Gravitational Waves in an A_4 Neutrino Mass Model

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Motivation

- The Standard Model leaves several questions about neutrino masses unanswered.
- A_4 symmetry predicts Tri-Bimaximal Mixing(TBM), close to observed angles.
- Spontaneous breaking of A4 creates energetically degenerate vacua, leading to domain walls.
- DW tend to dominate the energy density of the universe, so they need to annihilate via an energy bias.
- Annihilation leads to potentially detectable Gravitational Waves(GW)
- Energy bias typically explicitly breaks the symmetry, we show that we can annihilate DW without explicitly breaking A_4.
- We introduce a model invariant under $A_4 \times \mathbb{Z}_4$ that accurately describes neutrino mixing as well as solves the DW problem linking GW to neutrino physics.

Flavon Cross Couplings

In order to consider corrections to TBM, we must look at the cross-couplings between flavons $\phi \,\&\, \chi$. We get an additional part of the potential invariant under $A_4 \times \mathbb{Z}_4$: $V(\phi,\chi) = \frac{1}{2}\varepsilon_1(\chi\chi)_{\mathbf{1}}(\phi\phi)_{\mathbf{1}} + \frac{1}{4}\varepsilon_2(\chi\chi)_{\mathbf{1}'}(\phi\phi)_{\mathbf{1}''} + \frac{1}{4}\varepsilon_2^*(\chi\chi)_{\mathbf{1}''}(\phi\phi)_{\mathbf{1}'} + \frac{1}{2}\varepsilon_3\left[(\chi\chi)_{\mathbf{3}}(\phi\phi)_{\mathbf{3}}\right]_{\mathbf{1}} + \frac{1}{3}\varepsilon_4[(\chi\chi)_{\mathbf{3}}\phi]_{\mathbf{1}}$. Where ε are of $\mathcal{O}(0.1)$ which allows us to treat cross-couplings as perturbations to the vacua producing TBM:

$$\langle\chi\rangle = \begin{pmatrix} \frac{v_\chi}{\sqrt{3}} + \delta v_{\chi_1} \\ \frac{v_\chi}{\sqrt{3}} + \delta v_{\chi_2} \\ \frac{v_\chi}{\sqrt{3}} + \delta v_{\chi_2}^* \end{pmatrix}, \quad \langle\phi\rangle = \begin{pmatrix} v_\phi + \delta v_{\phi_1} \\ \delta v_{\phi_2} \\ \delta v_{\phi_2}^* \end{pmatrix}$$

with

$$\delta v_{\phi 1} = 0\,, \quad \delta v_{\phi 2} = v_{\phi\_}\varepsilon_\phi\,, \quad \delta v_{\phi 2}^* = v_{\phi\_}\varepsilon_\phi^*\,, \delta v_{\chi 1} = -2v_\chi\varepsilon_\chi\,, \quad \delta v_{\chi 2} = \delta v_{\chi 2}^* = v_\chi \varepsilon_\chi$$

We parametrize $\varepsilon_\phi = |\varepsilon_\phi| e^{i\theta_\phi}$ for convenience.

Gravitational Wave Results

In calculating perturbations of an additional energetically equivalent $\mathbb{Z}_2$ and $\mathbb{Z}_3$ vacua using our best fit neutrino parameters, we calculate a non-zero energy difference meaning domain walls are naturally unstable and thus collapse producing GW. To calculate the present day GW spectrum one must solve $\Omega h^2(f,t) = \frac{h^2}{\rho_c(t)}\frac{d\rho_{GW}(t)}{d\ln f}$, plotting for various domain wall tension we get the following spectra:

