Refining HFS of muonic hydrogen and Lepton $g-2$

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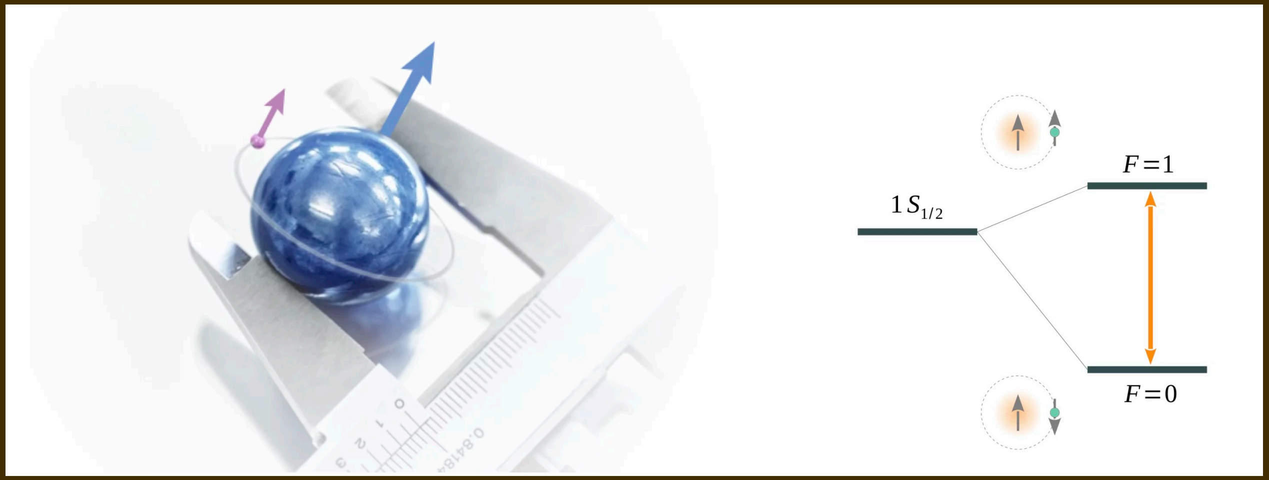
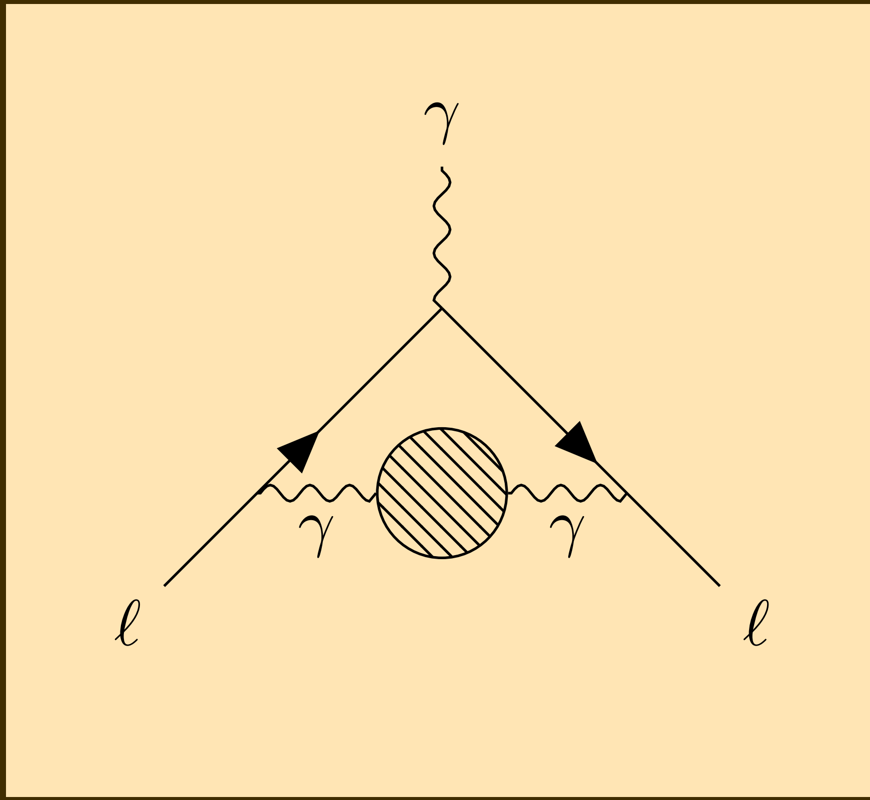
I. HFS



