Flavour models
in the era of precision neutrino physics

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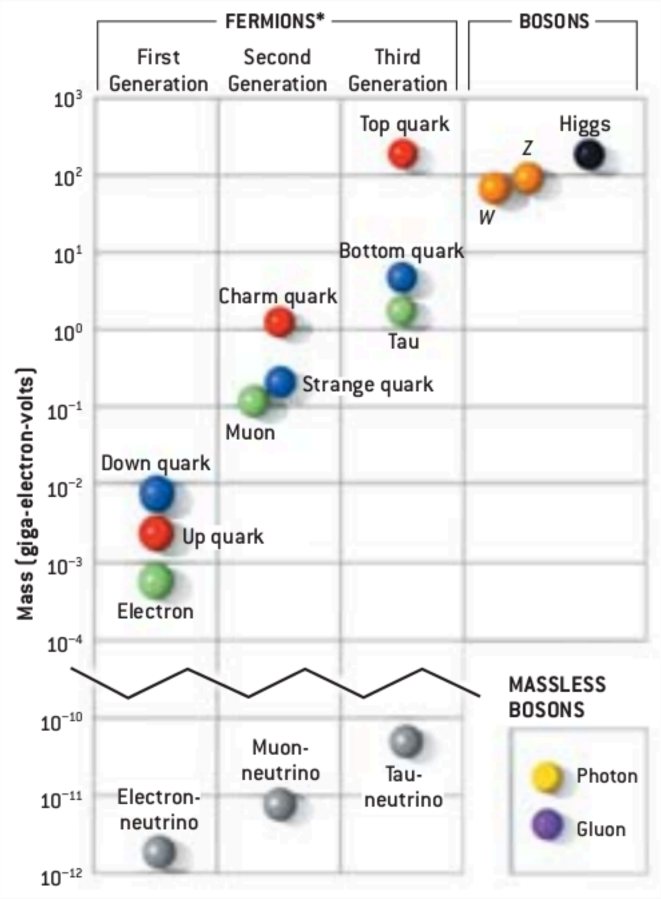
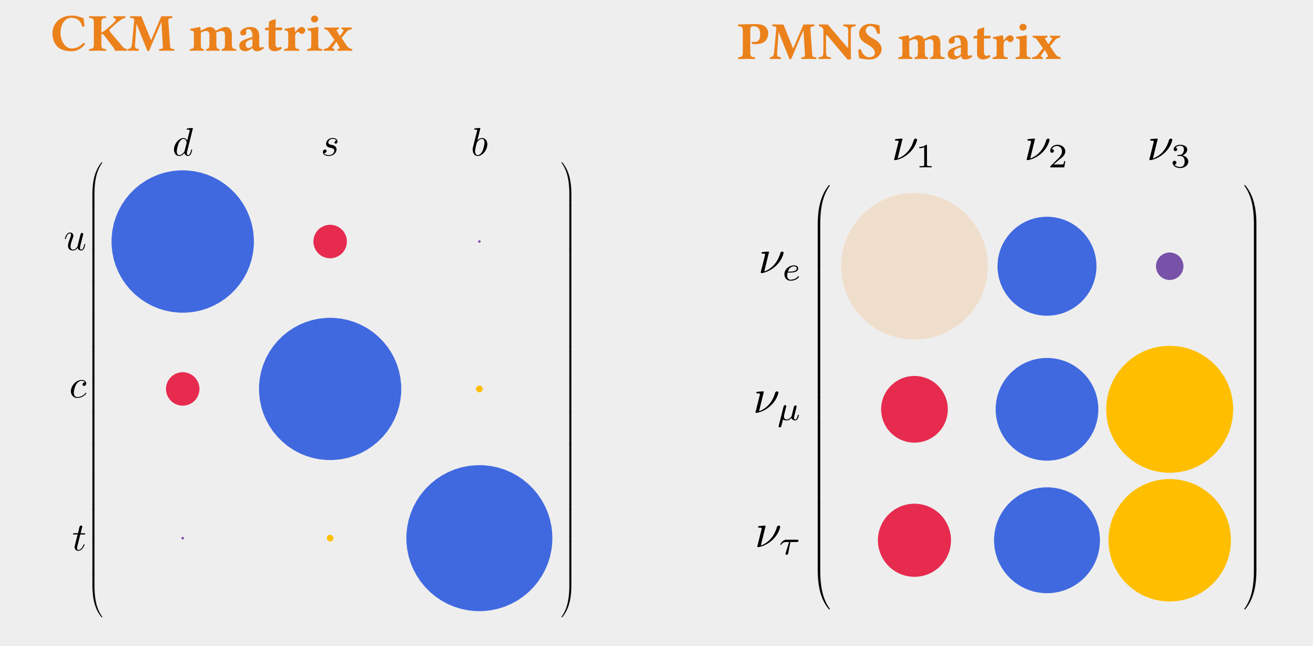
Flavour puzzle of the Standard Model



Quark vs lepton mixing



- Any explanation for three generations?
- Large charged fermion vs small neutrino mass hierarchy
- Explanation for different mixing for leptons and quarks?

Outline

Overview of lepton flavour models

- mixing patterns from symmetry breaking
- mixing sum rules
- residual symmetries
- CP symmetries
- symmetry breaking
- mass sum rules

Precision neutrino physics

- renormalization group (RG) corrections in the seesaw model
- RG correction in radiative neutrino mass models

G_f [eg. A_4]

G_ℓ [eg. Z_3]

charged lepton

$Q^\dagger M_\ell^\dagger M_\ell Q = M_\ell^\dagger M_\ell$

$U_e^\dagger Q U_e = \mathrm{diag}$

$Z_2(\times Z_2)$

neutrino

$Z^T M_\nu Z = M_\nu$

$U_\nu^\dagger Z U_\nu = \mathrm{diag}$

$$U = U_\ell^\dagger U_\nu$$

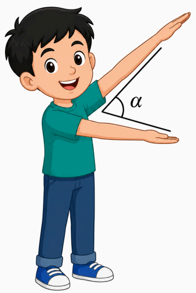
- $U_\ell \to U_\ell K_\ell$ removes 3 unphysical phases
- $U_\nu \to U_\nu K_\nu$ to obtain positive ν masses
- up to permutation of columns, $U_{\ell,\nu} \to U_{\ell,\nu} P_{\ell,\nu}$

Altarelli, Feruglio [hep-ph/0512103] He, Keum, Volkas [hep-ph/0601001]

Predictions

- **leptonic mixing angles** up to exchange of rows and columns
- **Dirac CP phase δ** up to π
- Majorana phases undetermined

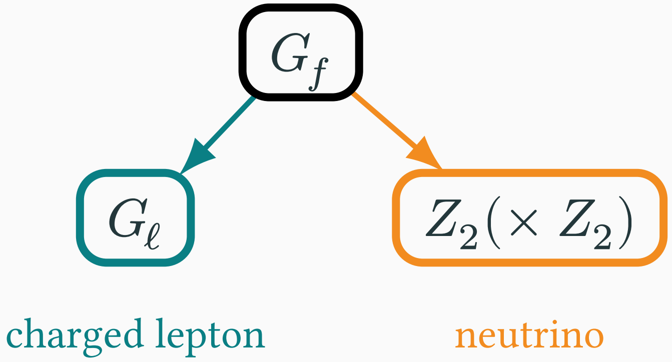
flavour symmetry **and** breaking pattern are important to obtain nontrivial mixing



poster 2#99 Cameron Moffett-Smith

poster 2#572 Harold Matias

How to probe flavour symmetries?



$G_\nu = Z_2 \times Z_2$

predictions of exact values

e.g. TBM in models based on $A_4, S_4, \ldots$

$$U_{\text{tbm}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$G_\nu = Z_2$

prediction of sum rules

e.g. *atmospheric mixing sum rule*

$$\sin\theta_{23} - \frac{1}{\sqrt{2}} = \lambda \sin\theta_{13}\cos\delta + \ldots$$

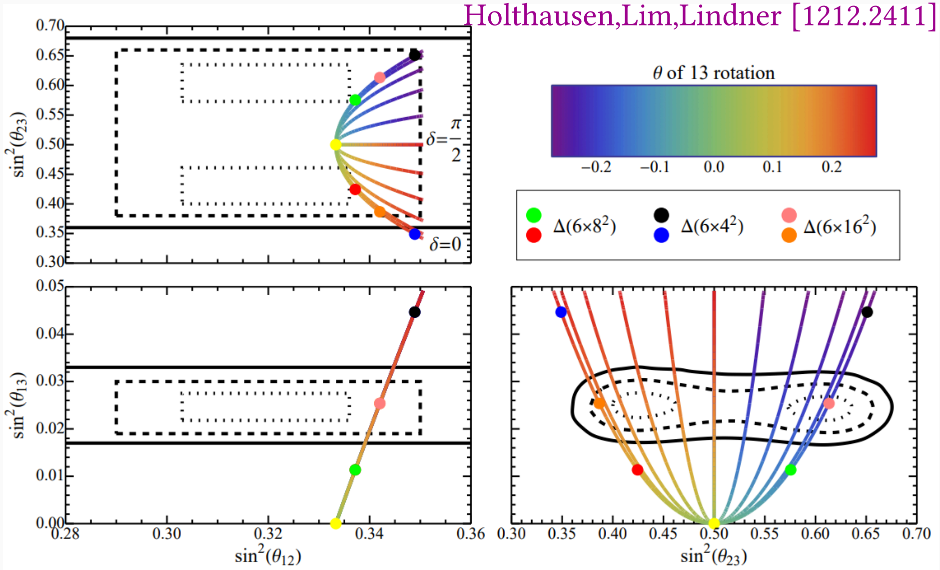
Common predictions $\lambda \simeq 1$ or $\lambda \simeq -\frac{1}{2}$

Residual symmetry G_ℓ - charged leptons

(approx.) conserved flavour charge

of charged leptons , e.g. lepton triality

Flavour models with $G_\nu = Z_2 \times Z_2$



Many groups have been proposed, see e.g. S_3 Fritzsch (1977) S_4 Pakvasa, Sugiwara (1979) A_4 Ma,Rajasekaran [hep-ph/0106291] Babu,Ma,Valle [hep-ph/0206292] A_5 Everett, Stuart [0812.1057] $\Delta(3n^2)$ Luhn,Nasri,Ramond [hep-th/0701188] $\Delta(6n^2)$ Escobar,Luhn [0809.0639] T_7 Luhn, Nasri, Ramond [0706.2341] Hagedorn,MS,Smirnov [0811.2955] T' Frampton,Kephart [hep-ph/9409330] $\Sigma(81)$ Ma [hep-ph/0612022] Hagedorn,MS,Smirnov [0811.2955] $Q_8 \rtimes A_4$ Holthausen,MS [1211.6953]

$G_\nu = Z_2 \times Z_2 \qquad \Delta(6n^2) : G_\ell = Z_3$

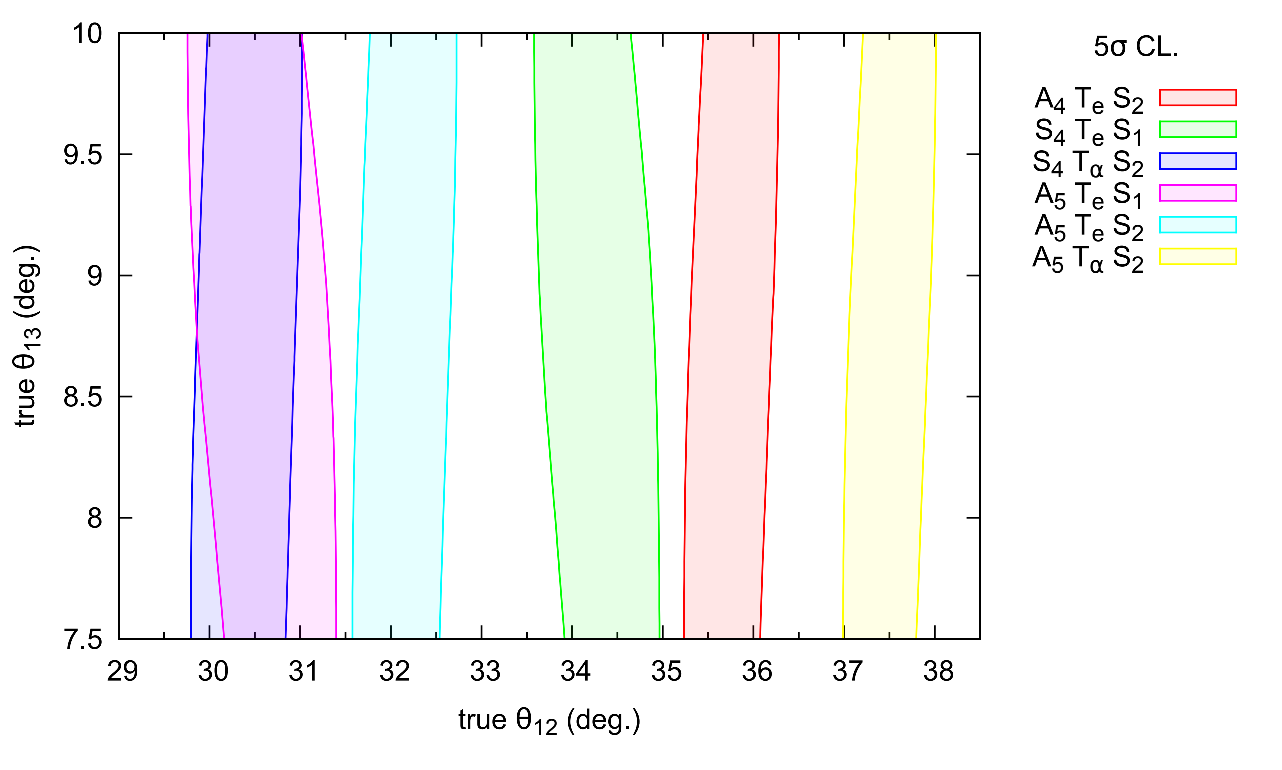
groups in scan predict TM2 pattern

$$U_{\rm tbm} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TM2

Larger groups like $\Delta(6n^2)$ and $\Delta(3n^2)$ predict exact values for mixing parameters consistent with data (incl $\theta_{13} \neq 0$)

