

Motivation

Neutrinos traverse Earth essentially unimpeded, offering a relay-free through-planet channel, immune to weather, geomagnetic storms, and jamming. Prior feasibility studies disagreed on the channel capacity C , the maximum reliable data rate in bits per second:

- **Neutrino beam from pion decay (NB-PD):** $C \ll 1$ bit/s [Saenz 1977, Silagadze 2008].
- **Neutrino beam from cooled muons (NB-CM),** a neutrino factory: $C \sim 1$ to 100 bit/s [Huber 2010].

This work reconciles both under a single framework: a unified rate calculation, a generalized information capacity valid for any slot count M , and energy per bit benchmarks for five NB-PD facilities against a 150 GeV NB-CM reference.

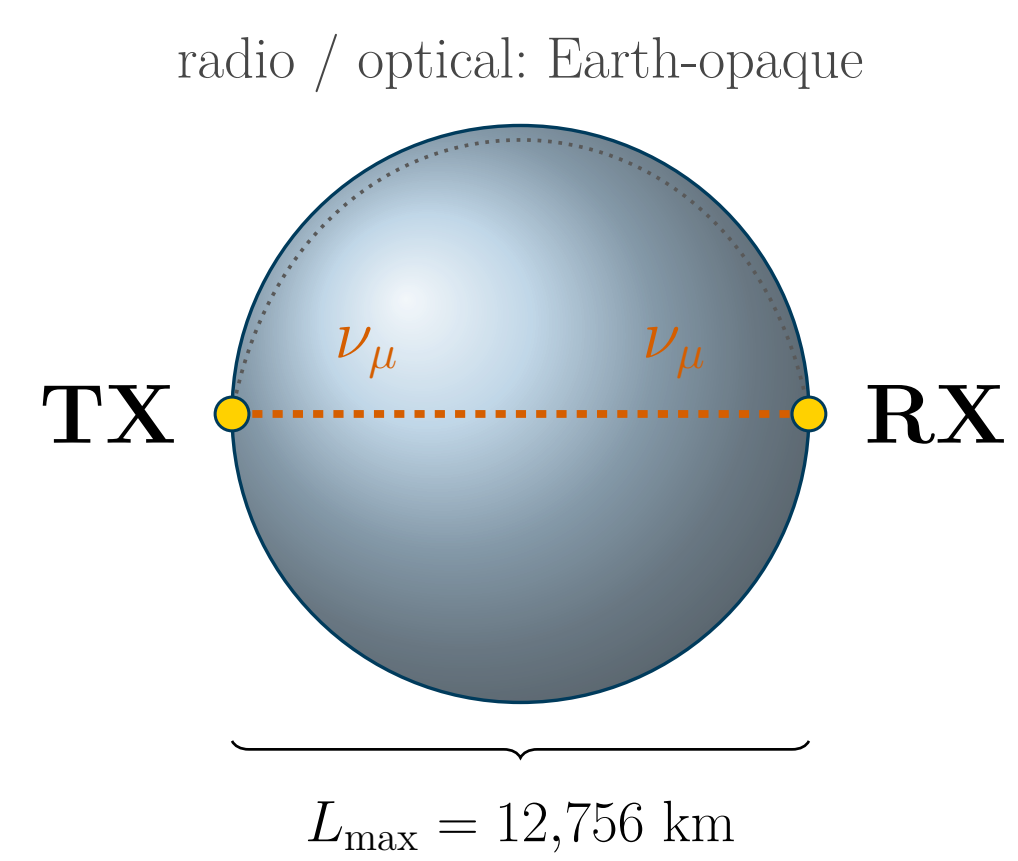


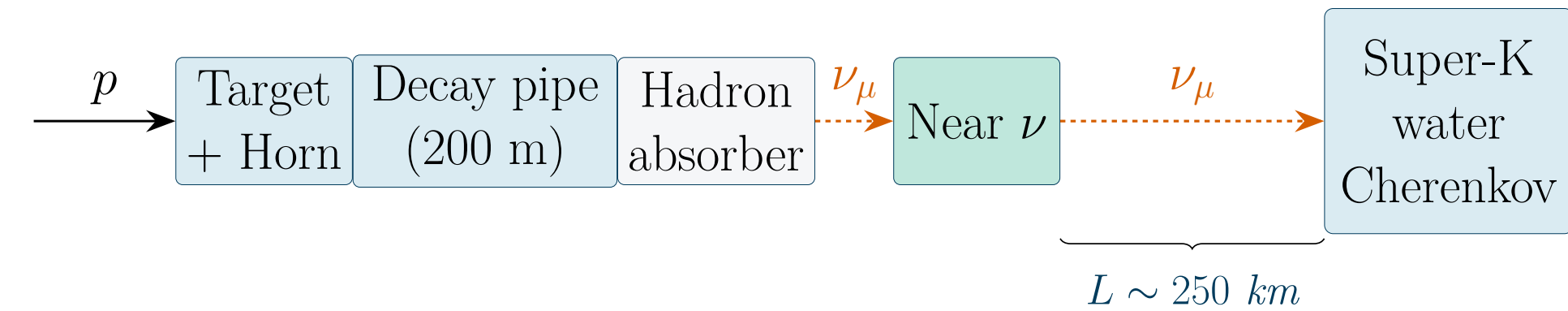
Fig. 1. Through-Earth neutrino link: ν_μ travels a nearly straight chord at light speed.

TX: transmitter, RX: receiver.

Feasibility target: $C \sim 1$ to 100 bit/s

Source Comparison: NB-PD vs NB-CM

NB-PD ($\pi^\pm \rightarrow \mu^\pm \nu_\mu$)

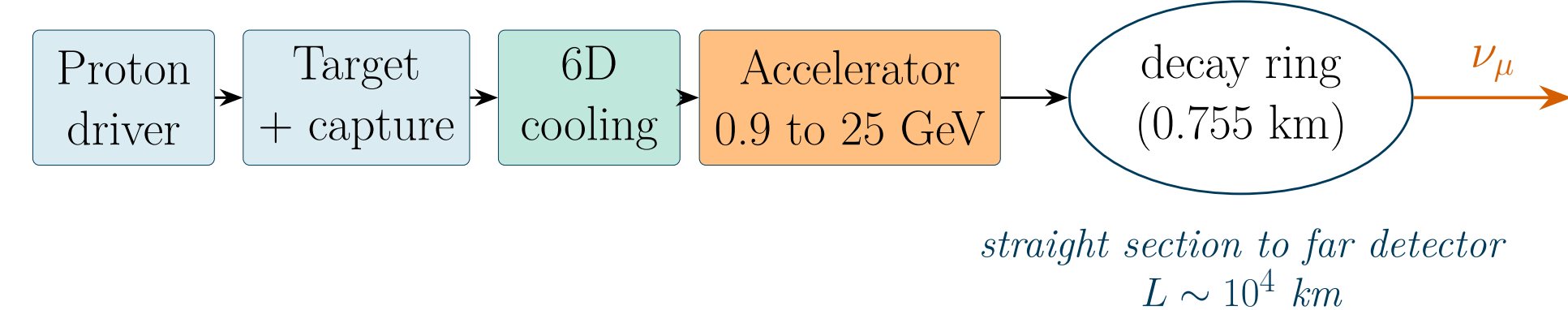


A π -monitor and downstream μ -monitor calibrate the on-axis neutrino yield. The near ν detector at ~ 300 m samples the beam before it propagates ~ 250 km through the Earth to Super-Kamiokande.

250 km baseline: K2K [Ahn 2006].