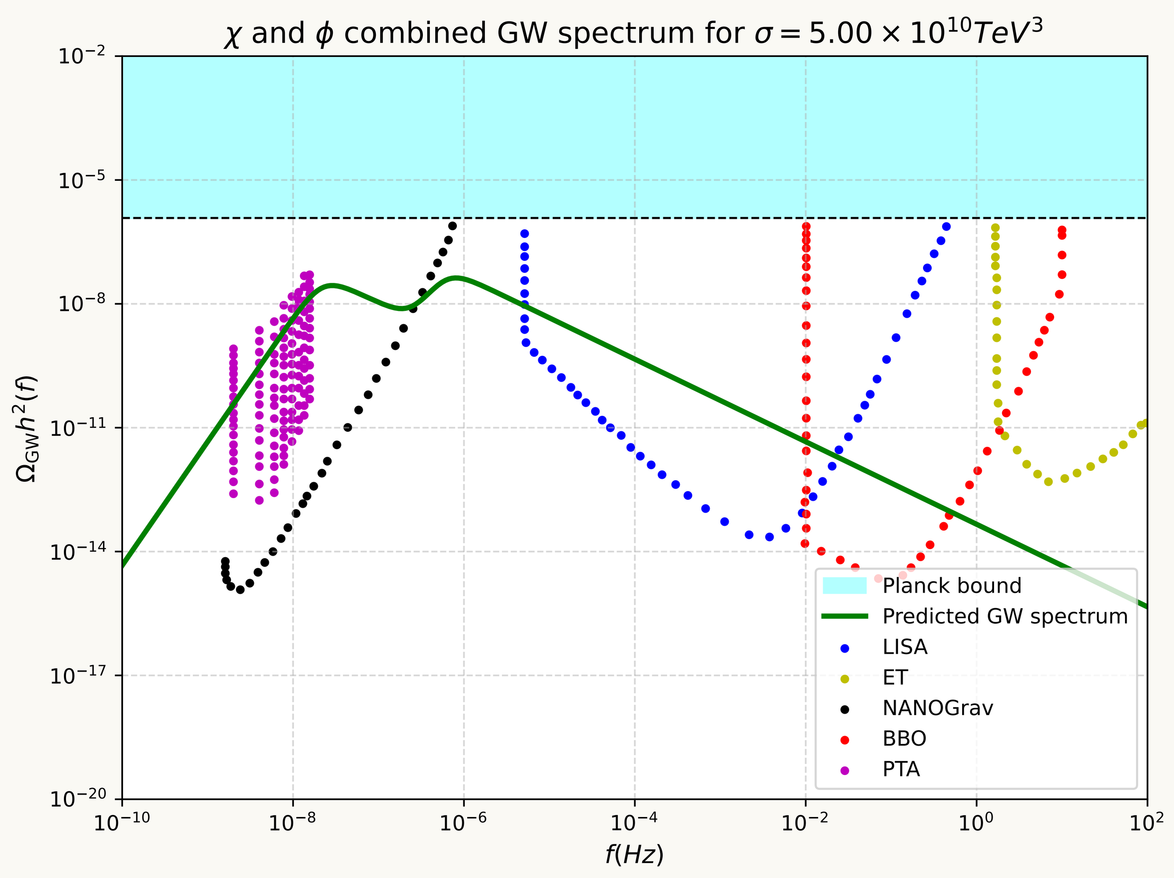

Figure 1. Gravitational Wave Spectra for $\sigma = 5 \times 10^{10}$TeV^3

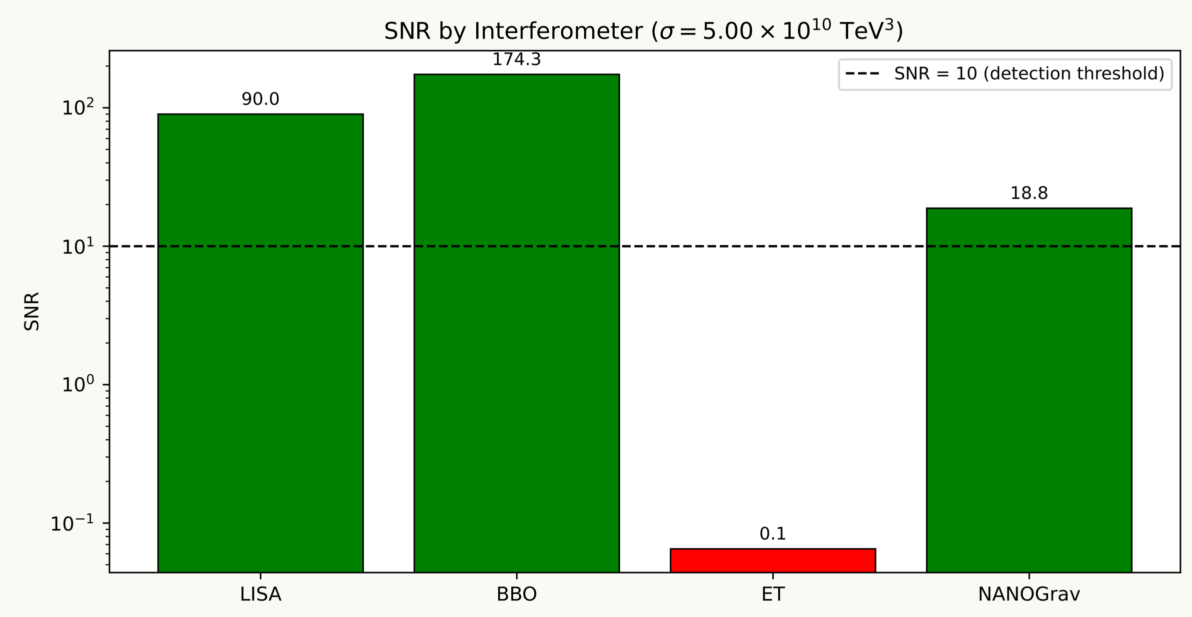

Figure 2. Signal-to-noise ratio for various interferometers. SNR≥10 constitutes a detection where

$$\text{SNR} \equiv \sqrt{2t_{\text{obs}}\int_{f_{\min}}^{f_{\max}} df \left(\frac{h^2\Omega_{\text{GW}}(f)}{h^2\Omega_{\text{EXP}}(f)}\right)^2}$$

