Scale-independent Relations between Neutrino Mass Parameters

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Introduction

- The neutrino flavor sector is still considered one of the most promising paths to discover physics beyond the Standard Model (BSM) [1].
- The origins of neutrino masses and mixing is still not fully understood in the Standard Model (SM) [1].
- Neutrino flavor model building seeks to answer these questions and gain a better understanding of the mass hierarchy and flavor mixing in the quark and lepton sectors [1].
- Models built from modular flavor symmetries have shown much promise and provide the context for where the invariants (in this discussion) were defined [2].
- Interestingly, the ingredients for these invariants were first discovered in top-down models [3].

Neutrino Mass Model

The model considered proposes that neutrino masses are described by the Weinberg operator.

Lepton masses in the SM amended by Weinberg operator [4]:

$$\mathscr{L}_{\text{lepton mass}} = -Y_e^{gf}\,\overline{\ell_{L,g}}e_{R,f}\cdot\phi - \frac{1}{4}\kappa_{gf}\ell^g\cdot\phi\ell^f\cdot\phi + \text{h.c.}$$

- Lepton doublets: $\ell_{L,f}$
- Right-handed charged lepton: $e_{R\,g}$
- SM Higgs: ϕ
- Charged lepton Yukawa couplings:
  $Y_e = \operatorname{diag}(y_e, y_\mu, y_\tau)$ with $y_f > 0$ for $f \in \{e,\mu,\tau\}$
- Effective neutrino mass (Weinberg) operator: κ

Specific to the neutrino flavor sector

- Neutrino mass matrix: $m_\nu = \kappa v_{\text{EW}}^2$
- 9 flavor parameters:

$$\{\xi_i\} = \{m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \varphi_1, \varphi_2\}$$

Invariants

In the basis chosen for the Yukawa couplings, the invariants are defined to be [2]

$$I_{fg} := \frac{(m_\nu)_{ff}\,(m_\nu)_{gg}}{\left((m_\nu)_{fg}\right)^2} = \frac{\kappa_{ff}\,\kappa_{gg}}{\left(\kappa_{fg}\right)^2}$$

Significance

- The invariants can be entirely expressed in terms of the flavor parameters.
- They are also complex, which implies that unless there are degeneracies, 6 independent linear combinations out of the 9 flavor parameters are described by them.
- In the framework of modular flavor symmetries, they have important properties. For example, in the Feruglio model [3], I_{12} = -2 and $I_{13}I_{23}$ = -32.

Research Question: How sensitive are these invariants to the flavor scale, specifically, what do their quantum corrections look like at the scale of our experiments?

Renormalization Group Equations

Since the Weinberg operator is symmetric, one possible form of writing its renormalization group equation would be [4]

$$\frac{\mathrm{d}}{\mathrm{d}t}\kappa = \sum_k \kappa^{(k)} := \sum_k \left(\frac{1}{16\pi^2}\right)^k \left[\alpha^{(k)}\kappa + P^{(k)}\kappa + \kappa\left(P^{(k)}\right)^\intercal + Q^{(k)}\kappa\left(Q^{(k)}\right)^\intercal\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t} := \mu\frac{\mathrm{d}}{\mathrm{d}\mu} \qquad t = \ln(\mu/\mu_0)$$

In similar fashion, we can write the loop expansion of the invariants as [4]

$$\dot{I}_{fg} := \frac{\mathrm{d}}{\mathrm{d}t}I_{fg} = \sum_k I_{fg}^{(k)}$$

and then expressing the renormalized invariants up to two loop [5], we find the expression on the bottom right.

$$I_{fg}^{(k)} = \frac{\kappa_{ff}^{(k)}\,\kappa_{gg}}{\left(\kappa_{fg}\right)^2} + \frac{\kappa_{ff}\,\kappa_{gg}^{(k)}}{\left(\kappa_{fg}\right)^2} - 2\frac{\kappa_{ff}\,\kappa_{gg}}{\left(\kappa_{fg}\right)^3}\kappa_{fg}^{(k)} = \frac{\kappa_{ff}\,\kappa_{gg}}{(16\pi^2)^k\kappa_{fg}^2}\left(Q_{ff}^{(k)} - Q_{gg}^{(k)}\right)^2$$

$$Q_{ff}^{(2)} = \sqrt{2}\,(Y_eY_e^\dagger)_{ff} = \sqrt{2}\,y_f^2$$

$$\frac{\mathrm{d}I_{fg}}{\mathrm{d}t} = \frac{2\left(y_f^2 - y_g^2\right)^2}{(16\pi^2)^2}I_{fg}$$

Results and Benchmark Values

(To the left) The results show the following [4]:

- If the invariants vanish at some scale, it will stay zero at all scales (applies to both the real and imaginary parts).
- Corrections to I_{12} are more suppressed than I_{13} and I_{23}.
- Generally, $y_\tau \sim 10^{-2}$, so for practical purposes, the invariants are also invariant under the renormalization group or similarly insensitive to the flavor scale.

(To the right) We estimate benchmark values to the quantum corrections choosing

- $y_e = 2 \times 10^{-6}$, $y_\mu = 5 \times 10^{-4}$, $y_\tau = 7 \times 10^{-3}$ and $\mu = 10^3$ GeV – 10^6 GeV
- (for the real part) I_{12} = -35, I_{13} = -10, and I_{23} = 2 [6]

$$\frac{\mathrm{d}I_{12}}{\mathrm{d}t} = \frac{2\left(y_e^2 - y_\mu^2\right)^2}{(16\pi^2)^2}I_{12}$$

$$\frac{\mathrm{d}I_{13}}{\mathrm{d}t} = \frac{2\left(y_e^2 - y_\tau^2\right)^2}{(16\pi^2)^2}I_{13}$$

$$\frac{\mathrm{d}I_{23}}{\mathrm{d}t} = \frac{2\left(y_\mu^2 - y_\tau^2\right)^2}{(16\pi^2)^2}I_{23}$$

$$\Delta I_{fg} \approx \frac{2\left(y_f^2 - y_g^2\right)^2}{(16\pi^2)^2}I_{fg}\Delta t$$

$$\Delta I_{12} \approx -1.2 \times 10^{-15}$$
$$\Delta I_{13} \approx -1.3 \times 10^{-11}$$
$$\Delta I_{23} \approx 2.6 \times 10^{-12}$$

Conclusion

We have studied the stability of the invariants (from modular flavor symmetries) under the renormalization group symmetries. We further show that up to two-loop, the quantum corrections at the flavor scale are at most on the order 10^{-10} [4], and thus, the invariants can be considered RG invariant.

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References

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