Atomic parity violation in highly charged ^{40,48}Ca and ^{208}Pb

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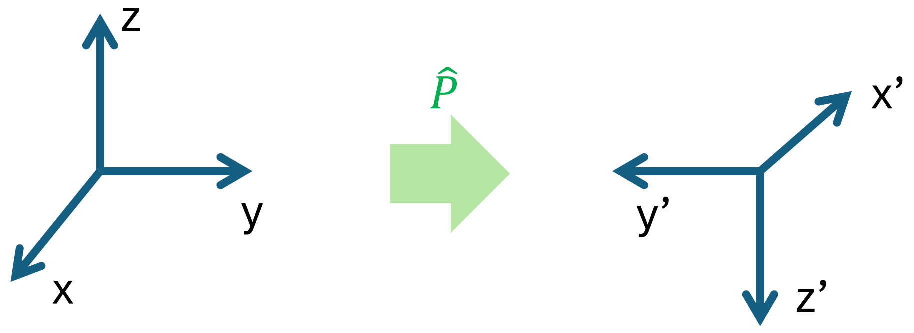
FPM

DFG Deutsche Forschungsgemeinschaft

Parity Violation

Parity operator: $\hat{P}\,\psi(\vec{r},t) = \psi(-\vec{r},t)$



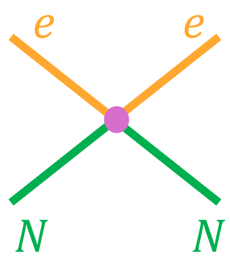
Parity *not* conserved in weak interaction.

Weak interaction bosons $W^{\pm}$ (m_W= 80 GeV), Z^0 (m_Z= 91 GeV) – heavy; pointlike interaction

Z^0-exchange between an electron and protons/neutrons => **atomic parity violation**.

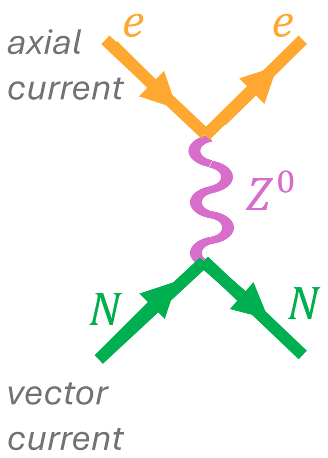
Atomic Parity Violation

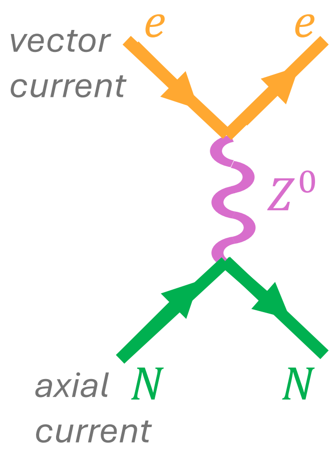
"Fermi theory": Z^0-boson exchange as a point-like interaction



$$G_F \approx 1.166 \times 10^{-5}\ (\hbar c)^3\ \mathrm{GeV}^{-2}$$ – Fermi constant

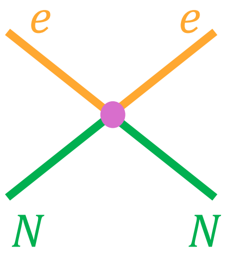
$$\hat{H}_{\mathrm{APV}} = \frac{G_F}{\sqrt{2}} \sum_N \underbrace{C_{1N}\, \bar{e}\, \gamma_\mu \gamma_5 e\ \bar{N} \gamma_\mu N}_{\text{Doesn't depend on nuclear spin}} + \underbrace{C_{2N}\, \bar{e}\, \gamma_\mu e\ \bar{N} \gamma_\mu \gamma_5 N}_{\text{Depends on nuclear spin}}$$





Atomic Parity Violation

"Fermi theory": Z^0-boson exchange as a point-like interaction



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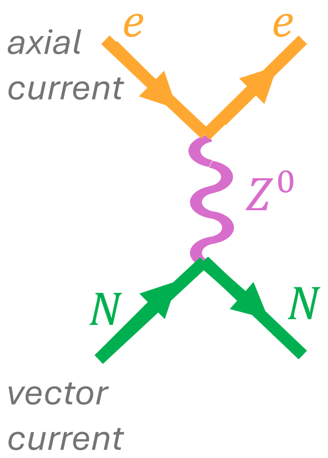
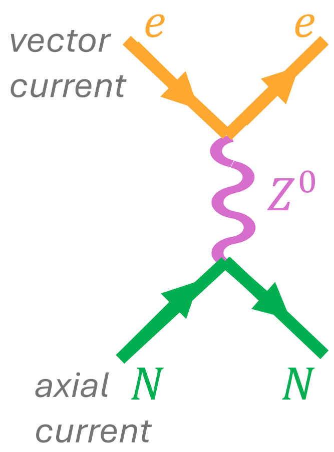
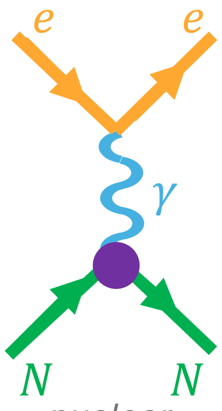
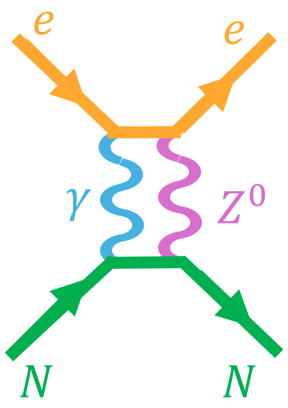
There are more spin-dependent APV effects:







nuclear anapole moment



HFS & Z^0

Spin-Independent APV

$$\hat{H}_{\text{SI}} = \frac{G_F}{\sqrt{2}} \sum_N \underbrace{C_{1N}\, \bar{e}\, \gamma_\mu \gamma_5 e\, \bar{N} \gamma_\mu N}_{\text{Doesn't depend on nuclear spin}}$$

(nucleons slow, non-relativistic)

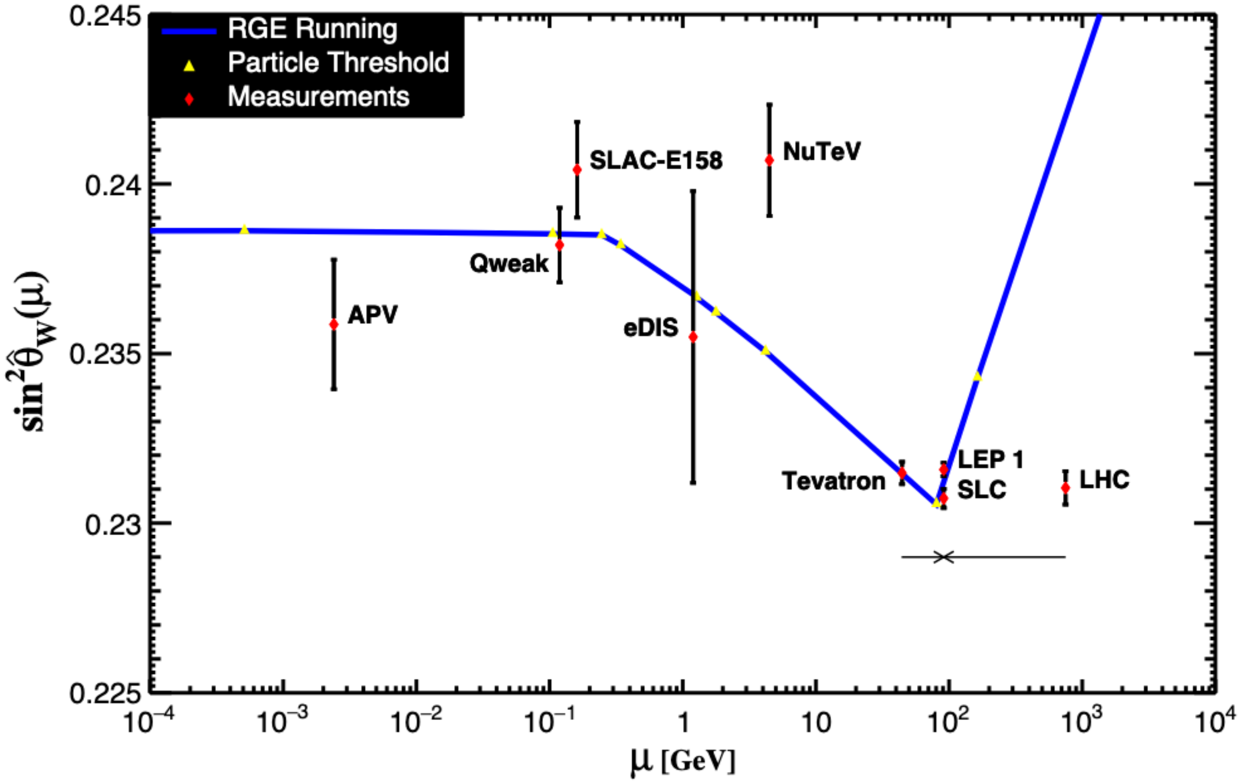
$$\hat{h}_{\text{SI}} = \frac{G_F}{\sqrt{2}} \left[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r}) \right] \gamma_5$$

