The background of the slide is a photograph of a modern university building. The building has a light-colored facade with large glass windows and doors. Two main entrances are visible, each with a dark awning. The entrance on the left is labeled 'STUDENT CENTER' in gold letters above the awning. The entrance on the right is labeled 'CONFERENCE CENTER' in gold letters above its awning. There are some trees and bushes in the foreground, and a paved area with a red curb is visible at the bottom. The sky is clear and blue.

Precise and Accurate $\beta\beta$ Matrix Elements

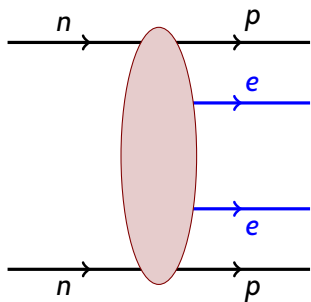
Sponsored by

@NDB: An NSF Focused Research Hub in Theoretical Physics

J. Engel

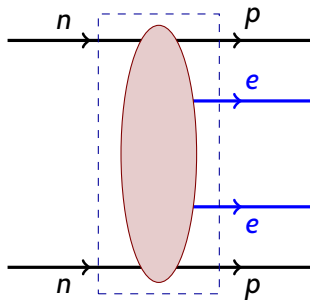
June 24, 2026

$0\nu\beta\beta$ Decay



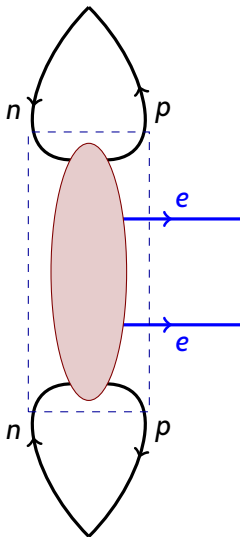
If it's observed ...

$0\nu\beta\beta$ Decay



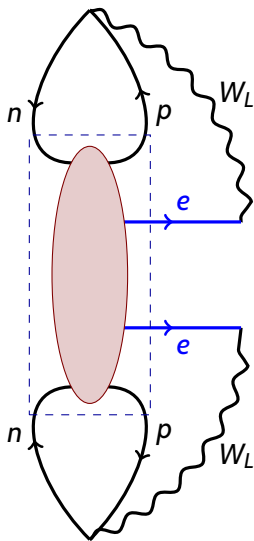
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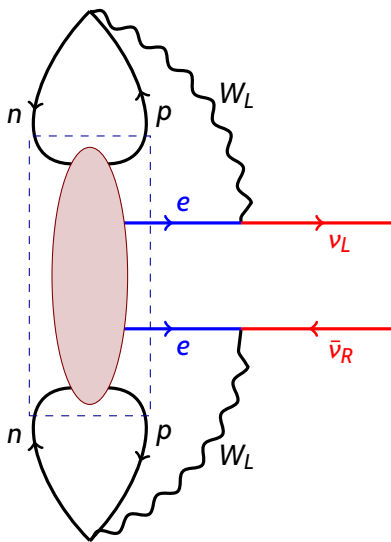
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$0\nu\beta\beta$ Decay



If it's observed ...

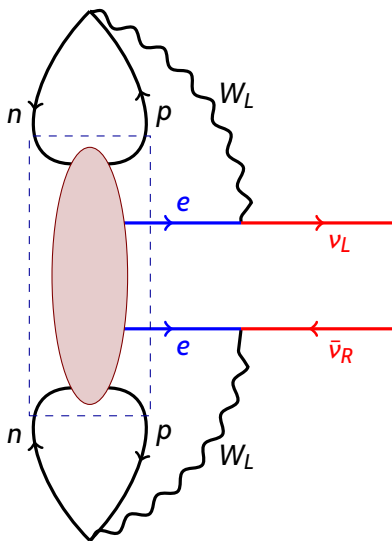
$0\nu\beta\beta$ Decay



If it's observed ...

Majorana neutrino propagator

$0\nu\beta\beta$ Decay

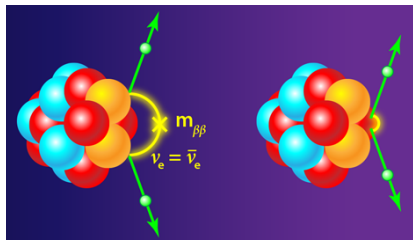


Majorana neutrino propagator

If it's observed ...

...neutrinos are Majorana particles (and lepton number is violated), no matter what physics is responsible.

The Problem

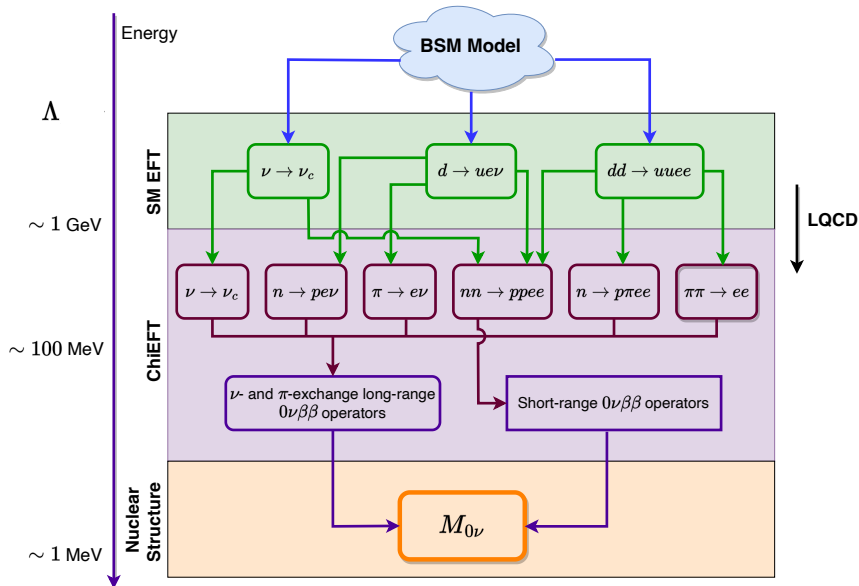


Rate depends on squares of unknown nuclear matrix elements. We need to compute them and **assign a believable (and not too large) uncertainty** so that experimentalists can better