5σ allowed regions after 6 years of data taking by JUNO



flavour group together with breaking pattern

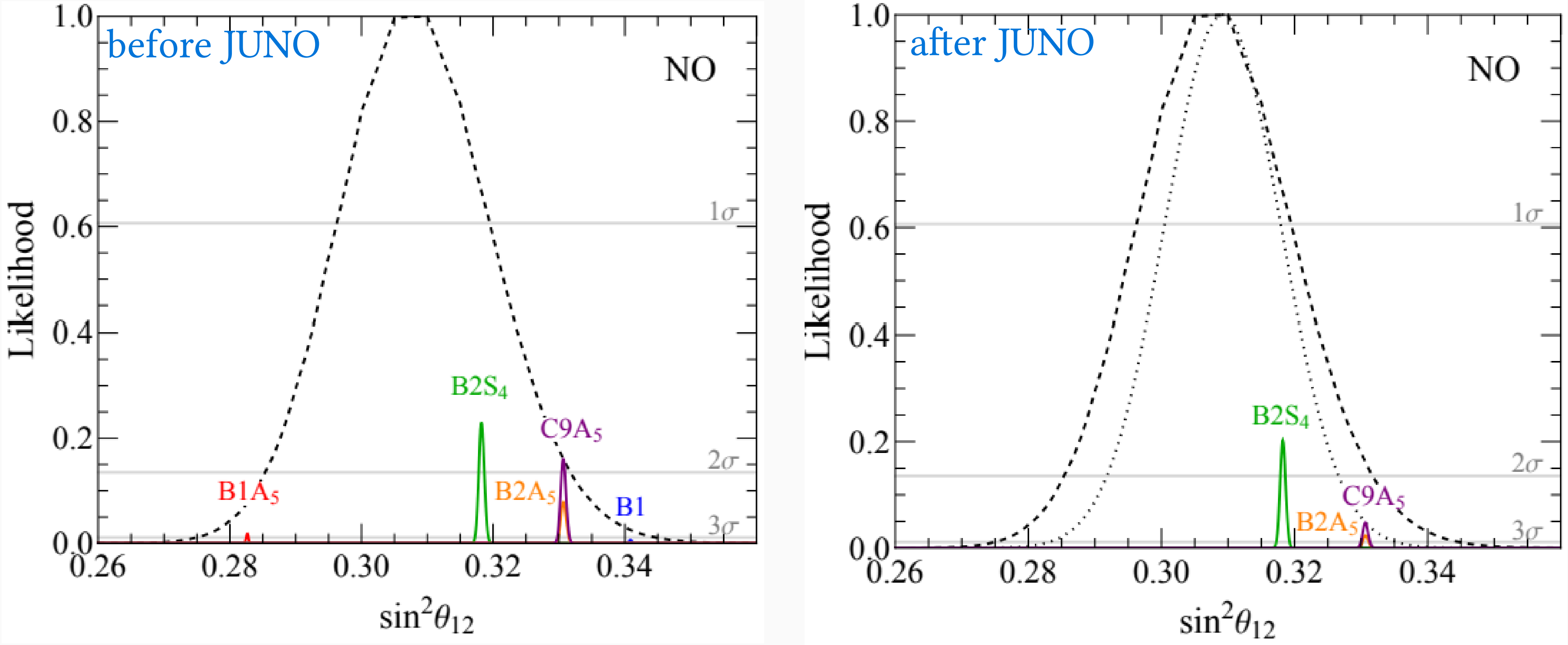
$A_4\, T_\alpha - S_2$	$s = \sqrt{\frac{2}{2-r^2}} - 1$
$S_4\, T_e - S_1$	$s = \sqrt{1 - \frac{2r^2}{2-r^2}} - 1$
$S_4\, T_\alpha - S_2$	$s = \sqrt{\frac{3}{2-2r^2}} - 1$
$A_5\, T_e - S_1$	$s = \sqrt{3 + \frac{6}{(3-\varphi)(r^2-2)}} - 1$
$A_5\, T_e - S_2$	$s = \sqrt{\frac{6}{(2-\varphi)(2-r^2)}} - 1$
$A_5\, T_\alpha - S_2$	$s = \sqrt{\frac{3\varphi}{(2-\varphi)(2-r^2)}} - 1$

$$s = \sqrt{3}s_{12} - 1, \quad r = \sqrt{2}s_{13}$$

Theory predictions clearly separated

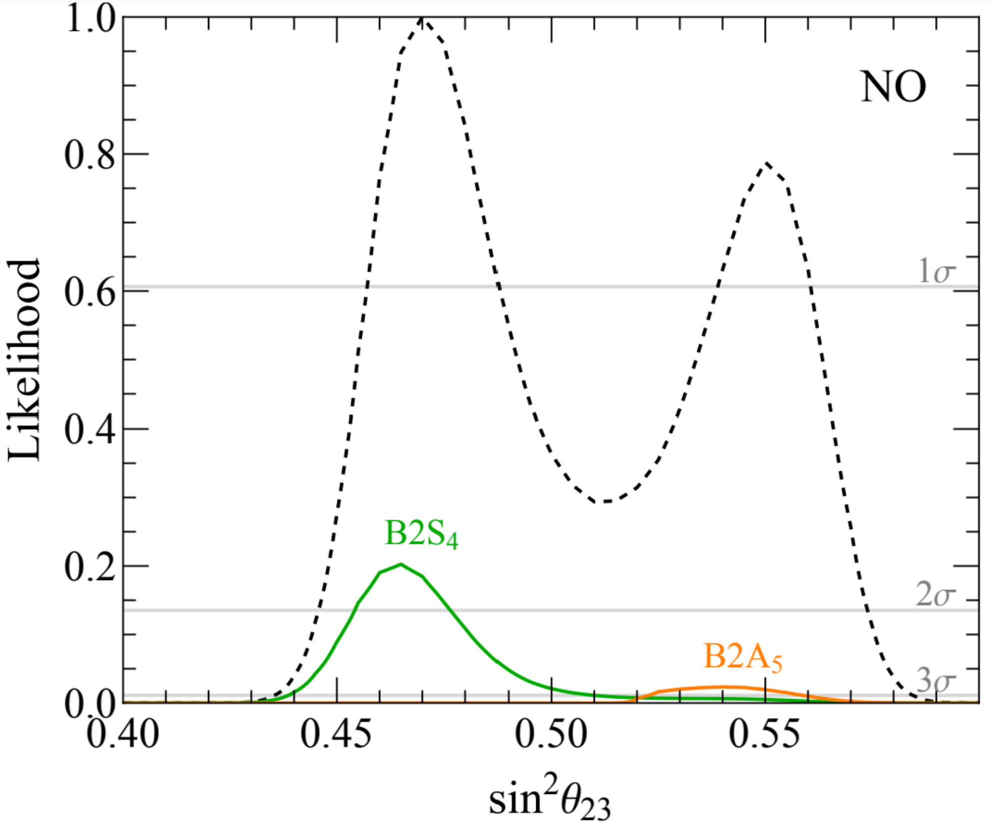
For a similar discussion of the atmospheric sum rule, see Ballett,King,Luhn,Pascoli [1308.4314]

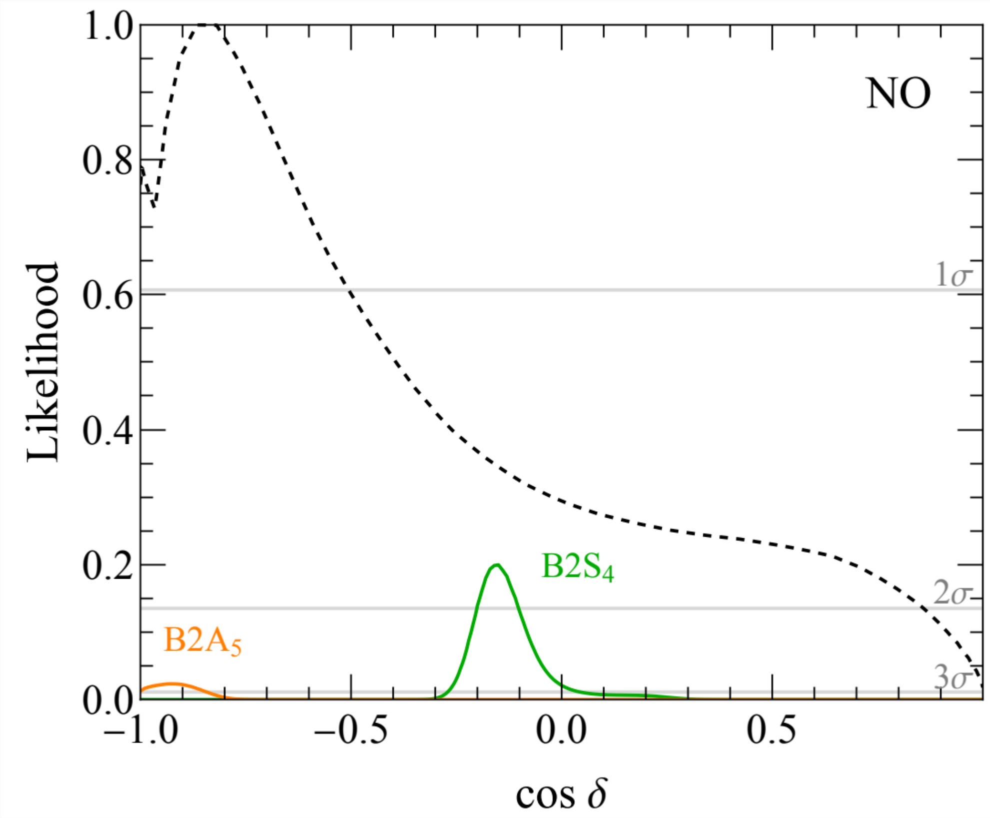
Petcov and Titov [2511.19408] carried out more general recent analysis. see also Petcov,Titov [1804.00182]



B2S_4 - $(S_4 : T_e, S_1)$ - TM1 B2A_5 - $G_\nu = \mathbb{Z}_2, G_\ell = \mathbb{Z}_2 \times \mathbb{Z}_2$ C9A_5 - $G_\nu = \mathbb{Z}_2, G_\ell = \mathbb{Z}_2$

B1 - $(A_4 : T_\alpha, S_2)$ - TM2 B1A_5 - $(A_5 : T_e, S_2)$ see also Ge+ [2511.15442] Zhang [2511.15654] Jiang+ [2511.16348] Dutta+ [2601.18397]





B2S_4 - $(S_4 : T_e, S_1)$ - TM1 B2A_5 - $G_\nu = \mathbb{Z}_2, G_\ell = \mathbb{Z}_2 \times \mathbb{Z}_2$

C9A_5: no constraints on $\sin^2\theta_{23}$ nor $\cos\delta$

Charged lepton corrections – $(2\ell, 2\nu)$

Ballett,King,Luhn,Pascoli,MS [1410.7573]

PMNS matrix

$$U = U_\ell^\dagger U_\nu$$

U_ℓ and U_ν not physical in SM

Flavour models fix flavour basis

quark-lepton complementarity $U_\ell \sim V_{\mathrm{CKM}}$ Minakata+ [hep-ph/0405088]

Pati-Salam unified model $G_{\mathrm{PS}} \times SO(3)_{\mathrm{fl}}$ King [hep-ph/0506297]

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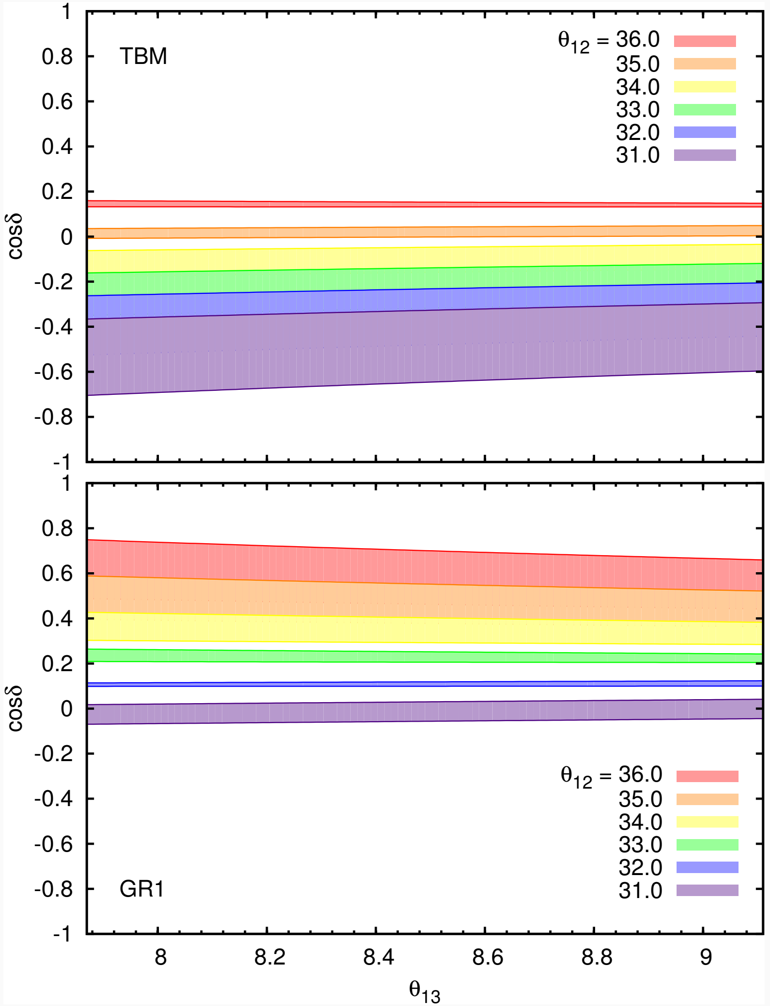
Solar mixing sum rule for $\theta^\nu_{13} = \theta^\ell_{13} = 0$

$\tan\theta^\nu_{12} = \frac{|U_{\tau 1}|}{|U_{\tau 2}|}$ results in

$$\cos\delta = \frac{t_{23} s_{12}^2 + s_{13}^2 c_{12}^2 / t_{23} - s_{12}^{\nu 2}(t_{23} + s_{13}^2/t_{23})}{s_{13}\sin 2\theta_{12}}$$

see also Marzocca,Petcov,Romanino,Sevilla [1302.0423] Petcov [1405.6006]

