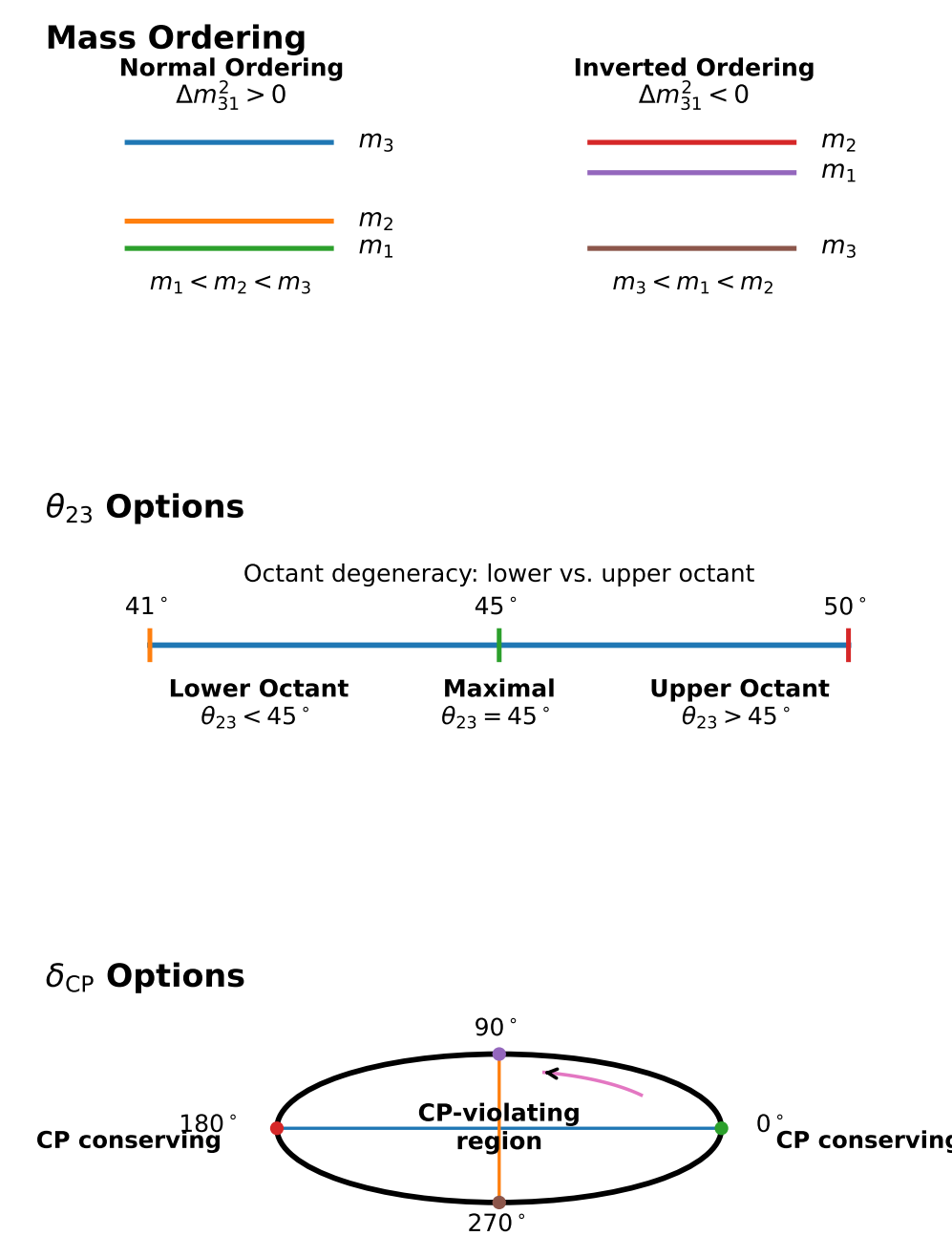


Motivation and Major Features

- The neutrino mass ordering (MO), the octant of θ_{23} , and the value of δ_{CP} remain key unknowns in three-flavor neutrino oscillation physics.
- Beyond-standard-model effects can affect neutrino oscillations and make the determination of these quantities more difficult through new degeneracies.
- We explore the effects of CPT-even Lorentz-invariance-violating potentials in the determination of unknown neutrino oscillation parameters.
- Can the combined analysis of accelerator, atmospheric, and reactor neutrinos help when LIV potentials are present?
- We consider DUNE (accelerator, atmospheric) and JUNO (reactor) simulated data for the analysis.