1. **plan their experiments**
2. **draw conclusions from their results**

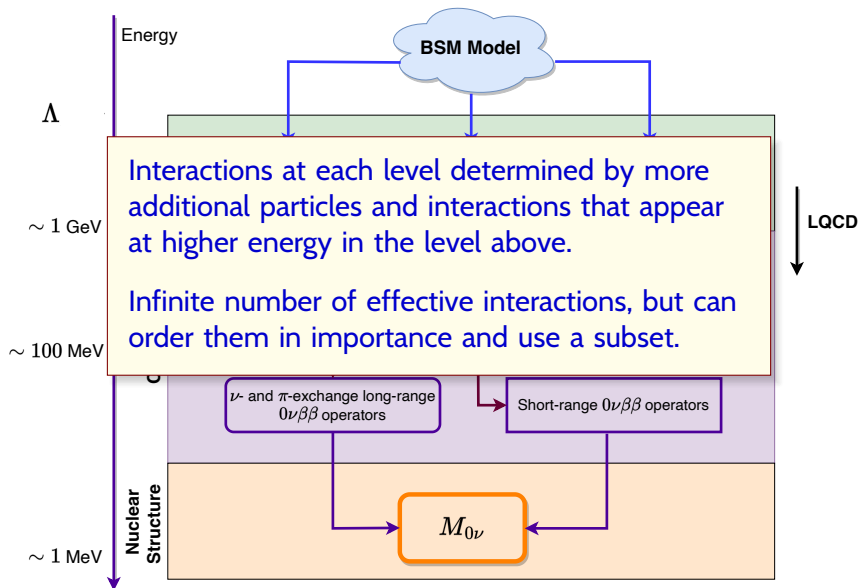
A Tower of EFTs

How we understand all the physics producing $\beta\beta$ decay



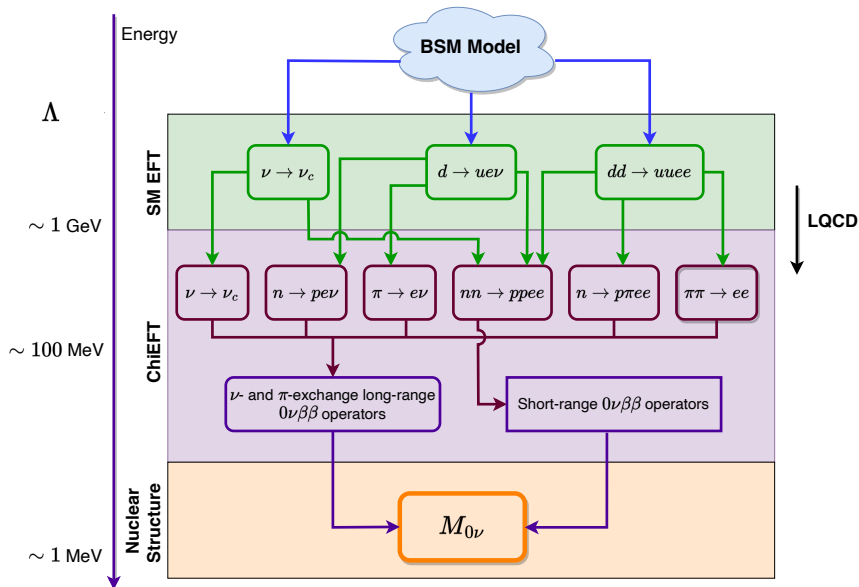
A Tower of EFTs

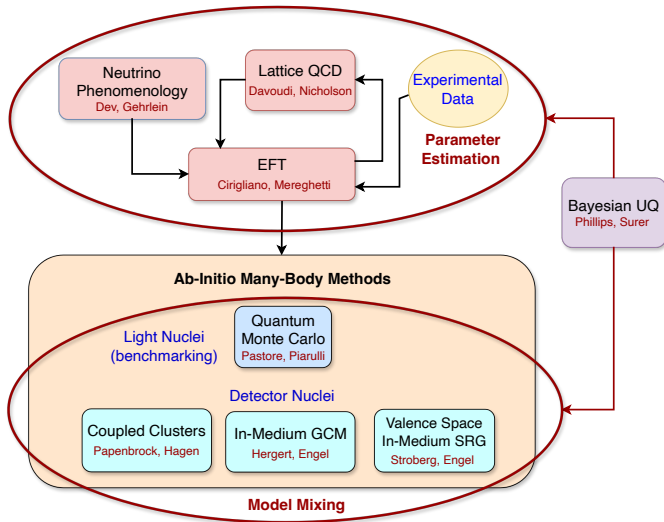
How we understand all the physics producing $\beta\beta$ decay



A Tower of EFTs

How we understand all the physics producing $\beta\beta$ decay





SM EFT: High Mass Scale Λ for New Physics

For Λ at GUT scale, lepton-number violation occurs only through dimension-5 “Weinberg operator,” leading to ν masses of

$$m \approx w \frac{v^2}{\Lambda} \approx 6 \times 10^{-3} w \text{ eV}.$$

v is the Higgs-field expectation value and w a Yukawa coupling

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and light- ν exchange is the whole story.

