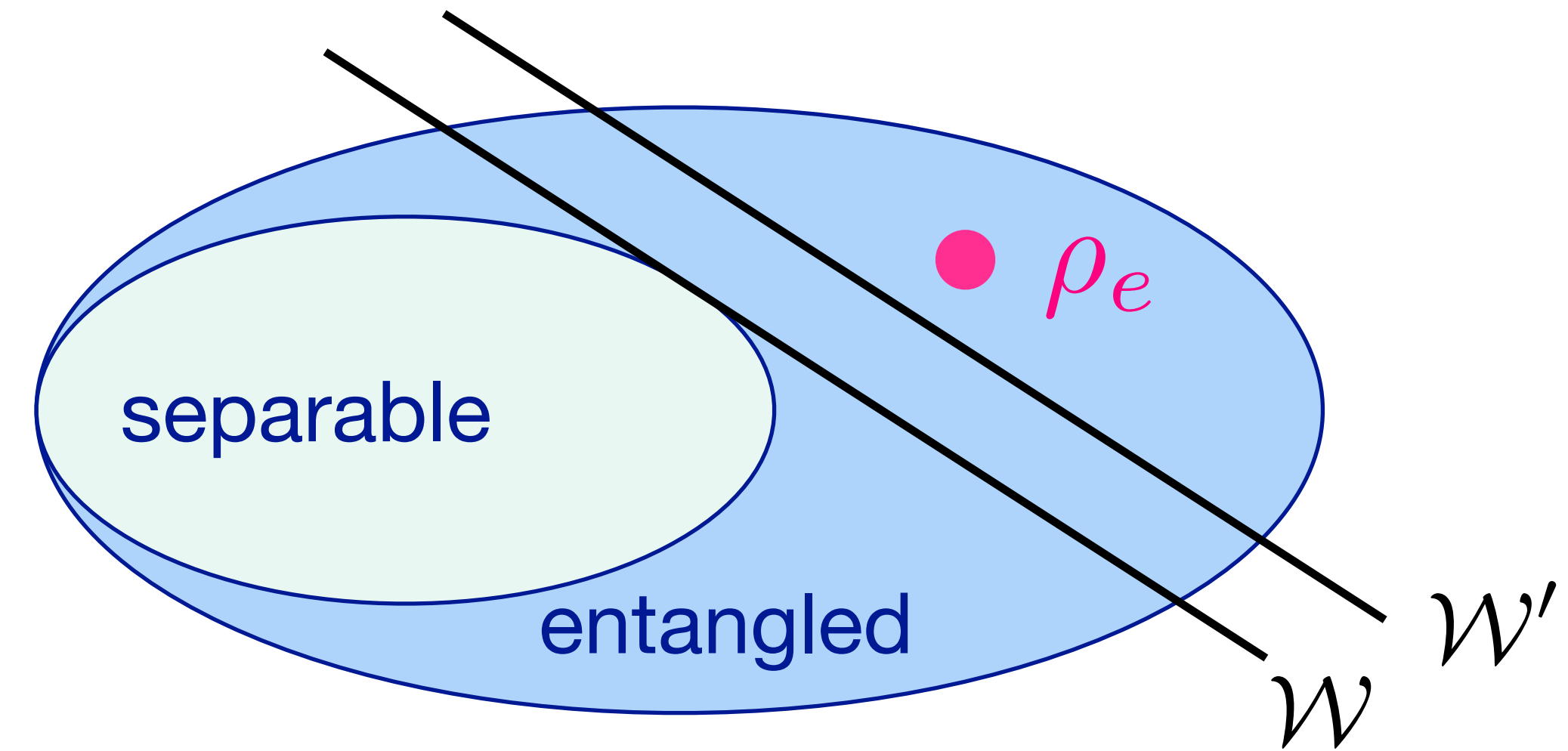
The Peres-Horodecki criterion assumes we know the density matrix ρ .
In practice, the reconstruction of the state, i.e. 15 Fano coefficients, is expensive.

Entanglement Witness:

The Hermitian $\mathcal{W}$ is called “*Witness*” if

$$\text{Tr}[\mathcal{W} \cdot \rho_{\text{sep}}] \geq 0, \quad \forall \rho_{\text{sep}}$$

$$\text{Tr}[\mathcal{W} \cdot \rho_e] < 0, \quad \text{for some entangled } \rho_e$$



Example: Pick a target entangled pure state $|\psi_e\rangle$, and define

$$\mathcal{W} = \alpha \mathbb{1} - |\psi_e\rangle\langle\psi_e| \quad \text{with} \quad \alpha = \max_{\text{all separable } |\phi\rangle} |\langle\phi|\psi_e\rangle|^2 < 1$$

Quantification of Entanglement:

⌘ LOCC breaks entanglement.

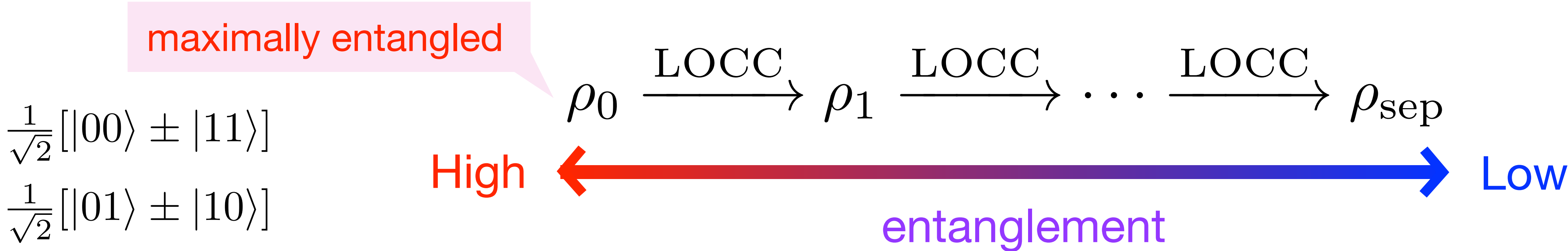
$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B \right]$$

Alice measures her qubit and finds 0

$|0\rangle_A \otimes |1\rangle_B$ (separable state)



⌘ LOCC introduces a partial ordering on quantum states.



A non-negative function $E(\rho) \geq 0$ is called “**entanglement measure (monotone)**” if

① Zero on separable states:

$$E(\rho_{\text{sep}}) = 0, \quad \forall \rho_{\text{sep}}$$

② Non-increasing under LOCC (monotonicity):

$$E(\Lambda_{\text{LOCC}}(\rho)) \leq E(\rho)$$

- $E(\rho)$ is called “**faithful**” if $E(\rho_e) > 0$ for all non-separable (entangled) states.
- LOCC $\ni$ **L**ocal **U**nitaries, $U_A \otimes U_B$, which are invertible.
- ② $\Rightarrow E(\rho)$ is invariant under **LU**

Entanglement measures for *pure* states:

⌘ If pure state is separable, the reduced state is pure:

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \longrightarrow \rho_A = \text{Tr}_B |\psi\rangle\langle\psi| = |\psi_A\rangle\langle\psi_A| \cdot \langle\psi_B|\psi_B\rangle = |\psi_A\rangle\langle\psi_A|$$

⌘ ENT = “*how much one subsystem becomes mixed when the other is traced out*”

Concurrence:

$$C(|\psi\rangle) \equiv \sqrt{2(1 - \text{Tr} \rho_A^2)}$$

Entanglement entropy:

$$E_F(|\psi\rangle) \equiv S_V(\rho_A) = -\text{Tr} [\rho_A \log_2 \rho_A]$$

Entanglement measures for *mixed* states:

⌘ Entanglement measure for the mixed state is defined by the **convex-roof**:

$$E(\rho) \equiv \min_{\{p_i, \psi_i\}} \sum_i p_i E(|\psi_i\rangle)$$

$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$$

$$p_i \geq 0, \sum_i p_i = 1$$

$$C(\rho) \equiv \min_{\{p_i, \psi_i\}} \sum_i p_i C(|\psi_i\rangle) = \min_{\{p_i, \psi_i\}} \sum_i p_i \sqrt{2(1 - \text{Tr}(\rho_A^i)^2)}$$

$$\rho_A^i = \text{Tr}_B |\psi_i\rangle \langle \psi_i|$$

$$E_f(\rho) \equiv \min_{\{p_i, \psi_i\}} \sum_i p_i E_f(|\psi_i\rangle) = \min_{\{p_i, \psi_i\}} \sum_i p_i S_V(\rho_A^i)$$

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$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$$
$$p_i \geq 0, \sum_i p_i = 1$$

⌘ For the **2-qubit** system, the **concurrence** has an **analytical formula**:

$$C(\rho) \equiv \min_{\{p_i, \psi_i\}} \sum_i p_i C(|\psi_i\rangle) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \quad \text{Wootters' (1998)}$$

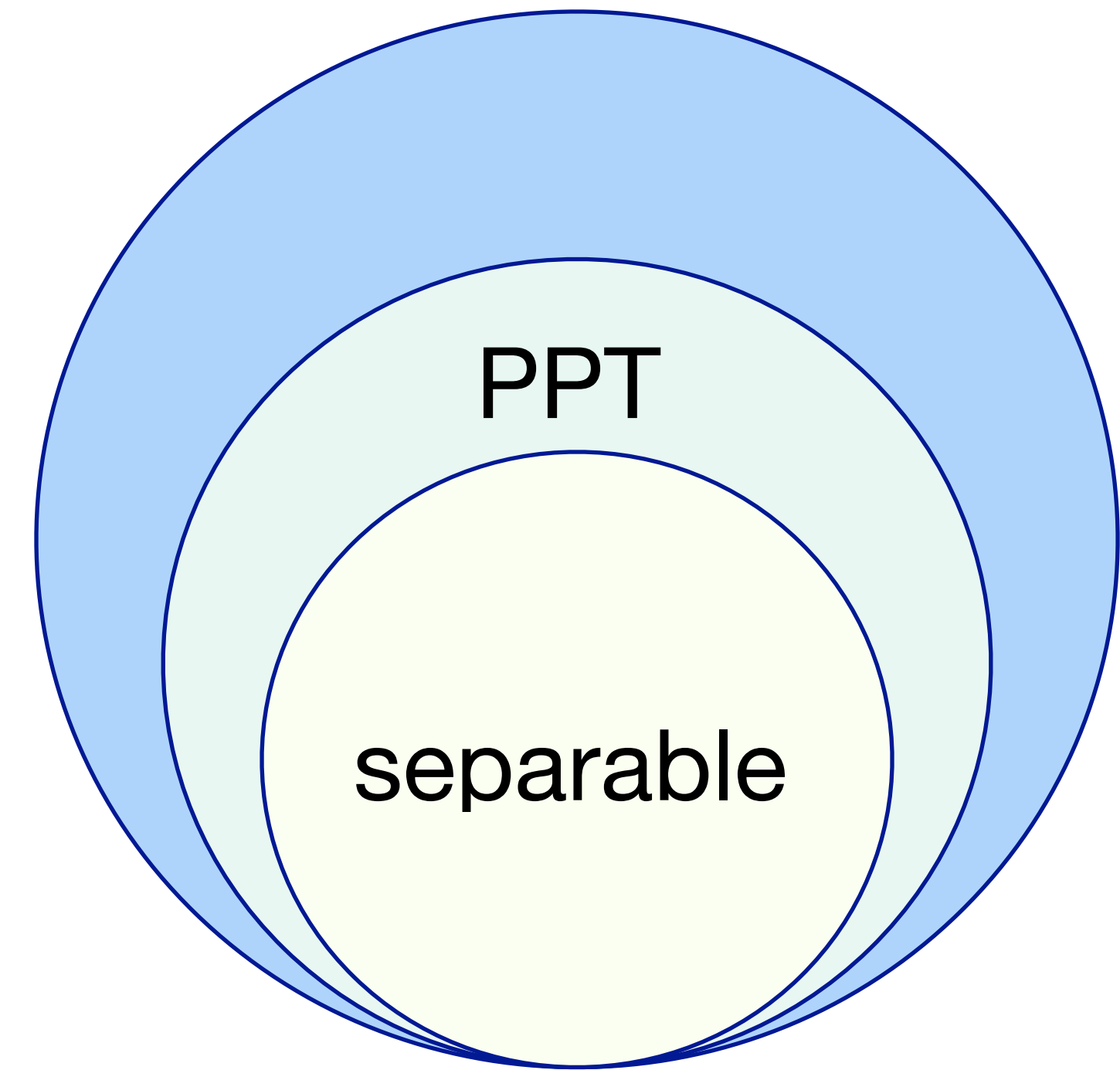
$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of $\sqrt{\rho \tilde{\rho}}$ with $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$.

- ⌘ A quantum state is called “**positive-partial-transpose (PPT)**” if $\rho^{T_B} \geq 0$ (PSD).
- ⌘ **All separable states are PPT.** The converse is true for 2×2 and 2×3 systems.

Negativity:

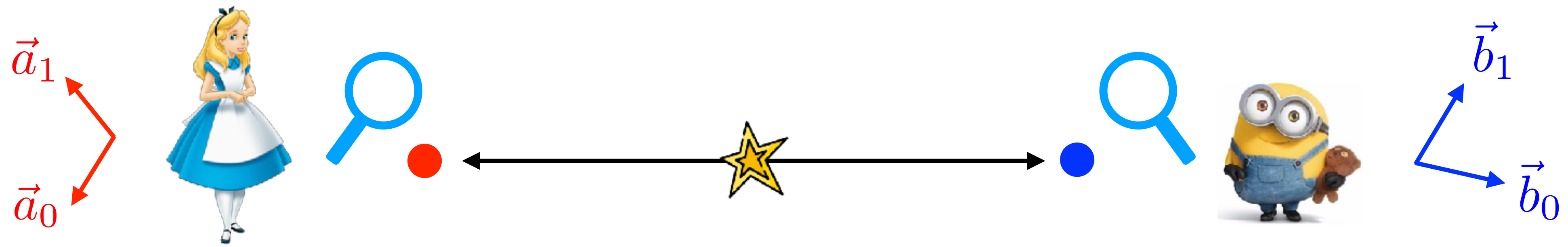
$$N(\rho) = \sum_{\mu; \lambda_{\mu} < 0} |\lambda_{\mu}|$$

λ_{μ} are the eigenvalues of ρ^{T_B}



- ⌘ Negativity is a **faithful** ENT measure only for **2×2** and **2×3** systems.

Bell Nonlocality



Bell's Experiment:

Alice and Bob have two spin-measurement axes to choose at each measurement. They repeat the measurement for many copies of the identical spin pair (state).

The result will look like:

Axes (x, y)	$(0, 1)$	$(1, 1)$	$(0, 0)$	$(1, 0)$	$(1, 1)$	$(1, 0)$	$(0, 1)$	$(0, 0)$	$\dots$
(A_x, B_y)	$(+, -)$	$(-, -)$	$(+, +)$	$(-, +)$	$(+, -)$	$(+, +)$	$(-, -)$	$(-, +)$	$\dots$

They compute the correlations:

$$\langle A_x B_y \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N A_x^{(k)} B_y^{(k)}$$

- They measure the following combination of correlations:

$$I(\{\vec{a}, \vec{b}\}) \equiv |\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle|$$



$$\langle A_x B_y \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N A_x^{(k)} B_y^{(k)}$$

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$$I(\{\vec{a}, \vec{b}\}) \equiv |\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle|$$

$$= |\langle A_0(B_0 + B_1) + A_1(B_0 - B_1) \rangle|$$

outcomes of $A_{0,1}$ are independent of Bob's choice b_0 or b_1 (**locality**)



$$\langle A_x B_y \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N A_x^{(k)} B_y^{(k)}$$

- They measure the following combination of correlations:

$$I(\{\vec{a}, \vec{b}\}) \equiv |\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle|$$

$$= |\langle A_0(B_0 + B_1) + A_1(B_0 - B_1) \rangle|$$

$$\leq 2$$

outcomes of $A_{0,1}$ are independent of Bob's choice b_0 or b_1 (**locality**)

all outcomes are predetermined before measurements (**reality**)



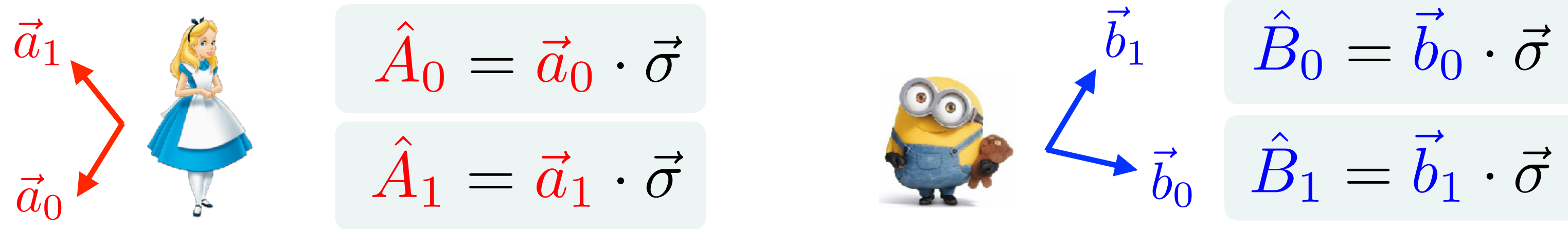
$$\mathcal{B}_{\text{CHSH}} \equiv \max_{\vec{a}, \vec{b}} I(\{\vec{a}, \vec{b}\}) \leq 2$$

CHSH inequality for Local Realism

Bell (1964), Clauser, Horne, Shimony, Holt (1969)

$$\langle A_x B_y \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N A_x^{(k)} B_y^{(k)}$$

- In **QM**, the observables are promoted to the operators:



The diagram illustrates the promotion of observables to operators in Quantum Mechanics. On the left, Alice is associated with red vectors $\vec{a}_0$ and $\vec{a}_1$. Next to her are two equations: $\hat{A}_0 = \vec{a}_0 \cdot \vec{\sigma}$ and $\hat{A}_1 = \vec{a}_1 \cdot \vec{\sigma}$. On the right, a Minion is associated with blue vectors $\vec{b}_0$ and $\vec{b}_1$. Next to him are two equations: $\hat{B}_0 = \vec{b}_0 \cdot \vec{\sigma}$ and $\hat{B}_1 = \vec{b}_1 \cdot \vec{\sigma}$.

$$\hat{I}(\{\vec{a}, \vec{b}\}) = \hat{A}_0 \hat{B}_0 + \hat{A}_0 \hat{B}_1 + \hat{A}_1 \hat{B}_0 - \hat{A}_1 \hat{B}_1$$

$$\mathcal{B}_{\text{CHSH}}(\rho) = \max_{\{\vec{a}, \vec{b}\}} \text{Tr}[\hat{I}(\{\vec{a}, \vec{b}\}) \cdot \rho] = 2\sqrt{\mu_1 + \mu_2}$$

Horodecki, Horodecki,
Horodecki (1995)

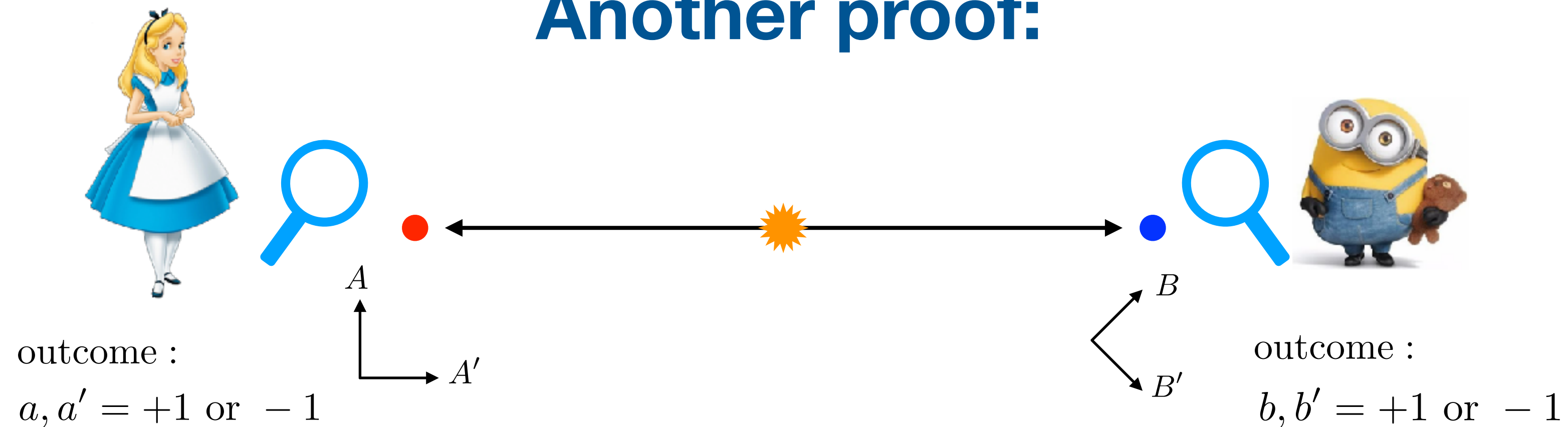
$\mu_1 \geq \mu_2 \geq \mu_3$ are the eigenvalues of $M = C^T C$, where $C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle_\rho = \text{Tr}[(\sigma_i \otimes \sigma_j) \cdot \rho]$

$0 \leq \mu_i \leq 1 \implies$ In **QM**, $\mathcal{B}_{\text{CHSH}}$ can go up to $2\sqrt{2}$



[J.F.Clauser, S.J.Freedman (1972)], [A.Aspect, J.Dalibard, G.Roger,(1982)], ...₄₅

Another proof:



$$\hat{\mathcal{B}} = |\hat{A}\hat{B} + \hat{A}'\hat{B} + \hat{A}\hat{B}' - \hat{A}'\hat{B}'|$$

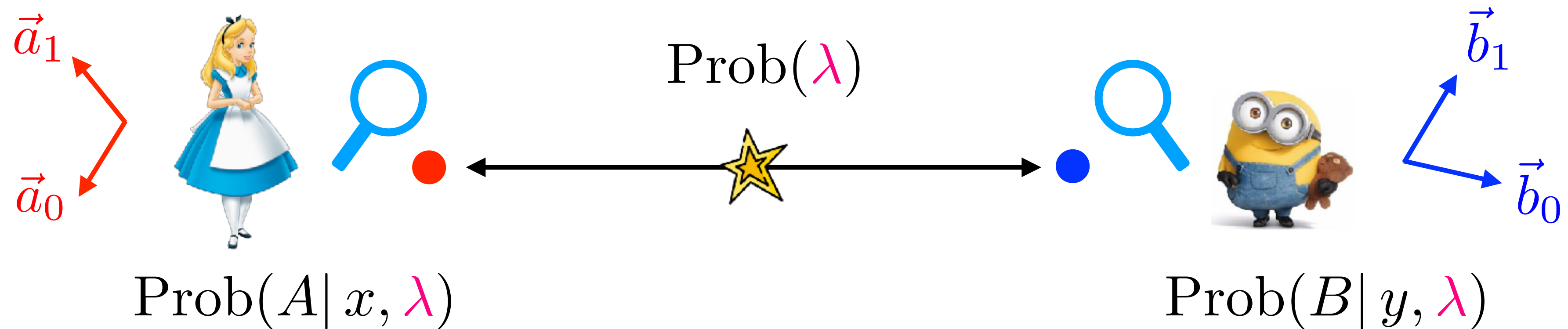
• In **Quantum Mechanics**,

$$\hat{\mathcal{B}}^2 = 4 - [\hat{A}, \hat{A}'][\hat{B}, \hat{B}'] \quad |[\hat{A}, \hat{A}']|, |[\hat{B}, \hat{B}']| \leq 2 \quad \leftarrow [\sigma_x, \sigma_y] = 2i\sigma_z$$

$$\Rightarrow \langle \mathcal{B}^2 \rangle_{\text{QM}} \leq 8 \quad \Rightarrow \quad \langle \mathcal{B} \rangle_{\text{QM}} \leq 2\sqrt{2} \quad \text{Tsirelson (1980)}$$

If this bound is violated, *QM will be excluded!*

Local Hidden Variable (LHV) Theory:



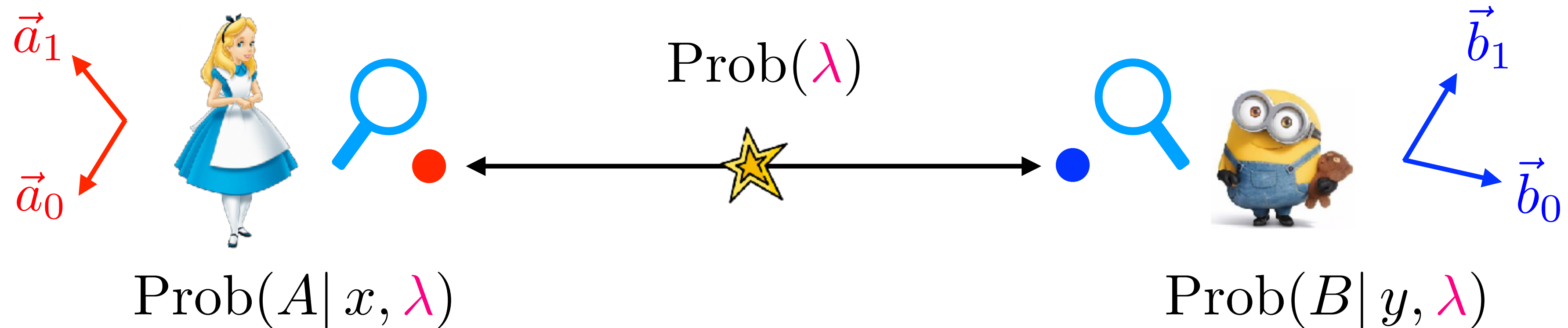
⌘ Joint conditional probability

LHV: $\text{Prob}(A, B|x, y) = \sum_{\lambda} \text{Prob}(\lambda) \cdot \text{Prob}(A|x, \lambda) \cdot \text{Prob}(B|y, \lambda)$

QM: $\text{Prob}(A, B|x, y) = \text{Tr}[(\hat{\Pi}_x \otimes \hat{\Pi}_y) \cdot \hat{\rho}]$

$$\hat{\Pi}_x = |a_x\rangle\langle a_x|$$

Local Hidden Variable (LHV) Theory:



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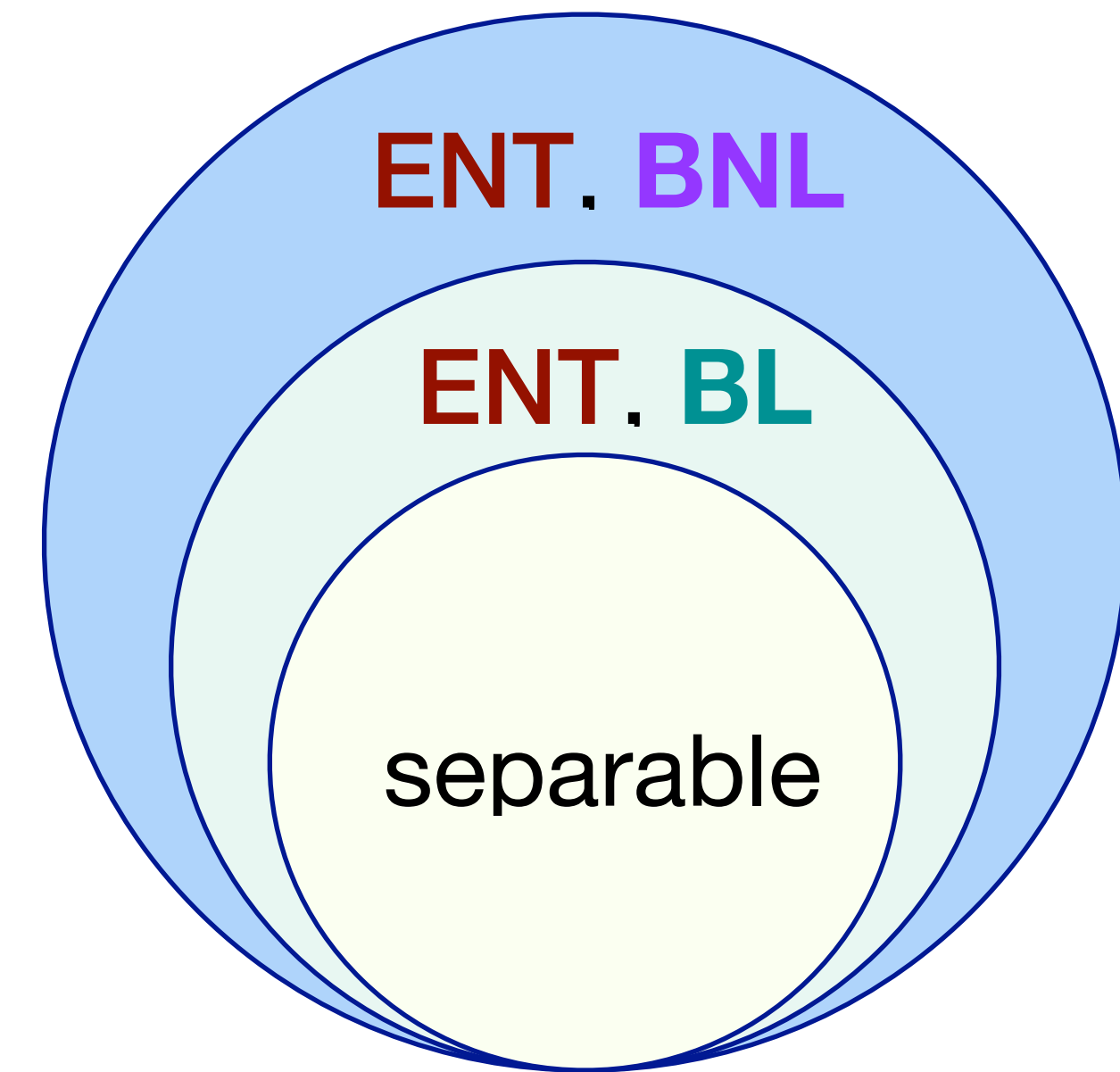
$\hat{\rho}_{\text{sep}} = \sum_k p_k \hat{\rho}_k^A \otimes \hat{\rho}_k^B$

$= \sum_k q_k \cdot \text{Tr}[\hat{\Pi}_x \hat{\rho}_k^A] \cdot \text{Tr}[\hat{\Pi}_y \hat{\rho}_k^B]$

➔ The results of separable states can be reproduced by LHV theories.

➔ **Separable** quantum states *cannot* violate **LHVT bound**.

$$\mathcal{B}_{\text{CHSH}}(\hat{\rho}_{\text{sep}}) \leq 2$$



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QM: $\text{Prob}(A, B | x, y) = \text{Tr}[(\hat{\Pi}_x \otimes \hat{\Pi}_y) \cdot \hat{\rho}]$

$\hat{\Pi}_x = |a_x\rangle\langle a_x|$

$$= \sum_k q_k \cdot \text{Tr}[\hat{\Pi}_x \hat{\rho}_k^A] \cdot \text{Tr}[\hat{\Pi}_y \hat{\rho}_k^B]$$

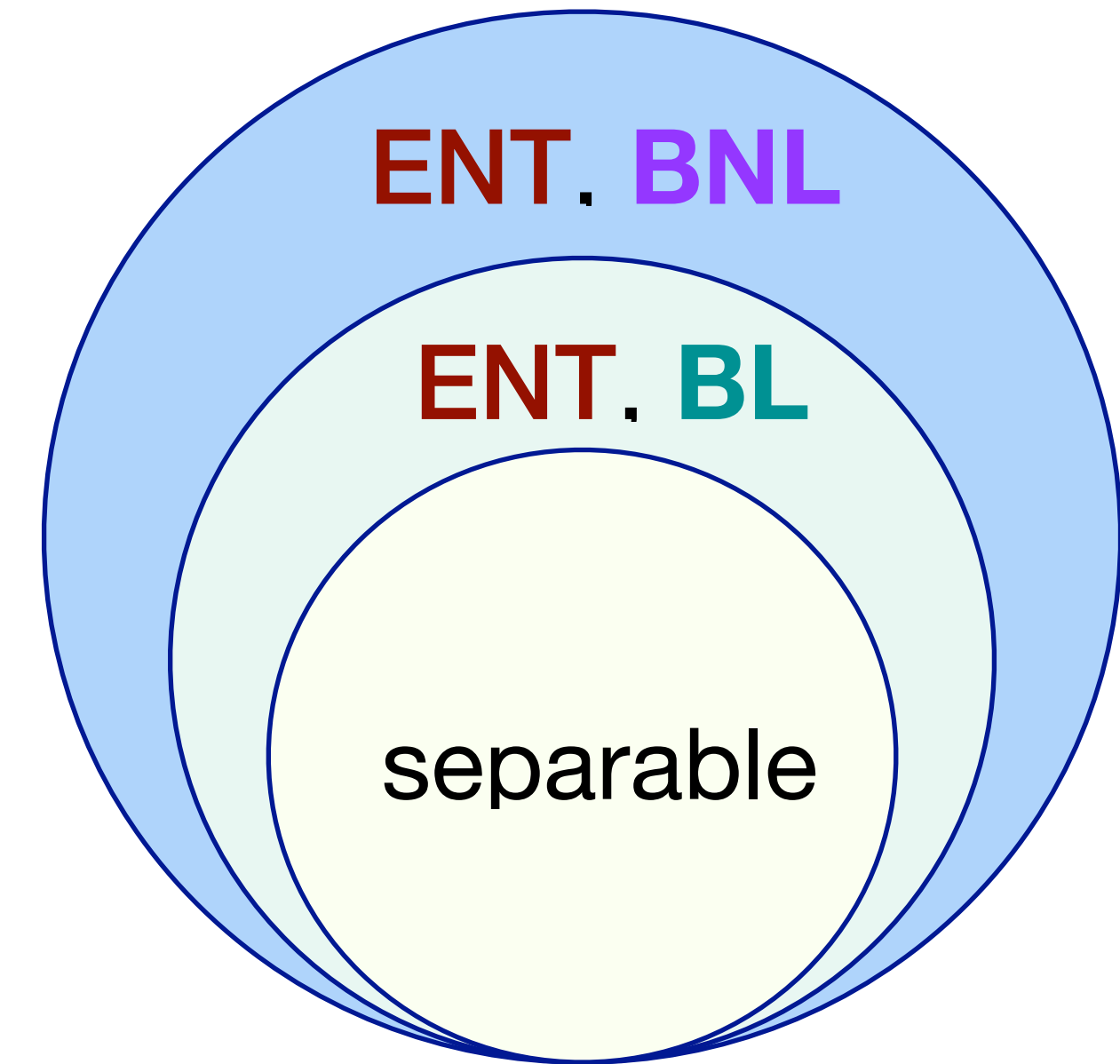
$$\hat{\rho}_{\text{sep}} = \sum_k p_k \hat{\rho}_k^A \otimes \hat{\rho}_k^B$$

- ➔ The results of separable states can be reproduced by LHV theories.
- ➔ **Separable** quantum states *cannot* violate **LHVT** bound.

$$\mathcal{B}_{\text{CHSH}}(\hat{\rho}_{\text{sep}}) \leq 2$$

Bell Nonlocal state

$$\mathcal{B}_{\text{CHSH}}(\rho_{\text{BNL}}) > 2$$

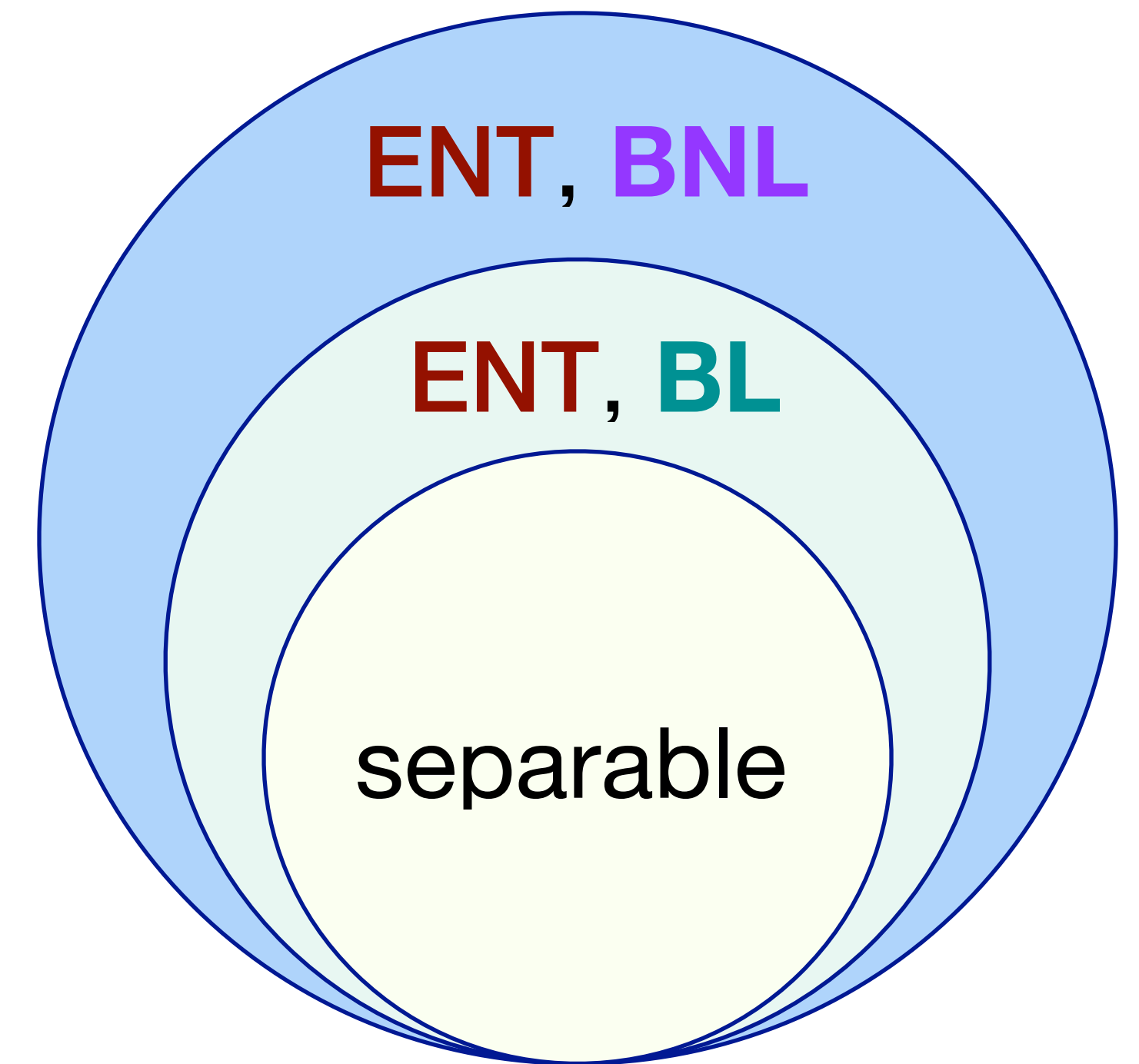
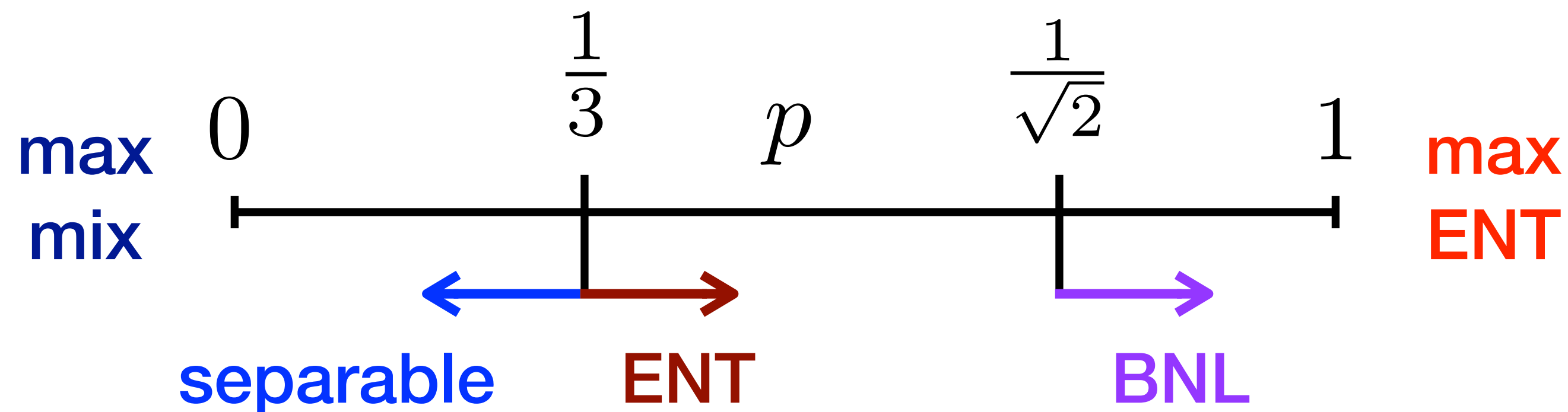


⌘ For mixed states, **entanglement** *does not* imply **Bell nonlocality**.

⌘ Werner state:

$$\rho(p) = p |\psi^-\rangle \langle \psi^-| + (1-p) \frac{\mathbb{1}}{4} \quad (0 \leq p \leq 1)$$

$$\text{with } |\psi^-\rangle = \frac{1}{\sqrt{2}} [|01\rangle - |10\rangle]$$



ENT: Entangled

BL: Bell Local ($\mathcal{B}_{\text{CHSH}} \leq 2$)

BNL: Bell Non-Local ($\mathcal{B}_{\text{CHSH}} > 2$)

- Is there conceivable distributions that **violate** the **QM bound**?

▶ We change the parameters to denote the outcomes:

$$A = \pm 1 = (-1)^a \quad B = \pm 1 = (-1)^b \quad a, b \in \{0, 1\}$$

▶ Popescu–Rohrlich box distribution:

$$\mathcal{P}_{\text{PR}}(a, b | x, y) = \begin{cases} \frac{1}{2}, & a \oplus b = xy, \\ 0, & \text{otherwise.} \end{cases}$$

Popescu, Rohrlich (1994)

▶ CHSH prediction:

$$\begin{aligned} \mathcal{B}_{\text{CHSH}} &= |\langle \hat{A}_0 \hat{B}_0 \rangle + \langle \hat{A}_0 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_0 \rangle - \langle \hat{A}_1 \hat{B}_1 \rangle| \\ &= 1 + 1 + 1 - (-1) = 4 \quad (\text{PR-box}) \quad \text{violating the QM bound } 2\sqrt{2} ! \end{aligned}$$

(x, y)	Possible (a, b) , each with probability 1/2
(0, 0)	(0, 0), (1, 1)
(0, 1)	(0, 0), (1, 1)
(1, 0)	(0, 0), (1, 1)
(1, 1)	(0, 1), (1, 0)

correlation

anti-correlation

- PR-box distribution is not so exotic:

$$\mathcal{P}_{\text{PR}}(a|x, y) = \sum_{b=0,1} \mathcal{P}_{\text{PR}}(a, b|x, y) = \frac{1}{2}$$

$$\mathcal{P}_{\text{PR}}(a, b|x, y) = \begin{cases} \frac{1}{2}, & a \oplus b = xy, \\ 0, & \text{otherwise.} \end{cases}$$

- Alice's local distribution is **independent of Bob's choice, y** .
- No information travels faster than the speed of light; **Causality OK**.

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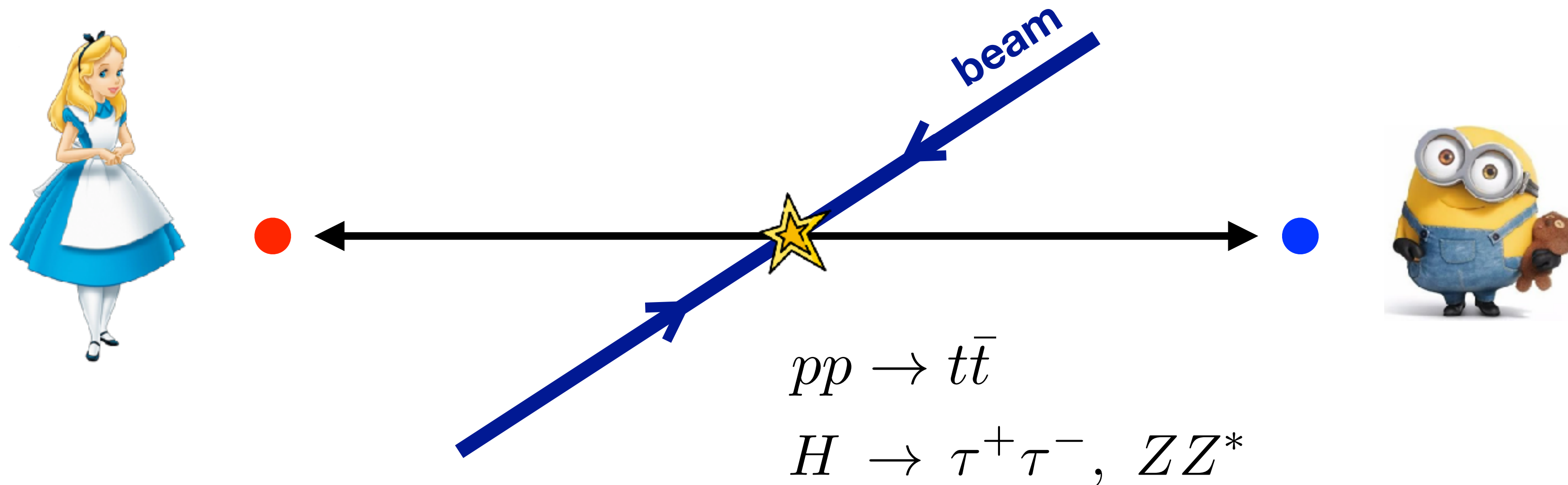
$$\mathcal{B}_{\text{CHSH}} \leq \begin{cases} 2 & \text{LHV, } \rho_{\text{sep}} \\ 2\sqrt{2} & \text{QM} \\ 4 & \text{beyond QM} \end{cases}$$

Bell's experiment can in principle **test** and **exclude QM!**

QI measurements @ Colliders

— $t\bar{t}$ focused —

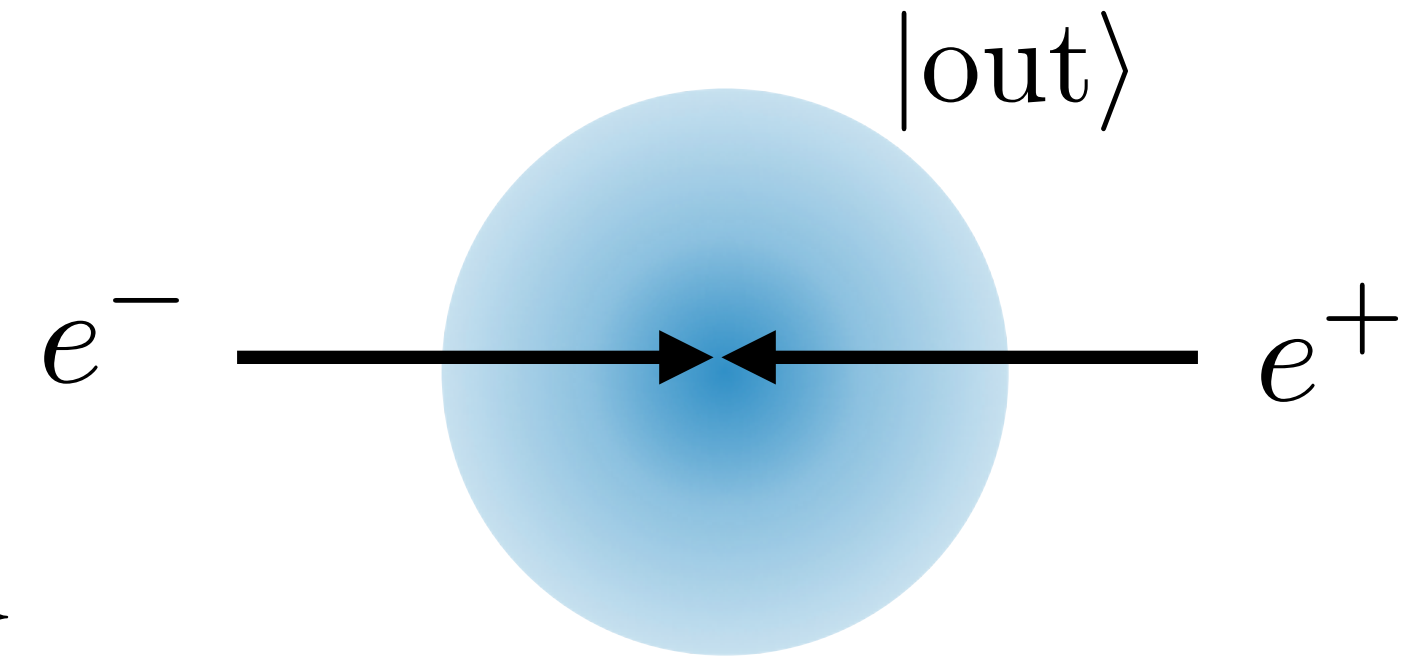
Can we measure various quantum properties at collider?



- Consider the collision of e^-e^+ with fixed polarisation:

initial data: $\{i\} = \{(p_{e^-}, \lambda_{e^-}), (p_{e^+}, \lambda_{e^+})\}$

possible final data: $\{f\} = \{(\eta_1, p_1, \lambda_1), (\eta_2, p_2, \lambda_2), \dots\}$



S-matrix:

$$S_{f,i} = \lim_{t \rightarrow \infty} \langle \{f\} | e^{-iHt} | \{i\} \rangle$$

- The **out-state** is a gigantic **superposition of all possible final states** (**pure**):

$$\begin{aligned} \sum_{\{f\}} |\{f\}\rangle \langle \{f\}| &= 1 & |\text{out}\rangle &\propto e^{-iHt} |\{i\}\rangle = \sum_{\{f\}} S_{f,i} |\{f\}\rangle \\ & & &= \mathcal{M}_{t\bar{t}} |t\bar{t}\rangle + \mathcal{M}_{\mu^+\mu^-} |\mu^+\mu^-\rangle + \mathcal{M}_{W^+W^-\gamma} |W^+W^-\gamma\rangle + \dots \end{aligned}$$

- If the detector registers the final state momenta consistent with $t\bar{t}$, we *project*

$$|\text{out}\rangle \propto \mathcal{M}_{t\bar{t}} |t\bar{t}\rangle + \mathcal{M}_{\mu^+\mu^-} |\mu^+\mu^-\rangle + \mathcal{M}_{W^+W^-\gamma} |W^+W^-\gamma\rangle + \dots$$

collapse
→

$$|\psi_{t\bar{t}}^{\text{spin}}\rangle = \sum_{\lambda_t, \lambda_{\bar{t}}} \mathcal{M}_{\lambda_t \lambda_{\bar{t}}}(\{p_f\}) |\lambda_t, \lambda_{\bar{t}}\rangle \in \mathbb{C}_t^2 \otimes \mathbb{C}_{\bar{t}}^2 \quad \mathcal{M}_{\lambda_t \lambda_{\bar{t}}} = \mathcal{M}_{\lambda_t \lambda_{\bar{t}}}^{\lambda_{e-}, \lambda_{e+}}(\{p_f\})$$

$$\rho^{\lambda_{e-}, \lambda_{e+}} = \frac{R}{\text{Tr}_{t\bar{t}}[R]} \quad R_{(\lambda_t, \lambda_{\bar{t}}), (\lambda'_t, \lambda'_{\bar{t}})}^{\lambda_{e-}, \lambda_{e+}} = \mathcal{M}_{\lambda_t, \lambda_{\bar{t}}} \mathcal{M}_{\lambda'_t, \lambda'_{\bar{t}}}^*$$

- If e^-e^+ beams are unpolarised,

$$\hat{\rho} = \sum_i q_i |\psi_i\rangle \langle \psi_i|$$

$$R^{\text{unpol}} = \sum_{\lambda_{e-}, \lambda_{e+}} \frac{1}{4} \cdot R^{\lambda_{e-}, \lambda_{e+}} = \frac{1}{4} [R^{++} + R^{+-} + R^{-+} + R^{--}]$$

mixed state

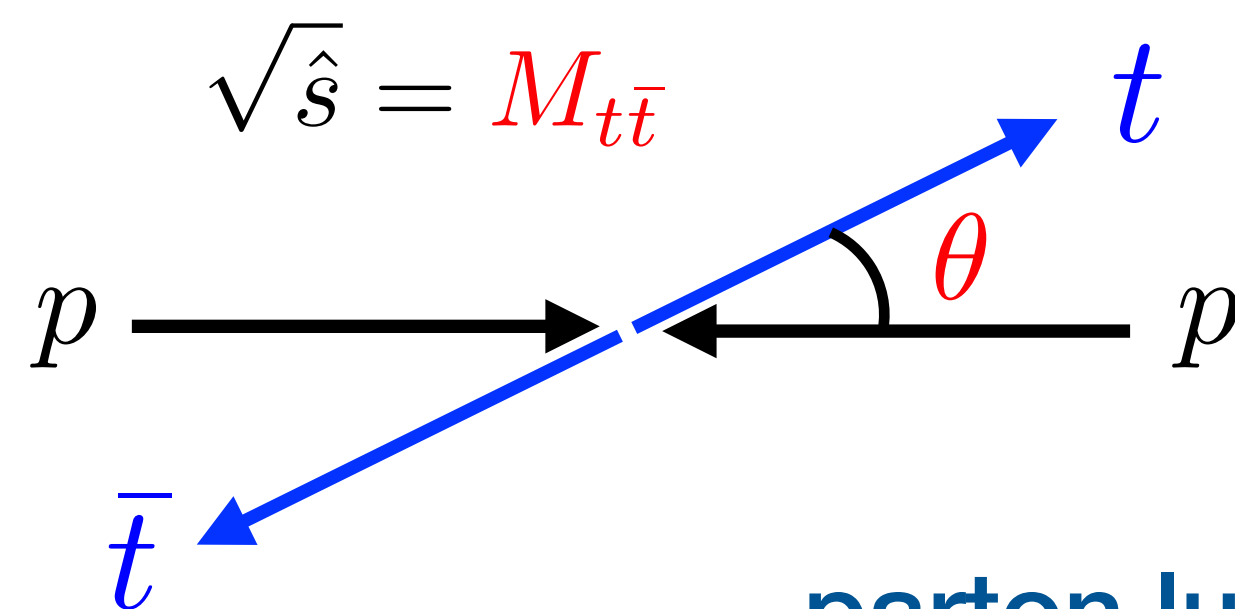
At a **hadron collider**, we also average over the **partonic initial states**:

Afik, Nova [2003.02280]

$$R_{ij} = \sum_{\text{parton pol.}}^{++,+-,-+,-} \frac{1}{4} \cdot \mathcal{M}_{ij \rightarrow t\bar{t}} \mathcal{M}_{ij \rightarrow t\bar{t}}^*$$

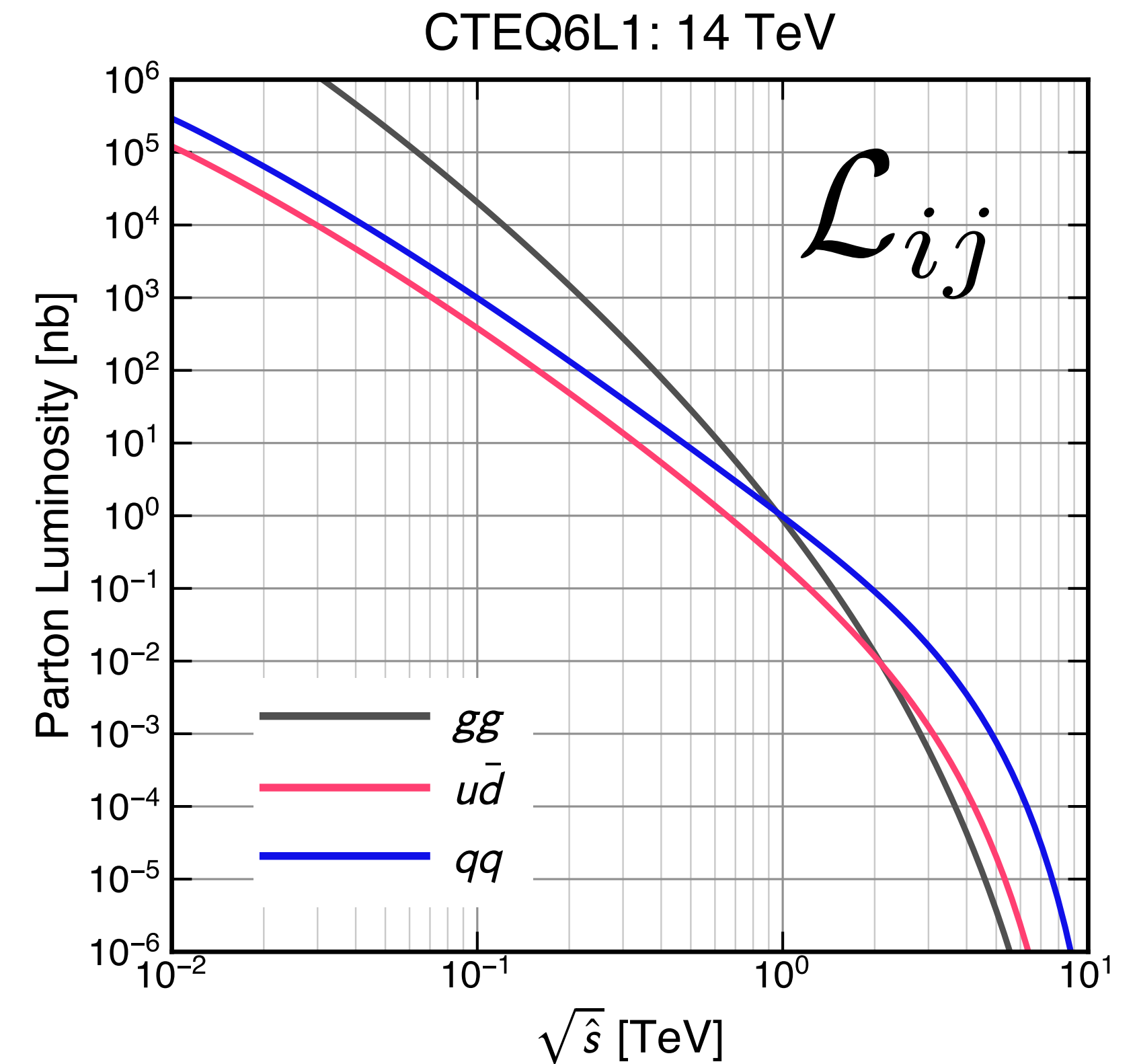
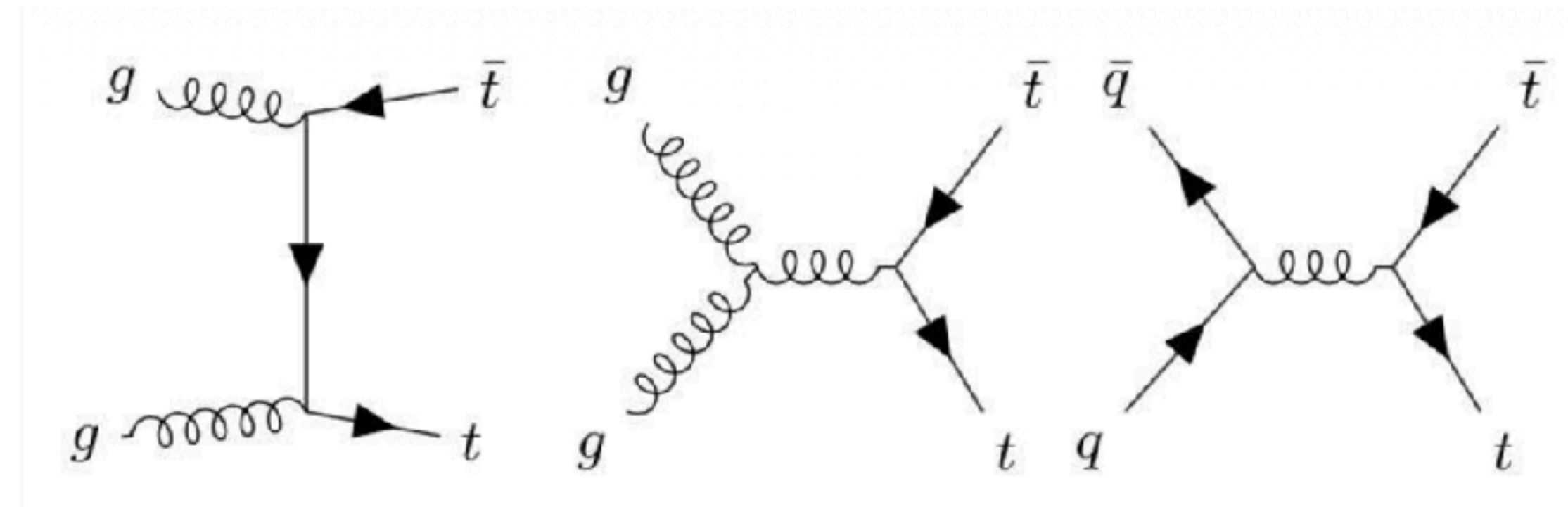
$$R(M_{t\bar{t}}, \theta) = \sum_{ij=gg, q\bar{q}} \mathcal{L}_{ij}(M_{t\bar{t}}) \cdot R_{ij}(M_{t\bar{t}}, \theta)$$

$$\rho(M_{t\bar{t}}, \theta) = R / \text{Tr } R$$



parton luminosity function:

$$\frac{d\mathcal{L}_{ij}}{d\hat{s}} = \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \delta(\hat{s} - sx_1x_2)$$



$$\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i (\sigma_i \otimes \mathbb{1}_2) + \bar{B}_j (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right)$$