with

Aldo Antognini
Franziska Hagelstein
Vadim Lensky

II. Lepton $g-2$



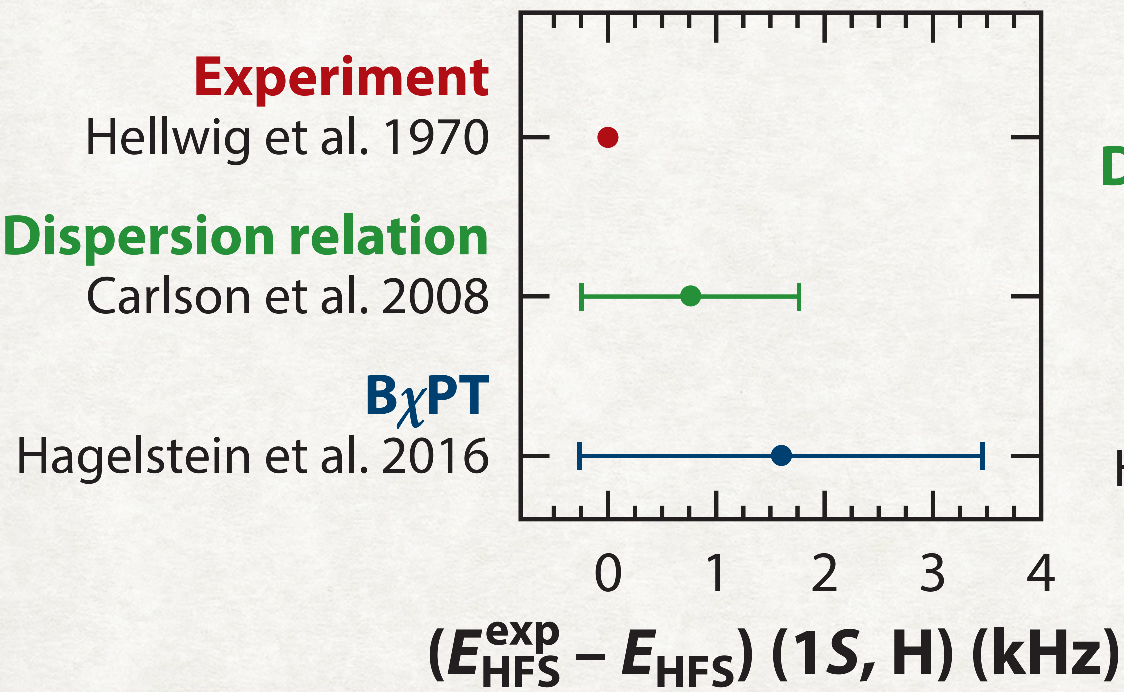
with

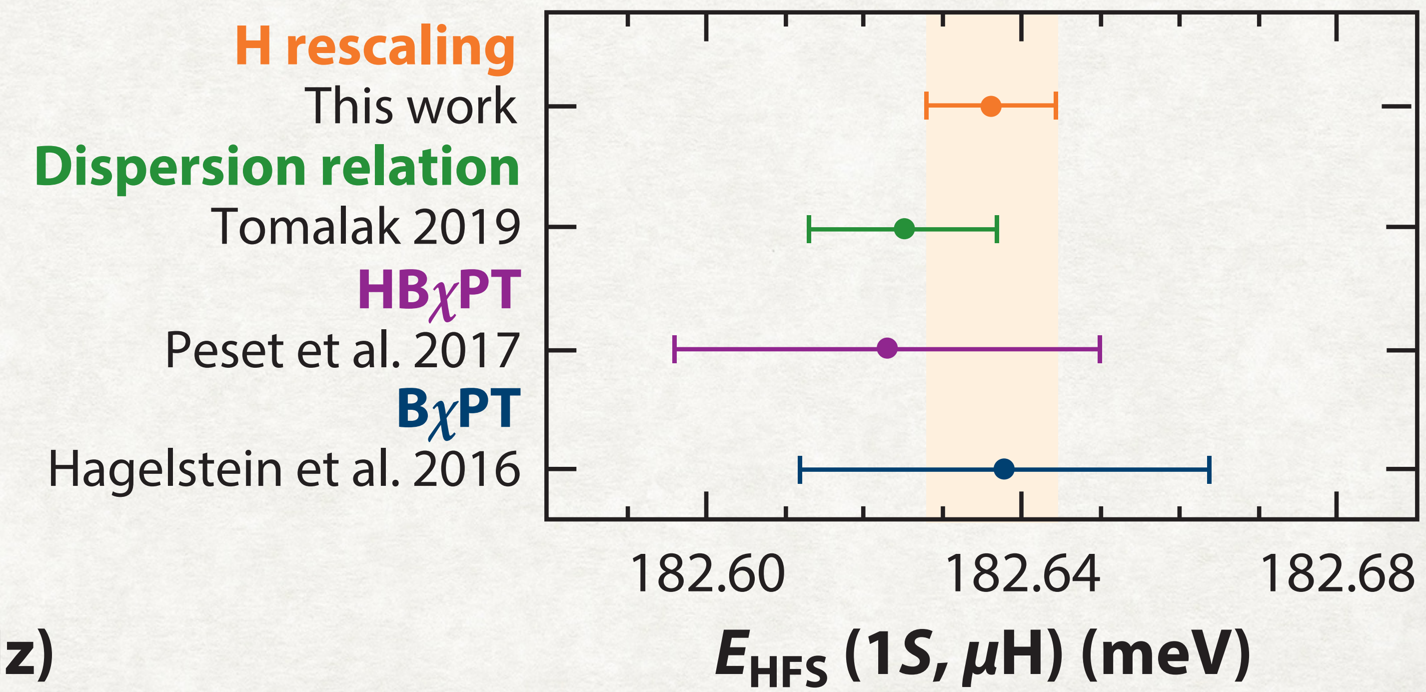
Siyuan Li
Maxim Pospelov
Harvey Meyer
Bogdan Malaescu
Volodymyr Biloshytskii

1S HFS SEARCH

by FAMU and CREMA Coll.

- Ongoing search for ground-state hyperfine splitting in μH
- When found — ground breaking (1ppm exp. precision)
- Theory is reaching 30 ppm at best (with H rescaling)





H rescaling (This work)

Annu. Rev. Nucl. Part. Sci. 2022. 72:389–418 Sec. 4.3

Annual Review of Nuclear and Particle Science

The Proton Structure in and out of Muonic Hydrogen

Aldo Antognini,[1,2] Franziska Hagelstein,[1,3]
and Vladimir Pascalutsa[3]

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[2]Institute for Particle Physics and Astrophysics, ETH Zurich, Zurich, Switzerland

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ZEMACH AND POLARIZABILITY CONTR.

1. Scaling (universal for short-range effects)

$$\frac{\Delta_i(\mathrm{H})}{m_r(\mathrm{H})} = \frac{\Delta_i(\mu\mathrm{H})}{m_r(\mu\mathrm{H})}, \qquad i = \mathrm{Z, pol.}$$

2. From Hydrogen 1S-HFS:

$$E^{\mathrm{Z+pol}}_{1S\text{-HFS}}(\mathrm{H}) = E_{\mathrm{F}}(\mathrm{H})\left[b_{1S}(\mathrm{H})\,\Delta_{\mathrm{Z}}(\mathrm{H}) + c_{1S}(\mathrm{H})\,\Delta_{\mathrm{pol}}(\mathrm{H})\right] = -54.900(71)\,\mathrm{kHz}$$

where $b_{1S}(\mathrm{H}) \simeq 1 + 2\times 10^{-5} + 0.01846 - 5\alpha/4\pi$ and $c_{1S}(\mathrm{H}) \simeq 1 + 2\times 10^{-5}$

radiative

- Obtain the effect in μH:

$$E^{\mathrm{Z+pol}}_{nS-\mathrm{hfs}}(\mu\mathrm{H}) = \frac{E_{\mathrm{F}}(\mu\mathrm{H})\,m_r(\mu\mathrm{H})\,b_{nS}(\mu\mathrm{H})}{n^3 E_{\mathrm{F}}(\mathrm{H})\,m_r(\mathrm{H})\,b_{1S}(\mathrm{H})} E^{\mathrm{Z+pol}}_{1S-\mathrm{hfs}}(\mathrm{H}) - \frac{E_{\mathrm{F}}(\mu\mathrm{H})}{n^3}\Delta_{\mathrm{pol}}(\mu\mathrm{H}) \times \underbrace{\left[c_{1S}(\mathrm{H})\frac{b_{nS}(\mu\mathrm{H})}{b_{1S}(\mathrm{H})} - c_{nS}(\mu\mathrm{H})\right]}_{=-6\times 10^{-5}\text{ for n = 1} \quad =-5\times 10^{-5}\text{ for n = 2}}$$

$$E^{\mathrm{Z+pol}}_{nS-\mathrm{hfs}}(\mu\mathrm{H}) \cong \frac{E_{\mathrm{F}}(\mu\mathrm{H})\, m_r(\mu\mathrm{H})\, b_{nS}(\mu\mathrm{H})}{n^3 E_{\mathrm{F}}(\mathrm{H})\, m_r(\mathrm{H})\, b_{1S}(\mathrm{H})} E^{\mathrm{Z+pol}}_{1S-\mathrm{hfs}}(\mathrm{H})$$