Fig. 2. Single-pass pion-decay beamline. Divergence $\theta_0 = pr/p_\mu \sim 5$ to 10 mrad.

NB-CM ($\mu^\pm \rightarrow e^\pm \nu_e \nu_\mu$)



straight section to far detector
 $L \sim 10^4$ km

IDS-NF design reaches 25 GeV. Huber's illustrative benchmark uses 150 GeV.

Neutrino factory design: IDS-NF [2011]; capacity estimate [Huber 2010].

Fig. 3. Muons are captured, 6D-cooled, and accelerated to $E_0 = 25$ to 150 GeV. Divergence

$\theta_0 = m_\mu/E_0 \sim 0.7$ mrad at 150 GeV, about 50 \times tighter than NB-PD.

Detection: CC Scattering and Cherenkov Tracking

Charged-current scattering. A neutrino from the beam interacts in the water target and produces a forward muon that carries the beam direction:

$$\nu_\mu + N \rightarrow \mu^- + X, \quad \sigma_{CC}^{\nu N} \simeq (0.67 \times 10^{-38} \text{ cm}^2) \frac{E_\nu}{\text{GeV}}.$$

Muon range in water (PDG CSDA):

$$R_\mu \approx (14.5 + 3.4 E_\mu/\text{GeV}) \text{ m}.$$

The secondary μ^- radiates Cherenkov light on a cone of half-angle θ_C around its track. The cone intersects the PMT wall as a Cherenkov ring, and reconstructing the ring fixes the muon direction that points back to the source. Directional tracking requires an effective area $A_{\text{eff}} \sim 10^3$ to 10^4 m² (KM3NeT, IceCube-Gen2).