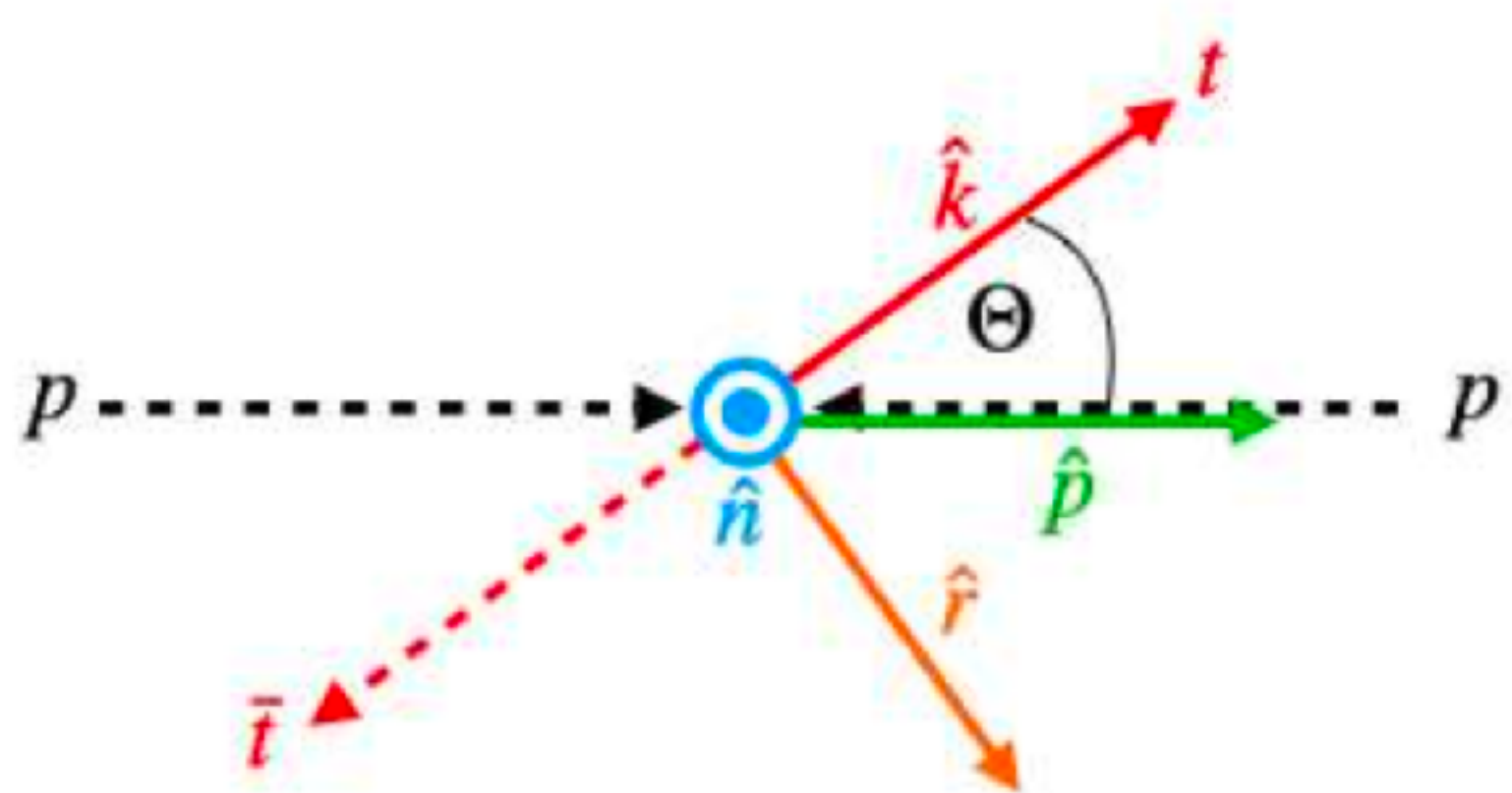
CP invariance: $C = C^T, B_i = \bar{B}_i \quad (t \leftrightarrow \bar{t})$

Reflection w.r.t $\hat{n}$: $B_r = B_k = C_{rn} = C_{kn} = 0$

LO QCD: $B_n = 0$

$$\left\{ \begin{array}{l} \sigma_n \rightarrow +\sigma_n \\ \sigma_r \rightarrow -\sigma_r \\ \sigma_k \rightarrow -\sigma_k \end{array} \right.$$

$(\hat{n}, \hat{r}, \hat{k})$ helicity basis



$$B_i = \bar{B}_i = 0 \quad (i = n, r, k)$$

$$C = \begin{pmatrix} C_{nn} & 0 & 0 \\ 0 & C_{rr} & C_{rk} \\ 0 & * & C_{kk} \end{pmatrix}$$

$$R = \tilde{A}I_4 + \sum_i \left(\tilde{B}_i^+ \sigma^i \otimes I_2 + \tilde{B}_i^- I_2 \otimes \sigma^i \right) + \sum_{i,j} \tilde{C}_{ij} \sigma^i \otimes \sigma^j$$

$$\tilde{A}^{q\bar{q}} = F_q(2 - \beta^2 \sin^2 \Theta)$$

$$\tilde{C}_{rr}^{q\bar{q}} = F_q(2 - \beta^2) \sin^2 \Theta$$

$$\tilde{C}_{nn}^{q\bar{q}} = -F_q \beta^2 \sin^2 \Theta$$

$$\tilde{C}_{kk}^{q\bar{q}} = F_q(2 \cos^2 \Theta + \beta^2 \sin^2 \Theta)$$

$$\tilde{C}_{rk}^{q\bar{q}} = \tilde{C}_{kr}^{q\bar{q}} = F_q \sqrt{1 - \beta^2} \sin 2\Theta$$

$$\tilde{A}^{gg} = F_g(\Theta) [1 + 2\beta^2 \sin^2 \Theta - \beta^4(1 + \sin^4 \Theta)]$$

$$\tilde{C}_{rr}^{gg} = -F_g(\Theta) [1 - \beta^2(2 - \beta^2)(1 + \sin^4 \Theta)]$$

$$\tilde{C}_{nn}^{gg} = -F_g(\Theta) [1 - 2\beta^2 + \beta^4(1 + \sin^4 \Theta)]$$

$$\tilde{C}_{kk}^{gg} = -F_g(\Theta) \left[1 - \beta^2 \frac{\sin^2 2\Theta}{2} - \beta^4(1 + \sin^4 \Theta) \right]$$

$$\tilde{C}_{rk}^{gg} = F_g(\Theta) \sqrt{1 - \beta^2} \beta^2 \sin 2\Theta \sin^2 \Theta$$

$$F_q = \frac{1}{18}, \quad F_g(\Theta) = \frac{7 + 9\beta^2 \cos^2 \Theta}{192(1 - \beta^2 \cos^2 \Theta)^2}$$

$$\beta = \sqrt{1 - 4m_t^2/M_{t\bar{t}}^2},$$

Entanglement

Wootters formula for the concurrence (2-qubit):

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

$$B_i = \overline{B}_i = 0 \quad (i = n, r, k)$$

$$C = \begin{pmatrix} C_{nn} & 0 & 0 \\ 0 & C_{rr} & C_{rk} \\ 0 & * & C_{kk} \end{pmatrix}$$

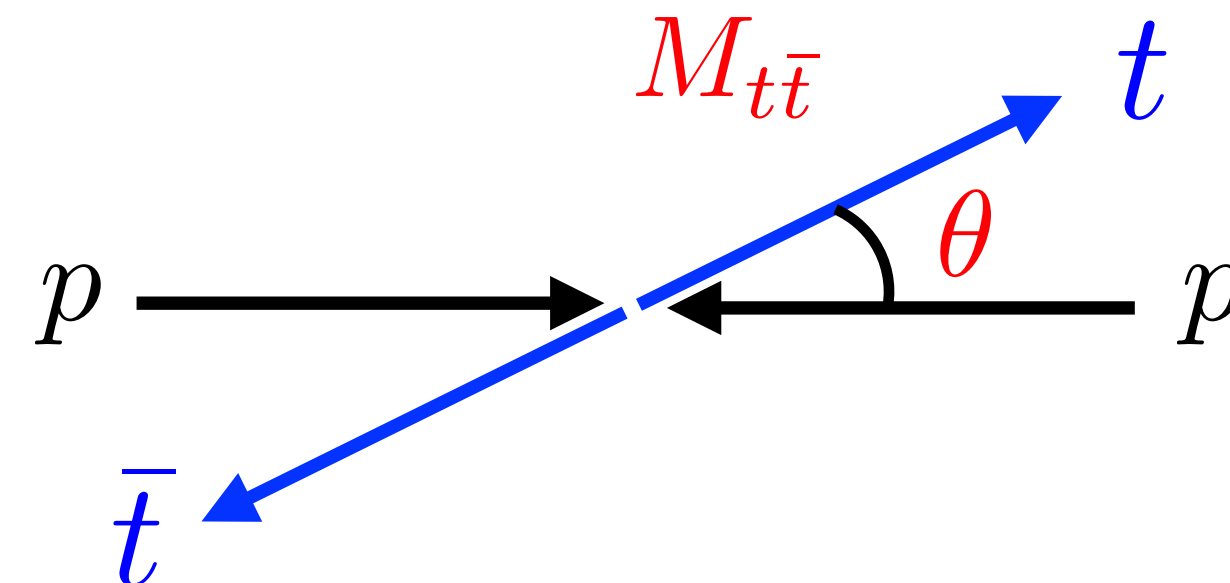
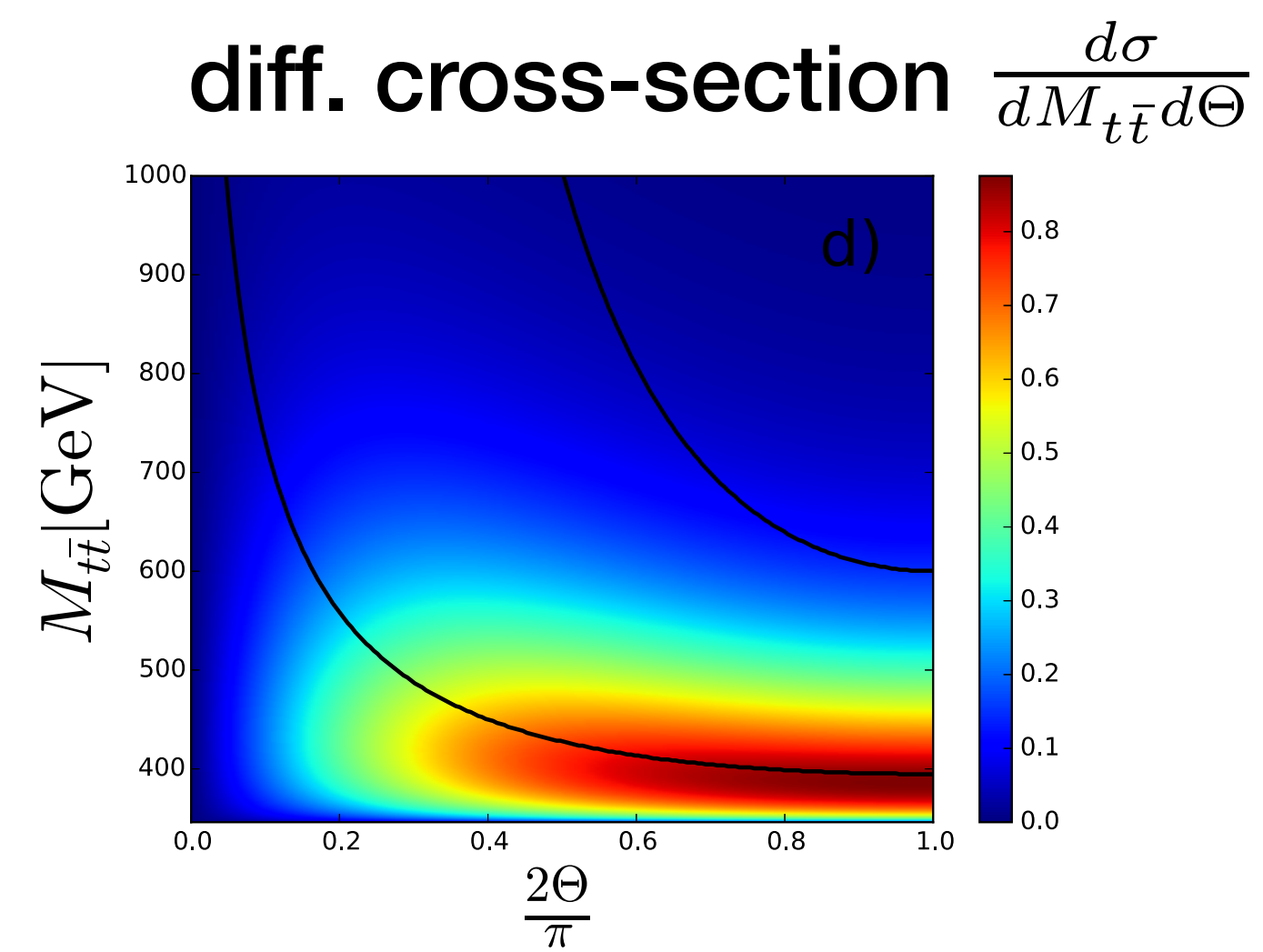
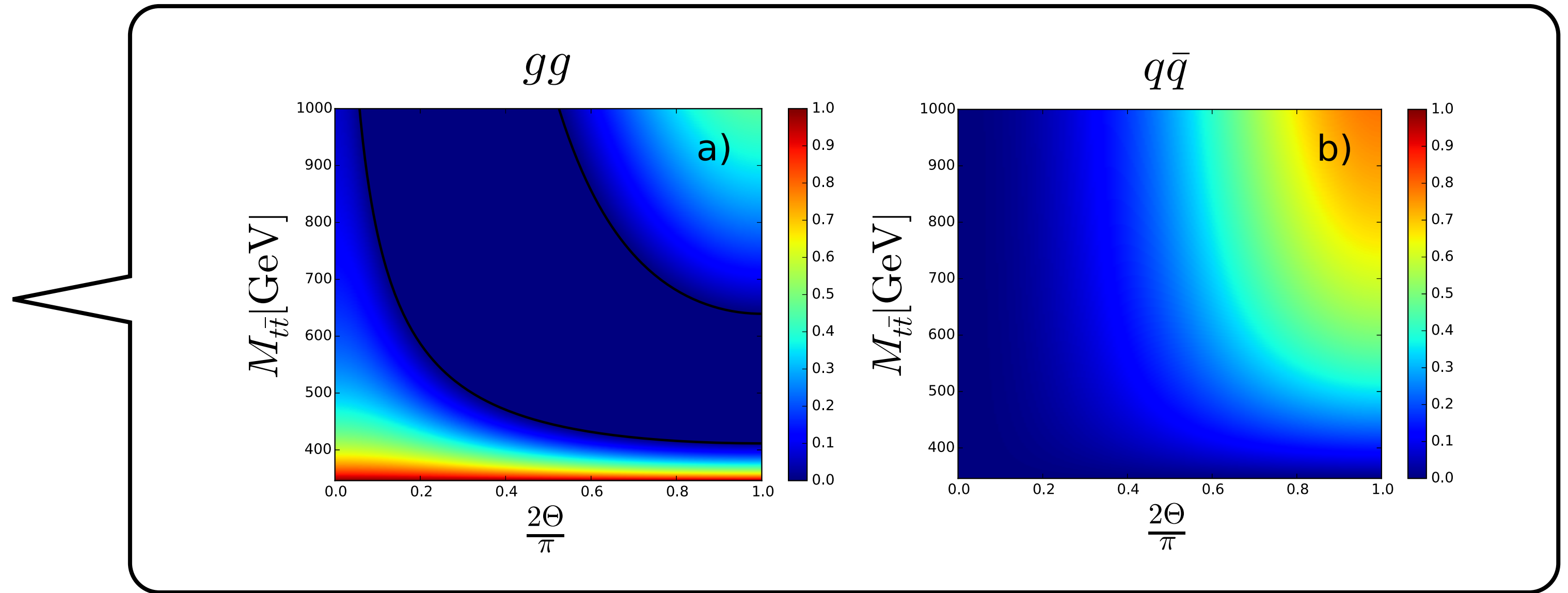
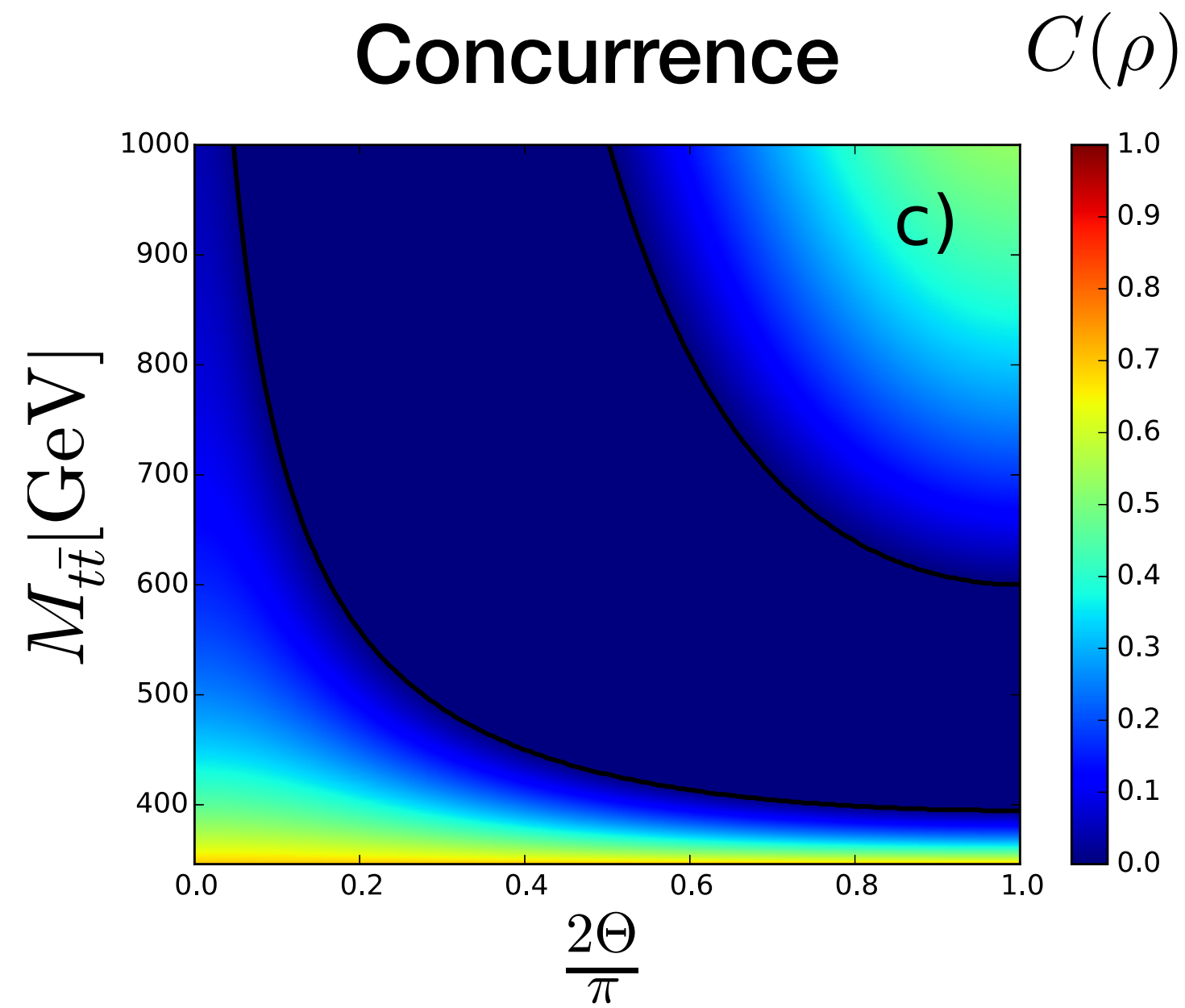
$$\tilde{\rho} = \rho$$



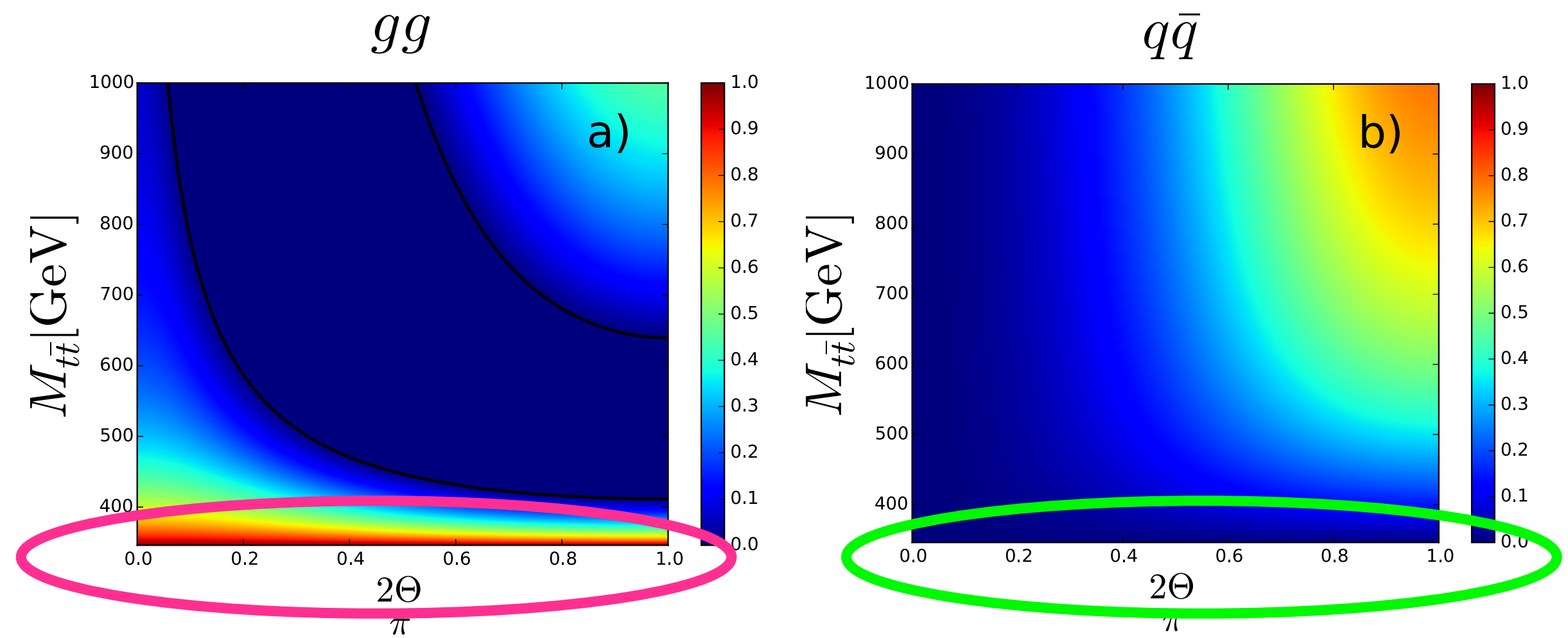
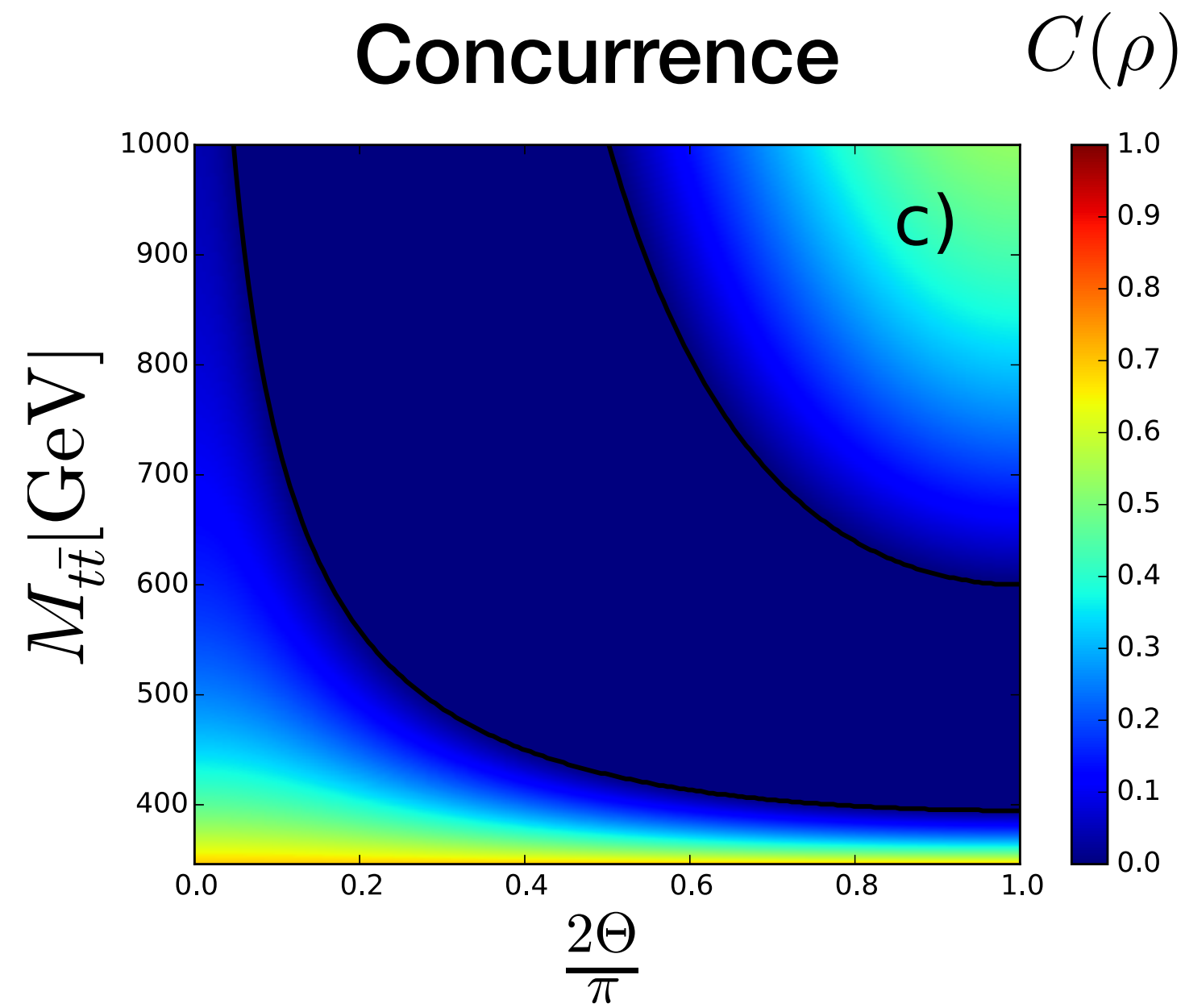
$$C(\rho) = \frac{\max(0, \Delta)}{2}$$

$$\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1$$

Concurrence

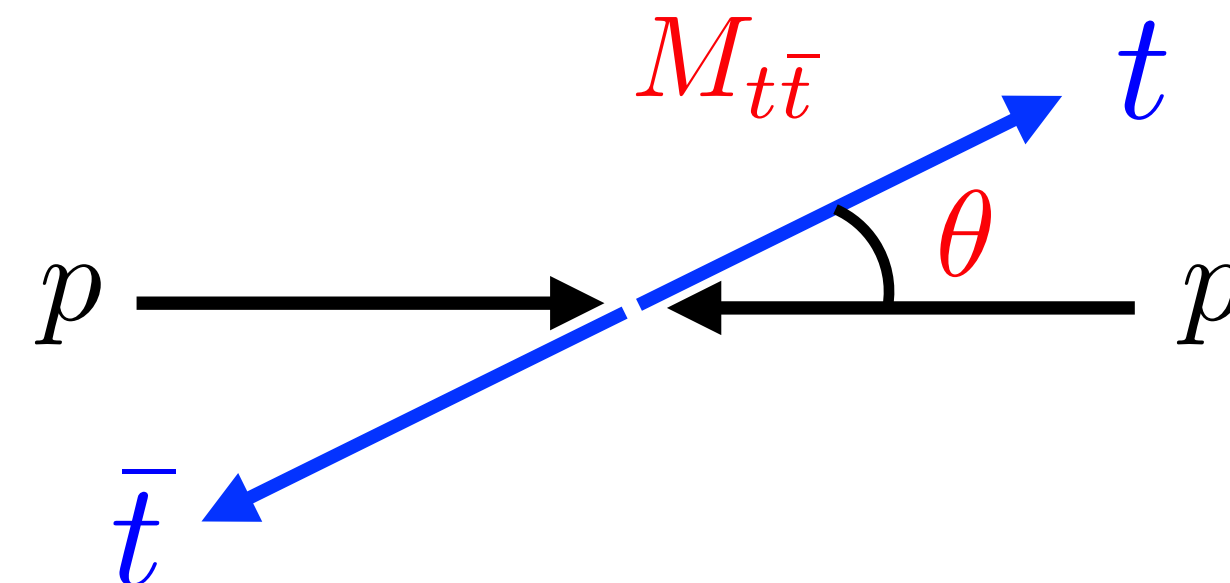
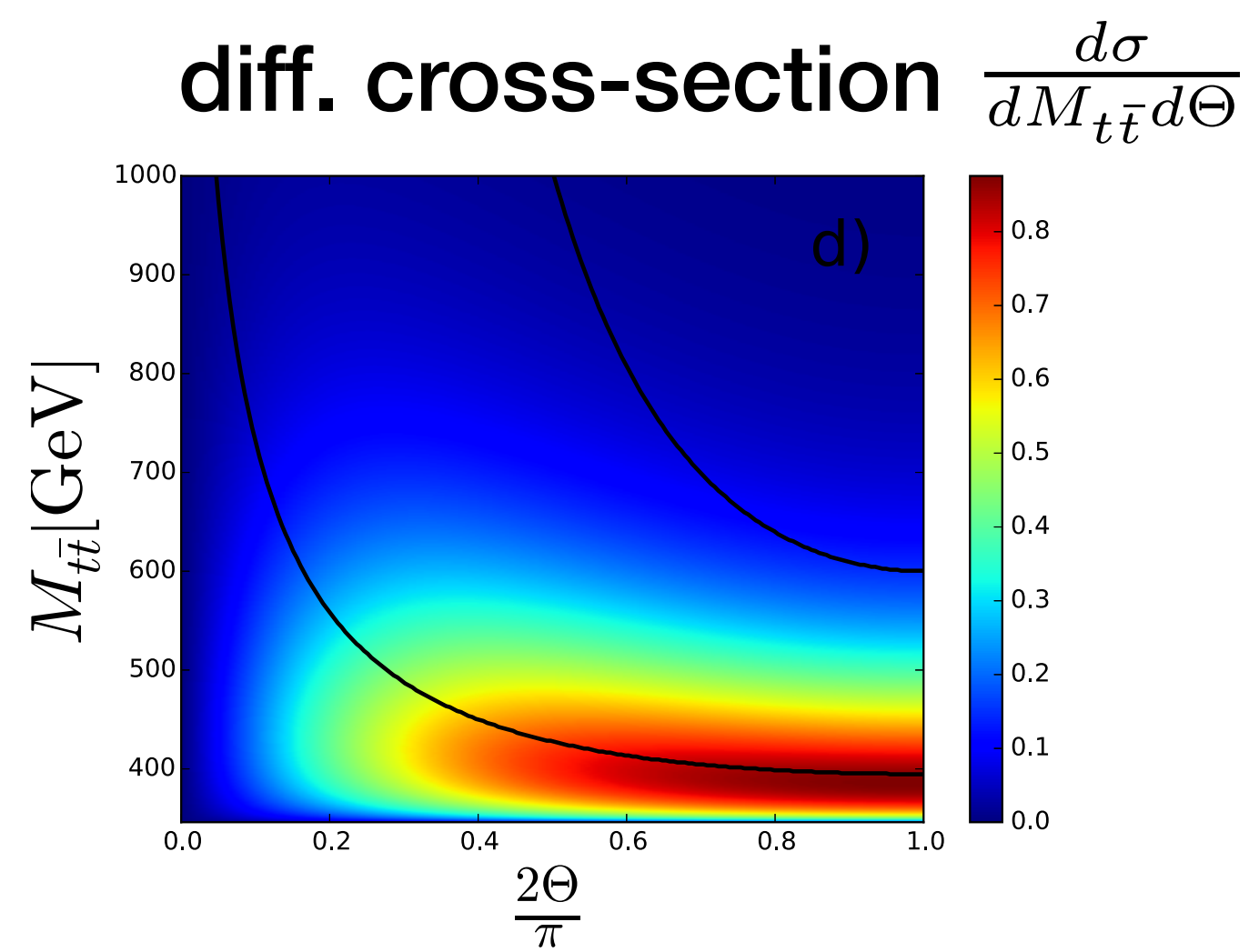


Concurrence



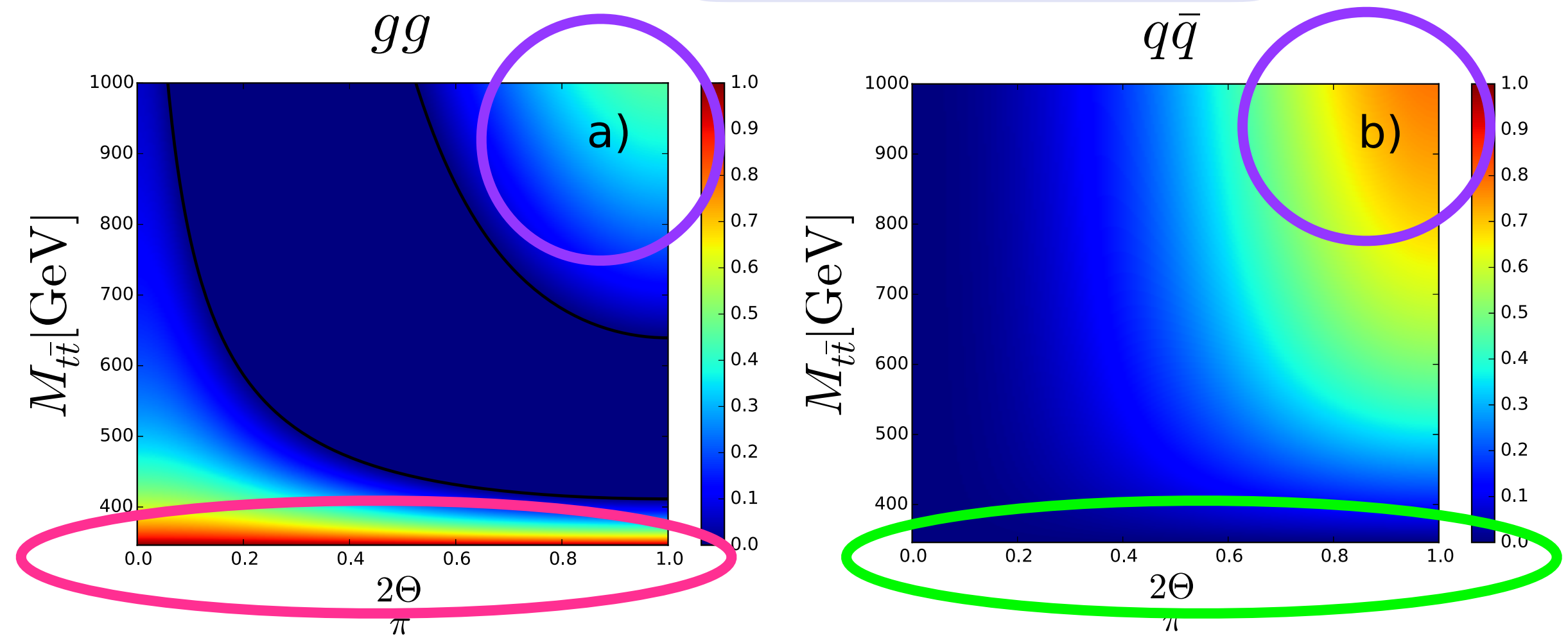
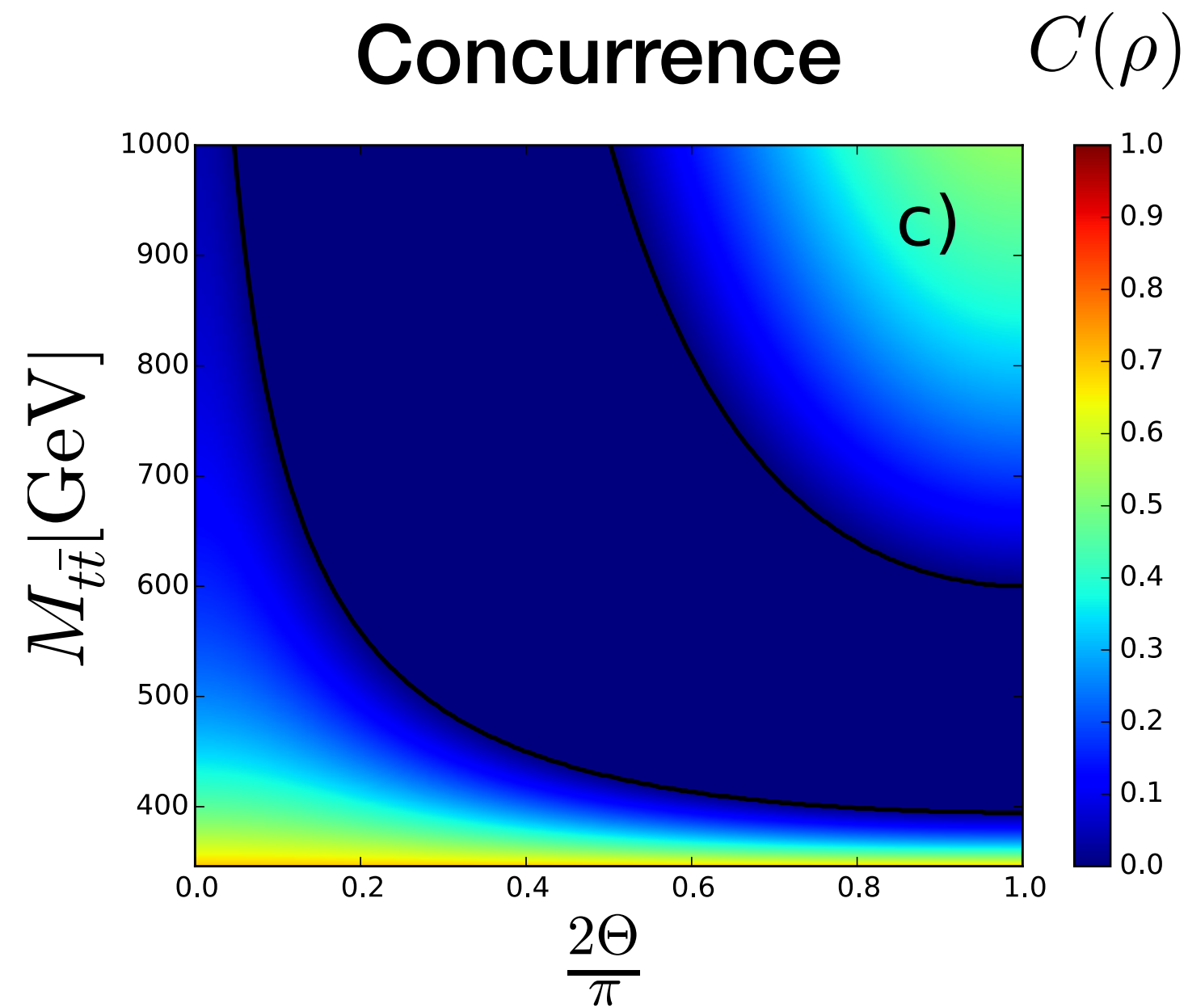
$$|\psi_0\rangle = \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

$$\rho_{\text{sep}} = \frac{1}{2} (|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|)$$



Concurrence

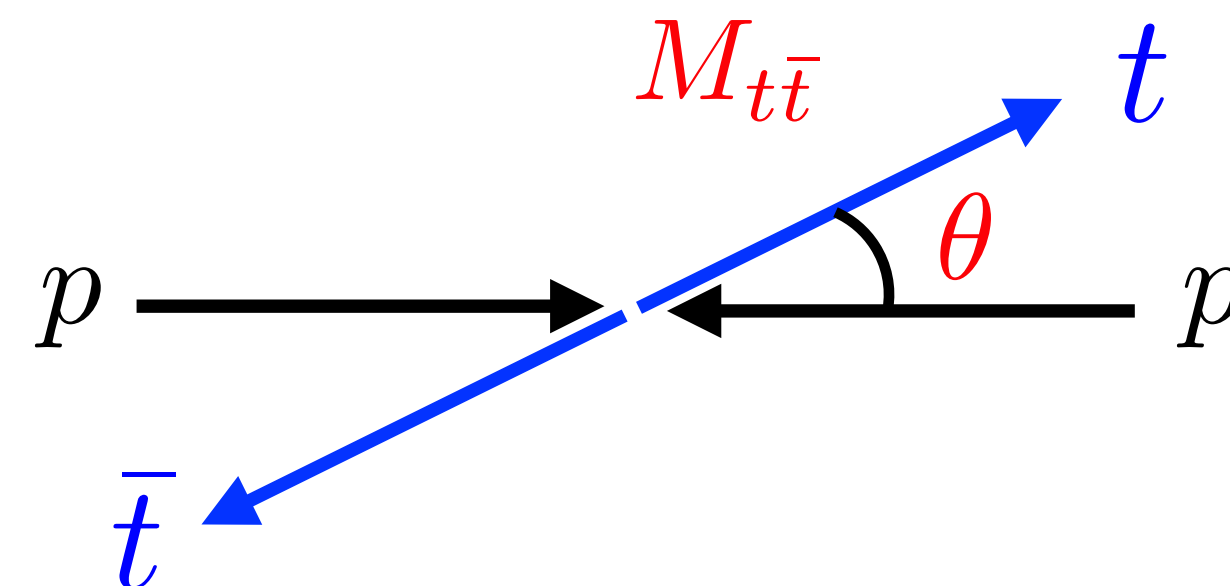
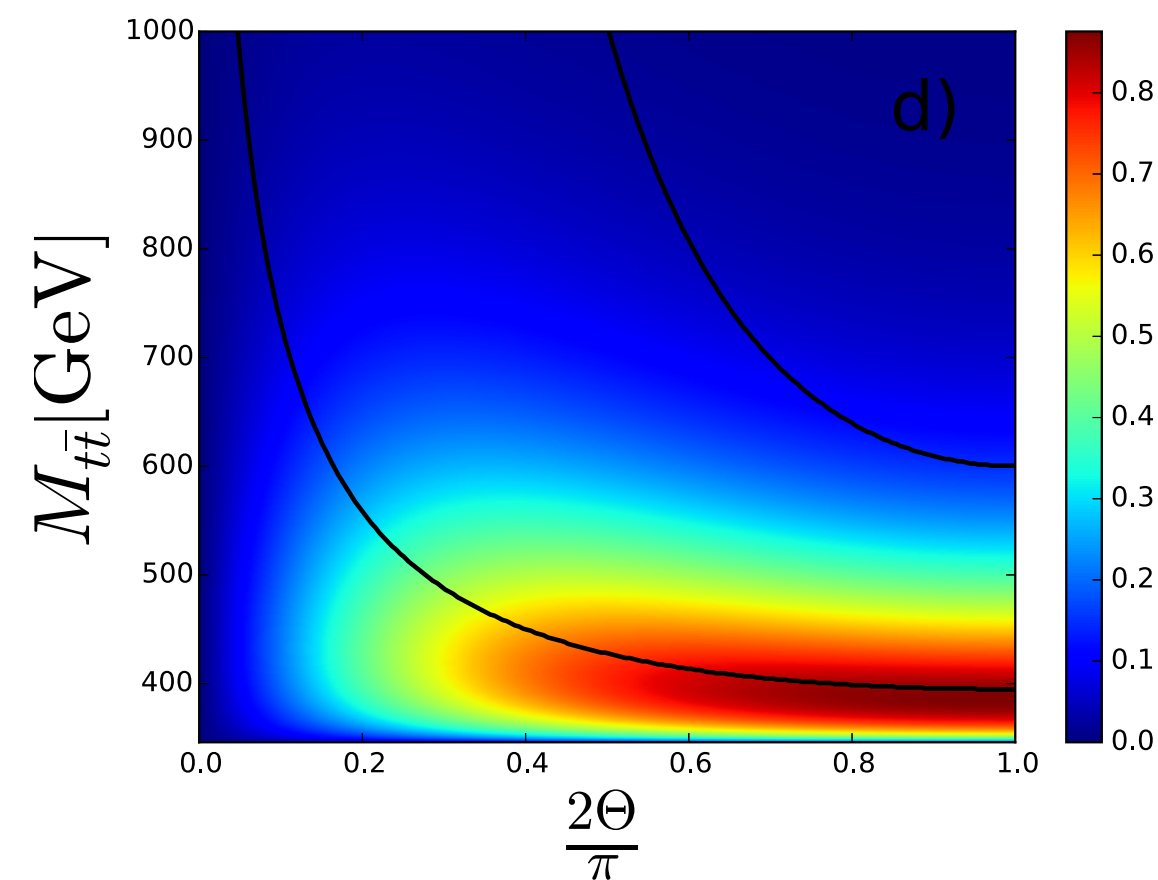
$$\frac{|\uparrow_{\hat{n}}\downarrow_{\hat{n}}\rangle + |\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle}{\sqrt{2}}$$



$$|\psi_0\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

$$\rho_{\text{sep}} = \frac{1}{2}(|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|)$$

diff. cross-section $\frac{d\sigma}{dM_{t\bar{t}}d\Theta}$



How to measure/reconstruct the spin density matrix from collider data?

⌘ Quantum State Tomography

$$\rho = \frac{1}{4} \left[\mathbb{1} + B_i^{(1)} \cdot (\sigma_i \otimes \mathbb{1}) + B_j^{(2)} (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

$$B_i^{(1)} = \text{Tr} [(\sigma_i \otimes \mathbb{1}) \rho] = \langle S_i^{(1)} \rangle_\rho \quad \longleftarrow \quad \text{top spin polarisation}$$

$$B_i^{(2)} = \text{Tr} [(\mathbb{1} \otimes \sigma_i) \rho] = \langle S_i^{(2)} \rangle_\rho \quad \longleftarrow \quad \text{anti-top spin polarisation}$$

$$C_{ij} = \text{Tr} [(\sigma_i \otimes \sigma_j) \rho] = \langle S_i^{(1)} S_j^{(2)} \rangle_\rho \quad \longleftarrow \quad \text{spin-spin correlation}$$

How to measure the spin and spin-correlation at colliders?

How to measure the spin?

$$\rho = |\uparrow_s\rangle\langle\uparrow_s| = \frac{1}{2}(\mathbb{1} + \hat{\mathbf{S}} \cdot \boldsymbol{\sigma})$$

- Consider an unstable spin-1/2 particle with spin in the $\hat{\mathbf{S}}$ direction at its rest frame.

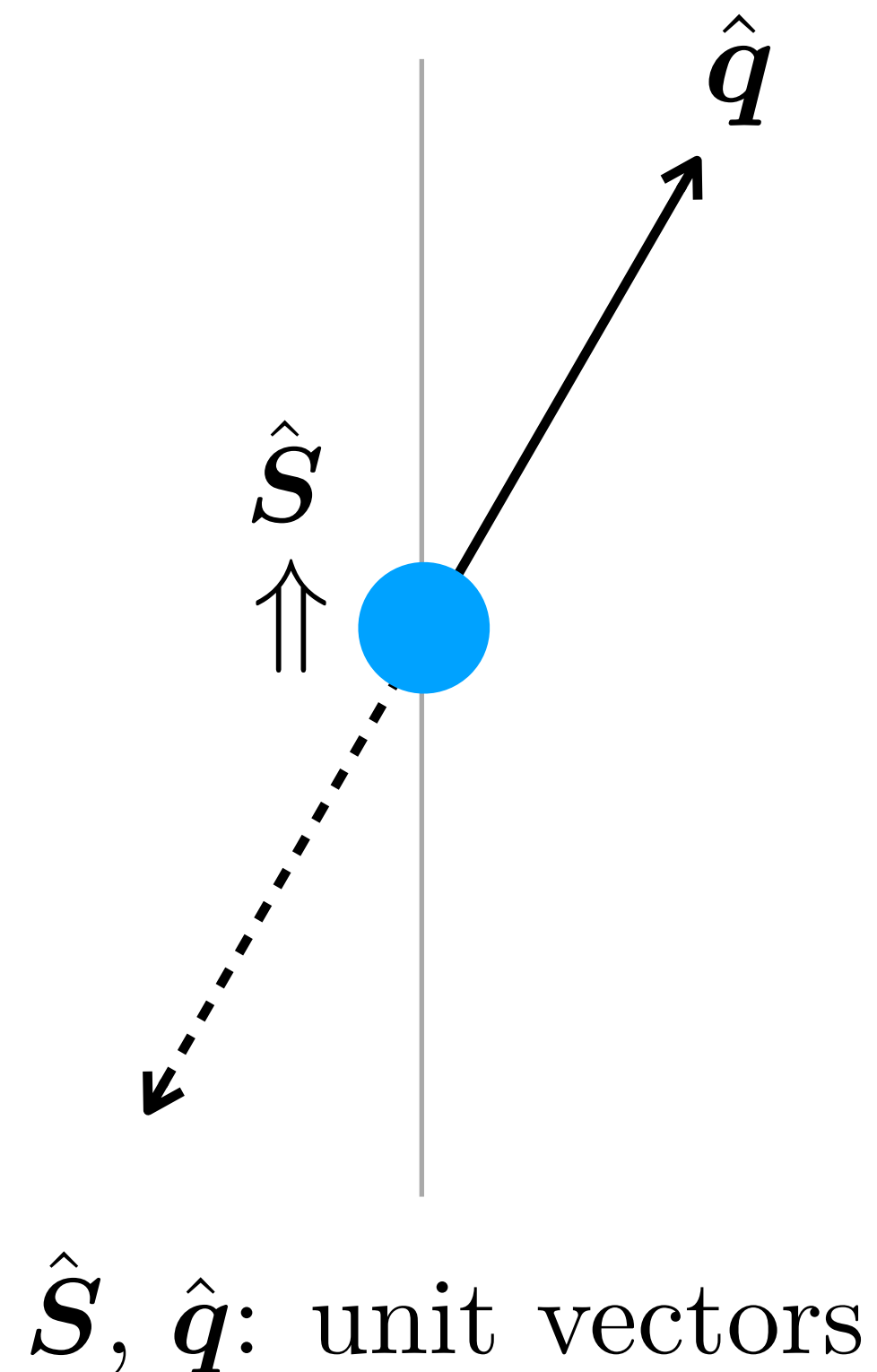
The conditional probability density of $\hat{\mathbf{q}}$ given $\hat{\mathbf{S}}$

$$\frac{dP(\hat{\mathbf{q}}|\hat{\mathbf{S}})}{d\Omega_q} = \frac{1 + \kappa \hat{\mathbf{q}} \cdot \hat{\mathbf{S}}}{4\pi} \quad -1 \leq \kappa \leq 1$$

- most general form by the symmetry, positivity, normalisation
- κ : **spin-analysing power** (memory of the parents' spin)

Channel	Analyser	κ (approx.)
$t \rightarrow bW^+$	b	-0.4
$t \rightarrow b\ell^+\nu$	ℓ^+	+ 1.0
$t \rightarrow bu\bar{d}$	$\bar{d}$	+0.93
$\tau^- \rightarrow \pi^- \nu$	π^-	+ 1.0
$\tau^- \rightarrow \rho^- \nu$	ρ^-	+0.45

$$\kappa_f = -\kappa_{\bar{f}} \\ \text{(C-odd)}$$



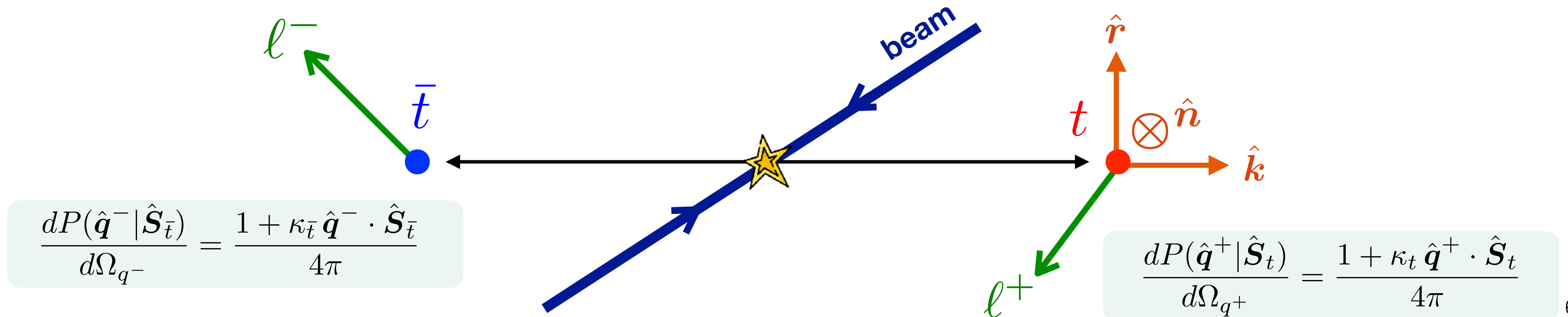
$$\rho = \frac{1}{4} \left[\mathbb{1} + B_i^{(1)} \cdot (\sigma_i \otimes \mathbb{1}) + B_j^{(2)} (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

$$B_i^{(1)} = \text{Tr} [(\sigma_i \otimes \mathbb{1}) \rho] = \langle S_i^{(1)} \rangle_\rho = \frac{3}{\kappa_1} \langle q_i^{(1)} \rangle$$

$$B_i^{(2)} = \text{Tr} [(\mathbb{1} \otimes \sigma_i) \rho] = \langle S_i^{(2)} \rangle_\rho = \frac{3}{\kappa_2} \langle q_i^{(2)} \rangle$$

$$C_{ij} = \text{Tr} [(\sigma_i \otimes \sigma_j) \rho] = \langle S_i^{(1)} S_j^{(2)} \rangle_\rho = \frac{9}{\kappa_1 \kappa_2} \langle q_i^{(1)} q_j^{(2)} \rangle$$

measurable at colliders



Quantum Information Science for Particle & Collider Physics

Lecture 2

Kazuki Sakurai

University of Warsaw

30th International Summer Institute (SI2026), 8 August 2026

Plan

Lecture 1:

- **QI Tools:** Density Matrix, Entanglement, Bell nonlocality
- **QI Measurements @ Collider:** ttbar, tau-tau, ZZ

Lecture 2:

- **Open Quantum Mechancis**
 - ◇ Reconstruction of Quantum Channel (probing NLQM)
 - ◇ Decoherence @ Colliders

- Quantum state = **Density operator**: $\hat{\rho} = \sum_i q_i |\psi_i\rangle \langle \psi_i|$ $q_i \geq 0$, $\sum_i q_i = 1$

- Concurrence**: (separable) $0 \leq C(\rho) \leq 1$ (maximally entangled)

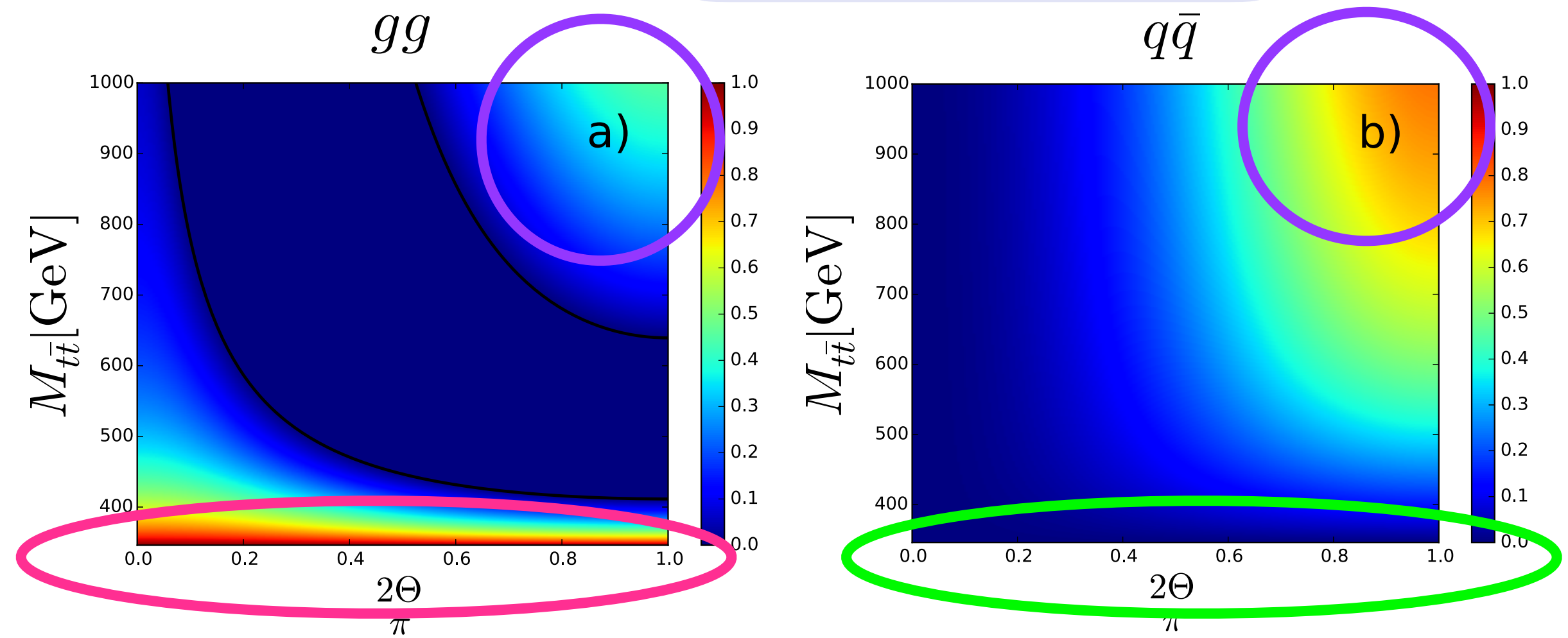
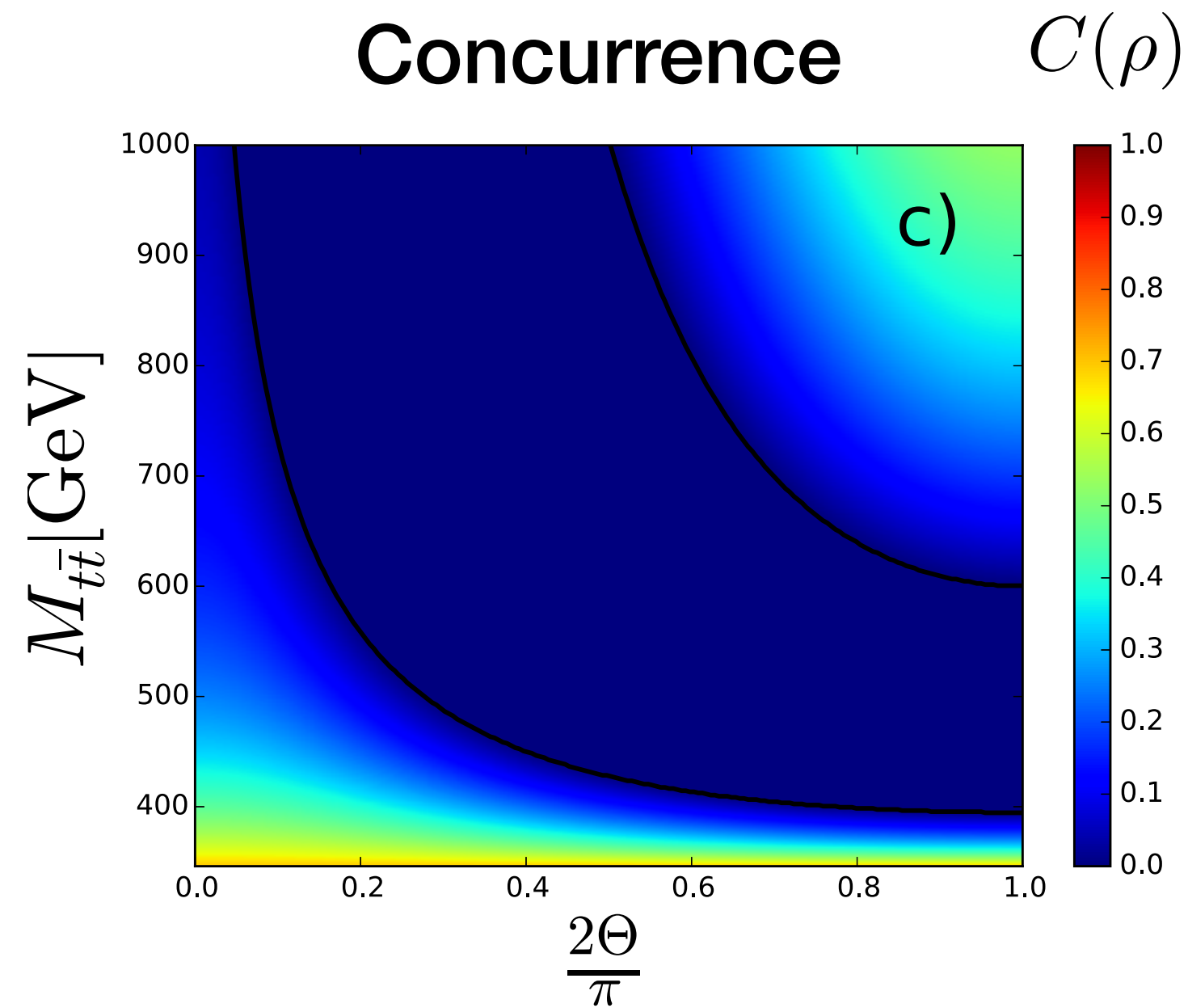
- Bell operator**:

$$\mathcal{B}_{\text{CHSH}} \leq \begin{cases} 2 & \text{LHV, } \rho_{\text{sep}} \\ 2\sqrt{2} & \text{QM} \\ 4 & \text{beyond QM} \end{cases}$$

$$\mathcal{B}_{\text{CHSH}} = \langle A_0 B_0 + A_0 B_1 + A_1 B_0 - A_1 B_1 \rangle$$