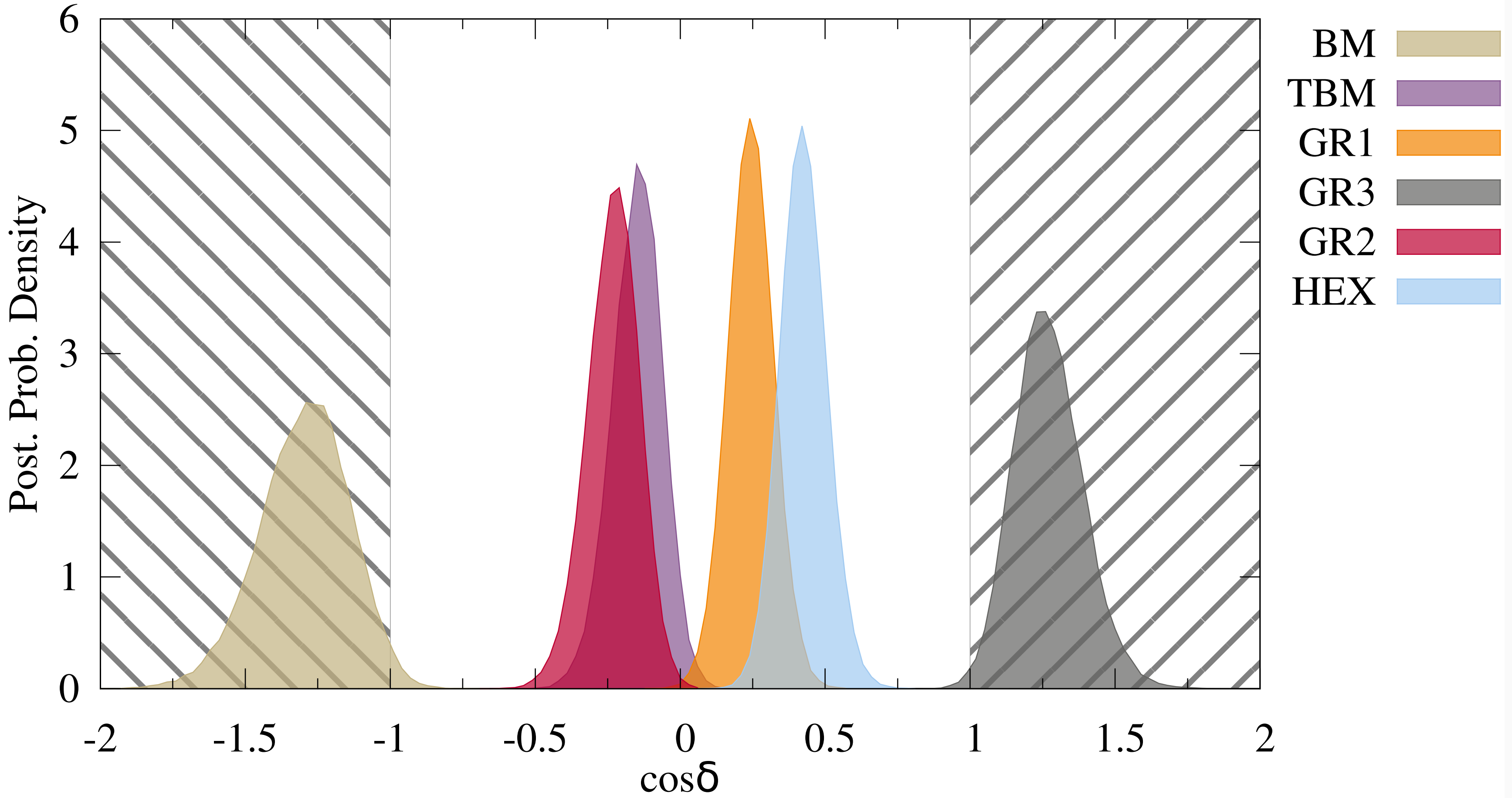
Girardi,Petcov,Titov [1410.8056] Ding,Valle [2402.16963]



Charged lepton corrections – $(2\ell, 2\nu)$

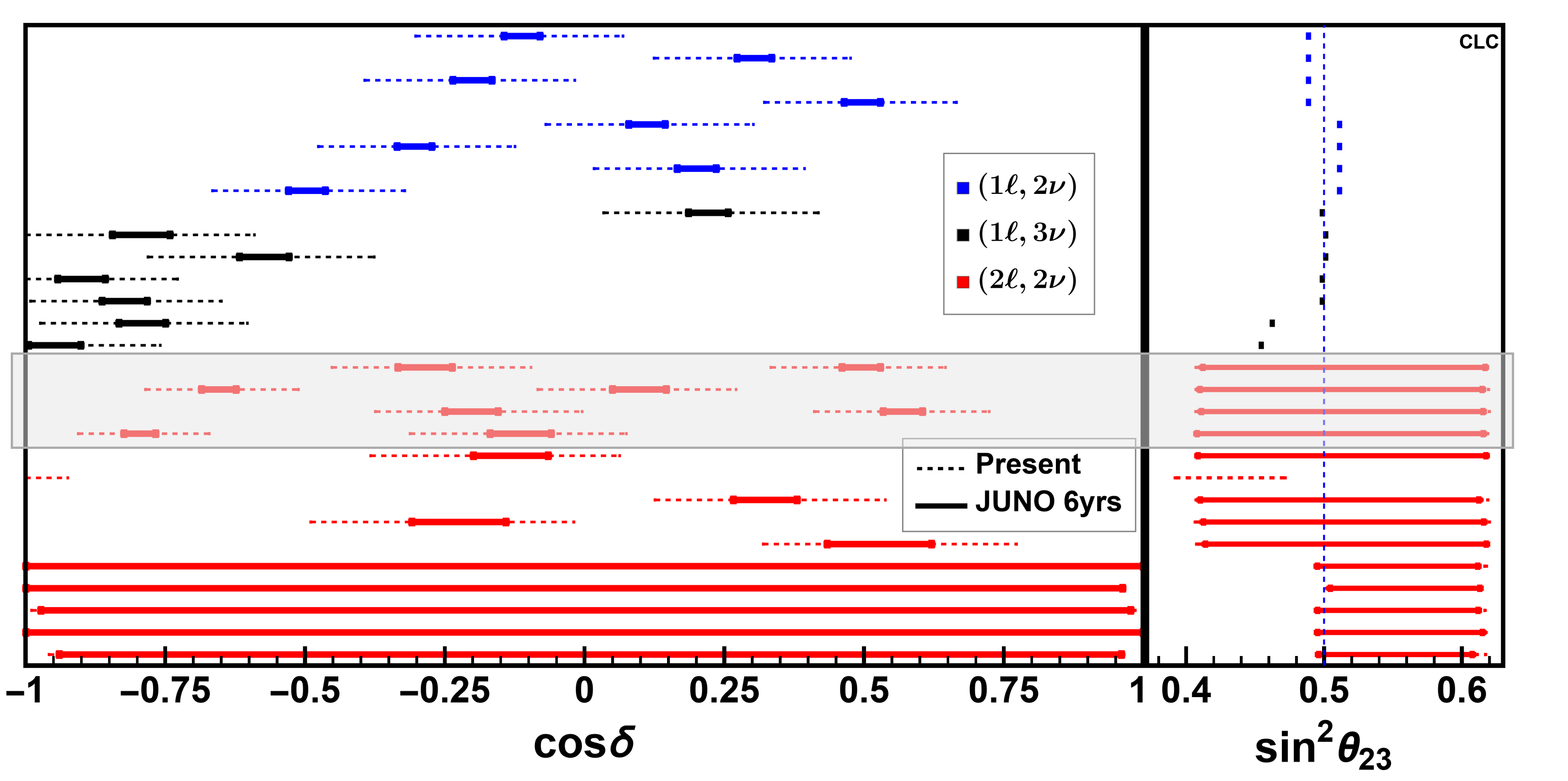
Ballett,King,Luhn,Pascoli,MS [1410.7573]

BiMaximal; TriBiMaximal; GoldenRatio; HEXagonal



based on NuFIT v2

Charged lepton corrections – after 1st JUNO result



Denton, Gehrlein, Truelson [2606.05291]

based on NuFIT v6.1 incl JUNO JUNO [2511.14593]

Residual symmetries G_ℓ

Calibbi,Hagedorn,MS,Vandeleur [2505.2435]

lepton triality Ma [1006.3524]

charged leptons carry flavour charge (so that they can be distinguished)

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \to T \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} = \begin{pmatrix} 1 & & \\ & \omega^2 & \\ & & \omega \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

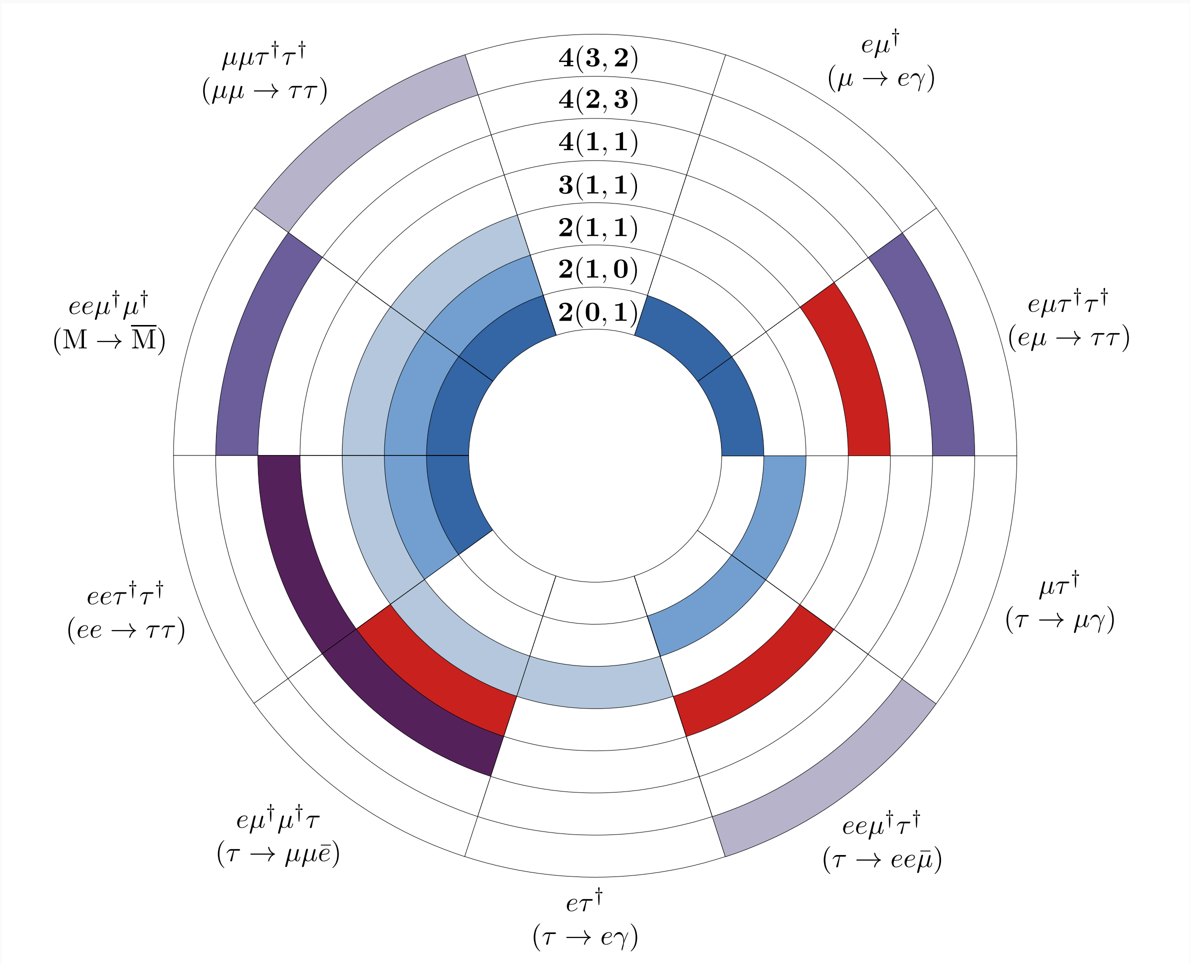
processes violating symmetry are suppressed/absent