proton density   *neutron density*

$$|C_{1n}| \gg |C_{1p}|$$

$$C_{1n} = -\frac{1}{2} \qquad C_{1p} = \frac{1}{2}(1 - 4\sin^2\theta_W)$$

Effect comes mostly from neutrons!



θ_W – Weinberg angle, $\sin^2\theta_W \approx 0.239$

(low energy SM prediction)

Spin-Independent APV

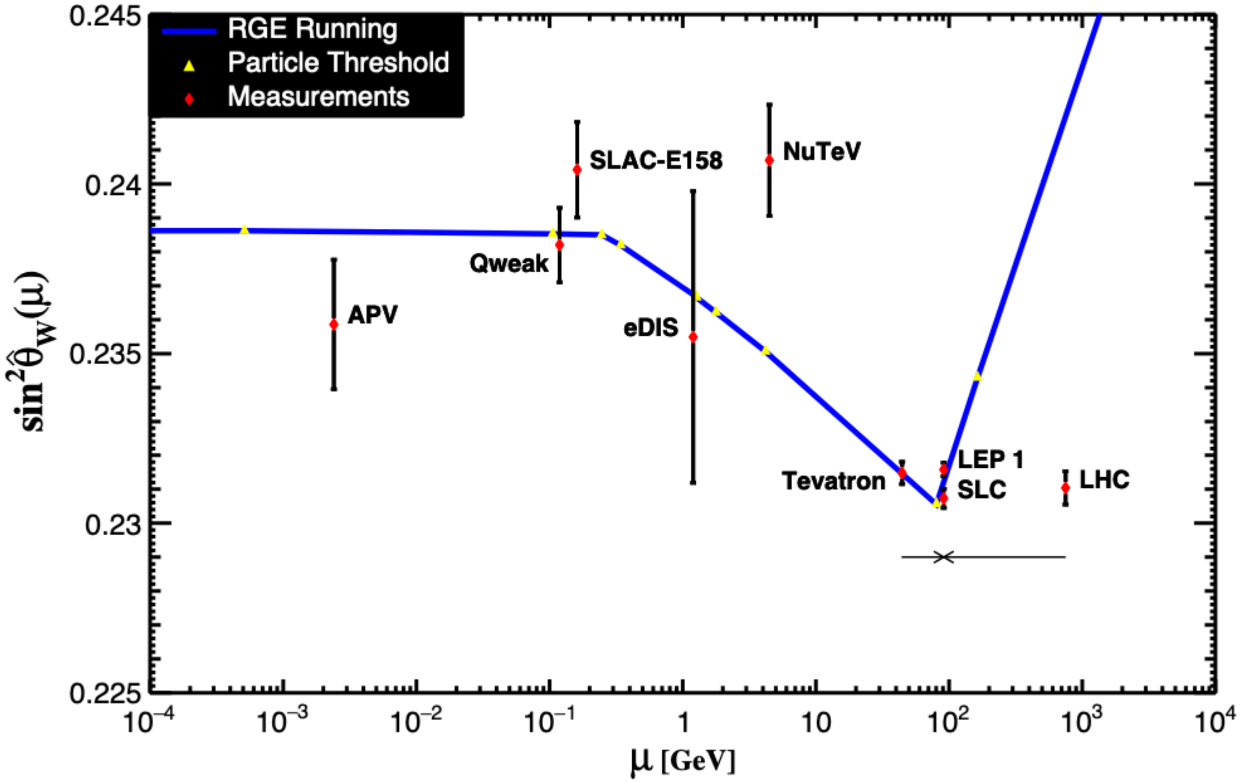
$$\hat{H}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}} \sum_N \underbrace{C_{1N}\, \bar{e}\, \gamma_\mu \gamma_5 e\, \bar{N} \gamma_\mu N}$$

Doesn't depend on nuclear spin

(nucleons slow, non-relativistic)

$$\hat{h}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}} \left[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r}) \right] \gamma_5$$

proton density *neutron density*

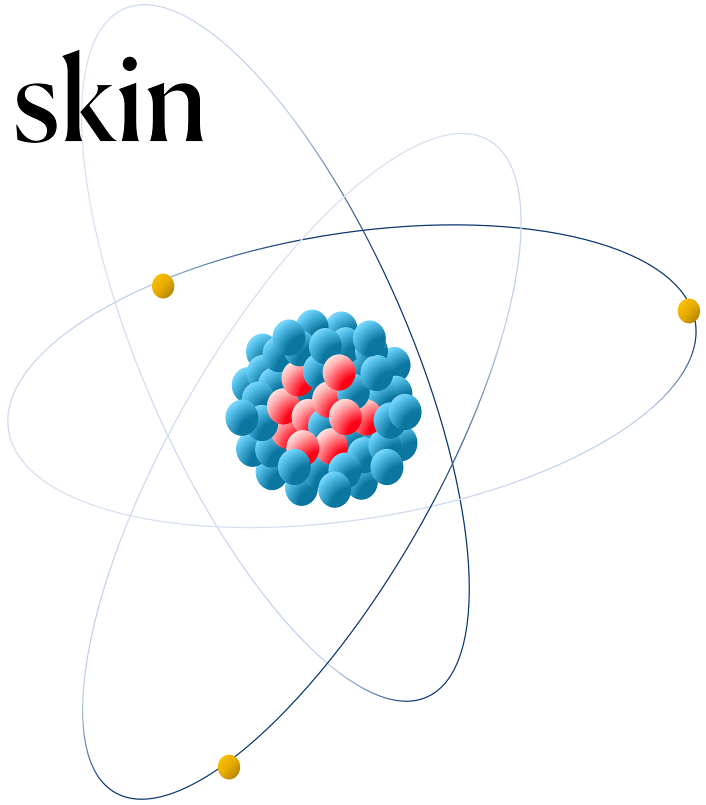


Assuming $\rho_p(\vec{r}) \approx \rho_n(\vec{r}) \approx \rho(\vec{r})$

$$\hat{h}_{\mathrm{SI}} = \frac{G_F}{2\sqrt{2}} Q_W\, \rho(\vec{r})\, \gamma_5 \qquad Q_W = Z(1 - 4\sin^2\theta_W) - N$$

weak charge

Spin-Independent APV: neutron skin



$$\hat{h}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}}\left[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r})\right]\gamma_5$$

proton density *neutron density*

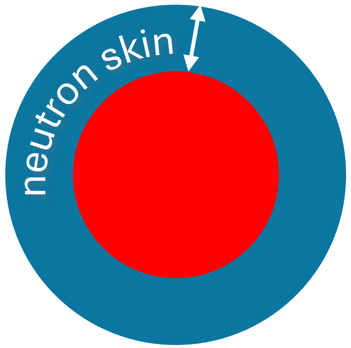
$$|C_{1n}| \gg |C_{1p}|$$

$$\hat{h}_{\mathrm{SI}} \sim \rho_n(\vec{r})$$

Neutron-skin correction to the weak charge Q_W :