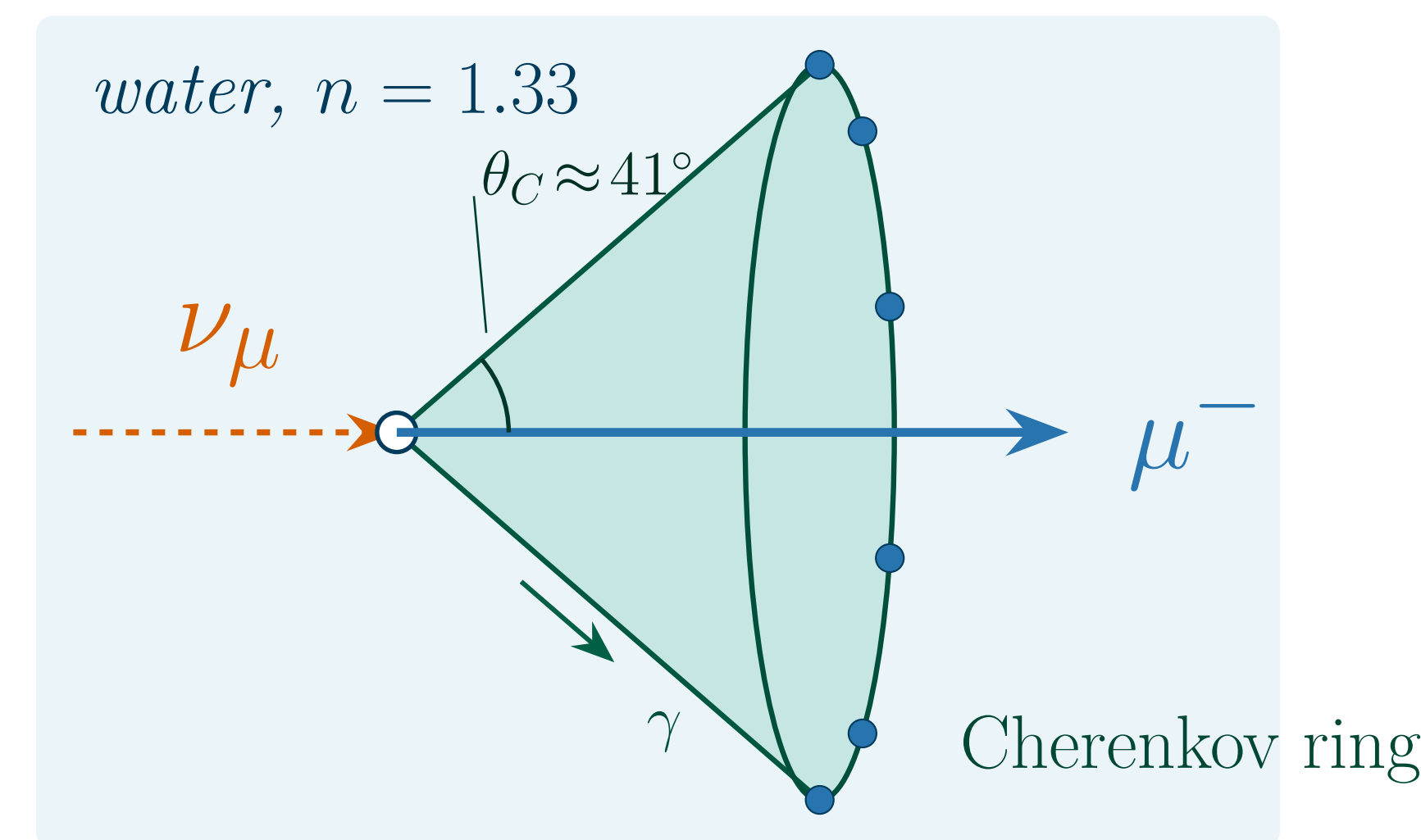


Fig. 4. Cherenkov cone of the secondary muon in water: $\cos \theta_C = 1/(n\beta_\mu)$ gives $\theta_C \approx 41^\circ$. The cone meets the PMT wall as a Cherenkov ring.