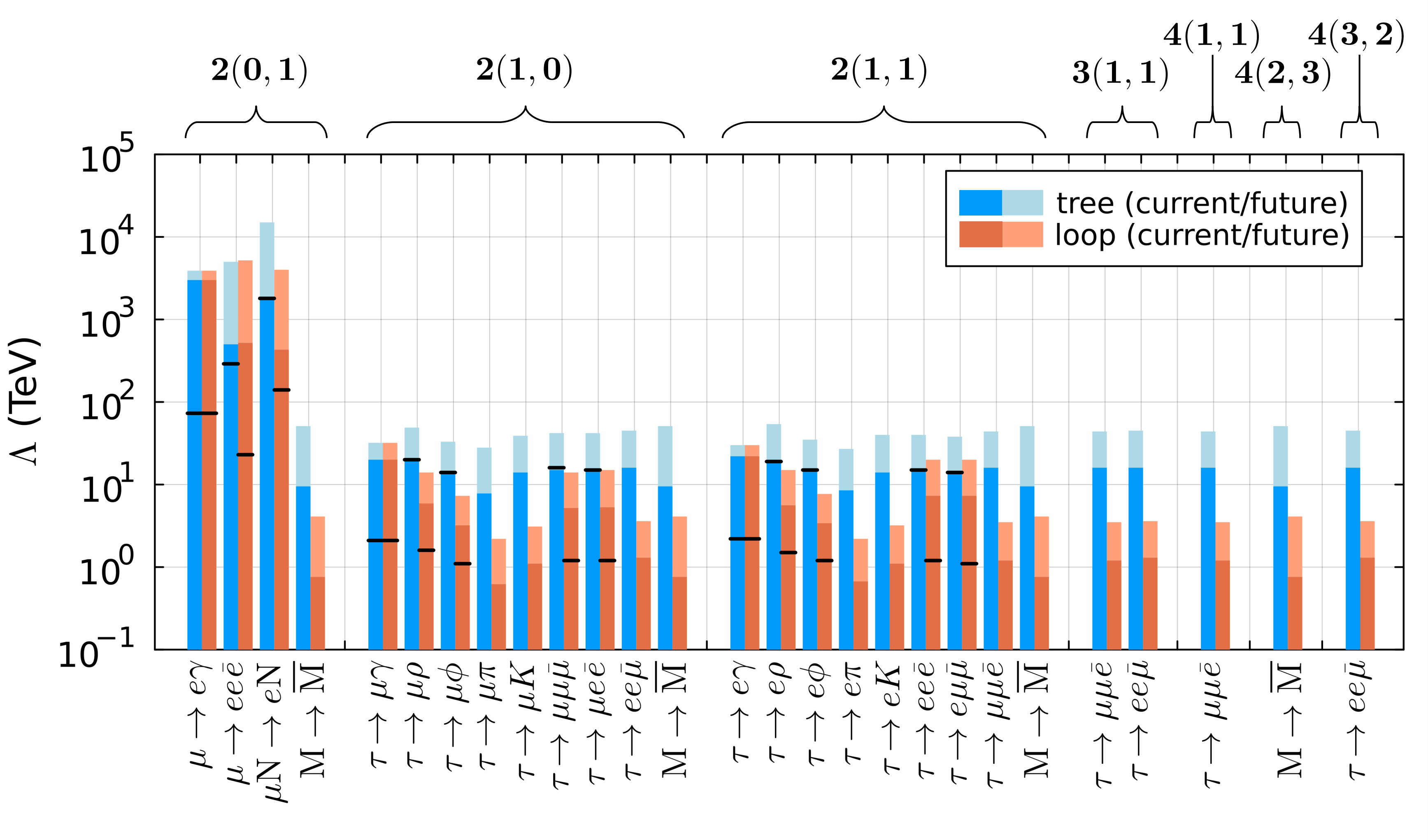
- ✗ $\mu \to e\gamma$, $\ell \to \ell'\gamma$, $\mu \to eee$
- ✓ $\tau \to \mu\mu\bar{e}$, $\tau \to ee\bar{\mu}$

phenomenology depends on G_ℓ and charge differences

many redundancies! $N(a,b)$ represents

$$Z_N[Q_\alpha = (q_e, q_e + a, q_e + b)]\ \forall q_e \in [1, N]$$





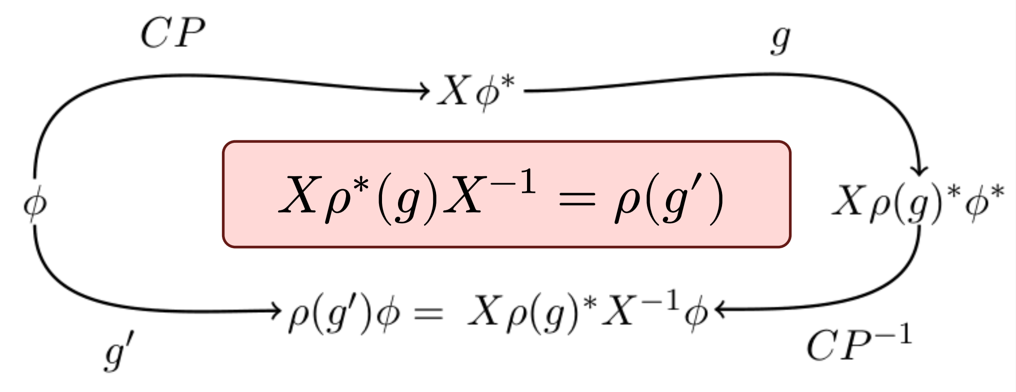
$\mathbf{2(0,1)}$
$\mathbf{2(1,0)}$
$\mathbf{2(1,1)}$
$\mathbf{3(1,1)}$
$\mathbf{4(1,1)}$
$\mathbf{4(2,3)}$
$\mathbf{4(3,2)}$
tree (current/future)
loop (current/future)
Λ (TeV)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
$\mu \to e\gamma$
$\mu \to ee\bar{e}$
$\mu N \to eN$
$M \to \overline{M}$
$\tau \to \mu\gamma$
$\tau \to \mu\rho$
$\tau \to \mu\phi$
$\tau \to \mu\pi$
$\tau \to \mu K$
$\tau \to \mu\mu\bar{\mu}$
$\tau \to \mu e\bar{e}$
$\tau \to ee\bar{\mu}$
$M \to \overline{M}$
$\tau \to e\gamma$
$\tau \to e\rho$
$\tau \to e\phi$
$\tau \to e\pi$
$\tau \to eK$
$\tau \to ee\bar{e}$
$\tau \to e\mu\bar{\mu}$
$\tau \to \mu\mu\bar{e}$
$M \to \overline{M}$
$\tau \to \mu\mu\bar{e}$
$\tau \to ee\bar{\mu}$
$\tau \to \mu\mu\bar{e}$
$M \to \overline{M}$
$\tau \to ee\bar{\mu}$

How to implement a CP symmetry in presence of a flavour symmetry?

Need to use generalised CP symmetry $\varphi \to X\varphi^*$

CP is an outer automorphism,
i.e. a symmetry of the flavour symmetry

Grimus,Rebelo [hep-ph/9506272] Feruglio,Hagedorn,Ziegler [1211.5560]
Holthausen,Lindner,MS [1211.6953]



Physical CP symmetries are class-inverting

Chen,Fallbacher,Mahanthappa,Ratz,Trautner [1402.0507]

i.e. mapping between particles and antiparticles.

CP phases from partially breaking CP

$G_f \rtimes H_{\rm CP}$

G_ℓ

$Z_2 \times H^\nu_{\rm CP}$

charged lepton

$Q^\dagger M_\ell^\dagger M_\ell Q = M_\ell^\dagger M_\ell$

$U_e^\dagger Q U_e = \text{diag}$

neutrino

$Z_2 : Z^T M_\nu Z = M_\nu$

$\text{CP} : X^T M_\nu X = M_\nu^*$

$X = \Omega_\nu \Omega_\nu^T$

$\Omega_\nu^\dagger Z \Omega_\nu = \text{diag}$

$$U = U_\ell^\dagger \Omega_\nu R(\theta) K_\nu$$

- $U_\ell \to U_\ell K_\ell$ removes 3 unphysical phases
- $\Omega_\nu^T M_\nu \Omega_\nu$ block diagonal → additional $R(\theta)$
- $U_\nu \to U_\nu K_\nu$ to obtain positive ν masses
- up to permutation of columns, $U_{\ell,\nu} \to U_{\ell,\nu} P_{\ell,\nu}$

Predictions

mixing angles and all CP phases as fct of θ up to exchange of rows/columns

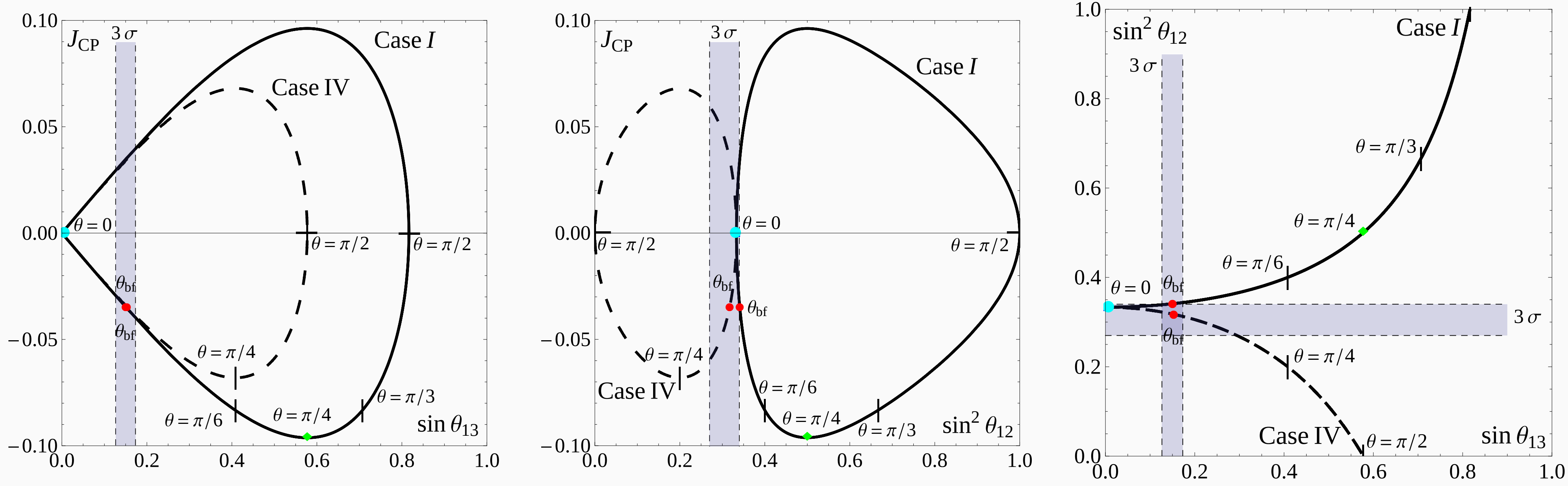
Applied to many groups

S_4 Feruglio,Hagedorn,Ziegler [1211.5560] [1303.7178] Ding,King,Luhn,Stuart [1303.6180] Ding,Zhou [1312.4401] Ding,Li [1312.4401] [1408.0785]

A_4 Ding,King,Stuart [1307.4212]

A_5 Ding,Li [1503.03711] Di Iura,Hagedorn,Meloni [1503.04140]Ballett,Pascoli,Turner [1503.07543]

$\Delta(3n^2), \Delta(6n^2)$ Hagedorn,Meroni,Molinaro [1408.7118] King,Neder [1403.1758] Ding,King,Neder [1409.8005]Ding,King [1403.5846] Ding,Zhou [1404.0592]



- 5 different embeddings of CP: $2\,|\sin\delta| = 1$, others 0
  see also Ding+ [1307.4212] [1303.6180] [1811.12340] Li+ [1312.4401] [1408.0785] [1608.01860] Feruglio+ [1303.7178] Luhn [1306.2358] Penedo+ [1705.00309] ...
- similar $A_4 \rtimes H_{CP}$ Feruglio+ [1211.5560] Ding+ [1307.4212] Li+ [1608.01860] Nishi [1601.00977]

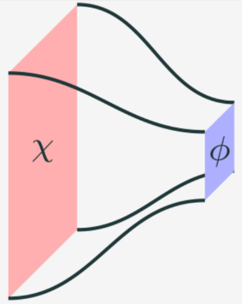
	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\lvert\sin\delta\rvert$	$\lvert J_{CP}\rvert$
I	$\frac{2}{3}\sin^2\theta$	$\frac{1}{2+\cos 2\theta}$	$\frac{1}{2}$	1	$\frac{\lvert\sin 2\theta\rvert}{6\sqrt{3}}$
IV	$\frac{1}{3}\sin^2\theta$	$\frac{\cos^2\theta}{2+\cos^2\theta}$	$\frac{1}{2}$	1	$\frac{\lvert\sin 2\theta\rvert}{6\sqrt{6}}$

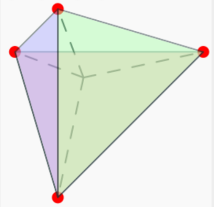
Non-trivial mixing requires flavons to take VEVs in different directions in field space

Extra dimensions

Brane-localized flavons $\phi, \chi \to$ enhanced symmetry $G_f \times G_f$

e.g. He, Keum, Volkas [hep-ph/0601001]





Explicit breaking via boundary conditions on orbifold

e.g. Kobayashi,Omura,Yoshioka [0809.3064]

Driving fields in SUSY

separate flavons using $U(1)_R$ symmetry and driving fields with R-charge 2 impose constraints Altarelli,Feruglio [hep-ph/0512103]

Common feature: **enhanced symmetry**

Group extension $\overline{G}_f$ of flavour group G_f

surjective homomorphism $\rho : \overline{G}_f \to G_f$

Holthausen,MS [1111.1730] Babu,Gabriel [1006.0203]

Example $Q_8 \rtimes A_4 \to A_4$

Holthausen,Lindner,MS [1211.5143]

Symmetry breaking in flavour models is often complicated!

