Fully Analytic Reconstruction of Observed Askaryan Radio Signals in Ice

Jordan C. Hanson[1] Damian Ibañez-Rodriguéz[1] Raymond Hartig[2]

[1]Department of Physics and Astronomy, Whittier College, [2]Department of Physics and Astronomy, California State Fullerton

ABSTRACT

The detection of ultra-high-energy neutrinos and cosmic rays (UHE-ν and UHECR, $E \gtrsim 10$ PeV) via the Askaryan effect with radio-frequency (RF) detectors in polar ice requires accurate modeling of Askaryan radiation. While existing analytic models describe the Askaryan field from particle cascades, the RF signal propagation and detector response are typically evaluated numerically. We present a new, fully analytic model for the observed RF voltage waveforms, accounting for Askaryan emission, ice propagation, and RF channel response. We validate the model using 100 PeV neutrino simulations generated with NuRadioMC. The correlation coefficients between model and simulation can exceed 0.94, and 99.99% of simulated events satisfy a correlation threshold of $\rho \geq 0.4$. Only 0.2 thermal-noise events are expected to exceed the threshold over five years assuming a 1 Hz background rate. Further, the model matches data from 13 UHECRs observed by the Askaryan Radio Array (ARA) with correlation coefficients between 0.69 and 0.86, with minimal fractional power residuals. These fits allow the measurement of the observed $\vec{E}$-field and provide strong evidence that the it originates from the Askaryan effect. Our model offers a fast, physical alternative to computationally intensive Monte Carlo techniques, enabling efficient UHE-ν identification in data from ARA, RNO-G, PUEO, and IceCube-Gen2.

KEY RESULTS

- Correlation coefficients between model and NuRadioMC exceed 0.94.
- 99.99% of simulated events satisfy a correlation threshold of $\rho \geq 0.4$.
- Only 0.2 thermal-noise events are expected to exceed $\rho \geq 0.4$ in five years assuming a 1 Hz background rate.
- Equations [A] are validated by NuRadioMC [B, C, and D]
- Equations match observed UHECR events from ARA [E and F]
- Our model fits 13 UHECR events [G] observed by ARA.

[A] EQUATIONS for UHE-ν and UHECR WAVEFORMS

Let the Askaryan *signal* from an in-ice UHECR cascade be $s(t)$. Let E_0 be the amplitude in V m^{-1} ns^{-1}, t to be time in ns relative to peak emission, and σ_t be the pulse width in ns. We have shown that $s(t)$ is [1][2]

$$s(t) = -E_0 t e^{-\frac{1}{2}(t/\sigma_t)^2} \tag{1}$$

To predict voltage traces from RF dipole antennas, $s(t)$ must be convolved with the RF channel response $r(t)$. Let R_0 be the amplitude of the response, with units of m ns^{-1}, f_0 be the resonance frequency of the dipole in GHz, and γ be the exponential decay constant in GHz. We have shown that $r(t)$ is [3]

$$r(t) = R_0 e^{-2\pi\gamma t}\cos(2\pi f_0 t) \tag{2}$$

Let $x = t/(\sqrt{2}\sigma_t)$, $z = (2\pi j f_0 - 2\pi\gamma)\sqrt{2}\sigma_t$, $q = -j\left(x + \frac{z}{2}\right)$, and $w(q)$ be the Faddeeva function. The convolution of $s(t)$ with $r(t)$ is