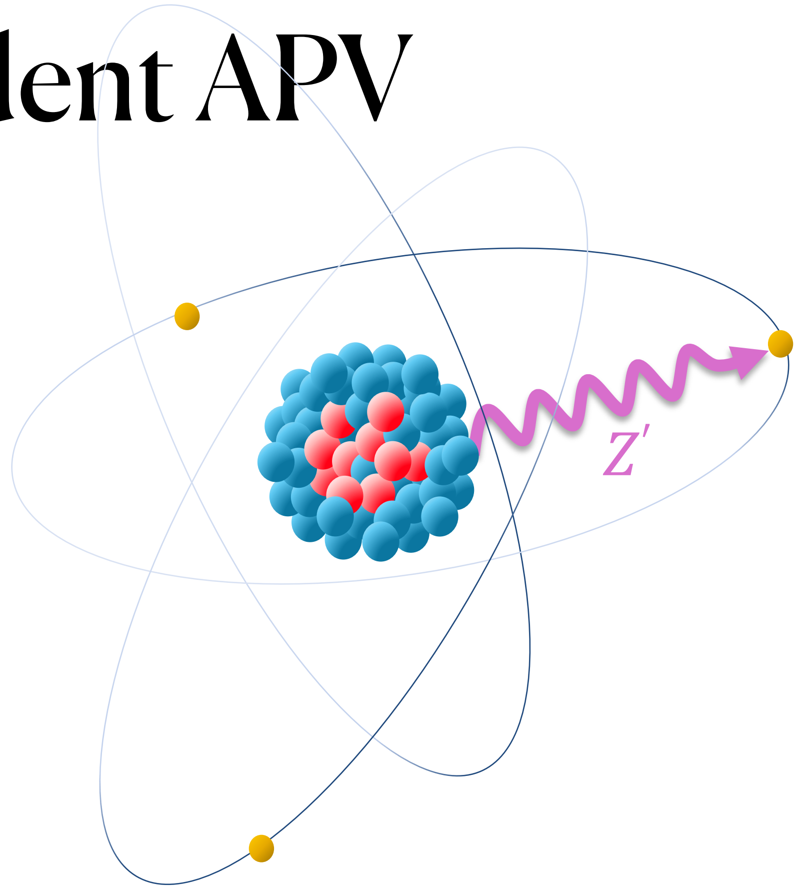
$$\hat{h}_{\mathrm{SI}} = \frac{G_F}{2\sqrt{2}} Q_W\, \rho(\vec{r})\, \gamma_5$$

$$Q_W = Z(1 - 4\sin^2\theta_W) - N + N\,\Delta\rho_{\mathrm{np}}$$



✨New Physics✨ Spin-Independent APV

Consider a Z' boson of mass $m_{Z'} \ll m_{Z^0}$.
Interaction between electrons and nucleons no longer pointlike.



Modelled with a Yukawa potential:

$$V(\vec{r}_e - \vec{R}_N) = \frac{g_{eN}}{4\pi} \frac{e^{-m_{Z'}|\vec{r}_e - \vec{R}_N|}}{|\vec{r}_e - \vec{R}_N|} \gamma_5$$

Integrated over nucleus:

$$\hat{h}_{Z'} = \frac{g_{eN}}{4\pi} N_N \gamma_5 \int \rho_{nuc}(\vec{R}_N) \frac{e^{-m_{Z'}|\vec{r}_e - \vec{R}_N|}}{|\vec{r}_e - \vec{R}_N|} d\vec{R}_N$$

Likewise, may be parametrized as a correction to the weak charge Q_W :

$$Q_W = Z(1 - 4\sin^2\theta_W) - N + N\Delta\rho_{\mathrm{np}} + Z\Delta Q_p^{\mathrm{new}} + N\Delta Q_n^{\mathrm{new}}$$

neutron skin | *proton 'new physics' APV* | *neutron 'new physics' APV*

Our Isotopes

$$Q_W = Z(1 - 4\sin^2\theta_W) - N + N\Delta\rho_{\mathrm{np}} + Z\Delta Q_p^{\mathrm{new}} + N\Delta Q_n^{\mathrm{new}}$$

neutron skin

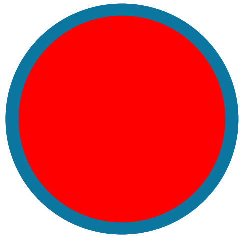
proton 'new physics' APV

neutron 'new physics' APV

^{40}Ca

$Z = N = 20$

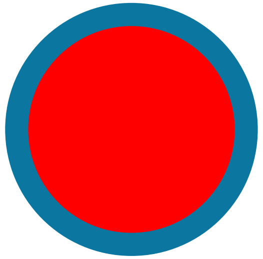
doubly magic 🦄 🦄



^{48}Ca

$Z = 20,\ N = 28$

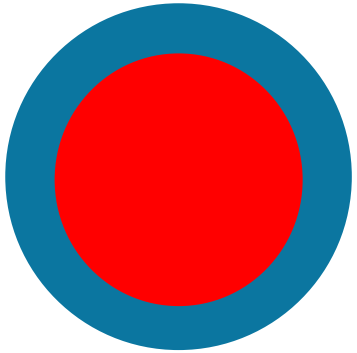
doubly magic 🦄 🦄



^{208}Pb

$Z = 82,\ N = 126$

doubly magic 🦄 🦄



Our Isotopes

$$Q_W = Z(1 - 4\sin^2\theta_W) - N + N\Delta\rho_{\text{np}} + Z\Delta Q_p^{\text{new}} + N\Delta Q_n^{\text{new}}$$

neutron skin — *proton 'new physics' APV* — *neutron 'new physics' APV*

^{40}Ca

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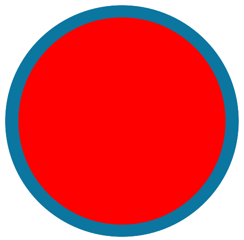


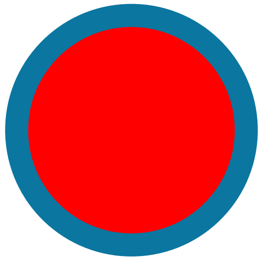
$r_p = 3.480$ fm

$\approx$

^{48}Ca

$Z = 20,\ N = 28$

doubly magic

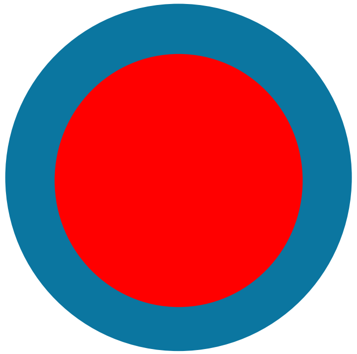


$r_p = 3.478$ fm

^{208}Pb

$Z = 82,\ N = 126$

doubly magic



neutron skin :

$\Delta r_{\text{np}} = -0.01(1)\ \text{fm} \approx 0$

$\Delta r_{\text{np}} = 0.121(35)$ fm

(CREX 2022)

$\Delta r_{\text{np}} = 0.283(71)$ fm

(PREX-II 2021)

Our Isotopes

$$Q_W = Z(1 - 4\sin^2\theta_W) - N + N\Delta\rho_{\mathrm{np}} + Z\Delta Q_p^{\mathrm{new}} + N\Delta Q_n^{\mathrm{new}}$$

neutron skin

proton 'new physics' APV

neutron 'new physics' APV

^{40}Ca

$Z = N = 20$

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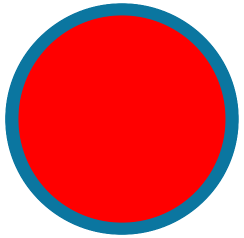

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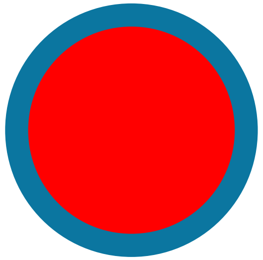

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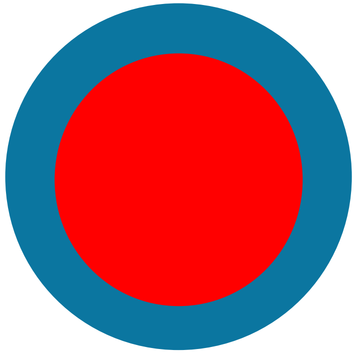

We consider **H- and Li-like** ions with these nuclei
& calculate matrix elements and APV amplitudes

APV in H- and Li-like ions

- APV-induced E1 amplitude between $a \to b$ atomic levels

$$\varepsilon_{\mathrm{E1,APV}}^{a\to b} = \sum_n \left(\frac{\langle b|\hat{h}_{\mathrm{APV}}|n\rangle\langle n|\hat{D}|a\rangle}{E_b - E_n} + \frac{\langle b|\hat{D}|n\rangle\langle n|\hat{h}_{\mathrm{APV}}|a\rangle}{E_a - E_n} \right)$$