Concurrence

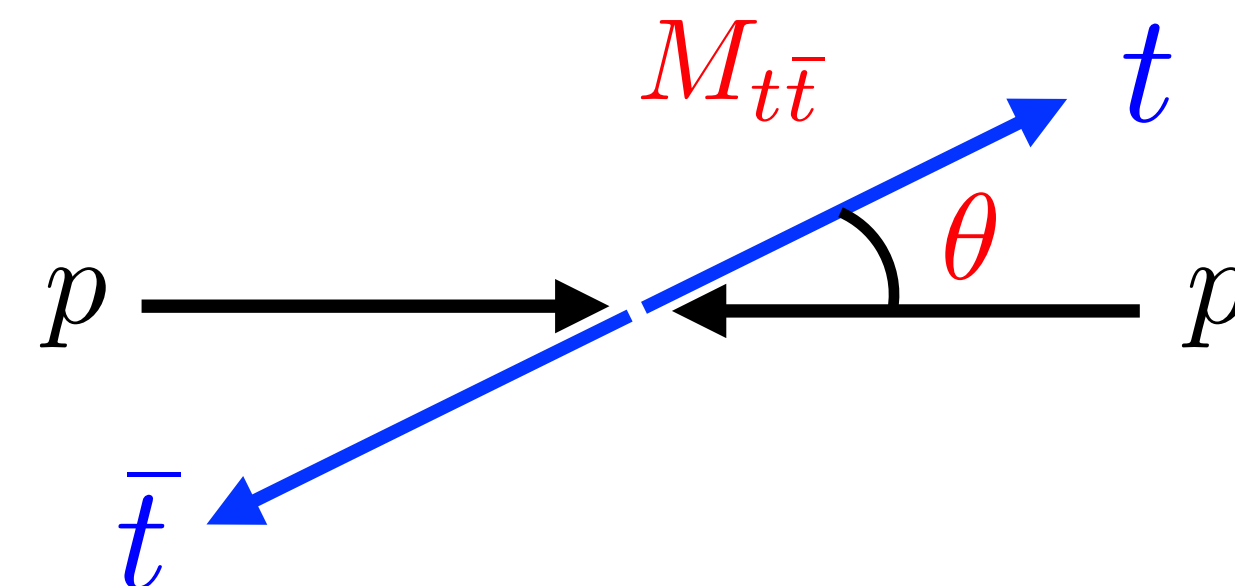
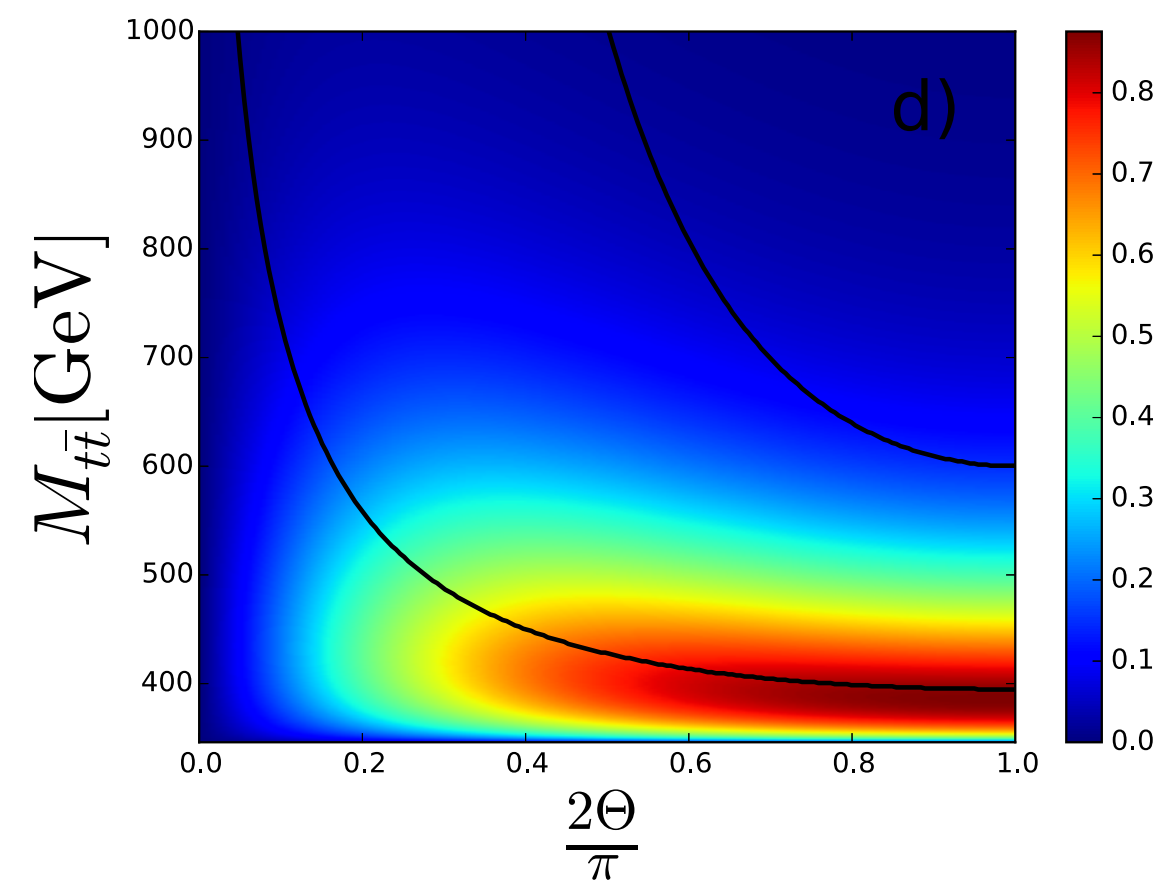
$$\frac{|\uparrow_{\hat{n}}\downarrow_{\hat{n}}\rangle + |\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle}{\sqrt{2}}$$



$$|\psi_0\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

$$\rho_{\text{sep}} = \frac{1}{2}(|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|)_z$$

diff. cross-section $\frac{d\sigma}{dM_{t\bar{t}}d\Theta}$



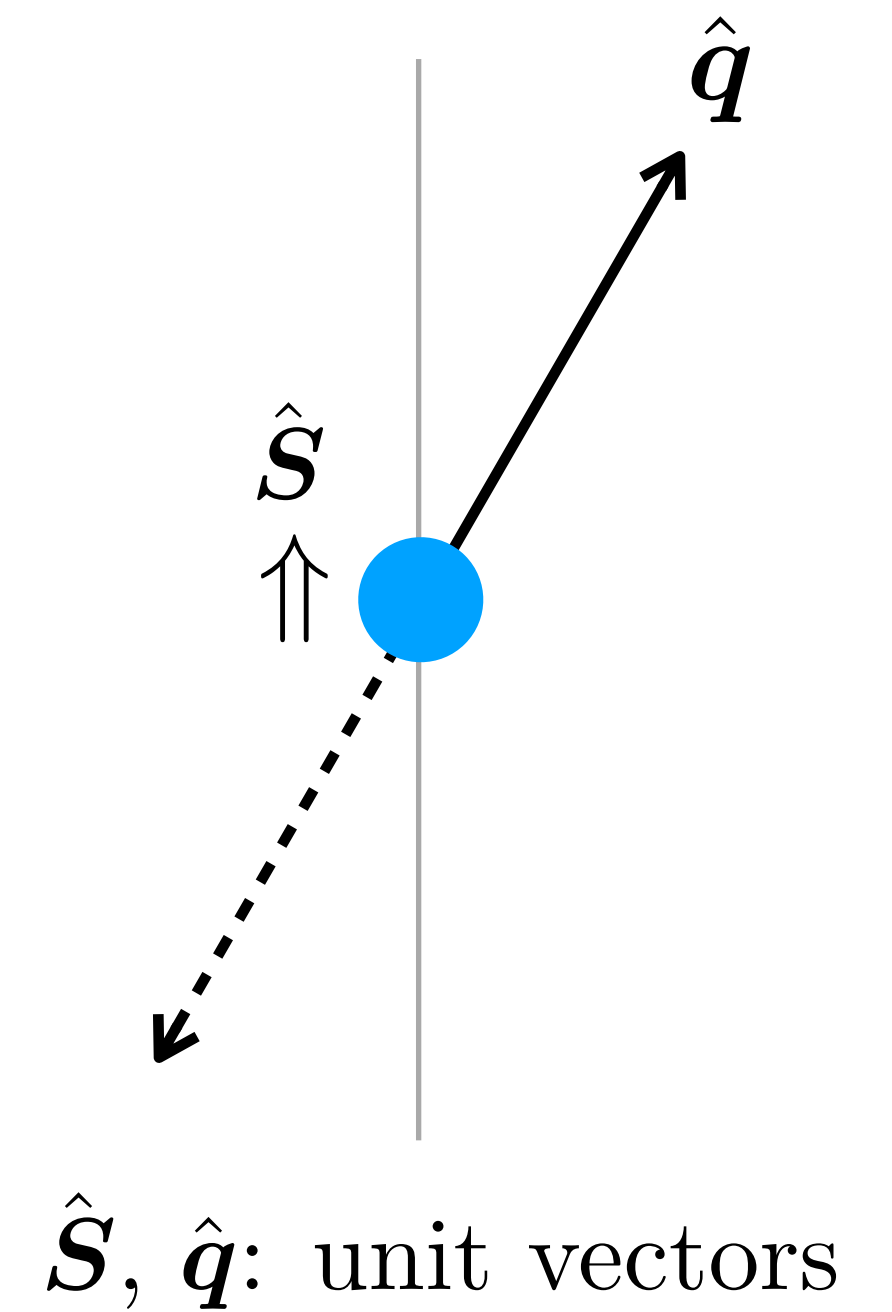
- **Probability rule:**

$$\frac{dP(\hat{q}|\hat{S})}{d\Omega_q} = \frac{1 + \kappa \hat{q} \cdot \hat{S}}{4\pi}$$

- **Spin analysing power, κ :**

Channel	Analyser	κ (approx.)
$t \rightarrow bW^+$	b	-0.4
$t \rightarrow b\ell^+\nu$	ℓ^+	+ 1.0
$t \rightarrow bu\bar{d}$	$\bar{d}$	+0.93
$\tau^- \rightarrow \pi^- \nu$	π^-	+ 1.0
$\tau^- \rightarrow \rho^- \nu$	ρ^-	+0.45

$$\kappa_f = -\kappa_{\bar{f}} \quad (\text{C-odd})$$



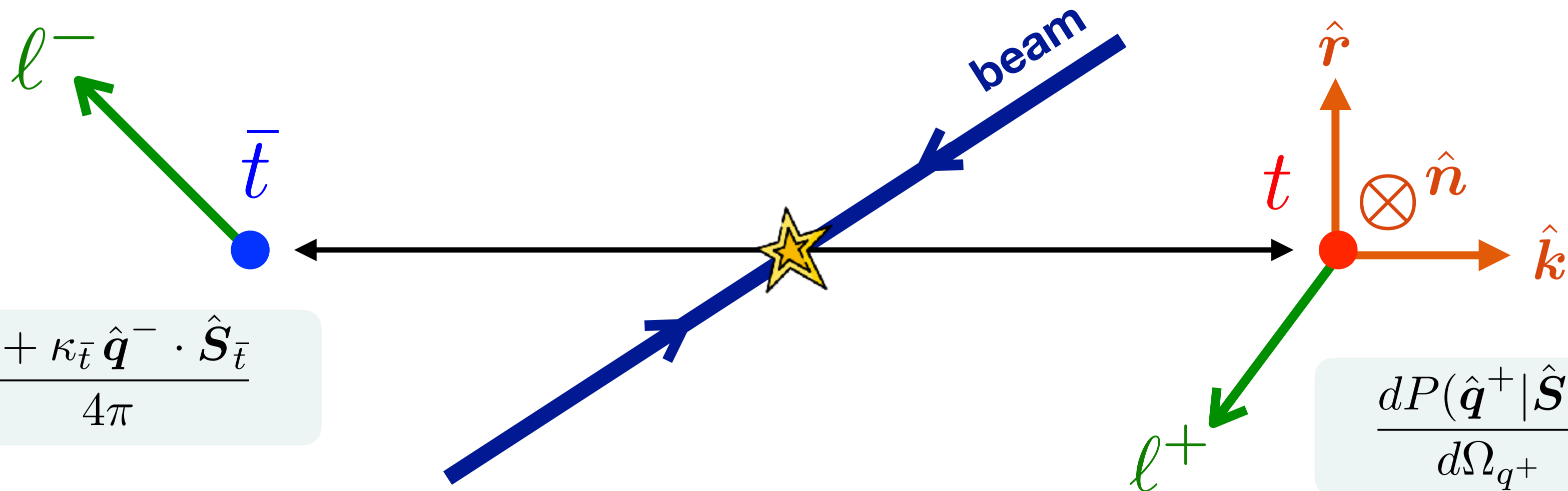
$$\rho = \frac{1}{4} \left[\mathbb{1} + B_i^{(1)} \cdot (\sigma_i \otimes \mathbb{1}) + B_j^{(2)} (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

$$B_i^{(1)} = \text{Tr} [(\sigma_i \otimes \mathbb{1}) \rho] = \langle S_i^{(1)} \rangle_\rho = \frac{3}{\kappa_1} \langle q_i^{(1)} \rangle$$

$$B_i^{(2)} = \text{Tr} [(\mathbb{1} \otimes \sigma_i) \rho] = \langle S_i^{(2)} \rangle_\rho = \frac{3}{\kappa_2} \langle q_i^{(2)} \rangle$$

$$C_{ij} = \text{Tr} [(\sigma_i \otimes \sigma_j) \rho] = \langle S_i^{(1)} S_j^{(2)} \rangle_\rho = \frac{9}{\kappa_1 \kappa_2} \langle q_i^{(1)} q_j^{(2)} \rangle$$

measurable at colliders



$$\frac{dP(\hat{\mathbf{q}}^- | \hat{\mathbf{S}}_{\bar{t}})}{d\Omega_{q^-}} = \frac{1 + \kappa_{\bar{t}} \hat{\mathbf{q}}^- \cdot \hat{\mathbf{S}}_{\bar{t}}}{4\pi}$$

$$\frac{dP(\hat{\mathbf{q}}^+ | \hat{\mathbf{S}}_t)}{d\Omega_{q^+}} = \frac{1 + \kappa_t \hat{\mathbf{q}}^+ \cdot \hat{\mathbf{S}}_t}{4\pi}$$

$$\rho = \frac{1}{4} \left[\mathbb{1} + B_i^{(1)} \cdot (\sigma_i \otimes \mathbb{1}) + B_j^{(2)} (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

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measurable at colliders

Spin measurement is possible only for weakly decaying particles:

$$t, \tau, Z, W^\pm$$

Observation of quantum entanglement with top quarks at the ATLAS detector

[The ATLAS Collaboration](#)

Entanglement is a key feature of quantum mechanics [1–3], with applications in fields such as metrology, cryptography, quantum information, and quantum computation [4–8]. It has been observed in a wide variety of systems and length scales, ranging from the microscopic [9–13] to the macroscopic [14–16]. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a center-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 fb^{-1} recorded with the ATLAS experiment. Spin entanglement is detected from the

Observation of quantum entanglement in top quark pair production in proton–proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

Abstract

Entanglement is an intrinsic property of quantum mechanics and is predicted to be exhibited in the particles produced at the Large Hadron Collider. A measurement of the extent of entanglement in top quark-antiquark ($t\bar{t}$) events produced in proton–proton collisions at a center-of-mass energy of 13 TeV is performed with the data recorded by the CMS experiment at the CERN LHC in 2016, and corresponding to an integrated luminosity of 36.3 fb^{-1} . The events are selected based on the presence of two leptons with opposite charges and high transverse momentum. An entanglement-sensitive observable D is derived from the top quark spin-dependent parts of the $t\bar{t}$ production density matrix and measured in the region of the $t\bar{t}$ production threshold. Values of $D < -1/3$ are evidence of entanglement and D is observed (expected) to be $-0.480^{+0.026}_{-0.029}$ ($-0.467^{+0.026}_{-0.029}$) at the parton level. With an observed significance of 5.1 standard deviations with respect to the non-entangled hypothesis, this provides observation of quantum mechanical entanglement within $t\bar{t}$ pairs in this phase space. This measurement provides a new probe of quantum mechanics at the highest energies ever produced.

Entanglement measurement in $pp \rightarrow t\bar{t}$

- The top is the only quark that decays before it hadronizes and depolarizes:

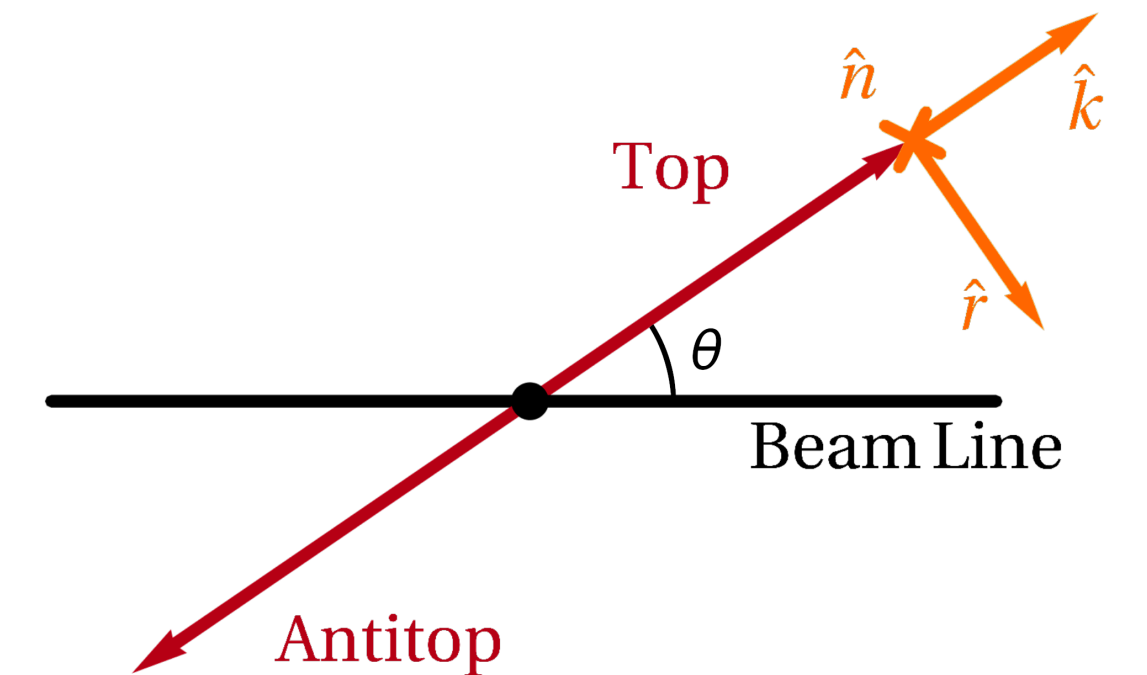
$$\tau_{\text{top}} \sim 10^{-25} \text{ s} \ll \tau_{\text{had}} \sim 10^{-23} \text{ s} \ll \tau_{\text{spin}} \sim 10^{-21} \text{ s}.$$

$$(\sim G_F m_t^3) \quad (\sim 1/\Lambda_{\text{QCD}}) \quad (\sim \Lambda_{\text{QCD}}^2/m_t)$$

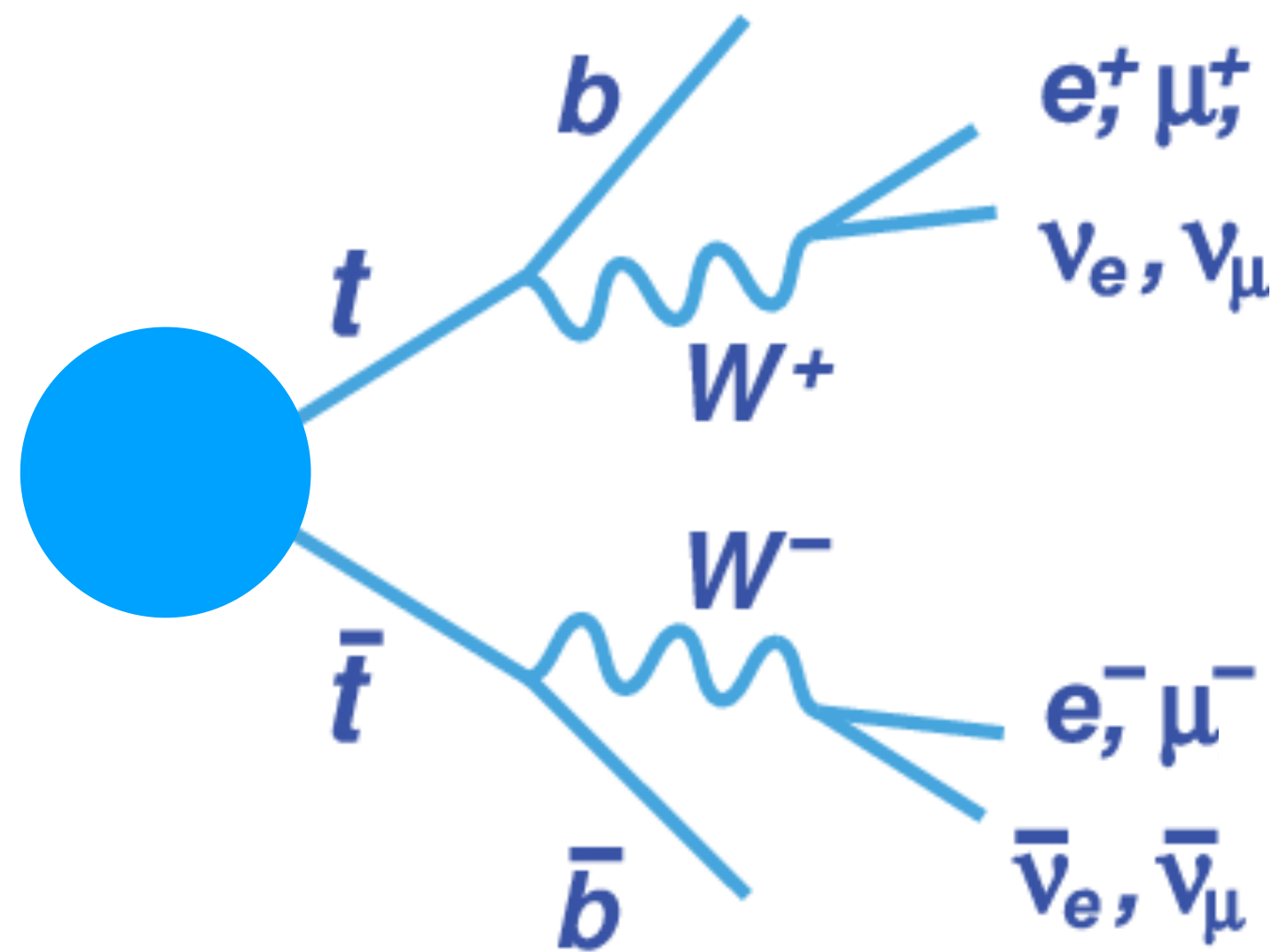
- $\kappa_t = -\kappa_{\bar{t}} = 1$ in the leptonic decay, $t\bar{t} \rightarrow (bW^+)(\bar{b}W^-) \rightarrow (b\ell^+\nu)(\bar{b}\ell^-\bar{\nu})$

- The quantum tomography, e.g., $C_{ij} = \frac{9}{\kappa_t \kappa_{\bar{t}}} \langle q_i^+ q_j^- \rangle = -9 \langle q_i^+ q_j^- \rangle$ requires the measurement of the product of the charged lepton directions **at the respective t and $\bar{t}$ rest frames**.

- Reconstruction of the top, antitop rest frames requires the neutrino momentum reconstruction.



Reconstruction of the neutrino momenta:



- 6 unknowns:

$$(p_{\nu,x}, p_{\nu,y}, p_{\nu,z}), (p_{\bar{\nu},x}, p_{\bar{\nu},y}, p_{\bar{\nu},z})$$

- 6 constraints:

$$m_W^2 = (p_\nu + p_{\ell^+})^2 \quad m_t^2 = (p_\nu + p_{\ell^+} + p_b)^2$$

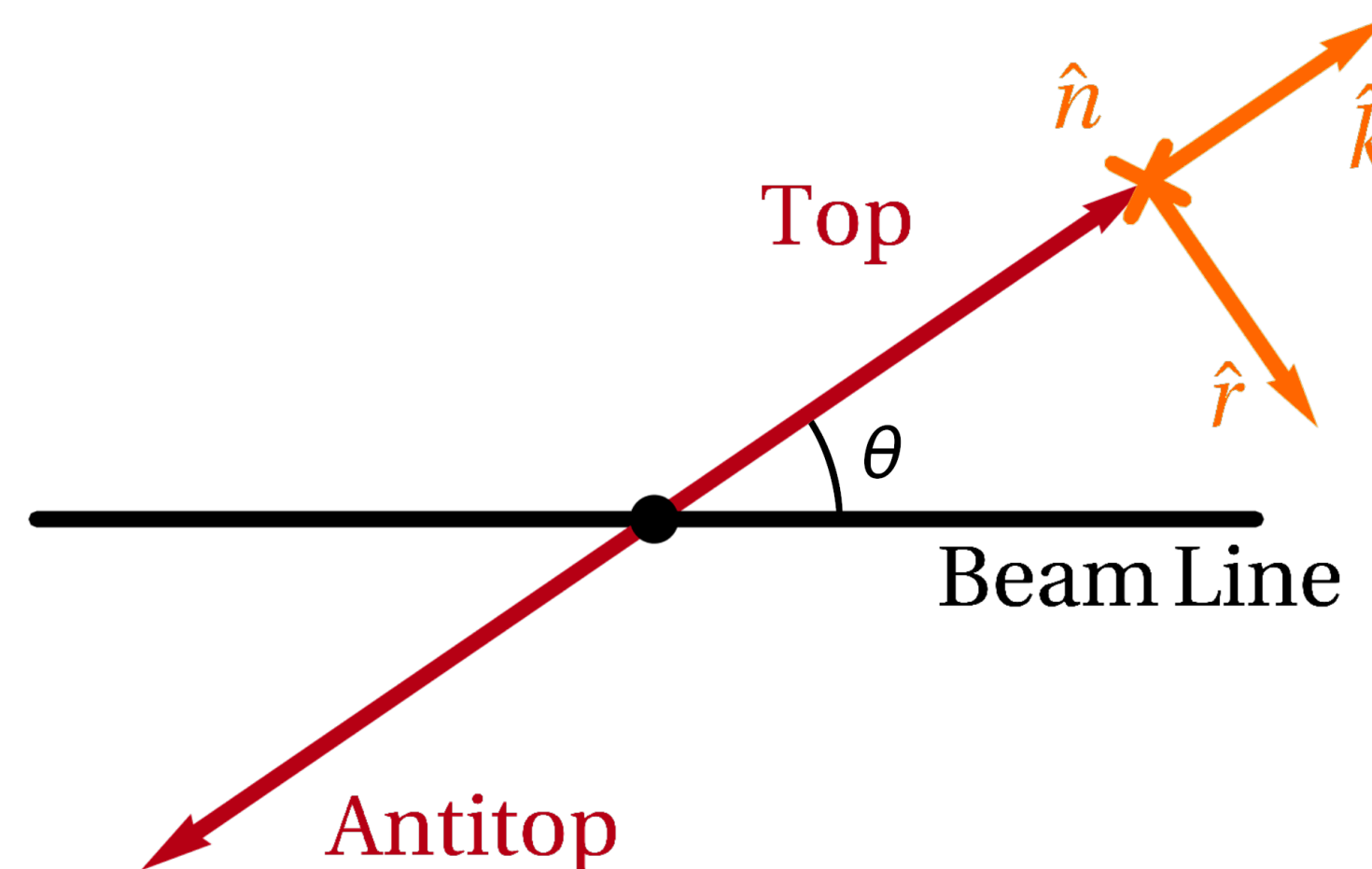
$$m_W^2 = (p_{\bar{\nu}} + p_{\ell^-})^2 \quad m_t^2 = (p_{\bar{\nu}} + p_{\ell^-} + p_{\bar{b}})^2$$

$$p_x^{\text{miss}} = p_{\nu,x} + p_{\bar{\nu},x} \quad p_y^{\text{miss}} = p_{\nu,y} + p_{\bar{\nu},y}$$

$\implies$ 2 solutions: $(\vec{p}_\nu, \vec{p}_{\bar{\nu}})_{\text{sol-1}}, (\vec{p}_\nu, \vec{p}_{\bar{\nu}})_{\text{sol-2}}$

- Choose one that gives smaller $M_{t\bar{t}}$

- We first boost all momenta to the rest frame of the $t\bar{t}$ system.
- We then boost ℓ^+ to the top's rest frame and ℓ^- to the antitop's rest frame to be ready to measure $\langle \hat{q}_i^+ \rangle, \langle \hat{q}_i^- \rangle, \langle \hat{q}_i^+ \hat{q}_j^- \rangle$
- In the helicity basis, the **basis vectors** are different from event to event.
The reconstructed density matrix is *not* a genuine quantum state, but a *fictitious state*.
Entanglement in a *fictitious state* $\Rightarrow$ **Entanglement** in the original **quantum state**.



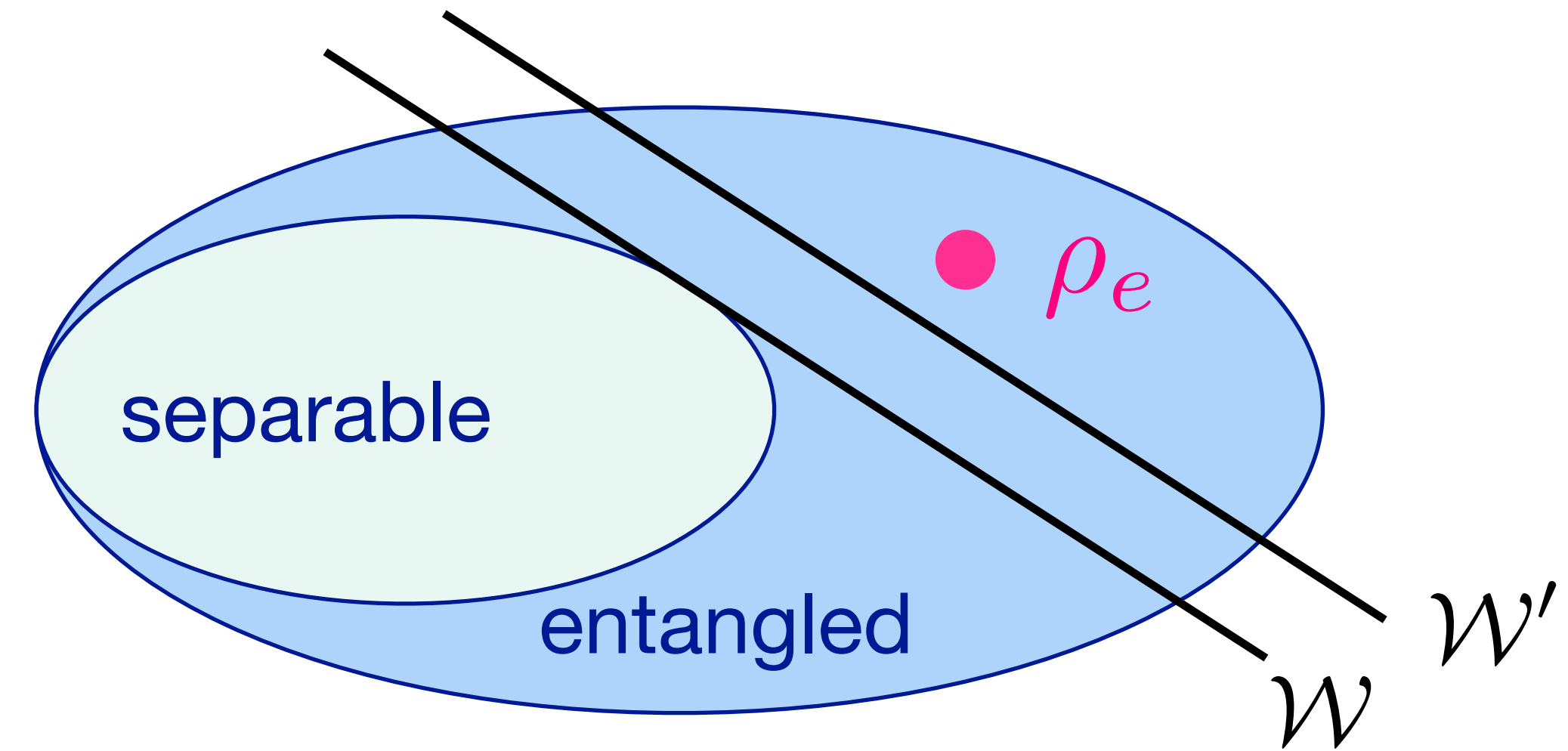
The Peres-Horodecki criterion assumes we know the density matrix ρ .
In practice, the reconstruction of the state, i.e. 15 Fano coefficients, is expensive.

Entanglement Witness:

The Hermitian $\mathcal{W}$ is called “*Witness*” if

$$\text{Tr}[\mathcal{W} \cdot \rho_{\text{sep}}] \geq 0, \quad \forall \rho_{\text{sep}}$$

$$\text{Tr}[\mathcal{W} \cdot \rho_e] < 0, \quad \text{for some entangled } \rho_e$$



Example: Pick a target entangled pure state $|\psi_e\rangle$, and define

$$\mathcal{W} = \alpha \mathbb{1} - |\psi_e\rangle\langle\psi_e| \quad \text{with} \quad \alpha = \max_{\text{all separable } |\phi\rangle} |\langle\phi|\psi_e\rangle|^2 < 1$$

- In the small $M_{t\bar{t}}$ region, the $t\bar{t}$ spin state is predicted to be $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
- No separable state overlaps with $|\psi^-\rangle$ more than $\alpha = \frac{1}{2}$ $\alpha \equiv \max_{\omega_{\text{sep}}} \langle \omega_{\text{sep}} | \psi^- \rangle$
- An entanglement witness operator:

$$\mathcal{W} \equiv \alpha \cdot \mathbb{1} - |\psi^-\rangle \langle \psi^-| \quad \text{with} \quad \alpha = \frac{1}{2}$$

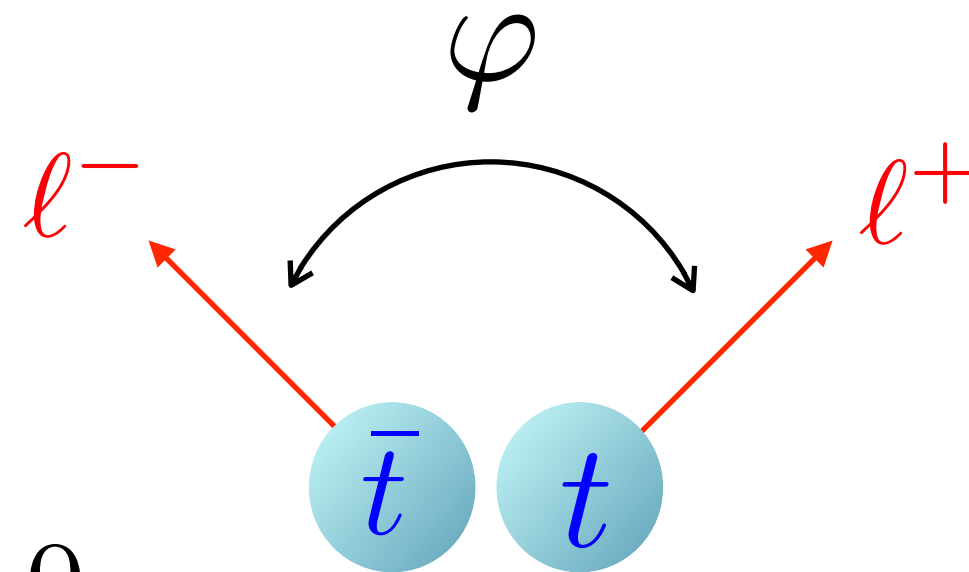
$$\text{Tr}[\mathcal{W} \cdot \rho_{\text{sep}}] \geq 0, \quad \forall \rho_{\text{sep}} \quad \text{Tr}[\mathcal{W} \cdot \rho_e] < 0, \quad \text{for some entangled } \rho_e$$

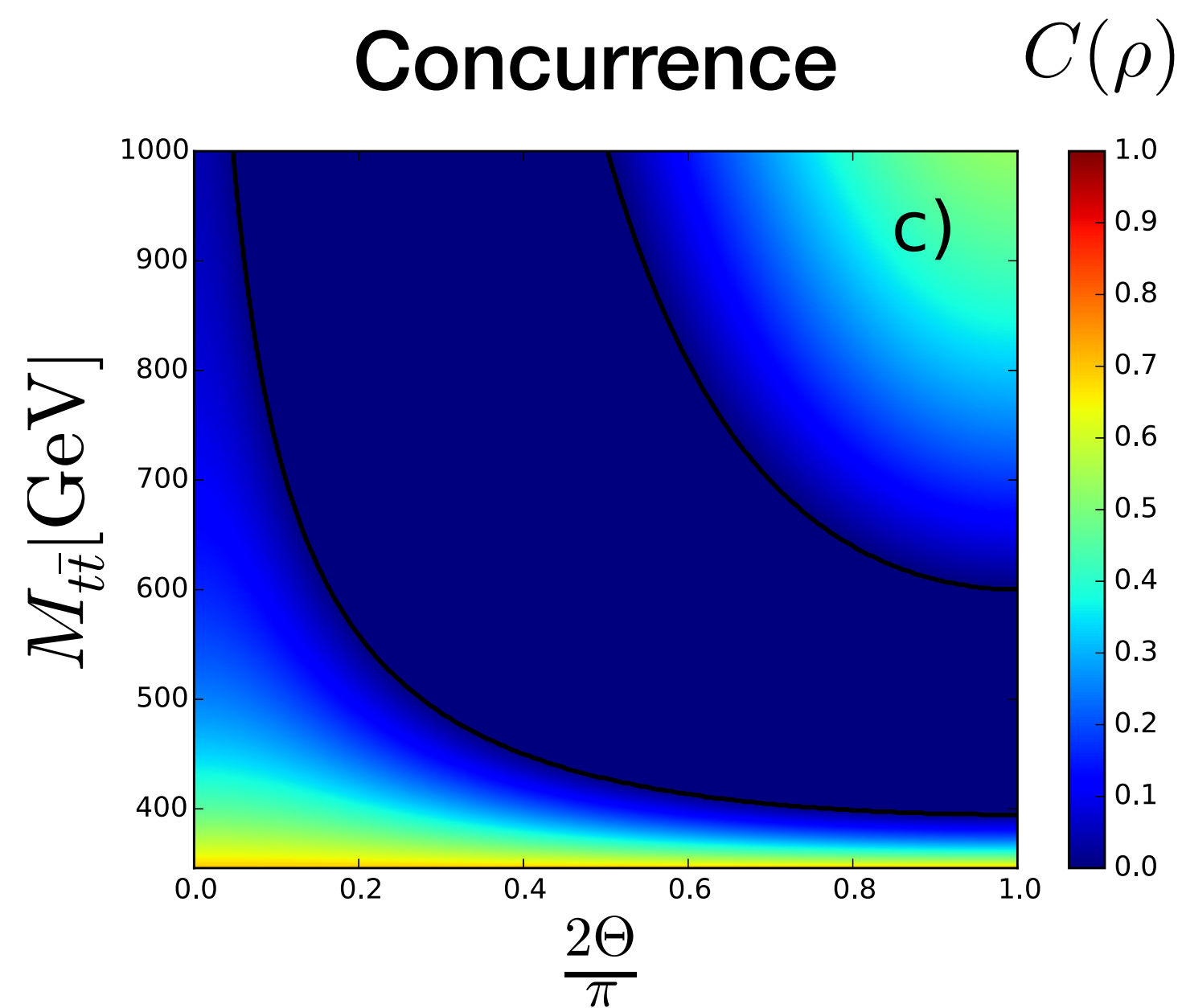
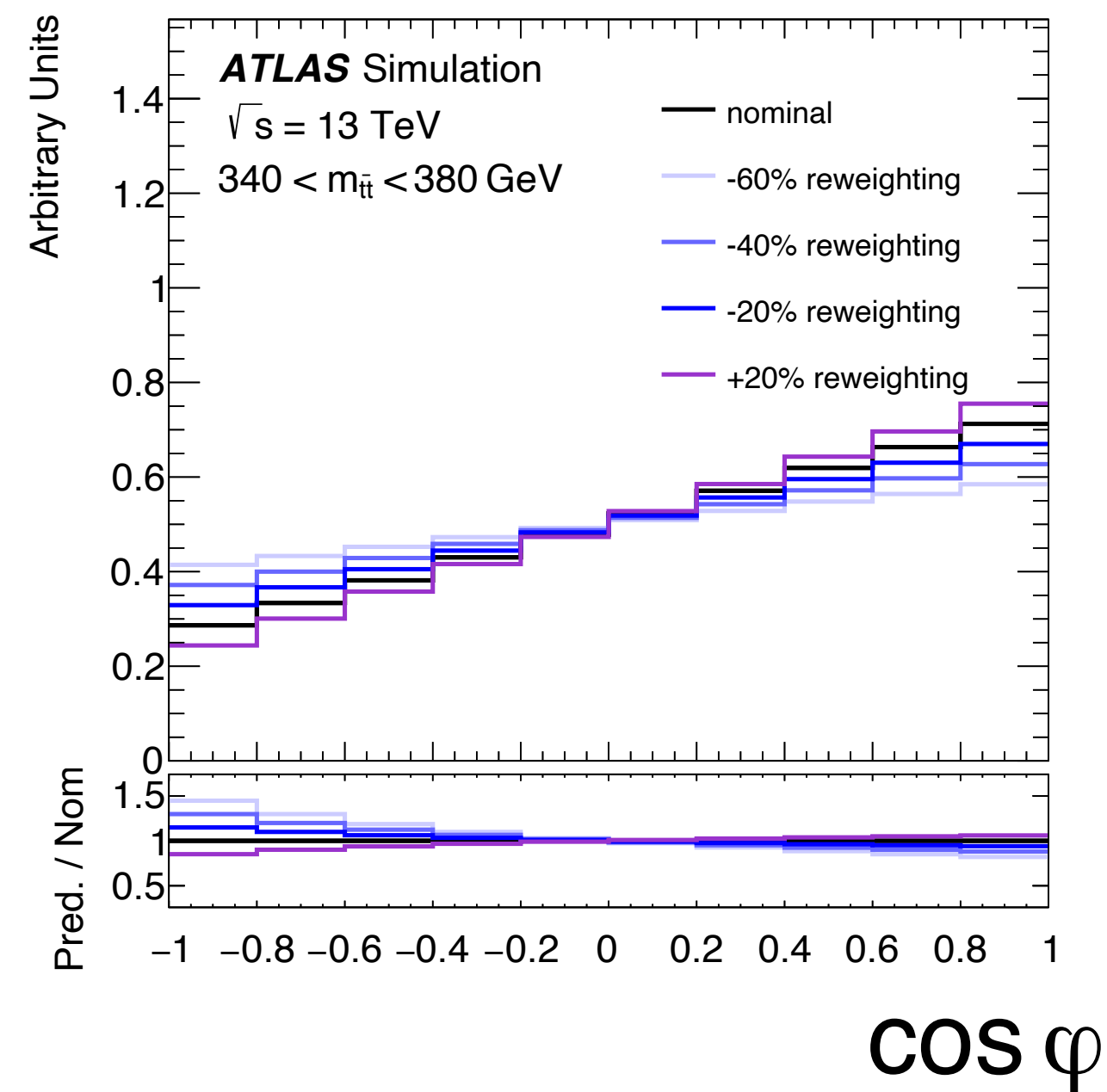
$$\mathcal{W} = \frac{1}{2} \cdot \mathbb{1} - |\psi^-\rangle \langle \psi^-| = \frac{1}{4} \left(\mathbb{1} + \sum_i \sigma_i \otimes \sigma_i \right)$$

$$\text{Tr}[\mathcal{W} \cdot \rho] = \frac{1}{4} \left(1 + \sum_i C_{ii} \right) = \frac{1}{4} \left(1 - 9 \langle \mathbf{q}^+ \cdot \mathbf{q}^- \rangle \right) = \frac{1}{4} \underbrace{\left(1 - 9 \langle \cos \varphi \rangle \right)}_{< 0} < 0$$

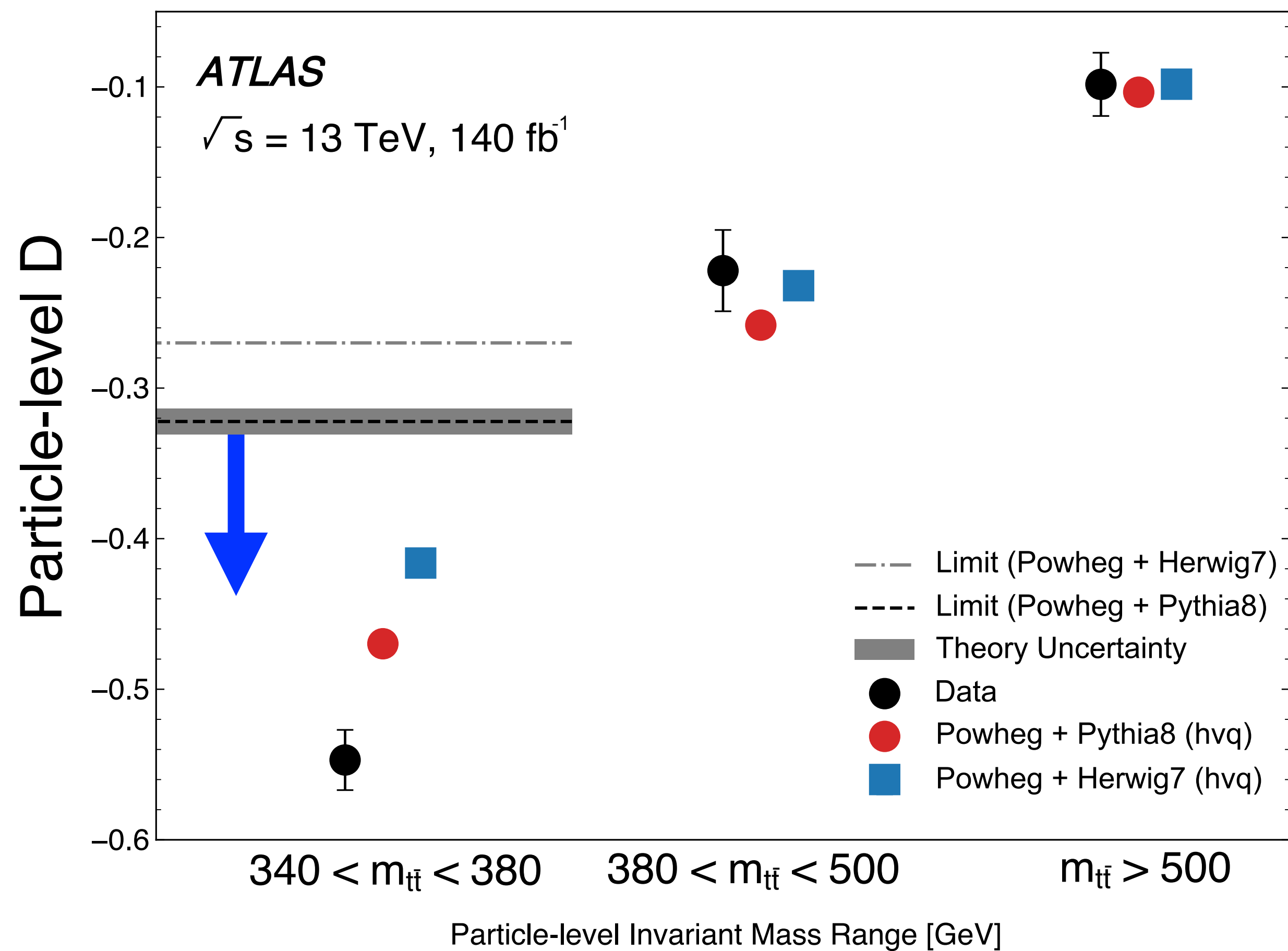
$$C_{ij} = -9 \langle q_i^+ q_j^- \rangle$$

$$D \equiv \frac{1}{3} \text{Tr} C = -3 \langle \cos \varphi \rangle < -\frac{1}{3} \rightarrow \text{Entanglement}$$





[ATLAS 2311.07288]

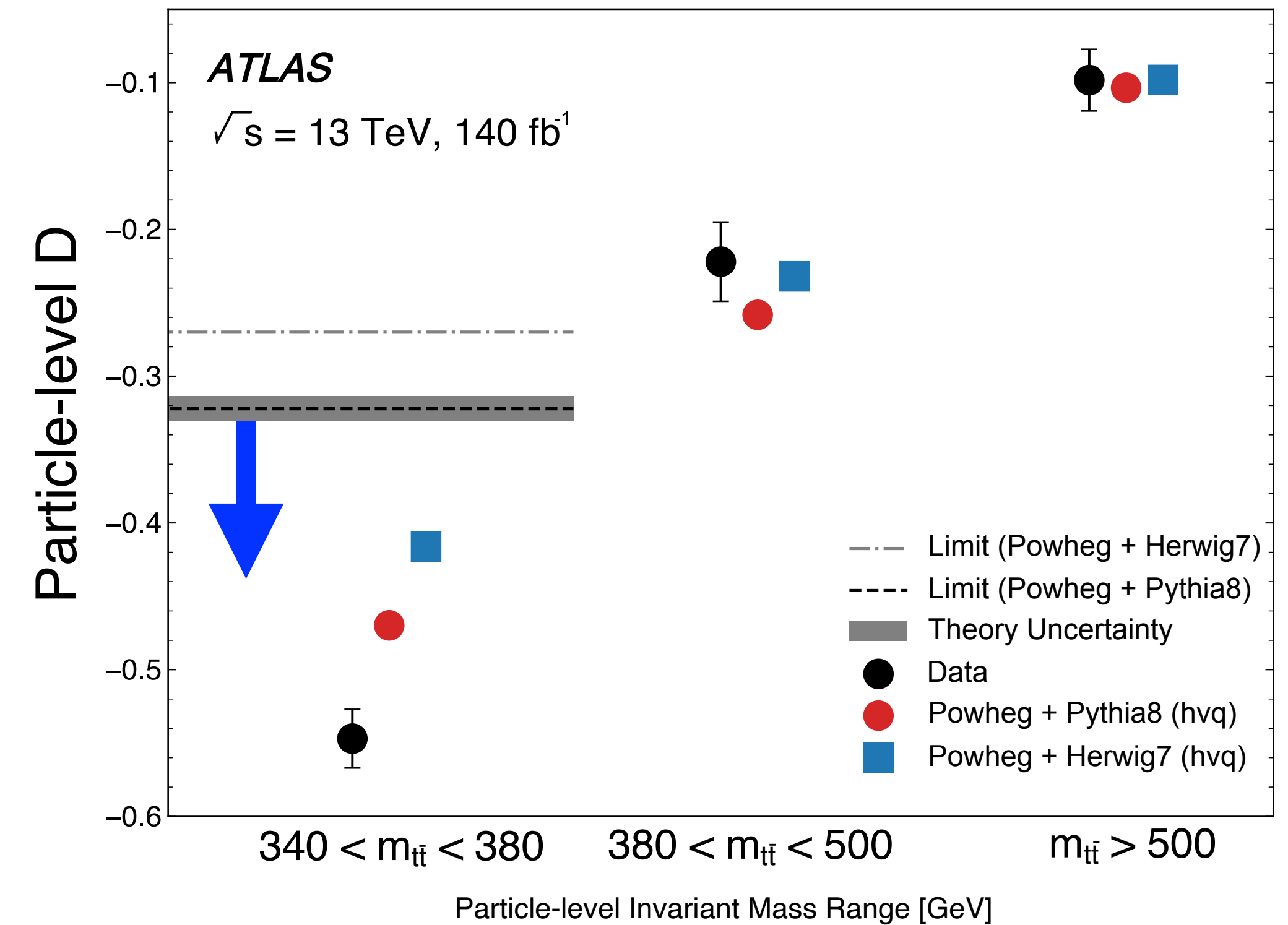
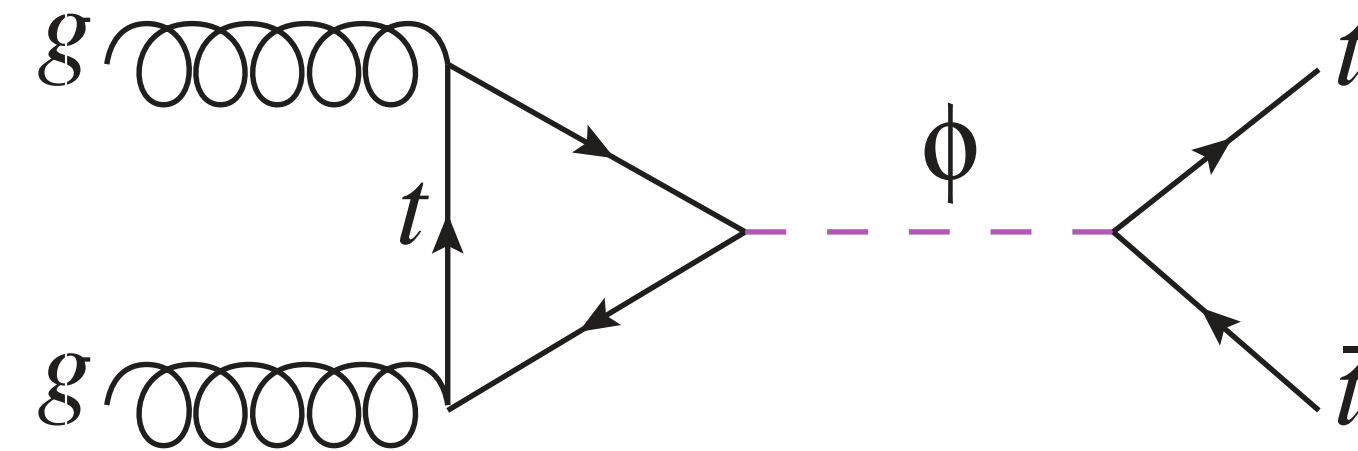
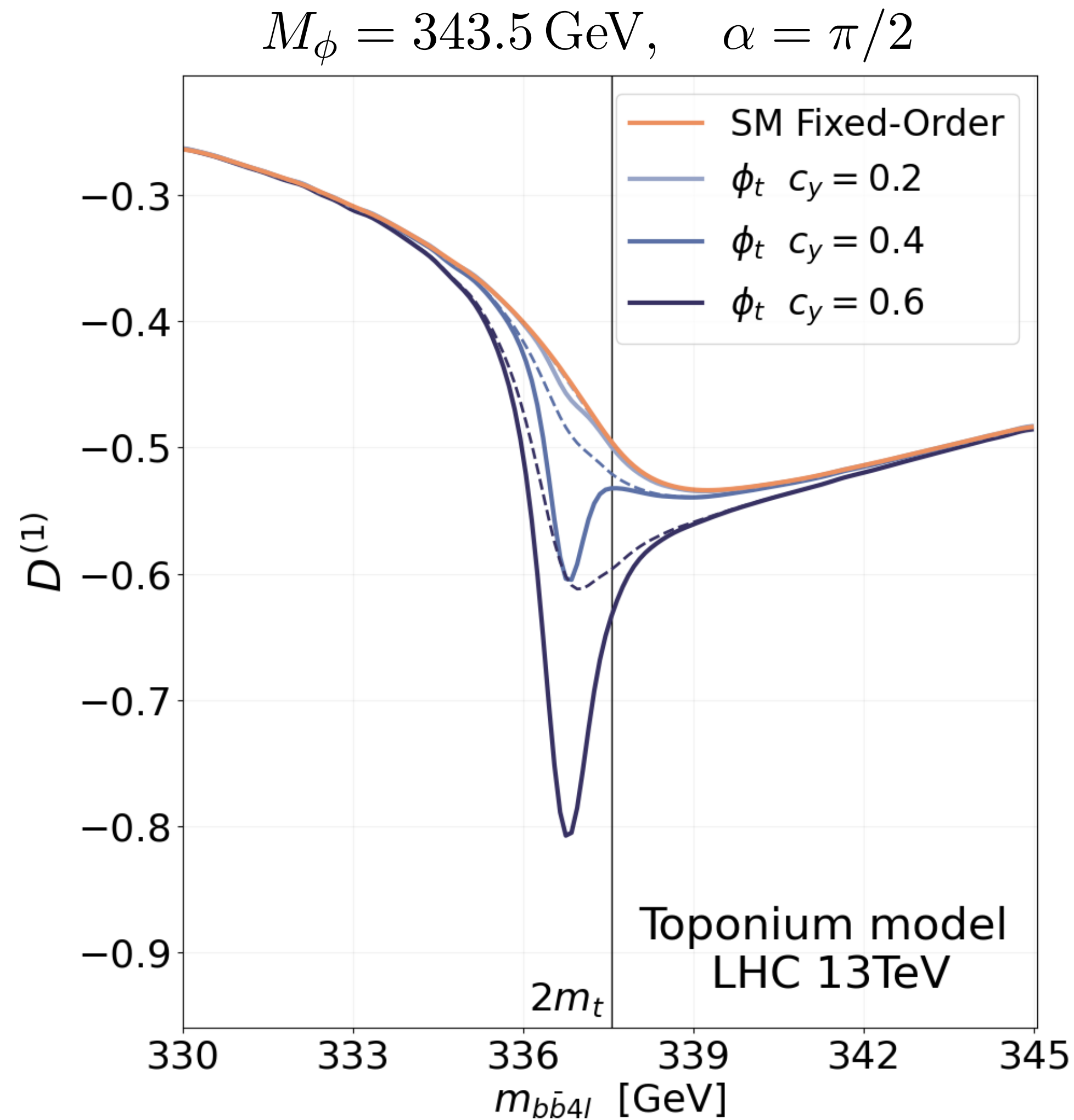


$$D \equiv \frac{1}{3} \text{Tr } C = -3 \langle \cos \varphi \rangle < -\frac{1}{3}$$

➡ Entanglement

Toponium-like particle: $\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{2}\phi(\partial^2 + M_\phi^2)\phi + c_y \frac{y_t}{\sqrt{2}} \phi \bar{t} (\cos \alpha + i\gamma^5 \sin \alpha) t.$

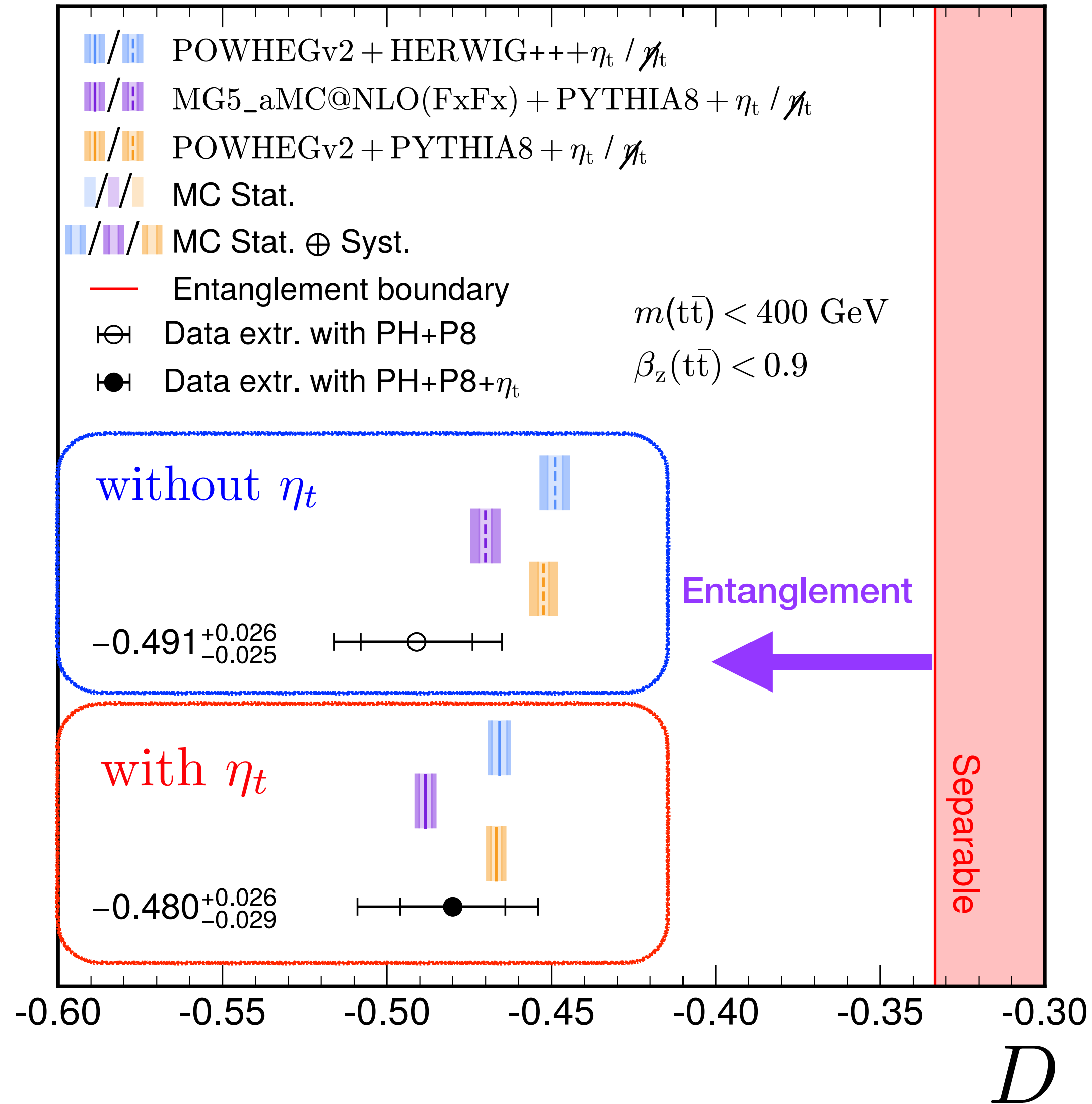
[Maltoni, Severi, Tentori, Vryonidou 2401.08751]



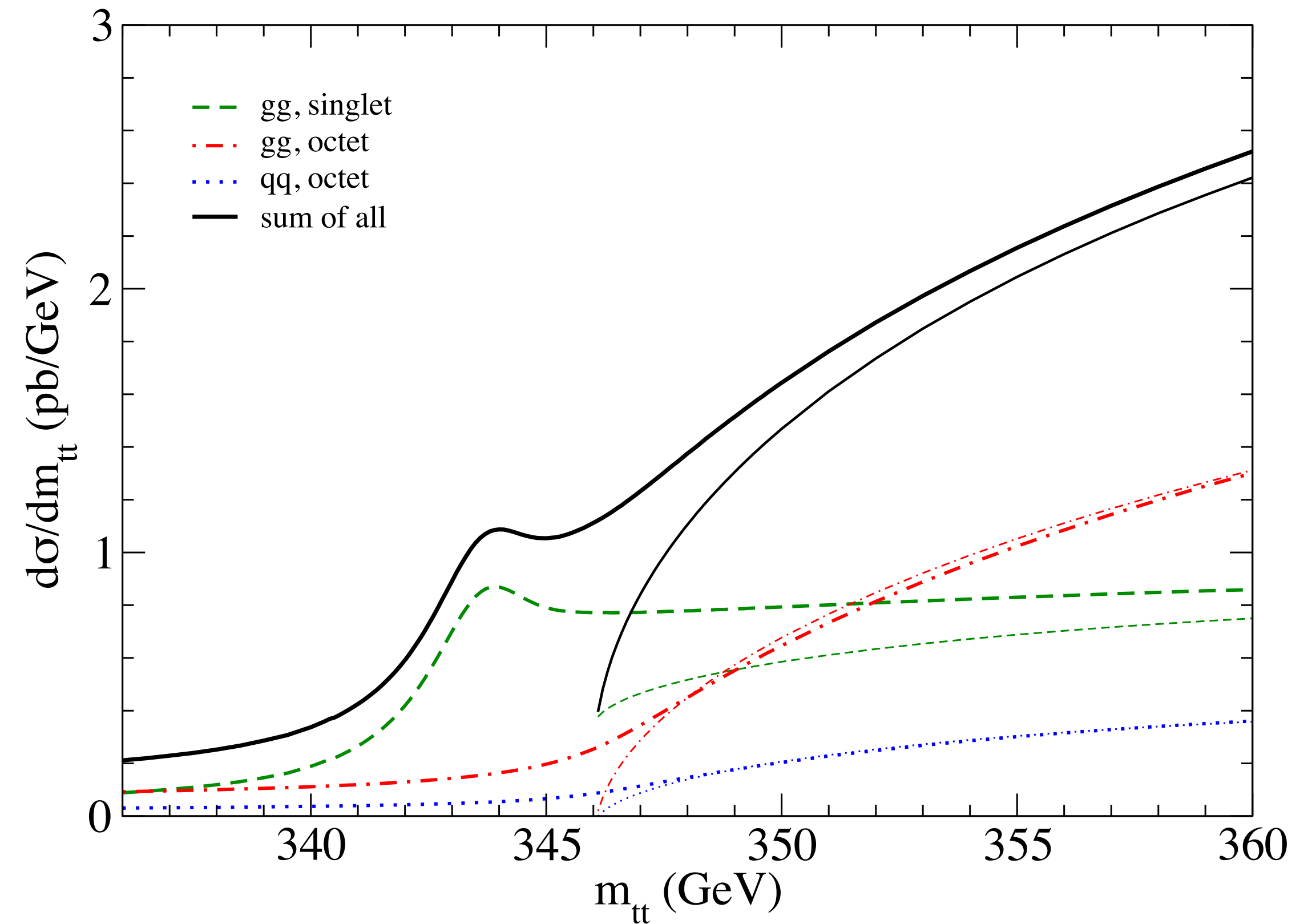
[CMS 2406.03976]

CMS

36.3 fb⁻¹ (13 TeV)



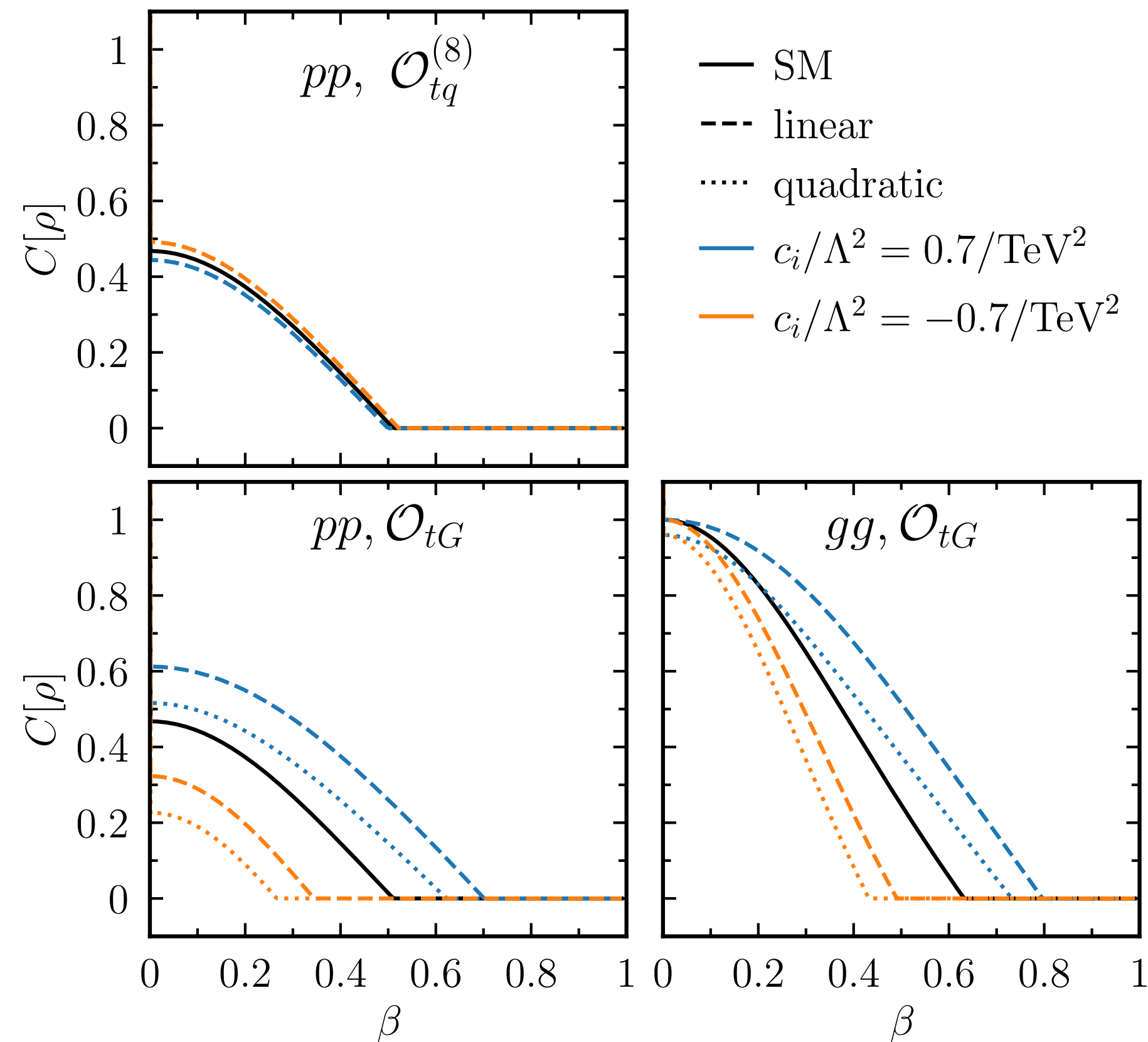
Hagiwara, Sumino, Yokoya [0804.1014]



Is entanglement sensitive to New Physics?