Key unknowns in three-flavor oscillations

Neutrinos under Lorentz Invariance Violations (LIV)

The total Hamiltonian in the presence of LIV is given by

$$H = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{bmatrix} a_{ee} & a_{e\mu} & a_{e\tau} \\ a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\ a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau} \end{bmatrix} - \frac{4}{3}E \begin{bmatrix} c_{ee} & c_{e\mu} & c_{e\tau} \\ c_{e\mu}^* & c_{\mu\mu} & c_{\mu\tau} \\ c_{e\tau}^* & c_{\mu\tau}^* & c_{\tau\tau} \end{bmatrix} \right], \quad (1)$$

$A = 2E\sqrt{2}G_F N_e = 7.6 \times 10^{-5} \rho(\text{g/cc})E(\text{GeV})\text{eV}^2$, G_F : Fermi Constant, N_e : No. density of e^- in matter, ρ : matter density, $a_{\alpha\beta}$: CPT odd, $c_{\alpha\beta}$: CPT even. $\hat{A} = A/\Delta m_{31}^2$, $\Delta = \Delta m_{31}^2 L/4E$

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}[4] \simeq 1 - \cos^2 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} + f_{21} \right) - \sin^2 2\theta_{13} \left[\cos^2 \theta_{12} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} + f_{31} \right) + \sin^2 \theta_{12} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} + f_{31} \right) \right], \quad (2)$$

$$P_{\nu_\mu \rightarrow \nu_e}[1] \simeq 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin[\hat{A}\Delta] \sin[(\hat{A}-1)\Delta]}{\hat{A}(\hat{A}-1)} \cos(\Delta + \delta_{CP}) + 4s_{13}^2 s_{23}^2 \frac{\sin^2[(\hat{A}-1)\Delta]}{(\hat{A}-1)^2} - \frac{8EL\Delta s_{13} \sin 2\theta_{23} \sin \Delta}{3} \sum_{\beta} \left[|c_{e\beta}| Z_{e\beta} \sin(\psi_{e\beta} + \delta_{CP}) + |c_{e\beta}| W_{e\beta} \cos(\psi_{e\beta} + \delta_{CP}) \right] \quad (3)$$

The functions $f_{21}, f_{31}[4]$ depend on LIV. $Z_{e\beta}, W_{e\beta}[1]$ depends on $\theta_{23}, \sin \Delta, \cos \Delta$.

Details of Experiments

We consider two experiments DUNE, JUNO and three different sources of neutrinos accelerator, atmospheric ($P_{\mu e}, P_{\mu\mu}$), and reactor (\bar{P}_{ee}).

Characteristic	DUNE acc.	DUNE atm.	JUNO reactor
Source	Beam neutrinos	Atmospheric neutrinos	Reactor $\bar{\nu}_e$
Channel	$\nu_\mu \rightarrow \nu_e, \nu_\mu \rightarrow \nu_\mu$	$\nu_\mu \rightarrow \nu_e, \nu_\mu \rightarrow \nu_\mu$	$\bar{\nu}_e \rightarrow \bar{\nu}_e$
Baseline	1300 km	10–10 ⁴ km	52.5 km
Energy	1–9 GeV	0.5–10.5 GeV	1.8–9.8 MeV
Exposure	40 kt × (1.5 + 1.5) yr	40 kt × 3 yr	20 kt × 3 yr
Energy resolution	DUNE smearing matrices	$\sigma_E = 15\%$	$\sigma_E = 3\%$
Angular resolution	–	$\sigma_\theta = 15^\circ$	–
Systematics	Signal/background norm., calibration	Flux 20%, zenith 5%, xsec 10%, syst. 5%, tilt	Flux norm., detector norm., energy scale, backgrounds
Physics role	MO, δ_{CP} and θ_{23}	MO, θ_{23}	MO, precision of θ_{13}, θ_{12}