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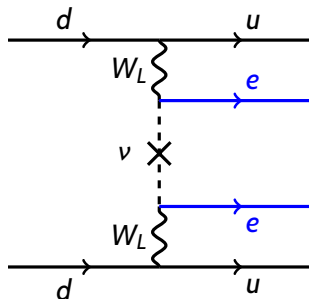


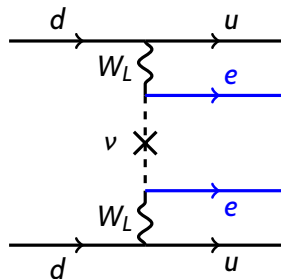
Diagram is proportional to weighted “Majorana mass” of light neutrinos,

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

U_{ei} is amount of mixing of electron flavor with i^{th} mass eigenstate.

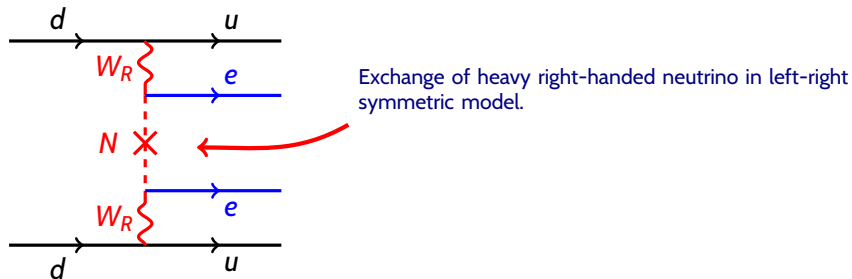
SM EFT: New Physics at Lower Scales

If $\Lambda = \mathcal{O}(1 - 100)$ TeV then presence of other particles at the weak scale means that higher-dimensional operators can be important.



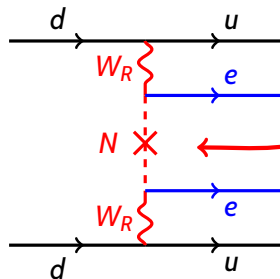
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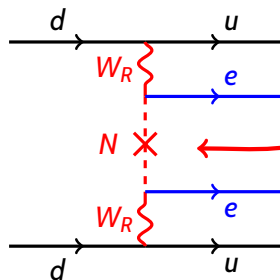
Exchange of heavy right-handed neutrino in left-right symmetric model.

Corresponds to dimension-9 operator below new-physics scale

This process can occur at the same rate as light- ν exchange (or even a larger rate) if $m_N \approx m_{W_R} \approx 1$ TeV.

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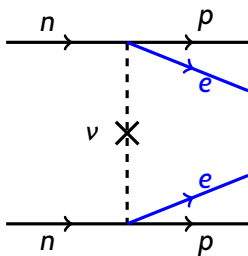
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Light sterile ν 's can also be exchanged with comparable rates.

Moving to nucleons: Chiral EFT

Light- ν Exchange at Leading Order

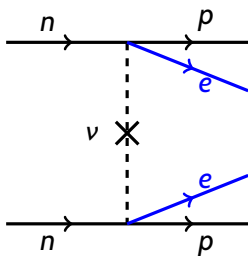


Usual long-range exchange

Older models typically include only this.

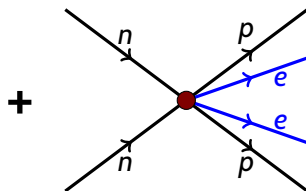
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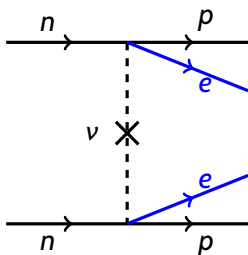
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Contact counter term from short short-range exchange at energies beyond breakdown scale of EFT.

Moving to nucleons: Chiral EFT

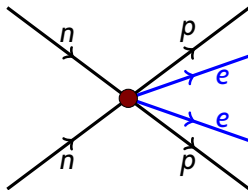
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+



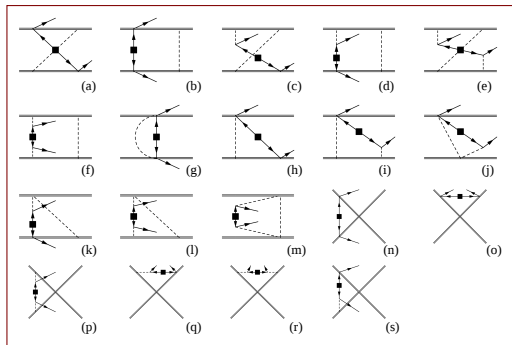
Contact counter term from short short-range exchange at energies beyond breakdown scale of EFT.

At present, coefficient of contact term estimated from large- N QCD or via sum rules.

Values of coefficients can be improved with lattice QCD.

Chiral EFT at Higher Order

$\approx 10\%$ effects



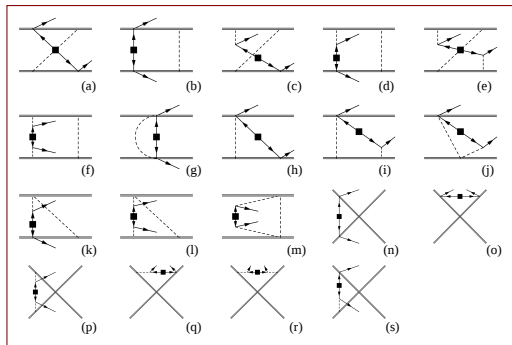
Some $N^2\text{LO}$ diagrams for $O_{\nu\beta\beta}$ operator in chiral EFT

Based on Cirigliano, Dekens, Mereghetti, and Walker-Loud, PRC 97 065501 (2018)

Some of these require contact counterterms with still unknown coefficients.