$\hat{h}_{\mathrm{APV}}$: standard model

$$\hat{h}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}}\left[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r})\right]\gamma_5$$

or ✨ new physics ✨

$$\hat{h}_{Z'} = \frac{g_{eN}}{4\pi} N_N \gamma_5 \int \rho_{nuc}(\vec{R}_N) \frac{e^{-m_{Z'}|\vec{r}_e - \vec{R}_N|}}{|\vec{r}_e - \vec{R}_N|} d\vec{R}_N$$

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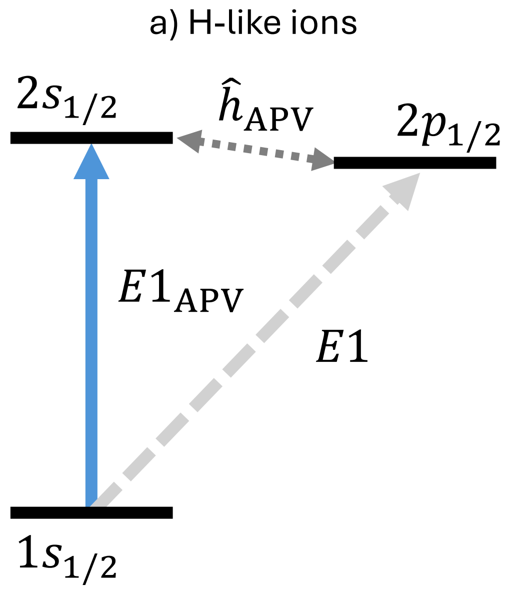
$$\hat{h}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}}\left[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r})\right]\gamma_5$$

or ✨ new physics ✨

$$\hat{h}_{Z\prime} = \frac{g_{eN}}{4\pi} N_N \gamma_5 \int \rho_{nuc}(\vec{R}_N) \frac{e^{-m_{Z\prime}|\vec{r}_e - \vec{R}_N|}}{|\vec{r}_e - \vec{R}_N|} d\vec{R}_N$$

- H-like ions ($1s \to 2s$):

  term with $\mathrm{m} = \langle 2s|\hat{h}_{\mathrm{APV}}|2p_{1/2}\rangle$ is dominant



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or ✨ new physics ✨

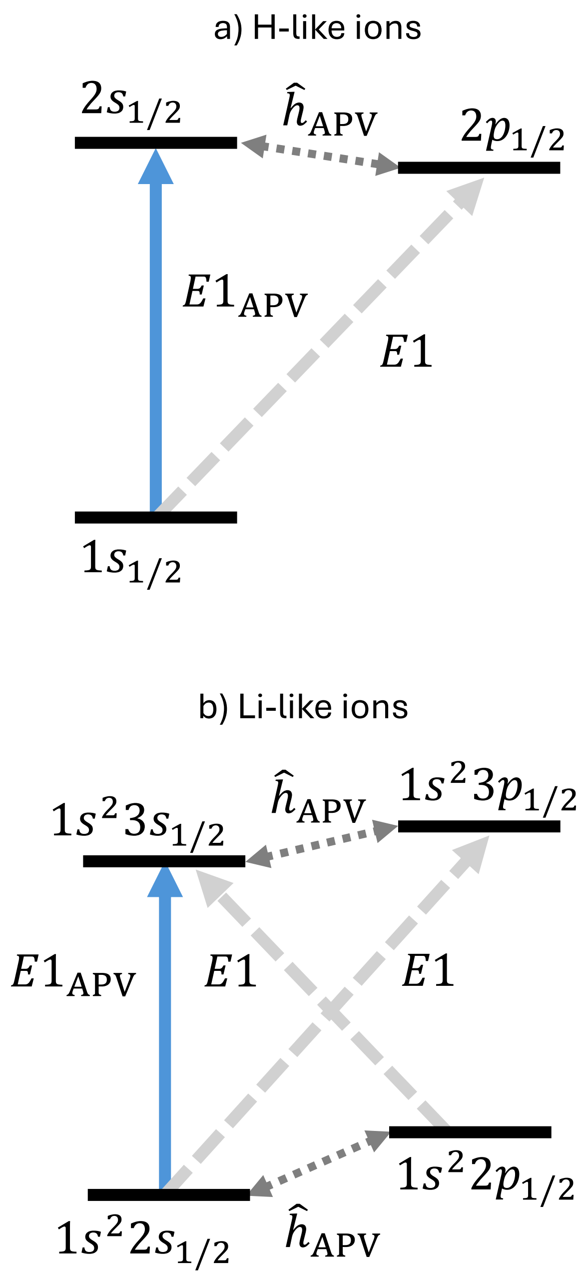
$$\hat{h}_{Z'} = \frac{g_{eN}}{4\pi} N_N \gamma_5 \int \rho_{nuc}(\vec{R}_N) \frac{e^{-m_{Z'}|\vec{r}_e - \vec{R}_N|}}{|\vec{r}_e - \vec{R}_N|} d\vec{R}_N$$

- H-like ions ($1s \to 2s$):

  term with $\mathrm{m} = \langle 2s|\hat{h}_{\mathrm{APV}}|2p_{1/2}\rangle$ is dominant

- Li-like ions ($1s^2 2s \to 1s^2 3s$):

  terms with $\langle 1s^2 2p_{1/2}|\hat{h}_{\mathrm{APV}}|1s^2 2s\rangle$ and $\langle 1s^2 3s|\hat{h}_{\mathrm{APV}}|1s^2 3p_{1/2}\rangle$ are dominant



APV Calculation: Input parameters

$$\varepsilon^{a\to b}_{\text{E1,APV}} = \sum_n \left(\frac{\langle b|\hat{h}_{\text{APV}}|n\rangle\langle n|\hat{D}|a\rangle}{E_b - E_n} + \frac{\langle b|\hat{D}|n\rangle\langle n|\hat{h}_{\text{APV}}|a\rangle}{E_a - E_n} \right)$$

incl. **frequency-dependent** corrections to dipole ME

H-like energy levels tabulated in Yerokhin & Shabaev, J. of Phys. and Chem. Ref. Data 44, 033103 (2015).

Li-like energy levels (CI-QEDMOD & GRASP):

Level	E_n (^{40}Ca), eV	ΔE_n (^{40,48}Ca), eV	E_n (Pb), eV
$1s^22s$	0	0	0
$1s^22p$	35.961(1)	0.0017(4)	230.65(5)
$1s^23s$	651.828(2)	0.0015(4)	14179.98(8)
$1s^23p$	661.783(2)	0.0019(4)	14241.51(8)
$1s^24p$	879.564(2)	0.0022(4)	19037.23(8)
$1s^25p$	980.2(1)	0.0023(4)	21215(2)
$1s^26p$	1035.7(1)	0.0024(4)	22439(2)
$1s^27p$	1067.1(2)	0.0025(4)	23068(64)
$1s^28p$	1088.3(2)	0.0025(4)	23514(64)

Nuclear Fermi parameters (analysis of experimental data):

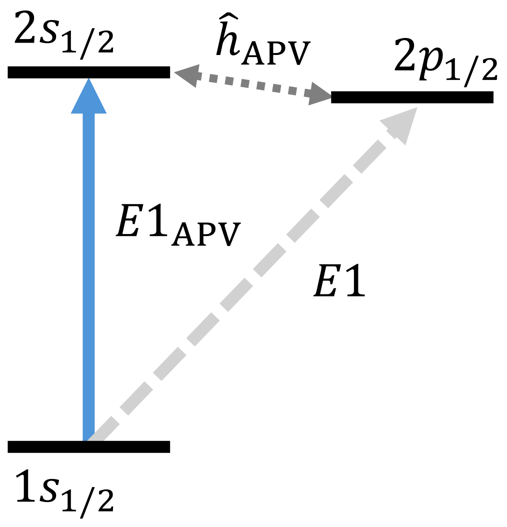
Isotope	c, fm	z, fm	r_{rms}, fm
^{40}Ca	3.629	0.552	3.480
	3.803	0.493	3.476
^{48}Ca	3.749	0.515	3.478
	3.739	0.518	3.478
^{208}Pb	6.663	0.513	5.502
	6.668	0.509	5.501