$$b_{1S}(\mathrm{H}) \simeq 1 + 2 \times 10^{-5} + 0.01846 - 5\alpha/4\pi$$

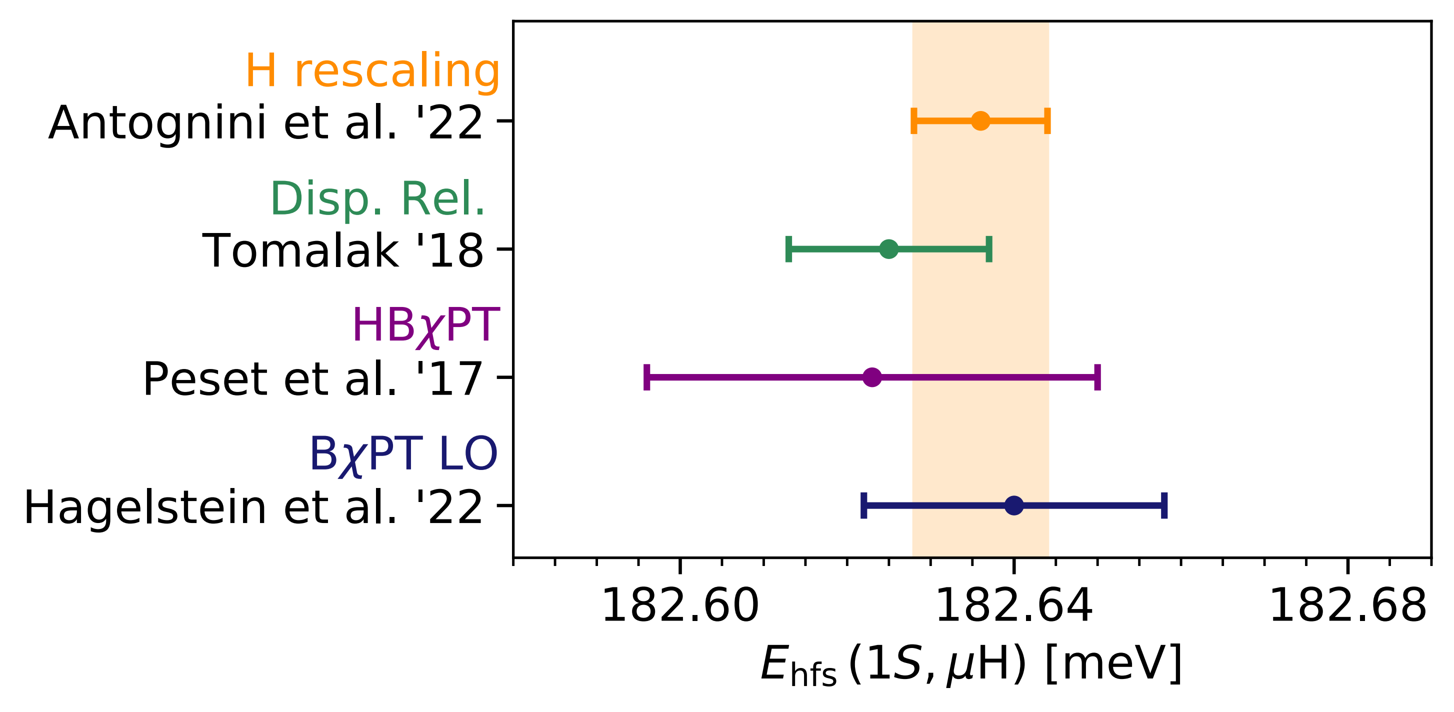
$$b_{1S}(\mu\mathrm{H}) \simeq 1 + 0.00402 + 0.01846 - 5\alpha/4\pi$$

eVP

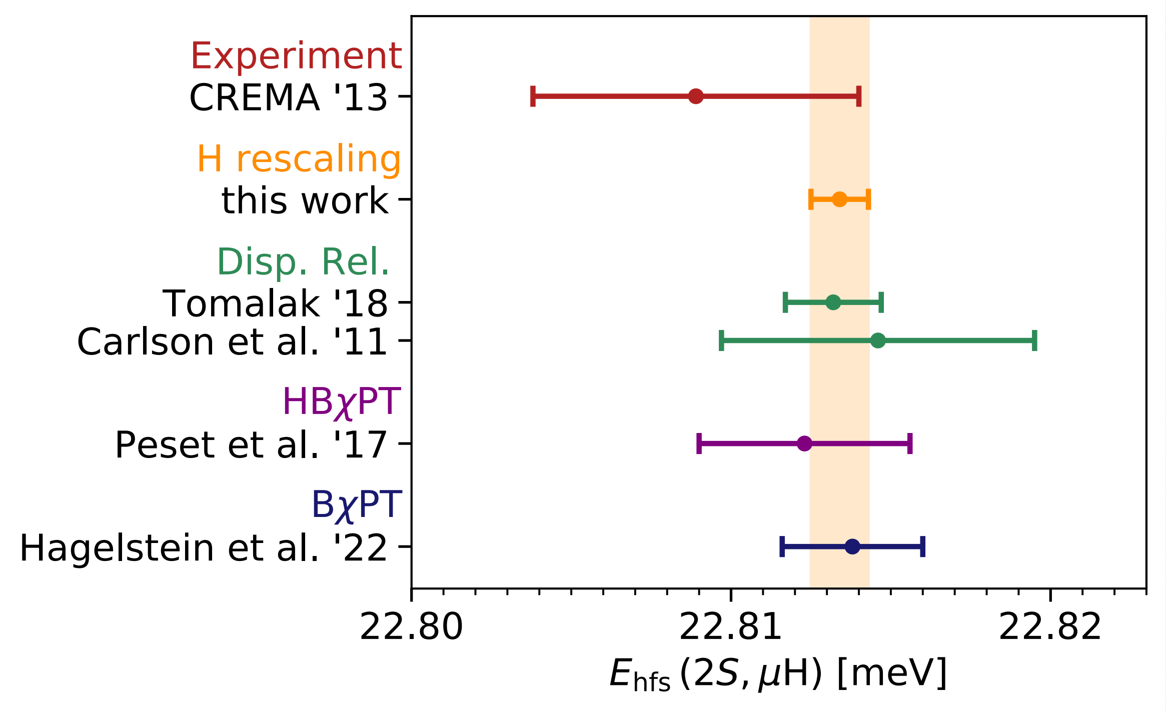
UNCERTAINTY DUE TO ZEMACH AND POLARISABILITY – ELIMINATED!