Sensitivity to $\theta_{23} - \delta_{CP}$ (same Mass Ordering)

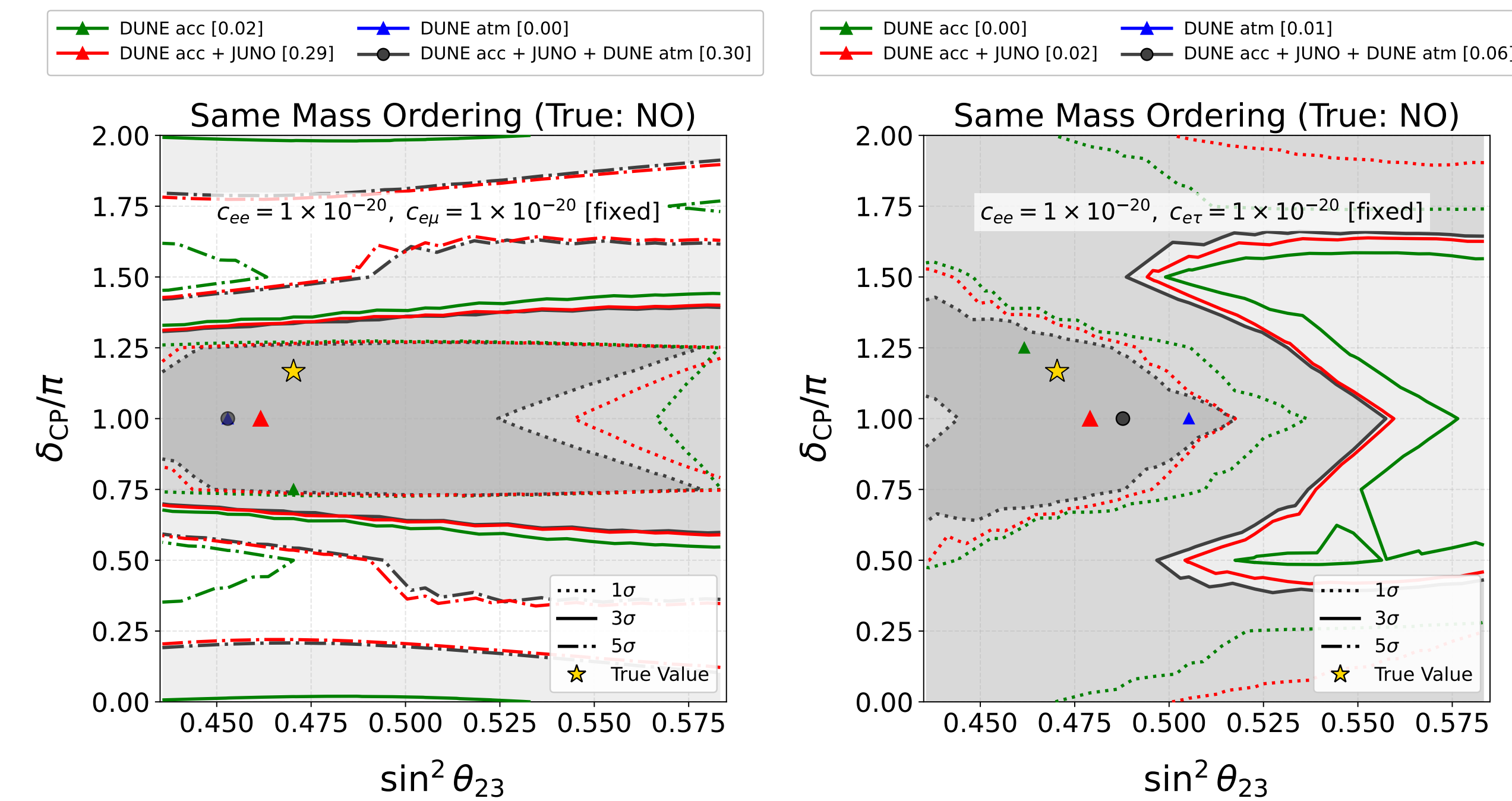


Figure 1. Sensitivity to NO for true NO in test $\sin^2 \theta_{23} - \delta_{CP}$ with fix $c_{ee} - c_{e\mu}$ [left], $c_{ee} - c_{e\tau}$ [right]