Chiral EFT at Higher Order

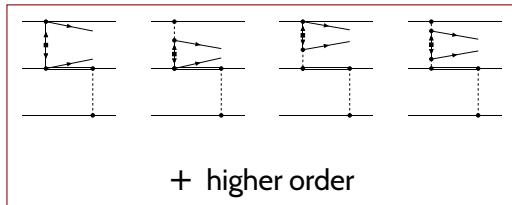
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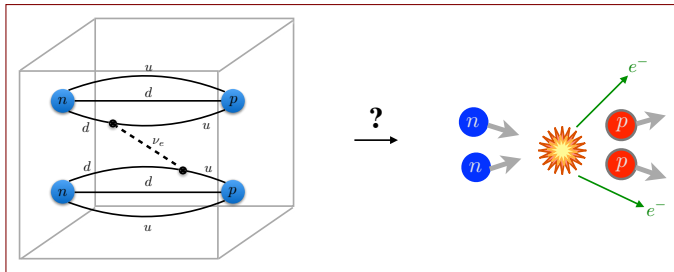


Three-nucleon diagrams for $O_{\nu\beta\beta}$ operator in chiral EFT with Δ 's

From G. Chambers-Wall, based on Chambers-Wall, Lieffers, King, Mereghetti, Pastore, Piarulli, and Wiringa, Physical Review C 113, 025502 (2026).

Lattice QCD

Deriving chiral-EFT coefficients

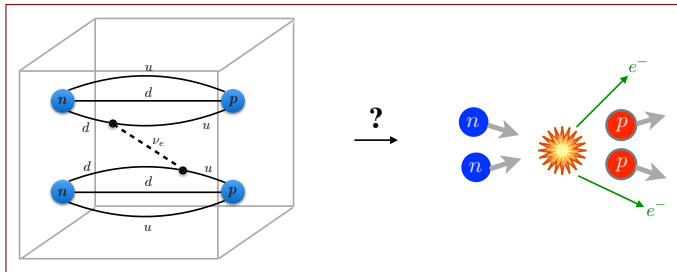


Chiral-EFT $O_{\nu\beta\beta}$
operator from
LQCD.

From Z. Davoudi

Lattice QCD

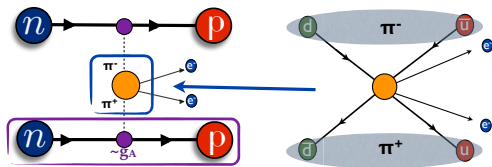
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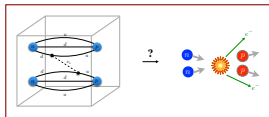
Treated already: Heavy-particle exchange between pions



From A. Nicholson

Lattice QCD

Deriving chiral-EFT coefficients

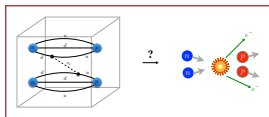


General case still requires a lot of work:

- ▶ LQCD calculations of anything involving two nucleons still limited to large pion mass

Lattice QCD

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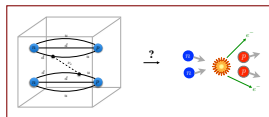


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 - ▶ General procedure for any EFT or just chiral EFT?
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Lattice QCD

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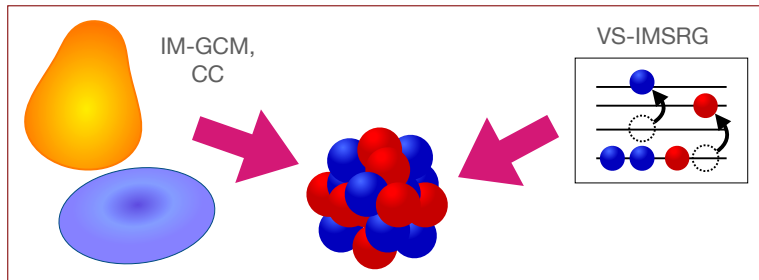


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@NDB is working hard on these issues.

Tower's Base: *Ab Initio* Nuclear-Structure Theory



Ab initio nuclear many-body methods — coupled clusters and two variants of IMSRG — schematically

Why *ab initio*? Because you can control uncertainty ...

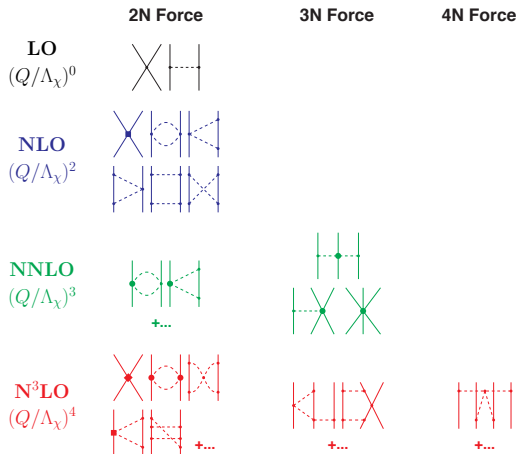
...at least in principle.