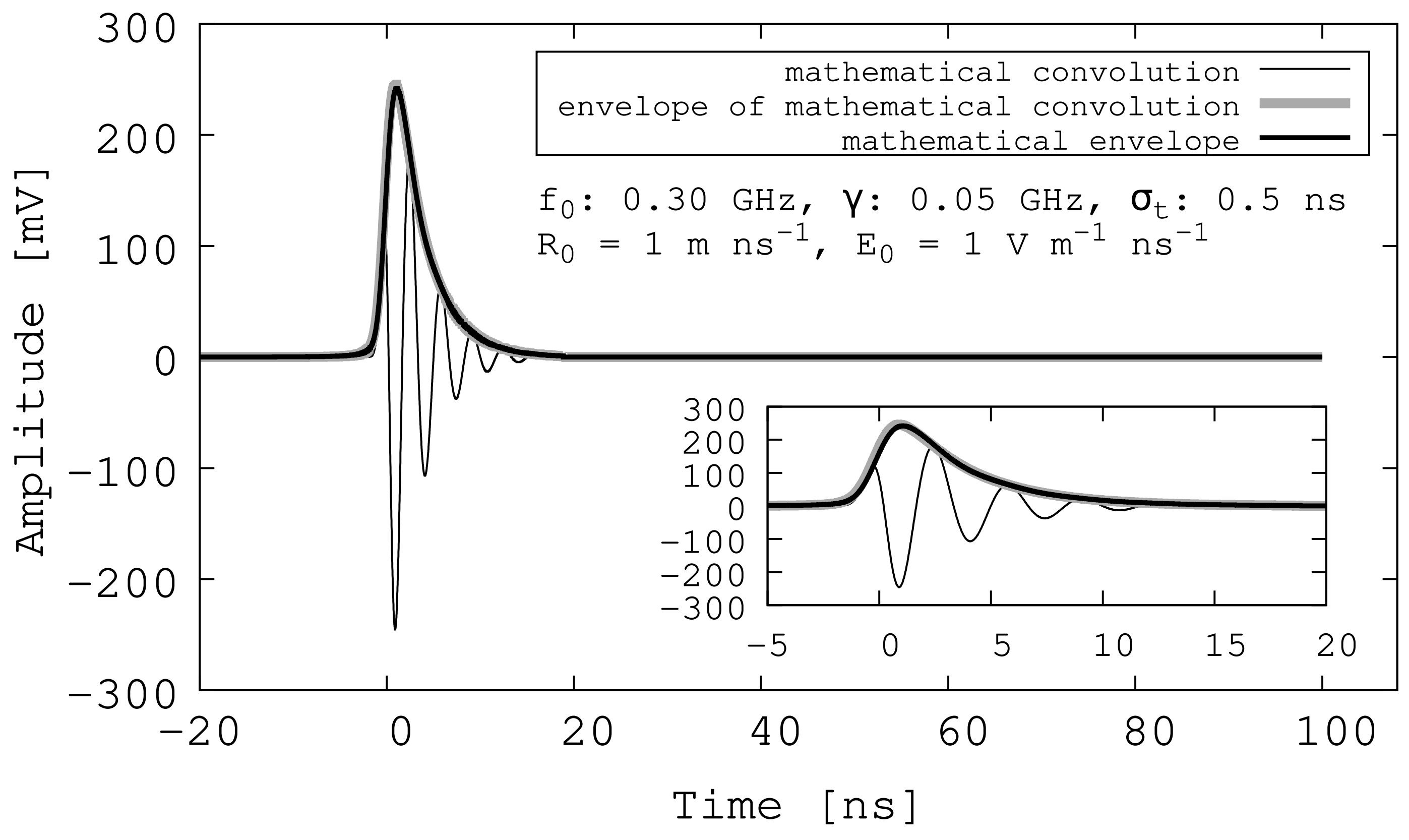
$$s * r = -\sqrt{\pi} R_0 E_0 \sigma_t^2 \left(x e^{-x^2} \Re\{w(q)\} - \frac{1}{2} e^{-x^2} \Re\left\{ -j \frac{dw(q)}{dq} \right\} \right) \tag{3}$$

Let $s_a(t)$ and $r_a(t)$ be the *analytic* signals of $s(t)$ and $r(t)$. The Hilbert envelope of the convolution of $s(t)$ with $r(t)$ is