RESULTING PREDICTIONS

1S



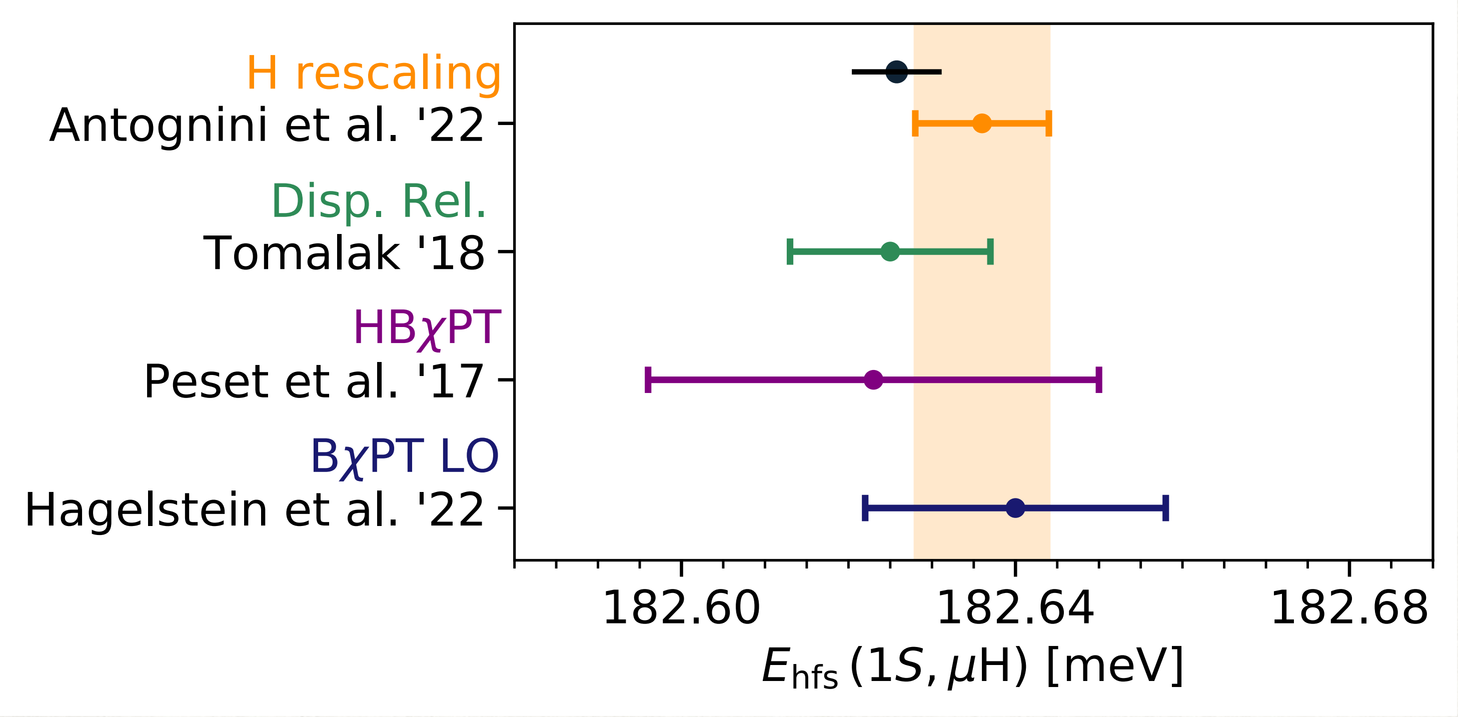
2S



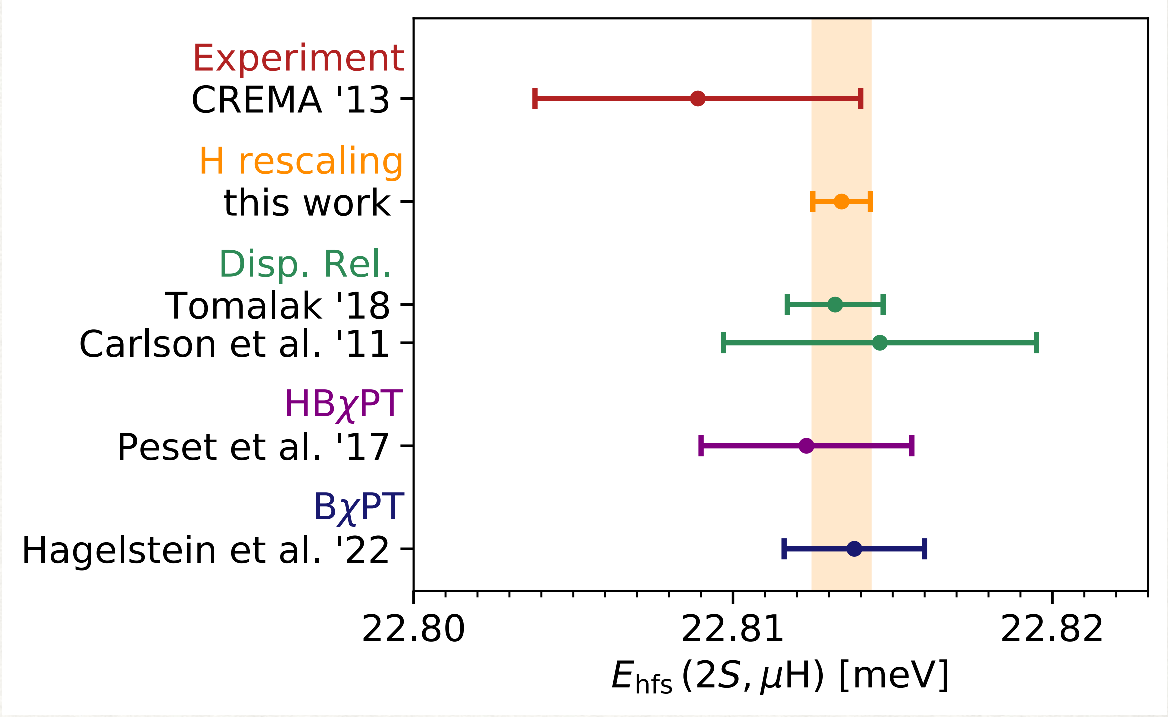
$$E_{1S\text{-HFS}}(\mu\mathrm{H}) = 182.636(8)\,\mathrm{meV}$$

RESULTING PREDICTIONS

1S



2S



$$E_{1S\text{-HFS}}(\mu\mathrm{H}) = 182.636(8)\,\mathrm{meV}$$

$$E_{\rm hfs} = 182\,626(5)\ \mu\mathrm{eV}.$$

[Maron, Pantak & Pachucki, arXiv:2606.06930]