Interactions

From chiral EFT

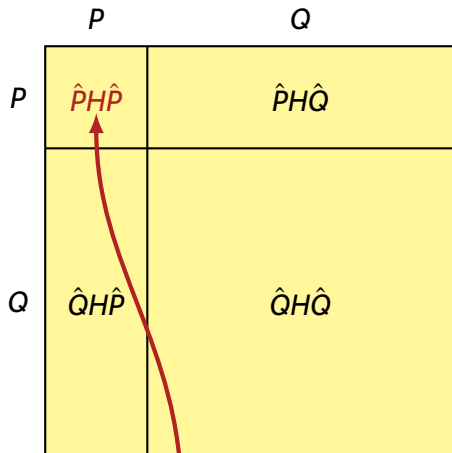
Expand interaction in powers of Q/Λ_χ .

$Q = m_\pi$ or typical nucleon momentum.



Ab Initio Nuclear-Structure for Heavy Nuclei

Partition of Full Hilbert Space



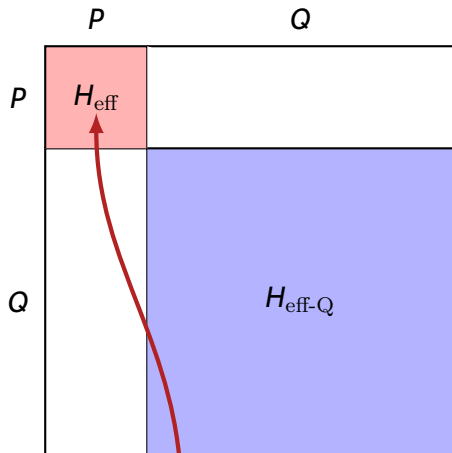
P = subspace you want
 Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q , with H_{eff} in P reproducing most important eigenvalues.

Simpler calculation done here.

Ab Initio Nuclear-Structure for Heavy Nuclei

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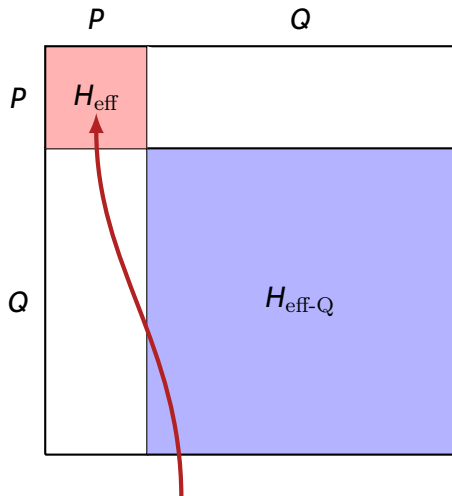
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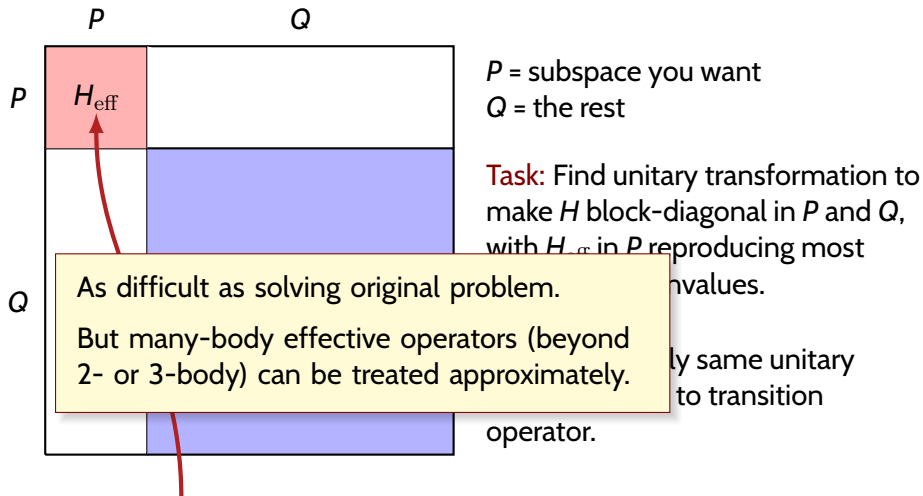
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Must must apply same unitary transformation to transition operator.

Simpler calculation done here.

Ab Initio Nuclear-Structure for Heavy Nuclei

Partition of Full Hilbert Space

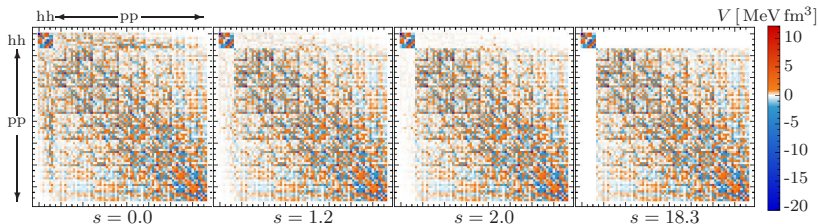


Simpler calculation done here.

In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian.
Gradually decouples selected set of states.



from H. Hergert

Trick is to keep all 1- and 2-body terms in effective Hamiltonian at each step (**IMSRG-2**, also includes “coherent” 3, 4-body ... terms).

If selected set is one state, end up with ground-state energy. If it's a valence space, get effective shell-model interaction and operators.