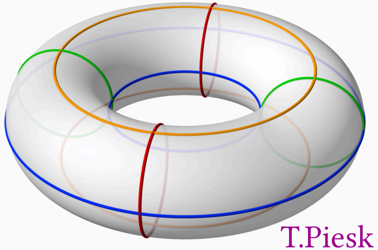
Feruglio [1706.08749]

poster 2#572
Harold
Matias

A single complex flavon τ (modulus) parametrizes shape of torus

modular action $\gamma \in \mathrm{SL}(2, \mathbb{Z})$

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$



T.Piesk

Principal congruence subgroup of level N

$$\Gamma(N) = \{\gamma \in \mathrm{SL}(2, \mathbb{Z} \mid \gamma = \mathbb{1} \bmod N\}$$

flavour group is finite modular group

$$G_f = \begin{cases} \Gamma_N \equiv \mathrm{SL}(2, Z)/\pm\Gamma(N) \\ \Gamma'_N \equiv \mathrm{SL}(2, Z)/\Gamma(N) \end{cases}$$

$\Gamma_3 \cong A_4$ $\Gamma_4 \cong S_4$ $\Gamma_5 \cong A_5$

Fields transform non-linearly

$$\psi \to (c\tau + d)^{-k}\rho(\gamma)\psi$$

weight $k \in \mathbb{Z}$ ρ unitary repr of $\Gamma_N^{(\prime)}$

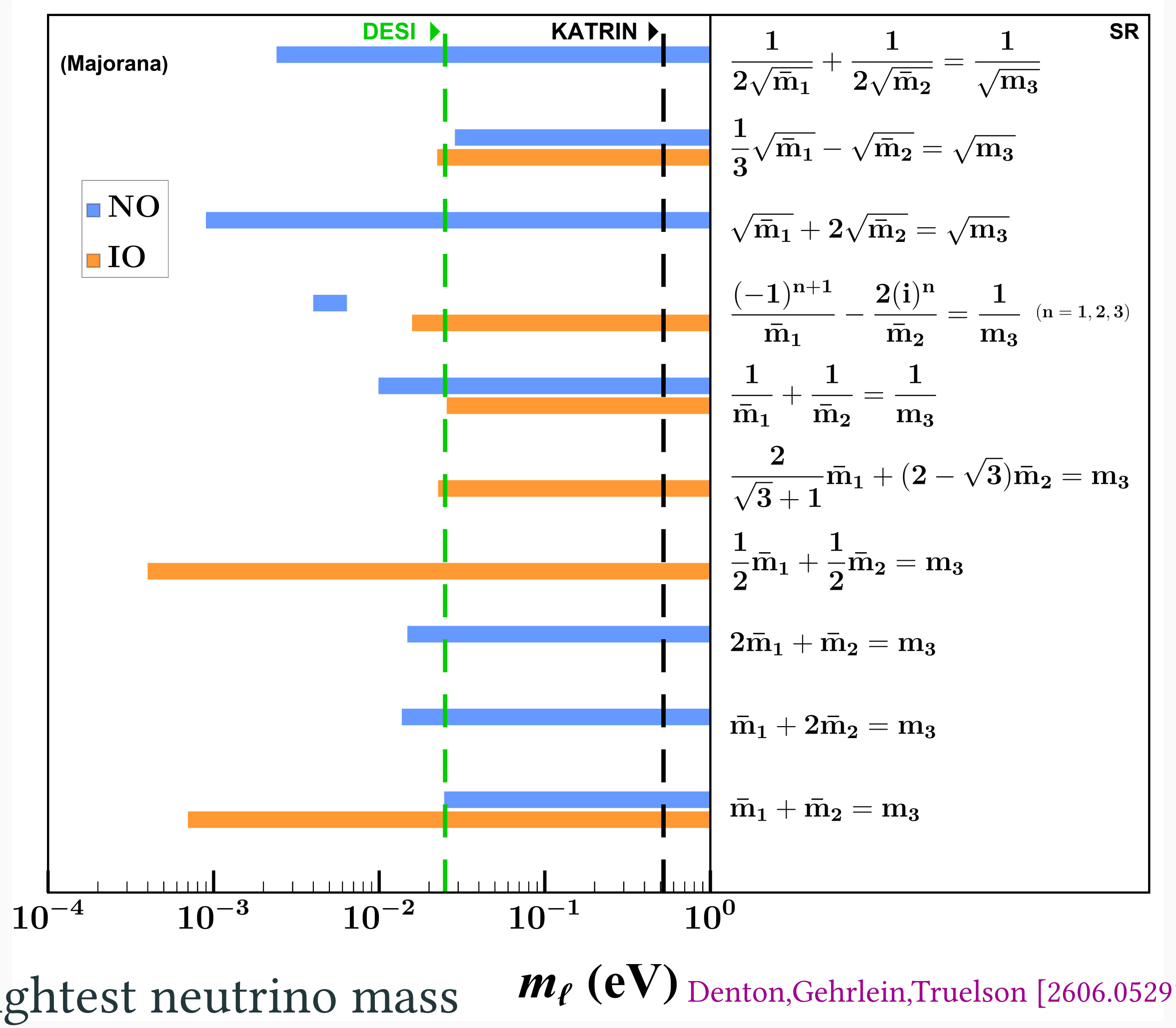
superpotential

$$\mathcal{W} = Y_{i_1 \ldots i_n}(\tau)\psi_{i_1} \ldots \psi_{i_n}$$

- Yukawa couplings functions of τ
- flavour breaking determined by expectation value of modulus τ

Kähler potential is not fixed by modular symmetry
→ introduces corrections Chen,Ramos-Sanchez,Ratz [1909.06910]

Neutrino mass sum rules – Majorana neutrinos



Denton,Gehrlein,Truelson [2606.05291]

Usually not origin of a flavour symmetry

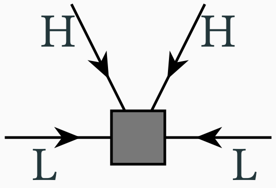
Flavour models reduce the number of free parameters

⇒ Mass sum rules arise when neutrino masses depend only on 2 free parameters

See Barry,Rodejohann [1007.5217] King,Merle,Stuart [1307.2901] Gehrlein,Spinrath [1704.02371] and references to explicit models

Weinberg operator

$$-\frac{1}{2}\frac{\kappa_{ij}}{\Lambda}(L_i\varepsilon H)^T C (L_j \varepsilon H)$$



violates lepton number $\Delta L = 2$

→ Majorana neutrino masses after EWSB

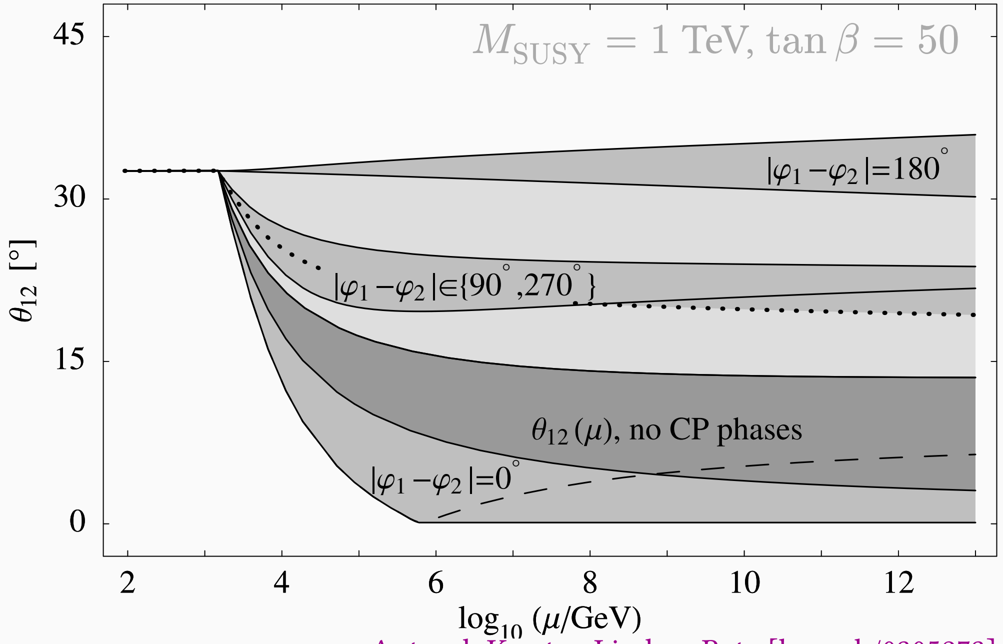
$$(M_\nu^M)_{ij} = \frac{\kappa_{ij}}{2}\frac{v^2}{\Lambda} \quad \text{with} \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

Tiny m_ν explained by high scale Λ

general form of RGE

$$\frac{\mathrm{d}\theta_{\mathrm{ij}}}{\mathrm{d}\ln\mu} \sim \frac{f(m_i, \delta, \varphi_1, \varphi_2)}{m_j^2 - m_i^2} F(Y_e, \theta_{12}, \theta_{13}, \theta_{23})$$

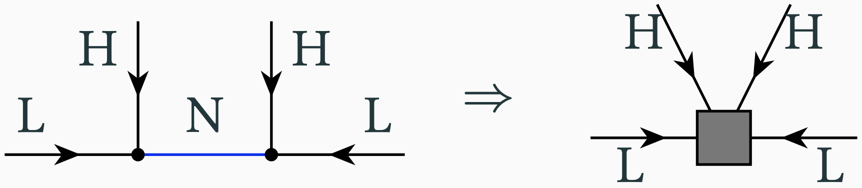
$$\frac{\mathrm{d}\theta_{12}}{\mathrm{d}\ln\mu} = -\frac{C y_\tau^2 (1+\tan^2\beta)}{32\pi^2} \sin 2\theta_{12} s_{23}^2 \frac{|m_1 e^{i\varphi_1} + m_2 e^{i\varphi_2}|^2}{\Delta m^2_{\mathrm{sol}}} + \mathcal{O}(\theta_{13})$$

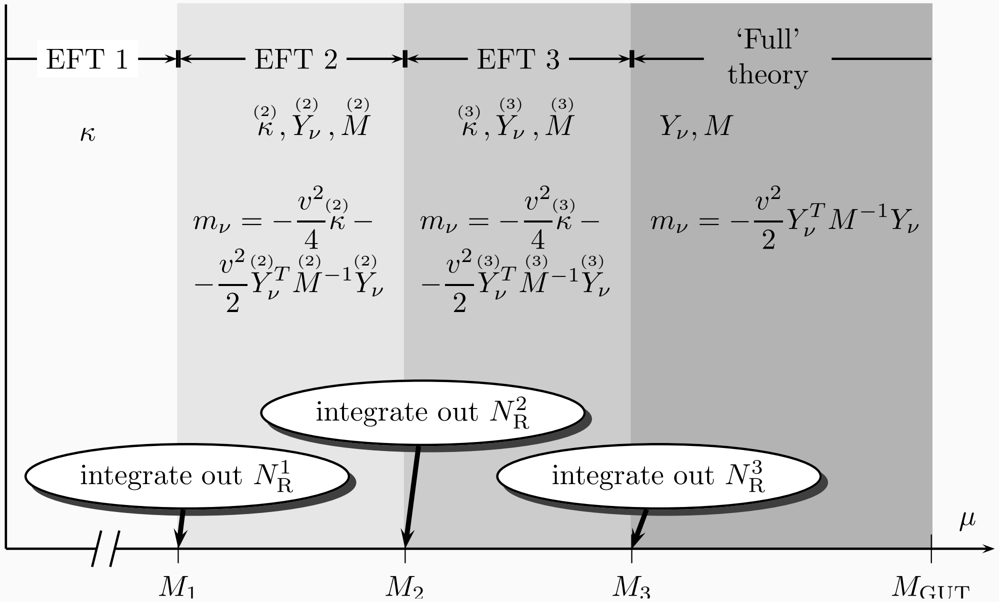


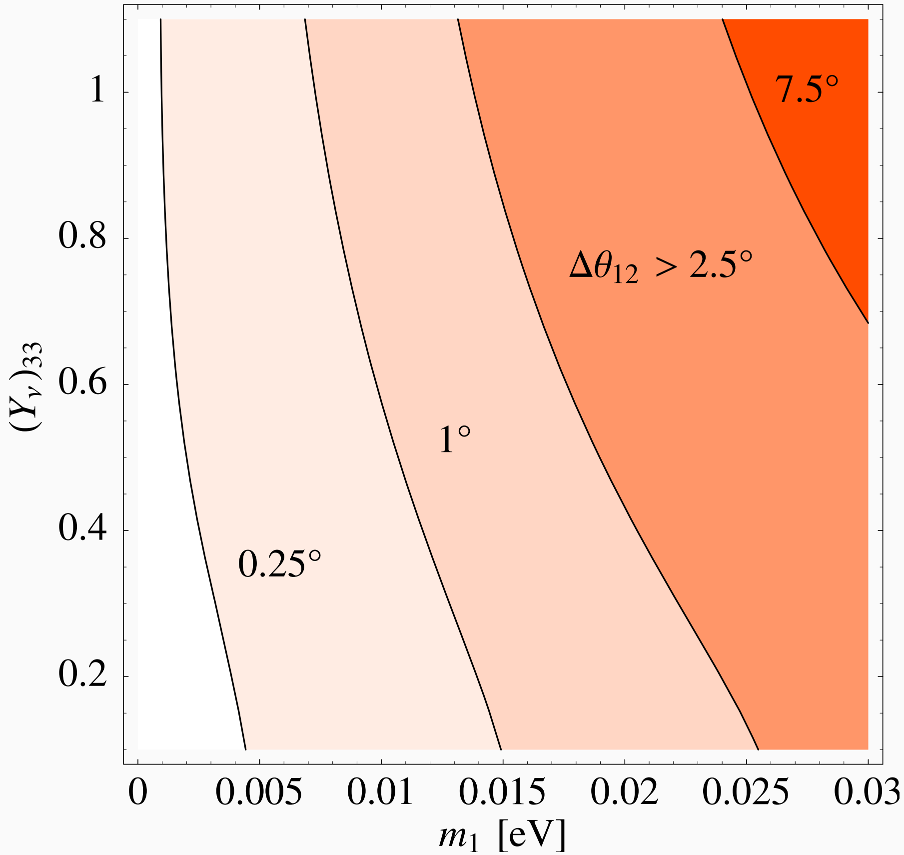
Antusch,Kersten,Lindner,Ratz [hep-ph/0305273]

poster 2#505
Shaheed Perez

Precision neutrino physics – seesaw model





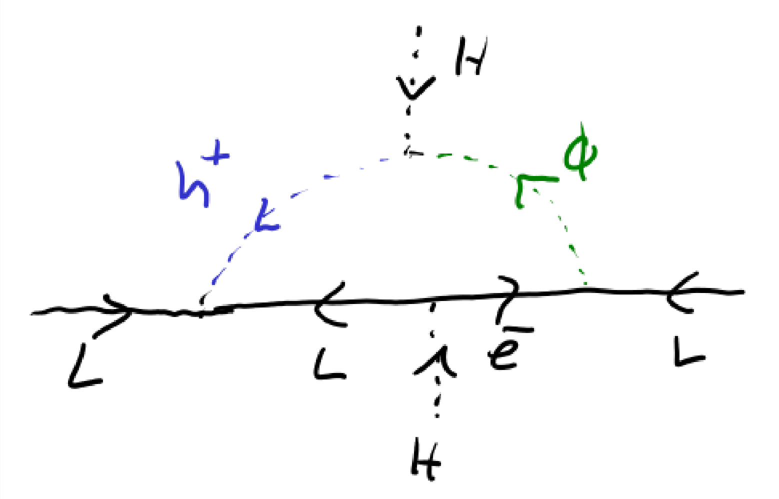


Antusch,Kersten,Lindner,Ratz,MS [hep-ph/0501272]

$M_{SUSY} = 1$ TeV, $\tan\beta = 20$, $\mu = 10^{14}$ GeV,
$\theta_{13} = 0$, $\theta_{23} = 45^o$, $\theta_{12} + \theta_c = 45^o$

see also recent work Zhang+ [2107.12133] Wang+ [2302.08140] Zhang [2504.00792], Ibarra+ [2411.08011]