Scalar Potential

We introduce two scalars $\phi \,\&\, \chi$ to be triplets of A_4 and write the most general renormalizable potential invariant under $A_4 \times \mathbb{Z}_4$: $V(\phi) = \frac{1}{2}\mu_\phi^2 I_1 + \frac{g_1}{4}I_1^2 + \frac{g_2}{2}I_2 + g_3\phi_1\phi_2\phi_3$ and $V(\chi) = \frac{1}{2}\mu_\chi^2\tilde{I}_1 + \frac{\tilde{g}_1}{4}\tilde{I}_1^2 + \frac{\tilde{g}_2}{2}\tilde{I}_2$ where $I_1 = \phi_1^2+\phi_2^2+\phi_3^2$, $I_2 = \phi_1^2\phi_2^2+\phi_1^2\phi_3^2+\phi_2^2\phi_3^2$ and the for $\tilde{I}$ replace ϕ with χ. The scalars called flavons obtain a vev in which $\phi \,\&\, \chi$ break A_4 into $\mathbb{Z}_3 \,\&\, \mathbb{Z}_2$ respectively. There are energetically equivalent vacuum alignments since there are multiple $\mathbb{Z}_2$ and $\mathbb{Z}_3$ subgroups of A_4 so domain walls will form.
To reproduce realistic mixing, we must consider cross couplings between flavons.

Corrections to TBM Results

Given the following particle content

Fields	η	ϕ	χ	L	N	H	e_R	μ_R	τ_R
A_4	**1**	**3**	**3**	**3**	**3**	**1**	**1**	**1**$''$	**1**$'$
$\mathbb{Z}_4$	-1	1	-1	i	i	1	i	i	i

Table 1. Particle Content and Representation Assignments

Implementing a type-I seesaw mechanism we write the most general Lagrangian invariant under $A_4 \times \mathbb{Z}_4$:

$$\mathcal{L} \supset y_D\left(\bar{L}N\right)_{\mathbf{1}}\tilde{H} \;+ y_N\left[\left(\bar{N}N^c\right)_{\mathbf{3}}\chi\right]_{\mathbf{1}} + \frac{1}{2}y_1\eta\bar{N}^cN + \frac{y_e}{\Lambda}\left(\phi\bar{L}\right)_{\mathbf{1}}e_RH \;+ \frac{y_\mu}{\Lambda}\left(\phi\bar{L}\right)_{\mathbf{1}'}\mu_RH + \frac{y_\tau}{\Lambda}\left(\phi\bar{L}\right)_{\mathbf{1}''}\tau_RH \;+ h.c.$$

We then expand all tensor products and plug in our perturbed vacua to derive the corrected TBM:

$$U_{PMNS} \approx \begin{pmatrix} 1 & -\varepsilon_\phi & -\varepsilon_\phi^* \\ \varepsilon_\phi^* & 1 & -\varepsilon_\phi \\ \varepsilon_\phi & \varepsilon_\phi^* & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & \sqrt{2}\varepsilon_\chi & 0 \\ -\sqrt{2}\varepsilon_\chi & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_\nu$$

. With P_ν being a matrix of Majorana phases. From this we can derive expressions for the mixing angles, derive a sumrule, and the Dirac CP phase and fitting to best fit values

$$\sin\theta_{13} \approx \sqrt{2}\,|\varepsilon_\phi \sin\theta_\phi| \approx 0.024$$
$$\sin\theta_{23} \approx \frac{1}{\sqrt{2}}(1 + |\varepsilon_\phi|\cos\theta_\phi) \approx 0.581$$
$$\sin\theta_{12} \approx \frac{1}{\sqrt{3}}\left(2\varepsilon_\chi - 2\sqrt{2}\sin\theta_{23} + 3\right) \approx 0.297$$
$$\delta_{CP} = \frac{3\pi}{2} - 2|\varepsilon_\phi|\sin\theta_\phi \approx 298.5^\circ$$

consistent within 3σ to data. We compute the mass squared differences $\Delta m_{21}^2 \approx 7.5 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 \approx 2.45 \times 10^{-3}$ eV^2 favoring normal ordering.

Conclusions and Acknowledgments

We can produce a realistic PMNS matrix and naturally solve the DW problem by including cross couplings between flavons. We found a set of parameters that produce realistic neutrino mixing and masses, favoring normal ordering, that produces a potentially detectible GW signal in near future experiments. We thank Anish Ghoshal for bringing to our attention this interesting topic and for discussions at the initial stage of the project. **CMS** acknowledges the support of the U.S. National Science Foundation (NSF) Graduate Research Fellowship Program (GRFP) and the Eugene Cota-Robles Fellowship provided by the University of California, Irvine. HM acknowledges the support of the U.S. Department of Education GAANN Fellowship. The work of M.-C.C. was partially supported by U.S. NSF under grant number PHY-2210283.

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