- Sensitivity improves significantly with the addition of JUNO and atmospheric ν 's over only the DUNE accelerator, as seen from shaded contours.
- Degeneracies in $\sin^2 \theta_{23} - \delta_{CP}$ plane still remain. Degeneracy in δ_{CP} is more in presence of $c_{ee} - c_{e\tau}$, whereas degeneracy in θ_{23} is greater in presence of $c_{ee} - c_{e\mu}$.
- More exposure of accelerator, atmospheric, and reactor neutrinos is needed to overcome the degeneracies.

Sensitivity to Opposite Mass Ordering

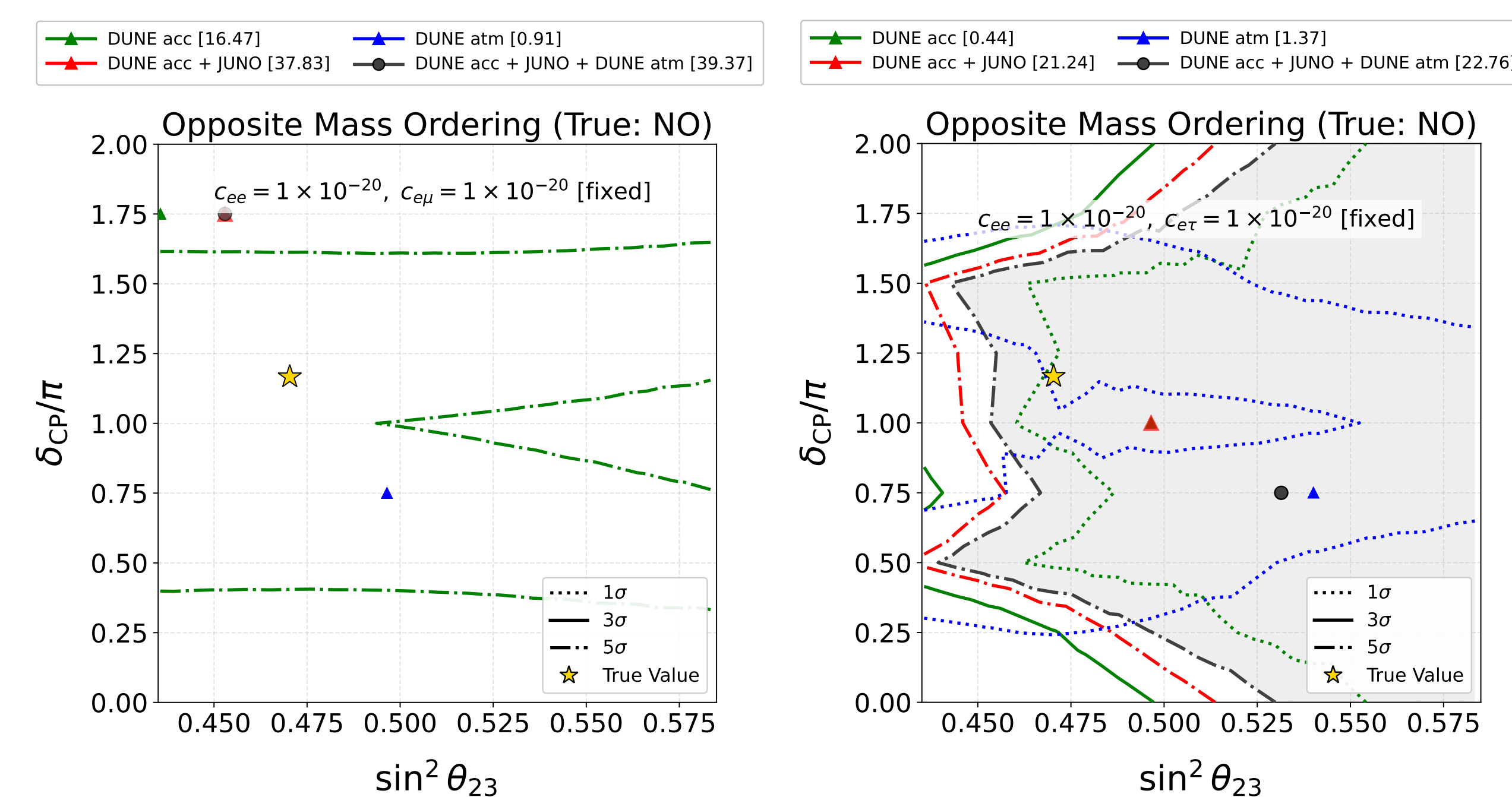


Figure 2. Sensitivity to IO for true NO in test $\sin^2 \theta_{23} - \delta_{CP}$ with fix $c_{ee} - c_{e\mu}$ [left], $c_{ee} - c_{e\tau}$ [right]

- $c_{ee} - c_{e\tau}$ (right) affects the sensitivity to mass ordering more than $c_{ee} - c_{e\mu}$ (left).
- The addition of different channels helps in overcoming the MO degeneracy completely in the presence of $c_{ee} - c_{e\mu}$ (left) in 5σ significance (as we don't see shaded contours bordered by black curves) with the lowest $\chi^2 = 39.37$.
- In the $c_{ee} - c_{e\tau}$ (right) case, only a small portion of the parameter space (on the left of the shaded contours bordered by black curves) has more than 5σ sensitivity to the opposite MO with a minimum $\chi^2 = 22.76$ in the combined scenario.
- To overcome MO degeneracy in the presence of $c_{ee} - c_{e\tau}$, will need more exposure from all the channels.