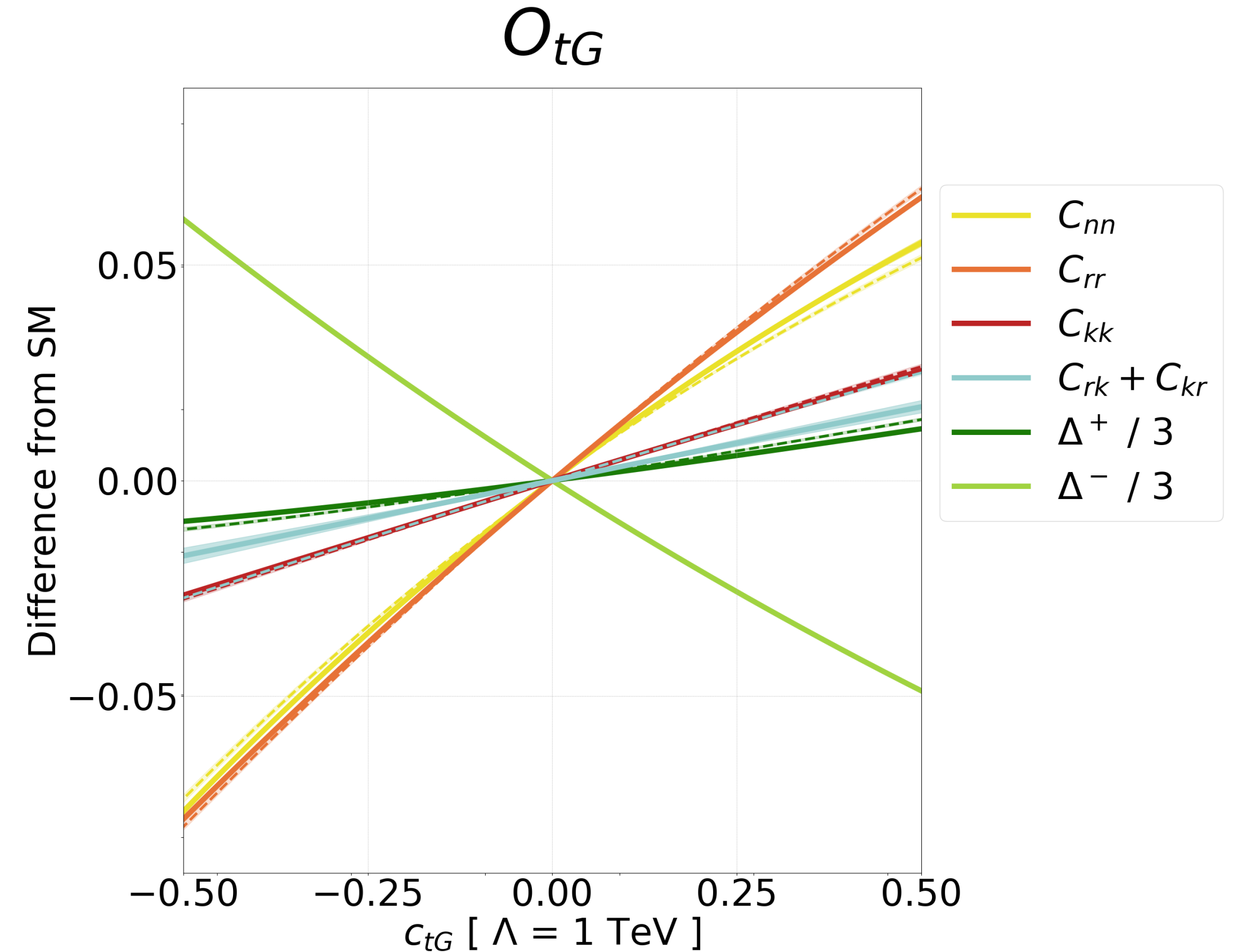
$$\mathcal{O}_{tG} = g_S \bar{Q} T_A \tilde{\varphi} \sigma^{\mu\nu} t G_{\mu\nu}^A \quad \mathcal{O}_{tq}^8 = \sum_{f=1}^2 (\bar{q}_f \gamma_\mu T_A q_f) (\bar{t} \gamma^\mu T^A t)$$

[Aoude, Madge, Maltoni, Mantani (2022)]



$\beta = \sqrt{1 - 4m_t^2/M_{t\bar{t}}^2}$: top velocity in the $t\bar{t}$ rest frame

[Severi, Vryonidou (2023)]



$$\Delta^\pm = \pm(C_{kk} + C_{rr}) - C_{nn} - 1$$

Is entanglement sensitive to New Physics?

rate: $c_{Qq}^{(8,3)} + c_{Qq}^{(8,1)} + c_{Qu}^8 + c_{tq}^8 + c_{tu}^8,$

spin $(B_k + \bar{B}_k)$: $-c_{Qq}^{(8,3)} - c_{Qq}^{(8,1)} - c_{Qu}^8 + c_{tq}^8 + c_{tu}^8.$

$t\bar{t}$ spin-correlation/entanglement only

Severi, Vryonidou (2023)

Operator	Run III Projection 300 fb ⁻¹ Differential	Current Global Fit
$\mathcal{O}_{tG}$	$[-0.03, 0.04]$	$[0.00, 0.11]$
$\mathcal{O}_{tu}^8$	$[-1.0, 0.7]$	$[-0.9, 0.3]$
$\mathcal{O}_{td}^8$	$[-1.3, 1.0]$	$[-1.3, 0.6]$
$\mathcal{O}_{tq}^8$	$[-0.7, 0.5]$	$[-0.5, 0.4]$
$\mathcal{O}_{Qu}^8$	$[-0.7, 0.6]$	$[-1.0, 0.5]$
$\mathcal{O}_{Qd}^8$	$[-0.9, 0.8]$	$[-1.6, 0.9]$
$\mathcal{O}_{Qq}^{(1,8)}$	$[-0.4, 0.3]$	$[-0.4, 0.3]$
$\mathcal{O}_{Qq}^{(3,8)}$	$[-1.1, 0.8]$	$[-0.5, 0.4]$

- spin-correlation measurement has a good sensitivity to new physics

- The SMEFiT analysis combined 317 measurements: **[105.00006]**
- 150 top-quark measurements:
 $t\bar{t}$ differential distributions, single-top, $t\bar{t}Z$, $t\bar{t}W$, tZ , tW , four-top, $t\bar{t}b\bar{b}$, etc.
 - 97 Higgs measurements.
 - 70 diboson measurements: 40 from LEP-2 and 30 from the LHC.

Observation of Bell nonlocal (BNL) state:

$$\hat{I}(\{\vec{a}, \vec{b}\}) \equiv |\hat{A}_0(\hat{B}_0 + \hat{B}_1) + \hat{A}_1(\hat{B}_0 - \hat{B}_1)|$$

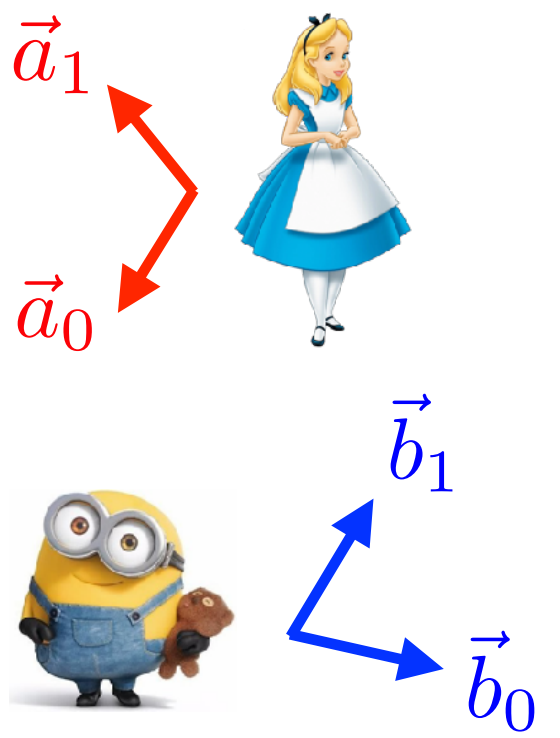
$$\langle \hat{I}(\{\vec{a}, \vec{b}\}) \rangle_{\rho_{\text{NL}}} = \text{Tr}[\hat{I}(\{\vec{a}, \vec{b}\}) \cdot \rho_{\text{NL}}] > 2 \quad \text{BNL}$$

$$\hat{A}_0 = \vec{a}_0 \cdot \vec{\sigma}$$

$$\hat{A}_1 = \vec{a}_1 \cdot \vec{\sigma}$$

$$\hat{B}_0 = \vec{b}_0 \cdot \vec{\sigma}$$

$$\hat{B}_1 = \vec{b}_1 \cdot \vec{\sigma}$$



Observation of Bell nonlocal (BNL) state:

$$\hat{I}(\{\vec{a}, \vec{b}\}) \equiv |\hat{A}_0(\hat{B}_0 + \hat{B}_1) + \hat{A}_1(\hat{B}_0 - \hat{B}_1)|$$

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- For general state: $\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i (\sigma_i \otimes \mathbb{1}_2) + \bar{B}_j (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right)$

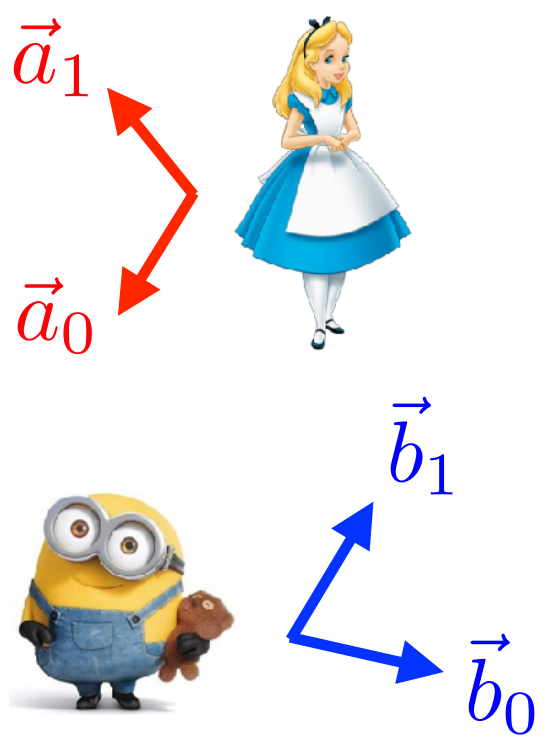
$$\text{Tr}[\hat{I} \cdot \rho] = |(a_0)_i C_{ij} (b_0 + b_1)_j + (a_1)_i C_{ij} (b_0 - b_1)_j|$$

$$\hat{A}_0 = \vec{a}_0 \cdot \vec{\sigma}$$

$$\hat{A}_1 = \vec{a}_1 \cdot \vec{\sigma}$$

$$\hat{B}_0 = \vec{b}_0 \cdot \vec{\sigma}$$

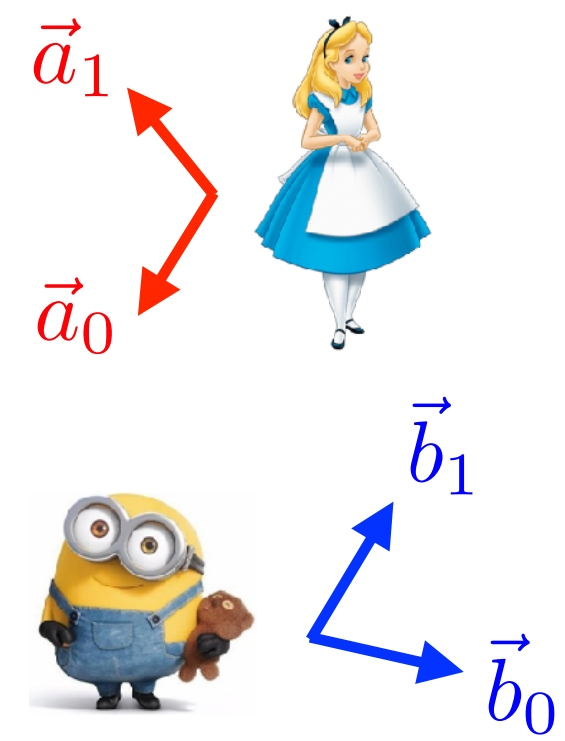
$$\hat{B}_1 = \vec{b}_1 \cdot \vec{\sigma}$$



Observation of Bell nonlocal (BNL) state:

$$\hat{A}_0 = \vec{a}_0 \cdot \vec{\sigma}$$

$$\hat{A}_1 = \vec{a}_1 \cdot \vec{\sigma}$$



$$\hat{I}(\{\vec{a}, \vec{b}\}) \equiv |\hat{A}_0(\hat{B}_0 + \hat{B}_1) + \hat{A}_1(\hat{B}_0 - \hat{B}_1)|$$

$$\hat{B}_0 = \vec{b}_0 \cdot \vec{\sigma}$$

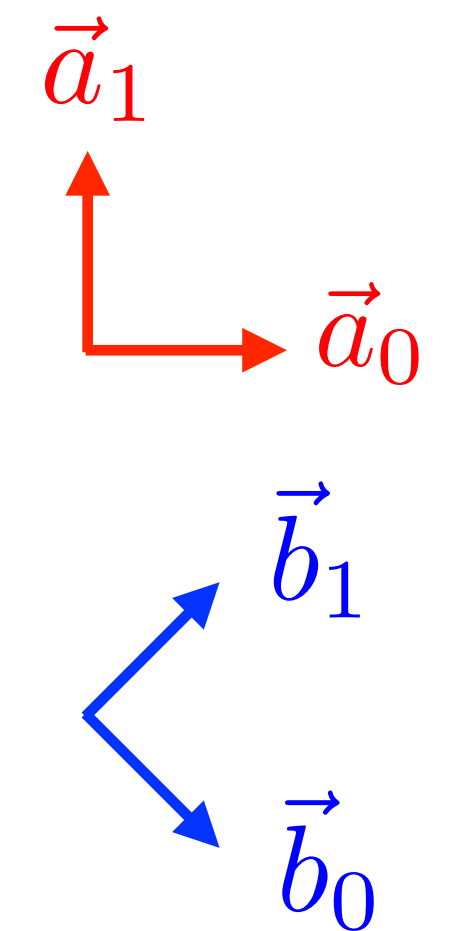
$$\hat{B}_1 = \vec{b}_1 \cdot \vec{\sigma}$$

$$\langle \hat{I}(\{\vec{a}, \vec{b}\}) \rangle_{\rho_{\text{NL}}} = \text{Tr}[\hat{I}(\{\vec{a}, \vec{b}\}) \cdot \rho_{\text{NL}}] > 2 \quad \text{BNL}$$

- For general state: $\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i (\sigma_i \otimes \mathbb{1}_2) + \bar{B}_j (\mathbb{1} \otimes \sigma_j) + C_{ij} (\sigma_i \otimes \sigma_j) \right)$

$$\text{Tr}[\hat{I} \cdot \rho] = |(a_0)_i C_{ij} (b_0 + b_1)_j + (a_1)_i C_{ij} (b_0 - b_1)_j| = \sqrt{2} |C_{ii} \pm C_{jj}|$$

$$\vec{a}_0 = \hat{i}, \quad \vec{a}_1 = \hat{j}, \quad b_0 = \pm \frac{1}{\sqrt{2}}(\hat{i} + \hat{j}), \quad \vec{b}_1 = \pm \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$



Aguilar-Saavedra, Casas [2205.00542]

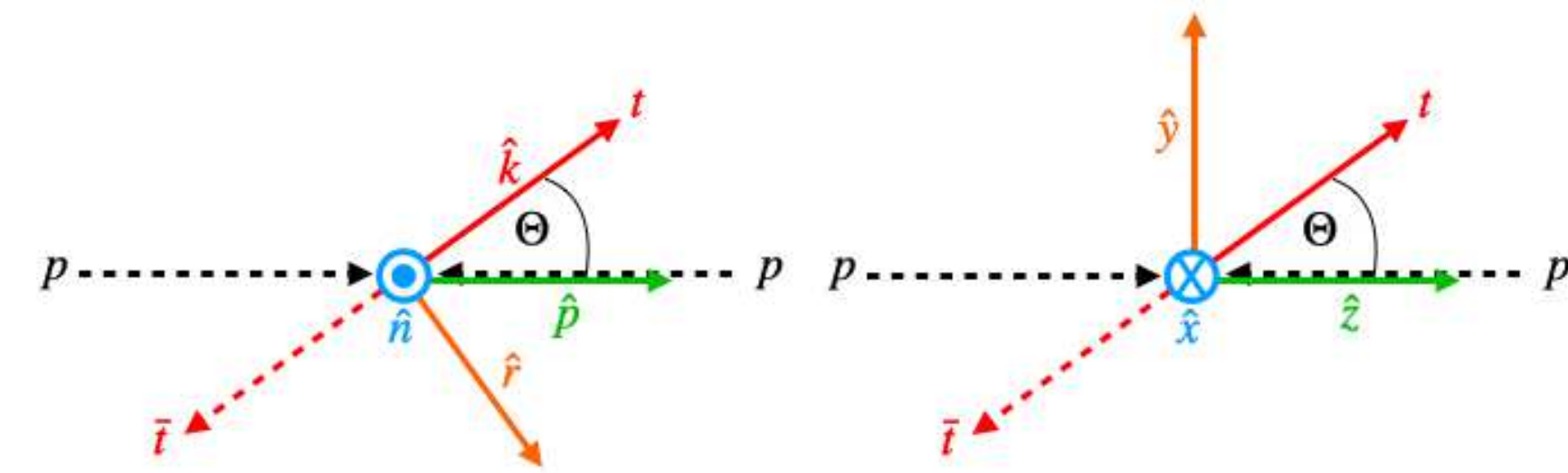
$$|C_{ii} + C_{jj}| > \sqrt{2} \quad \text{or} \quad |C_{ii} - C_{jj}| > \sqrt{2}$$

➡ the state is **BNL**

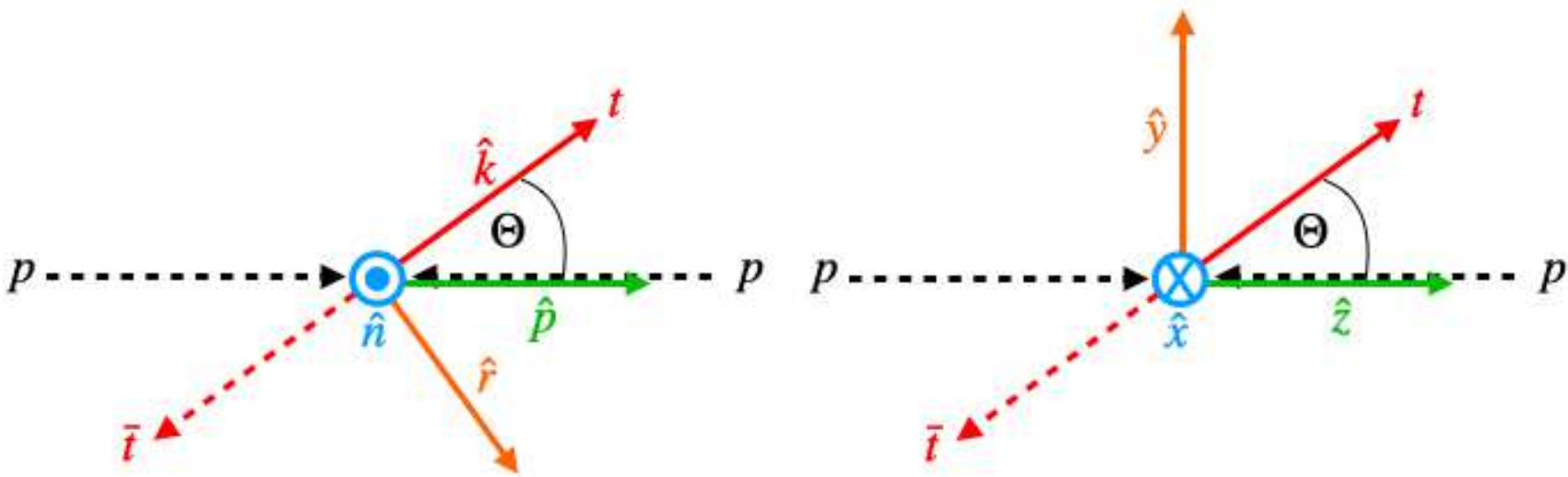
Experimental Reconstruction / Prospects

$$\varphi_+ = \frac{1}{2}(\varphi_a + \varphi_b), \quad \varphi_- = \frac{1}{2}(\varphi_a - \varphi_b) \quad a = \ell^+, \quad b = \ell^-$$

$$\frac{1}{\sigma} \frac{d\sigma}{d\varphi_+ d\varphi_-} = \frac{1}{2\pi^2} - \frac{1}{32} \left[\frac{C_{11} + C_{22}}{2} \cos 2\varphi_- + \frac{C_{11} - C_{22}}{2} \cos 2\varphi_+ + \frac{C_{12} + C_{21}}{2} \sin 2\varphi_+ + \frac{C_{21} - C_{12}}{2} \sin 2\varphi_- \right]$$



Experimental Reconstruction / Prospects

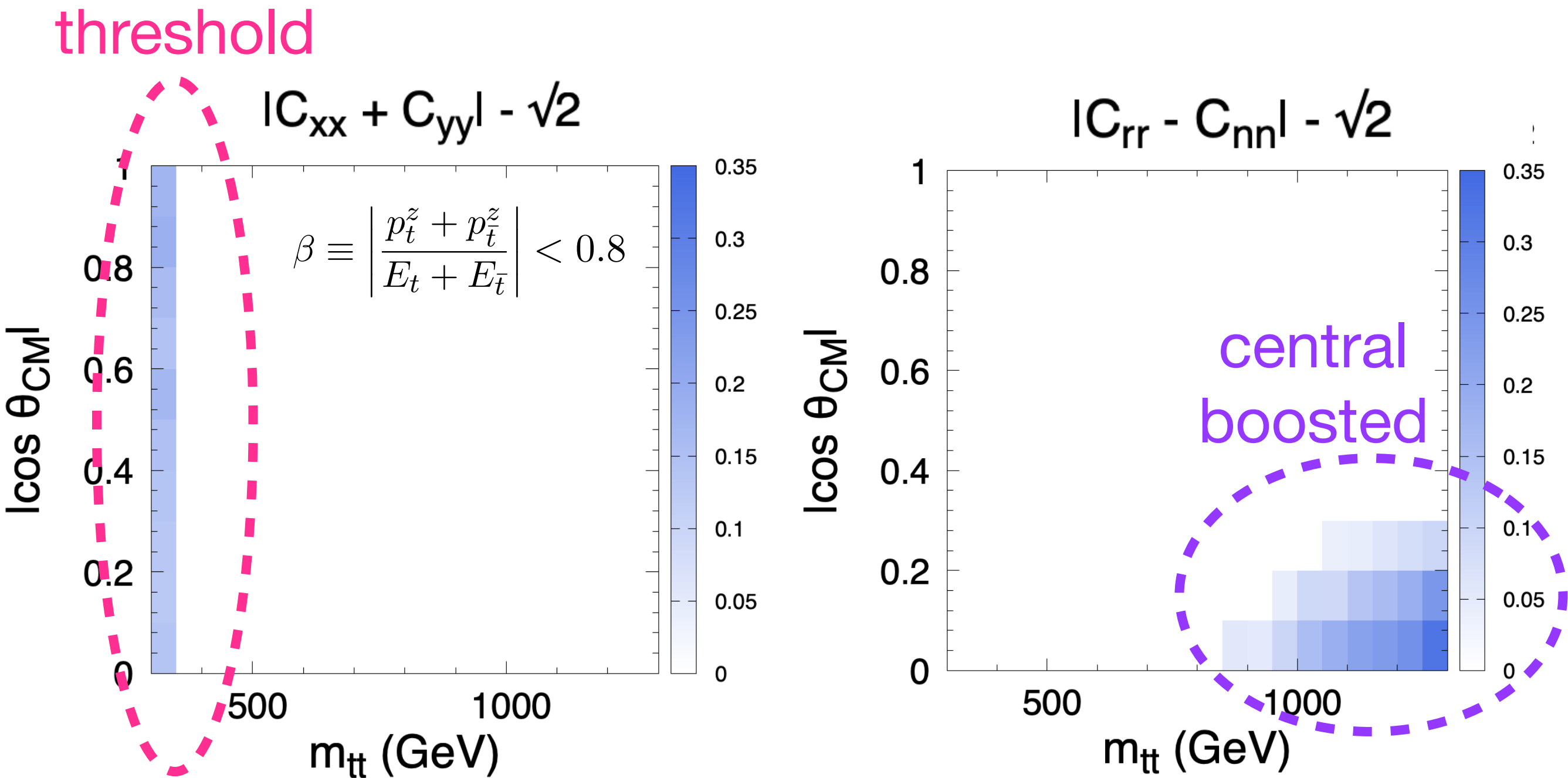


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$$A_{\pm} = \frac{N(\cos(\varphi_a \mp \varphi_b) > 0) - N(\cos(\varphi_a \mp \varphi_b) < 0)}{N(\cos(\varphi_a \mp \varphi_b) > 0) + N(\cos(\varphi_a \mp \varphi_b) < 0)}$$

$$C_{11} \pm C_{22} = -\frac{16}{\pi} A_{\pm}$$

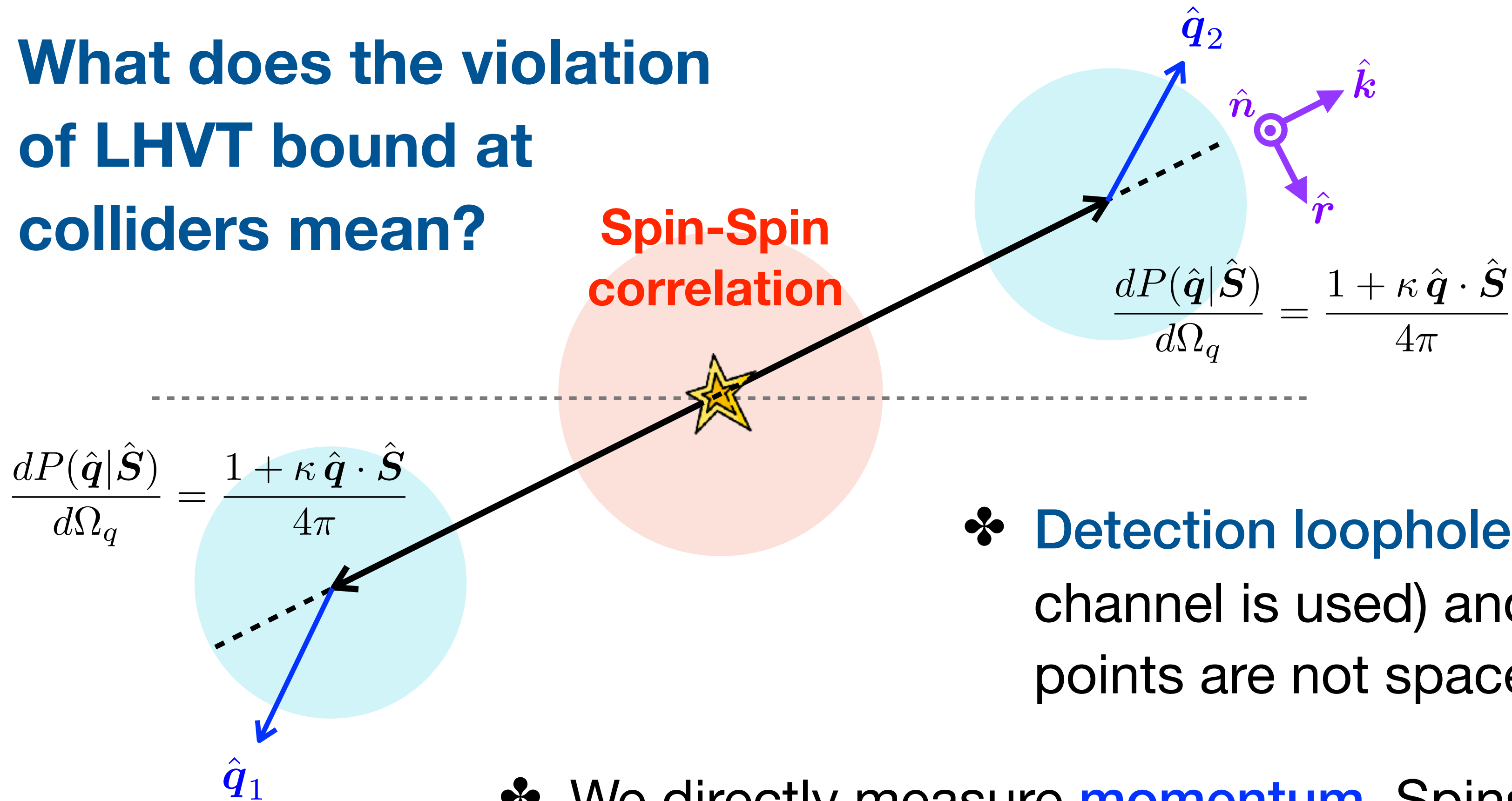


Aguilar-Saavedra, Casas [2205.00542]

selection	indicator	300 fb ⁻¹	3000 fb ⁻¹
$m_{t\bar{t}} \leq 353, \beta \leq 0.8$	$ C_{xx} + C_{yy} $	2.7σ	9.5σ
$m_{t\bar{t}} \geq 1000,$ $ \cos \theta_{\text{CM}} \leq 0.2$	$ C_{rr} - C_{nn} $	1.7σ	5.8σ

Bell Nonlocality can be seen with
> 5σ at HL-LHC

What does the violation of LHVT bound at colliders mean?



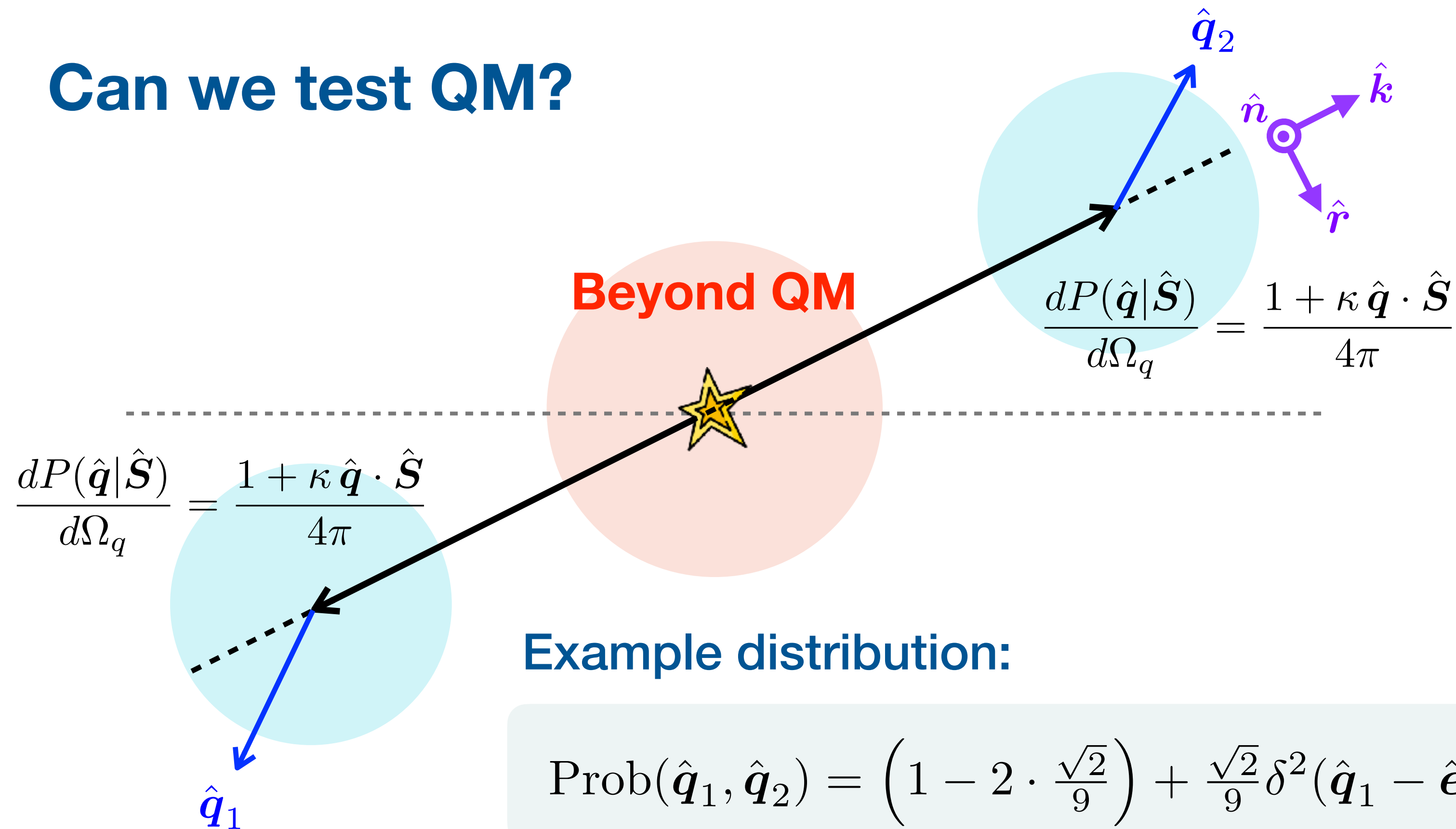
❖ **Detection loophole** (only a part of the leptonic channel is used) and **locality loophole** (two decay points are not space-like separated).

❖ We directly measure **momentum**. Spins are only inferred from them. The observed momentum dist can be explained by LHVT.

❖ LHVT is excluded by low-energy experiments. What is the point?

Abel Dittmar, Dreiner (1992), Dreiner, Ubaldi [2507.15949], Bechtle, Breuning, Dreiner, Duhr [2507.15947], M.Low [2508.10979], Aguilar-Saavedra, Casas [2205.00542], Aguilar-Saavedra, Casas Moreno [2603.19389]

Can we test QM?



Example distribution:

$$\hat{e}_{\pm} = (\hat{r} \pm \hat{n})/\sqrt{2}$$

$$\text{Prob}(\hat{q}_1, \hat{q}_2) = \left(1 - 2 \cdot \frac{\sqrt{2}}{9}\right) + \frac{\sqrt{2}}{9} \delta^2(\hat{q}_1 - \hat{e}_+) \delta^2(\hat{q}_2 - \hat{e}_+) + \frac{\sqrt{2}}{9} \delta^2(\hat{q}_1 - \hat{e}_-) \delta^2(\hat{q}_2 - \hat{e}_-)$$

$$\langle \hat{A}_x \hat{B}_y \rangle = \frac{9}{\kappa_1 \kappa_2} \langle (\hat{q}_1 \cdot \vec{a}_x) (\hat{q}_2 \cdot \vec{b}_y) \rangle \quad \vec{a}_0 = \hat{r} \quad \vec{a}_1 = \hat{n} \quad \vec{b}_0 = \hat{e}_+ \quad \vec{b}_1 = \hat{e}_-$$

$$\kappa_1 = -\kappa_2 = 1 \quad \langle I \rangle = \underbrace{\langle \hat{A}_0 \hat{B}_0 \rangle}_{+1} + \underbrace{\langle \hat{A}_0 \hat{B}_1 \rangle}_{+1} + \underbrace{\langle \hat{A}_1 \hat{B}_0 \rangle}_{+1} - \underbrace{\langle \hat{A}_1 \hat{B}_1 \rangle}_{-1} = 4 > 2\sqrt{2} \quad (\text{QM bound})$$

Detection of QM violation is in principle conceivable!

Beyond entanglement

CMS Physics Analysis Summary

Contact: cms-pag-conveners-top@cern.ch

2025/03/12

Observation of magic states of top quark pairs produced in proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

Abstract

An observation of magic states of top quark-antiquark ($t\bar{t}$) systems produced in
proton-proton collisions at the LHC at $\sqrt{s} = 13$ TeV is presented. Magic, defined

Quantum Magic

- Quantum computation promises substantial speedup compared to the classical counterpart.
- **What is the essence of the quantum advantage?** → entanglement (?)

Quantum Magic

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- **What is the essence of the quantum advantage?** → entanglement (?)

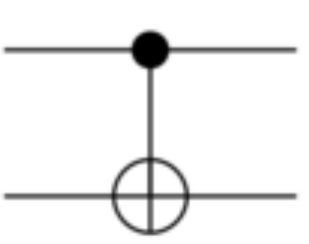
Gottesman-Knill theorem: Any quantum circuit consisting only of

- **Clifford gates** (H, S, CNOT)
- Computational Input states, e.g., $|0\rangle^{\otimes n}$

D. Gottesman (1998)

can be **simulated** on a **classical computer** in **polynomial time**.

Clifford gates

Hadamard		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
CNOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

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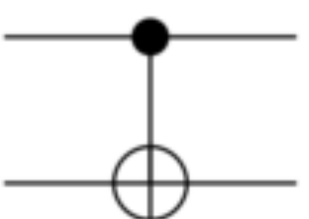
D. Gottesman (1998)

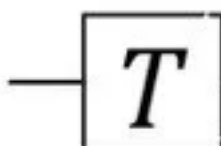
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Universal gates

Clifford gates

Hadamard		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
----------	---	--

CNOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
------	---	--

T Gate		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i \cdot \frac{\pi}{4}} \end{bmatrix}$
--------	---	--

Quantum Magic

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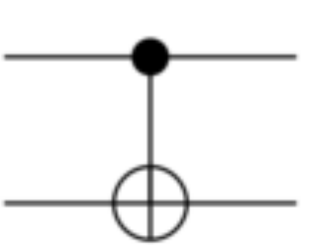
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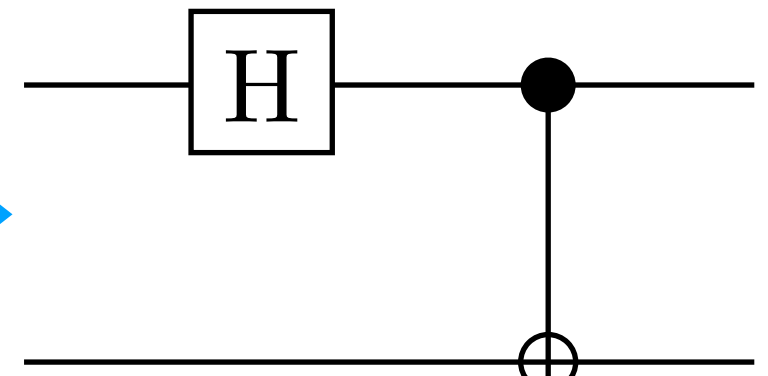
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Clifford gates *can create* maximally **entangled** states:

$$|00\rangle \xrightarrow{\text{Circuit}} \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \quad \left. \vphantom{\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)} \right\} \text{Entanglement is *not* the essence of quantum advantage!}$$




- The essence of quantum advantage is non-Cliffordness, called “**magic**” (non-stabiliserness)

- A useful measure of magic is the **stabiliser Rényi entropy** (SRE).

L.Leone, S.Oliviero, A.Hamma (2022)

- For probabilities $p_i \geq 0$, $\sum_i p_i = 1$, the **Rényi entropy of order $\alpha > 0$** is defined as

$$H_\alpha(\{p\}) = \frac{1}{1-\alpha} \log \left(\sum_i p_i^\alpha \right)$$

- In the limit $\alpha \rightarrow 1$, the Rényi entropy approaches the **Shannon entropy**:

$$\lim_{\alpha \rightarrow 1} H_\alpha(\{p\}) = - \sum_i p_i \log p_i$$

- For any $|\psi\rangle$ of the n -qubit system, the **Pauli spectrum** forms a probability distribution.

$$\Xi_P \equiv \frac{1}{d} \langle \psi | P | \psi \rangle^2 \geq 0 \implies \sum_{P \in \mathcal{P}_n} \Xi_P = 1 \quad (d = \dim \mathcal{H} = 2^n)$$

Pauli string:

$$\mathcal{P}_n = \{ \sigma_{\mu_1} \otimes \cdots \otimes \sigma_{\mu_n} \}$$

$$\sigma_{\mu} = \{ \mathbb{1}, \sigma_1, \sigma_2, \sigma_3, \}$$

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- The **SRE** is defined as the **RE** ($\alpha = 2$) of the Pauli spectrum:

$$M_2(|\psi\rangle) \equiv -\log \sum_{P \in \mathcal{P}_n} \Xi_P^2 - \log d = -\log \left[\frac{1}{d} \sum_{P \in \mathcal{P}_n} \langle \psi | P | \psi \rangle^4 \right]$$

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- Because $\langle 0 | \sigma_{\mu} | 0 \rangle = (1, 0, 0, 1)$, for $|\psi\rangle = |0\rangle^{\otimes n}$, $\langle \psi | P | \psi \rangle = 1$ (for 2^n terms) and 0 for others.

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- Clifford unitary sends a Pauli string to another Pauli string (normaliser)

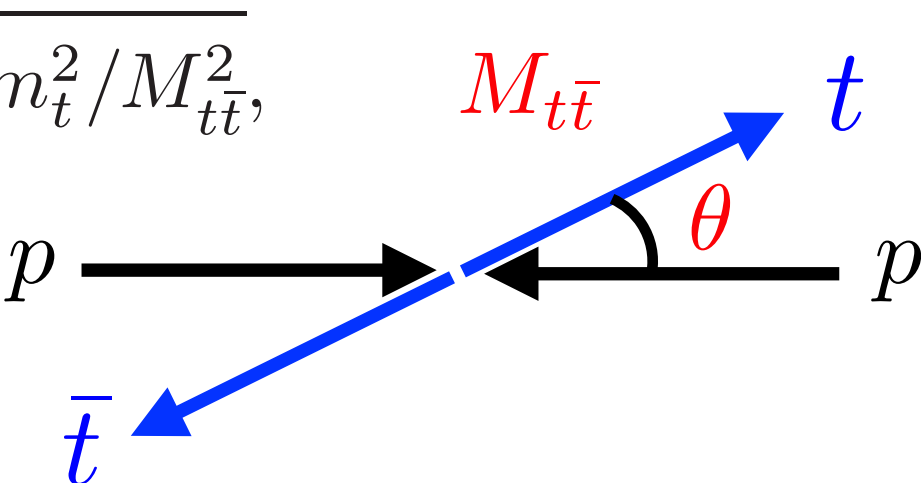
$$\mathcal{C} = \{U \mid U^\dagger \mathcal{O} U \in \mathcal{P}_n, \forall \mathcal{O} \in \mathcal{P}_n\} \longrightarrow \text{SRE is invariant under the Clifford unitaries}$$

The properties of SRE:

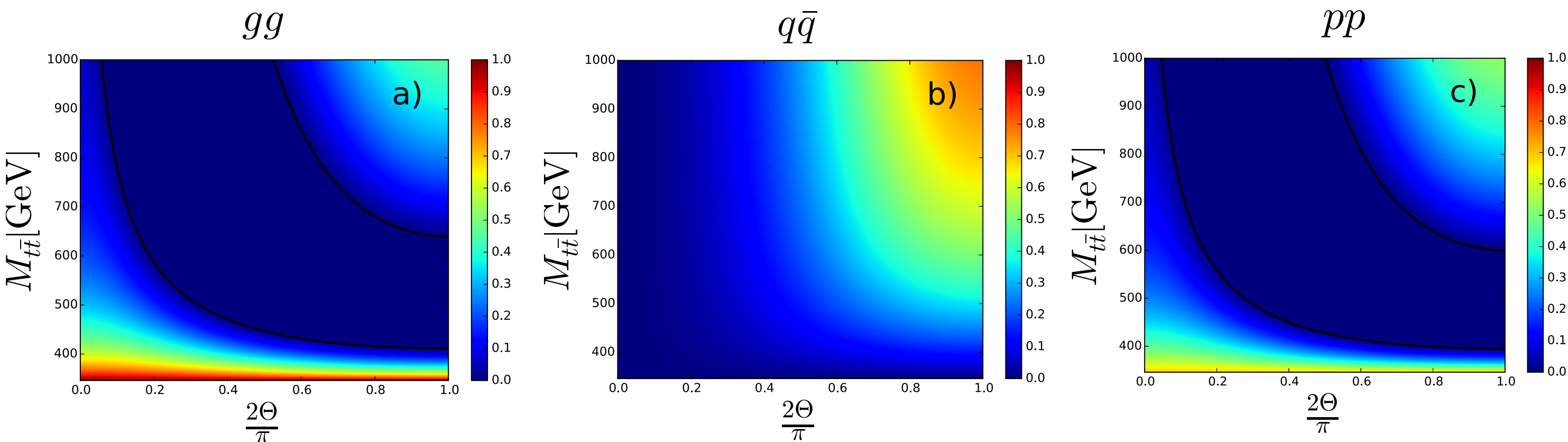
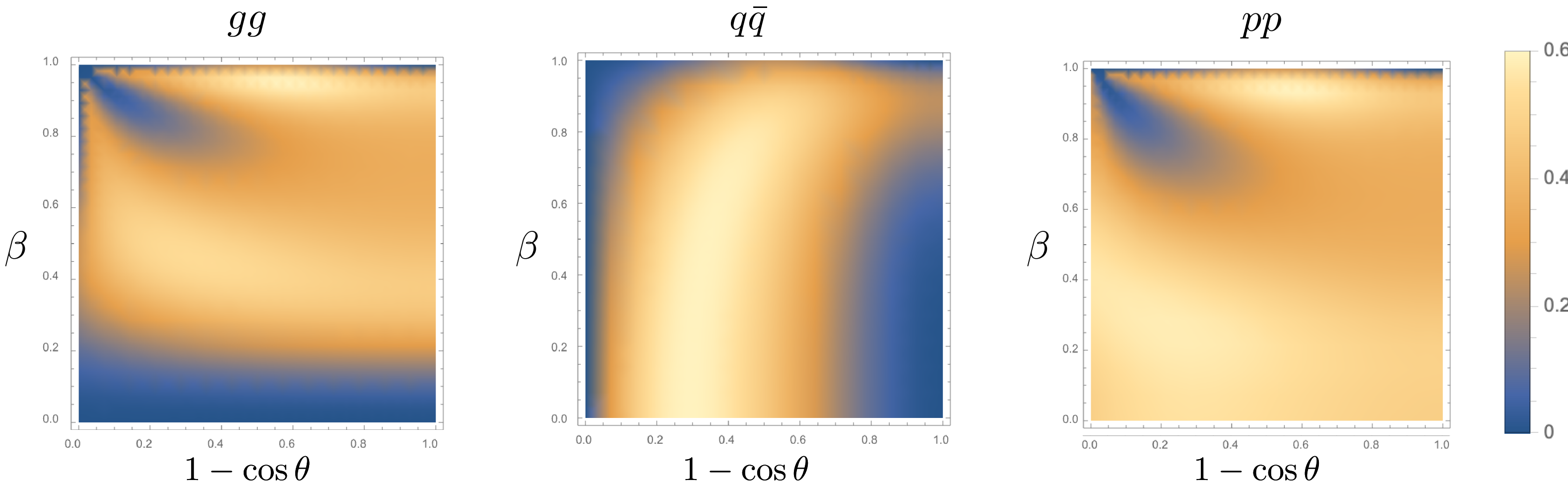
L.Leone, S.Oliviero, A.Hamma (2022)

- Invariant under Clifford unitaries:
- $M_\alpha(|\psi\rangle) = 0$ iff $|\psi\rangle$ is a stabiliser state; reachable by Clifford unitaries from $|0\rangle^{\otimes n}$.
- Resource monotone: it does not increase under the *stabiliser protocol*.
(Clifford unitaries + Pauli measurements + stabiliser ancillas)

Magic of top quarks :

$$\beta = \sqrt{1 - 4m_t^2/M_{t\bar{t}}^2},$$


White, White [2406.07321]



SRE-2 for mixed states:

$$\tilde{M}_2(\rho) = -\log_2 \left(\frac{\sum_{P \in \mathcal{P}_n} \text{Tr}^4(\rho P)}{\sum_{P \in \mathcal{P}_n} \text{Tr}^2(\rho P)} \right)$$

2-qubit maximum:

$$M_2^{\max} = \log_2 \frac{16}{7} \simeq 1.193$$

Q.Liu, I.Low, Z.Yin
[2502.17550]

Magic is largely
complementary to
Entanglement

Afik, Nova [2003.02280]

magic:

ENT:

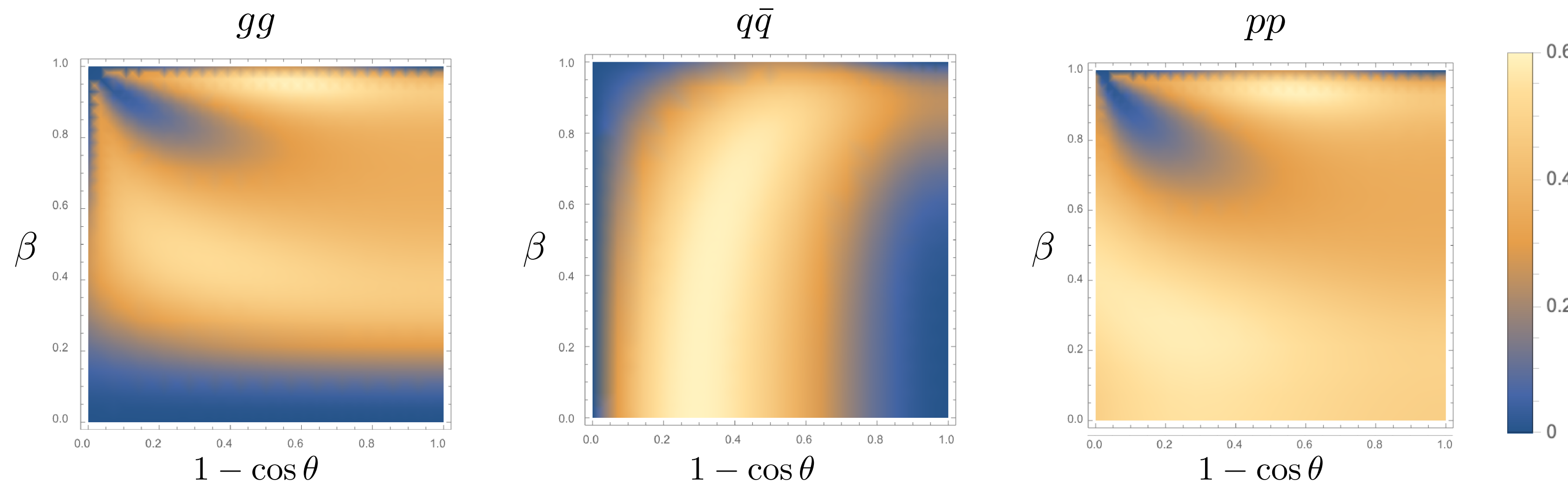
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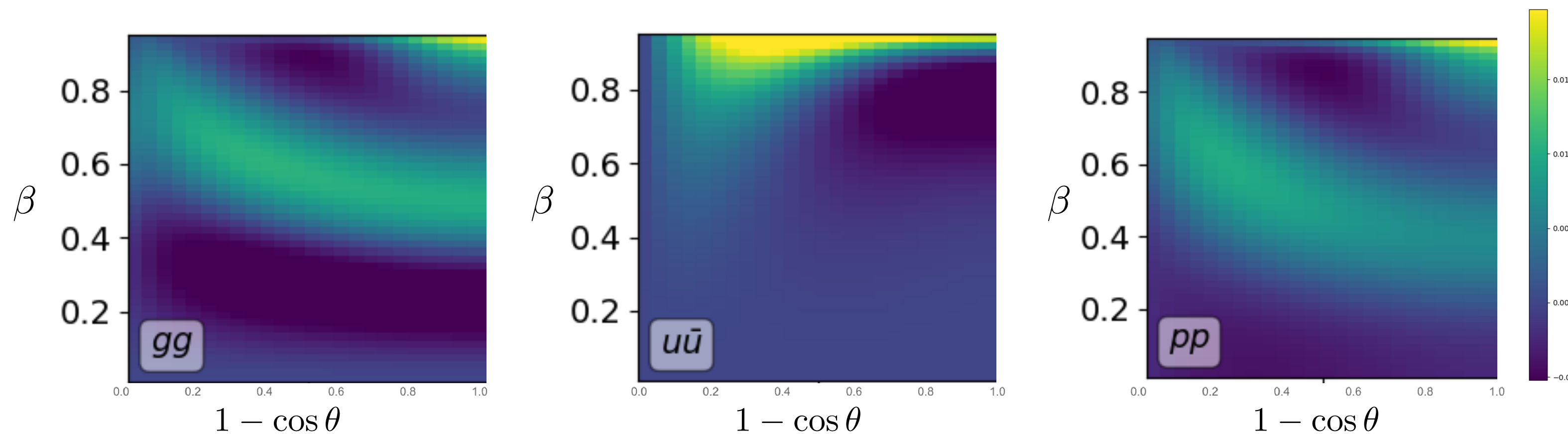
White, White [2406.07321]



magic:

Magic is potentially
useful to probe
New Physics

SMEFT - SM



$$c_{tG}/\Lambda^2 = 0.1 \text{ TeV}^{-2}$$

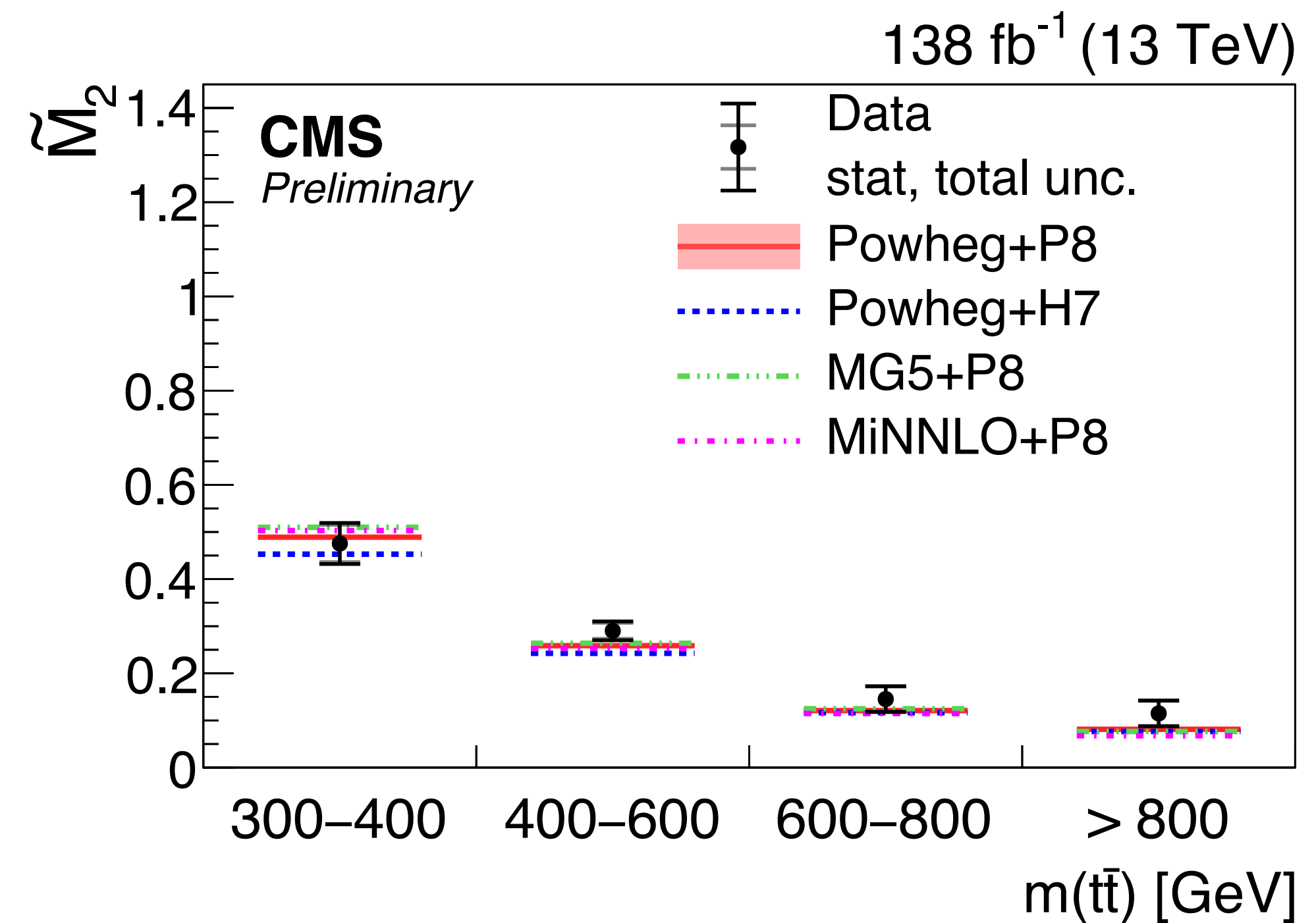
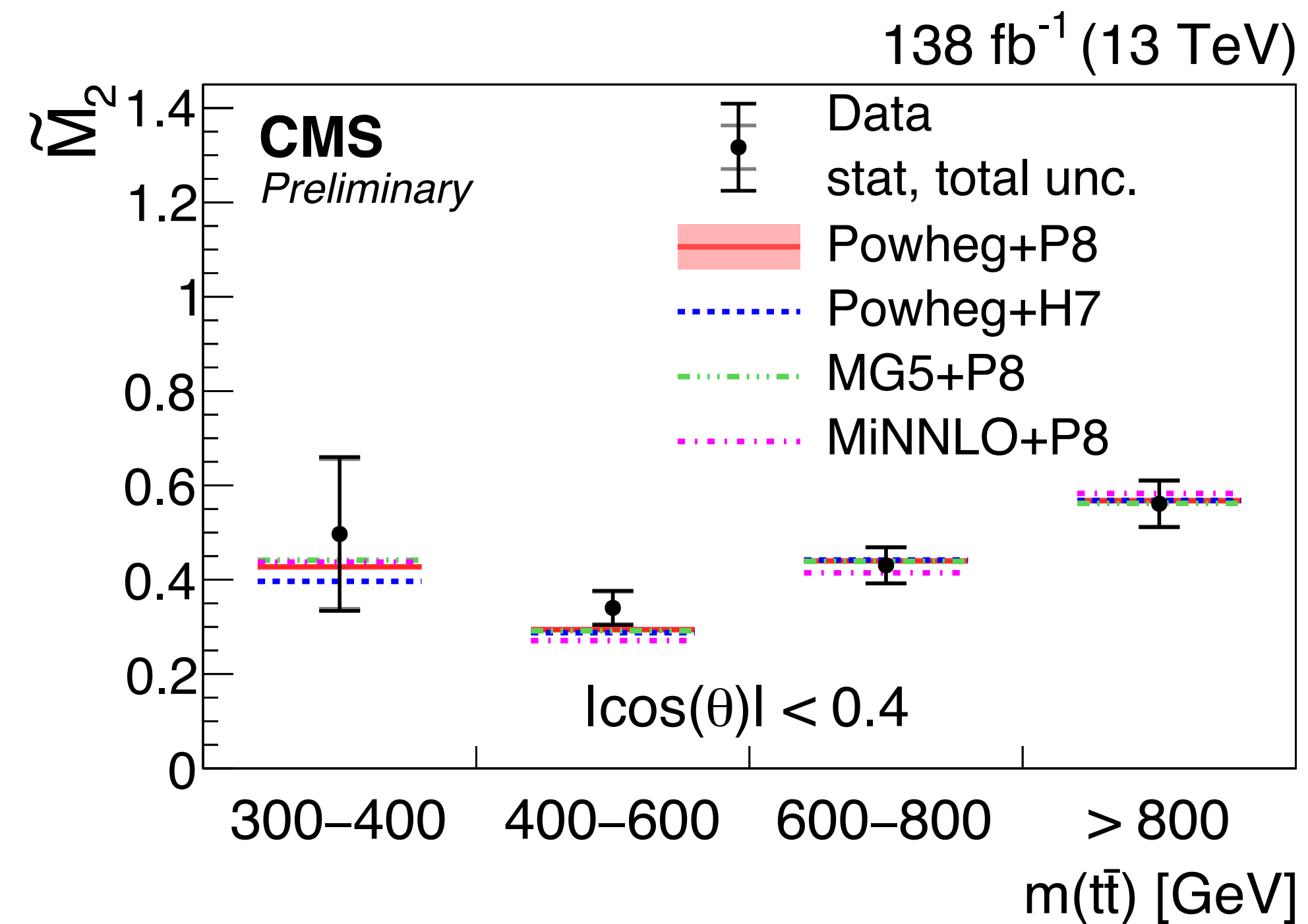
$$\mathcal{O}_{tG} = g_S \bar{Q} T_A \tilde{\varphi} \sigma^{\mu\nu} t G_{\mu\nu}^A$$

Aoude, Banks, White, White
[2505.12522]

Magic at the LHC:

- Recently, the CMS reported a first measurement of magic in top quarks

CMS PAS TOP-25-001



Good agreement between theory and data.