Coupled-Cluster Theory

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T |\varphi_0\rangle \quad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \dots$$

m,n > F *i,j < F*

Slater determinant

Here the Hamiltonian is transformed in a non-unitary way:

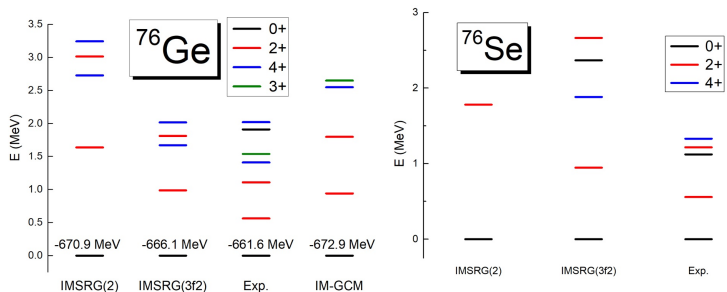
$$H \longrightarrow \tilde{H} \equiv e^{-T} H e^T$$

so that $|\varphi_0\rangle$ is its ground state. Must solve algebraic equations for the t 's.

Excited states, states in closed-shell + a few nucleons, constructed from simple excitations of $|\varphi_0\rangle$.

IMSRG in Ge-Se System

Spectrum of ^{76}Ge and ^{76}Se

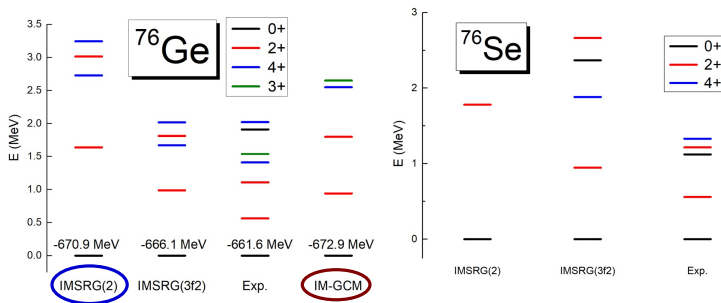


Belley et al., Phys. Rev. Lett. 132, 182502 (2024)

From talk by B. He

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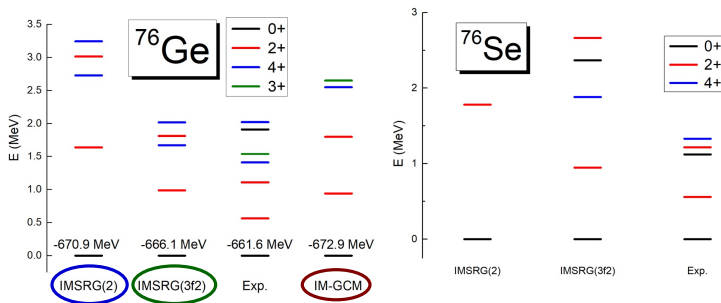
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Two variants of IMSRG: valence-space version and “In-Medium Generator Coordinate Method,” with deformed reference state.

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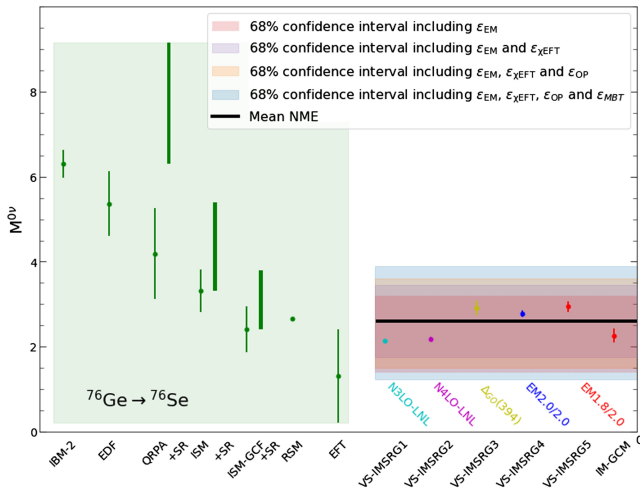
Two variants of IMSRG: valence-space version and “In-Medium Generator Coordinate Method,” with deformed reference state.

For the valence version, have approximation to IMSRG-3.

IMSRG Matrix Elements: ^{76}Ge

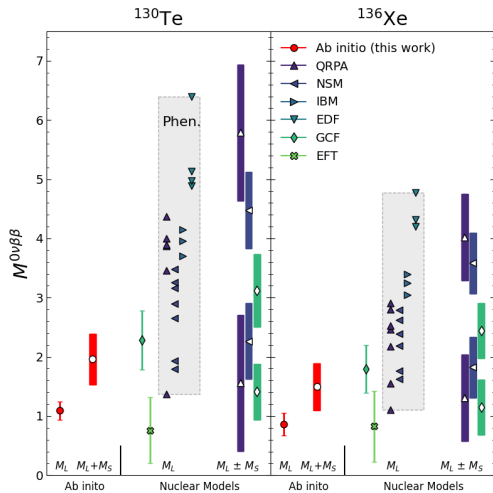
^{76}Ge with Two Versions of In-Medium Similarity Renormalization Group

A. Belley et al. PRL 132, 182502 (2024)

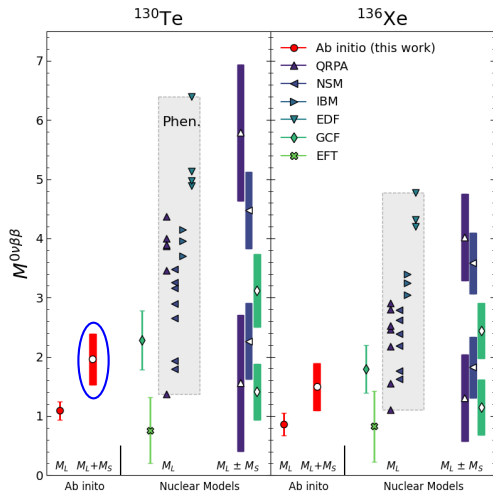


Matrix elements are on the small side (compared to phenomenological ones) but still quite uncertain.