Signal Rate Scaling Law: E_ν^2 vs E_0^4

Combining $\sigma_{CC} \propto E_\nu$, $P_{\text{det}} \propto R_\mu \propto E_\mu$, and the on-axis flux, the signal rate S (detected μ /s) is the integral

$$S = \int_0^{E_0} \sigma_{CC}(E_\nu) P_{\text{det}}(E_\nu) \frac{d\phi}{dE_\nu} dE_\nu$$

which collapses to two forms. A pion-decay beam has no γ^2 boost and no straight section:

$$S_{\text{NB-PD}} \propto \frac{N_p}{L^2} E_\nu^2 f_{\text{horn}} A_{\text{eff}}$$

the cooled-muon factory adds both:

$$S_{\text{NB-CM}} \propto N_\mu \frac{E_0^4}{L^2} f_{\text{straight}} A_{\text{eff}}$$

Origin of the extra E_0^2 :

- tight collimation $\theta_0 = m_\mu/E_0 \Rightarrow \text{flux} \propto E_0^2$,
- muon range $R_\mu \propto E_\mu \propto E_\nu$, combined with $\sigma_{CC} \propto E_\nu \Rightarrow \text{detection} \propto E_\nu^2$.

This single scaling law accounts for the entire difference.

Information Theory: Pulse Position Modulation (PPM)

PPM splits each 1 s frame into M slots with P pulses, with erasure $\varepsilon = 1 - (1 - e^{-S/P})^P$. Huber's empirical fit, valid only at $M = 2^{14}$:

$$C(S) = 6.61 S^{0.74} \text{ bit/s}.$$

This work (paper Eqns 17, 18): arbitrary M ,

$$C_{\text{frame}}(S) \approx S y (1 - e^{-1/y}) \log_2(M/Sy)$$

with $y = P/S$ from the implicit optimality condition

$$(1 - e^{-1/y} - \frac{1}{y} e^{-1/y}) \ln(M/Sy) = 1 - e^{-1/y}.$$

Growth is quasi-linear in S and only logarithmic in M , so large M cannot compensate for small S . For the NB-CM benchmark $S = 2.2 \text{ s}^{-1}$: $C \approx 11.85 \text{ bit/s}$.

Master Parameter Comparison

Parameter	MINOS	K2K	T2K	NOvA	DUNE	NB-CM 150
E_0 [GeV]	10 to 15	3 to 4	1.4	4	10 to 20	150
E_ν [GeV]	3	1.3	0.6	2	1 to 5	50
θ_0 [deg]	0.6	0.6	0.6	0.6	0.6	0.04
σ_{CC} [10^{-38} cm^2]	2.0	0.9	0.4	1.3	2.0	34
R_μ in water [m]	6	3 to 4	2	5	10	100
A_{eff} [m ²]	10^3	10^3	10^3	10^3	10^4	10^3
Baseline L [km]	735	250	295	810	1300	10^4
Signal rate S [s^{-1}]	2×10^{-5}	7×10^{-7}	10^{-5}	10^{-4}	4×10^{-4}	2.2
Capacity C [bps]	5×10^{-4}	2×10^{-5}	3×10^{-4}	2×10^{-3}	6×10^{-3}	~ 10
Wall-plug [MW]	3	0.05	5	7	12	65
E_{bit} [J/bit]	6.5×10^9	2.6×10^9	2×10^{10}	4×10^9	2×10^9	6.5×10^6

Table 1. Source, detector, and information-theory parameters. E_0 is the average pion energy for NB-PD, and the stored muon energy for NB-CM.

Ocean Coverage

Fraction of Earth's ocean area over which a 150 GeV NB-CM achieves the target capacity, for surface (day), through-hull, and surface (night) operation [Huber 2010]:

C target Day Hull Night

1 bit/s 90% 97% $\sim 100\%$

10 bit/s 65% 85% $\sim 100\%$

100 bit/s 12% 30% 40%

Table 2. Daytime solar muons increase the fake-track background. Night operation recovers

near-complete coverage at 1 to 10 bit/s.