Sensitivity in $\theta_{13} - \Delta m_{31}^2$

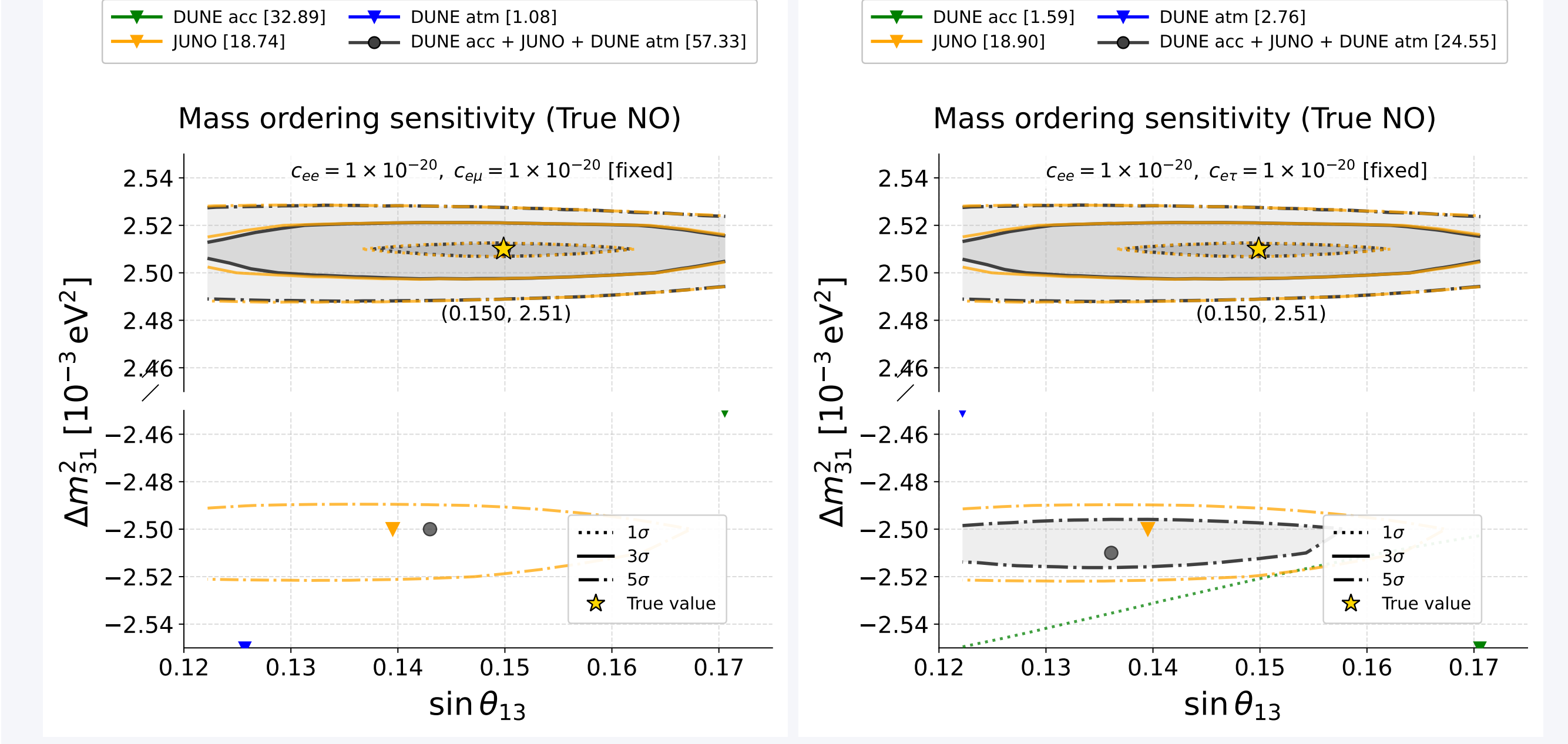


Figure 3. Sensitivity to MO for true NO in test $\sin \theta_{13} - \Delta m_{31}^2$ with fix $c_{ee} - c_{e\mu}$ [left], $c_{ee} - c_{e\tau}$ [right]

- $c_{ee} - c_{e\tau}$ (right) affects the sensitivity to MO (bottom panels) more than $c_{ee} - c_{e\mu}$ (left).
- Sensitivity in $\theta_{13} - \Delta m_{31}^2$ (top panels) is narrower in the combined case but needs to be improved with more exposure from different channels when LIV ($c_{ee}, c_{e\mu}, c_{e\tau}$) is present.

Numerical Analysis

We perform pulled chi-square analyses using GLOBES[2, 3] for the DUNE accelerator and JUNO reactor analyses, and use an in-house code for atmospheric neutrinos at DUNE. The total χ^2 is obtained by marginalizing over the oscillation parameters ω and the systematic pull variables ξ ,

$$\chi^2 = \min \left[2 \sum_i \left[N_i^{\text{test}} - N_i^{\text{true}} + N_i^{\text{true}} \ln \left(\frac{N_i^{\text{true}}}{N_i^{\text{test}}} \right) \right] + \sum_{r=1}^4 \xi_r^2 \right]. \quad (4)$$