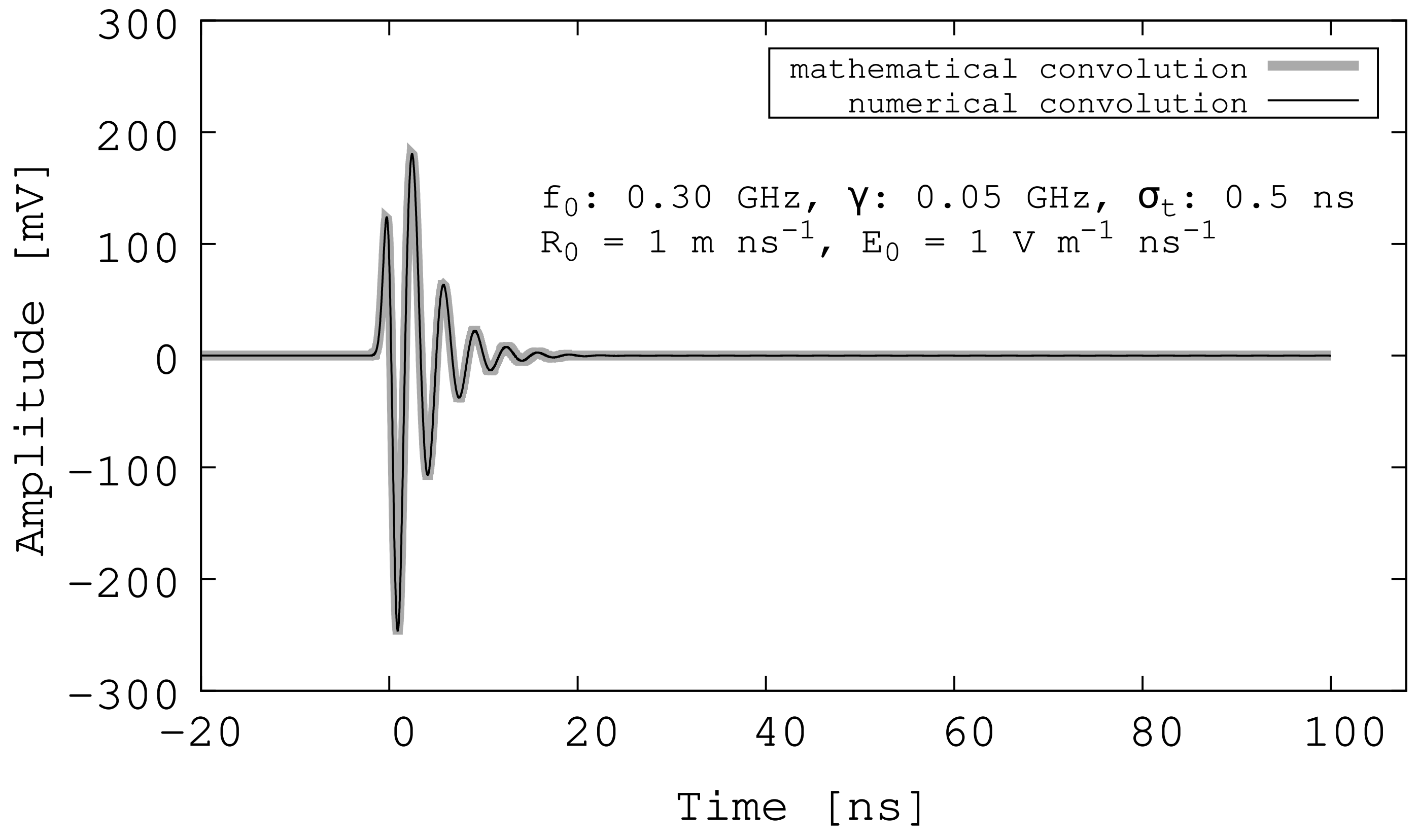
$$\mathscr{E}_{r*s}(t) = \frac{1}{2}|r_a(t) * s_a(t)| \tag{4}$$

The full analyic solution to Eq. 4 is provided in our papers [2][3].