Results

H-like ions

Standard model: $\hat{h}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}}\left[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r})\right]\gamma_5$

p,n separate: $\hat{h}_{\mathrm{SI},p,n} = \frac{G_F}{2\sqrt{2}}\,\rho_{p,n}(\vec{r})\,\gamma_5$



$$\mathrm{m}_{p,n} = \langle 2s|\hat{h}_{\mathrm{SI},p,n}|2p_{1/2}\rangle$$

$$\varepsilon^{a\to b}_{\mathrm{PV},p,n} = \sum_n \left(\frac{\langle b|\hat{h}_{\mathrm{SI},p,n}|n\rangle\langle n|\hat{D}|a\rangle}{E_b - E_n} + \frac{\langle b|\hat{D}|n\rangle\langle n|\hat{h}_{\mathrm{SI},p,n}|a\rangle}{E_a - E_n} \right)$$

	^{40}Ca	^{48}Ca	^{208}Pb	
$\mathrm{m}_p^{(\mathrm{SM})}$	$-372.14(2)$	$-372.122(3)$	$-826378(24)$	$i\alpha^2 m_e c^2$
$\mathrm{m}_n^{(\mathrm{SM})}$	$-372.15(3)$	$-371.98(4)$	$-819257(1792)$	
$\varepsilon_{\mathrm{PV},p}^{(\mathrm{SM})}$	$-950.12(6)$	$-950.08(4)$	$-2667(3)$	$10^{-15} i e a_B$
$\varepsilon_{\mathrm{PV},n}^{(\mathrm{SM})}$	$-950.15(6)$	$-949.7(1)$	$-2644(7)$	
$\mathcal{M}^{(\mathrm{SM})}$	$6.8327(5) \times 10^3$	$9.772(1) \times 10^3$	$9.72(2) \times 10^7$	$i\alpha^2 m_e c^2$
$\mathcal{E}_{\mathrm{PV}}^{(\mathrm{SM})}$	$1.7445(1) \times 10^4$	$2.4950(3) \times 10^4$	$3.139(8) \times 10^5$	$10^{-15} i e a_B$

~ 1.5x larger than ^{40}Ca

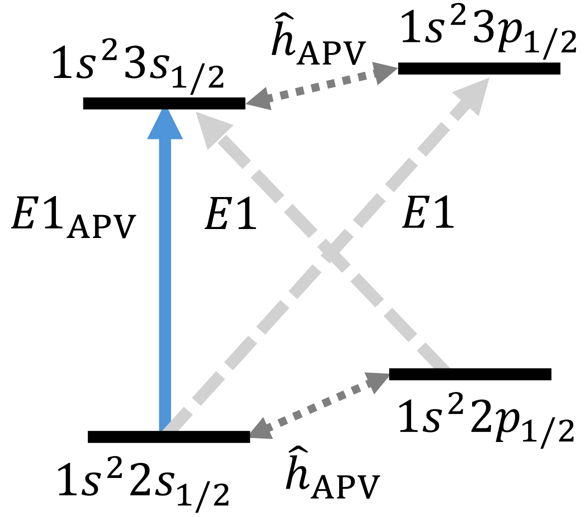
Results

Li-like ions

Standard model: $\hat{h}_{\mathrm{SI}} = \frac{G_F}{\sqrt{2}}[C_{1p} Z \rho_p(\vec{r}) + C_{1n} N \rho_n(\vec{r})]\gamma_5$

p,n separate: $\hat{h}_{\mathrm{SI},p,n} = \frac{G_F}{2\sqrt{2}} \rho_{p,n}(\vec{r})\, \gamma_5$

$$\varepsilon^{a\to b}_{\mathrm{PV},p,n} = \sum_n \left(\frac{\langle b|\hat{h}_{\mathrm{SI},p,n}|n\rangle\langle n|\hat{D}|a\rangle}{E_b - E_n} + \frac{\langle b|\hat{D}|n\rangle\langle n|\hat{h}_{\mathrm{SI},p,n}|a\rangle}{E_a - E_n} \right)$$



	^{40}Ca	^{48}Ca	^{208}Pb
$\varepsilon^{(\mathrm{SM})}_{\mathrm{PV},p}$	9.129(4)	9.128(4)	946(2)
$\varepsilon^{(\mathrm{SM})}_{\mathrm{PV},n}$	9.129(4)	9.125(4)	938(3)
$\mathcal{E}^{(\mathrm{SM})}_{\mathrm{PV}}$	−167.61(7)	−239.7(1)	−111413(369)