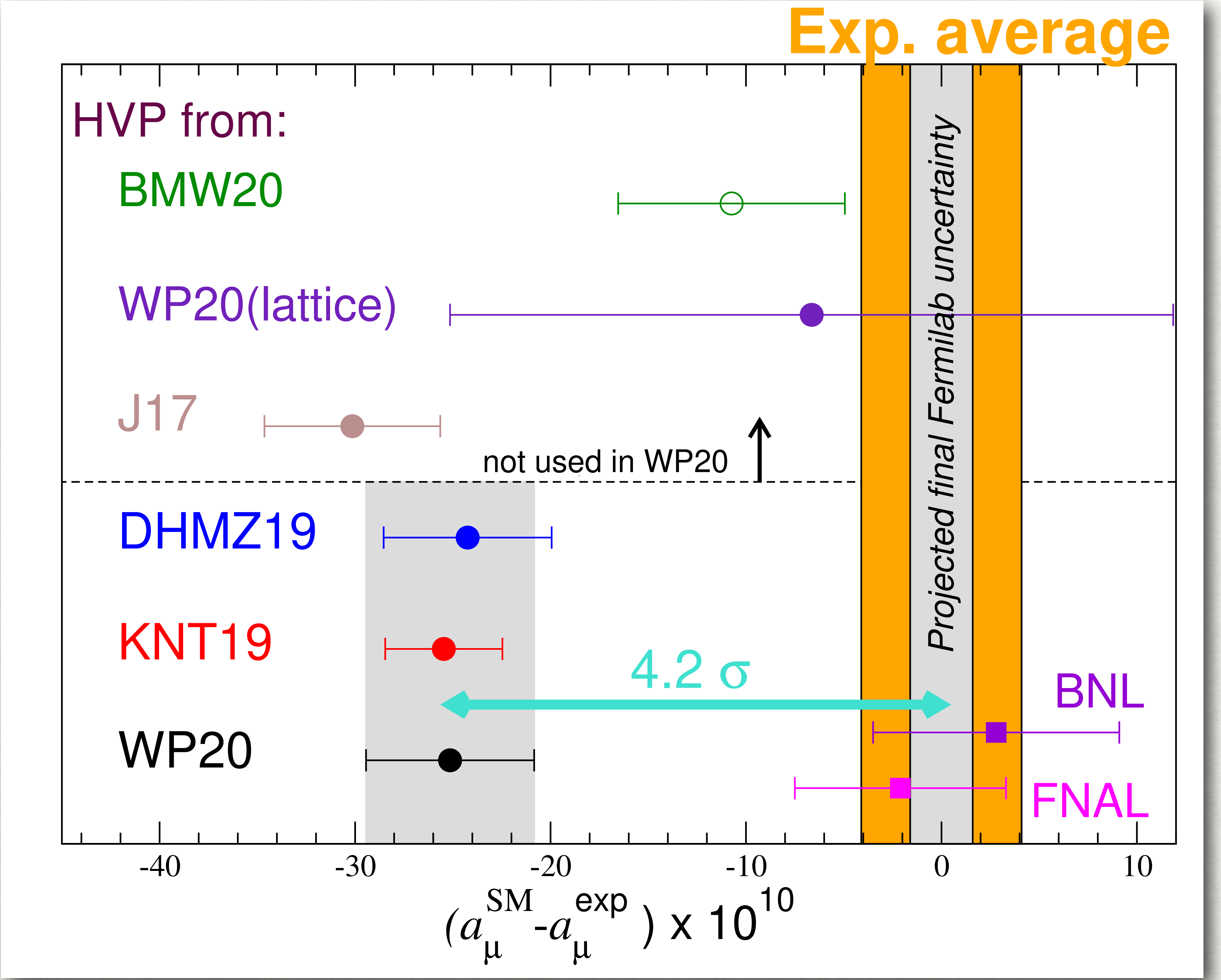
See talk by Pachucki, poster by Maron

PART II

MASS SCALING IN LEPTON $g-2$

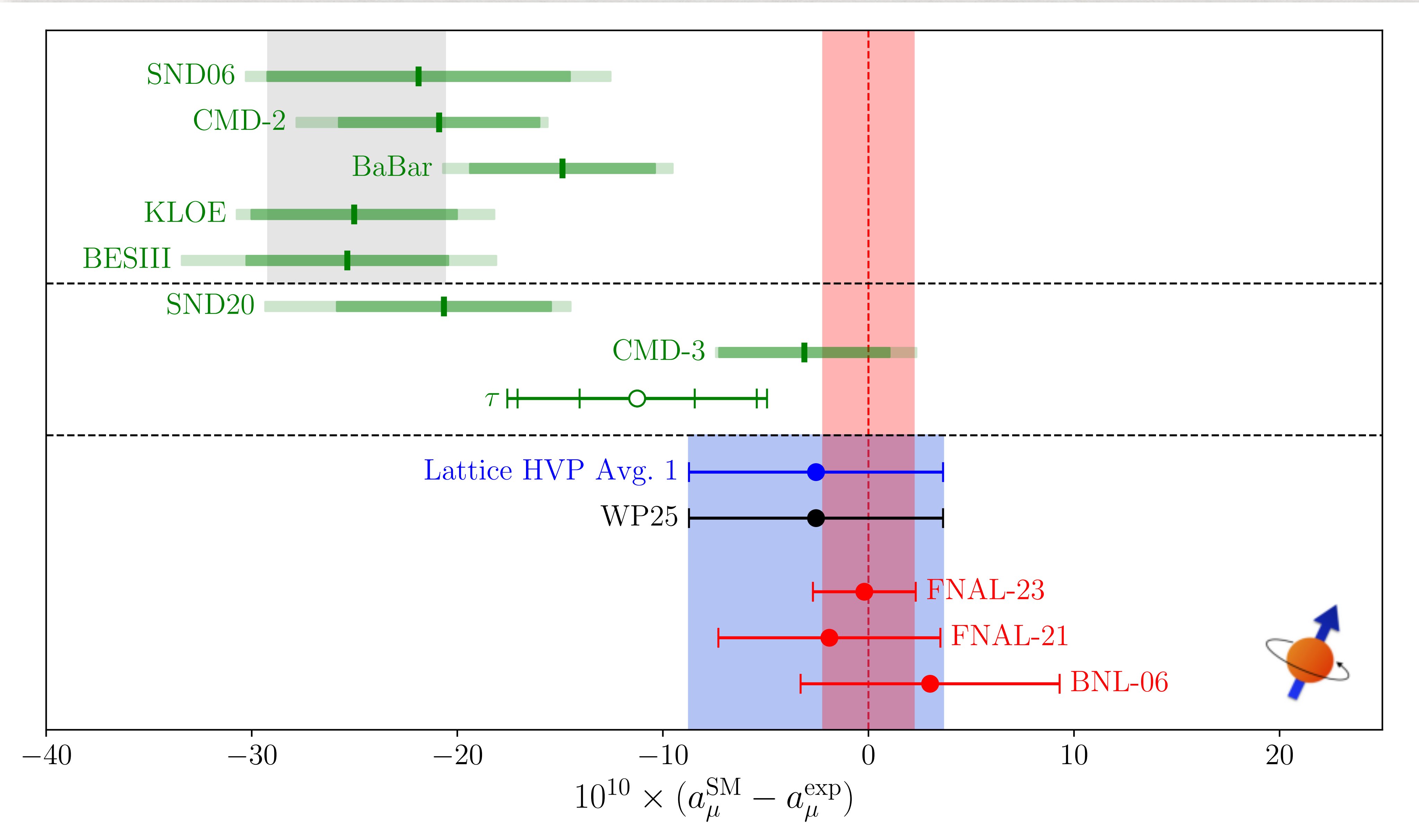
MUON ANOMALY

"THEORY INITIATIVE" WP 2020 VS 2025



MUON ANOMALY

"THEORY INITIATIVE" WP 2020 VS 2025



To appear: **The muon $g-2$ versus the electron as a natural window quantity**

Siyuan Li and Vladimir Pascalutsa
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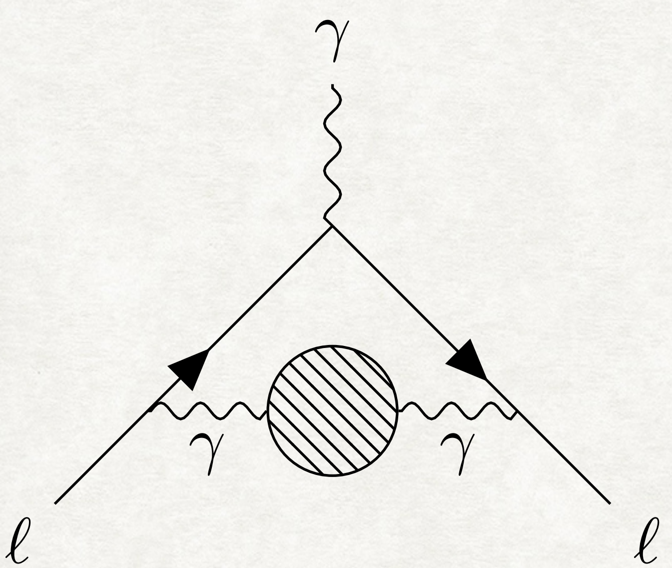
Maxim Pospelov
William I. Fine Theoretical Physics Institute, University of Minnesota, MN 55455, United States

$$a_{\mu-e} \equiv a_\mu - \frac{m_\mu^2}{m_e^2} a_e$$



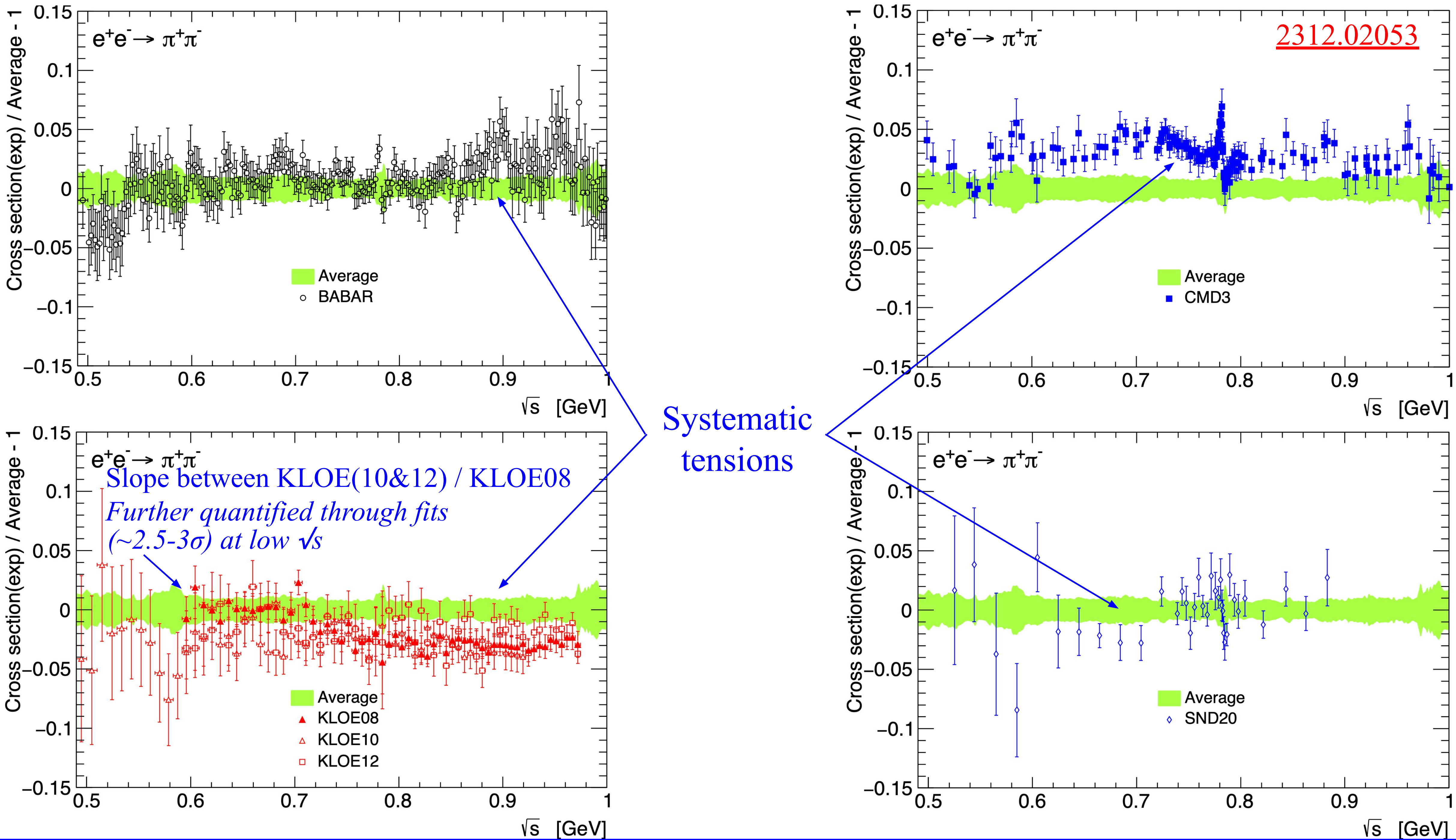
LO HVP	$a_\mu\,(10^{-10})$	$a_e\,(10^{-14})$	$a_{\mu-e}\,(10^{-10})$
lattice QCD	713.2(6.1) [6]	189.3(8.2) [8]	−96.1(29.0)
data-driven	692.8(2.4) [9]	186.10(66) [9]	−102.8(4)
discrepancy	20.4(6.6)	3.2(8.2)	6.7(29.0)

$$a_e^{\mathrm{HVP}} \cong \frac{\alpha m_e^2}{3\pi}\Pi'(0) \equiv \frac{\alpha}{3\pi^2} m_e^2 \int_{s_0}^{\infty} \mathrm{d}s\, \frac{\mathrm{Im}\,\Pi(s)}{s^2}$$

[8] S. Borsanyi *et al.* (Budapest-Marseille-Wuppertal), Hadronic vacuum polarization contribution to the anomalous magnetic moments of leptons from first principles, Phys. Rev. Lett. **121**, 022002 (2018), arXiv:1711.04980 [hep-lat].
[9] A. Keshavarzi, D. Nomura, and T. Teubner, $g-2$ of charged leptons, $\alpha(M_Z^2)$, and the hyperfine splitting of muonium, Phys. Rev. **D 101**, 014029 (2020),