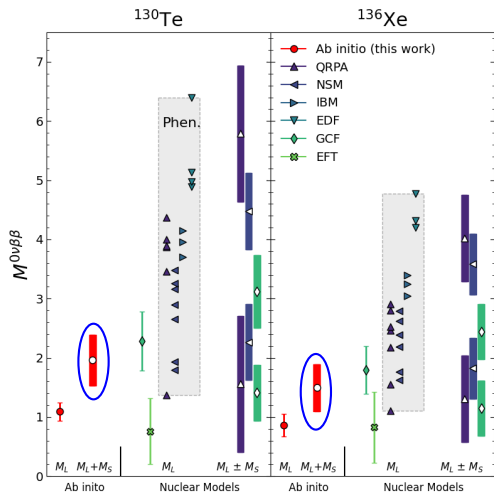
IMSRG Matrix Elements: Heavier Nuclei



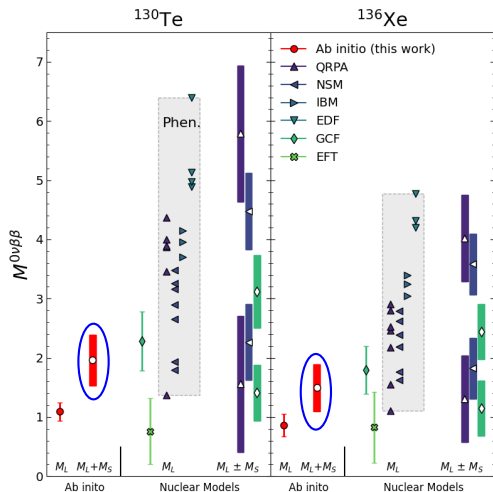
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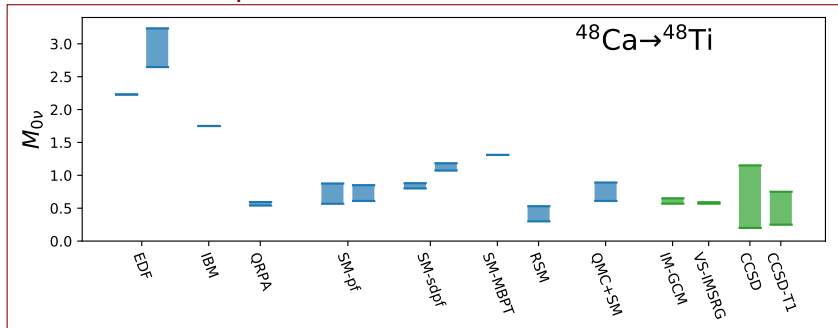


Again, results are on low end of those produced by models.

Lots of UQ still to do here.

Coupled-Clusters Matrix Element for ^{48}Ca

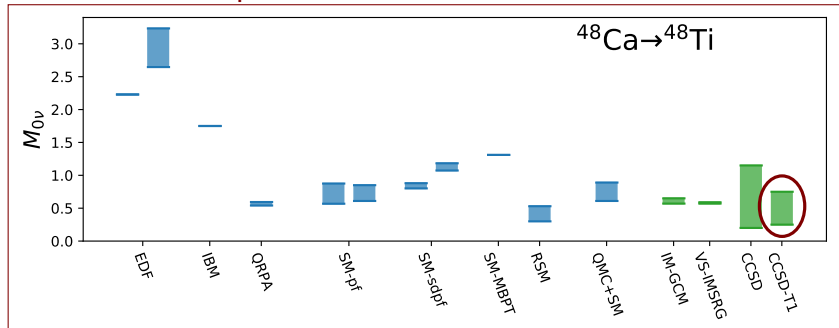
Comparison of all methods: no contact



Ab initio results are leading order in χEFT

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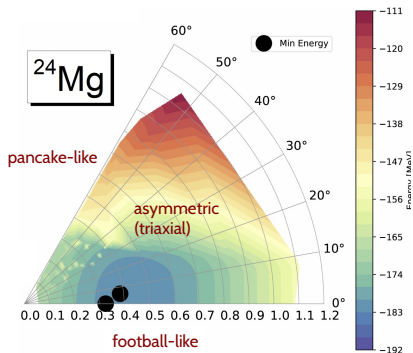


Ab initio results are leading order in χEFT

Coupled Clusters for ^{76}Ge

^{76}Ge is triaxial.

First triaxial-nucleus structure calculations have been done:

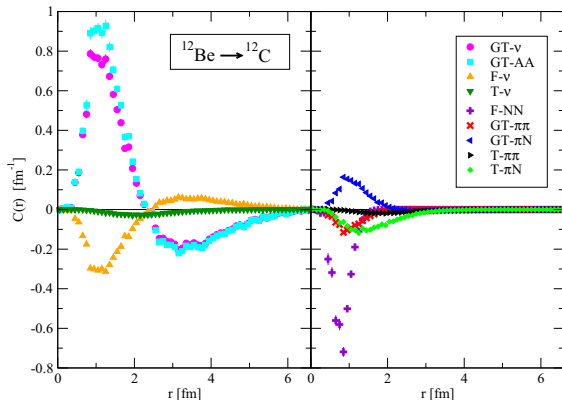


From talk last summer by B. He

Now scaling up to heavier nuclei, moving to $\beta\beta$ decay.