$10^{-15} i e a_B$

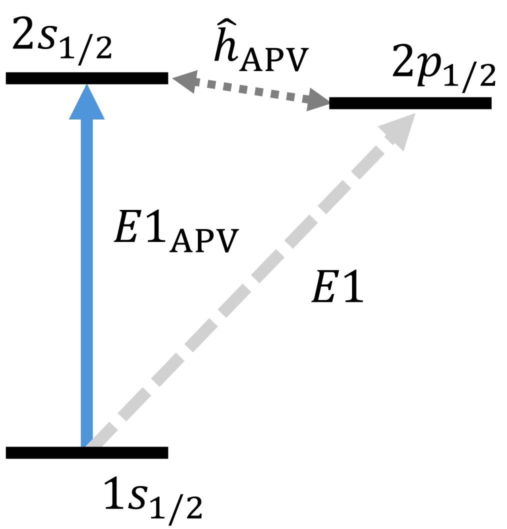
~ 1.5x larger than ^{40}Ca

Results

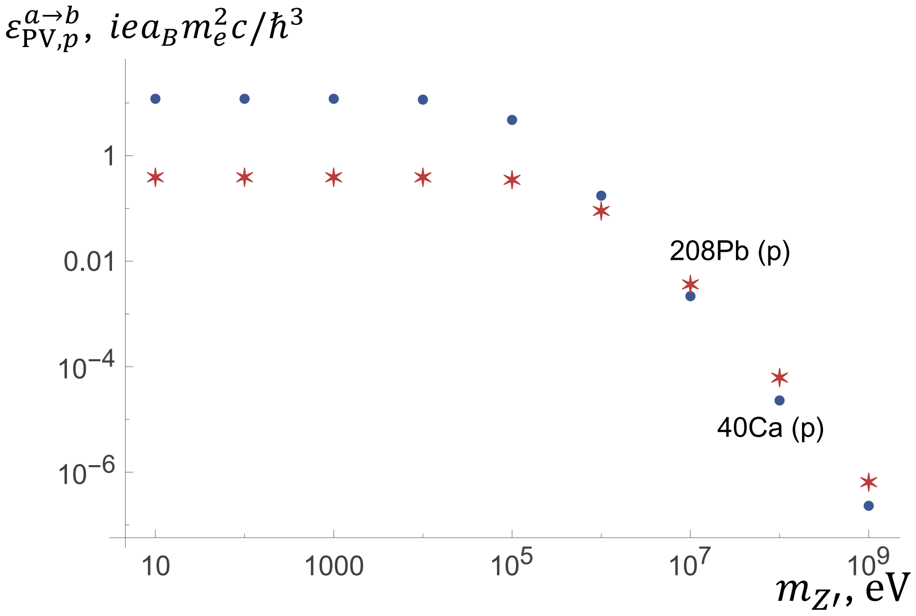
H-like ions

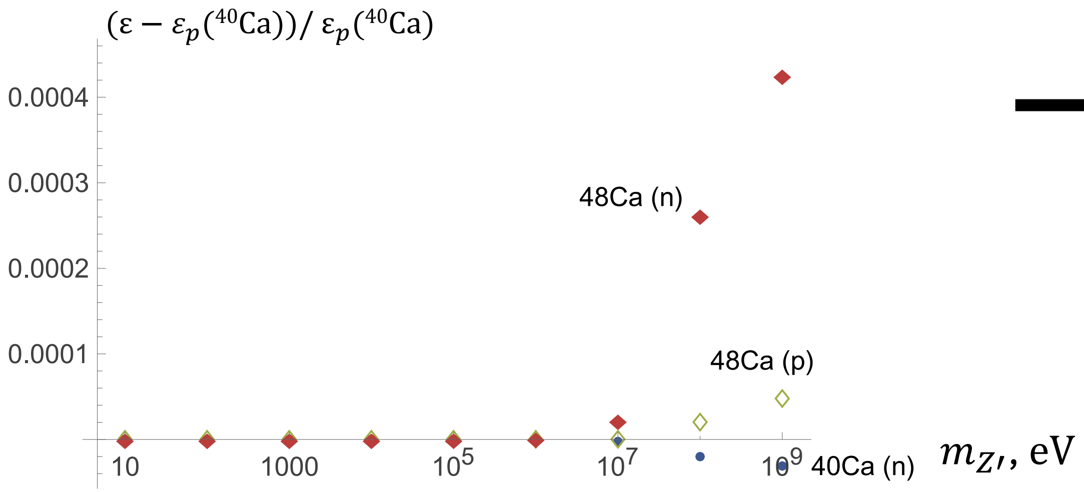
✨New Physics:✨

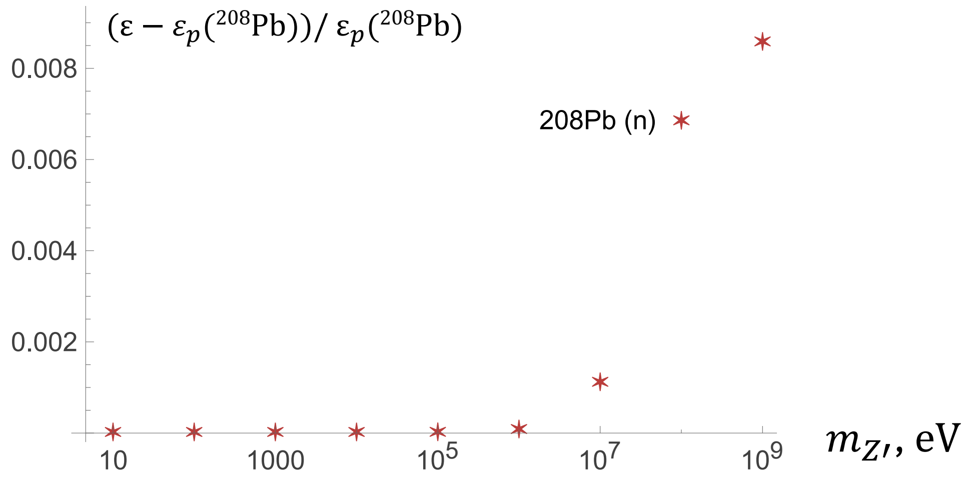
$$\hat{h}_{Z'} = \frac{1}{4\pi}\left(\frac{m_{Z'}c}{\hbar}\right)^2 \gamma_5 \int \rho_{nuc}(\vec{R}_N)\frac{e^{-m_{Z'}|\vec{r}_e-\vec{R}_N|}}{|\vec{r}_e-\vec{R}_N|}d\vec{R}_N$$



$$\varepsilon^{a\to b}_{\mathrm{PV},p,n} = \sum_n\left(\frac{\langle b|\hat{h}_{Z'}|n\rangle\langle n|\hat{D}|a\rangle}{E_b-E_n}+\frac{\langle b|\hat{D}|n\rangle\langle n|\hat{h}_{Z'}|a\rangle}{E_a-E_n}\right)$$