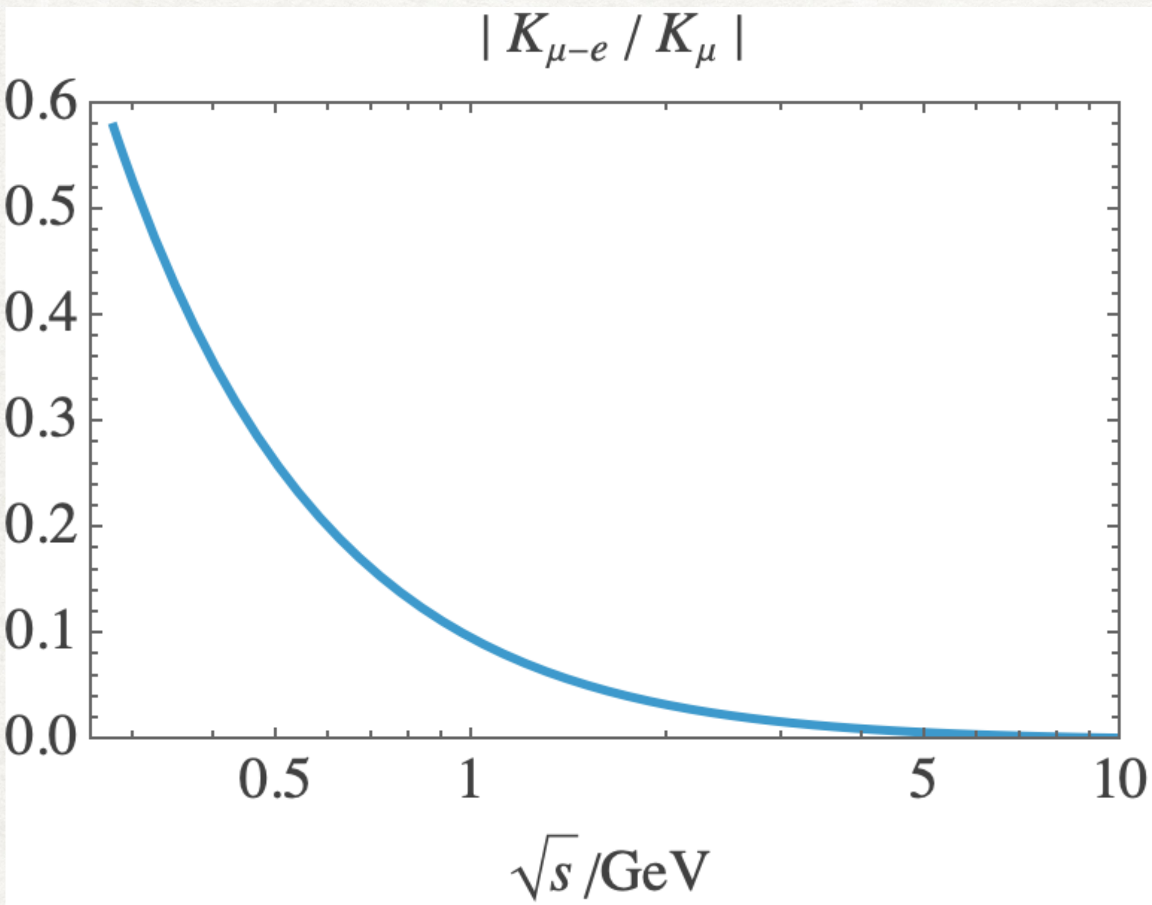
DATASET INCONSISTENCIES

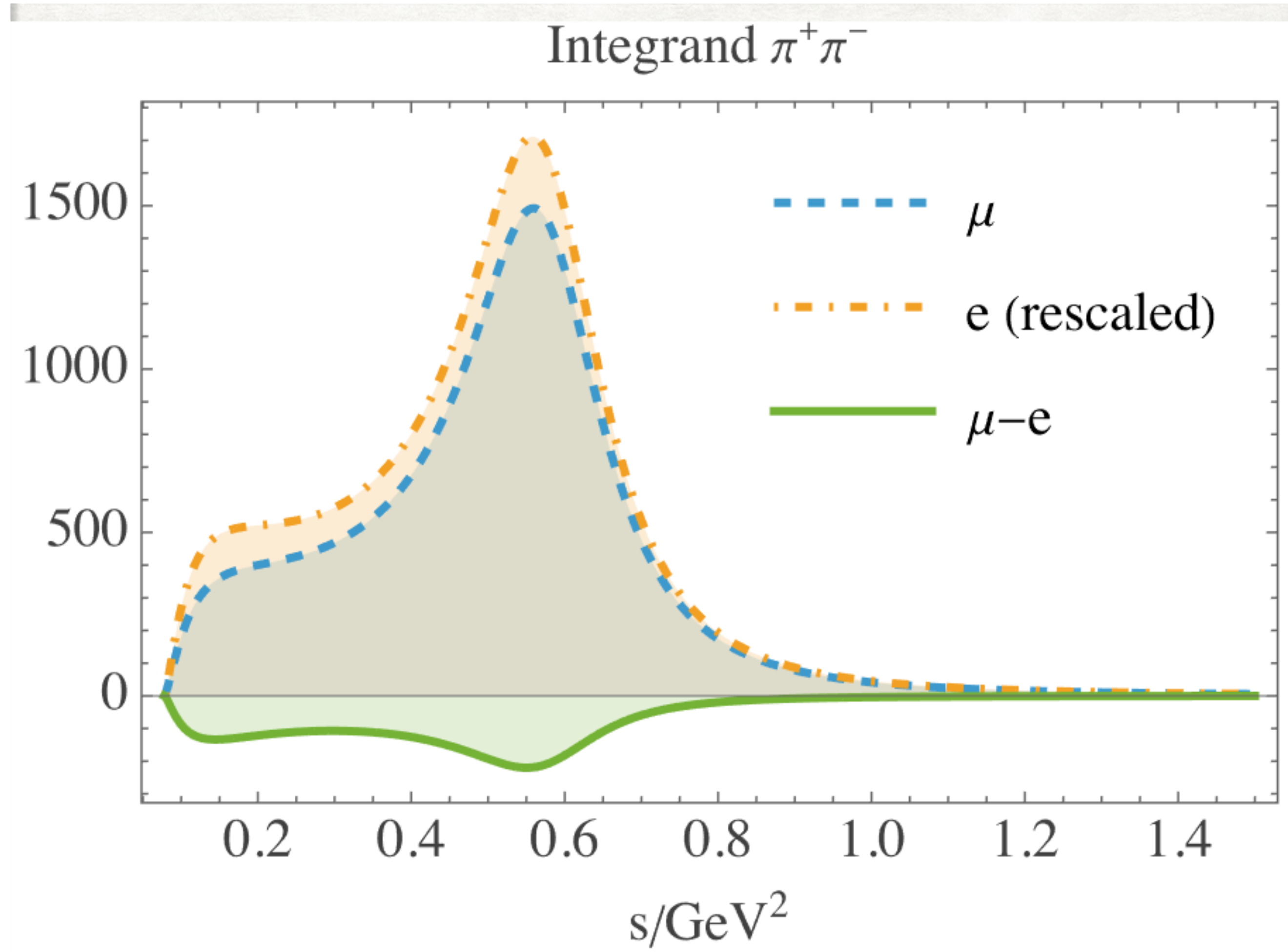
Combining the $e^+e^- \rightarrow \pi^+\pi^-$ data: relative differences