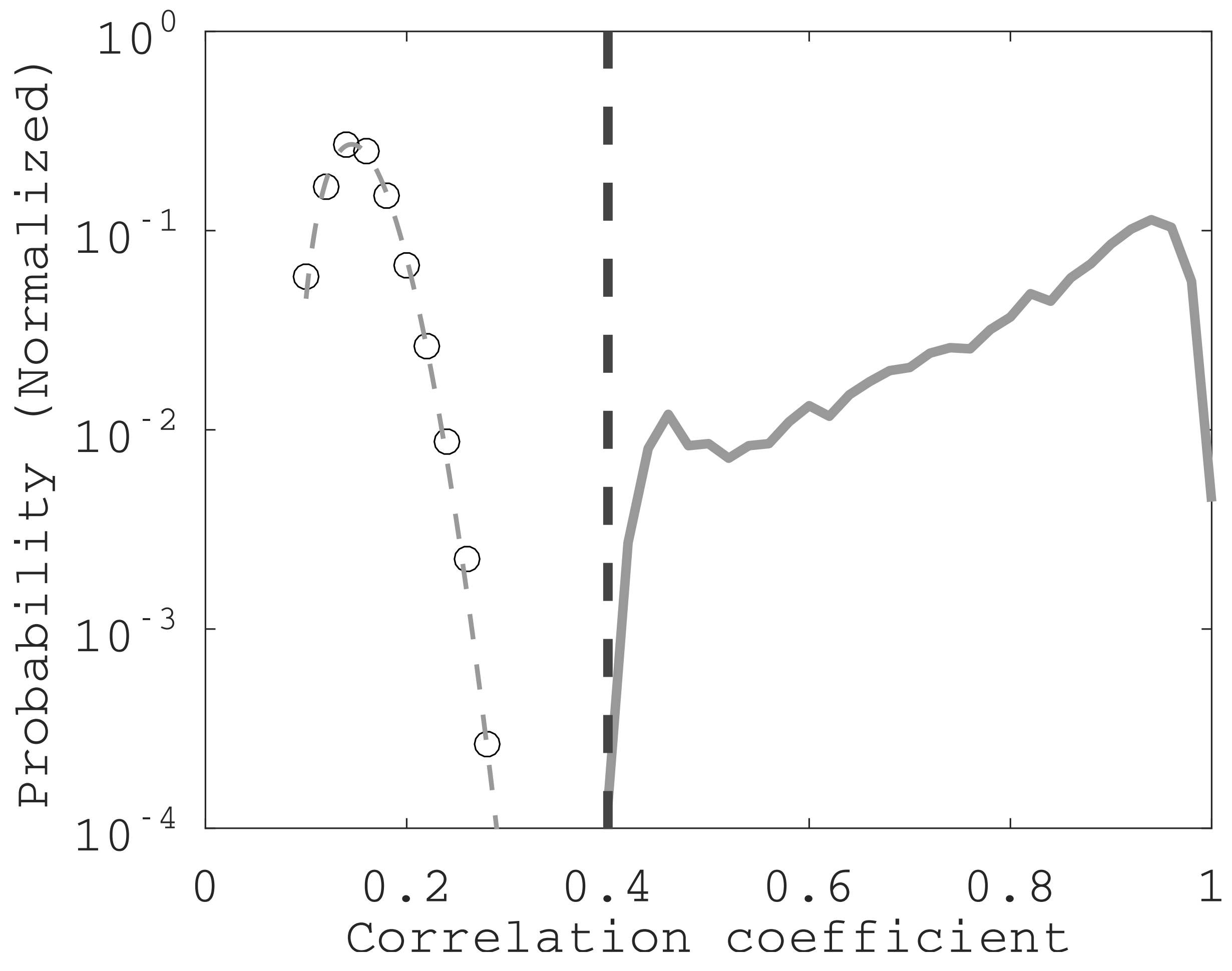
[B] EQ. 3, ENVELOPE of EQ. 3, and EQ. 4 MATCH