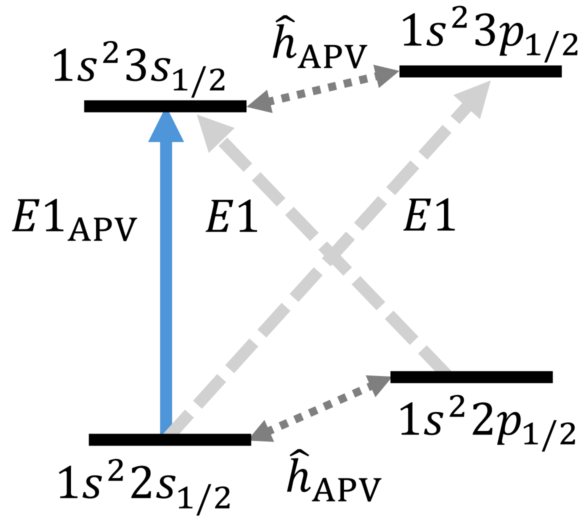


Results

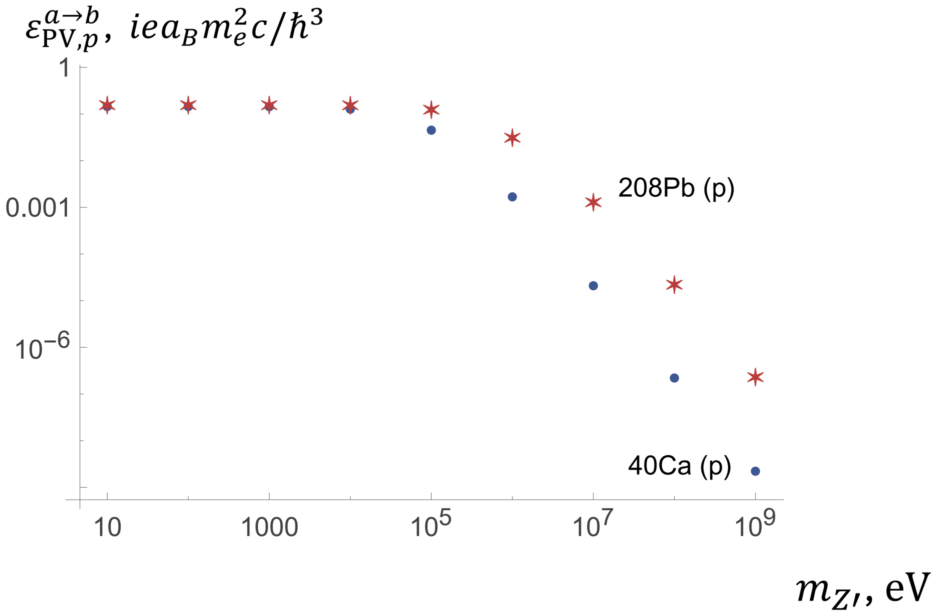
Li-like ions

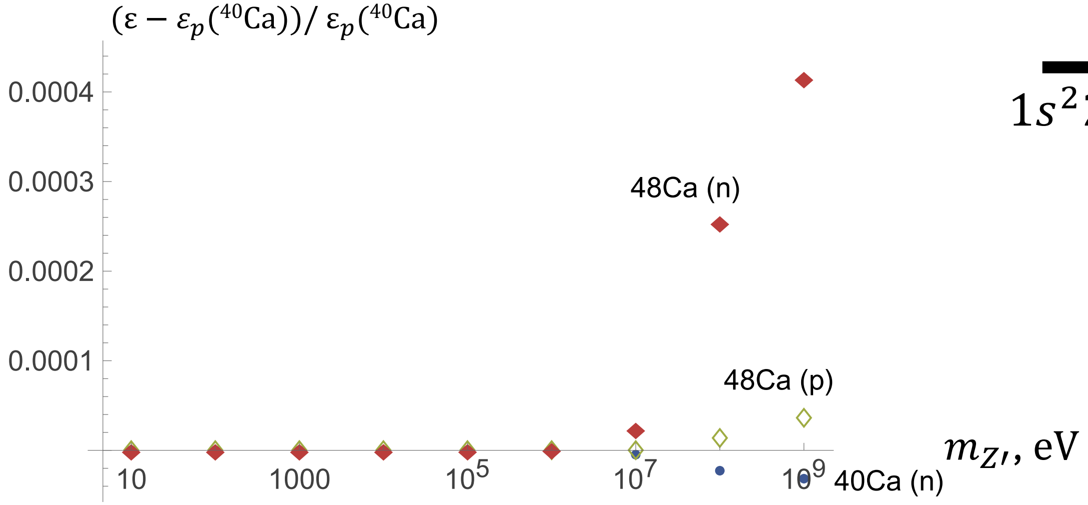
✨New Physics:✨

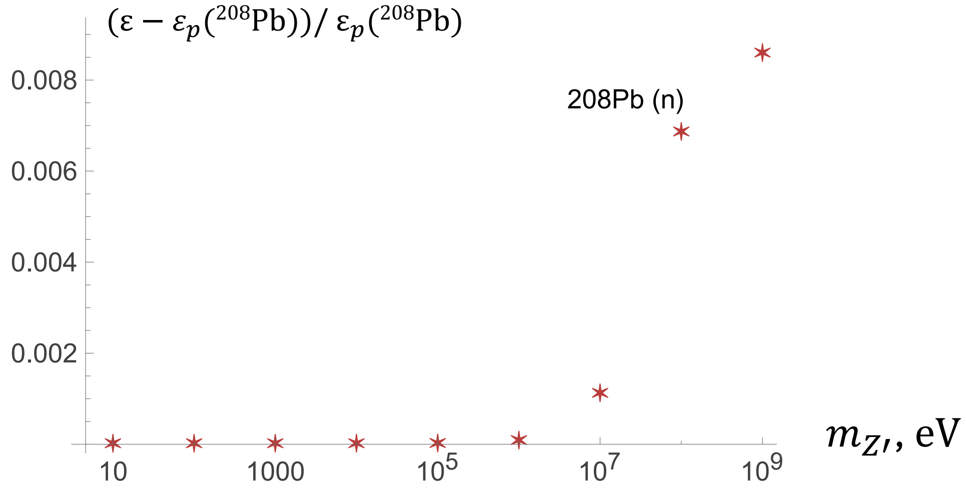
$$\hat{h}_{Z'} = \frac{1}{4\pi}\left(\frac{m_{Z'}c}{\hbar}\right)^2 \gamma_5 \int \rho_{nuc}(\vec{R}_N)\frac{e^{-m_{Z'}|\vec{r}_e-\vec{R}_N|}}{|\vec{r}_e-\vec{R}_N|}d\vec{R}_N$$



$$\varepsilon^{a\to b}_{\mathrm{PV},p,n} = \sum_n \left(\frac{\langle b|\hat{h}_{Z'}|n\rangle\langle n|\hat{D}|a\rangle}{E_b - E_n} + \frac{\langle b|\hat{D}|n\rangle\langle n|\hat{h}_{Z'}|a\rangle}{E_a - E_n}\right)$$







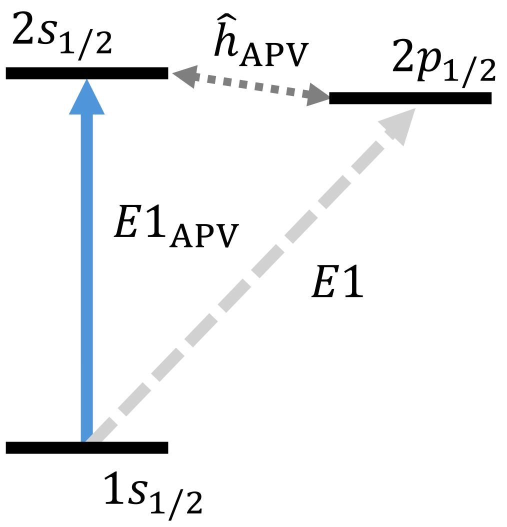
Thank you!

Results

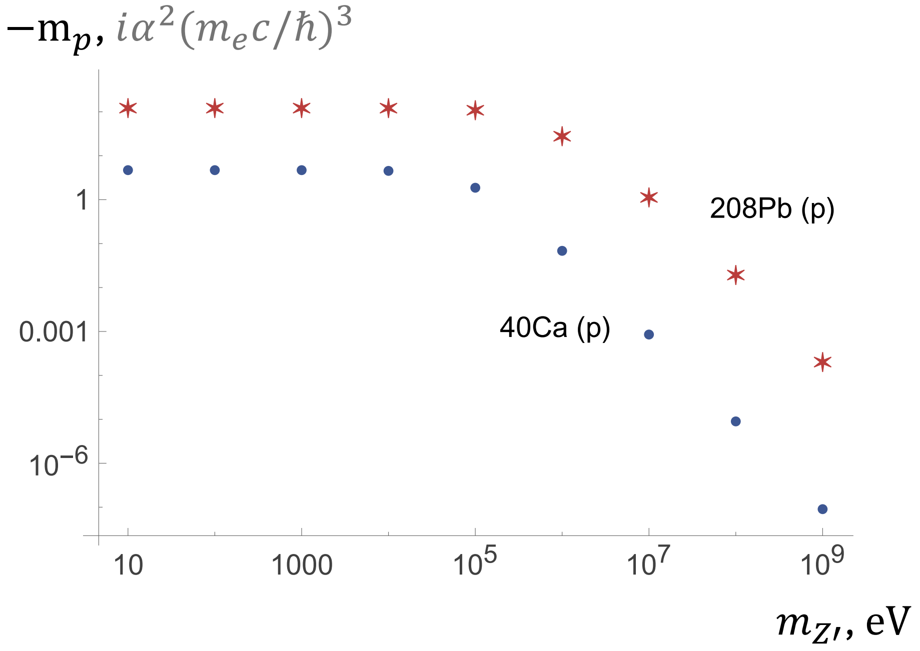
H-like ions

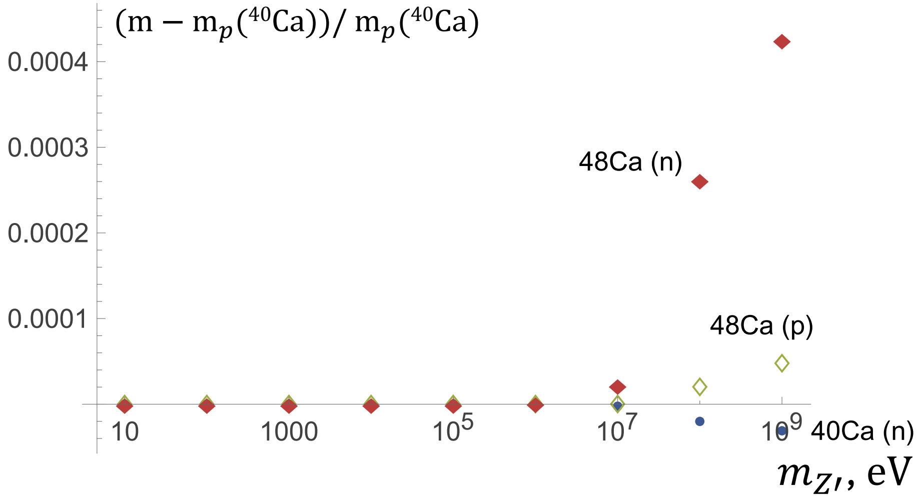
✨New Physics:✨

$$\hat{h}_{Z'} = \frac{1}{4\pi}\left(\frac{m_{Z'}c}{\hbar}\right)^2 \gamma_5 \int \rho_{nuc}(\vec{R}_N)\frac{e^{-m_{Z'}|\vec{r}_e-\vec{R}_N|}}{|\vec{r}_e-\vec{R}_N|}d\vec{R}_N$$



$$\mathrm{m}_{p,n} = \langle 2s | \hat{h}_{Z',p,n} | 2p_{1/2} \rangle$$





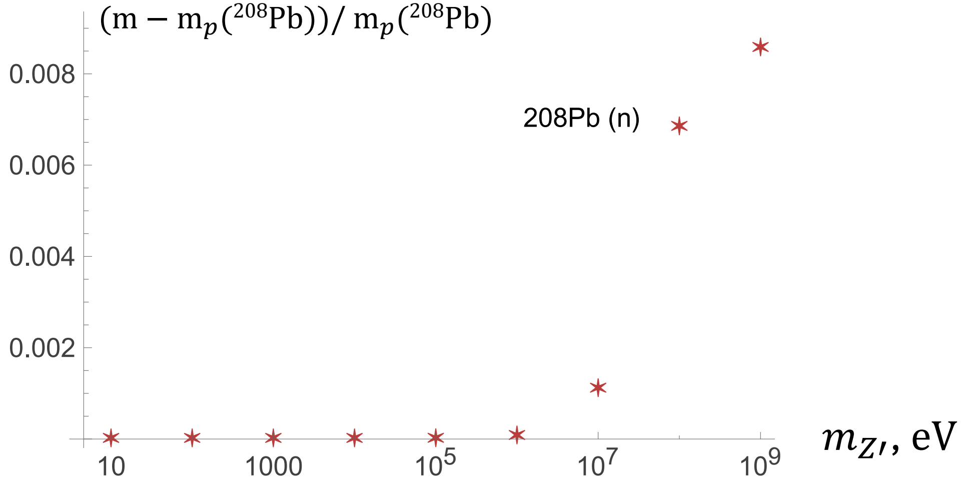


TABLE II. Energy levels $1s^2nl$ of Li-like ^{40}Ca and ^{208}Pb. Levels with $n \leq 6$ are calculated using CI-QEDMOD method [27], including numerical uncertainties and isotope shifts in ^{48}Ca with respect to ^{40}Ca. Energies for $n = 7, 8$ are found using **GRASP2K** [28]; we estimate the errors for $n = 7, 8$ in ^{40}Ca by comparing our results to NIST data [29], and in ^{208}Pb by comparing to an **AMBiT** [30] calculation.