Parameter	True Value	Test Values
θ_{12}	33.76°	33.76°
θ_{13}	8.62°	7.0° – 10.0°
θ_{23}	43.3°	41.3° – 49.8°
δ_{CP}	210°	0° – 360°
Δm_{31}^2	$7.54 \times 10^{-5} \text{eV}^2$	$7.54 \times 10^{-5} \text{eV}^2$
Δm_{31}^2 (same MO)	$2.51 \times 10^{-3} \text{eV}^2$	$[2.45 - 2.55] \times 10^{-3} \text{eV}^2$
Δm_{31}^2 (opposite MO)	$2.51 \times 10^{-3} \text{eV}^2$	$[-2.55 - 2.45] \times 10^{-3} \text{eV}^2$
$c_{ee}, c_{e\mu}, c_{e\tau}$	1×10^{-20}	1×10^{-20}

Table 1. True and test values of the parameters

Take-home Message

- LIV can create degeneracies in θ_{23}, δ_{CP} , and MO determination.
- JUNO improves the combined analysis through strong sensitivity in Δm_{31}^2 and MO.
- DUNE accelerator neutrinos provide strong constraints on δ_{CP} and θ_{23} through $P_{\mu e}$ and $P_{\mu\mu}$.
- Atmospheric neutrinos provide events through their wide energy and baseline coverage.
- Complementarity between different channels allows robust sensitivity when combined.

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