QI measurements @ Colliders

— beyond $t\bar{t}$ —

$$h \rightarrow \tau^- \tau^+$$

- Yukawa interaction with the CP-phase δ : $\mathcal{L} \ni -\frac{m_\tau}{v_{\text{SM}}} \kappa H \bar{\psi}_\tau (\cos \delta + i\gamma_5 \sin \delta) \psi_\tau$
- The spin state in the final state, $h \rightarrow \tau^- \tau^+$, is pure and momentum indep.:

$$|\Psi_{H \rightarrow \tau\tau}(\delta)\rangle = \frac{1}{\sqrt{2}} (|+, -\rangle + e^{i2\delta} |-, +\rangle)$$

- This state is maximally entangled regardless of δ :

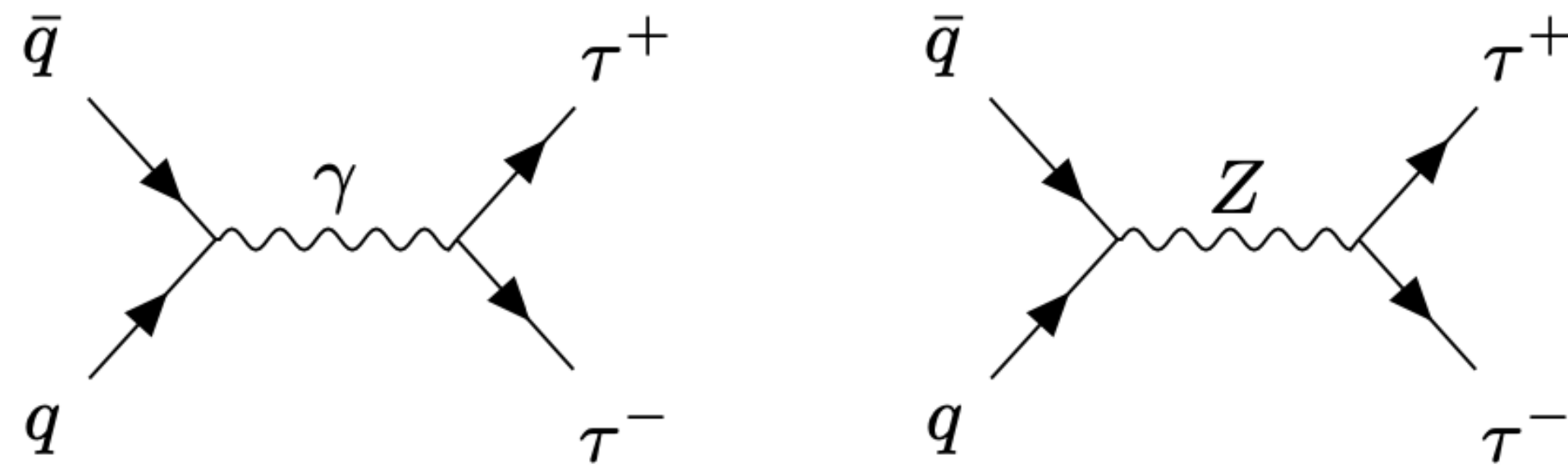
$$C(|\psi\rangle) = 1, \quad \mathcal{B}_{\text{CHSH}}(|\psi\rangle) = 2\sqrt{2}$$

- The spin correlation matrix carries the CP-phase δ

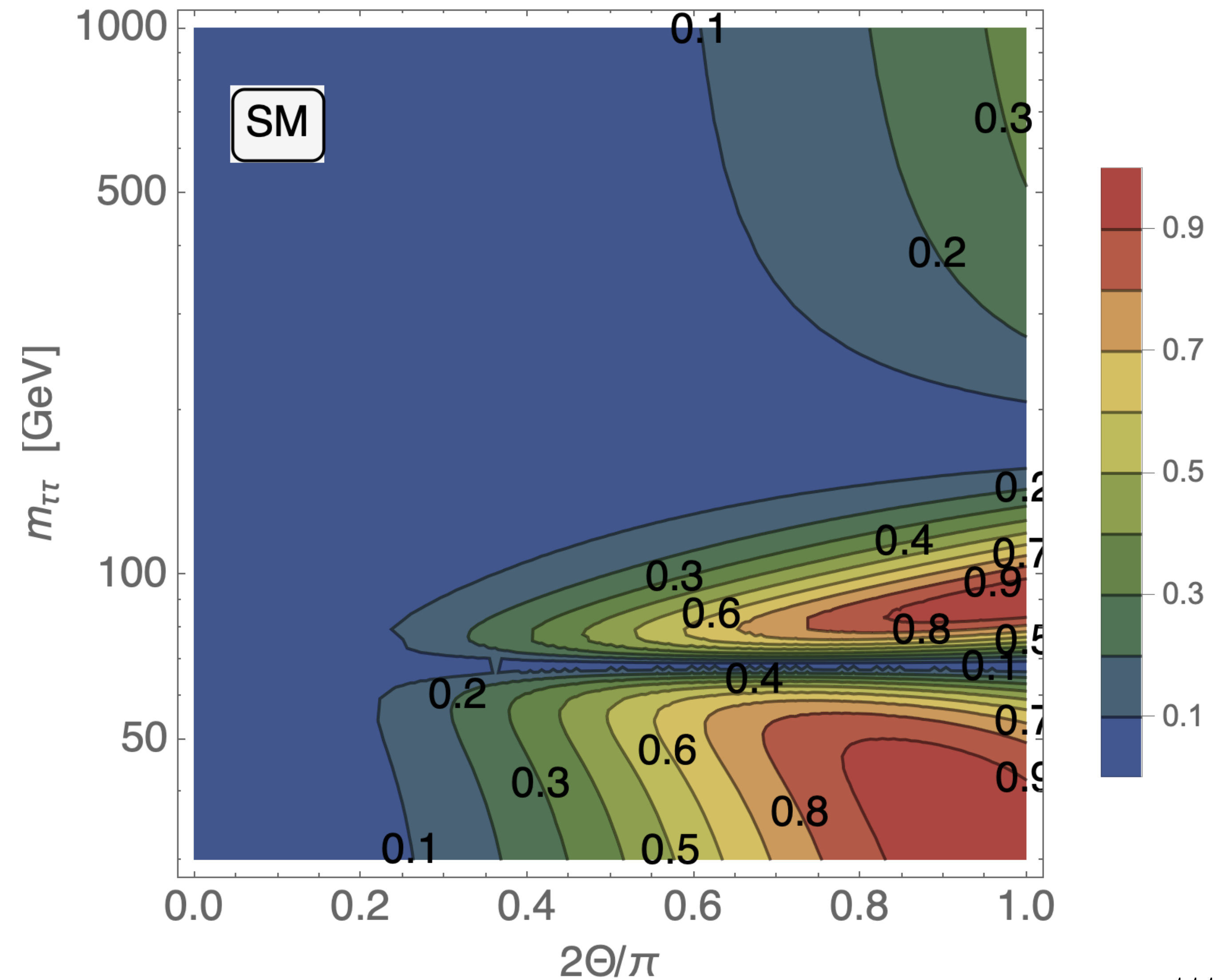
$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Drell-Yan:

Fabbrichesi, Floreanini, Gabrielli, [2208.11723]



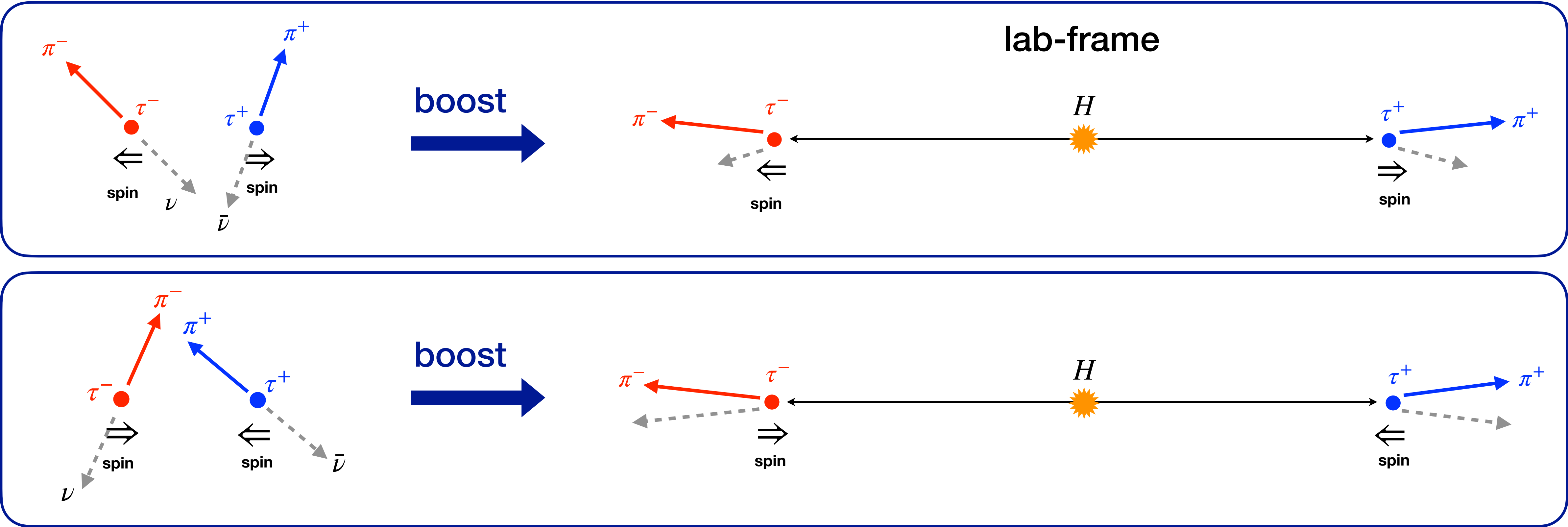
Concurrence



Challenge in the experimental reconstruction:

- QST requires the reconstruction of the $\tau^\pm$ **rest frames**.
- $m_\tau \ll m_h \Rightarrow$ Taus are highly boosted at the **lab-frame**.

Decay	κ	BR
$\tau^- \rightarrow \pi^- \nu$	+1	10.8 %
$\tau^- \rightarrow \rho^- \nu$	+0.45	25.5 %
$\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$	-1/3	35.2 %



- Different configurations at the rest frames look very similar at the lab-frame.

QIS studies for $\tau^-\tau^+$

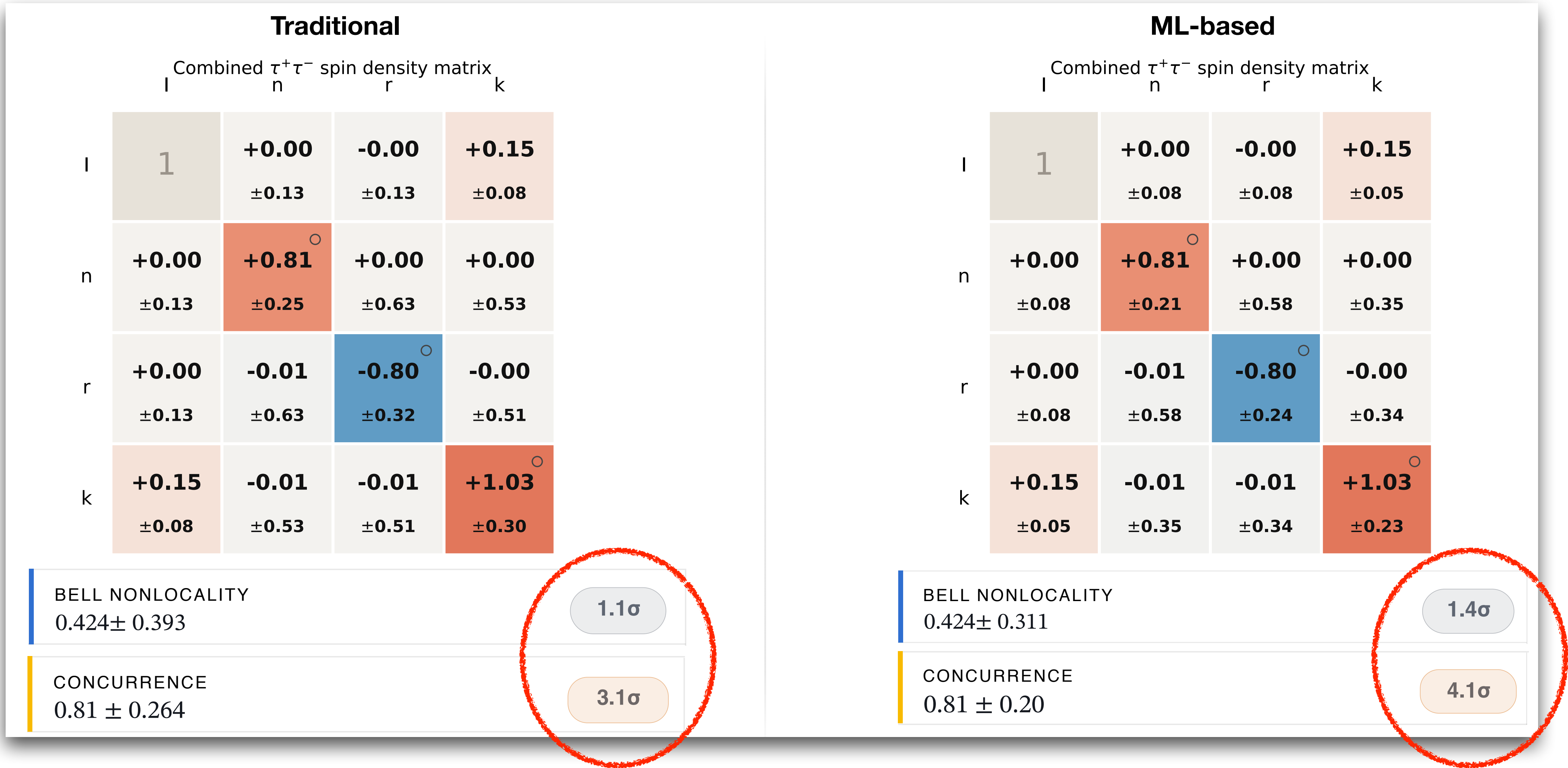
- Reconstruction of $\tau^+\tau^-$ spin state is challenging at the LHC. See, however, [Zhang et.al. \[2504.01496\]](#)
- Promissing for lepton colliders.

Facility, process, sample	Entanglement	Bell / CHSH	What actually limits it
Belle II $\tau\tau$ continuum, 2×10^8 pairs <i>on tape</i>	463σ	87σ	Fast sim only: no backgrounds, no systematics. Read as “ $\gg 5\sigma$, data in hand.”
BESIII $\psi(2S)$, 3.5×10^6 pairs	$> 5\sigma$ if syst. $\leq 1\%$; 3.7σ at 2%	no sensitivity	Threshold correlations are <i>small</i> : systematics-limited, not statistics-limited.
BEPC-II $\sqrt{s} = 5.6$ GeV, 20 fb^{-1}	$> 5\sigma$; 2.7% at 2% syst.	4.7σ at 2%	Best Bell prospect below the Higgs — <i>if</i> 2% is reachable.
BEPC-II scan, 4.0–5.6 GeV, 20 points	$> 5\sigma$; 4% precision	1.9σ at 2%; $> 5\sigma$ at 0.5%	Spreading luminosity <i>costs</i> Bell reach: concentrate at top energy.
ILC-250 $H \rightarrow \tau\tau$, 3 ab^{-1} , 385 sig. + 20 bkg.	$> 5\sigma$	$< 1\sigma$	Steering $\sim 4\sigma$; CP phase to $ \delta < 7.9^\circ$.
FCC-ee-240 $H \rightarrow \tau\tau$, 5 ab^{-1} , 663 sig. + 36 bkg.	$\gg 5\sigma$	$\sim 3\sigma$	Steering $\gg 5\sigma$; $ \delta < 5.4^\circ$. The <i>only</i> Higgs-factory τ Bell test.

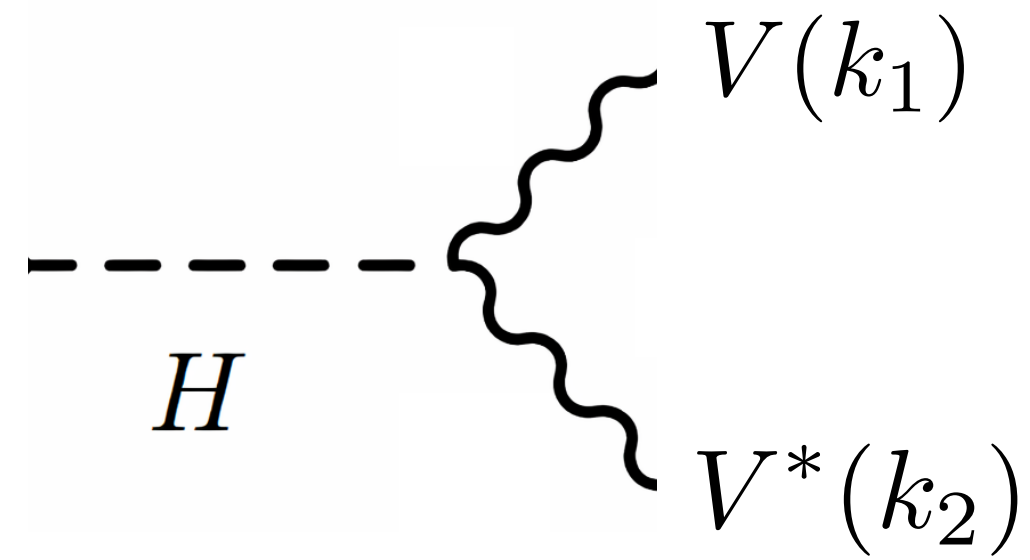
[Altakach, Lamba, Maltoni, Mawatari, KS \[2211.10513\]](#), [Ehatäht, Fabbrichesi, Marzola, Veelken \[2311.17555\]](#),
[D.Jeans \[2603.12609\]](#), [Han, Low, Su \[2501.04801\]](#), [Ma, Li \[2309.08103\]](#)

Re-analysis of LEP (DELPHI) data:

Chen-Hua Hsu's talk at BOOST 2026 conference



$$H \rightarrow V V^*$$

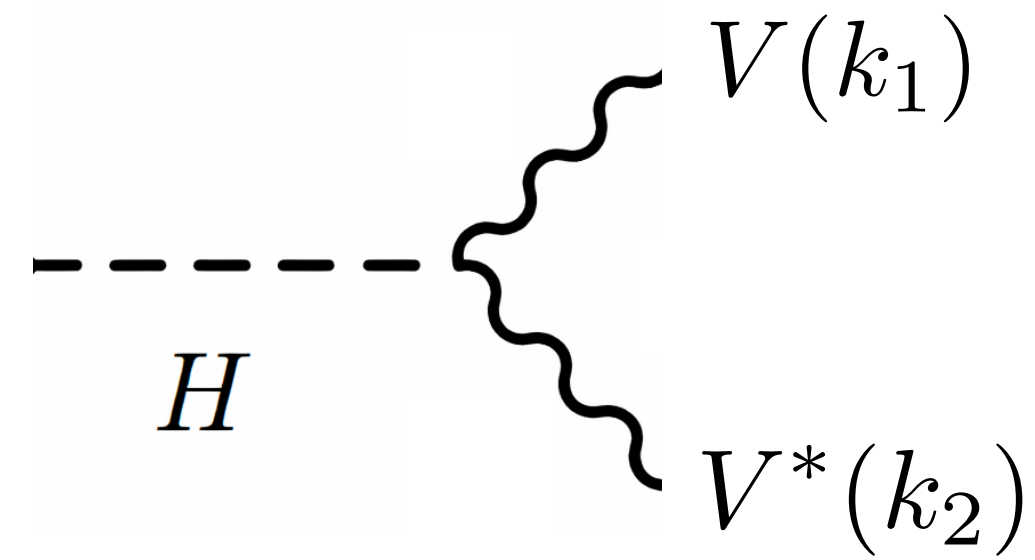


$$\mathcal{M}_{\lambda_1, \lambda_2} \propto \epsilon_{\lambda_1}(k_1) \cdot \epsilon_{\lambda_2}(k_2) = \begin{cases} 1 & (\lambda_1 \lambda_2) = (+-, -+) \\ -\beta \equiv -\frac{k_1 \cdot k_2}{m_1 m_2} & (\lambda_1 \lambda_2) = (0, 0) \\ 0 & \text{otherwise} \end{cases}$$

$$|\psi_{VV}\rangle = \frac{1}{\sqrt{2 + \beta^2}} (|+-\rangle - \beta|00\rangle + |-+\rangle)$$

pure 2-qutrit

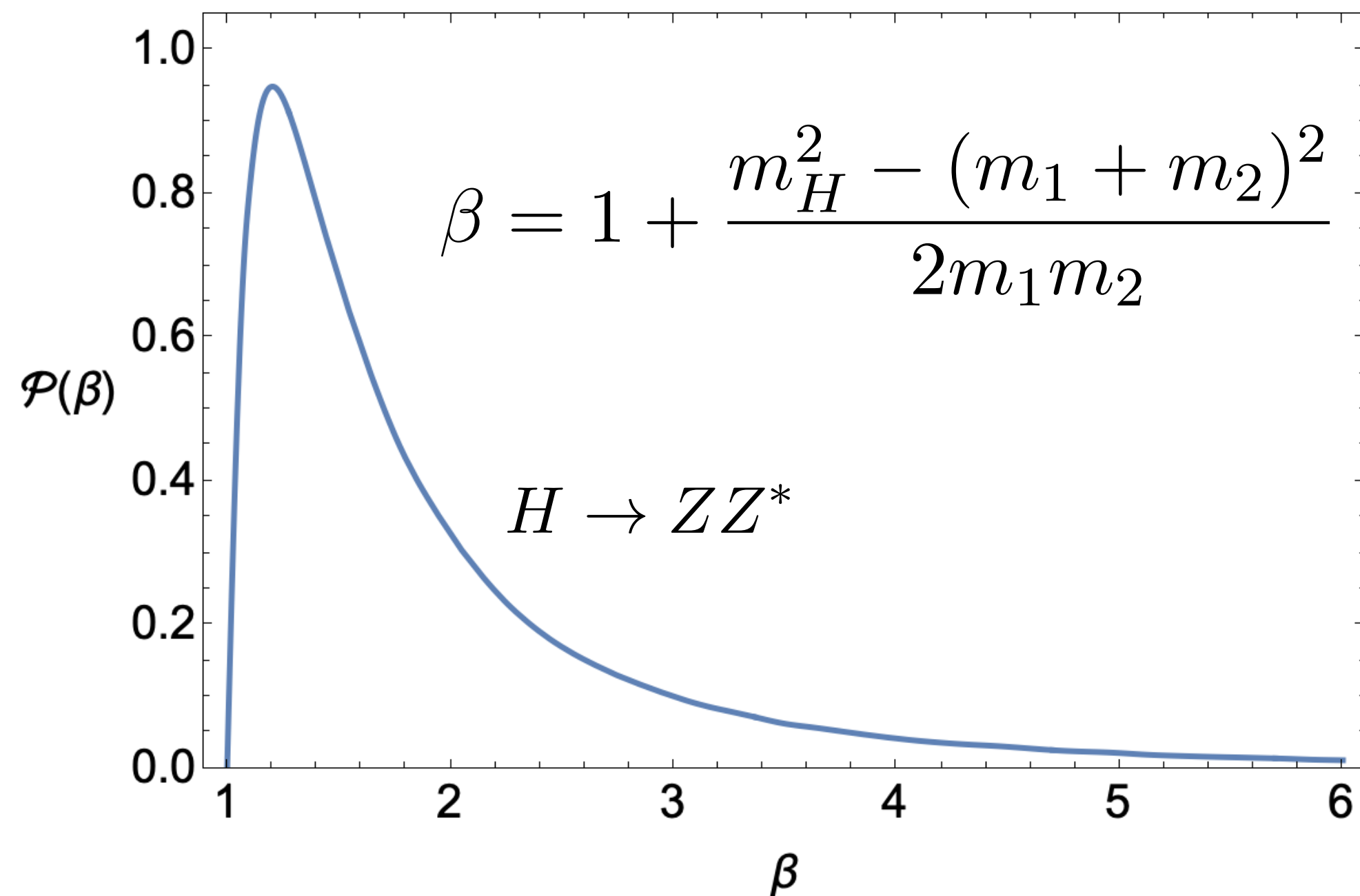
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pure 2-qutrit



$$\beta = 1 \implies \text{Max. Ent.: } \frac{1}{\sqrt{3}} (|+-\rangle - |00\rangle + |-+\rangle)$$

$$\beta \rightarrow \infty \implies \text{separable: } |00\rangle$$

A.Barr [2106.01377],
Aguilar-Saavedra, Bernal, Casas, Moreno [2209.13441]

Bell test for 2-qutrit:

Alice's choice: $x \in \{1, 2\}$ Alice's outcome: $A_x \in \{0, 1, 2\}$
 Bob's choice: $y \in \{1, 2\}$ Bob's outcome: $B_y \in \{0, 1, 2\}$

- The probability that Bob's result differs from Alice's by $k \bmod 3$.

$$P(A_a = B_b + k) \equiv \sum_{j=0}^2 P(A_a = j, B_b = j + k \pmod{3}).$$

maximally entangled

$\neq$

maximally nonlocal

- Consider the following combination of probabilities:

$$I_3 = \left[P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \right] \\ - \left[P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1) \right]$$

CGLMP ineq:

Collins, Gisin, Linden,
Massar, Popescu (2002)

$$I_3 \leq \begin{cases} 2 & \text{LHV, } \rho_{\text{sep}} \\ 1 + \sqrt{\frac{11}{3}} \simeq 2.91 & \text{QM} \\ 4 & \text{beyond QM} \end{cases}$$

$$|\psi_{\text{NL}}^{\text{max}}\rangle = \frac{|+-\rangle - \gamma|00\rangle + |-+\rangle}{\sqrt{2 + \gamma^2}} \\ \gamma = \frac{\sqrt{11} - \sqrt{3}}{2} \simeq 0.792$$

- The density matrix of **1-qudit** is 3 x 3; $\rho_{mm'}$ ($m = -1, 0, +1$; a vector of $SO(3)$)
- We consider the multipole decomposition: $1 \otimes 1 = 0 \oplus 1 \oplus 2$.
- The basis operators T_M^L ($-L \leq M \leq L$)

$$(L = 0) \quad T_0^0 = \mathbb{1}_3,$$

$$J_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad J_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad J_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(L = 1) \quad T_0^1 = \sqrt{\frac{3}{2}} J_z, \quad T_{\pm 1}^1 = \mp \frac{\sqrt{3}}{2} (J_x \pm iJ_y),$$

$$(L = 2) \quad T_0^2 = \frac{\sqrt{2}}{3} (T_1^1 T_{-1}^1 + T_{-1}^1 T_1^1 + 2(T_0^1)^2) \quad T_{\pm 1}^2 = \sqrt{\frac{2}{3}} (T_{\pm 1}^1 T_0^1 + T_0^1 T_{\pm 1}^1) \quad T_{\pm 2}^2 = \frac{2}{\sqrt{3}} (T_{\pm 1}^1)^2,$$

- Orthonormalisation, conjugation relation

$$\text{Tr} \left\{ T_M^L (T_{M'}^{L'})^\dagger \right\} = 3 \delta_{LL'} \delta_{MM'}, \quad (T_M^L)^\dagger = (-1)^M T_{-M}^L.$$

- Any single-qutrit density matrix is written as

$$\rho = \frac{1}{3} \left[\mathbb{1}_3 + \sum_{L=1}^2 \sum_{M=-L}^L A_{LM} T_M^L \right]$$

$$\begin{array}{c} \rho^\dagger = \rho \\ \downarrow \\ A_{LM} = (-1)^M (A_{L,-M})^*, \end{array}$$

- The distribution of the decay product of the qutrit is given by

Y_L^M spherical harmonics

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \left[1 + \sum_{L,M} A_{LM} B_L Y_L^M(\theta, \varphi) \right]$$

$$\begin{aligned} B_0 &= 1, \\ B_1 &= -\sqrt{2\pi} \eta_\ell, \\ B_2 &= \sqrt{\frac{2\pi}{5}}. \end{aligned}$$

$$\eta = (g_L^2 - g_R^2)/(g_L^2 + g_R^2) \quad \eta_\ell = \frac{1 - 4s_W^2}{1 - 4s_W^2 + 8s_W^4} \simeq 0.13, \quad (Z \rightarrow \ell^+ \ell^-)$$

- Quantum State Tomography

$$\int Y_L^M Y_{L'}^{M'*} d\Omega = \delta_{LL'} \delta_{MM'}$$

$$A_{LM} = \frac{4\pi}{B_L} \int d\Omega \left(\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \right) Y_L^{M*}(\theta, \varphi) = \frac{4\pi}{B_L} \langle Y_L^{M*} \rangle_{\text{events}}.$$

2-qutrit case

$$L = 1, 2 \text{ and } -L \leq M \leq L$$

$$\rho = \frac{1}{9} \left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A_{LM}^1 T_M^L \otimes \mathbb{1}_3 + A_{LM}^2 \mathbb{1}_3 \otimes T_M^L + C_{L_1 M_1 L_2 M_2} T_{M_1}^{L_1} \otimes T_{M_2}^{L_2} \right]$$

- The decay distribution:

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2} \left[1 + B_L A_{LM}^1 Y_L^M(\Omega_1) + B_L A_{LM}^2 Y_L^M(\Omega_2) + B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2) \right]$$

- Quantum State Tomography: Aguilar-Saavedra, Bernal, Casas, Moreno [2209.13441]

$$\int \frac{1}{\sigma} \frac{d\sigma}{d\Omega_j} Y_L^M(\Omega_j) d\Omega_j = \frac{B_L}{4\pi} A_{LM}^j$$

$$\int \frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2) d\Omega_1 d\Omega_2 = \frac{B_{L_1} B_{L_2}}{(4\pi)^2} C_{L_1, M_1, L_2, M_2}$$

Simplification 0⁺⁺:

- ⌘ Assumption 1: **Higgs is a scalar** $\Rightarrow J_z = 0$ along the ZZ production axis
 $\Rightarrow$ only $|+-\rangle, |00\rangle, |-+\rangle$ can be populated.
 $\Rightarrow$ only a 3 x 3 block of the 9 x 9 density matrix survives.
- ⌘ Assumption 2: **Parity even** $\Rightarrow$ all $L = 1$ coefficients vanish

- Only 6 elements remain:
(3 parameters)

$$\begin{aligned}\rho_{+-,+-} &= \rho_{-+,-+} = \frac{1}{6} (\sqrt{2} A_{2,0}^1 + 2), & \rho_{00,00} &= \frac{1}{3} (1 - \sqrt{2} A_{2,0}^1), \\ \rho_{+-,00} &= \rho_{00,-+} = \frac{1}{3} C_{2,1,2,-1}, & \rho_{+-,-+} &= \frac{1}{3} C_{2,2,2,-2},\end{aligned}$$

- Peres-Horodecki condition: $\rho^{T_B} < 0$

$$ZZ \text{ pair is entangled} \iff C_{2,1,2,-1} \neq 0 \text{ or } C_{2,2,2,-2} \neq 0.$$

- From 0^{++} assumption:

$$I_3 = \frac{1}{36} \left(18 + 16\sqrt{3} - \sqrt{2} \left(9 - 8\sqrt{3} \right) A_{2,0}^1 - 8 \left(3 + 2\sqrt{3} \right) C_{2,1,2,-1} + 6 C_{2,2,2,-2} \right)$$

- SM prediction:

$$I_3^{\text{SM}} = 2.60$$

- Prospects at the LHC: Aguilar-Saavedra, Bernal, Casas, Moreno [2209.13441]

LHC Run-3 (300 fb⁻¹)

	min m_{Z_2}			
	0	10 GeV	20 GeV	30 GeV
N	450	418	312	129
$C_{2,1,2,-1}$	-0.98 ± 0.31	-0.97 ± 0.33	-1.05 ± 0.38	-1.06 ± 0.61
$C_{2,2,2,-2}$	0.60 ± 0.37	0.64 ± 0.38	0.74 ± 0.43	0.82 ± 0.63
I_3	2.66 ± 0.46	2.67 ± 0.49	2.82 ± 0.57	2.88 ± 0.89

Entanglement $\sim 3\sigma$

Non-locality $\sim 2\sigma$

HL-LHC(3000 fb⁻¹)

	min m_{Z_2}			
	0	10 GeV	20 GeV	30 GeV
N	4500	4180	3120	1290
$C_{2,1,2,-1}$	-0.95 ± 0.10	-1.00 ± 0.10	-1.04 ± 0.12	-1.04 ± 0.19
$C_{2,2,2,-2}$	0.60 ± 0.12	0.64 ± 0.12	0.74 ± 0.14	0.83 ± 0.20
I_3	2.63 ± 0.15	2.71 ± 0.16	2.81 ± 0.18	2.84 ± 0.28

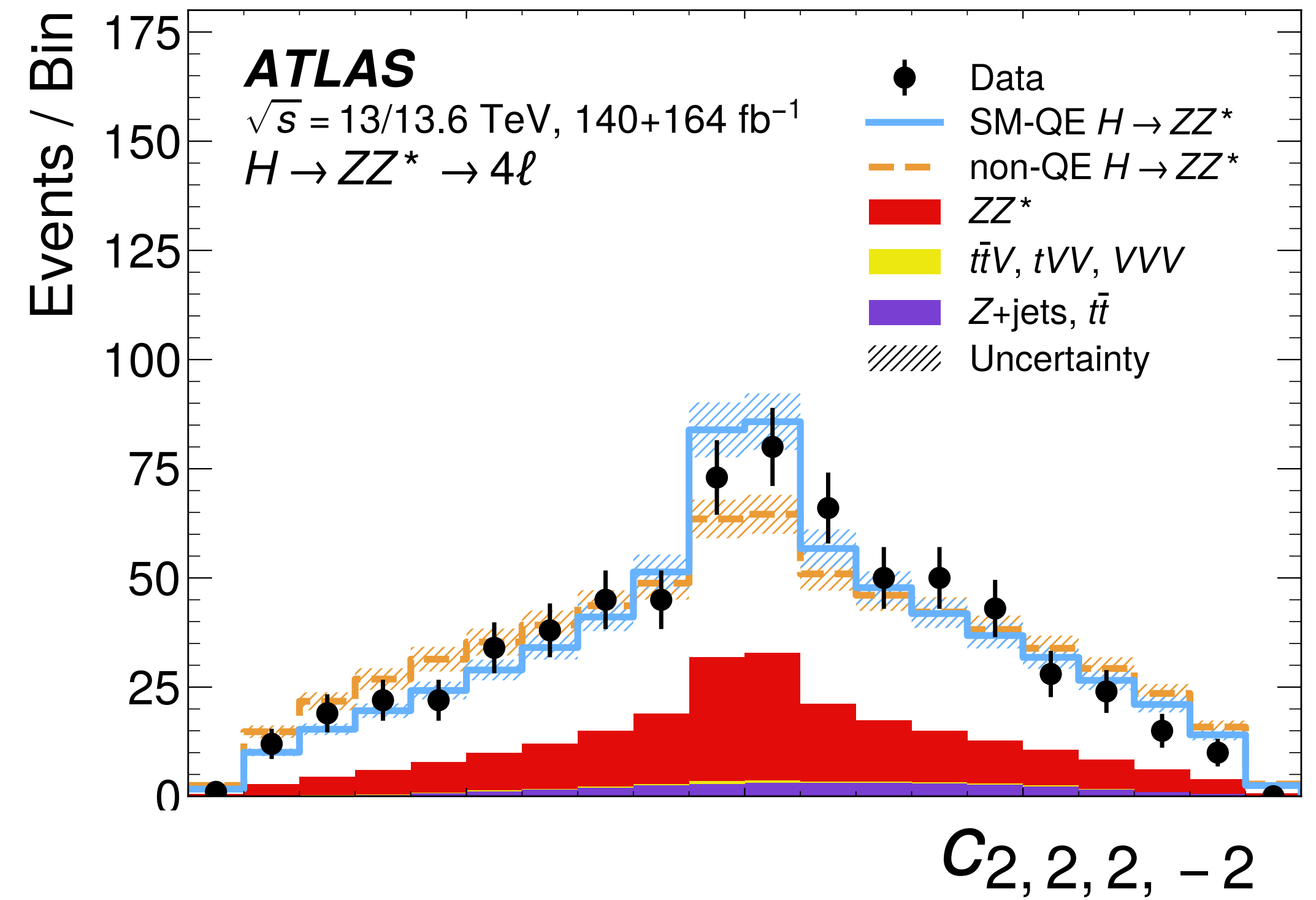
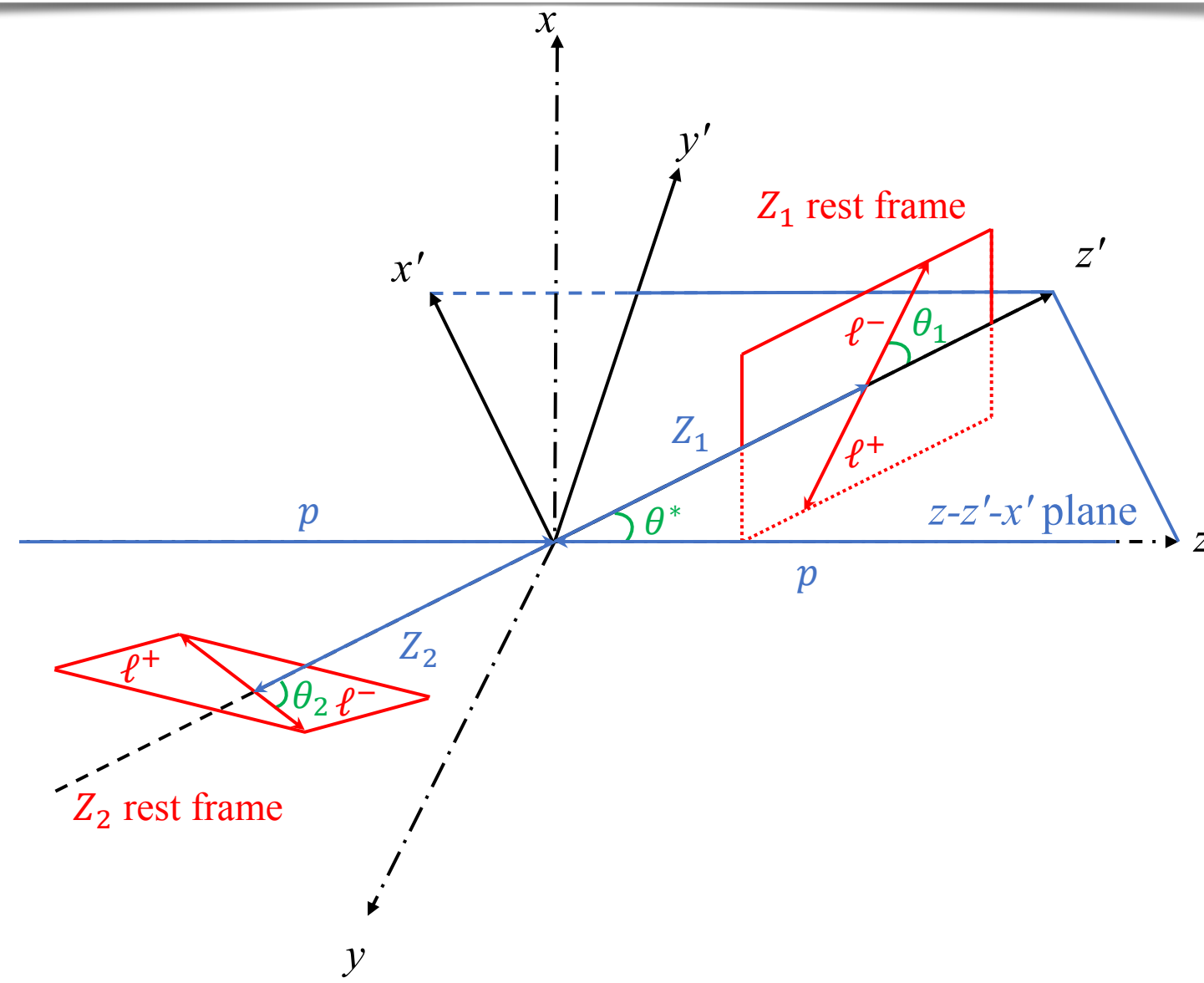
Entanglement $> 5\sigma$

Non-locality $\sim 4.5\sigma$

Measurements of Z-boson pair entanglement in decays of Higgs bosons at the ATLAS experiment

The ATLAS Collaboration

Entanglement is a key property of quantum systems. In this Letter the first measurements of quantum entanglement between spins in pairs of Z bosons are reported, using proton–proton collision data from the Large Hadron Collider (LHC) at center-of-mass energies of 13 TeV and 13.6 TeV, recorded with the ATLAS detector. Measurements of angular observables sensitive to ZZ^* spin-density-matrix elements in the $H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \ell^+ \ell^-$ process yield coefficients $C_{2,1,2,-1} = -0.71 \pm 0.45$ and $C_{2,2,2,-2} = 0.08 \pm 0.44$, consistent with their Standard Model predictions. A complementary hypothesis test using the full angular distribution, and relying on several Standard Model assumptions in the decays, provides substantially higher sensitivity to quantum correlations and disfavors the separable-state hypothesis at a significance of 4.7 standard deviations (expected 4.9σ) relative to the entangled Standard Model hypothesis. These results provide strong evidence of quantum entanglement between massive bosons (spin qutrits) at the electroweak scale.



$$c_{2 M_1 2 M_2} = 40\pi \Re \left(Y_2^{M_1}(\theta_1, \phi_1) \times Y_2^{M_2}(\theta_2, \phi_2) \right)$$

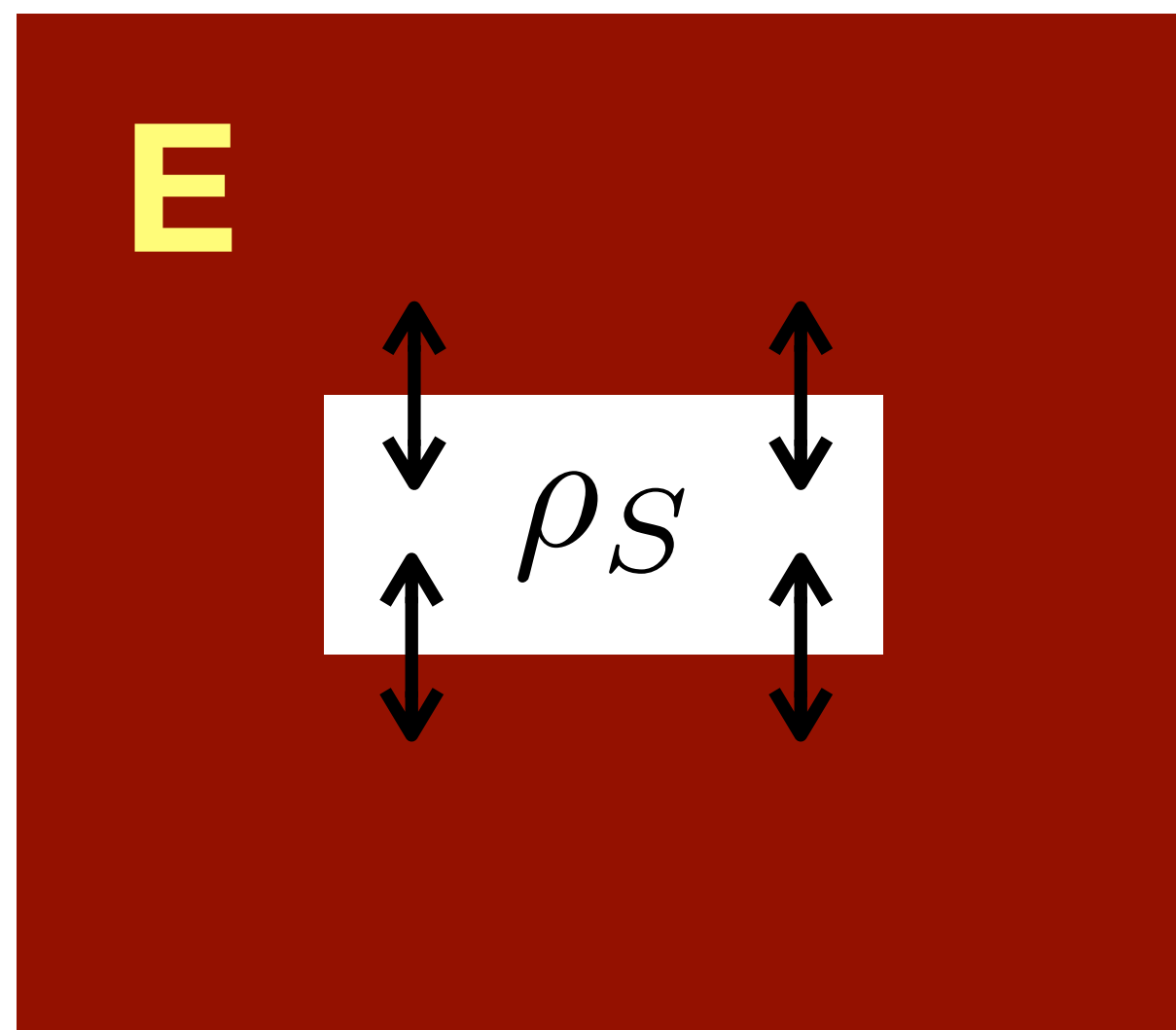
likelihood-ratio test; SM vs separable $|00\rangle$ hypotheses

Entanglement $\sim 4.7\sigma$

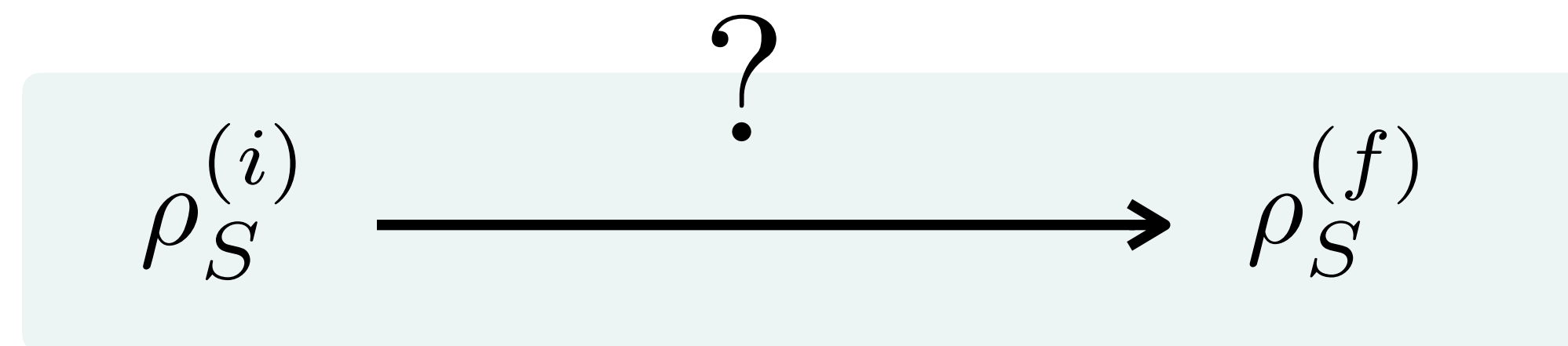
CMS reports a similar result [CMS PAS HIG-25-011]

Open Quantum Mechanics

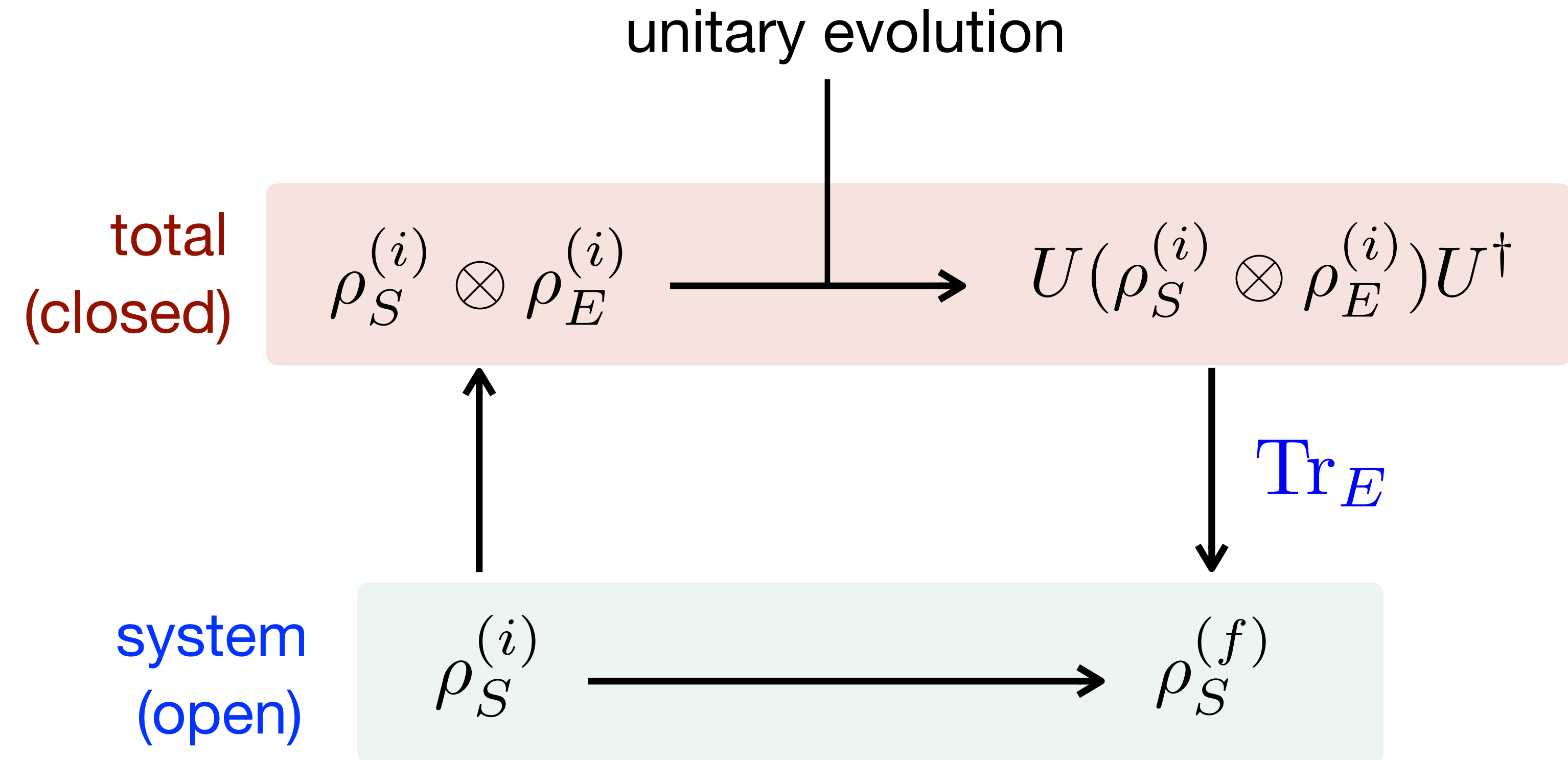
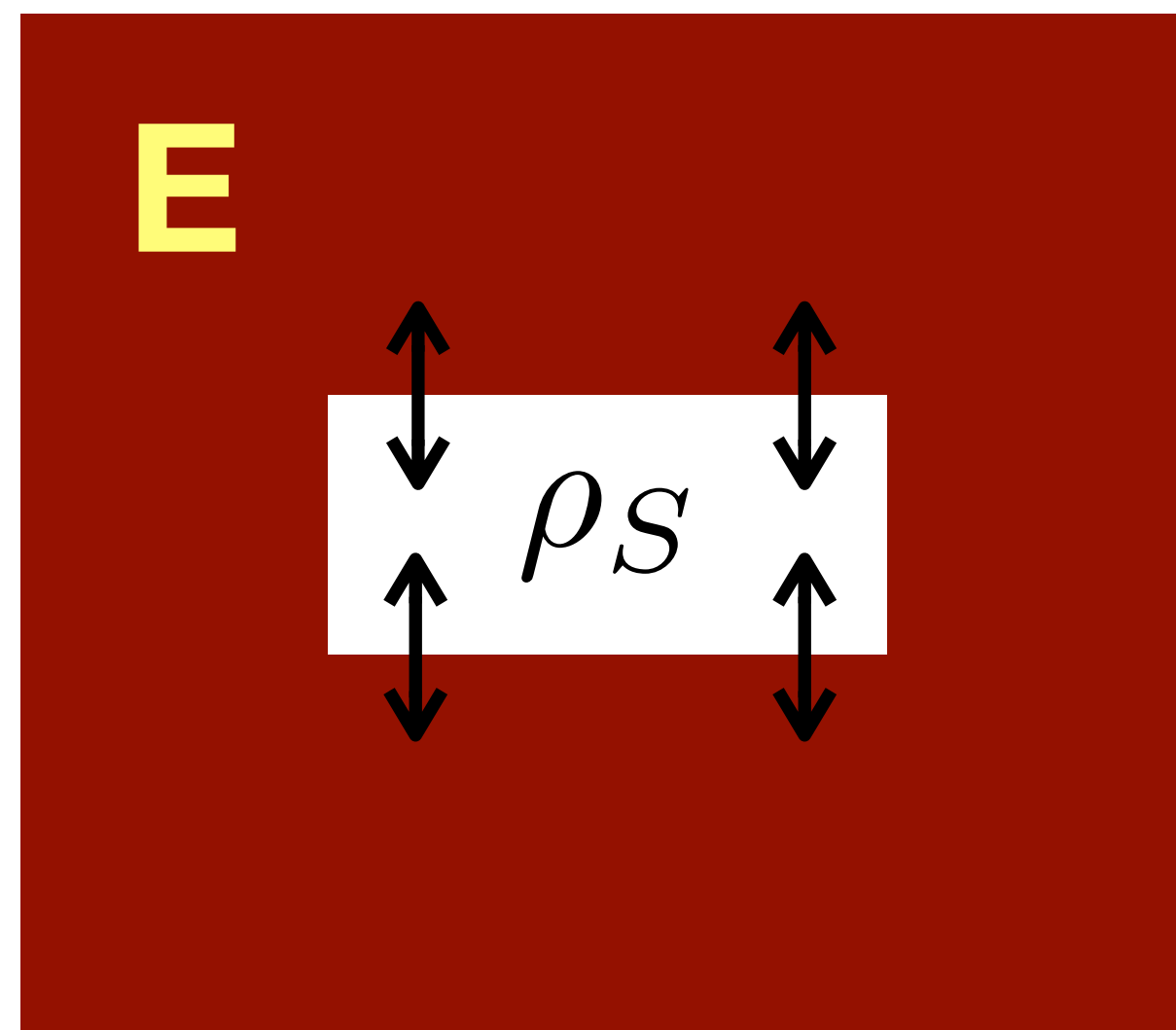
How does the system evolve in an environment?



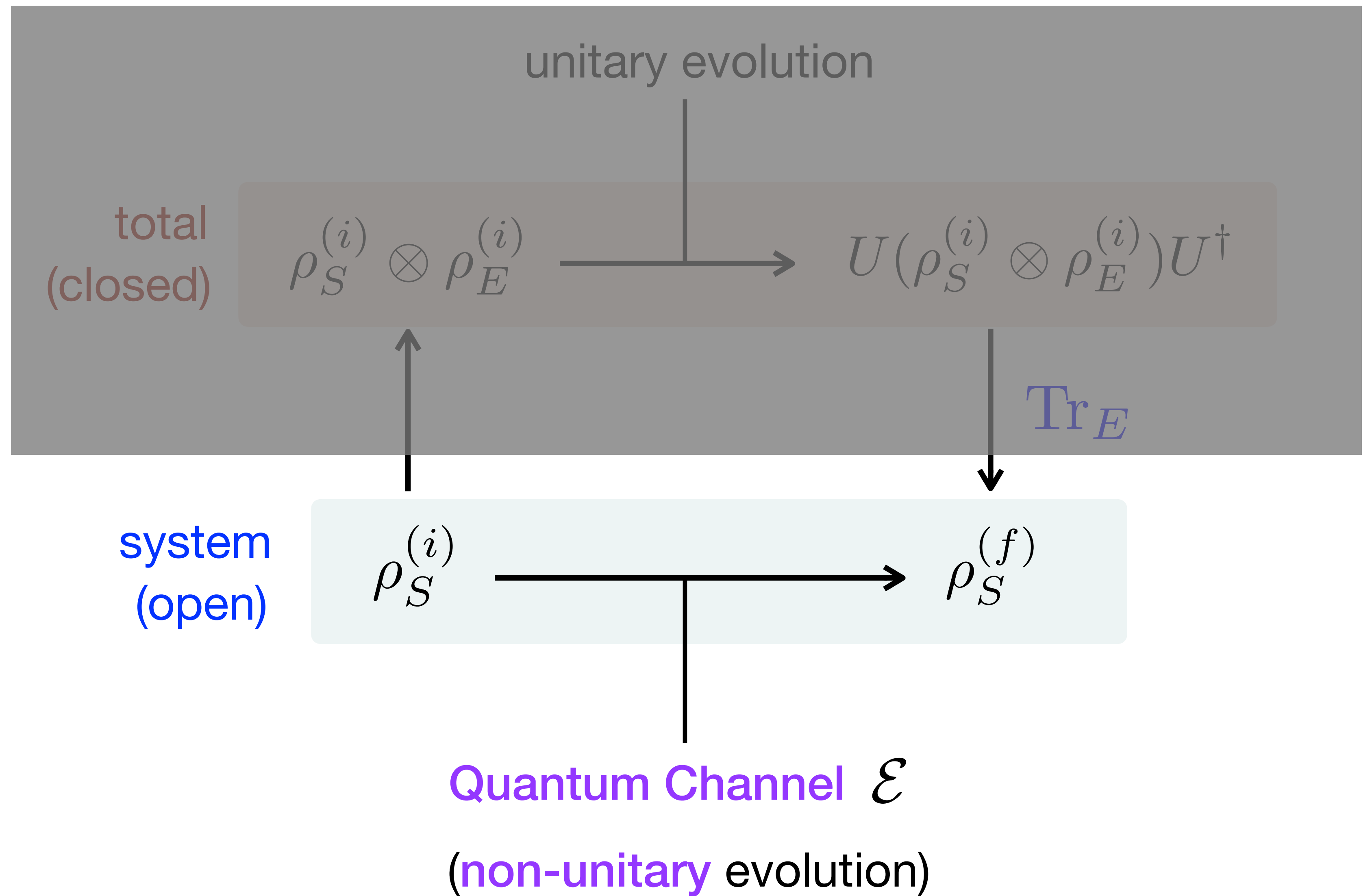
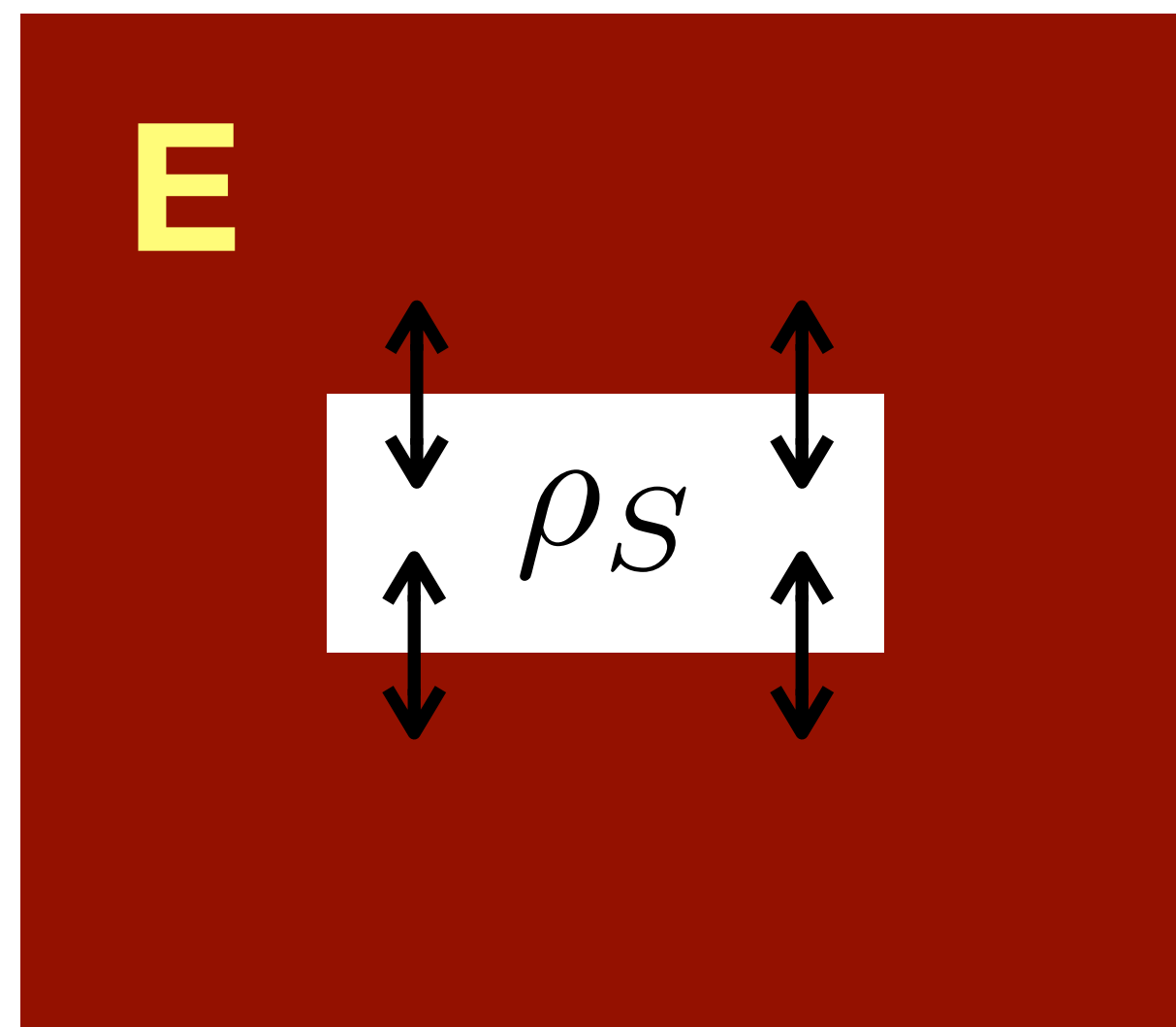
system
(open)



How does the system evolve in an environment?



How does the system evolve in an environment?