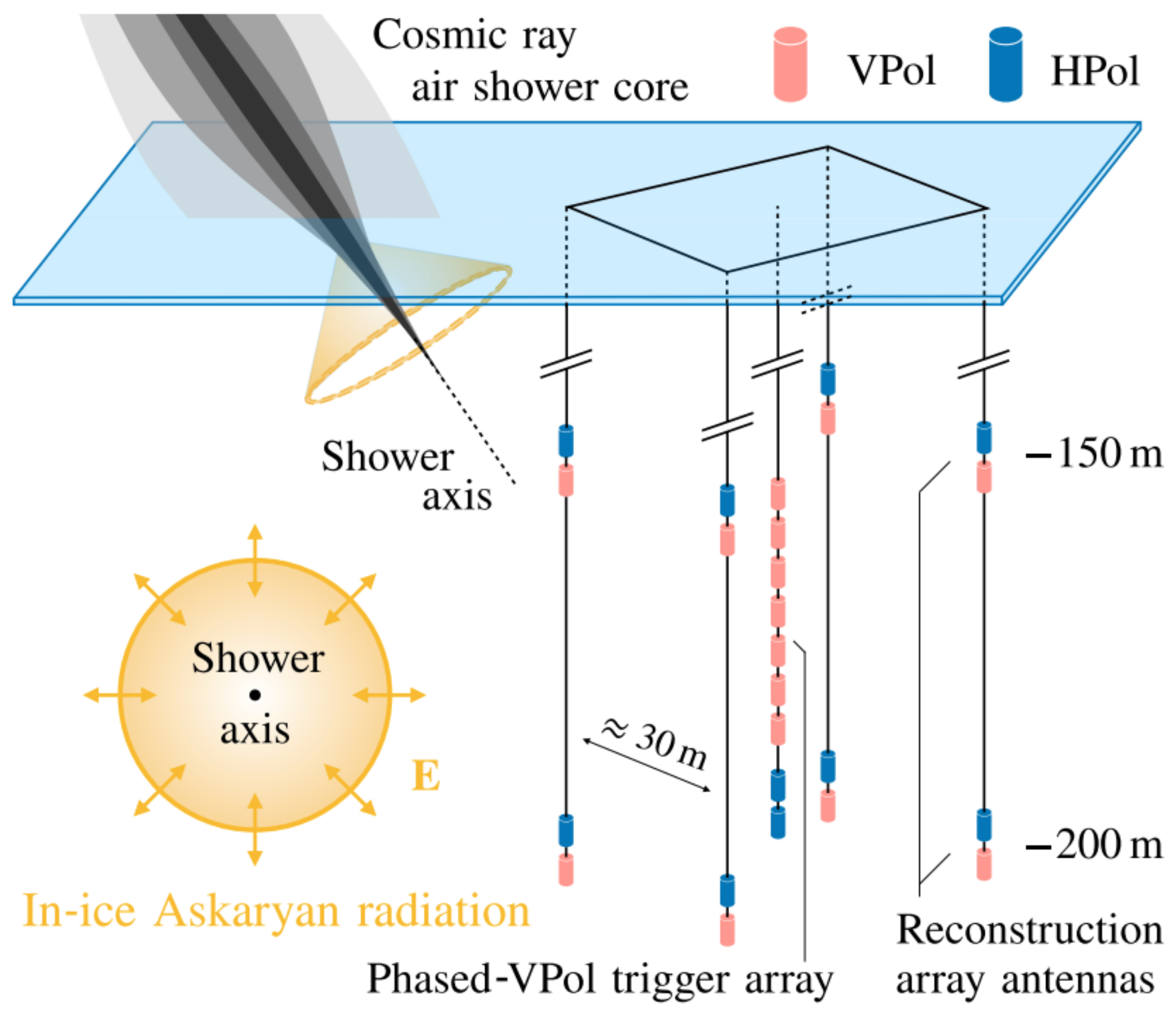
[C] EQ. 3 and the CONVOLUTION of EQS. 1 and 2 MATCH



