Chiral Anomalies in Models of Neutrino Masses as Modular Forms

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Introduction

- Properties of fermions like their masses and their mixings with other generations are determined by the Yukawa sector.
- The lack of an organizing principle in the Yukawa sector and of a robust model for neutrino masses accounts for the proliferation of the 20 to 22 free parameters needed to account for the flavor structure of the SM.
- We present a model for neutrino masses in which they are given by modular forms. The lepton sector is governed by a modular symmetry which captures lepton masses and mixings.
- Chiral anomalies arising from modular transformations should be cancelled since they are reparametrizations (gauge transformations) of a background field called the modulus τ.
- We consider chiral anomalies in various models based on the modular symmetry $\Gamma_3 \cong A_4$ and argue for different anomaly cancellation mechanisms based on the bottom-up vs top-down assumptions for those models.

$\Gamma_3 \cong A_4$ Model with Weinberg Operator

The symmetry of this model is given by the finite modular group Γ_3 which is isomorphic to A_4.

The superpotential for charged leptons and neutrinos is given by

$$\mathscr{W}_l = \alpha\, E_1^c (L\phi_T)_1 H_d + \beta\, E_2^c (L\phi_T)_{1'} H_d + \gamma\, E_3^c (L\phi_T)_{1''} H_d$$

$$\mathscr{W}_\nu = \frac{\kappa}{\Lambda}(L H_u L H_u Y_3^{(2)}(\tau))_1$$

The charge assignments for the model are given in the table below.

	E_1^c	E_2^c	E_3^c	L	H_u	H_d	φ_T
$SU(2)_L$	**1**	**1**	**1**	**2**	**2**	**2**	**1**
Y^2_{GUT}	3/5	3/5	3/5	3/20	3/20	3/20	0
$\Gamma_3 \simeq A_4$	**1**	**1″**	**1′**	**3**	**1**	**1**	**3**
Modular weight	$k_{E_1^c}$	$k_{E_2^c}$	$k_{E_3^c}$	k_L	k_{H_u}	k_{H_d}	k_{φ_T}

Modular Transformations and Modular Forms

The full modular group is given by SL(2,Z). The action of $\gamma \in$ SL(2,Z) on the modulus τ is blind to the negatives of γ. Thus, the effective action on τ is given by PSL(2,Z).

Chiral superfields obey the modularity condition and thus transform as

$$\Phi^{(j)}(\gamma\tau) = (c\tau + d)^{-k_j} \rho_j(\gamma) \Phi^{(j)}(\tau)$$

Modular forms not only obey the modularity condition but are also holomorphic. They transform as

$$Y(\gamma\tau) = (c\tau + d)^{k_Y}\, Y(\tau)$$

Linear Realization at Fixed Points and Anomalies

At fixed points of the modulus $\gamma\tau^* = \tau^*$, however, certain transformations become linearly realized on the chiral superfields and modular forms. These points are

$$\mathbb{Z}_4^{(\mathrm{S})} = \{\mathbb{1}, \mathrm{S}, \mathrm{S}^2, \mathrm{S}^3\} \qquad \text{at } \tau = \mathrm{i}\ ,$$

$$\mathbb{Z}_3^{(\mathrm{ST})} = \{\mathbb{1}, \mathrm{ST}, (\mathrm{ST})^2\} \qquad \text{at } \tau = \omega$$

where S and T are generators of the transformations Γ_3 .

This allows us to relate discrete charges of the linearly realized discrete modular transformations and modular weights by

$$q^{(\Phi)} = k^{(\Phi)} + \frac{N}{2\pi}\arg(\rho) \bmod \mathrm{N}$$

We can relate modular weights with chiral discrete anomalies using the equation above and thus impose restrictions on modular weights by imposing anomaly universality and anomaly cancellation.

$$A_{G-G-\mathbb{Z}_N} = \sum_{\boldsymbol{r}^{(f)}} \ell(\boldsymbol{r}^{(f)})\,(q^{(f)} - R) + \ell(\mathrm{adj}) \cdot R$$

with R = 1 for the R case and R = 0 for the non-R case. For $U(1)_Y$ symmetries, the anomalies are

$$A_{\mathrm{U(1)_Y - U(1)_Y} - \mathbb{Z}_N} = \sum_f d_{(f)} Y^2_{(f)} (q^{(f)} - R)$$

Anomalies for R and non-R Symmetries

Invoking an axion/modulus sector in a bottom-up non-R scenario would be highly unmotivated (no natural target-space modular symmetry story with local SUSY and R-symmetries), we should thus cancel any anomalies that arise in non-R cases with the already existing chiral fermion spectrum.

The $SU(2)_L$ anomalies at both fixed points are given by

$$2A^{\mathrm{non-R}}_{\mathrm{SU(2)_L - SU(2)_L} - \mathbb{Z}_4^{(\mathrm{S})}} = \frac{3}{2}k_L + \frac{1}{2}k_{H_u} + \frac{1}{2}k_{H_d} \bmod 4$$

$$2A^{\mathrm{non-R}}_{\mathrm{SU(2)_L - SU(2)_L} - \mathbb{Z}_3^{(\mathrm{ST})}} = \frac{3}{2}k_L + \frac{1}{2}k_{H_u} + \frac{1}{2}k_{H_d} \bmod 3$$

The $U(1)_Y$ anomalies at both fixed points are given by

$$10A^{\mathrm{non-R}}_{\mathrm{U(1)_Y - U(1)_Y} - \mathbb{Z}_4^{(\mathrm{S})}} = 9k_L + 18k_{E^c} + 3k_{H_u} + 3k_{H_d} \bmod 12$$

$$10A^{\mathrm{non-R}}_{\mathrm{U(1)_Y - U(1)_Y} - \mathbb{Z}_3^{(\mathrm{ST})}} = 9k_L + 18k_{E^c} + 3k_{H_u} + 3k_{H_d} \bmod 3$$

Anomaly Universality and Cancellation

Anomaly cancellation is determined by $A \cong 0 \bmod \frac{N}{2}$

while anomaly universality for two anomalies of mod N and mod M implies

$$N A_M \cong M A_N \bmod MN$$

Feruglio's model allows for anomaly cancellation of $SU(2)_L$ anomalies but only a nonzero universal $U(1)_Y$ anomaly at both fixed points. The modular weight assignment corresponding to these conditions is given by

$k_{\mathscr{W}}$	k_L	k_{H_u}	k_{H_d}	k_ϕ	$k_{E_i^c}$
0	1	0	−3	−3	5

Conclusion

Relations between modular weights and discrete charges at fixed points allows us to use anomalies to restrict modular weight assignments in modular flavor symmetry models. Anomalies in non-R cases should be cancelled by the existing chiral fermions in the theory like the quarks themselves, while in the R-case, the GS mechanism can be naturally invoked.

References

Lee et al., *A Unique Z_4^R Symmetry for the MSSM*, Phys. Lett. B 694 (2011).
Feruglio, *Are Neutrino Masses Modular Forms?*, arXiv:1706.08749.
Ibáñez and Lüst, *Duality Anomaly Cancellation...*, Nucl. Phys. B 382 (1992).