Quantum corrections in a radiative neutrino mass model – Zee model

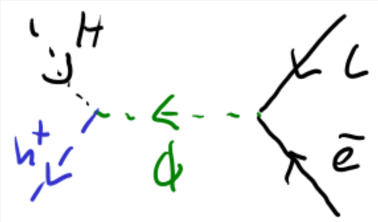


Two additional scalar fields Zee [PLB 93(1980)389]

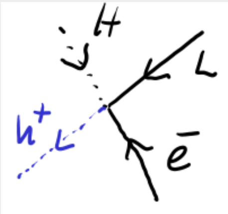
- charged scalar $h^+ \sim (1,1,1)$
- isodoublet scalar $\varphi \sim (1,2,\frac{1}{2})$

$$m_\nu = \frac{f m_E Y_2}{8\pi^2} \frac{\mu \langle H \rangle \ln\left(\frac{m_\varphi^2}{m_h^2}\right)}{m_h^2 - m_\varphi^2} + (\dots)^T$$

Tree-level matching – $m_\varphi \gg m_h$



$E^2 \overset{=}{\ll} m_\varphi^2$



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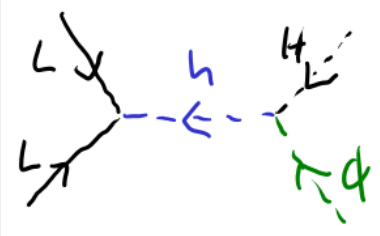


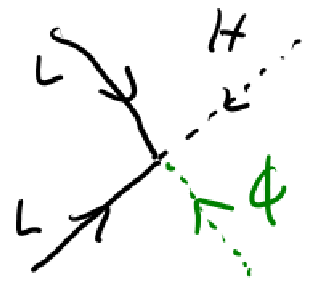
$E^2 \overset{=}{\ll} m_h^2$

$\Delta L = 2$ dim-5 operator at tree level

MS, Vandeleur [2512.17235]

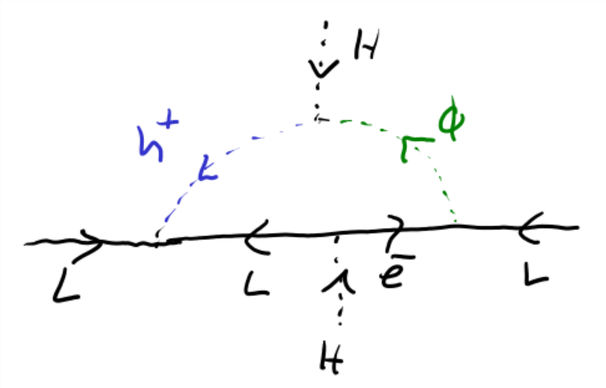
- Neutrino mass in the full theory

$$m_\nu = \frac{f m_E Y_2}{8\pi^2} \frac{\mu \langle H \rangle \ln\left(\frac{m_\varphi^2}{m_h^2}\right)}{m_h^2 - m_\varphi^2} + (\dots)^T$$



$$C_{LLH\varphi} = \frac{Y_2 \mu}{m_\varphi^2}$$

- In EFT neutrino mass dominantly generated through RG corrections sourced by $C_{LLH\varphi}$ → consistent with $\ln\left(\frac{m_\varphi^2}{m_h^2}\right)$ term



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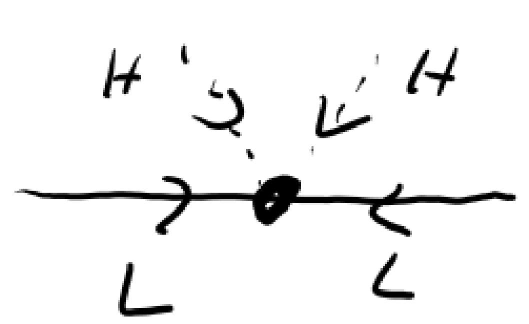

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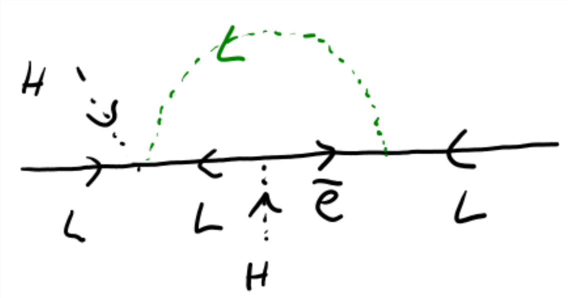
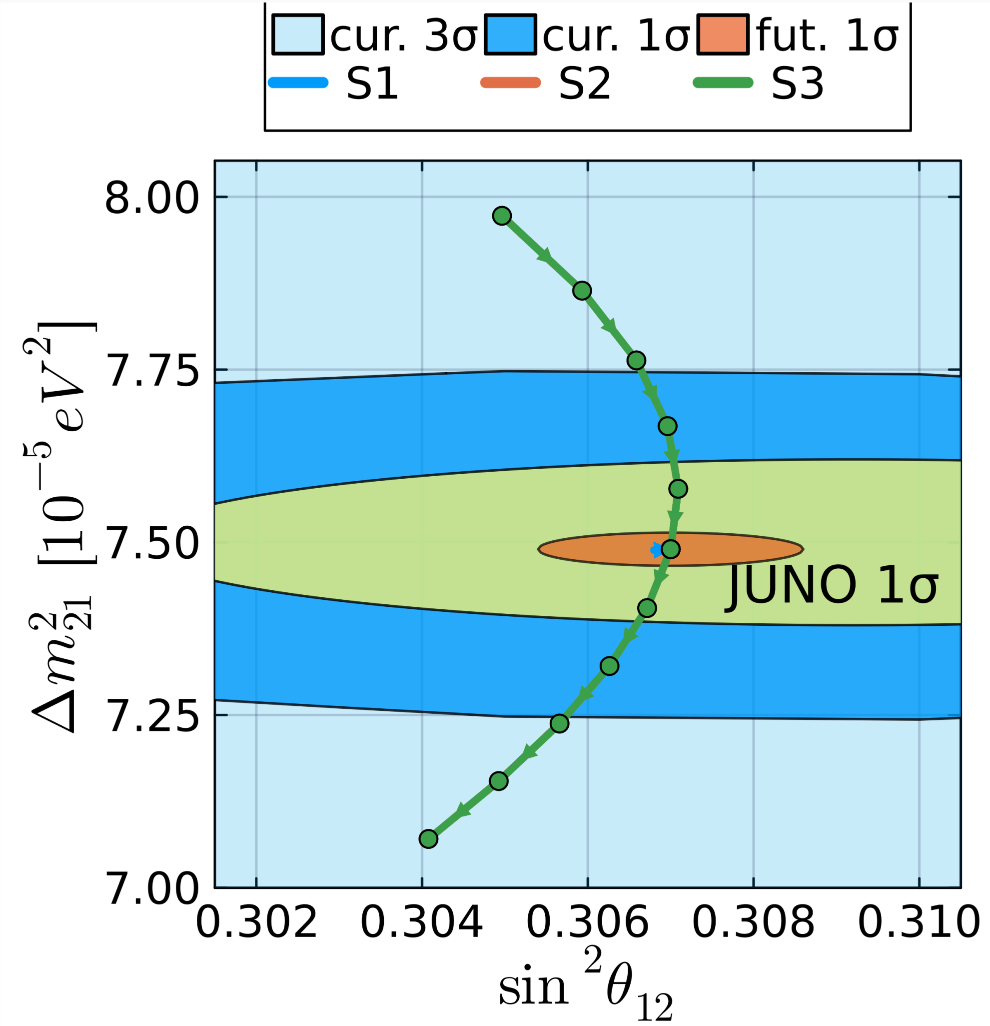


How large are quantum corrections?



Neutrino mass in the full theory

$$m_\nu = \frac{f m_E Y_2}{8\pi^2} \frac{\mu \langle H \rangle \ln\left(\frac{m_\varphi^2}{m_h^2}\right)}{m_h^2 - m_\varphi^2} + (\ldots)^T$$

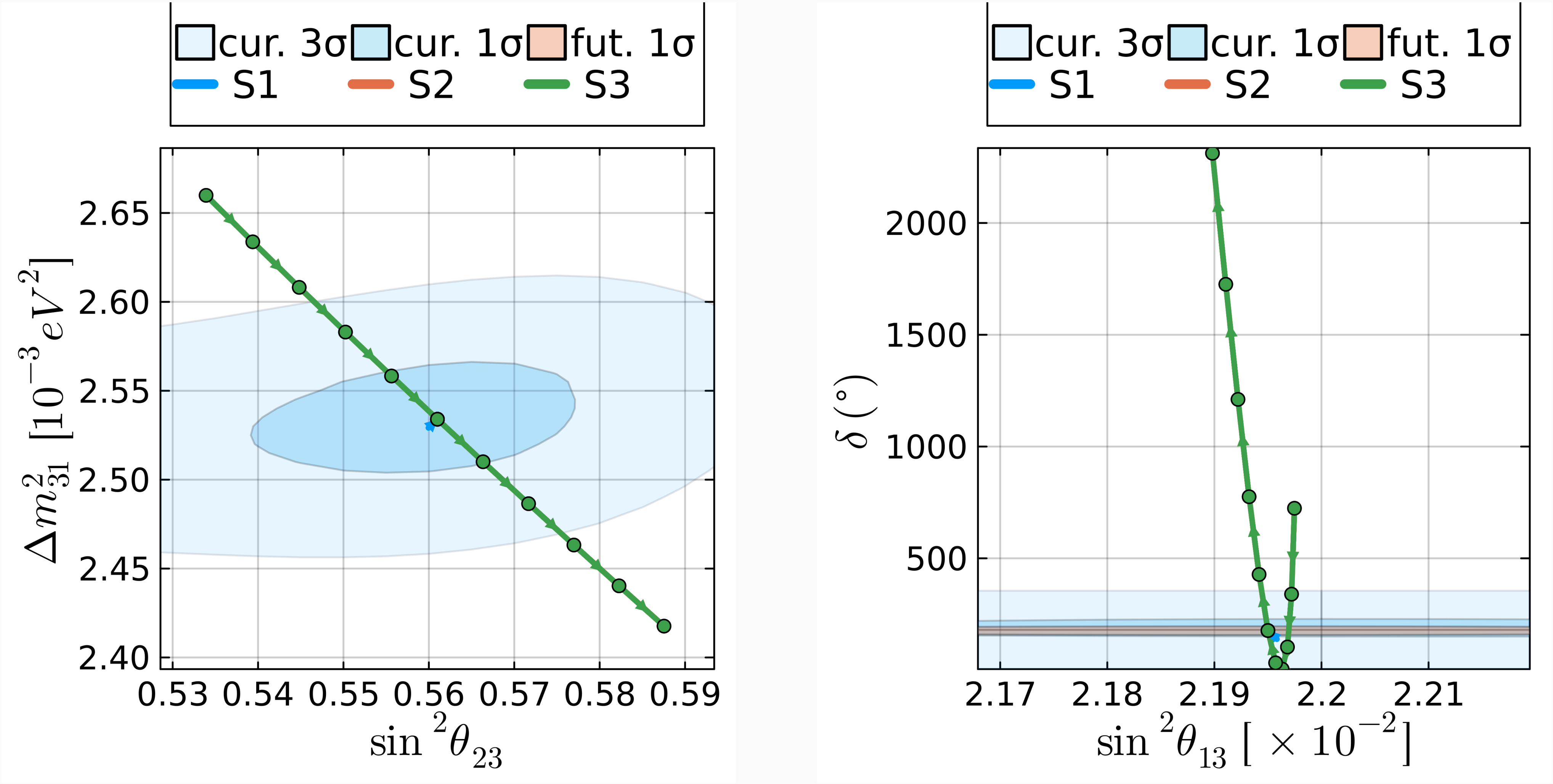
Example: $m_h = 100$ TeV, $m_\varphi = 1$ TeV, $m_E \frac{\sqrt{2}}{v} \ll Y_2 \sim \mathcal{O}(0.1 - 0.5)$, $Y_{u2} = Y_u, Y_{d2} = Y_d$

Rescaling $f \to \gamma f$ with $\gamma \in [0.95, 1.05]$

- S1: $Y_{e1,2} \to \gamma^{-\frac{1}{2}} Y_{e1,2}$
- S2: $Y_{e2} \to \gamma^{-1} Y_{e2}$
- S3: $Y_{e1} \to \gamma^{-1} Y_{e1}$

leaving *full theory* m_ν^{FT} *invariant*, but different predictions in EFT

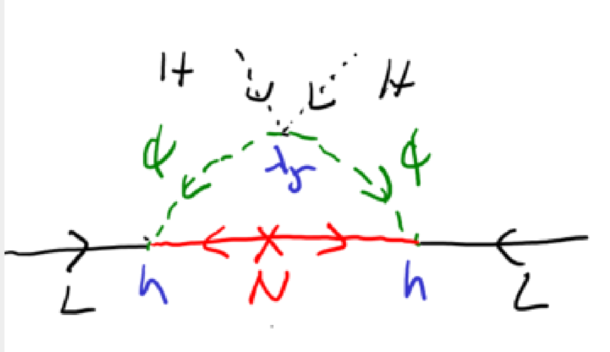
Quantum corrections in Zee model – atmospheric mixing



Comments on quantum corrections in the radiative seesaw model

Model

Tao [hep-ph/9603309] Ma [hep-ph/0601225]



- $\mathbb{Z}_2$ parity
- SM singlet fermions $N \sim (1,1,0)\_$
- isodoublet scalar $\varphi \sim \left(1,2,\frac{1}{2}\right)\_$

$$m^{\nu}_{ij} \simeq \frac{\lambda_5 \langle H\rangle^2}{16\pi^2} \frac{h_{ik} M_k h_{jk}}{m_\varphi^2} f\left(\frac{M_k^2}{m_\varphi^2}\right)$$