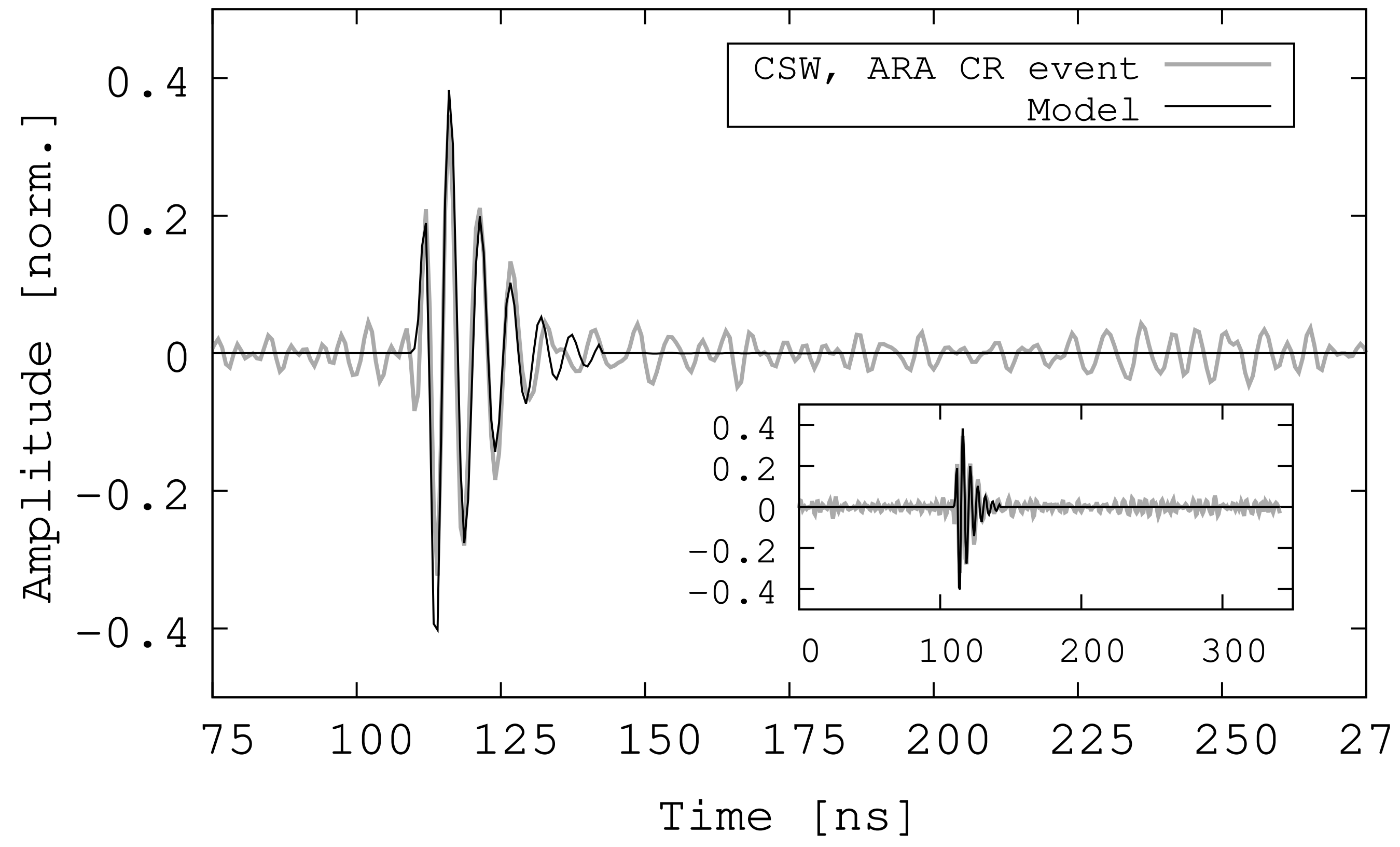
[D] THERMAL NOISE DISTRIBUTION (black circles), MC UHE-ν CORRELATION with EQ. 4 using NuRadioMC



[E] EVENT GEOMETRY FOR UHECR in ARA



[F] EXAMPLE OF MATCH between EQ. 3 and UHECR EVENT



[G] CONCLUSIONS

Our model predicts the actual waveforms observed by UHE-ν detectors. The model matches NuRadioMC with $\rho \geq 0.4$, and peaks at 0.94. The model rejects all but 0.2 thermal background events in 5 years of operations. The model matches UHECR events observed by ARA with $0.66 \leq \rho \leq 0.84$ (table at right). The model will be matched in the future to observed UHE-ν events.

ID	$\sigma_{t,1}$ (ns)	ρ
1915-26288	0.5 ± 0.1	0.83
1957-13330	0.5 ± 0.1	0.66
2171-31805	0.5 ± 0.1	0.775
2250-20189	0.6 ± 0.1	0.80
2352-85489	0.6 ± 0.1	0.83
2375-17342	0.5 ± 0.1	0.69
2529-09767	0.6 ± 0.1	0.80
2716-58611	0.8 ± 0.1	0.84
2782-00106	0.6 ± 0.1	0.815
2955-47449	0.6 ± 0.1	0.68
2961-98361	0.6 ± 0.1	0.79
2978-29412	0.5 ± 0.1	0.78
3352-89556	0.5 ± 0.1	0.78

REFERENCES

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2 J.C. Hanson and R. Hartig, Physical Review D **113**, 083043 (2026)
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