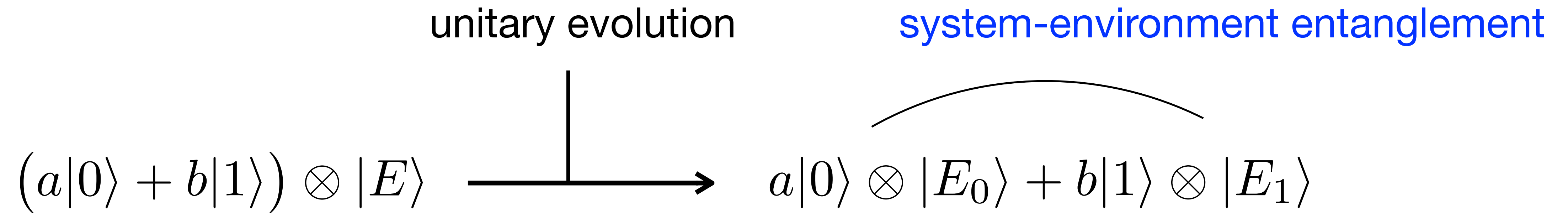


Decoherence: Interacting with an environment, the system becomes **more classical**.

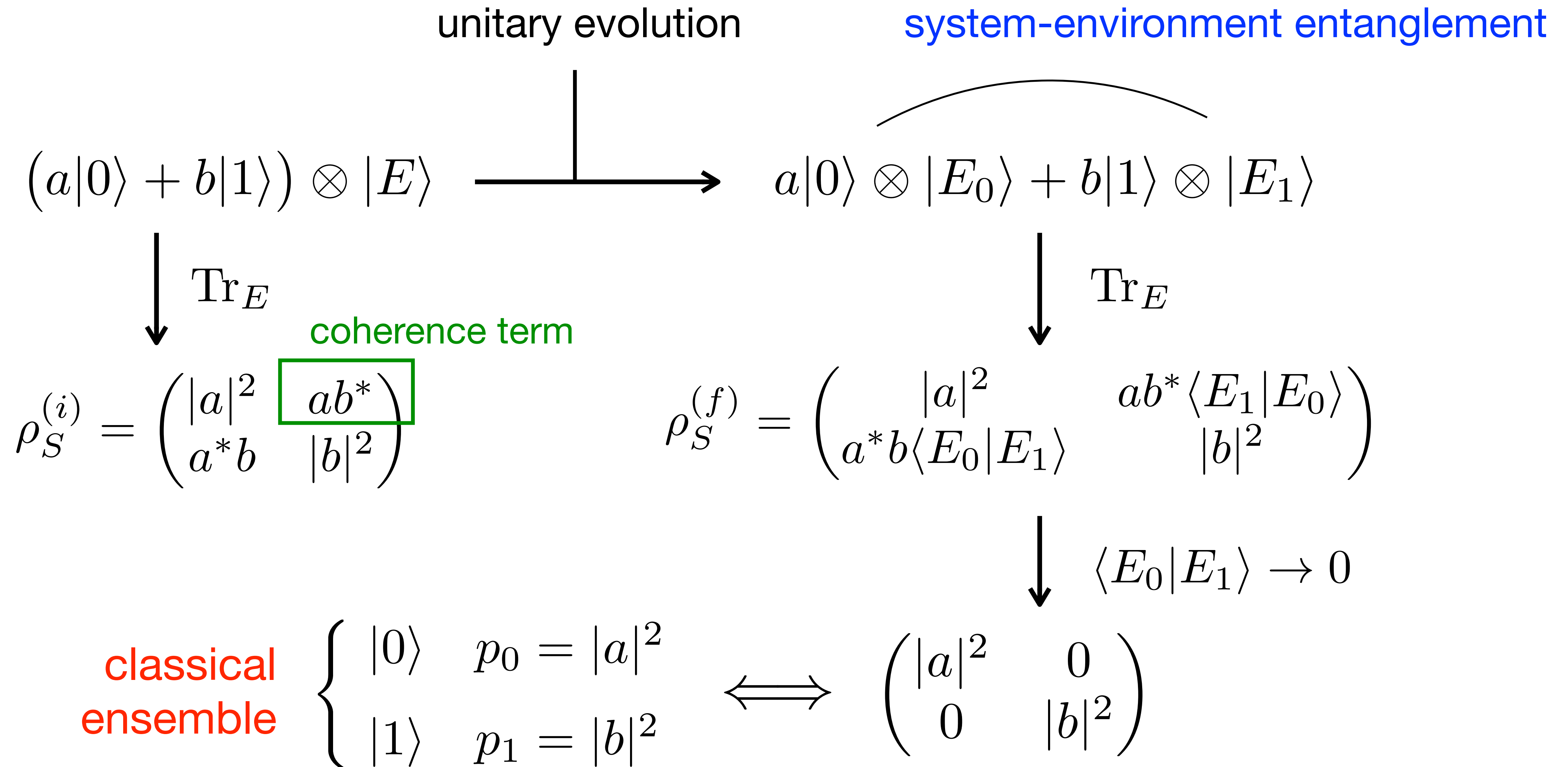
$$(a|0\rangle + b|1\rangle) \otimes |E\rangle$$

Decoherence: Interacting with an environment, the system becomes **more classical**.

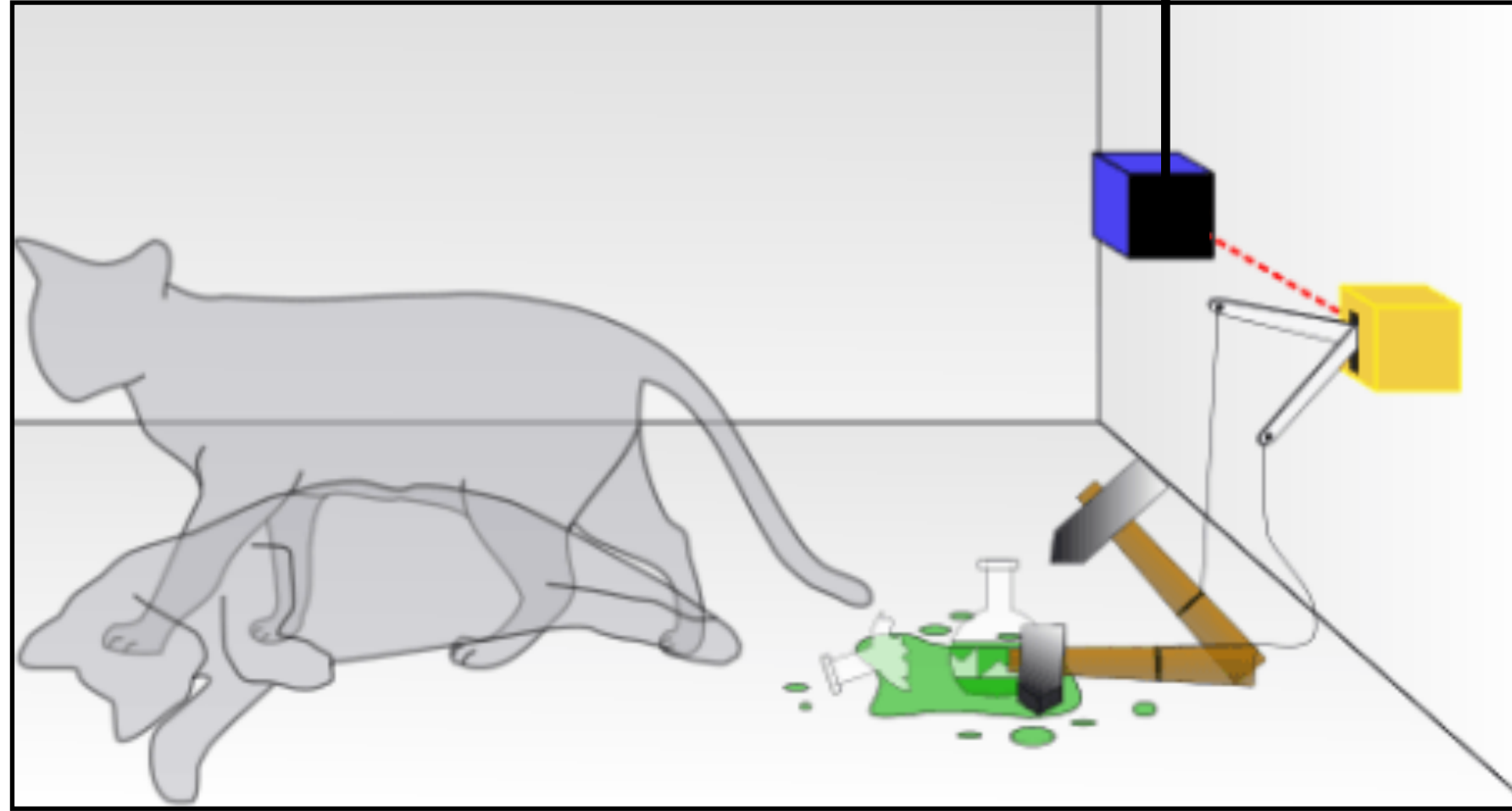
unitary evolution system-environment entanglement

$$(a|0\rangle + b|1\rangle) \otimes |E\rangle \xrightarrow{\quad} a|0\rangle \otimes |E_0\rangle + b|1\rangle \otimes |E_1\rangle$$


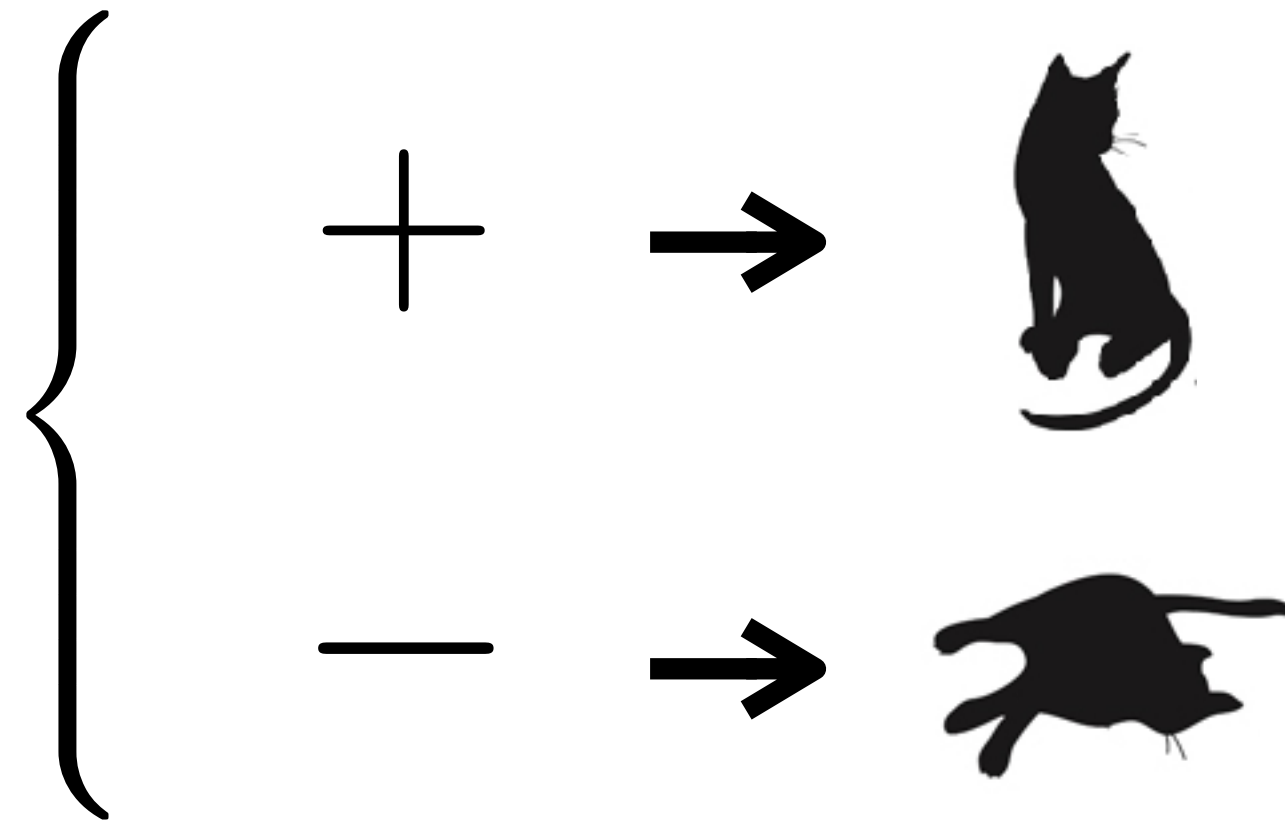
Decoherence: Interacting with an environment, the system becomes **more classical**.



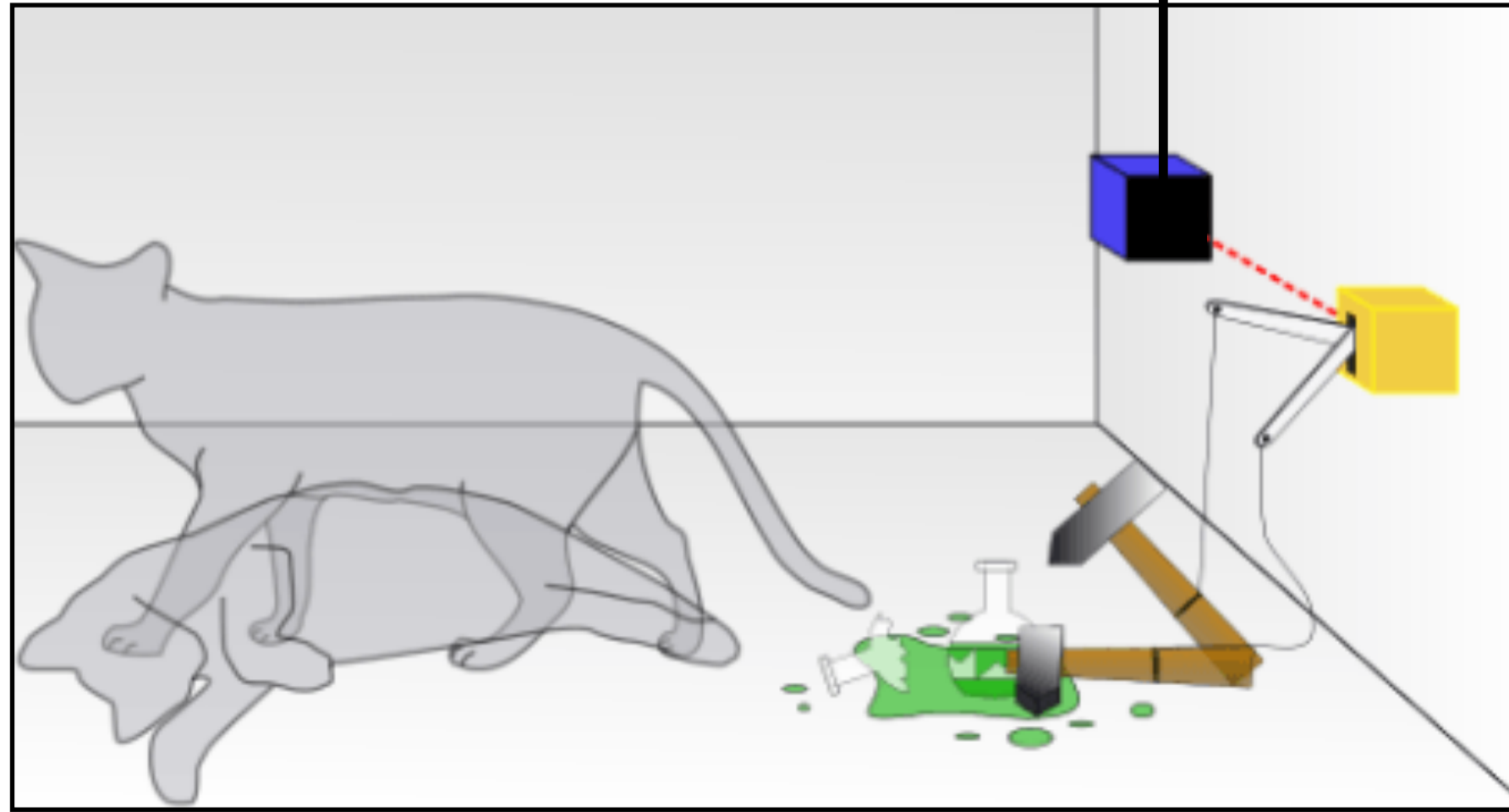
$$\frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$$



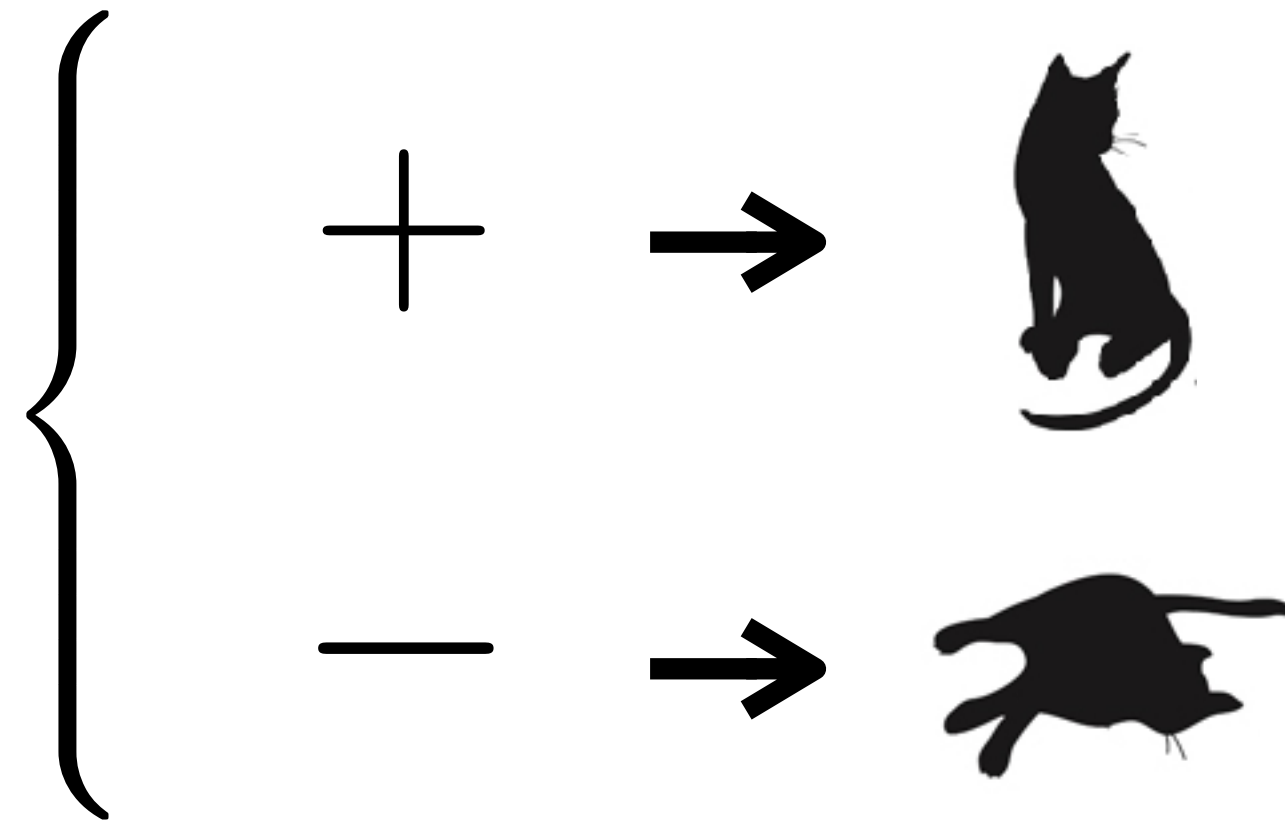
measure spin-z component



$$\frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$$



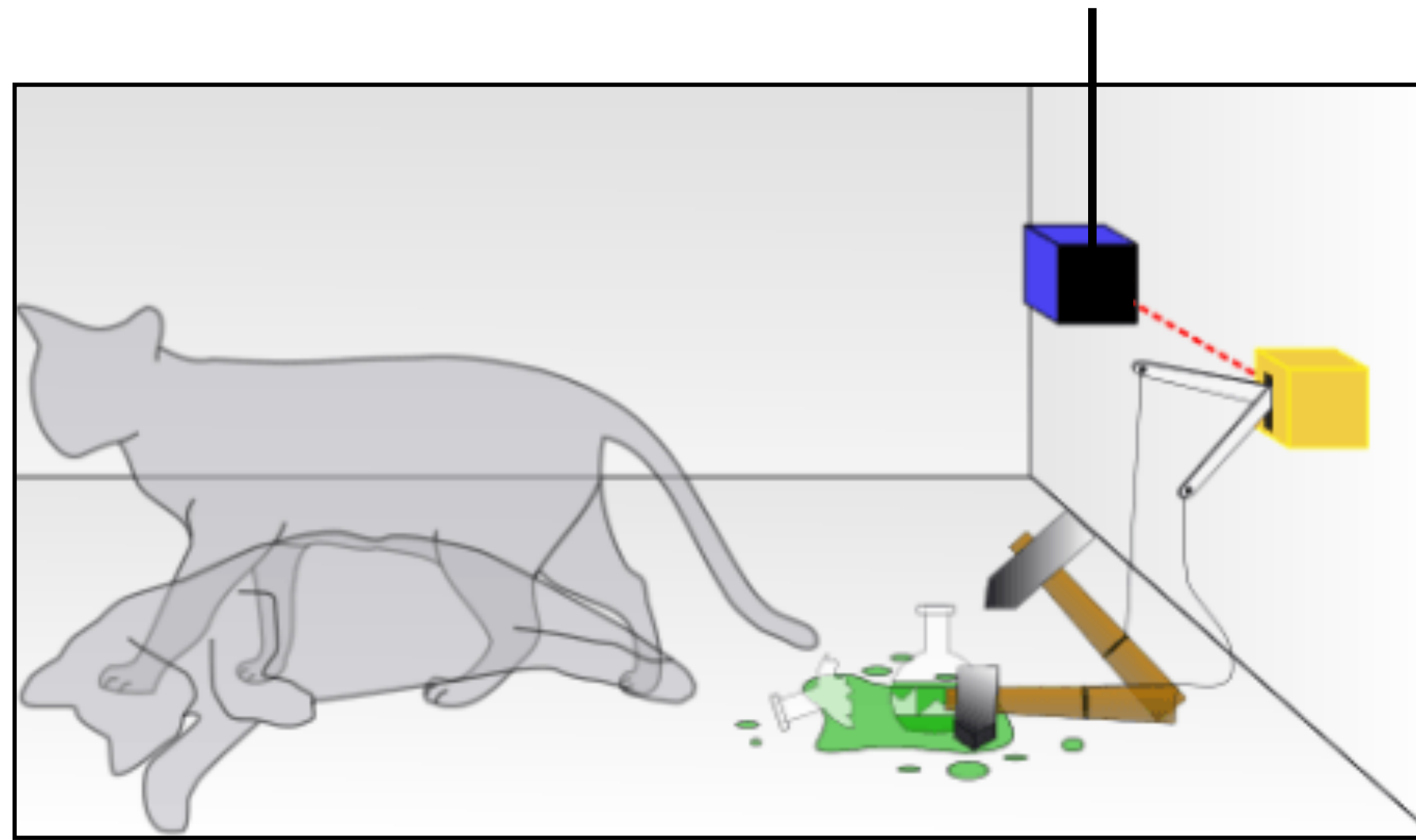
measure spin-z component



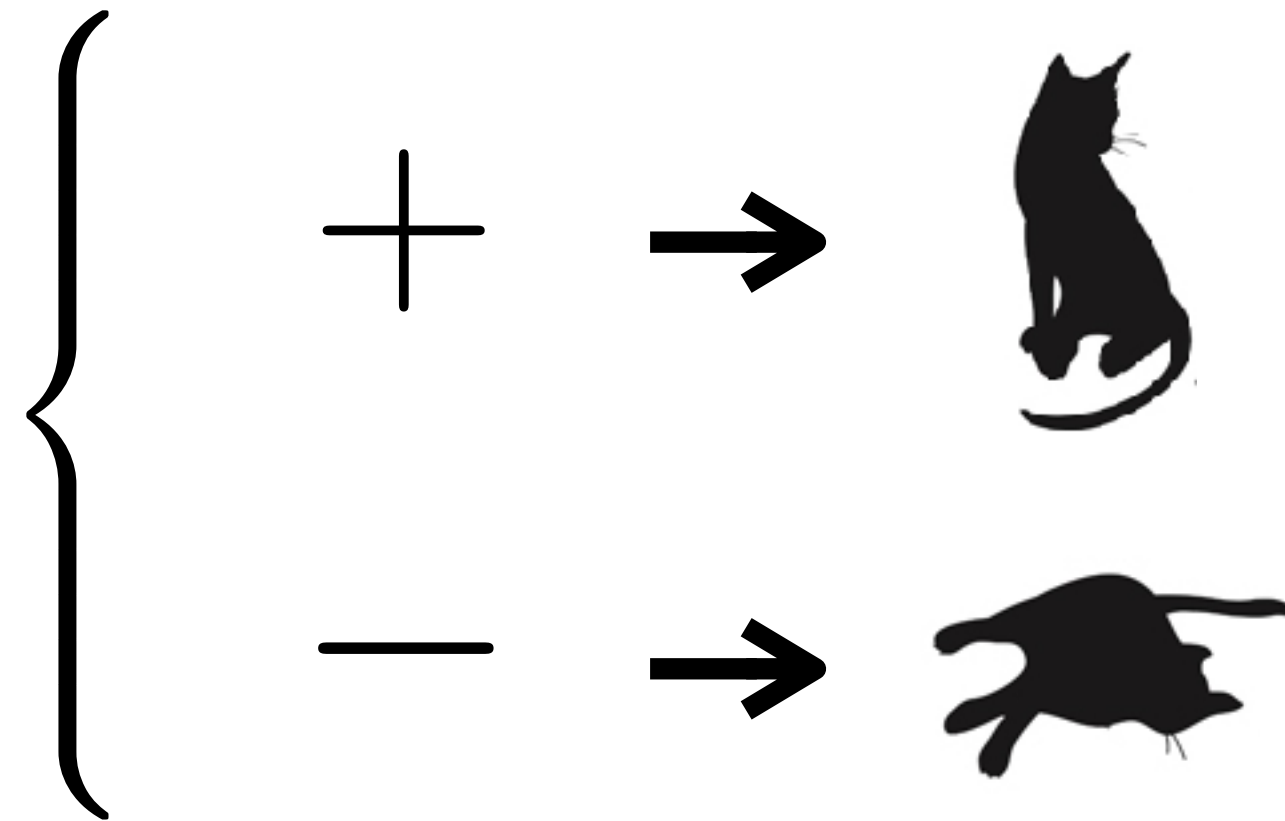
$$|\psi^f\rangle = \frac{1}{\sqrt{2}} \left[|+\rangle \otimes |\text{cat sitting}\rangle + |-\rangle \otimes |\text{cat lying}\rangle \right]$$

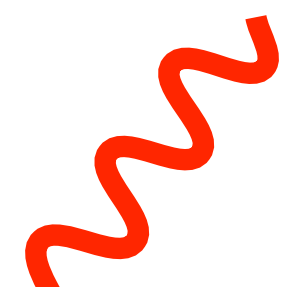
coherent superposition of a wake and sleep cat?

$$\frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$$



measure spin-z component

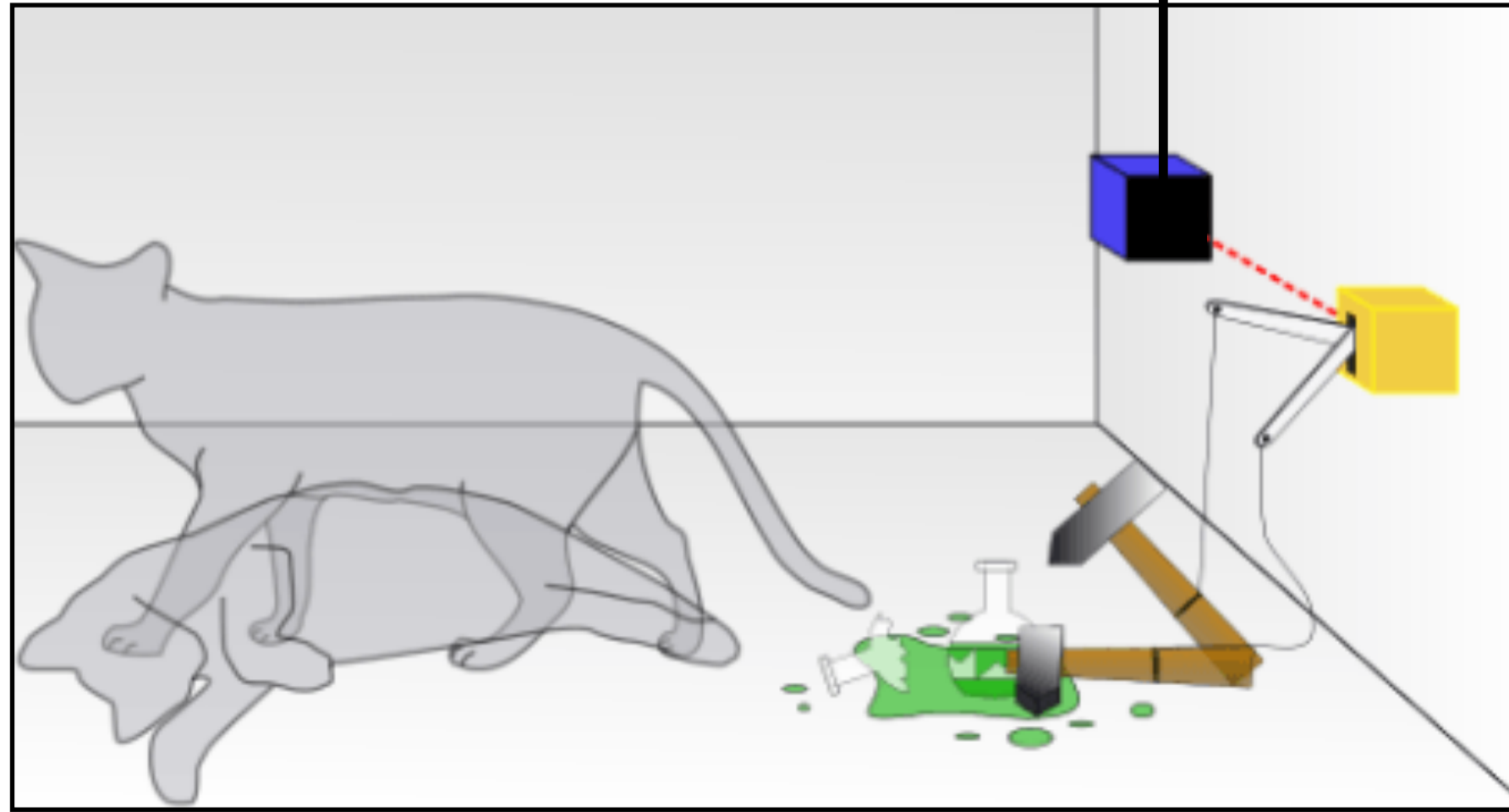


γ 

$$|\psi^f\rangle = \frac{1}{\sqrt{2}} \left[|+\rangle \otimes \left| \text{cat sitting} \right\rangle + |-\rangle \otimes \left| \text{cat lying} \right\rangle \right]$$

coherent superposition of a wake and sleep cat?

$$\frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$$



measure spin-z component

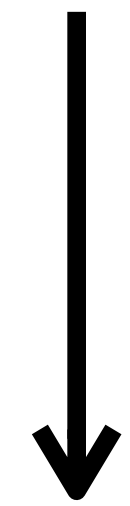
$$\left\{ \begin{array}{l} + \rightarrow \text{cat sitting} \rightarrow \text{Env}(0\gamma) \\ - \rightarrow \text{cat lying} \rightarrow \text{Env}(1\gamma) \end{array} \right.$$

γ wavy

~~$$|\psi^f\rangle = \frac{1}{\sqrt{2}} [|+\rangle \otimes |\text{cat sitting}\rangle + |-\rangle \otimes |\text{cat lying}\rangle]$$~~

$$|\psi^f\rangle = \frac{1}{\sqrt{2}} \left[|+\rangle \otimes |\text{cat sitting}\rangle \otimes |\text{Env}(0\gamma)\rangle + |-\rangle \otimes |\text{cat lying}\rangle \otimes |\widetilde{\text{Env}(1\gamma)}\rangle \right]$$

$$|\psi^f\rangle = \frac{1}{\sqrt{2}} \left[|+\rangle \otimes \left| \text{cat} \right\rangle \otimes |\text{Env}(0\gamma)\rangle + |-\rangle \otimes \left| \text{dog} \right\rangle \otimes |\widetilde{\text{Env}}(1\gamma)\rangle \right]$$



Tr_E

$$A = \langle \widetilde{\text{Env}}(1\gamma) | \text{Env}(0\gamma) \rangle = 0$$

$$\rho_S^f = \begin{array}{c} \langle + \text{cat} | \quad \langle - \text{dog} | \\ \left| + \text{cat} \right\rangle \\ \left| - \text{dog} \right\rangle \end{array} \begin{pmatrix} \frac{1}{2} & \frac{1}{2}A \\ \frac{1}{2}A^* & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

classical ensemble

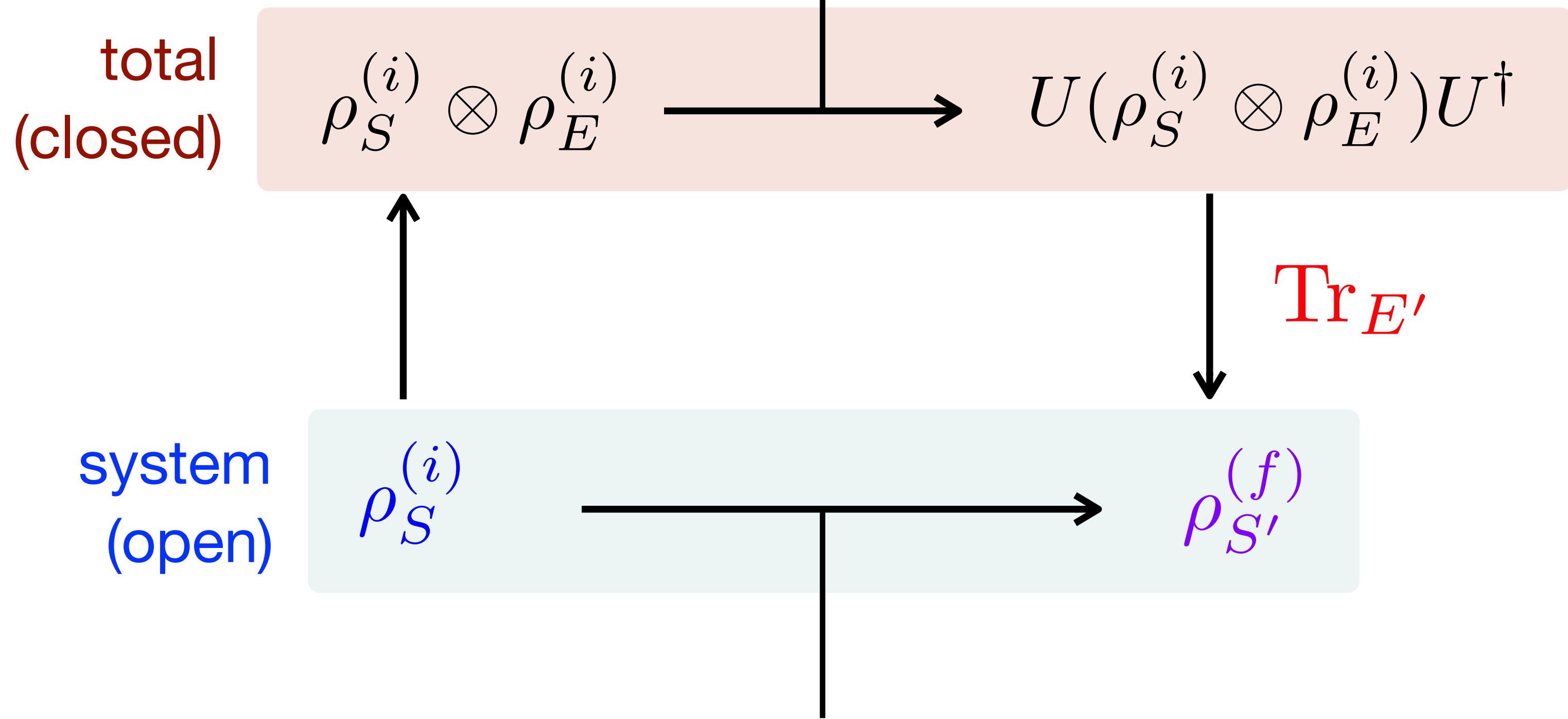
$$\left\{ \begin{array}{ll} + \text{cat} & 50\% \\ - \text{dog} & 50\% \end{array} \right.$$

Quantum Channel: Evolution of the system in an environment

$$\rho_{S'}^{(f)} = \text{Tr}_{E'} \left[U(\rho_S^{(i)} \otimes |0\rangle\langle 0|_E)U^\dagger \right]$$

unitary evolution

$\rho_E^{(i)} = |0\rangle\langle 0|_E$



E' (S') may not be the same as E (S).

Quantum Channel $\mathcal{E} : \mathcal{H}_S \mapsto \mathcal{H}_{S'}$
(non-unitary evolution)

Quantum Channel: Evolution of the system in an environment

$$\begin{aligned}\rho_{S'}^{(f)} &= \text{Tr}_{E'} \left[U(\rho_S^{(i)} \otimes |0\rangle\langle 0|_E) U^\dagger \right] \quad \{ |k\rangle_{E'} \}: \text{basis of the traced out Env.} \\ &= \sum_k \langle k|_{E'} \left(U(\rho_S^{(i)} \otimes |0\rangle\langle 0|_E) U^\dagger |k\rangle_{E'} \right)\end{aligned}$$

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Quantum Channel: Evolution of the system in an environment

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 &= \sum_k \langle k|_{E'} \left(U(\rho_S^{(i)} \otimes |0\rangle\langle 0|_E) U^\dagger |k\rangle_{E'} \right) \\
 &= \sum_k K_k \rho_S K_k^\dagger
 \end{aligned}$$

$\{|k\rangle_{E'}\}$: basis of the traced out Env.

$K_k = \langle k|_{E'} U |0\rangle_E$
 ↑
 operator : $\mathcal{H}_S \rightarrow \mathcal{H}_{S'}$

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Quantum Channel: Evolution of the system in an environment

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$K_k = \langle k|_{E'} U |0\rangle_E$
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 operator : $\mathcal{H}_S \rightarrow \mathcal{H}_{S'}$

- $\{K_k\}$ are the **Kraus operators**, satisfying

$$\sum_k K_k^\dagger K_k = \sum_k \langle 0|_E U^\dagger |k\rangle_{E'} \langle k|_{E'} U |0\rangle_E = \langle 0|_E U^\dagger U |0\rangle_E = \mathbb{1}$$

The **quantum channel** $\mathcal{E}$ is a map $\mathcal{E} : \mathcal{H}_S \rightarrow \mathcal{H}_{S'}$

$$\rho_S^{(i)} \rightarrow \rho_{S'}^{(f)} = \mathcal{E}[\rho_S^{(i)}] = \sum_k K_k \rho_S^{(i)} K_k^\dagger \quad \sum_k K_k^\dagger K_k = \mathbb{1}$$

The **quantum channel** $\mathcal{E}$ is a map $\mathcal{E} : \mathcal{H}_S \rightarrow \mathcal{H}_{S'}$

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Properties:

- Linear
- Trace-preserving
- Completely-positive

$$\text{Tr}[\rho_{S'}^{(f)}] = \text{Tr}\left(\sum_k K_k^\dagger K_k \rho_S^{(i)}\right) = \text{Tr}[\rho_S^{(i)}]$$

$$\rho_{S'}^{(f)} = \sum_k K_k \sqrt{\rho_S^{(i)}} \left[K_k \sqrt{\rho_S^{(i)}} \right]^\dagger$$

For $\tilde{\rho}^{(i)} = \rho_{\text{ext}} \otimes \rho_S^{(i)}$ and $\tilde{K}_k = \mathbb{1}_{\text{ext}} \otimes K_k$, the extended map $\tilde{\rho}^{(f)} = \sum_k \tilde{K}_k \tilde{\rho}^{(i)} \tilde{K}_k^\dagger$ is positive.

⌘ Any linear **C**ompletely-**P**ositive **T**race-**P**reserving (**CPTP**) map is a **quantum channel**.

Single-qubit examples:

- **Phase-flip** $K_0 = \sqrt{1-p} \mathbb{1}, \quad K_1 = \sqrt{p} \sigma_z$

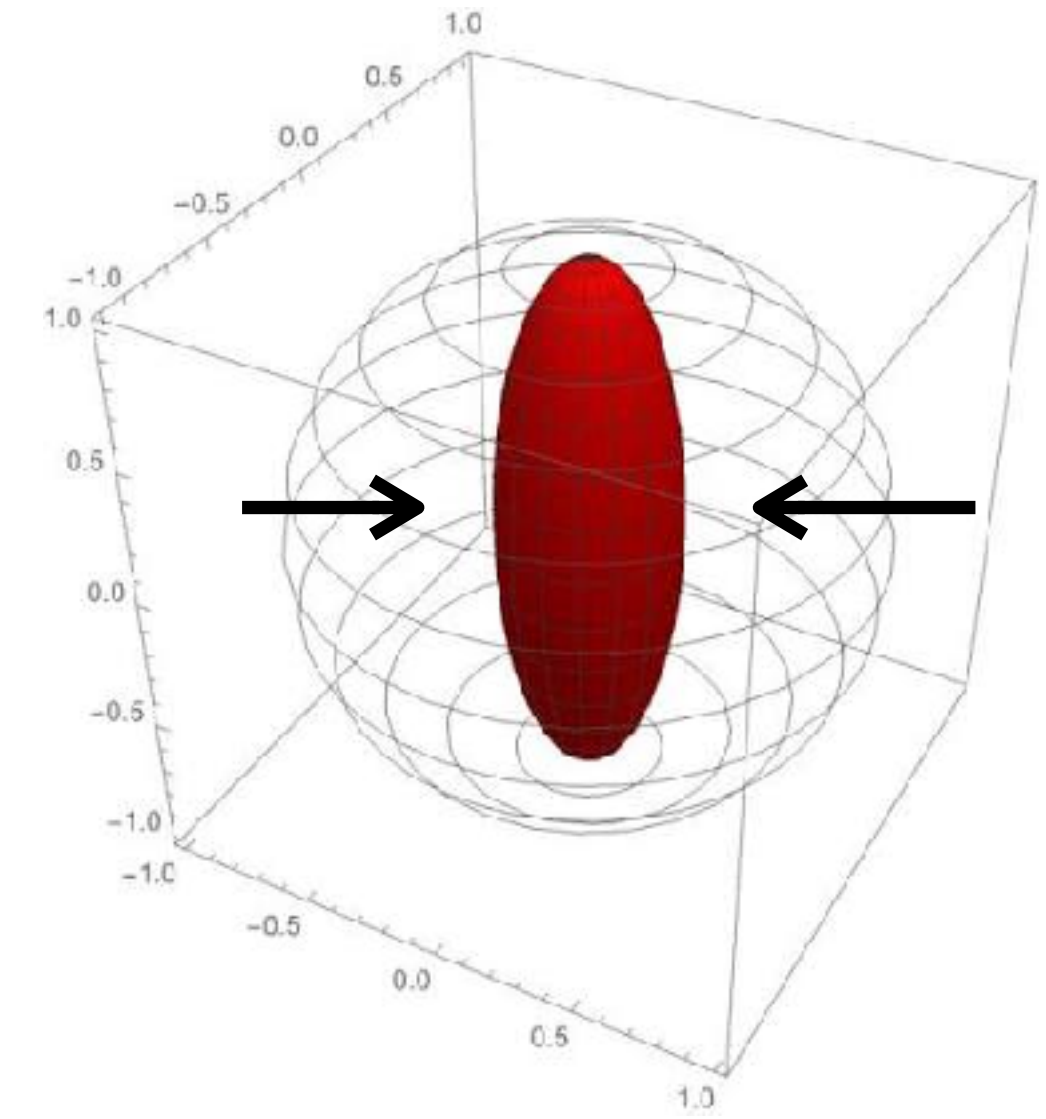
$$\rho = \frac{1}{2} [\mathbb{1} + B_i \sigma_i] \longrightarrow \mathcal{E}(\rho) = (1-p)\rho + p \sigma_z \rho \sigma_z$$

Single-qubit examples:

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$$= \frac{1}{2}[\mathbb{1} + B'_i \sigma_i] \quad \left\{ \begin{array}{l} B'_x = (1-2p)B_x \\ B'_y = (1-2p)B_y \\ B'_z = B_z \end{array} \right.$$

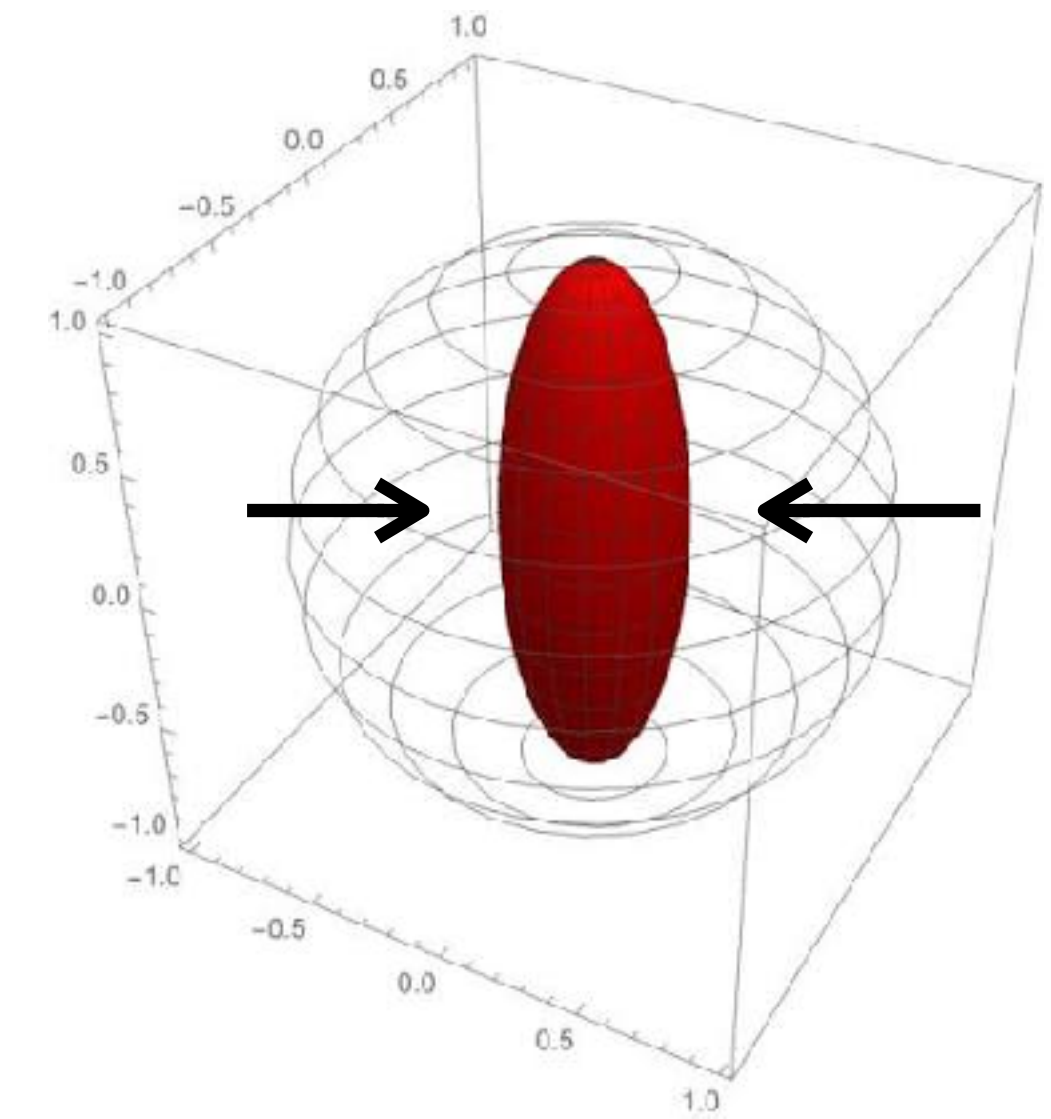


Single-qubit examples:

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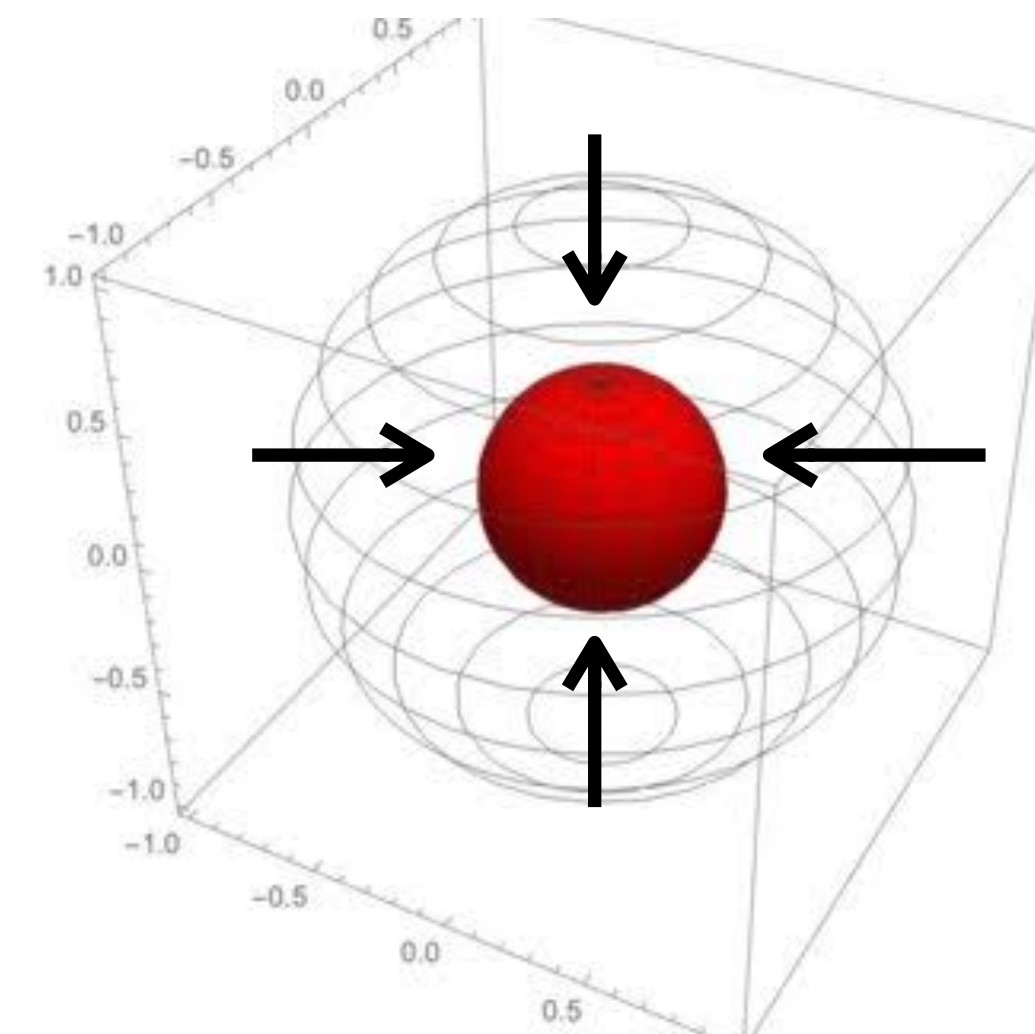
$$= \frac{1}{2}[\mathbb{1} + B'_i \sigma_i] \quad \begin{cases} B'_x = (1-2p)B_x \\ B'_y = (1-2p)B_y \\ B'_z = B_z \end{cases}$$



- **Depolarising** $K_0 = \sqrt{1 - \frac{3p}{4}} \mathbb{1}, \quad K_i = \sqrt{\frac{p}{4}} \sigma_i,$

$$\rho \longrightarrow \mathcal{E}(\rho) = (1-p)\rho + p \frac{\mathbb{1}}{2}$$

$$\sigma_i \rho \sigma_i = 2\mathbb{1} - \rho$$



Choi Matrix: A quantum channel can be described by a single matrix.

- Consider an input state: $\rho_{\text{in}} = \sum_{i,j} \rho_{ij} |i\rangle \langle j|$

$$d_{\text{in}} = \dim \mathcal{H}_{\text{in}}$$

$$d_{\text{out}} = \dim \mathcal{H}_{\text{out}}$$

$$D = d_{\text{in}} \cdot d_{\text{out}}$$

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$$D = d_{\text{in}} \cdot d_{\text{out}}$$

- $\mathcal{E}(|i\rangle \langle j|)$ is an operator in $\mathcal{H}_{\text{out}}$ $\longrightarrow [\mathcal{E}(|i\rangle \langle j|)]_{\alpha\beta} = \langle \alpha | \mathcal{E}(|i\rangle \langle j|) | \beta \rangle$

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- $\mathcal{E}(|i\rangle\langle j|)$ is an operator in $\mathcal{H}_{\text{out}}$ $\longrightarrow$ $[\mathcal{E}(|i\rangle\langle j|)]_{\alpha\beta} = \langle\alpha|\mathcal{E}(|i\rangle\langle j|)|\beta\rangle$
- Define the $D \times D$ **Choi matrix** as

$$d_{\text{in}} = \dim \mathcal{H}_{\text{in}}$$

$$d_{\text{out}} = \dim \mathcal{H}_{\text{out}}$$

$$D = d_{\text{in}} \cdot d_{\text{out}}$$

$$\tilde{\mathcal{E}} \equiv \frac{1}{d_{\text{in}}} \begin{pmatrix} \mathcal{E}(|1\rangle\langle 1|) & \mathcal{E}(|1\rangle\langle 2|) & \cdots & \mathcal{E}(|1\rangle\langle d_{\text{in}}|) \\ \mathcal{E}(|2\rangle\langle 1|) & \mathcal{E}(|2\rangle\langle 2|) & \cdots & \mathcal{E}(|2\rangle\langle d_{\text{in}}|) \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{E}(|d_{\text{in}}\rangle\langle 1|) & \mathcal{E}(|d_{\text{in}}\rangle\langle 2|) & \cdots & \mathcal{E}(|d_{\text{in}}\rangle\langle d_{\text{in}}|) \end{pmatrix}$$

each block is a $d_{\text{out}} \times d_{\text{out}}$ matrix

- The Choi matrix has **complete information** about the quantum channel.

$$\tilde{\mathcal{E}} \equiv \frac{1}{d_{\text{in}}} \begin{pmatrix} \mathcal{E}(|1\rangle\langle 1|) & \mathcal{E}(|1\rangle\langle 2|) & \cdots & \mathcal{E}(|1\rangle\langle d_{\text{in}}|) \\ \mathcal{E}(|2\rangle\langle 1|) & \mathcal{E}(|2\rangle\langle 2|) & \cdots & \mathcal{E}(|2\rangle\langle d_{\text{in}}|) \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{E}(|d_{\text{in}}\rangle\langle 1|) & \mathcal{E}(|d_{\text{in}}\rangle\langle 2|) & \cdots & \mathcal{E}(|d_{\text{in}}\rangle\langle d_{\text{in}}|) \end{pmatrix} \quad [\mathcal{E}(|i\rangle\langle j|)]_{\alpha\beta} = \langle\alpha|\mathcal{E}(|i\rangle\langle j|)|\beta\rangle$$

- The Choi matrix is Hermitian $\tilde{\mathcal{E}}^\dagger = \tilde{\mathcal{E}}$
 - QC is **C**ompletely-**P**ositive $\Leftrightarrow \tilde{\mathcal{E}}$ is PSD
 - QC is **T**race-**P**reserving $\Leftrightarrow \text{Tr } \tilde{\mathcal{E}} = 1$
- } $\tilde{\mathcal{E}}$ is a state in $\mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$
- “Channel–State duality”**

Choi–Jamiołkowski isomorphism

Jamiołkowski (1972), Choi (1975)

Time evolution of *open* quantum systems

- Consider a short time interval δt

$$\rho(t + \delta t) = K_0 \rho K_0^\dagger + \sum_{j \geq 1} K_j \rho K_j^\dagger$$

$$K_0 = \mathbb{1} + G \delta t$$

$$K_j = L_j \sqrt{\delta t} \quad (j \geq 1)$$

Time evolution of *open* quantum systems

- Consider a short time interval δt

$$\rho(t + \delta t) = K_0 \rho K_0^\dagger + \sum_{j \geq 1} K_j \rho K_j^\dagger = \rho + \left[G \rho + \rho G^\dagger + \sum_{j \geq 1} L_j \rho L_j^\dagger \right] \delta t$$

$$K_0 = \mathbb{1} + G \delta t$$

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- Trace-Preservation condition

$$\mathbb{1} \stackrel{!}{=} K_0^\dagger K_0 + \sum_{j \geq 1} K_j^\dagger K_j = \mathbb{1} + \underbrace{\left[G + G^\dagger + \sum_{j \geq 1} L_j^\dagger L_j \right] \delta t}_{=0} \Rightarrow G = -iH - \frac{1}{2} \sum_{j \geq 1} L_j^\dagger L_j$$

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The **GKSL** (Lindblad master) **equation**:

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_j \left[L_j \rho L_j^\dagger - \frac{1}{2} \{ L_j^\dagger L_j, \rho \} \right]$$

Gorini, Kossakowski,
Sudarshan (1976),
Lindblad (1976)

Single-qubit example:

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_j \left[L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\} \right]$$

- Dephasing

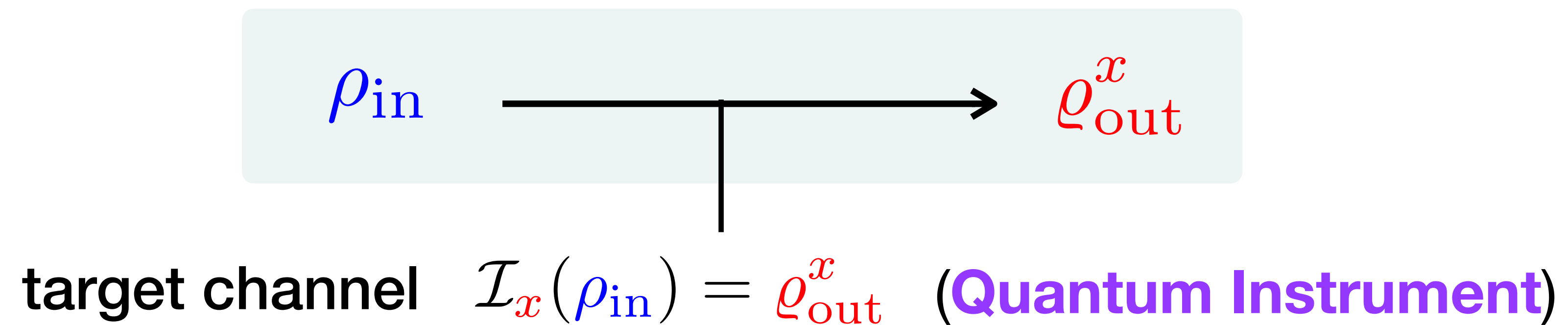
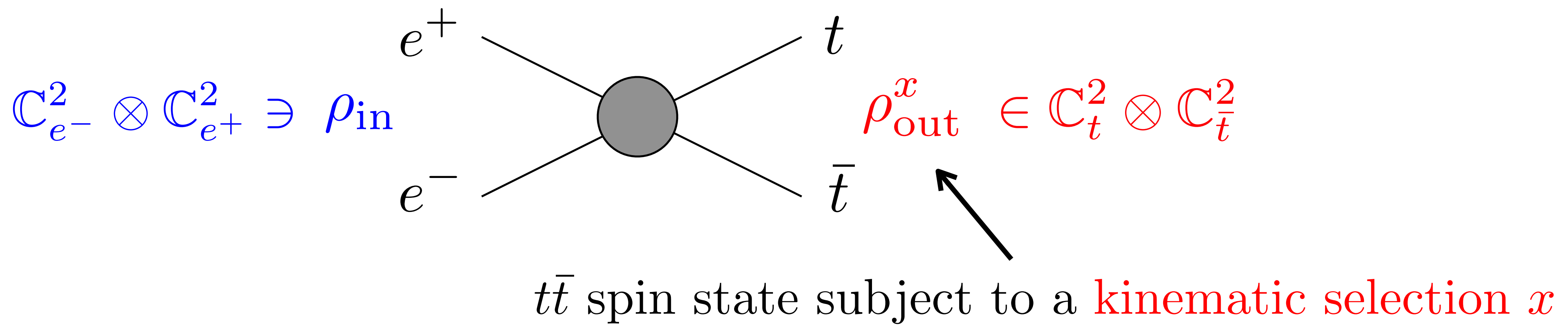
$$L = \sqrt{\frac{\gamma}{2}} \sigma_z \Rightarrow \dot{\rho} = \frac{\gamma}{2} (\sigma_z \rho \sigma_z - \rho) \Rightarrow \begin{cases} \dot{\rho}_{00} = \dot{\rho}_{11} = 0 \\ \dot{\rho}_{01} = -\gamma \cdot \rho_{01} \end{cases}$$