Level	E_n (^{40}Ca), eV	ΔE_n (^{40,48}Ca), eV	E_n (Pb), eV
$1s^2 2s$	0	0	0
$1s^2 2p$	35.961(1)	0.0017(4)	230.65(5)
$1s^2 3s$	651.828(2)	0.0015(4)	14179.98(8)
$1s^2 3p$	661.783(2)	0.0019(4)	14241.51(8)
$1s^2 4p$	879.564(2)	0.0022(4)	19037.23(8)
$1s^2 5p$	980.2(1)	0.0023(4)	21215(2)
$1s^2 6p$	1035.7(1)	0.0024(4)	22439(2)
$1s^2 7p$	1067.1(2)	0.0025(4)	23068(64)
$1s^2 8p$	1088.3(2)	0.0025(4)	23514(64)

TABLE I. Fermi distribution parameters [see Eq. (35)] for the nuclear charge densities of ^{40,48}Ca and ^{208}Pb. They are chosen to reproduce the Barret moments given in [20] and the ratio of 2nd to 4th moment from two different methods. For ^{40}Ca, we either use the sum-of-Gaussians (SOG) or Fourier-Bessel (FB) parametrizations. For ^{48}Ca, we consider the SOG parametrization for ^{40}Ca and the isotopic difference from either the upper or the lower curve in Fig. 3 of Ref. [17]. For ^{208}Pb, we consider either the parametrization in Ref. [21] or that in [22], respectively.

Isotope	c, fm	z, fm	$r_{\rm rms}$, fm	Method
^{40}Ca	3.629	0.552	3.480	SOG [21, 23]
	3.803	0.493	3.476	FB [21]
^{48}Ca	3.749	0.515	3.478	SOG [23], Ref. [17]
	3.739	0.518	3.478	SOG [23], Ref. [17]
^{208}Pb	6.663	0.513	5.502	[21, 24]
	6.668	0.509	5.501	[22]

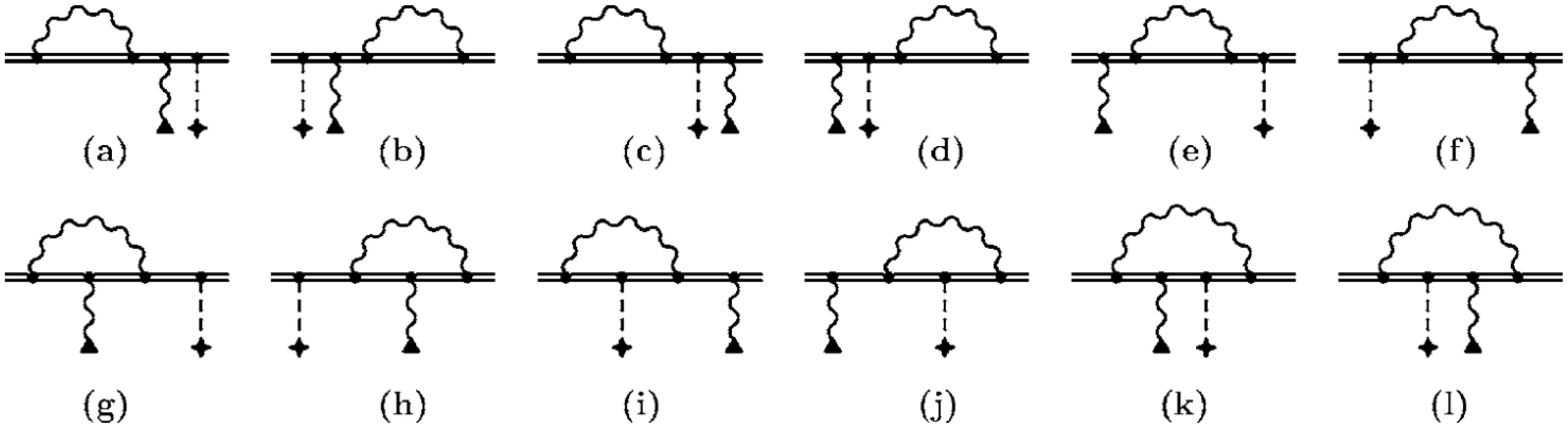
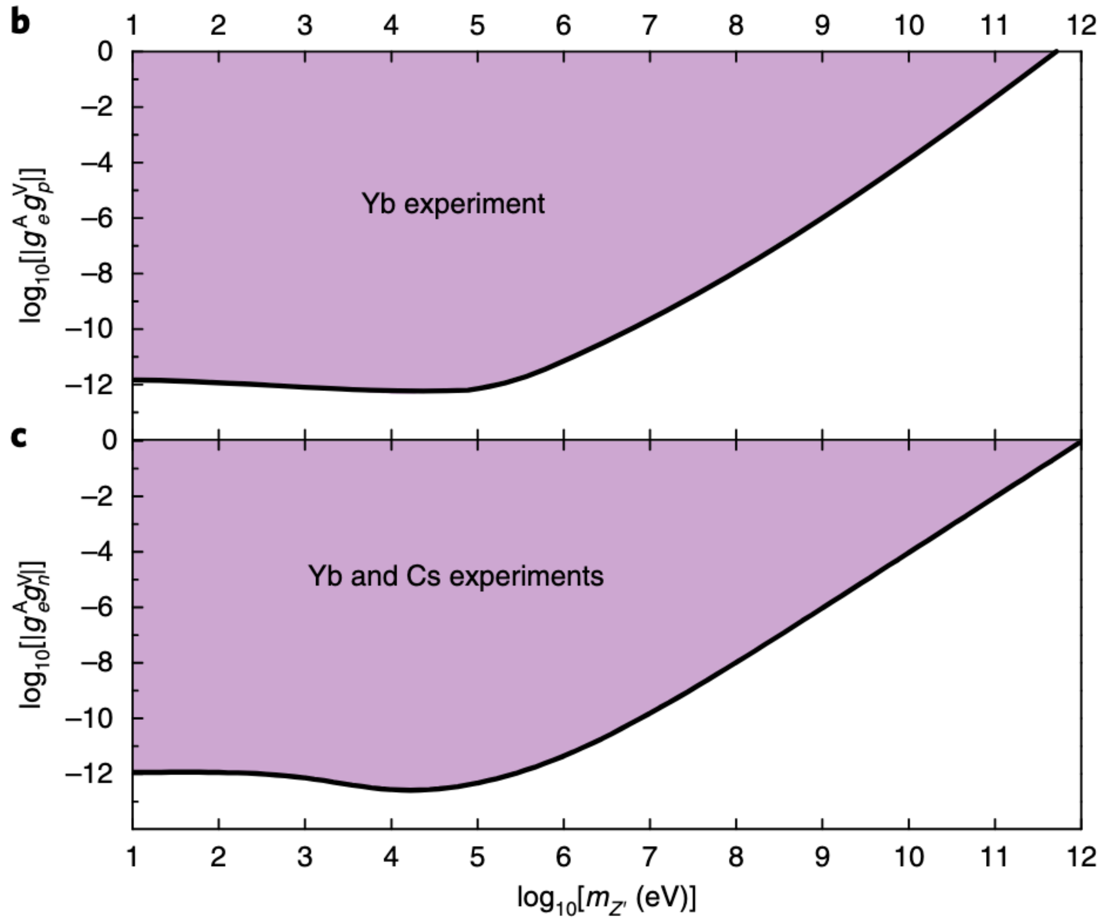


FIG. 2. Feynman diagrams for the SE corrections to the PNC transition amplitude. The wavy line terminated with a triangle indicates the absorbed photon. The dashed line terminated with a cross indicates the electron-nucleus weak interaction.



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