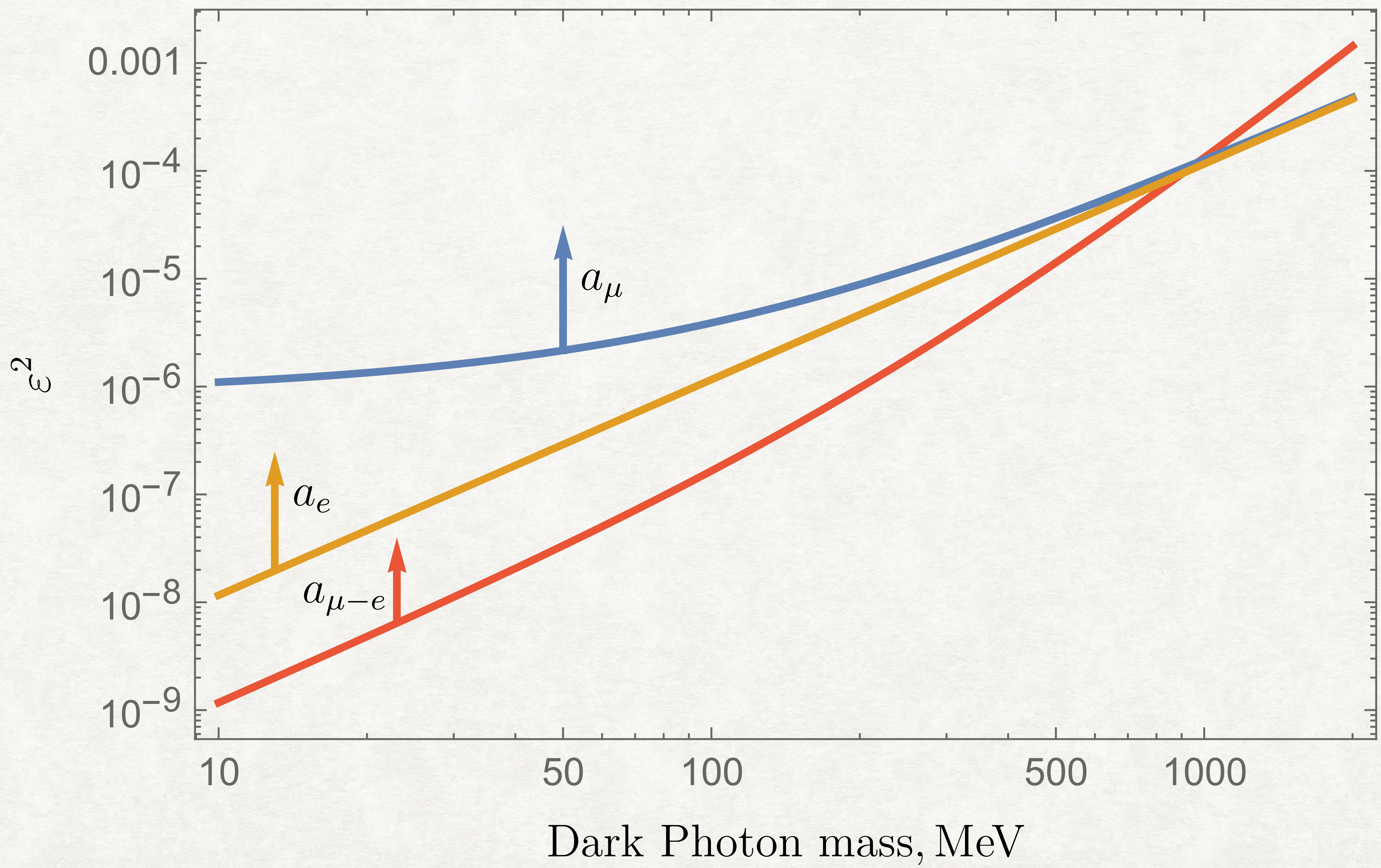
$a_{\mu-e}$ PROVIDES A SUB-GEV WINDOW

$$a_\ell^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{s_0}^{\infty} \mathrm{d}s\, \frac{\mathrm{Im}\,\Pi(s)}{s} K_\ell(s)$$



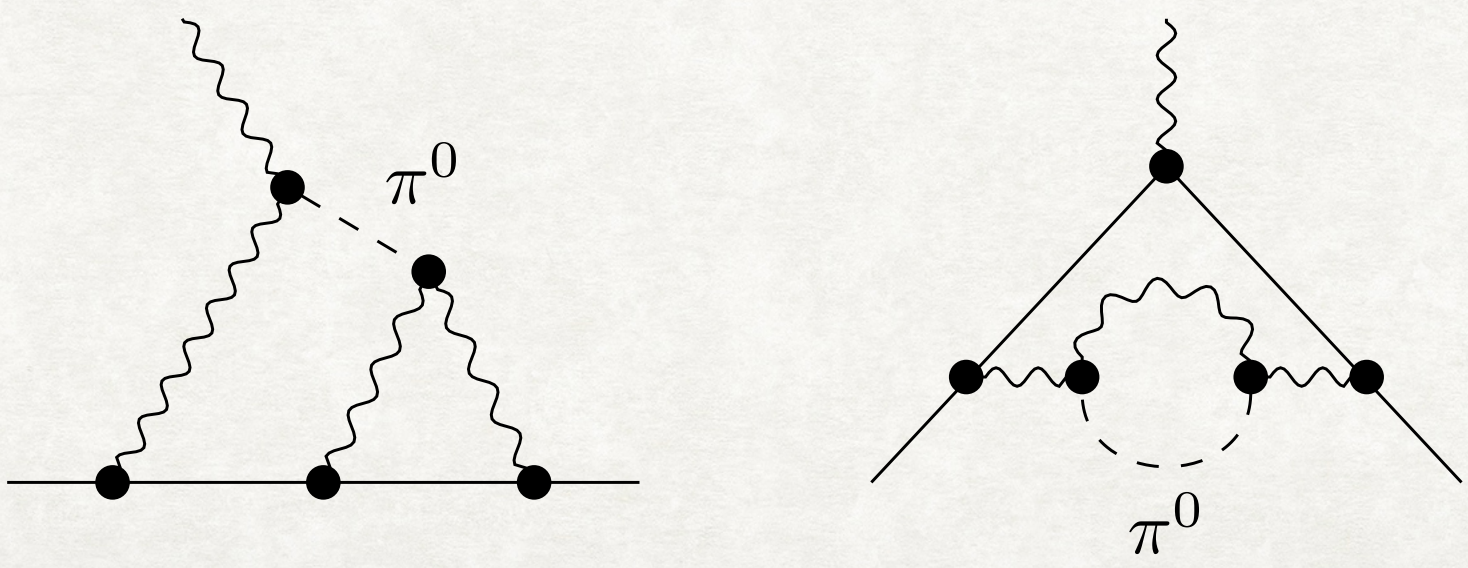


ALSO FOR NEW-PHYSICS SEARCHES



EFFECTIVE-FIELD-THEORY (EFT) PREDICTIONS FOR $a_{\mu-e}$

- π^0, η, η' contributions via light-by-light
- $\pi^0\gamma$ contr. (isospin breaking)
- ALP contributions (Barr-Zee, etc.)



CONCLUSION

- $\mu - e$ quantities are useful for μH HFS search
- sorting out $(g-2)_\mu$
- and New Physics searches... — stay tuned!