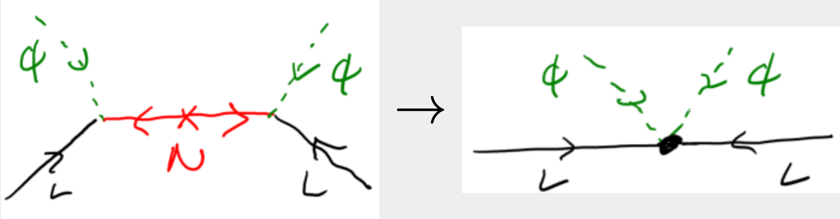
for $\lambda_5 \langle H\rangle^2 \ll M_k^2, m_\varphi^2$

$M_k \gg m_\varphi$

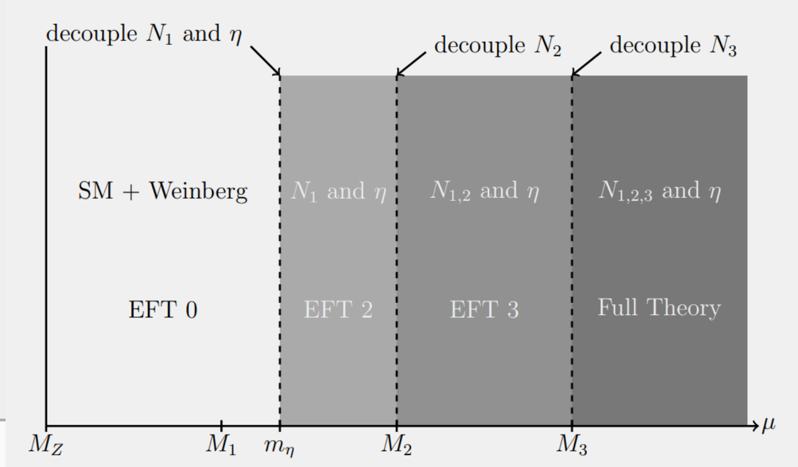
Bouchand,Merle [1205.0008] Merle,Platscher [1507.06314]

D5 operators generated at $\mu = M_k$:

- $LL\varphi\varphi$ via at tree level

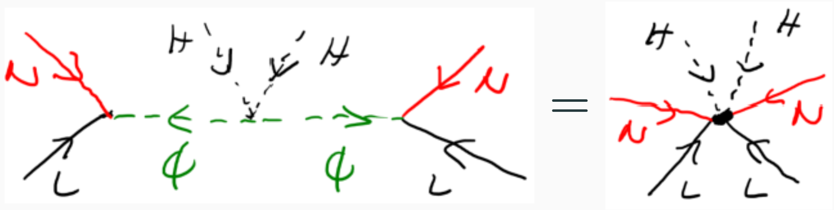


- $LLHH$ at one-loop level

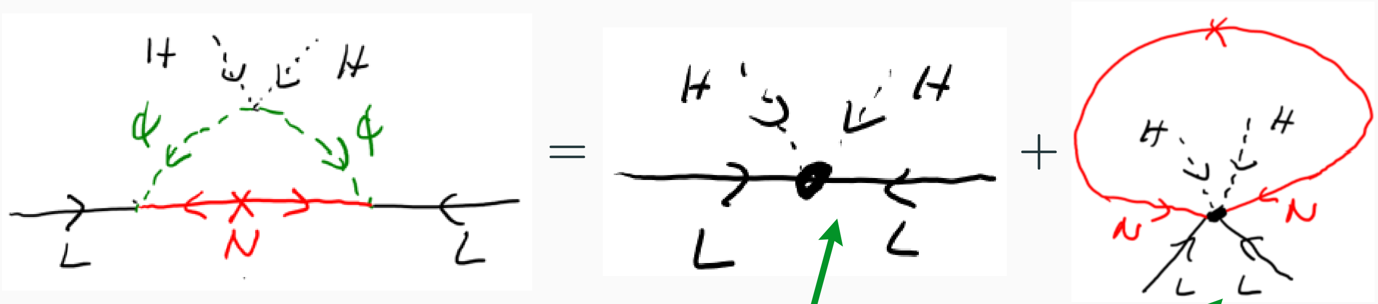


Dimension-8 operator $LNHLNH$ generated at tree level

tree level



loop level



Neutrino mass

$$m^\nu_{ij} \simeq \frac{\lambda_5 \langle H\rangle^2}{16\pi^2}\frac{h_{ik}M_k h_{jk}}{m_\varphi^2}\left[1 + \frac{M_k^2}{m_\varphi^2}\left(1 + \ln\left(\frac{M_k^2}{m_\varphi^2}\right)\right)\right]$$

EFT power counting identifies relevant contributions

for $\lambda_5\langle H\rangle^2 \ll M_k^2 \ll m_\varphi^2$

Take-home messages

flavour models

- flavour (and CP) symmetries are predictive
- JUNO is already probing solar mixing sum rules and **excluded flavour models**
- Measurement of the Dirac CP phase and more precise determination of θ_{23} at long baseline neutrino oscillation experiments will be important to test flavour symmetries
- mass measurements probe mass sum rules

precise predictions require effective field theory treatment

- quantum corrections important for comparison of theory predictions to upcoming precision data
- **EFT power counting** identifies relevant contributions

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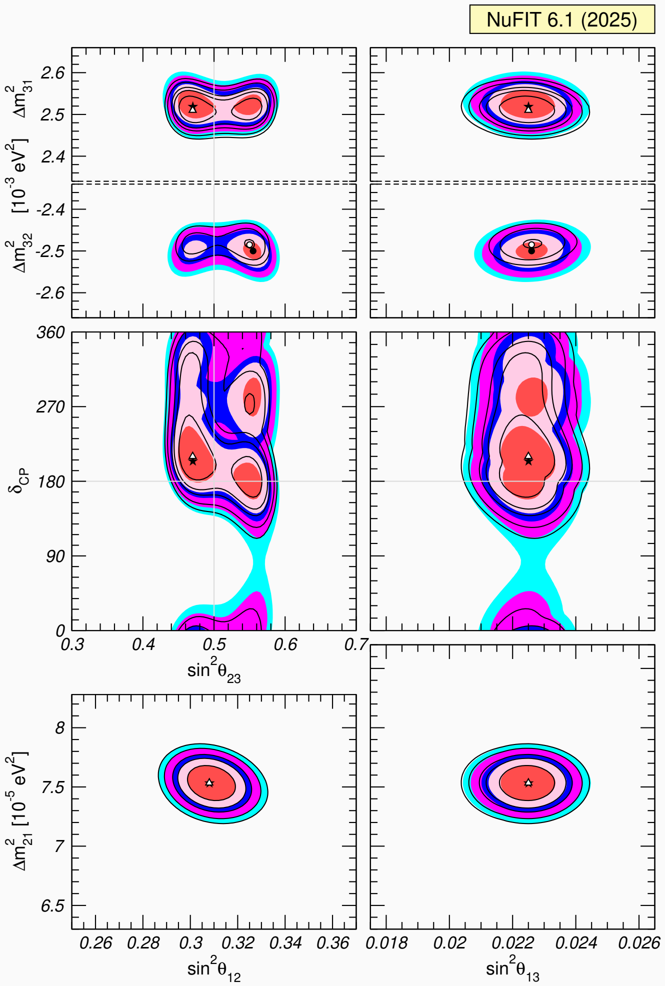
Thank you!

Backup slides

Current status: global fit to neutrino oscillation data



Leptonic (PMNS) mixing matrix

$$U_{\mathrm{PMNS}} = R_{23}(\theta_{23})\, U_{13}(\theta_{13},\delta)\, R_{12}(\theta_{12})$$

with $U_{13}(\theta_{13},\delta) = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$ $R_{12}(\theta_{12}) = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\sin^2\theta_{12}$	$0.2893 - 0.3295$	$\approx \frac{1}{3}$	0.3036 ± 0.0064
$\sin^2\theta_{23}$	$0.432 - 0.590$	$\approx \frac{1}{2}$	[Yi-Fang Wang's talk]
$\sin^2\theta_{13}$	$0.02070 - 0.02420$	≈ 0	
$\delta[^\circ]$	$114 - 405$		

Symmetries work well for gauge interactions.

Is there a symmetry in the lepton sector?

Flavour symmetries - wish list

- Continuous vs. **discrete** because there are more group invariants
- Abelian vs. **non-Abelian** in order to get correlations between parameters
- Subgroup of largest flavour group in the SM, $G_f < U(3)^5$

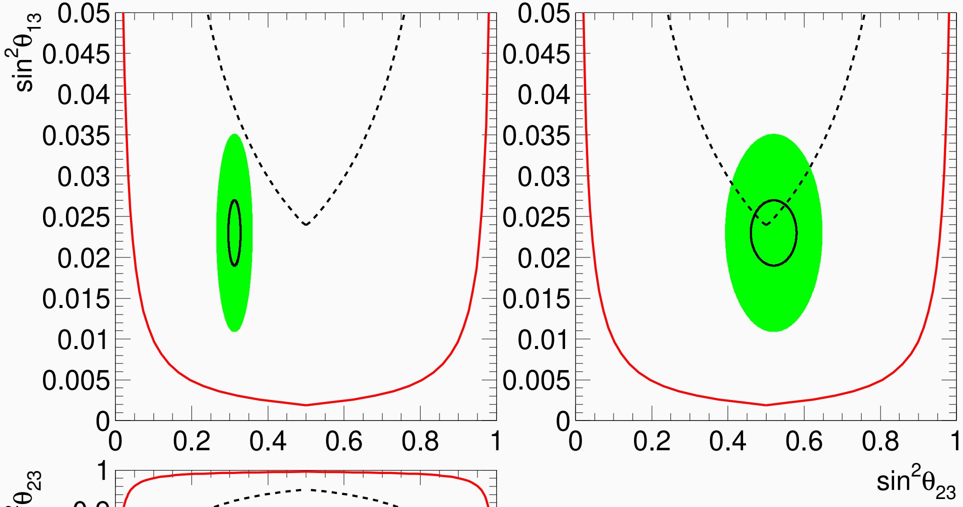
  $\rightarrow$ usually a discrete subgroup of $U(3)$ (or $SU(3)$)
- with **3-dimensional** representation to explain 3 generations
- Flavour symmetry G_f must be **broken at low energies**, either explicitly or spontaneously

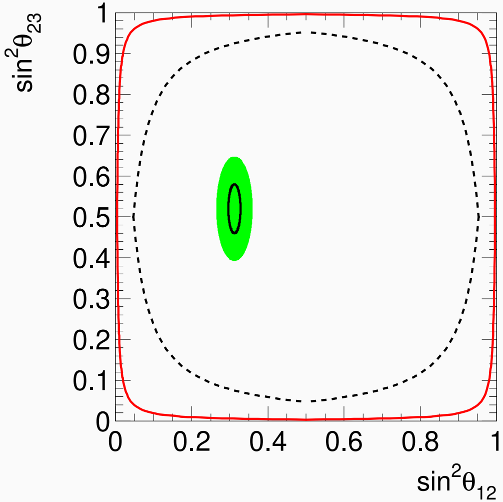
- All mixing parameters are random variables

  Hall, Murayama, Weiner [hep-ph/9911341] Haba, Murayama [hep-ph/0009174] de Gouvea, Murayama [hep-ph/0301050] [1204.1249]

- Probability dist'n invariant under basis change. Distributions are flat in s_{12}^2, s_{23}^2, c_{13}^4 due to Haar measure and mutually independent

- Kolmogorov-Smirnov test shows that predictions are consistent with data at 2σ





large CP phase δ distribution peaked at

$$\sin\delta \simeq \pm 1 \rightarrow \delta \simeq \pm\frac{\pi}{2}$$

[1204.1249]

Symmetry vs Anarchy

- All mixing parameters are random variables

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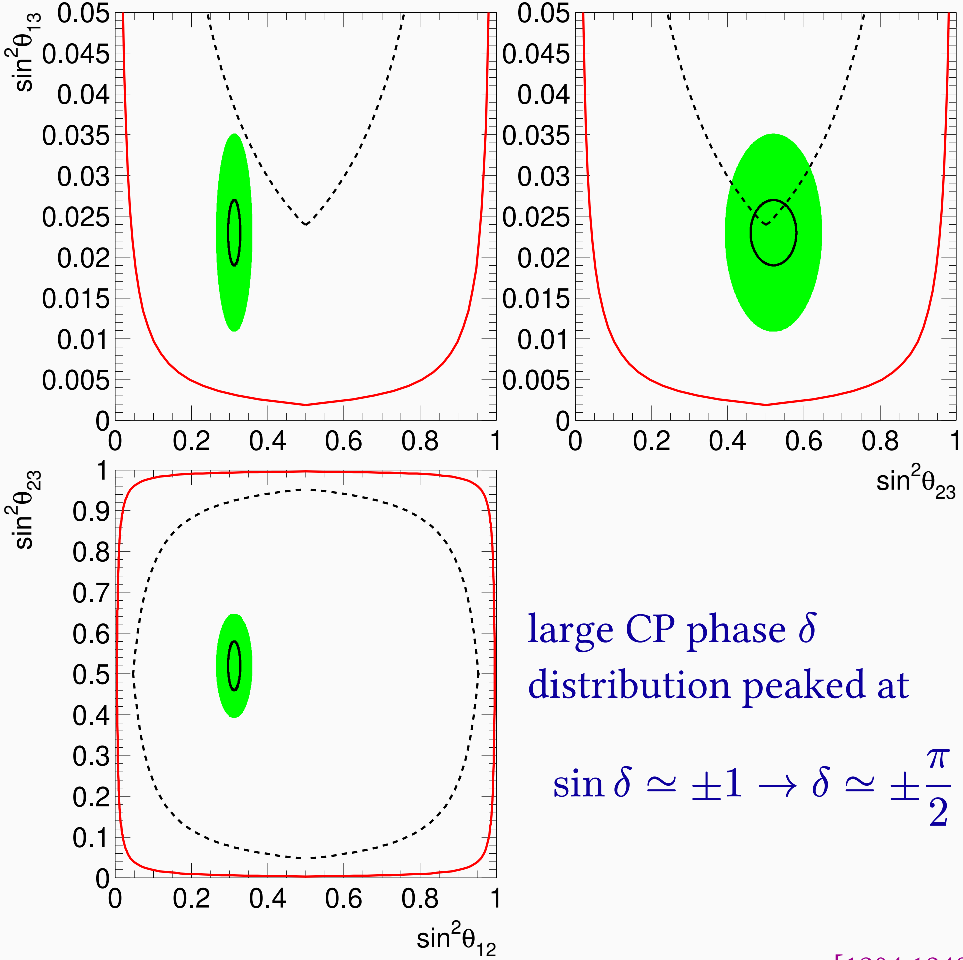
- Probability dist'n invariant under basis change. Distributions are flat in s^2_{12}, s^2_{23}, c^4_{13} due to Haar measure and mutually independent