Feasibility Plot: Rate and Capacity vs Energy

Scaling relations plotted:

$$S_{\text{NB-CM}} \propto E_0^4, \quad S_{\text{NB-PD}} \propto E_0^2, \quad C \approx 6.61 S^{0.74} \text{ bit/s}.$$

(For NB-PD, $E_0^3 \propto E_\nu^2$)

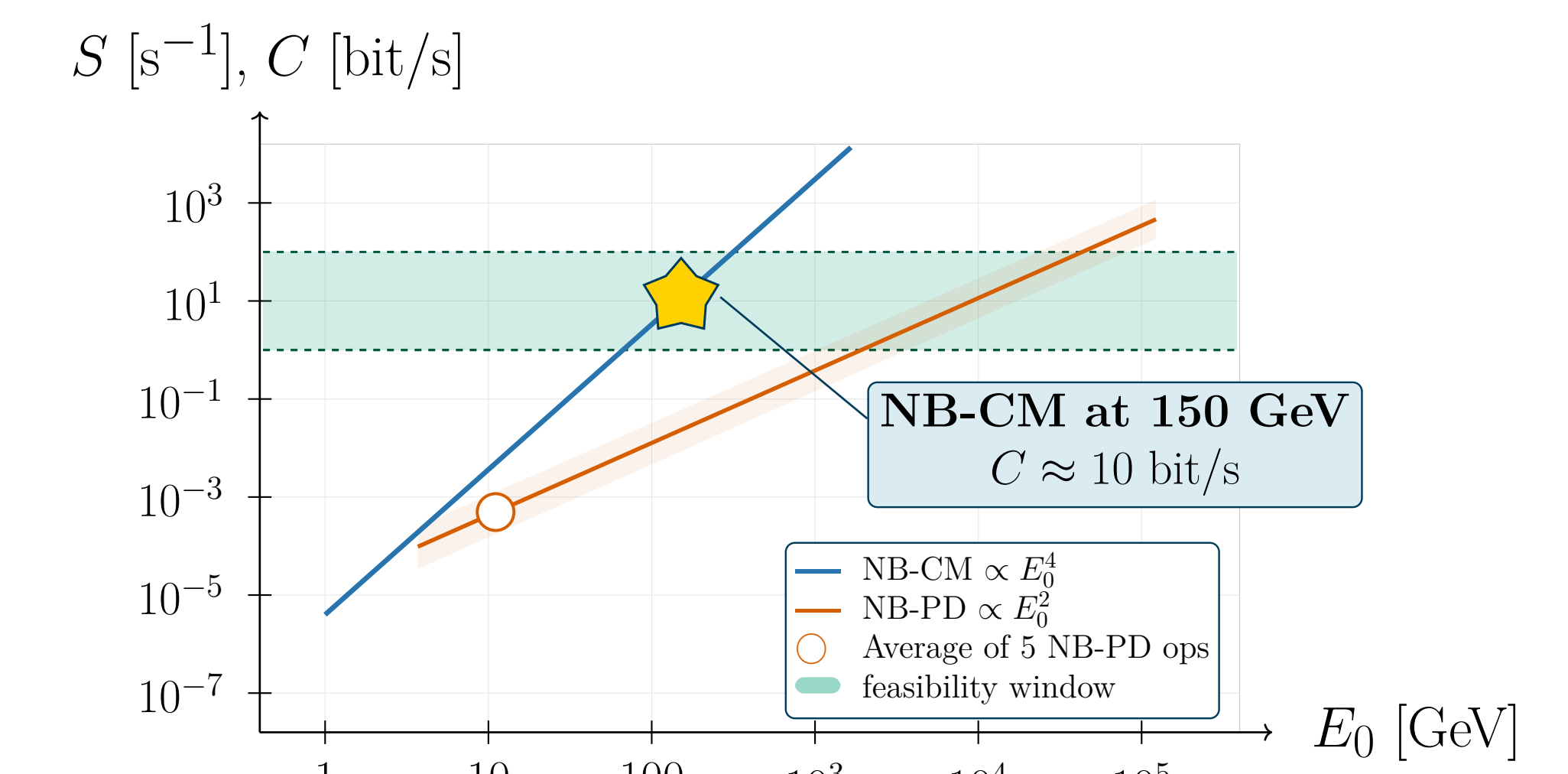


Fig. 5. The NB-CM line crosses the feasibility window at $E_0 \approx 60$ GeV. Every NB-PD facility lies 3 to 5 orders of magnitude below feasibility.