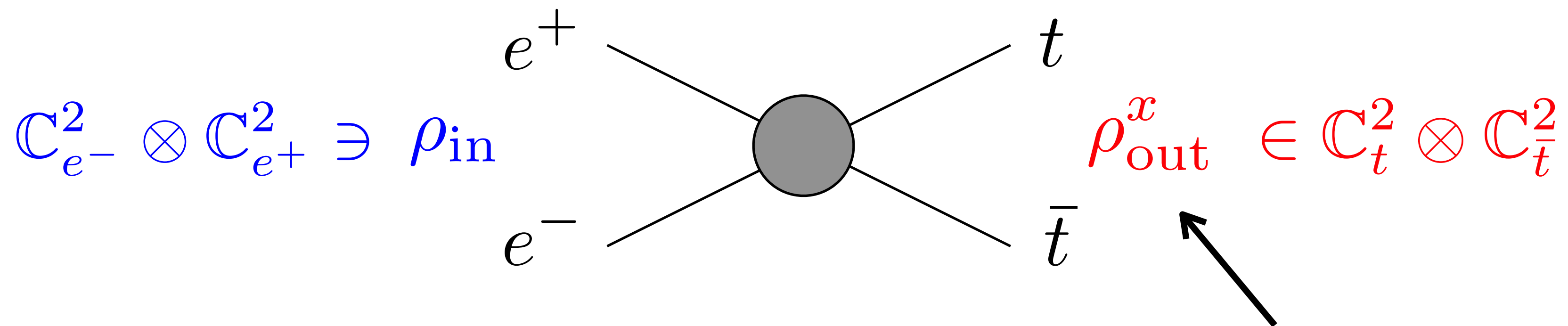
$$\Rightarrow \rho(t) = \begin{pmatrix} \rho_{00}(0) & e^{-\gamma t} \rho_{01}(0) \\ e^{-\gamma t} \rho_{10}(0) & \rho_{11}(0) \end{pmatrix}$$

coherence (off-diagonal) terms **decay** exponentially

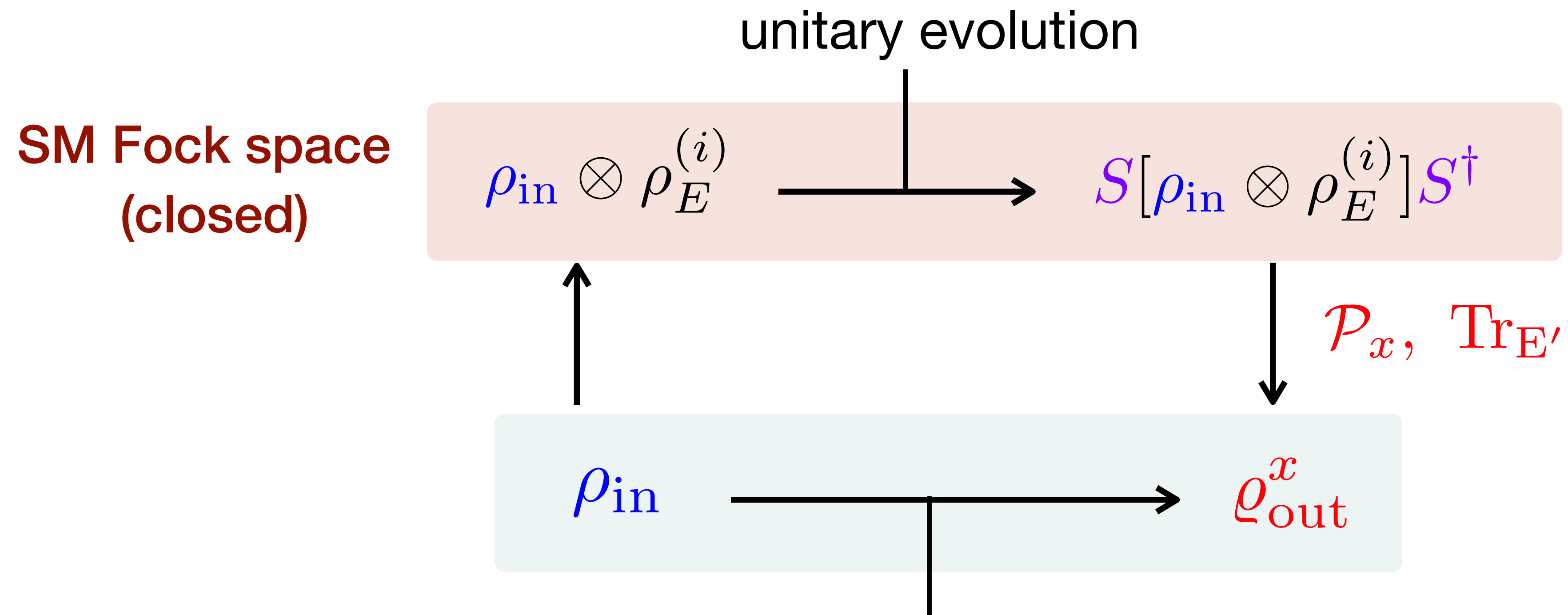
Application

- Reconstruction of QC; probing NLQM
- Decoherence as radiation effects





$t\bar{t}$ spin state subject to a **kinematic selection** x



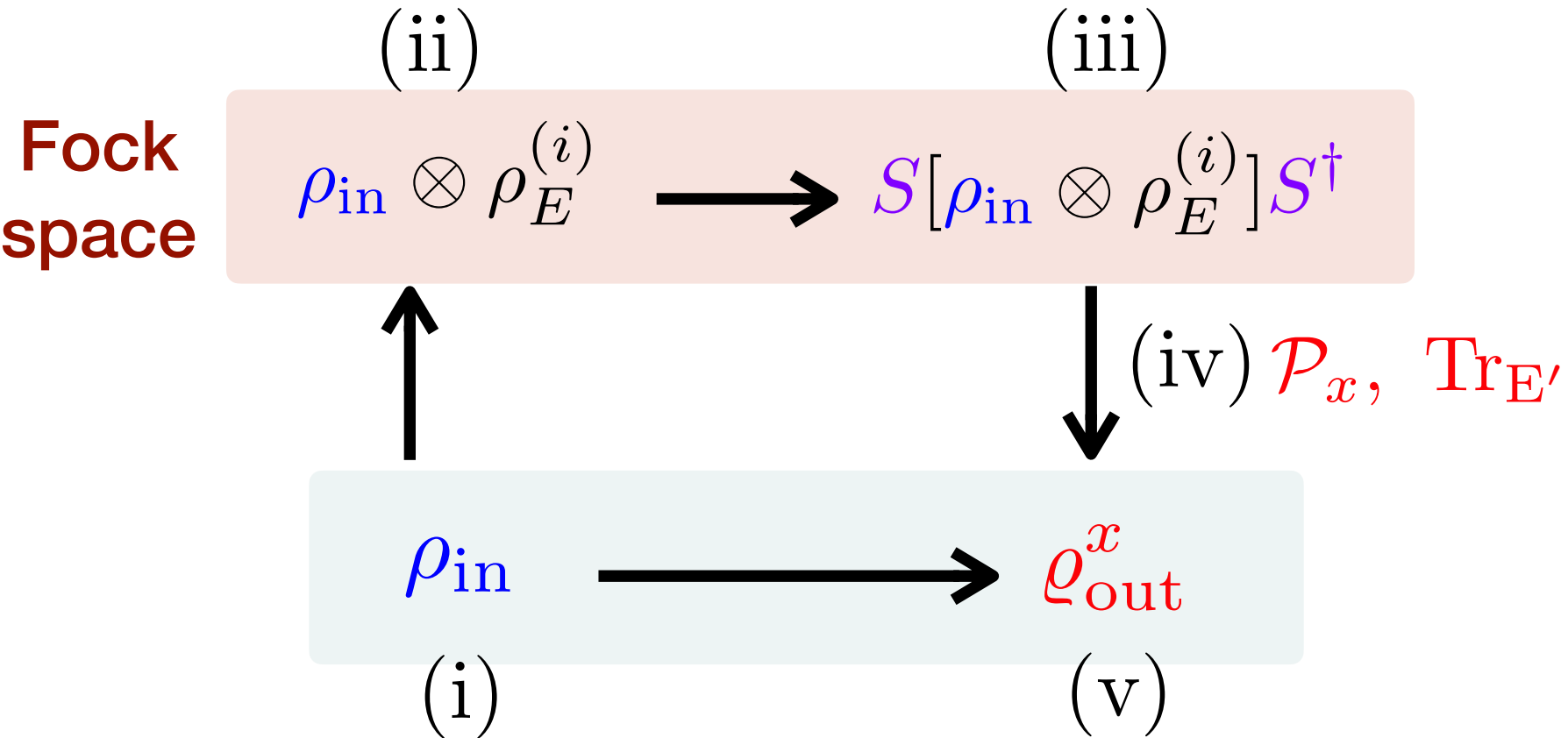
target channel $\mathcal{I}_x(\rho_{\text{in}}) = \mathcal{Q}_{\text{out}}^x$ (**Quantum Instrument**)

$$\text{(i)} \quad \rho_{\text{in}} = \sum_{I,J,K,L} \rho_{\text{in}[I,J],[K,L]} |I,J\rangle \langle K,L|$$

$$e^-e^+ \text{ spin basis}$$

$$|I,J\rangle = |I\rangle_{e^-} \otimes |J\rangle_{e^+}$$

$$I,J,K,L \in \{+,-\}$$



(i)

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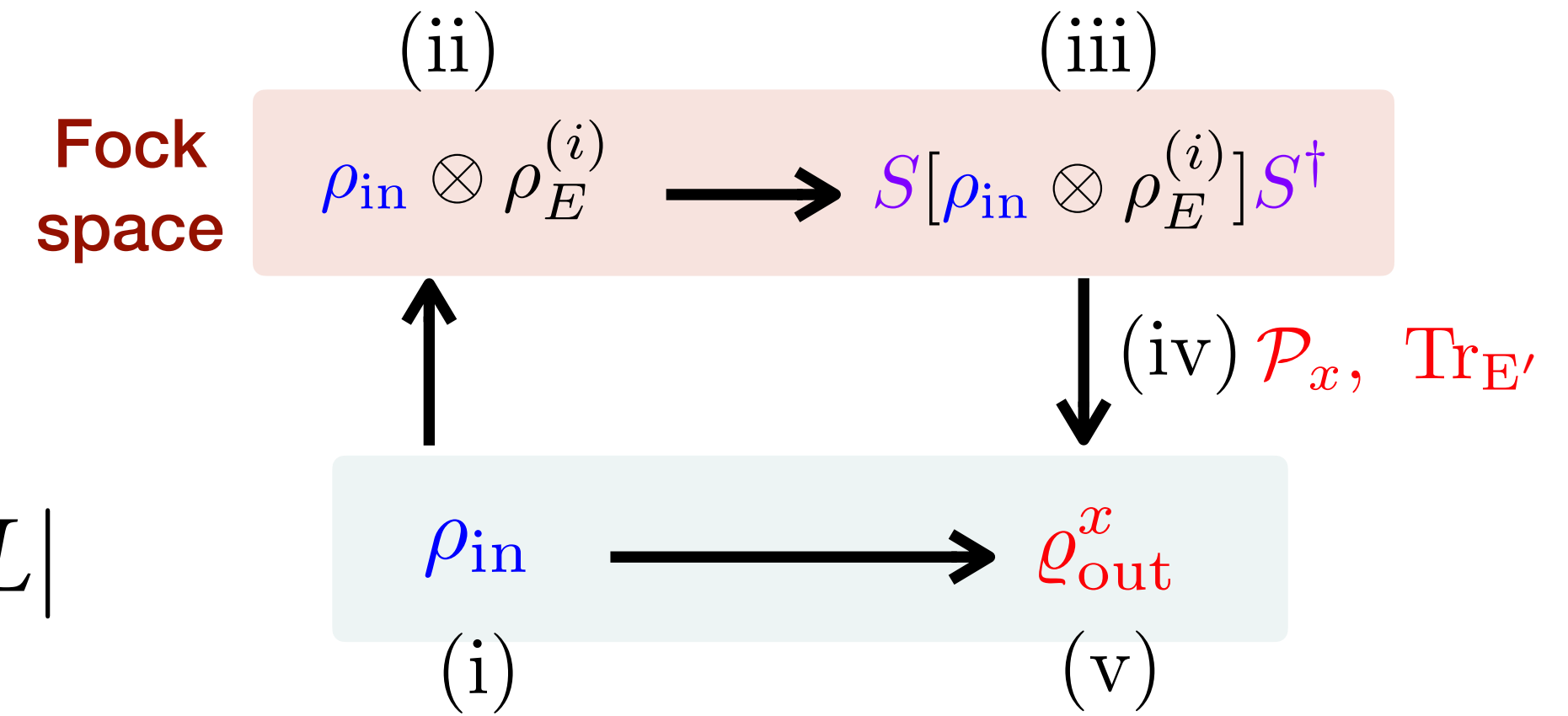
(ii)

$$\bar{\rho}_{\text{in}} = |\tilde{p}_{\text{in}}\rangle \langle \tilde{p}_{\text{in}}| \otimes \rho_{\text{in}} = \sum_{I,J,K,L} \rho_{\text{in}[I,J],[K,L]} |\tilde{p}_{\text{in}}; I,J\rangle \langle \tilde{p}_{\text{in}}; K,L|$$

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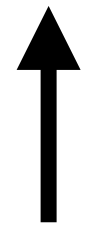
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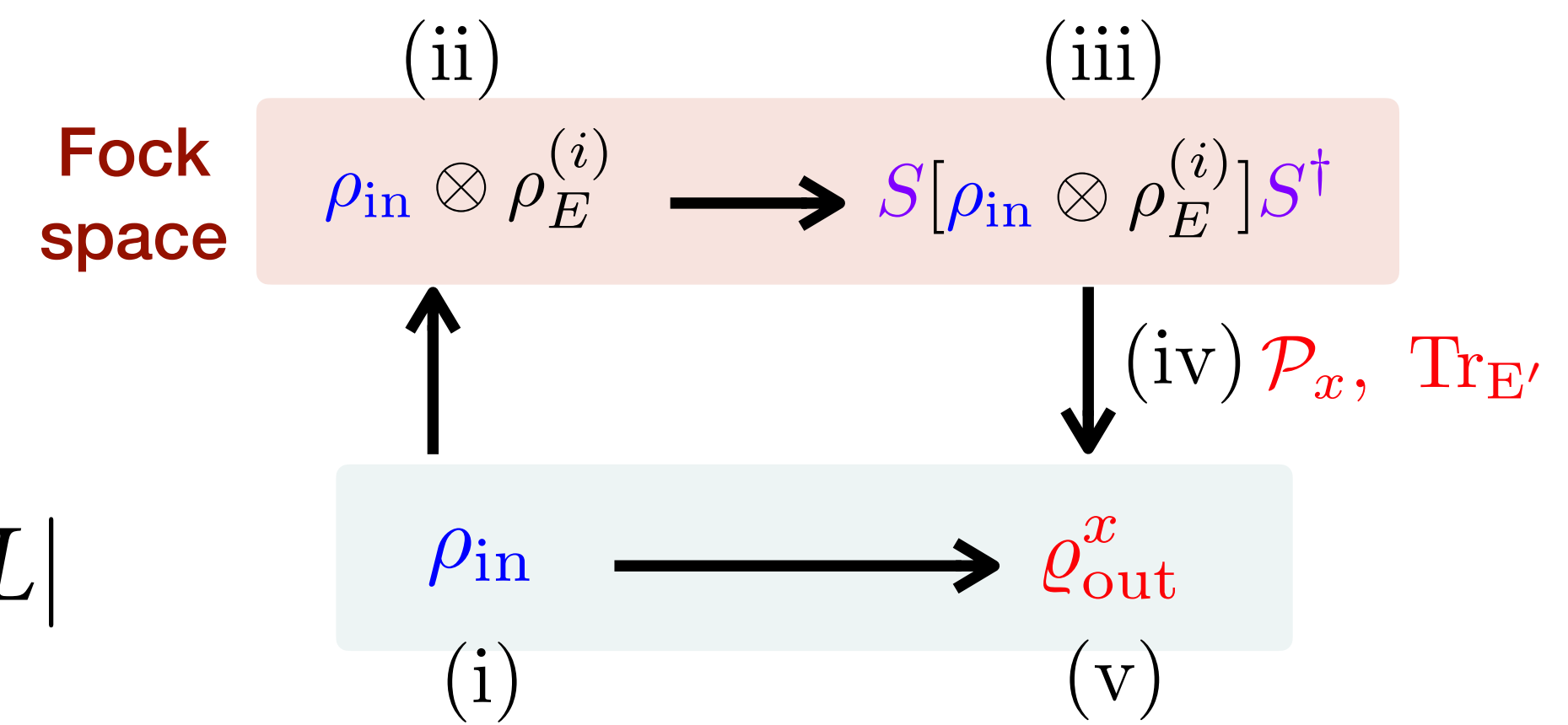
$$(ii) \quad \bar{\rho}_{\text{in}} = |\tilde{p}_{\text{in}}\rangle \langle \tilde{p}_{\text{in}}| \otimes \rho_{\text{in}} = \sum_{I,J,K,L} \rho_{\text{in}[I,J],[K,L]} |\tilde{p}_{\text{in}}; I, J\rangle \langle \tilde{p}_{\text{in}}; K, L|$$

$$(iii) \quad \bar{\rho}_{\text{out}} = S \bar{\rho}_{\text{in}} S^\dagger$$



a superposition involving

$$t\bar{t}, \quad W^+W^-, \quad \mu^+\mu^-, \quad \mu^+\mu^-\gamma, \dots$$



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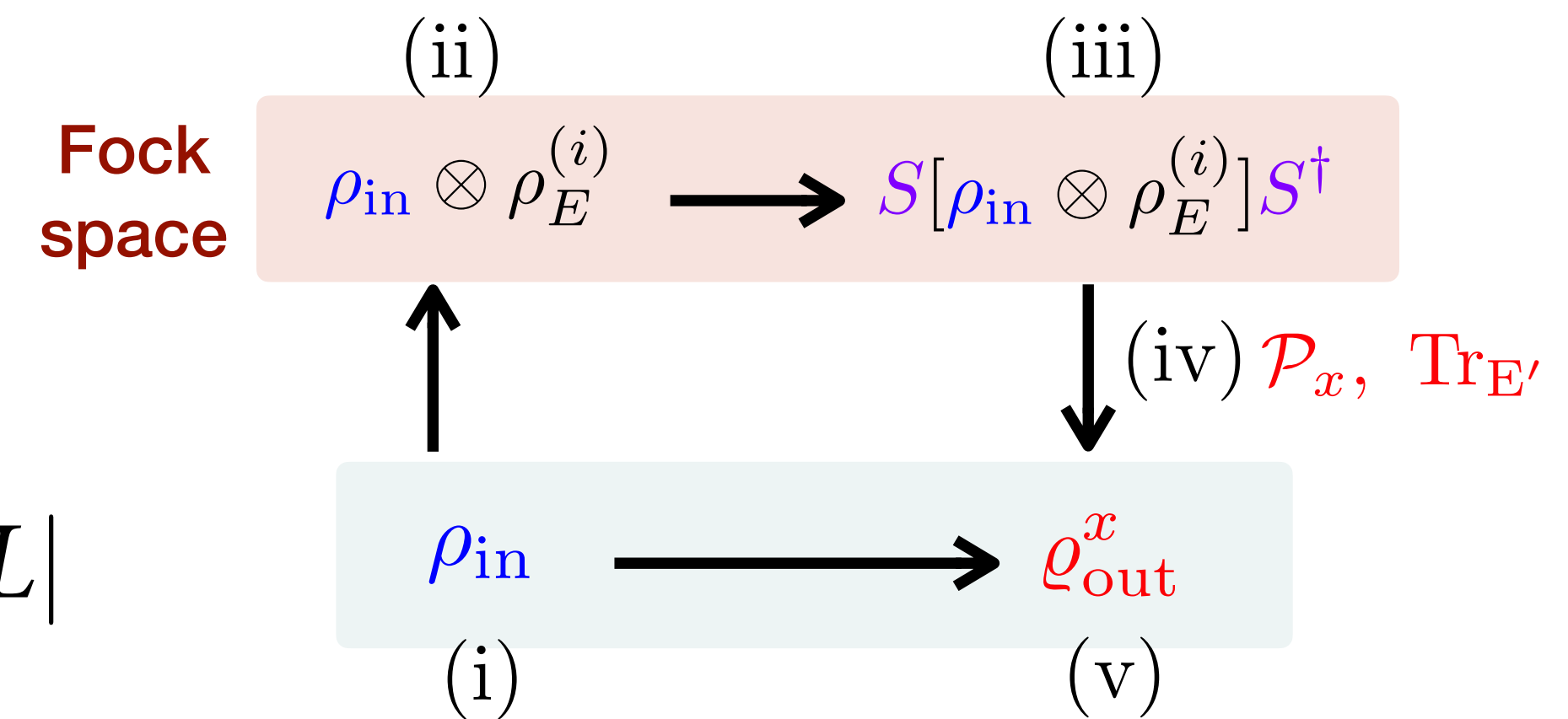
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a superposition involving

$$t\bar{t}, \quad W^+W^-, \quad \mu^+\mu^-, \quad \mu^+\mu^-\gamma, \dots$$



$$(iv) \quad \mathcal{P}_x = \sum_{A,B} \int_x d\Pi_{\gamma\delta} |p_f; A, B\rangle \langle p_f; A, B|$$

$$d\Pi_i = \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i}$$

$$t\bar{t} \text{ spin labels}$$

$$A, B, C, D \in \{0, 1\}$$

e^-e^+ spin basis

$$|I, J\rangle = |I\rangle_{e^-} \otimes |J\rangle_{e^+}$$

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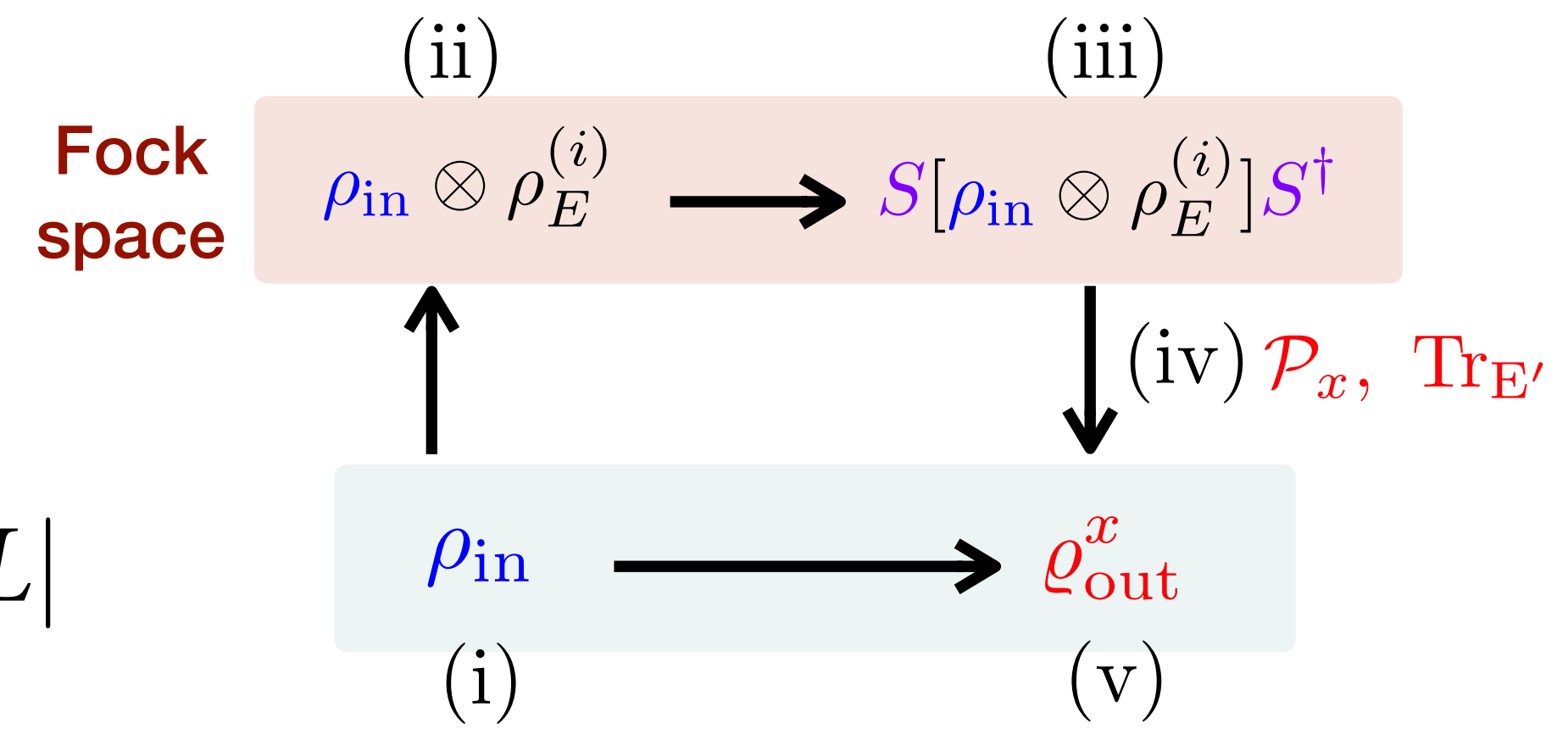
$$(ii) \quad \bar{\rho}_{\text{in}} = |\tilde{p}_{\text{in}}\rangle \langle \tilde{p}_{\text{in}}| \otimes \rho_{\text{in}} = \sum_{I,J,K,L} \rho_{\text{in}[I,J],[K,L]} |\tilde{p}_{\text{in}}; I, J\rangle \langle \tilde{p}_{\text{in}}; K, L|$$

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$$(v) \quad \varrho'_x = \text{Tr}_P [\mathcal{P}_x \bar{\rho}_{\text{out}} \mathcal{P}_x]$$

$$= \sum_{I,J,K,L} \sum_{A,B,C,D} \rho_{\text{in}[I,J],[K,L]} \int_x d\Pi_{\gamma\delta} \langle p_f; A, B | S | \tilde{p}_{\text{in}}; I, J \rangle \times$$

$$\times \langle \tilde{p}_{\text{in}}; K, L | S^\dagger | p_f; C, D \rangle |A, B\rangle \langle C, D|$$



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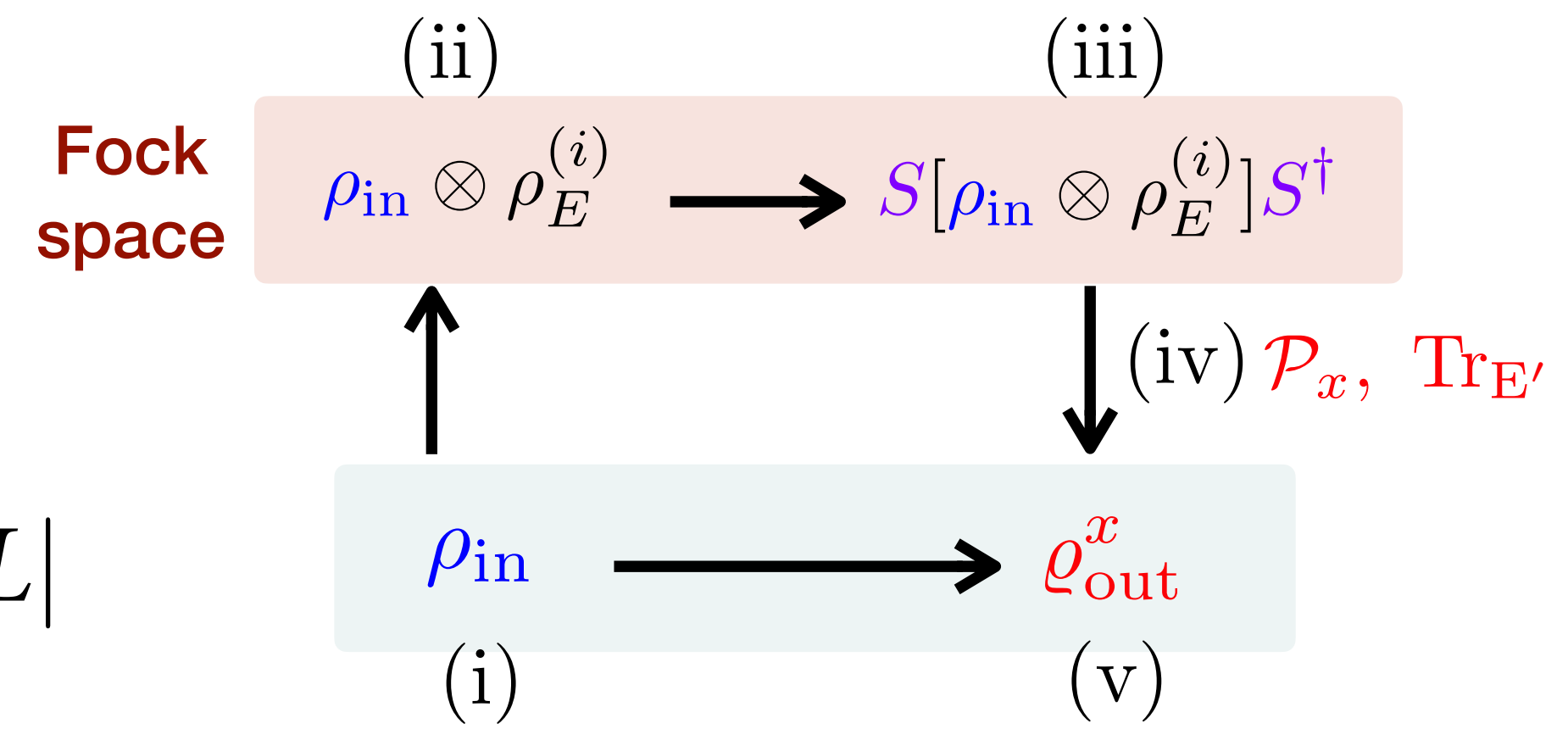
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$$\times \langle \tilde{p}_{\text{in}}; K, L | S^\dagger | p_f; C, D \rangle |A, B\rangle \langle C, D|$$

$$\propto \mathcal{M}_{A,B}^{I,J}$$



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$t\bar{t}$ spin labels
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$$|I, J\rangle = |I\rangle_{e^-} \otimes |J\rangle_{e^+}$$

$$I, J, K, L \in \{+, -\}$$

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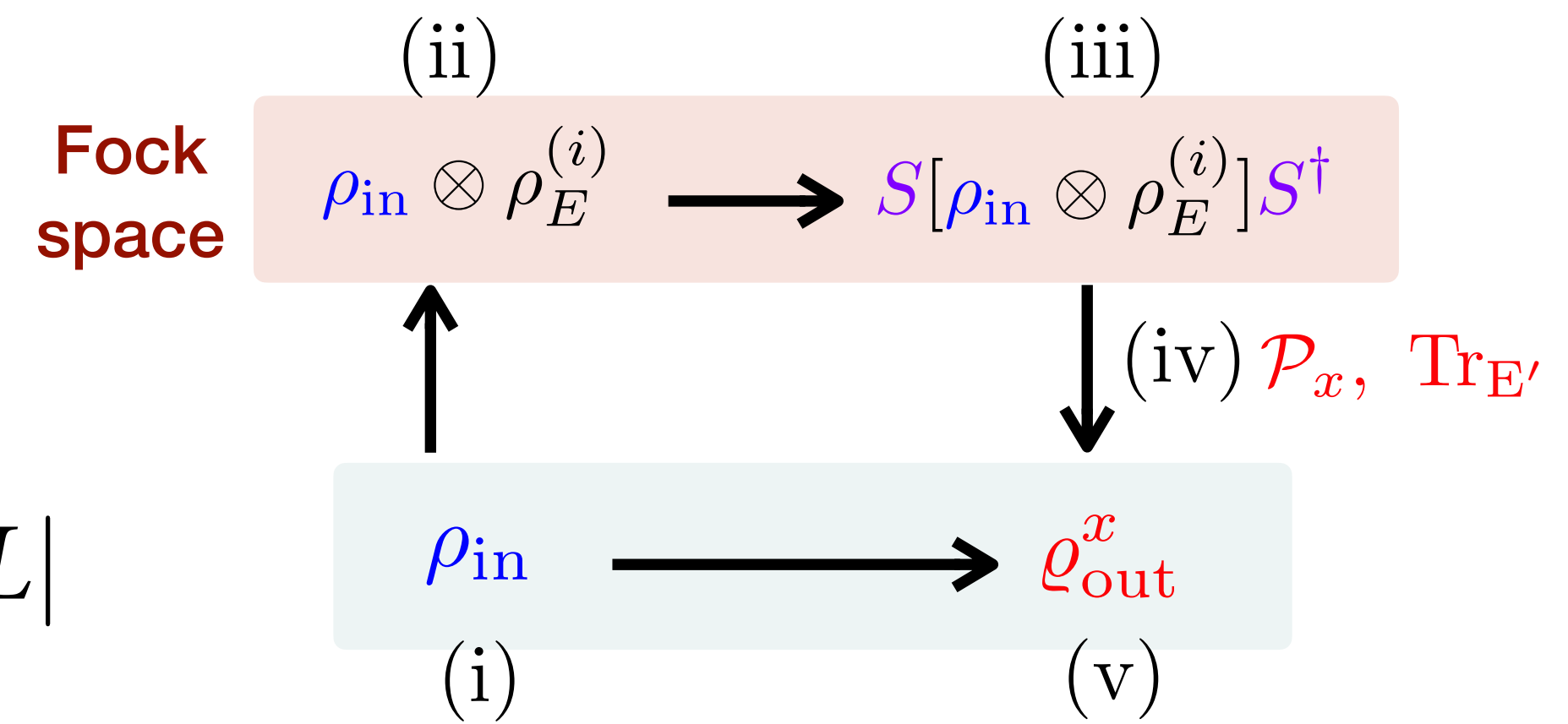
$$(v) \quad \varrho'_x = \text{Tr}_P [\mathcal{P}_x \bar{\rho}_{\text{out}} \mathcal{P}_x]$$

$$= \sum_{I,J,K,L} \sum_{A,B,C,D} \rho_{\text{in}[I,J],[K,L]} \int_x d\Pi_{\gamma\delta} \langle p_f; A, B | S | \tilde{p}_{\text{in}}; I, J \rangle \times$$

$$\times \langle \tilde{p}_{\text{in}}; K, L | S^\dagger | p_f; C, D \rangle |A, B\rangle \langle C, D|$$

$$= \int_x d\Pi_{\text{LIPS}} E(p_f) \rho_{\text{in}} E^\dagger(p_f)$$

**Kraus
operators**



$$(iv) \quad \mathcal{P}_x = \sum_{A,B} \int_x d\Pi_{\gamma\delta} |p_f; A, B\rangle \langle p_f; A, B|$$

$$d\Pi_i = \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i}$$

$t\bar{t}$ spin labels

$$A, B, C, D \in \{0, 1\}$$

$$E(p_f) = \frac{1}{\sqrt{2s\sigma_{\mathcal{N}}}} \sum_{I,J,A,B} \mathcal{M}_{A,B}^{I,J} |A, B\rangle \langle I, J|$$

Choi matrix

$$\mathcal{I}_x(|I, J\rangle\langle K, L|)_{[A, B], [C, D]} = \frac{1}{d_{\text{in}}} \frac{1}{\sigma_{\mathcal{N}}} \frac{1}{2s} \int_x d\Pi_{\text{LIPS}} \mathcal{M}_{A, B}^{I, J} \left(\mathcal{M}_{C, D}^{K, L} \right)^*$$

$$(iii) \quad \bar{\rho}_{\text{out}} = S \bar{\rho}_{\text{in}} S^\dagger$$

$$(iv) \quad \mathcal{P}_x = \sum_{A, B} \int_x d\Pi_{\gamma\delta} |p_f; A, B\rangle\langle p_f; A, B|$$

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$$d\Pi_i = \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i}$$

$t\bar{t}$ spin labels

$$A, B, C, D \in \{0, 1\}$$

$$\propto \mathcal{M}_{A, B}^{I, J}$$

$$\times \langle \tilde{p}_{\text{in}}; K, L | S^\dagger | p_f; C, D \rangle |A, B\rangle\langle C, D|$$

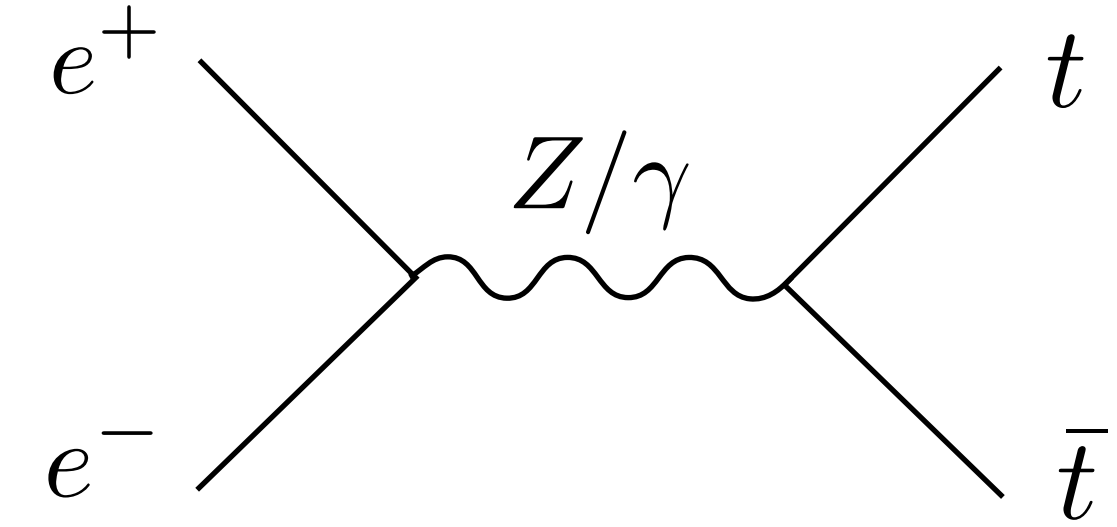
$$= \int_x d\Pi_{\text{LIPS}} E(p_f) \rho_{\text{in}} E^\dagger(p_f)$$

**Kraus
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$$E(p_f) = \frac{1}{\sqrt{2s\sigma_{\mathcal{N}}}} \sum_{I, J, A, B} \mathcal{M}_{A, B}^{I, J} |A, B\rangle\langle I, J|$$

Standard Model prediction (LO):

$$\begin{aligned}
 \mathcal{M}_{00}^{++} &= \mathcal{M}_{11}^{++} = e^{i\phi} \sum_i \frac{s}{2\Lambda_i^2} \gamma^{-1} c_R^i \sin \theta (d_L^i + d_R^i), \\
 \mathcal{M}_{01}^{++} &= -e^{i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_R^i (1 + \cos \theta) [d_L^i (1 - \beta) + d_R^i (1 + \beta)], \\
 \mathcal{M}_{10}^{++} &= -e^{i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_R^i (1 - \cos \theta) [d_L^i (1 + \beta) + d_R^i (1 - \beta)], \\
 \mathcal{M}_{00}^{--} &= \mathcal{M}_{11}^{--} = -e^{-i\phi} \sum_i \frac{s}{2\Lambda_i^2} \gamma^{-1} c_L^i \sin \theta (d_L^i + d_R^i), \\
 \mathcal{M}_{01}^{--} &= -e^{-i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_L^i (1 - \cos \theta) [d_L^i (1 - \beta) + d_R^i (1 + \beta)], \\
 \mathcal{M}_{10}^{--} &= -e^{-i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_L^i (1 + \cos \theta) [d_L^i (1 + \beta) + d_R^i (1 - \beta)],
 \end{aligned}$$



$$\mathcal{M}_{A,B}^{+-} = \mathcal{M}_{A,B}^{-+} = 0$$

$$\mathcal{I}_x (|I, J\rangle \langle K, L|)_{[A,B],[C,D]} = \frac{1}{d_{\text{in}}} \frac{1}{\sigma_{\mathcal{N}}} \frac{1}{2s} \int_x d\Pi_{\text{LIPS}} \mathcal{M}_{A,B}^{I,J} \left(\mathcal{M}_{C,D}^{K,L} \right)^*$$

$$\sigma_{\mathcal{N}} = \frac{N_C}{128\pi^2} \frac{q}{s\sqrt{s}} \int \sum_{I,J,A,B} \left| \mathcal{M}_{A,B}^{I,J} \right|^2 d\Omega$$

The SM Choi matrix ($\rho_{\text{in}} \rightarrow Q_{\text{out}}^x$) :

$$\tilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++ , ++)} & 0 & 0 & I_x^{(++ , --)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(-- , ++)} & 0 & 0 & I_x^{(-- , --)} \end{pmatrix}$$

$$I_x^{(-- , ++)} = [I_x^{(++ , --)}]^\dagger$$

$$\frac{dI^{(++ , ++)}}{d \cos \theta d\phi} = \begin{pmatrix} a_{11}^{(+)} s_\theta^2 & a_{12}^{(+)} s_\theta (1 + c_\theta) & a_{13}^{(+)} s_\theta (1 - c_\theta) & a_{14}^{(+)} s_\theta^2 \\ a_{21}^{(+)} s_\theta (1 + c_\theta) & a_{22}^{(+)} (1 + c_\theta)^2 & a_{23}^{(+)} (1 - c_\theta^2) & a_{24}^{(+)} s_\theta (1 + c_\theta) \\ a_{31}^{(+)} s_\theta (1 - c_\theta) & a_{32}^{(+)} (1 - c_\theta^2) & a_{33}^{(+)} (1 - c_\theta)^2 & a_{34}^{(+)} s_\theta (1 - c_\theta) \\ a_{41}^{(+)} s_\theta^2 & a_{42}^{(+)} s_\theta (1 + c_\theta) & a_{43}^{(+)} s_\theta (1 - c_\theta) & a_{44}^{(+)} s_\theta^2 \end{pmatrix},$$

$$\frac{dI^{(++ , --)}}{d \cos \theta d\phi} = e^{i2\phi} \begin{pmatrix} a_{11}^{(+-)} s_\theta^2 & a_{12}^{(+-)} s_\theta (1 - c_\theta) & a_{13}^{(+-)} s_\theta (1 + c_\theta) & a_{14}^{(+-)} s_\theta^2 \\ a_{21}^{(+-)} s_\theta (1 + c_\theta) & a_{22}^{(+-)} (1 - c_\theta^2) & a_{23}^{(+-)} (1 + c_\theta)^2 & a_{24}^{(+-)} s_\theta (1 + c_\theta) \\ a_{31}^{(+-)} s_\theta (1 - c_\theta) & a_{32}^{(+-)} (1 - c_\theta)^2 & a_{33}^{(+-)} (1 - c_\theta^2) & a_{34}^{(+-)} s_\theta (1 - c_\theta) \\ a_{41}^{(+-)} s_\theta^2 & a_{42}^{(+-)} s_\theta (1 - c_\theta) & a_{43}^{(+-)} s_\theta (1 + c_\theta) & a_{44}^{(+-)} s_\theta^2 \end{pmatrix}$$

$$\frac{dI^{(-- , --)}}{d \cos \theta d\phi} = \begin{pmatrix} a_{11}^{(-)} s_\theta^2 & a_{12}^{(-)} s_\theta (1 - c_\theta) & a_{13}^{(-)} s_\theta (1 + c_\theta) & a_{14}^{(-)} s_\theta^2 \\ a_{21}^{(-)} s_\theta (1 - c_\theta) & a_{22}^{(-)} (1 - c_\theta)^2 & a_{23}^{(-)} (1 - c_\theta^2) & a_{24}^{(-)} s_\theta (1 - c_\theta) \\ a_{31}^{(-)} s_\theta (1 + c_\theta) & a_{32}^{(-)} (1 - c_\theta^2) & a_{33}^{(-)} (1 + c_\theta)^2 & a_{34}^{(-)} s_\theta (1 + c_\theta) \\ a_{41}^{(-)} s_\theta^2 & a_{42}^{(-)} s_\theta (1 - c_\theta) & a_{43}^{(-)} s_\theta (1 + c_\theta) & a_{44}^{(-)} s_\theta^2 \end{pmatrix}.$$

$$a^{(+)}|_{\sqrt{s}=370 \text{ GeV}} = \begin{pmatrix} 0.503 & -1.442 & 0.364 & 0.503 \\ -1.442 & 4.137 & -1.043 & -1.442 \\ 0.364 & -1.043 & 0.263 & 0.364 \\ 0.503 & -1.442 & 0.364 & 0.503 \end{pmatrix} \cdot 10^{-2},$$

$$a^{(+-)}|_{\sqrt{s}=370 \text{ GeV}} = \begin{pmatrix} -0.800 & 0.264 & -1.979 & -0.800 \\ 2.293 & -0.758 & 5.676 & 2.293 \\ -0.578 & 0.191 & -1.431 & -0.578 \\ -0.800 & 0.264 & -1.980 & -0.800 \end{pmatrix} \cdot 10^{-2},$$

$$a^{(-)}|_{\sqrt{s}=370 \text{ GeV}} = \begin{pmatrix} 1.271 & -0.420 & 3.146 & 1.271 \\ -0.420 & 0.139 & -1.040 & -0.420 \\ 3.146 & -1.040 & 7.788 & 3.146 \\ 1.271 & -0.420 & 3.146 & 1.271 \end{pmatrix} \cdot 10^{-2},$$

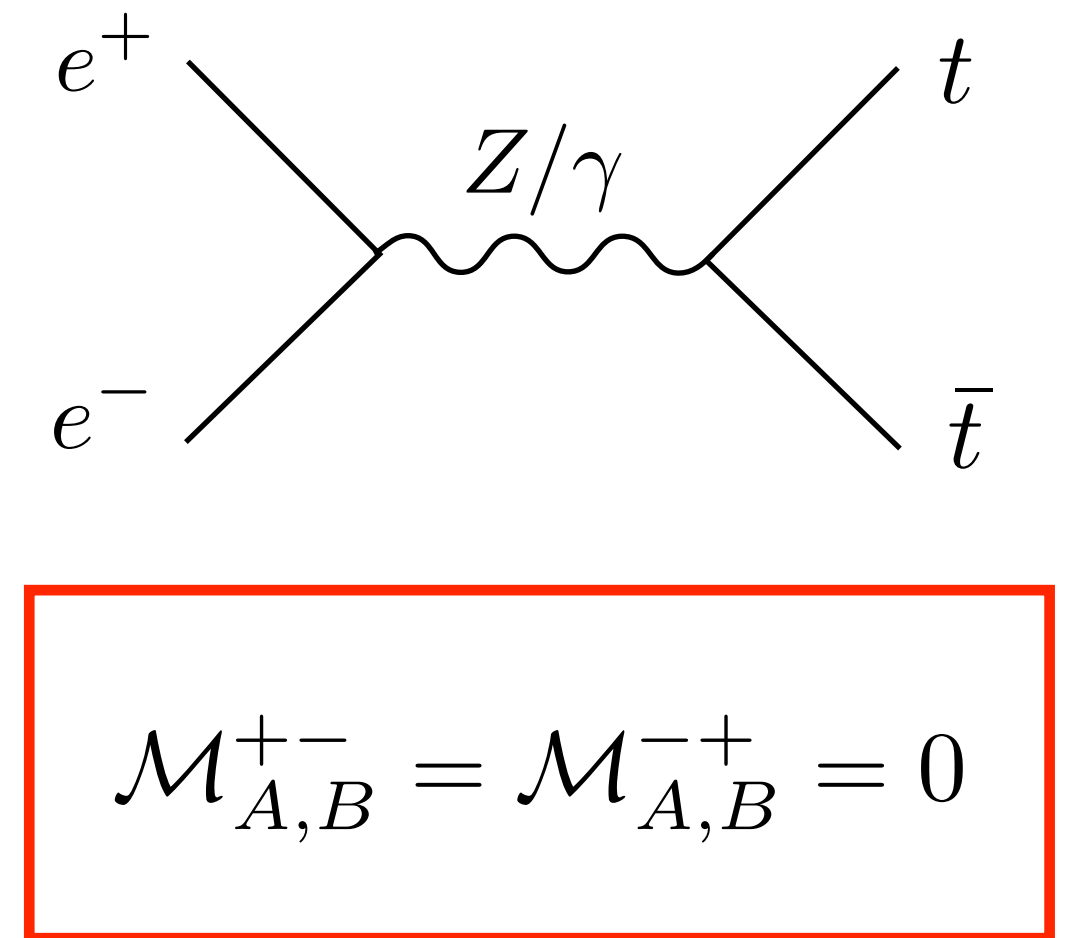
$$a^{(+)}|_{\sqrt{s}=1 \text{ TeV}} = \begin{pmatrix} 0.219 & -0.777 & -0.494 & 0.219 \\ -0.777 & 2.755 & 1.751 & -0.777 \\ -0.494 & 1.751 & 1.113 & -0.494 \\ 0.219 & -0.777 & -0.494 & 0.219 \end{pmatrix} \cdot 10^{-2},$$

$$a^{(+-)}|_{\sqrt{s}=1 \text{ TeV}} = \begin{pmatrix} -0.340 & -0.810 & -1.162 & -0.340 \\ 1.205 & 2.870 & 4.117 & 1.205 \\ 0.766 & 1.824 & 2.617 & 0.766 \\ -0.340 & -0.810 & -1.162 & -0.340 \end{pmatrix} \cdot 10^{-2},$$

$$a^{(-)}|_{\sqrt{s}=1 \text{ TeV}} = \begin{pmatrix} 0.527 & 1.256 & 1.802 & 0.527 \\ 1.256 & 2.990 & 4.290 & 1.256 \\ 1.802 & 4.290 & 6.154 & 1.802 \\ 0.527 & 1.256 & 1.802 & 0.527 \end{pmatrix} \cdot 10^{-2},$$

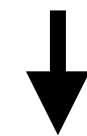
The SM Choi matrix ($\rho_{\text{in}} \rightarrow Q_{\text{out}}^x$):

$$\tilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++ , ++)} & 0 & 0 & I_x^{(++ , --)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(-- , ++)} & 0 & 0 & I_x^{(-- , --)} \end{pmatrix}$$



- This zero structure is broken by **new physics**:

s-channel scalar



$$\frac{c_S}{\Lambda^2} [\bar{\psi}_e \psi_e] [\bar{\psi}_t \psi_t],$$

s-channel tensor



$$\frac{c_T}{\Lambda^2} [\bar{\psi}_e \sigma_{\mu\nu} \psi_e] [\bar{\psi}_t \sigma^{\mu\nu} \psi_t],$$

t-channel scalar



$$\frac{c_t}{\Lambda^2} [\bar{\psi}_e \psi_t] [\bar{\psi}_t \psi_e], \dots$$

LO of **new physics** can directly competes with **NLO** of **SM**.

Experimental reconstruction of the Choi matrix (Quantum Process Tomography):

- Need an informationally-complete set of beam polarisation settings:

$$|I, J\rangle\langle K, L| \quad I, J, K, L \in \{+, -\} \quad \longleftarrow \quad 2^4 = 16 \text{ basis operators}$$

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$$4 \text{ } e^- \text{ polarisations: } \rho_{\text{in},e^-}^+ = \eta(q, +), \quad \rho_{\text{in},e^-}^- = \eta(q, -), \quad \rho_{\text{in},e^-}^{\vec{x}} = \eta(q, \vec{x}), \quad \rho_{\text{in},e^-}^{\vec{y}} = \eta(q, \vec{y})$$

$$4 \text{ } e^+ \text{ polarisations: } \rho_{\text{in},e^+}^+ = \eta(\bar{q}, +), \quad \rho_{\text{in},e^+}^- = \eta(\bar{q}, -), \quad \rho_{\text{in},e^+}^{\vec{x}} = \eta(\bar{q}, \vec{x}), \quad \rho_{\text{in},e^+}^{\vec{y}} = \eta(\bar{q}, \vec{y})$$

$$\eta(q, \mathbf{n}) = q|\mathbf{n}\rangle\langle\mathbf{n}| + \frac{1-q}{2} \mathbf{1} = \frac{1}{2} [(1+q)|\mathbf{n}\rangle\langle\mathbf{n}| + (1-q)|-\mathbf{n}\rangle\langle-\mathbf{n}|] = \sum_{I,K=\pm} C_{(q,\mathbf{n})}^{(I,K)} |I\rangle\langle K|$$

↑ ↑ polarisation axis

↑ purity of polarisation ($0 \leq q \leq 1$)

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↑ ↑ polarisation axis

purity of polarisation ($0 \leq q \leq 1$)

$$|\vec{x}\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$$

- Inversion (electron beam):

$$|I\rangle\langle K| = \sum_a X_a^{(I,K)} \rho_{e^-}^a \quad X = \frac{1}{2q} \begin{pmatrix} 1+q & -(1-q) & 0 & 0 \\ -(1+i) & -(1+i) & 2 & 2i \\ -1+i & -1+i & 2 & -2i \\ -(1-q) & 1+q & 0 & 0 \end{pmatrix}$$

$$|\vec{y}\rangle = \frac{1}{\sqrt{2}} (|+\rangle + i|-\rangle)$$

Experimental reconstruction of the Choi matrix (Quantum Process Tomography):

- Inversion (electron-positron beams):

$$\rho_{\text{in}}^{(a,b)} = \rho_{\text{in},e^-}^a \otimes \rho_{\text{in},e^+}^b \quad Y = X|_{q \rightarrow \bar{q}}$$

$$|I, J\rangle\langle K, L| = \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} \rho_{\text{in}}^{a,b}$$

The reconstruction of all 16 initial basis operators from the 16 physical beam polarisation states.

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The reconstruction of all 16 initial basis operators from the 16 physical beam polarisation states.

$$\mathcal{I}_x(|I, J\rangle\langle K, L|) = \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} \mathcal{I}_x(\rho_{\text{in}}^{a,b}) = \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} Q_x^{(a,b)}$$

↑
reconstruct

QPT

↑
reconstruct

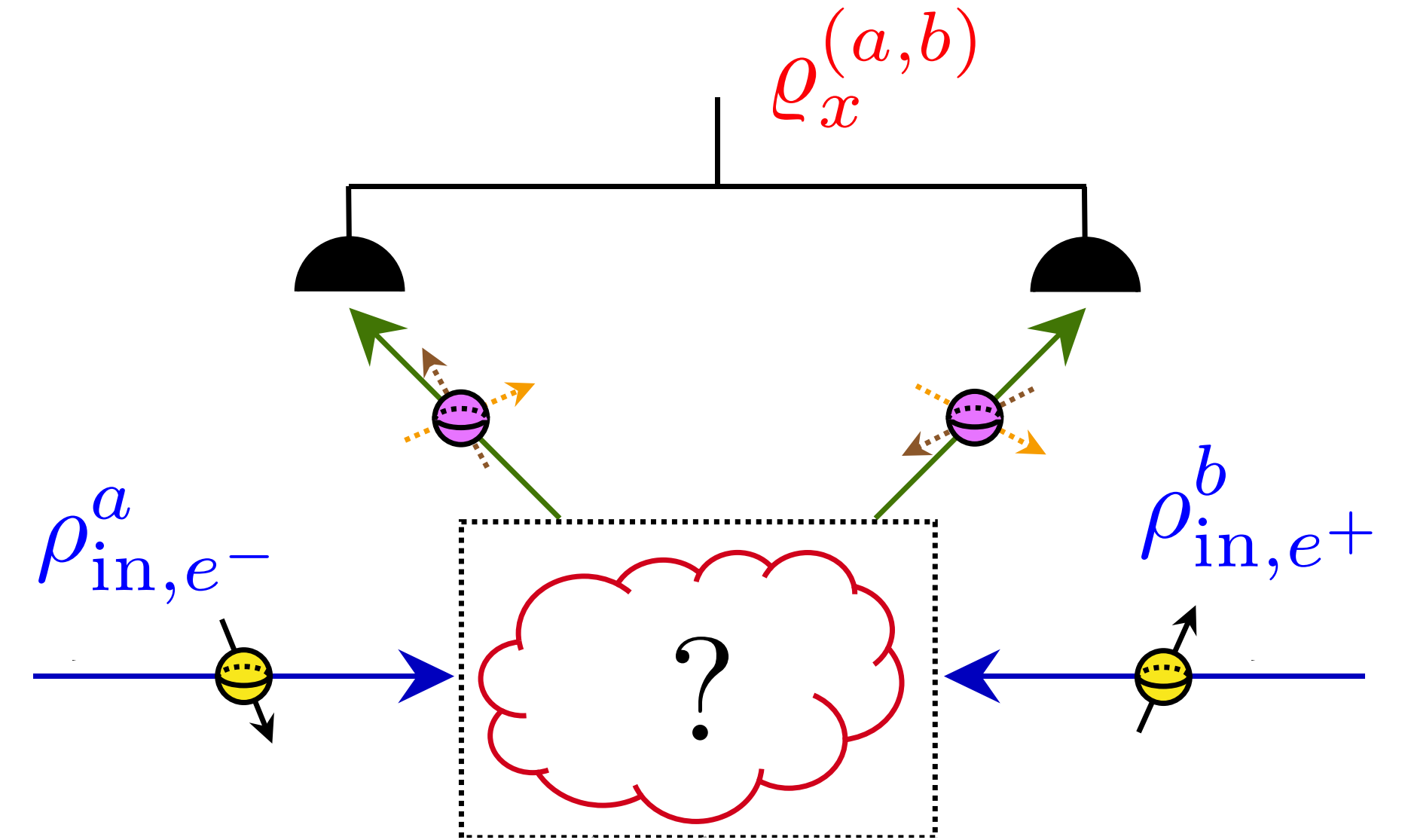
QST

Experimental reconstruction of the Choi matrix (Quantum Process Tomography):

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$$\rho_{\text{in}}^{(a,b)} = \rho_{\text{in},e^-}^a \otimes \rho_{\text{in},e^+}^b \quad Y = X|_{q \rightarrow \bar{q}}$$

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reconstruct

QPT

reconstruct

QST

$$\tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A, B], [C, D]}$$

Precision test of SM

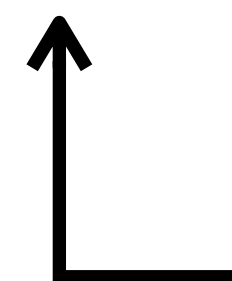
$$\tilde{\mathcal{I}}_x^{\text{SM}} = \begin{pmatrix} I_x^{(++ , ++)} & 0 & 0 & I_x^{(++ , --)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(-- , ++)} & 0 & 0 & I_x^{(-- , --)} \end{pmatrix}$$

Positivity test of QM

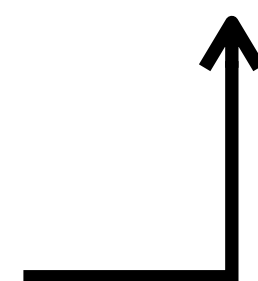
$$\lambda_{\min}(\tilde{\mathcal{I}}_x) = \min \{ \lambda \mid \det(\tilde{\mathcal{I}}_x - \lambda) = 0 \}$$

Linearity test of QM

$$[Q_x]_{[A, B], [C, D]}^{\text{prediction}} = \sum_{[I, J], [K, L]} \rho_{[I, J], [K, L]}^{\text{in, new}} \tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A, B], [C, D]}$$



check
experimentally



QPT = reconstruction of the *theory*

High Energy test of QM

Born rule:

$$\langle \hat{A} \rangle_\psi = \langle \psi | \hat{A} | \psi \rangle$$



Bell test

Linear time-evolution:

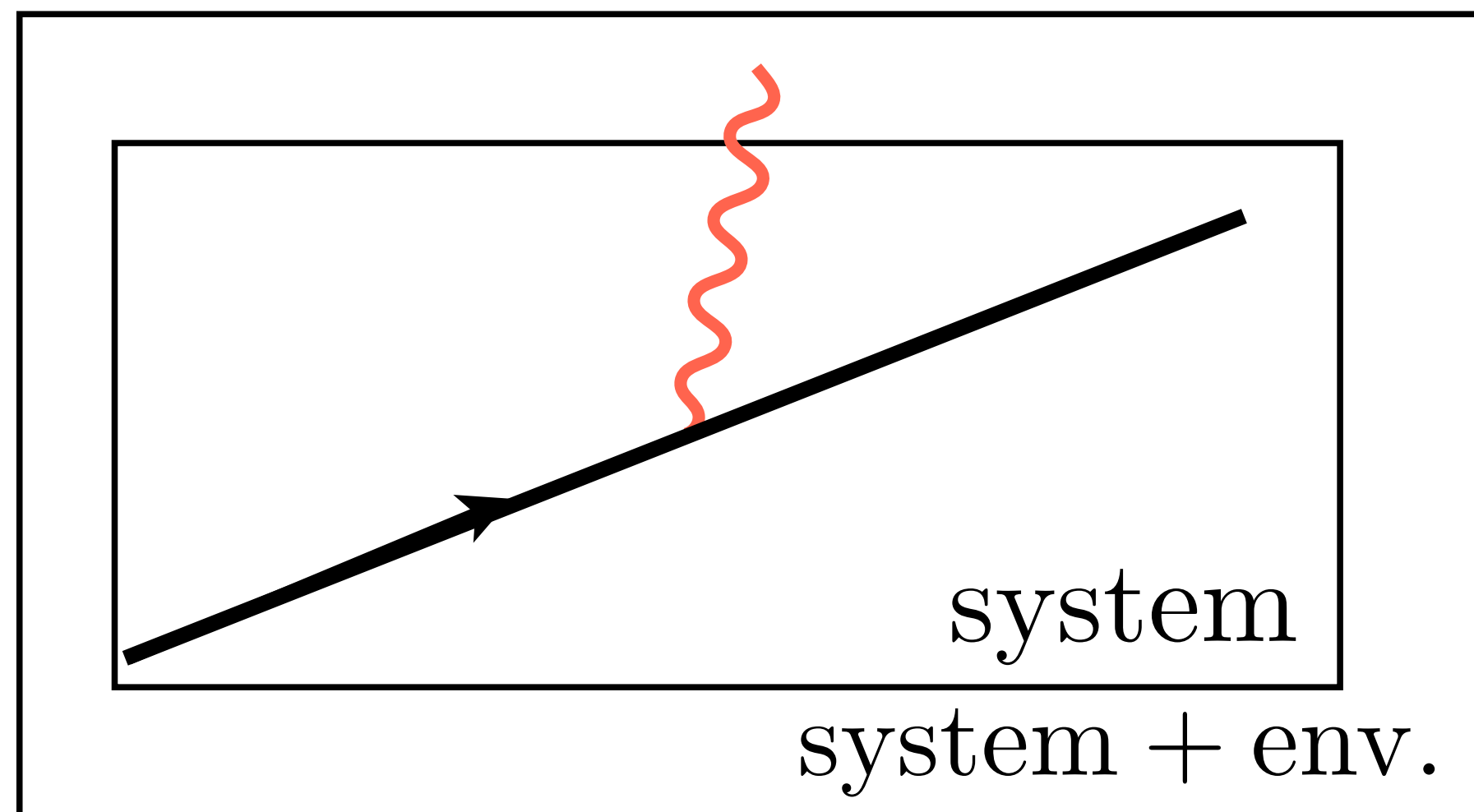
$$|\psi(t)\rangle = \hat{U}(t) |\psi(0)\rangle$$