- Kolmogorov-Smirnov test shows that predictions are consistent with data at 2σ

- Combination of (small) symmetry with anarchy

  Antonelli, Caravaglios, Ferrary, Picariello [hep-ph/0207347] Altarelli, Feruglio, Masina [hep-ph/0210342]



large CP phase δ distribution peaked at

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[1204.1249]

Popular mixing patterns

Tribimaximal mixing

Harrison,Perkins,Scott [hep-ph/0202074]

$s^2_{13} = 0$, $s^2_{12} = \frac{1}{3}$, $s^2_{23} = \frac{1}{2}$

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Prediction of $A_4, S_4, \ldots$

trimaximal mixing:

TM1: only first column

TM2: only second column

Golden ratio mixing

Datta, Ling, Ramond [hep-ph/0306002]

$s^2_{13} = 0$, $s^2_{23} = \frac{1}{2}$

$$\begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ -\frac{1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

with $\varphi = \frac{1+\sqrt{5}}{2}$

Prediction of $A_5, \ldots$

GR1: $t_{12} = \frac{1}{\varphi}$ Datta+ [hep-ph/0306002]

GR2: $\theta_{12} = \frac{\pi}{5}$ Rodejohann [0810.5239]

GR3: $c_{12} = \frac{\varphi}{\sqrt{3}}$ Lam [1104.0055]

Bimaximal mixing

Fukugita,Tanimoto,Yanagida [hep-ph/9709388]

$s^2_{13} = 0$, $s^2_{12} = s^2_{23} = \frac{1}{2}$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Prediction of $S_4, \ldots$

among others, e.g.:

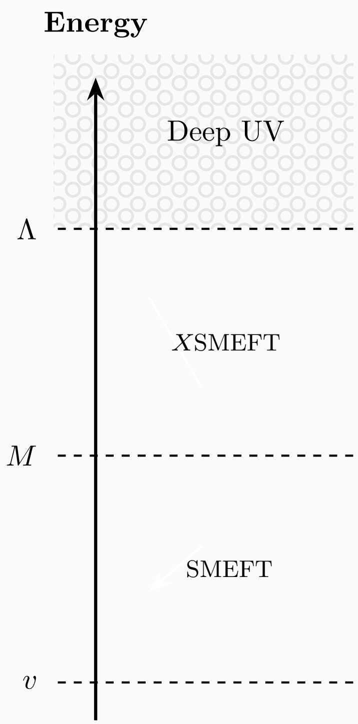
HEXagonal $\theta_{12} = \frac{\pi}{6}$

Albright,Dueck, Rodejohann [1004.2798]

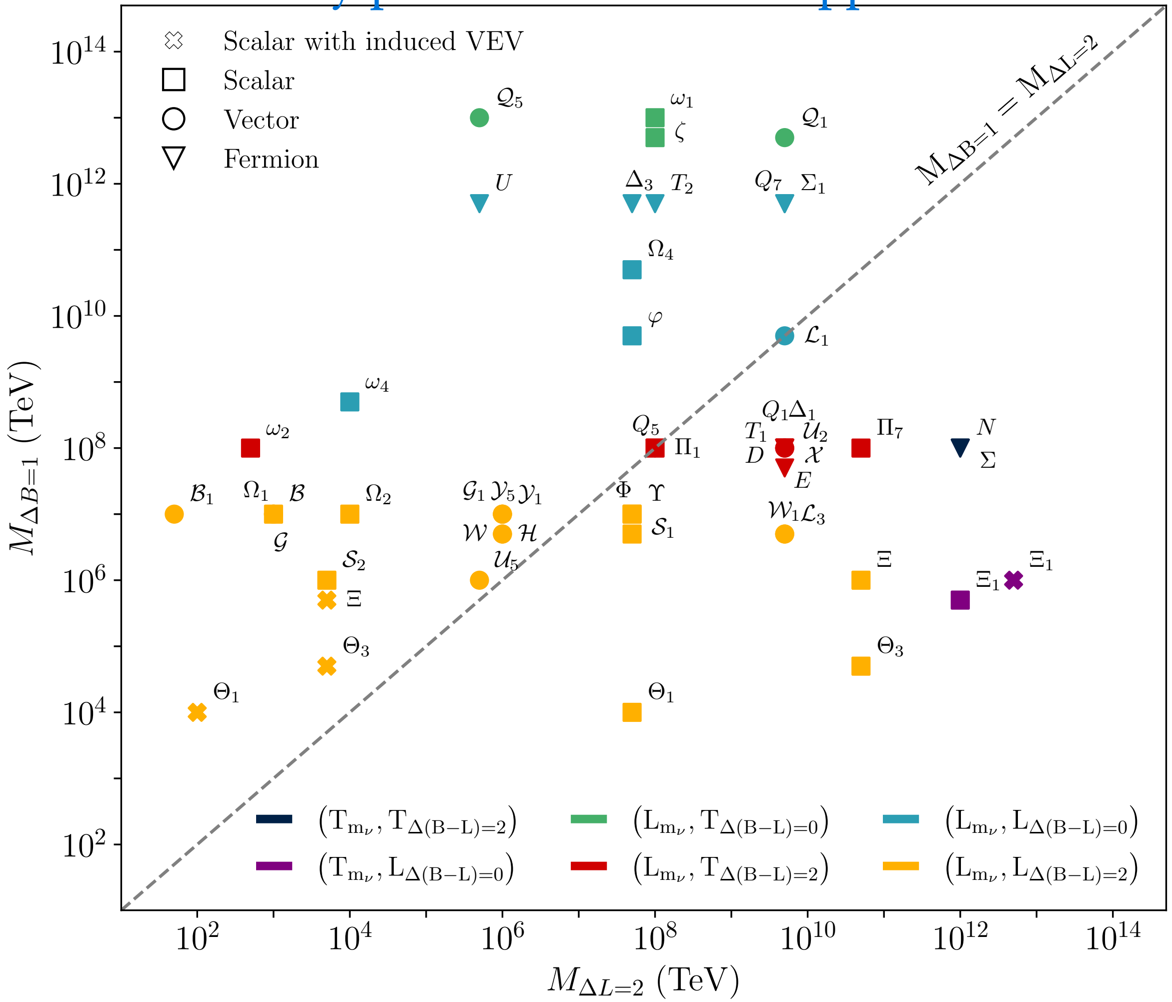
Prediction of $D_{12}, \ldots$

All three patterns ruled out, but can be good starting points.

- SM extended by one particle
- with mass gap to heavier ones



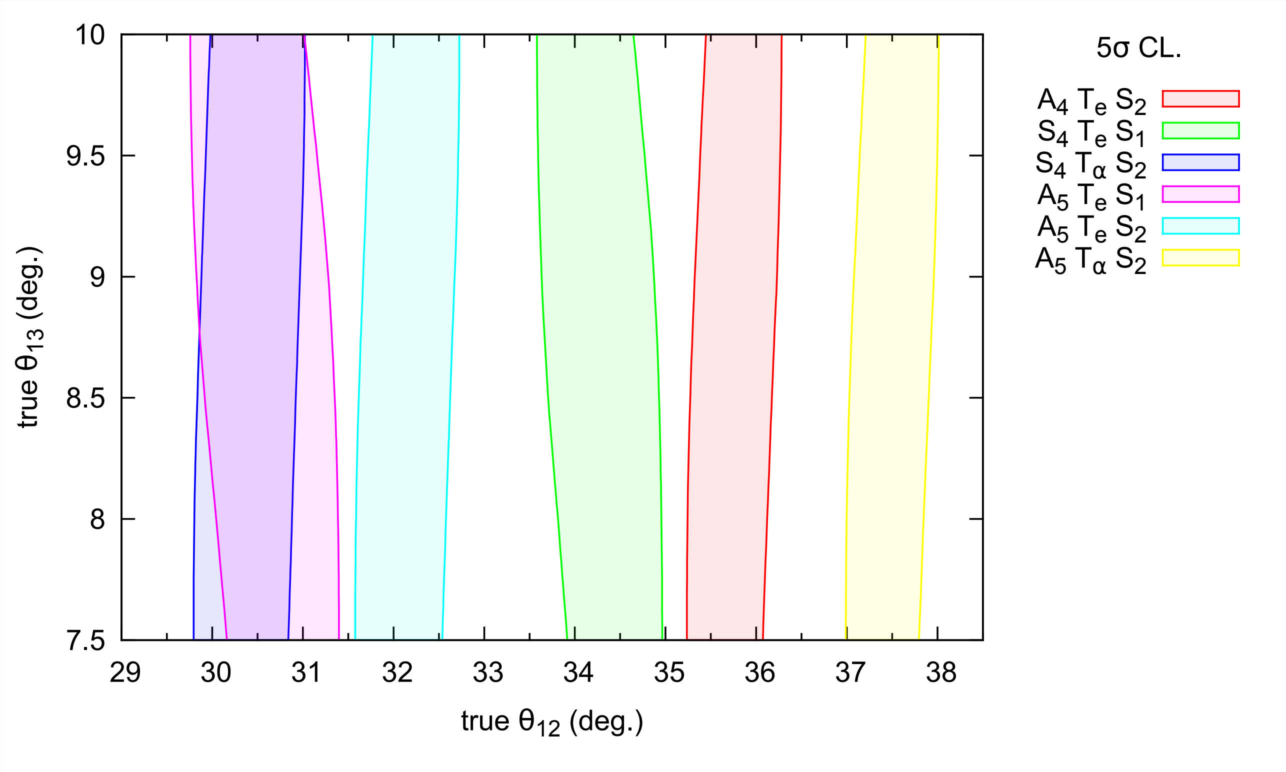
Sensitivity plot - conservative upper bounds on M



Solar mixing sum rule

Ballett, King, Luhn, Pascoli, MS [1406.0308]

5σ allowed regions after 6 years of data taking by JUNO



flavour group together with breaking pattern

$A_4\,T_\alpha - S_2$	$s = \sqrt{\frac{2}{2-r^2}} - 1$
$S_4\,T_e - S_1$	$s = \sqrt{1 - \frac{2r^2}{2-r^2}} - 1$
$S_4\,T_\alpha - S_2$	$s = \sqrt{\frac{3}{2-2r^2}} - 1$
$A_5\,T_e - S_1$	$s = \sqrt{3 + \frac{6}{(3-\varphi)(r^2-2)}} - 1$
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$$s = \sqrt{3}s_{12} - 1, \quad r = \sqrt{2}s_{13}$$

Theory predictions clearly separated

For a similar discussion of the atmospheric sum rule, see Ballett,King,Luhn,Pascoli [1308.4314]

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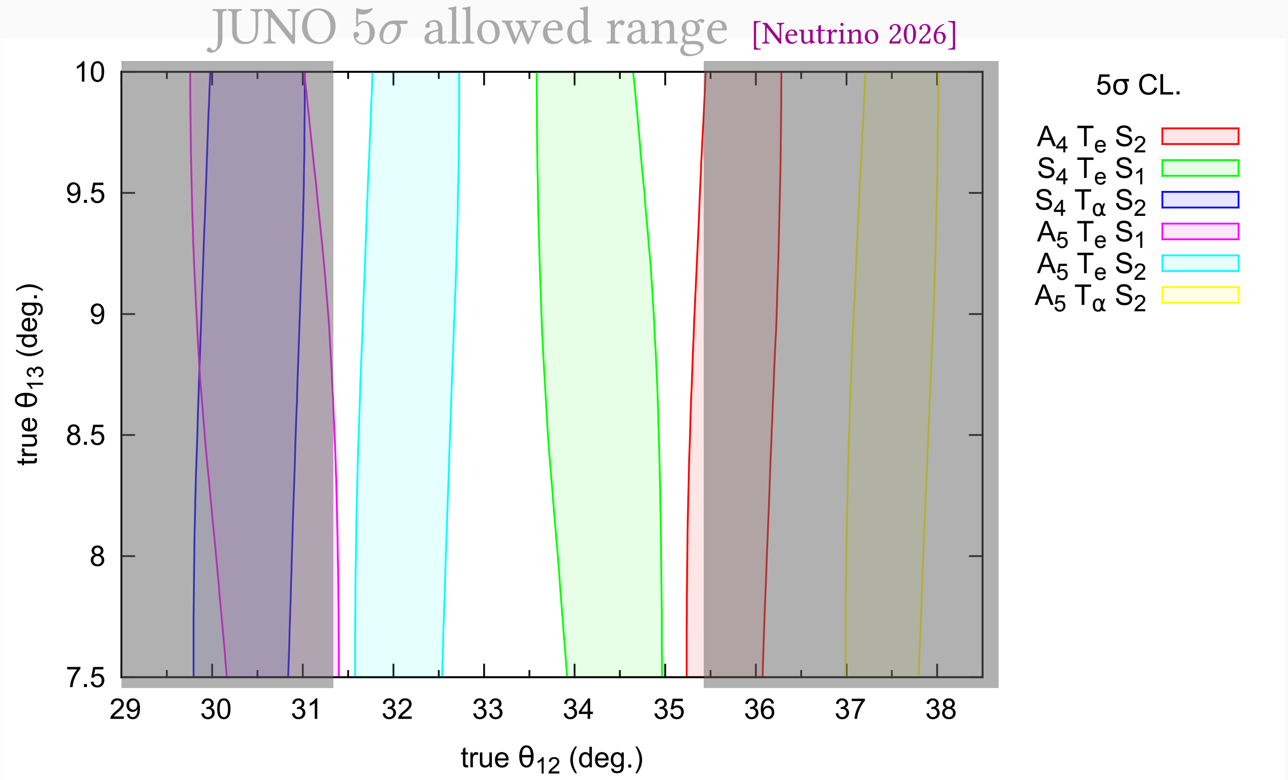

→ JUNO already excluded several flavour models

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