Energy per Bit and Conclusions

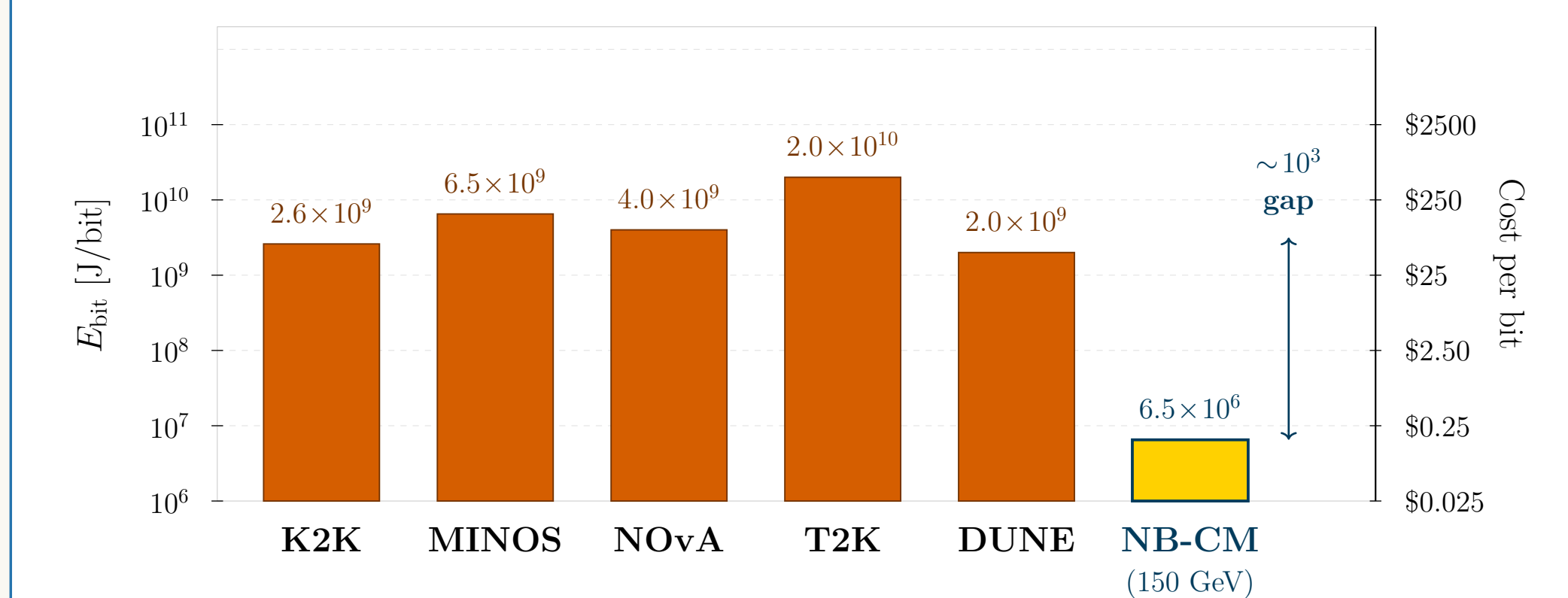


Fig. 6. $E_{\text{bit}} = P_{\text{wall}}/C$ across the five NB-PD facilities (Table 1) versus the 150 GeV NB-CM.

NB-PD sources cluster at 10^9 to 10^{10} J/bit, with the NB-CM approximately three orders of magnitude lower. The right axis converts to cost per bit at the US industrial electricity price

$\$0.09/\text{kWh}$ (2.5×10^{-8} $\$/\text{J}$).

Results established by this work:

- **Derived a unified rate integral** putting NB-PD and NB-CM on equal footing: $S_{\text{NB-CM}} \propto N_\mu E_0^4/L^2$ versus $S_{\text{NB-PD}} \propto N_p E_\nu^2/L^2$. The extra E_0^2 alone accounts for the full 10^3 to 10^5 capacity gap.
- **Generalized Huber's empirical PPM capacity** (fixed at $M = 2^{14}$) to arbitrary M . Large M cannot rescue NB-PD because C grows only logarithmically in M .
- **Benchmarked five NB-PD facilities** (MINOS, K2K, T2K, NOvA, DUNE) against a 150 GeV NB-CM reference. Every NB-PD source lies 10^3 to 10^5 below $C = 1$ bit/s, while a 150 GeV NB-CM reaches $C \approx 10$ bit/s at $E_{\text{bit}} \approx 6.5 \times 10^6$ J/bit, approximately three orders of magnitude lower in energy cost than any NB-PD.
- **Showed feasibility is driven by accelerator physics, not detector upgrades.** Reaching $C = 1$ bit/s with a pion-decay beam alone requires E_0 of order 1 TeV (parent pion energy), which is hard to steer and needs a decay region far longer than existing beams.
- **Identified the near-term path:** a dedicated NB-CM front-end that reuses muon-collider cooling and acceleration R&D sustains $C \sim 1$ to 10 bit/s over most of Earth's oceans at night.