Quantum Process Tomography

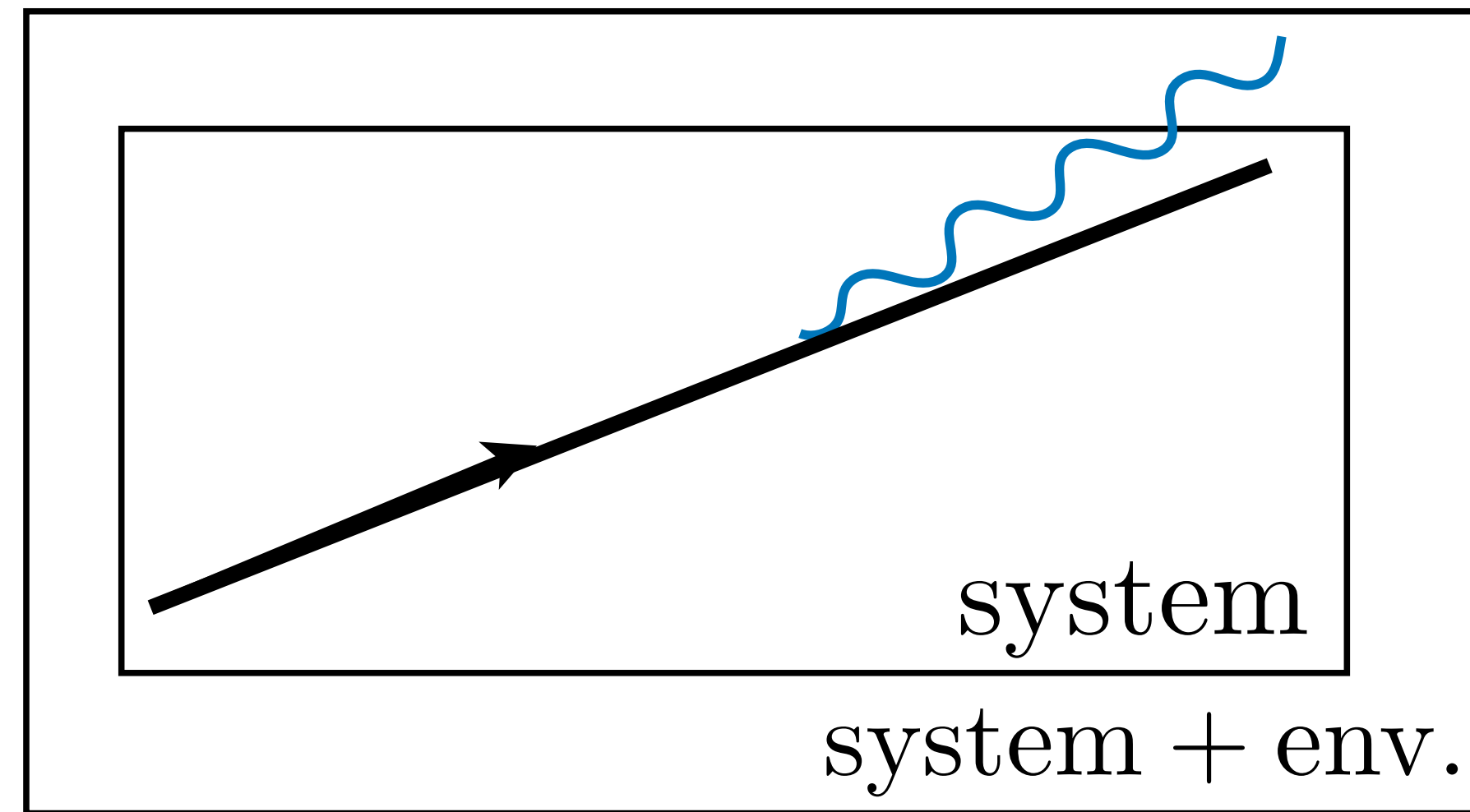
Decoherence as the radiation effect

soft



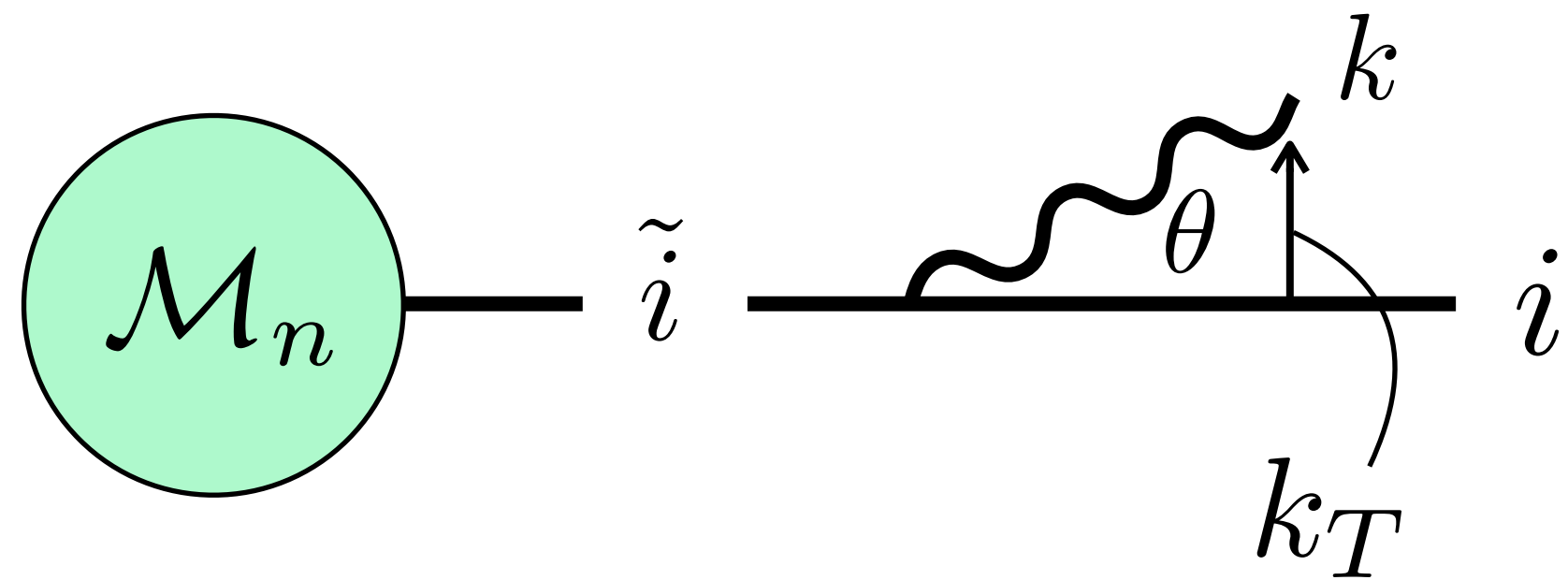
+

collinear



Collinear emission:

splitting function



$$\begin{array}{c} \tilde{i} \quad i \quad k \\ \downarrow \downarrow \downarrow \\ \mathcal{F}^{h,h,\lambda}(z) = \frac{1}{\sqrt{1-z}} \begin{cases} 1, & \lambda = h, \\ z, & \lambda = -h, \end{cases} \quad h, \lambda = \pm, \end{array}$$

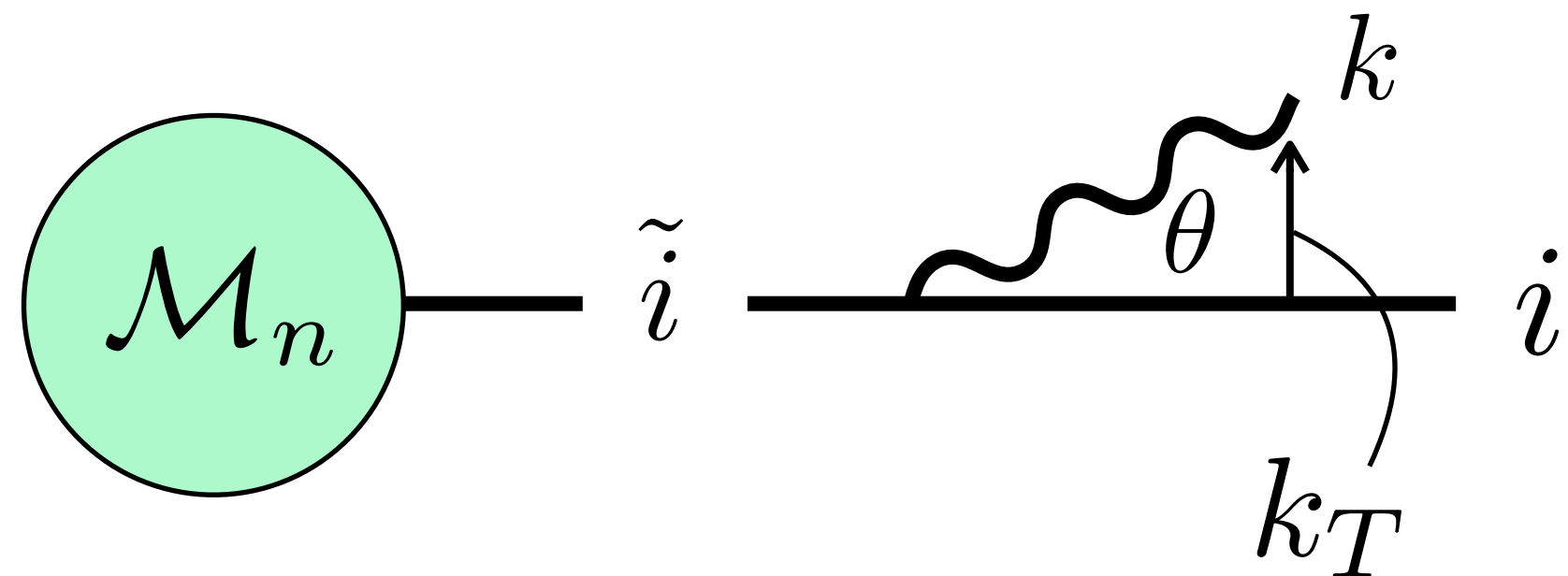
$$E_i \sim z E_{\tilde{i}}$$

$$E_k \sim (1-z) E_{\tilde{i}}$$

$$k_T \sim (1-z) E_{\tilde{i}} \theta$$

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➔ $K_+(z) = \frac{1}{\sqrt{1-z}} \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix}$

$$K_-(z) = \frac{1}{\sqrt{1-z}} \begin{pmatrix} z & 0 \\ 0 & 1 \end{pmatrix}$$

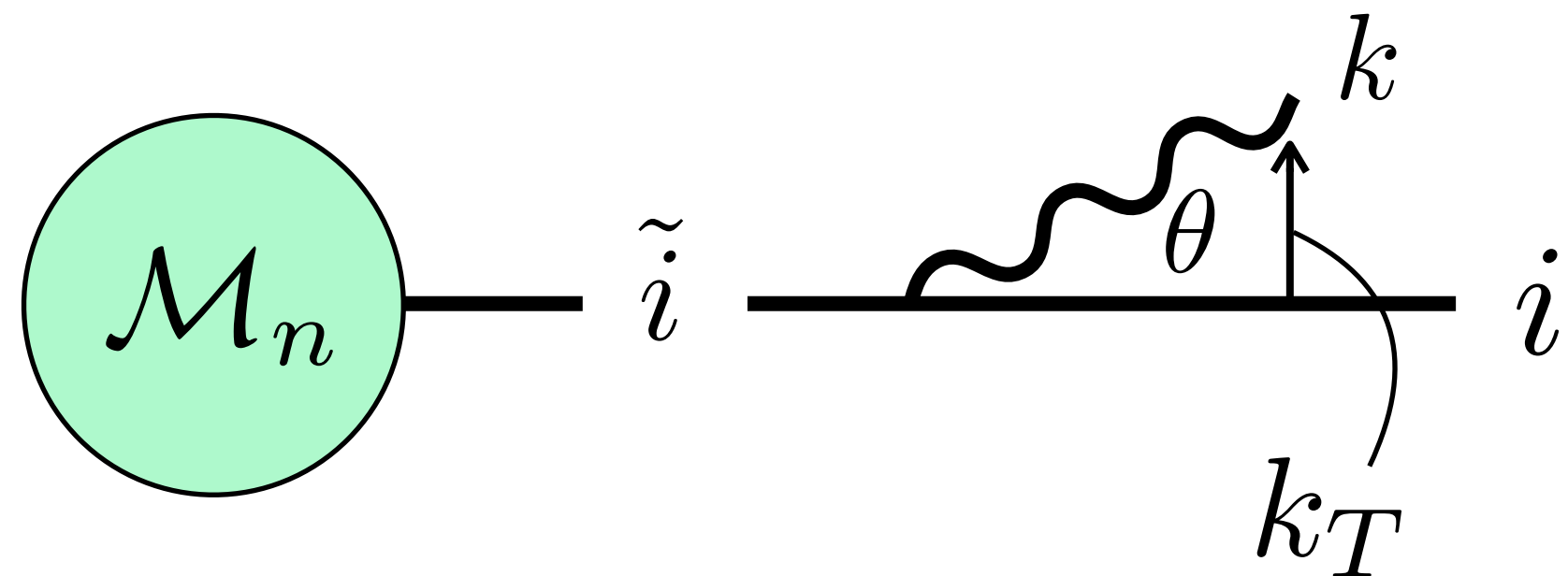
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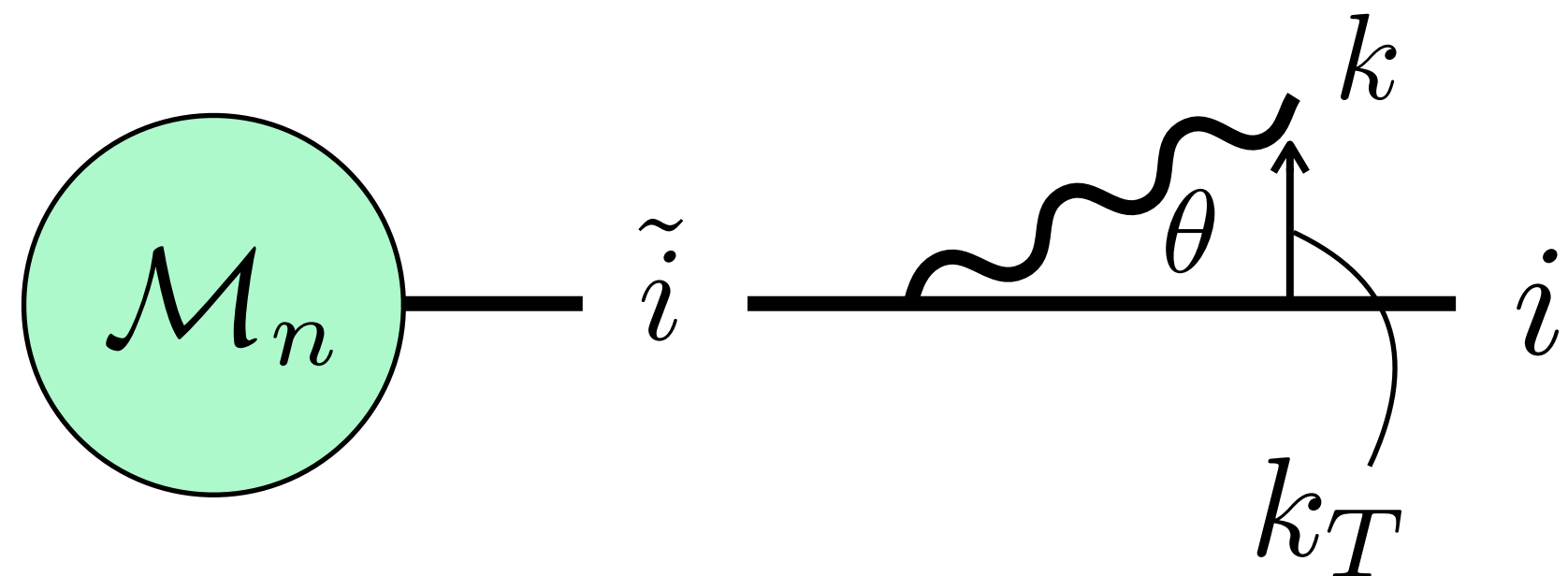
$$k_T \sim (1-z) E_{\tilde{i}} \theta$$

$$\rightarrow K_{\pm}(z) = a(z) \mathbb{1} \pm b(z) \sigma_3, \quad a(z) = \frac{1+z}{2\sqrt{1-z}}, \quad b(z) = \frac{\sqrt{1-z}}{2}.$$

phase-flip channel

Collinear emission:

splitting function



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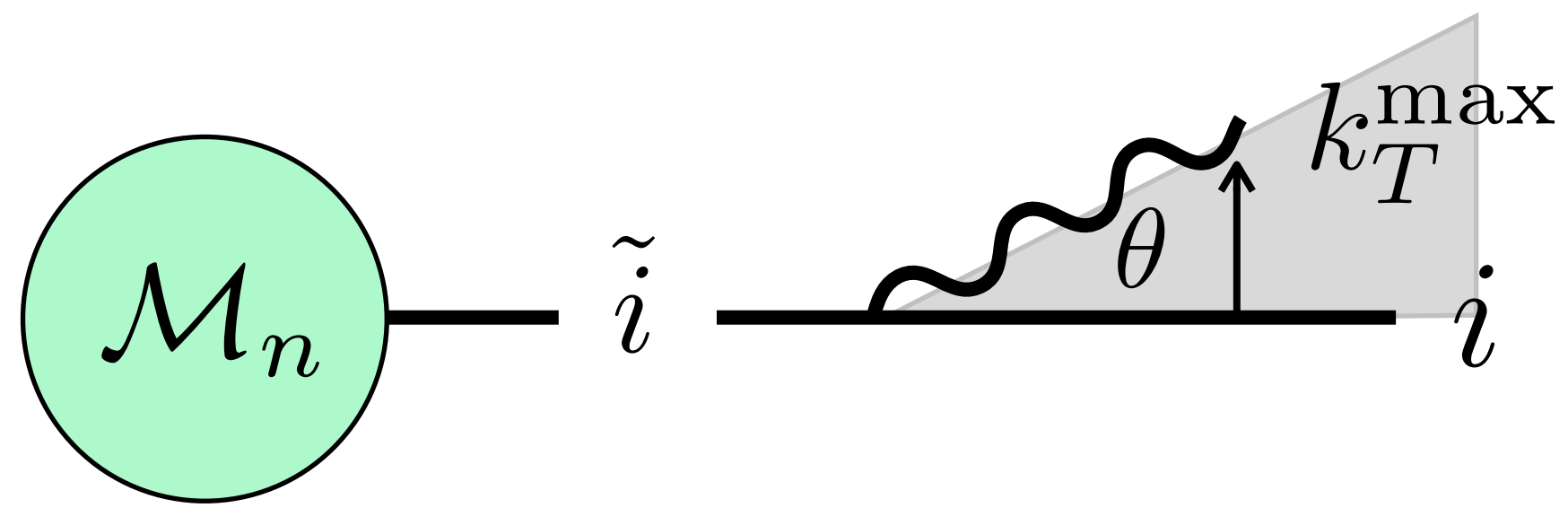
$$\rightarrow K_+(z) = \frac{1}{\sqrt{1-z}} \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix} \quad K_-(z) = \frac{1}{\sqrt{1-z}} \begin{pmatrix} z & 0 \\ 0 & 1 \end{pmatrix}$$

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phase-flip channel

$$\rightarrow \sum_{\lambda=\pm} K_{\lambda}(z) \rho K_{\lambda}^{\dagger}(z) = P_U(z) \rho + \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) \quad P_U(z) \equiv \frac{1+z^2}{1-z}$$



$k_T^{\max} = Q\delta$ ($\delta \ll 1$): emission below this k_T is unresolved.
 $Q \sim \sqrt{\hat{s}}$: hard scattering scale

- We integrate out the radiation below $k_T^{\max} \sim Q\delta$
- The splitting probability is given by

$$d\mathcal{P}(z) = \frac{|M_{n+1}|^2 d\Phi_{n+1}}{|M_n|^2 d\Phi_n} = \frac{\alpha}{\pi} P_U(z) dz dt$$

- This leads to

$$d\rho_{\text{real}} = \frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz \left[P_U(z) \rho + \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) \right]$$

$$\begin{array}{ccc}
 Q\delta & \xrightarrow{\mu \sim k_T} & m_f \text{ (IR cut-off)} \\
 0 & \xrightarrow{t \equiv \ln \frac{Q\delta}{\mu}} & \ln \frac{Q\delta}{m_f}
 \end{array}$$

$$P_U(z) \equiv \underbrace{\frac{1+z^2}{1-z}}$$

soft singularity
at $z \rightarrow 1$

$$\left| \text{Feynman diagram} \right|^2 \quad \mathrm{d}\rho_{\mathrm{real}} = \frac{\alpha}{\pi} \mathrm{d}t \int_0^{1-\epsilon} \mathrm{d}z \left[P_U(z) \, \rho + \frac{1-z}{2} \left(\sigma_3 \rho \, \sigma_3 - \rho \right) \right]$$

$$\left| \text{diagram} \right|^2$$

$$\left[\text{diagram}_1 \times \text{diagram}_2 \right]$$

$$d\rho_{\text{real}} = \frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz \left[P_U(z) \rho + \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) \right]$$

$$d\rho_{\text{virt}} = -\frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz P_U(z) \rho$$

$$\left| \text{diagram} \right|^2$$

$$d\rho_{\text{real}} = \frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz \left[\cancel{P_U}(z) \rho + \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) \right]$$

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$$d\rho_{\text{virt}} = -\frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz \cancel{P_U}(z) \rho$$

$$\frac{d\rho}{dt} = \frac{\alpha}{\pi} \int_0^1 dz \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) = \frac{\alpha}{4\pi} (\sigma_3 \rho \sigma_3 - \rho)$$

$$\left| \text{diagram} \right|^2$$

$$d\rho_{\text{real}} = \frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz \left[\cancel{P_U(z)} \rho + \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) \right]$$

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$$d\rho_{\text{virt}} = -\frac{\alpha}{\pi} dt \int_0^{1-\epsilon} dz \cancel{P_U(z)} \rho$$

RG $\Leftrightarrow$ GKSL

$$\left\{ \begin{aligned} \frac{d\rho}{dt} &= \frac{\alpha}{\pi} \int_0^1 dz \frac{1-z}{2} (\sigma_3 \rho \sigma_3 - \rho) = \frac{\alpha}{4\pi} (\sigma_3 \rho \sigma_3 - \rho) \\ &= L \rho L^\dagger - \frac{1}{2} \{L^\dagger L, \rho\} \end{aligned} \right.$$

$$L = \sqrt{\frac{\alpha}{4\pi}} \sigma_3$$

$$\left| \text{diagram} \right|^2$$

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decay of coherence

$$\Rightarrow \rho(t) = \begin{pmatrix} \rho_{++}(0) & \boxed{e^{-\frac{\alpha}{2\pi} t} \rho_{+-}(0)} \\ e^{-\frac{\alpha}{2\pi} t} \rho_{-+}(0) & \rho_{--}(0) \end{pmatrix}$$

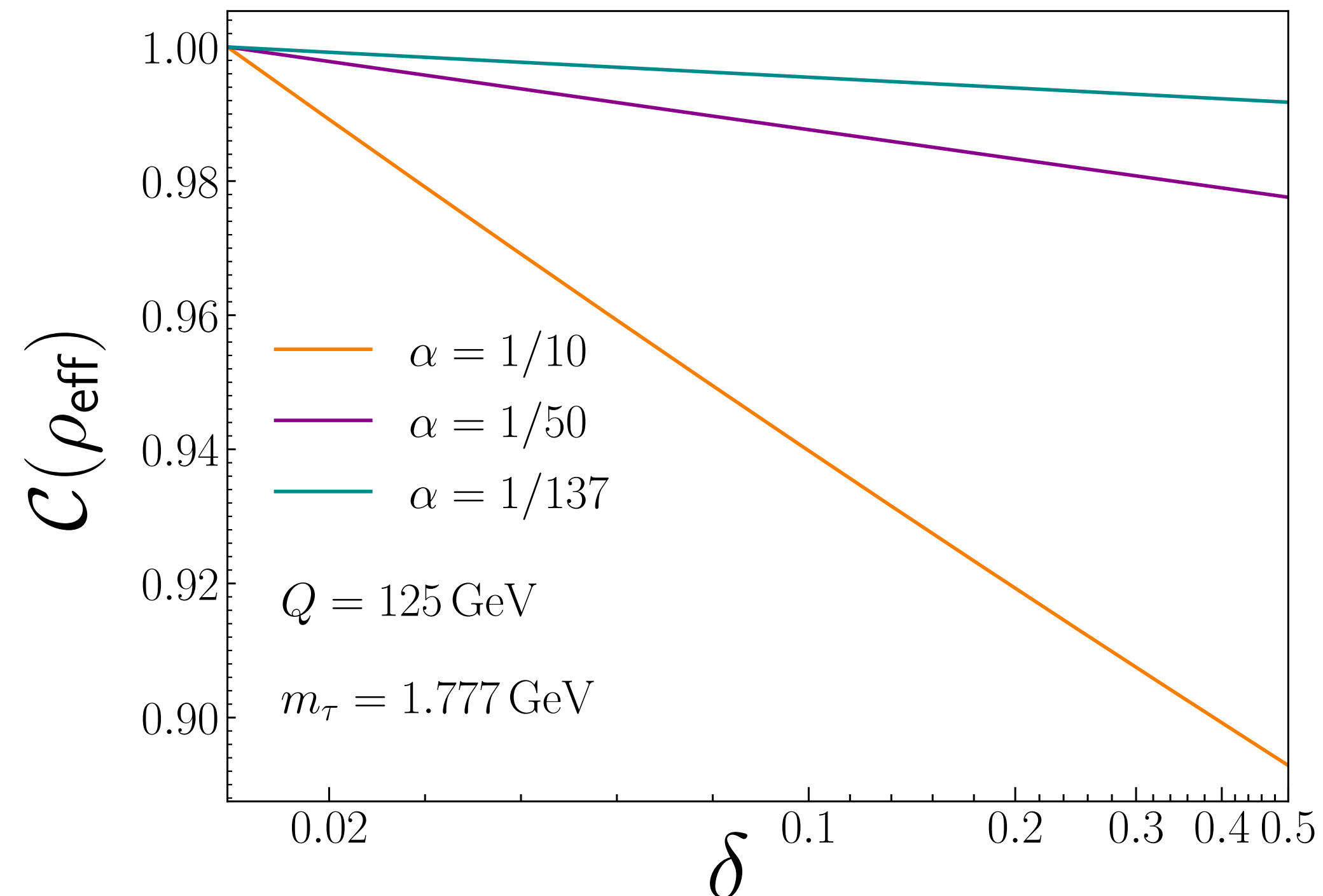
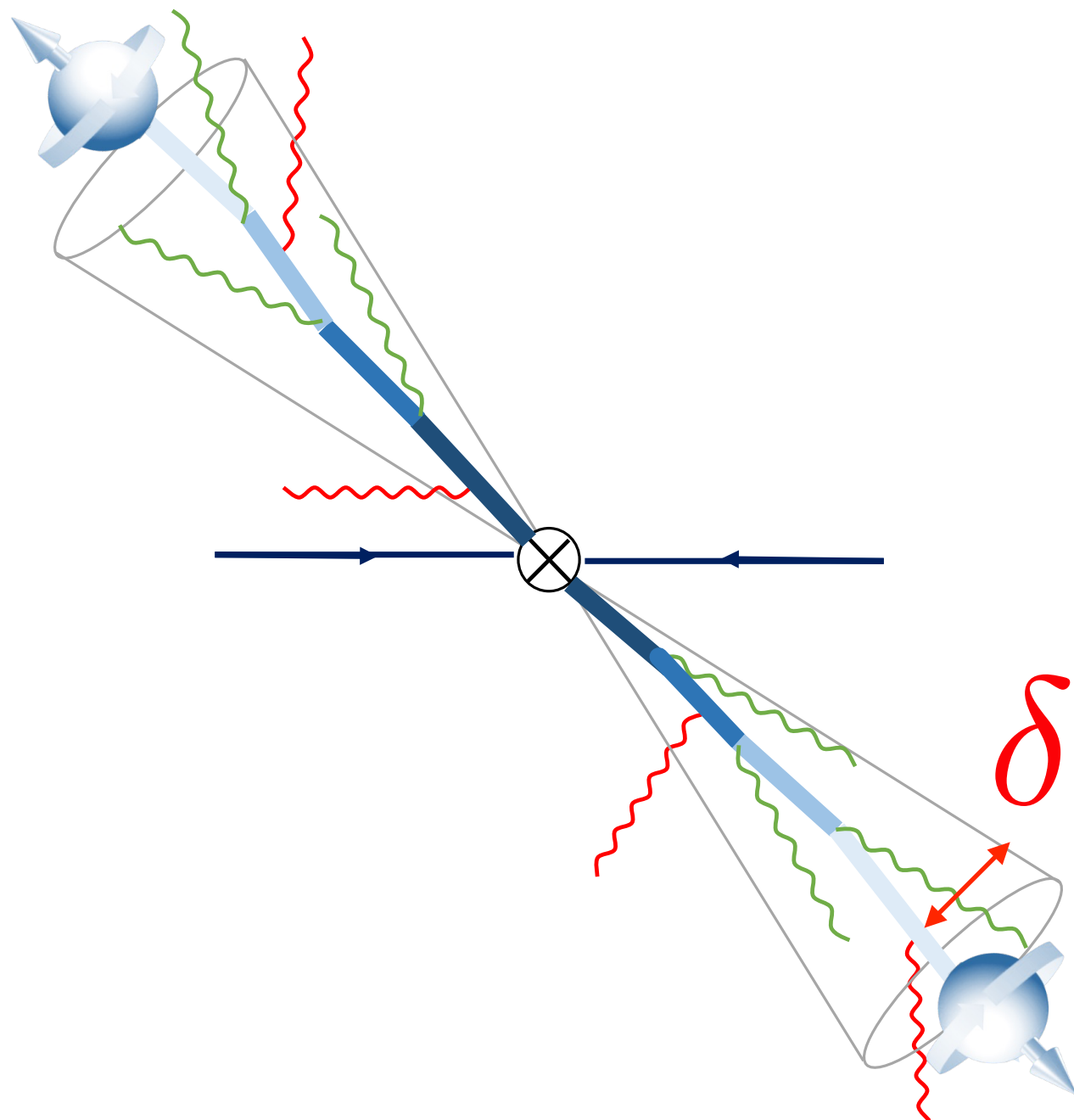
$$L = \sqrt{\frac{\alpha}{4\pi}} \sigma_3$$

Decoherence by radiation:

Lindblad (GKSL) Master Equation = RGE

$$\frac{d\hat{\rho}_{\text{eff}}}{dt} = -\frac{\alpha}{2\pi}\hat{\rho}_{\text{eff}} + \frac{\alpha}{4\pi} \left[(\hat{\sigma}_3 \otimes \hat{I}) \hat{\rho}_{\text{eff}} (\hat{\sigma}_3 \otimes \hat{I}) + (\hat{I} \otimes \hat{\sigma}_3) \hat{\rho}_{\text{eff}} (\hat{I} \otimes \hat{\sigma}_3) \right]$$

$t \equiv \log(Q\delta/\mu)$ Q : Hard scale, μ : RGE scale, δ : Angular resolution

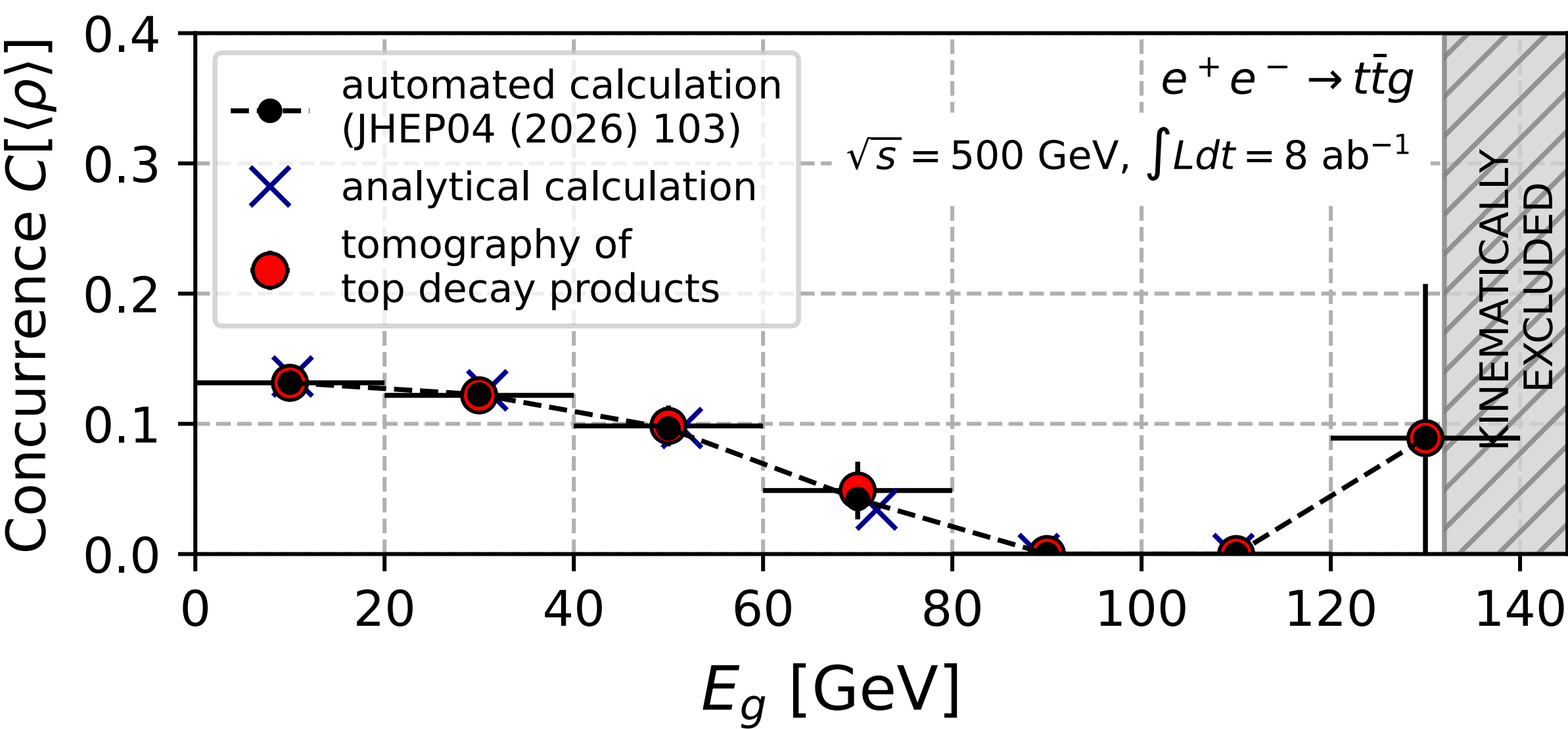


Aoude, Barr, Maltoni,
Satrioni [2504.07030],
Lin, Shao, Wang, Yang
[2510.1395]

Experimental prospects:

- Decoherence detection @ LHC and ILC is promising

Aoude, Camacho, Durupt, Garcia Mir, Maltoni, Llacer, KS, Satrioni, Vos [2604.16268]



Machine	$\sqrt{s}$ [TeV]	$\int \mathcal{L} dt$ [fb ⁻¹]	inclusive		+ radiation			$R = \frac{C_{t\bar{t}+jet}}{C_{t\bar{t}}}$	$\mathcal{S} = \frac{R-1}{\sigma_R}$
			$t\bar{t}$		$t\bar{t} + jet$				
			# events	C[ρ]	Cut	# events	C[ρ]		
e^+e^- collisions									
linear e^+e^-	0.5	8 ^{ab⁻¹}	4.38·10 ⁶	0.133	$p_T^g > 30$ GeV	0.14·10 ⁶	0.04	0.31± 0.23 / 0.11	dilep. 3.0 σ / all ch. 8.2 σ
LCF [44]	1.0	8 ^{ab⁻¹}	1.32·10 ⁶	0.249	$p_T^g > 120$ GeV	0.10·10 ⁵	0.08	0.33± 0.15 / 0.07	dilep. 4.6 σ / all ch. 12 σ
pp collisions (Fiducial region: $ \cos \Theta_t^{t\bar{t}RF} < 0.4, m_{t\bar{t}} > 800$ GeV)									
LHC Run 2	13	140	0.57·10 ⁶	0.405	$p_T^j > 250$ GeV	0.06·10 ⁶	0.12	0.29± 0.12 / 0.04	dilep. 5.8 σ / all ch. 16 σ
HL-LHC [64]	14	3000	14.9·10 ⁶	0.431	$p_T^j > 250$ GeV	1.77·10 ⁶	0.10	0.22± 0.06 / 0.02	dilep. 12 σ / all ch. 34 σ

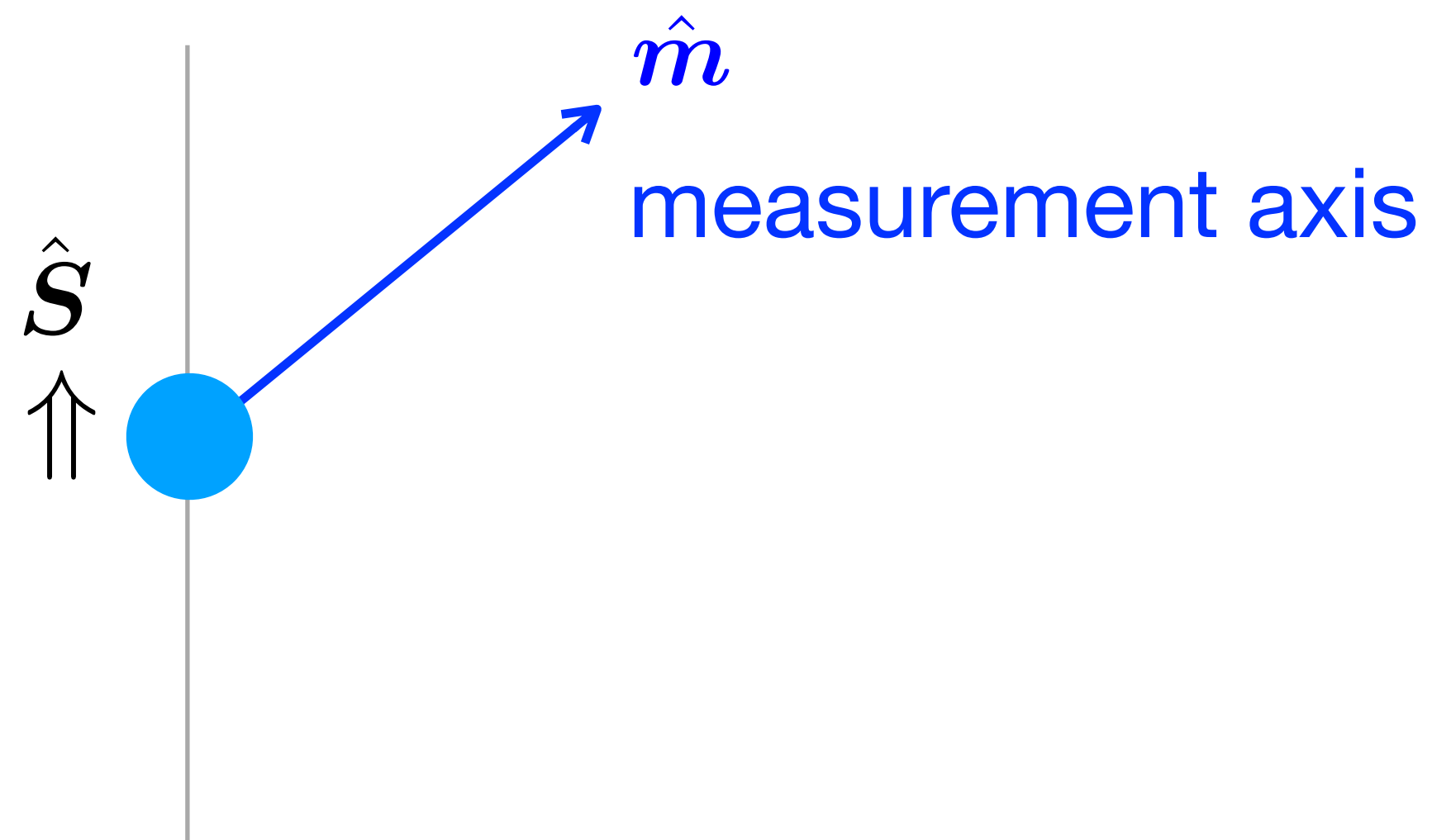
decoherence detection

Thank you for listening

Back up

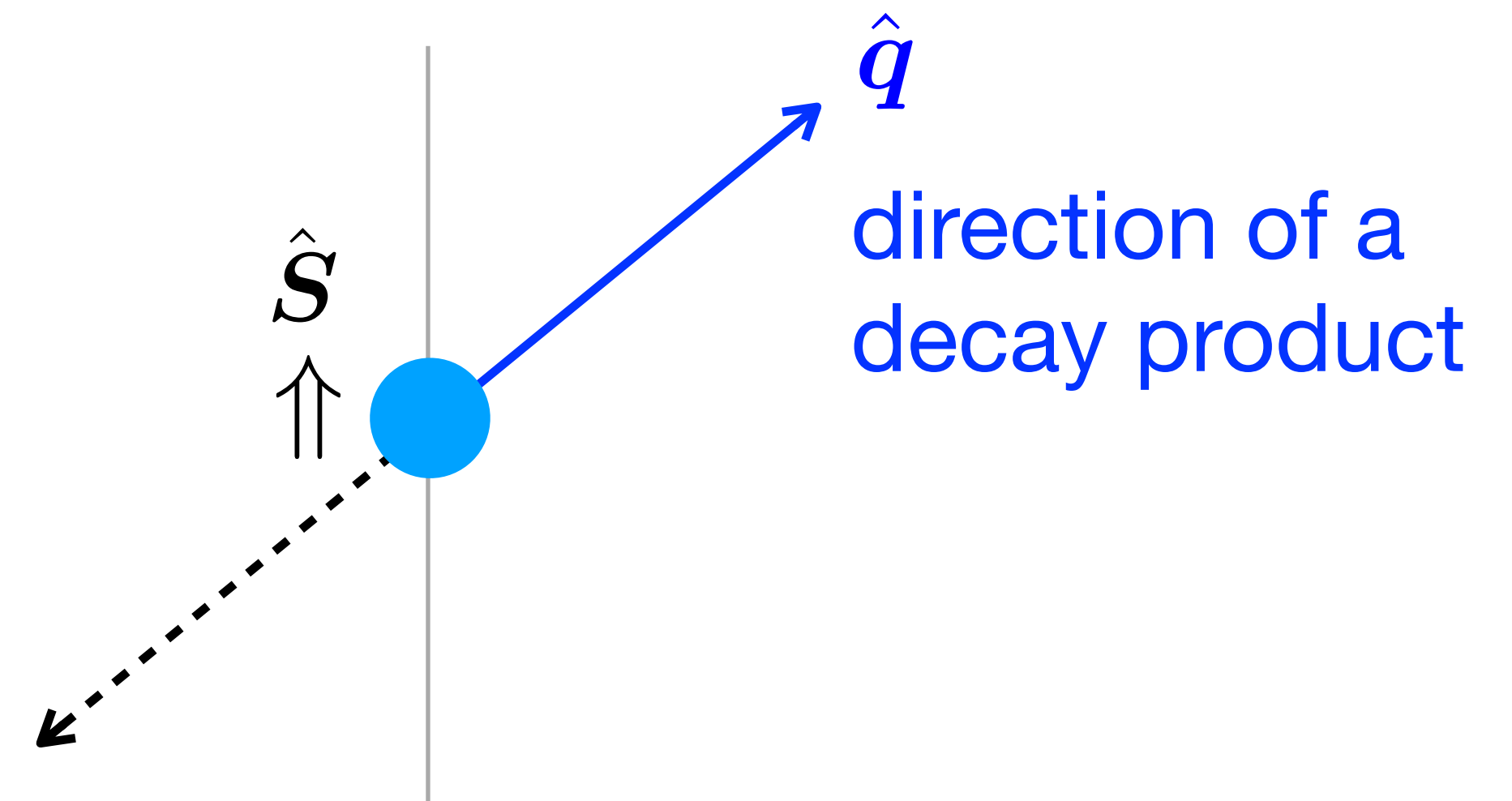
Weak decay = self spin-measurement:

Spin measurement in QM:



$$\text{Prob}(+|\hat{m}) = |\langle +\hat{m} | +\hat{S} \rangle|^2 = \frac{1 + \hat{m} \cdot \hat{S}}{2}$$

Weak decays:



$$\frac{2\pi}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{1 + \kappa \hat{q} \cdot \hat{S}}{2}$$

Works for all weakly decaying particles in the SM: $t, \tau, W^\pm, Z$

Conditional prob.: $\frac{dP(\hat{\mathbf{q}}|\hat{\mathbf{S}})}{d\Omega_q} = \frac{1 + \kappa \hat{\mathbf{q}} \cdot \hat{\mathbf{S}}}{4\pi}$

Normalisation : $\int d\Omega_q \frac{dP(\hat{\mathbf{q}}|\hat{\mathbf{S}})}{d\Omega_q} = 1$

Prob. density for $\hat{\mathbf{S}}$: $P(\hat{\mathbf{S}})$

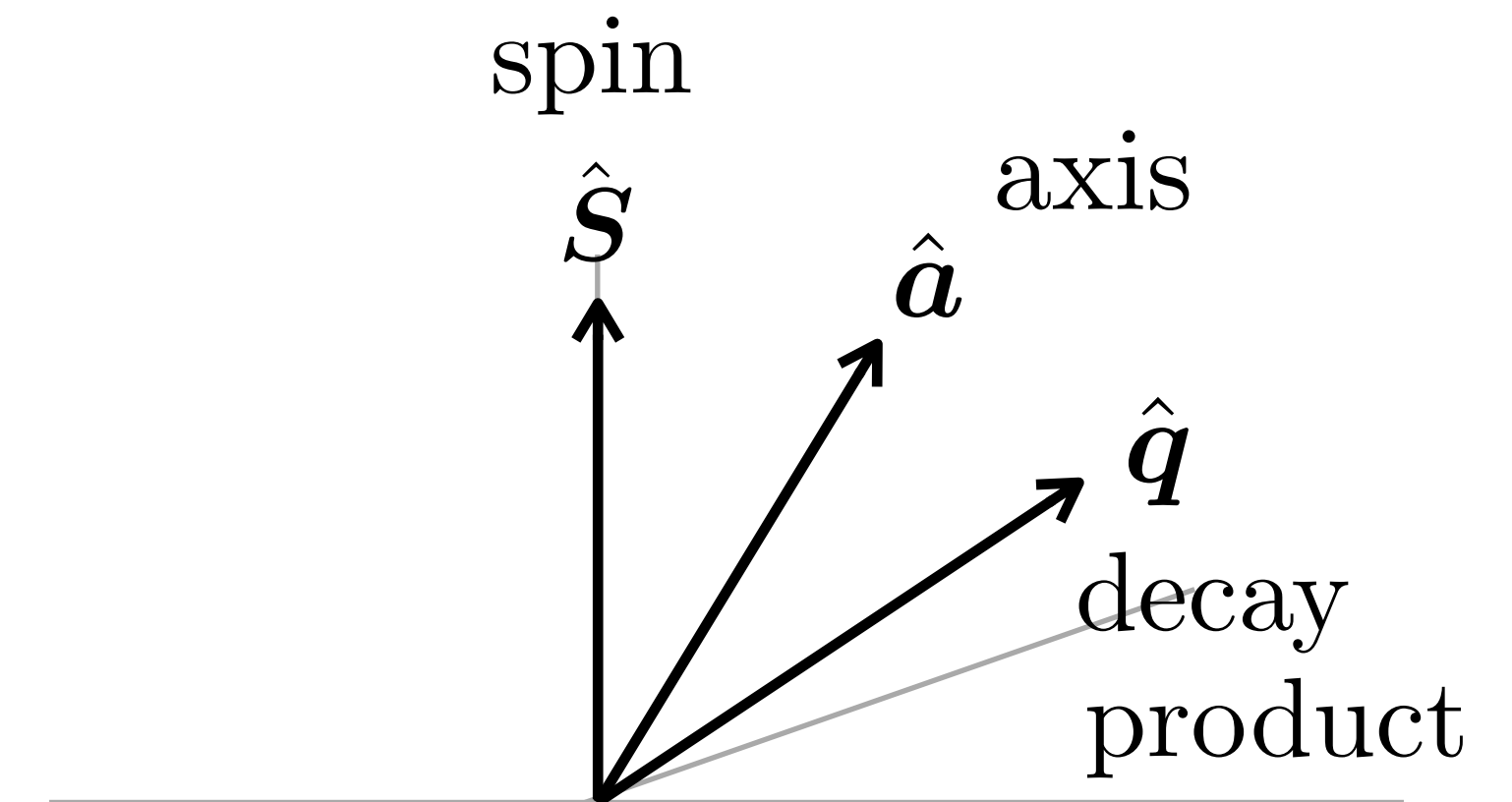
Avrg of S_a : $\langle S_a \rangle = \int \frac{d\Omega_s}{4\pi} (\hat{\mathbf{a}} \cdot \hat{\mathbf{S}}) P(\hat{\mathbf{S}})$

Avrg of q_a : $\langle q_a \rangle = \int \frac{d\Omega_s}{4\pi} \int d\Omega_q (\hat{\mathbf{a}} \cdot \mathbf{q}) \frac{dP(\hat{\mathbf{q}}|\hat{\mathbf{S}})}{d\Omega_q} P(\hat{\mathbf{S}})$

$$= \int \frac{d\Omega_s}{4\pi} \int \frac{d\Omega_q}{4\pi} (a_x s_\theta c_\phi + a_y s_\theta s_\phi + a_z c_\theta) (1 + \kappa \hat{\mathbf{q}} \cdot \hat{\mathbf{S}}) P(\hat{\mathbf{S}})$$

$$= \int \frac{d\Omega_s}{4\pi} a_z P(\hat{\mathbf{S}}) \int_{-1}^1 \frac{dc_\theta}{2} c_\theta (1 + \kappa c_\theta) = \frac{\kappa}{3} \langle S_a \rangle$$

$$\langle S_a \rangle = \frac{3}{\kappa} \langle q_a \rangle$$



$$\hat{\mathbf{S}} = (0, 0, 1)$$

$$\hat{\mathbf{a}} = (a_x, a_y, a_z)$$

$$\hat{\mathbf{q}} = (s_\theta c_\phi, s_\theta s_\phi, c_\theta)$$

Joint prob.: $P(\hat{\mathbf{S}}, \hat{\mathbf{S}}')$

$$\frac{dP(\hat{\mathbf{q}}|\hat{\mathbf{S}})}{d\Omega_q} = \frac{1 + \kappa \hat{\mathbf{q}} \cdot \hat{\mathbf{S}}}{4\pi}$$

$$\frac{dP(\hat{\mathbf{q}}'|\hat{\mathbf{S}}')}{d\Omega_{q'}} = \frac{1 + \kappa' \hat{\mathbf{q}}' \cdot \hat{\mathbf{S}}'}{4\pi}$$

$$\langle S_a S'_b \rangle = \int \frac{d\Omega_s}{4\pi} \frac{d\Omega_{s'}}{4\pi} (\hat{\mathbf{a}} \cdot \hat{\mathbf{S}})(\hat{\mathbf{b}} \cdot \hat{\mathbf{S}}') P(\hat{\mathbf{S}}, \hat{\mathbf{S}}')$$

$$\langle q_a q'_b \rangle = \int \frac{d\Omega_s}{4\pi} \frac{d\Omega_{s'}}{4\pi} d\Omega_q d\Omega_{q'} (\hat{\mathbf{a}} \cdot \hat{\mathbf{q}})(\hat{\mathbf{b}} \cdot \hat{\mathbf{q}}') \frac{dP(\hat{\mathbf{q}}|\hat{\mathbf{S}})}{d\Omega_q} \frac{dP(\hat{\mathbf{q}}'|\hat{\mathbf{S}}')}{d\Omega_{q'}} P(\hat{\mathbf{S}}, \hat{\mathbf{S}}')$$

$$= \int \frac{d\Omega_s}{4\pi} \frac{d\Omega_{s'}}{4\pi} \frac{d\Omega_q}{4\pi} \frac{d\Omega_{q'}}{4\pi} (a_x s_\theta c_\phi + a_y s_\theta s_\phi + a_z c_\theta)(b_x s_{\theta'} c_{\phi'} + b_y s_{\theta'} s_{\phi'} + b_z c_{\theta'}) \\ (1 + \kappa c_\theta)(1 + \kappa' c_{\theta'}) P(\hat{\mathbf{S}}, \hat{\mathbf{S}}')$$

$$= \int \frac{d\Omega_s}{4\pi} \frac{d\Omega_{s'}}{4\pi} a_z b_z P(\hat{\mathbf{S}}, \hat{\mathbf{S}}') \int_{-1}^1 \frac{dc_\theta}{2} \frac{dc_{\theta'}}{2} c_\theta c_{\theta'} (1 + \kappa c_\theta)(1 + \kappa' c_{\theta'}) = \frac{\kappa \kappa'}{9} \langle S_a S'_b \rangle$$

$$\langle S_a S'_b \rangle = \frac{9}{\kappa \kappa'} \langle q_a q'_b \rangle$$

- Consider a measurement operator for the spin axis $\hat{\mathbf{P}}$:

$$M(\hat{\mathbf{P}}) \equiv \frac{1}{4\pi} (\mathbb{1} + \kappa \hat{\mathbf{P}} \cdot \hat{\boldsymbol{\sigma}})$$

$|\kappa| = 1$: projective

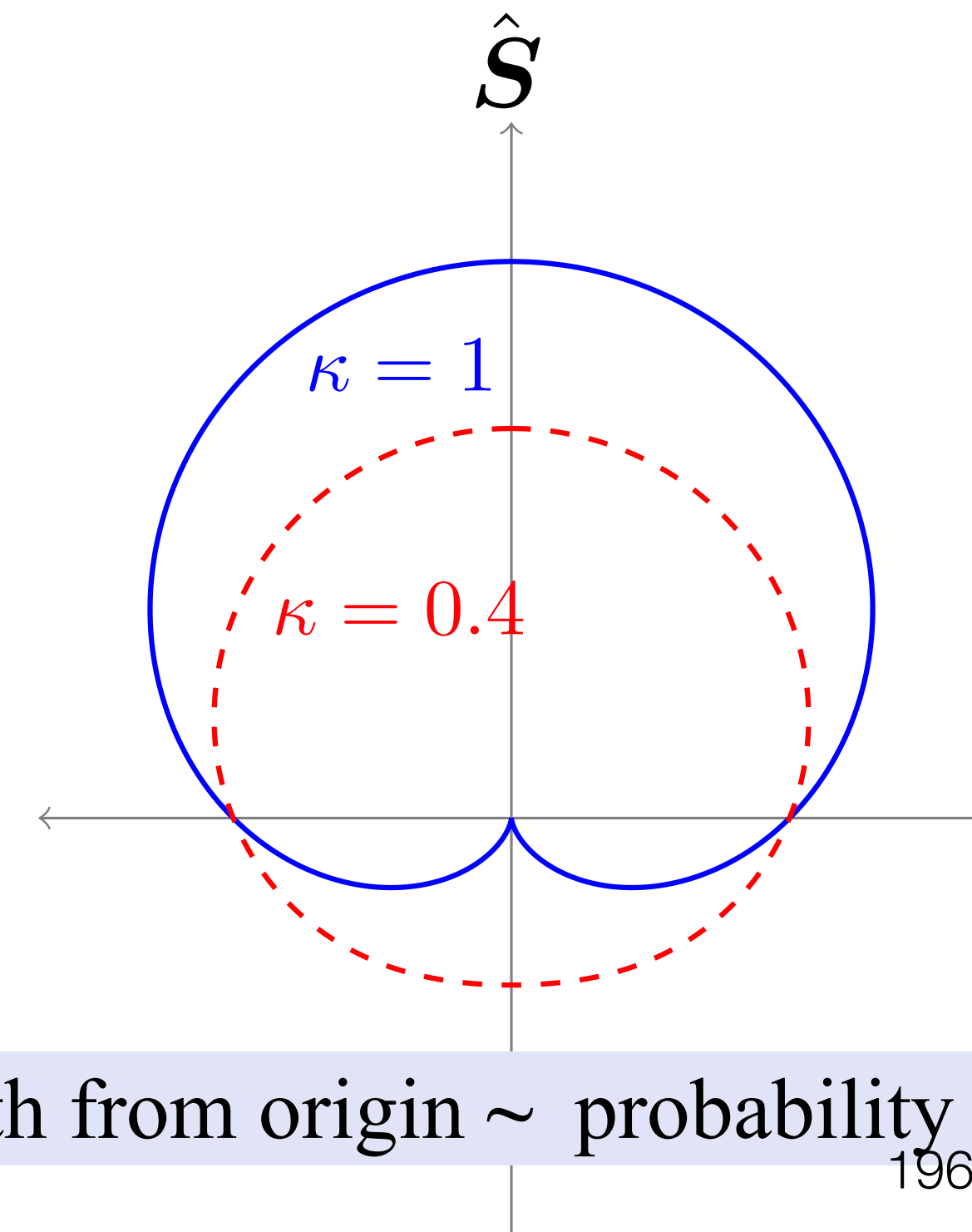
$|\kappa| < 1$: weak (POVM)

$$\left. \begin{array}{l} \text{positive: } M(\hat{\mathbf{P}}) \geq 0 \quad (|\kappa| \leq 1) \\ \text{complete: } \int d\Omega M(\hat{\mathbf{P}}) = \mathbb{1} \end{array} \right\} \text{Tr}[M(\hat{\mathbf{P}}) \cdot \rho] \leftarrow \text{probability}$$

- For $\rho = |\uparrow_s\rangle\langle\uparrow_s| = \frac{1}{2}(\mathbb{1} + \hat{\mathbf{S}} \cdot \boldsymbol{\sigma})$, we find

$$\widetilde{\text{Prob}}(\hat{\mathbf{P}}|\rho) = \text{Tr}[M(\hat{\mathbf{P}}) \cdot \rho] = \frac{1}{4\pi} (1 + \kappa \hat{\mathbf{S}} \cdot \hat{\mathbf{P}}) = \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} \Big|_{\hat{\mathbf{q}}=\hat{\mathbf{P}}}$$

$\Rightarrow$ probability dist. = decay dist.



- Experimental average of j -th component the decay product direction:

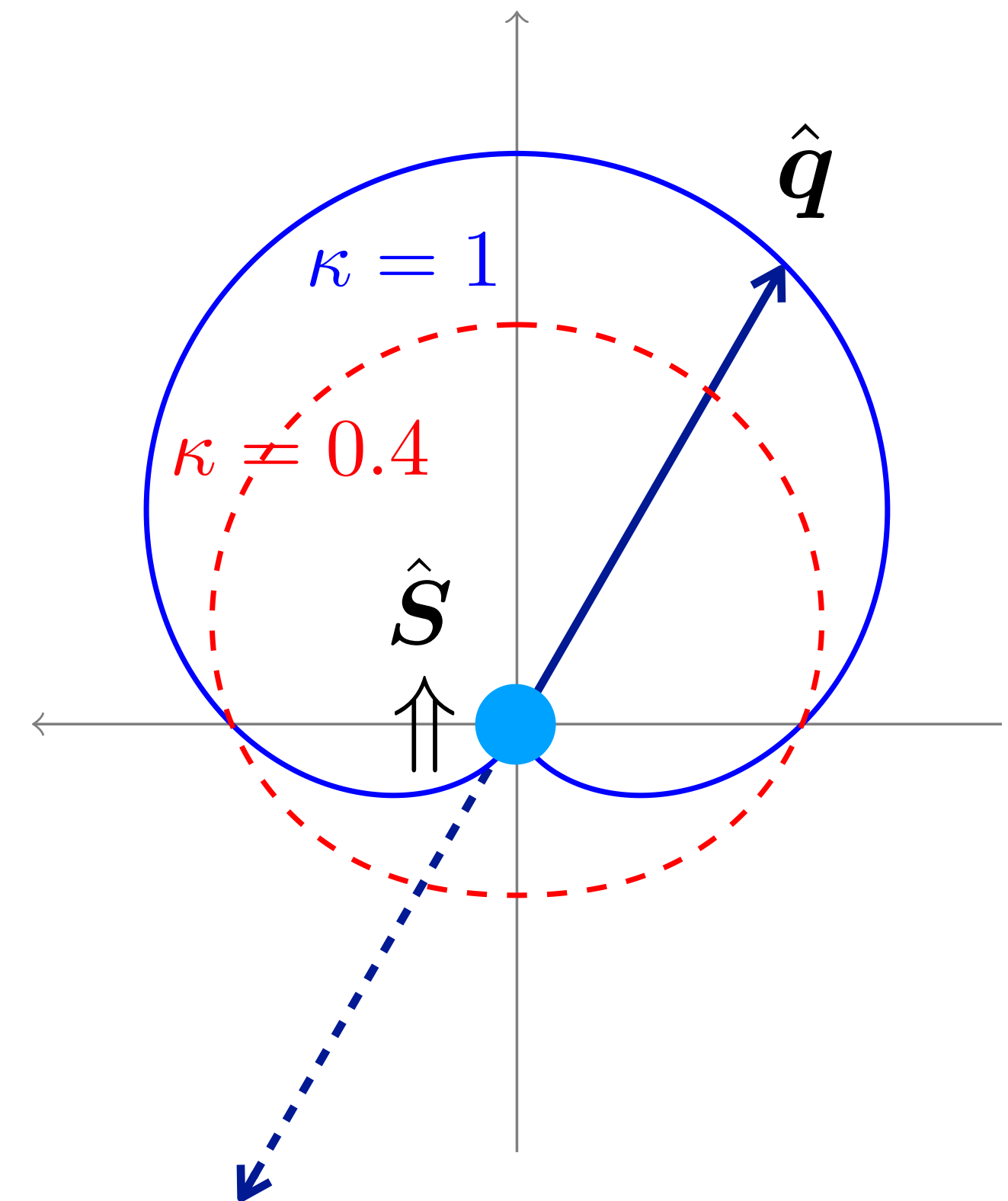
$$\langle q_j \rangle = \int d\Omega \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} q_j = \int \frac{d\Omega}{4\pi} \left(1 + \kappa S_i q_i \right) q_j = \frac{\kappa}{3} S_j$$

$$\int \frac{d\Omega}{4\pi} q_i = 0$$

$$\int \frac{d\Omega}{4\pi} q_i q_j = \frac{1}{3} \delta_{ij}$$

Experimental reconstruction of the spin:

$$S_j = \frac{3}{\kappa} \langle q_j \rangle \quad (j = 1, 2, 3)$$



$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2} (1 + \kappa_1 \mathbf{B}_1 \cdot \hat{\mathbf{q}}_1 + \kappa_2 \mathbf{B}_2 \cdot \hat{\mathbf{q}}_2 + \kappa_1 \kappa_2 \hat{\mathbf{q}}_1 \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_2)$$

$$\langle q_{1i} \rangle = \int d\Omega_1 d\Omega_2 \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega_1 d\Omega_2} q_{1i} = \int \frac{d\Omega_1}{4\pi} \kappa_1 B_{1k} q_{1k} q_{1i} = \frac{\kappa_1}{3} B_{1i}$$

$$\langle q_{1i} q_{2j} \rangle = \int d\Omega_1 d\Omega_2 \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega_1 d\Omega_2} q_{1i} q_{2j} = \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} \kappa_1 \kappa_2 C_{kl} q_{1k} q_{2l} q_{1i} q_{2j} = \frac{\kappa_1 \kappa_2}{9} C_{ij}$$

Experimental reconstruction of the density matrix:

$$B_{1i} = \frac{3}{\kappa_1} \langle q_{1i} \rangle, \quad B_{2i} = \frac{3}{\kappa_2} \langle q_{2i} \rangle, \quad C_{ij} = \frac{9}{\kappa_1 \kappa_2} \langle q_{1i} q_{2j} \rangle.$$

$$\int \frac{d\Omega}{4\pi} q_i = 0$$

$$\int \frac{d\Omega}{4\pi} q_i q_j = \frac{1}{3} \delta_{ij}$$

“Quantum State Tomography”

Details of neutrino reconstruction

	ATLAS (2311.07288)	CMS (2406.03976)
Channel	$e\mu$ only, 140 fb^{-1}	$ee, \mu\mu, e\mu$, 36.3 fb^{-1}
Algorithm	Ellipse method [1], analytic	Kinematic reconstruction [3], analytic + smearing
Fixed inputs	$m_W = 80.4$, $m_t = 172.5 \text{ GeV}$	$m_W = 80.4$ (smeared, rel. Breit–Wigner $\Gamma = 2.1 \text{ GeV}$), $m_t = 172.5 \text{ GeV}$
Resolution	single pass on measured objects	jet and lepton energies <i>and directions</i> smeared by their resolutions, 100 smearings per event
Output per event	one solution	weighted <i>average</i> of the smeared solutions, weights from the expected $m(\ell b)$ spectrum
Choice among roots	lowest $m_{t\bar{t}}$	lowest $m(t\bar{t})$
Jet–lepton pairing	both assignments tried; at particle level minimize $ m_t - m(W_1 b_{1/2}) + m_t - m(W_2 b_{2/1}) $	all suitable b -jet pairs and both assignments; the one with the largest summed weight wins
b -tagging fallback	if only one b -tag, use the leading untagged jet	untagged combinations tried only if no solution is found
On failure	Neutrino Weighting [2] ($\sim 5\%$ of events, scanning η_ν and weighting by agreement with $\vec{p}_T^{\text{miss}}$); if that also fails, each lepton is paired with its nearest b -jet as a <i>proxy</i> for the top direction, with no neutrino at all	event rejected ($\sim 9\%$); efficiency parameterized in lepton p_T, η , N_b and p_T^{miss} , with scale factors derived separately for $ee, \mu\mu, e\mu$
Correction	calibration curve, detector $\rightarrow$ <i>particle</i> level (fiducial)	profile likelihood fit with templates, to <i>parton</i> level

[1] Betchart, Demina, Harel: 1305.1878, [2] D0 Collaboration: hep-ex/9706014, [3] CMS Collaboration: 1505.04480