Benchmarking

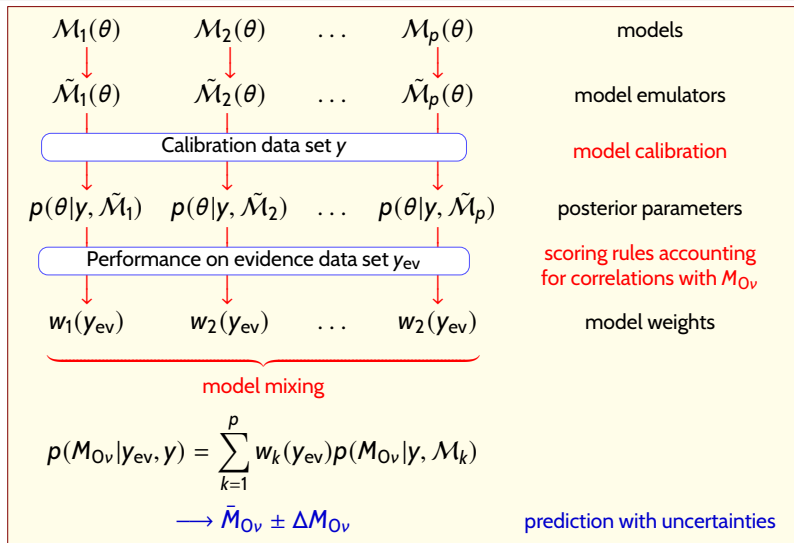
Contributions to matrix elements as function of internucleon separation



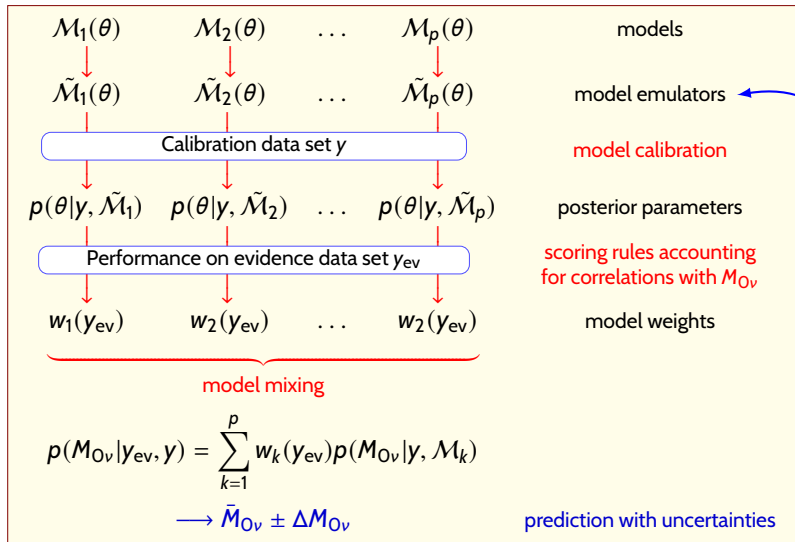
From S. Pastore et al., PRC 97, 014606 (2018).

Quantum Monte Carlo used to compute $\beta\beta$ matrix elements in light nuclei. Results will be benchmarks for coupled-clusters and IMSRG approaches.

Quantifying Uncertainty

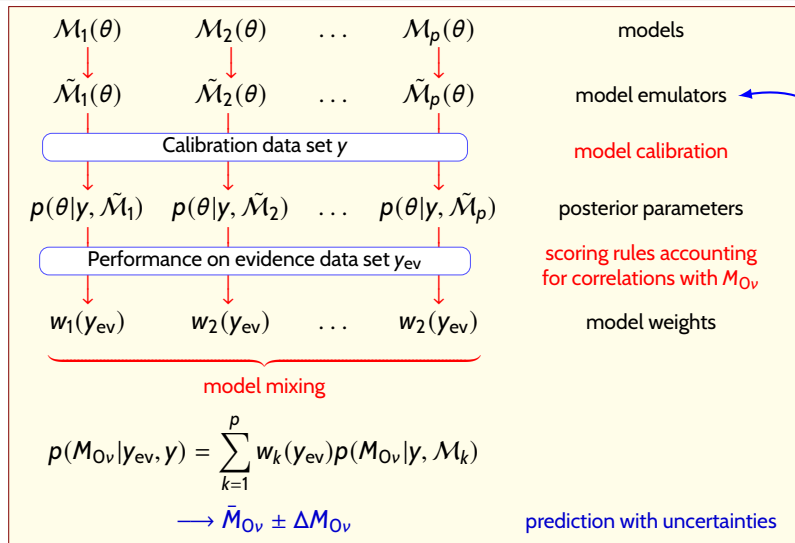


Quantifying Uncertainty



Fast approximate computations

Quantifying Uncertainty



- Repeat for all $\beta\beta$ nuclei
- Repeat with calibration data set refined for $Ov\beta\beta$.

Fast approximate computations

Quantifying Uncertainty

Important UQ work within this plan:

- ▶ **Emulators:** Used for Bayesian parameter variation. Already exist for IMSRG and coupled-clusters methods; work underway on efficient calibration.

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Design of procedure for choosing y_{ev} and scoring models a major challenge.

To Conclude ...

Lots done already to go from underlying theories of lepton-number violation to nuclear matrix elements.

Still a lot to do, but matrix elements in the important nuclei, with real uncertainty estimates, are coming!

To Conclude ...

Lots done already to go from underlying theories of lepton-number violation to nuclear matrix elements.

Still a lot to do, but matrix elements in the important nuclei, with real uncertainty estimates, are coming!

That's all.
Thanks for listening!