Spin-1 Quantum Electrodynamics and New Physics

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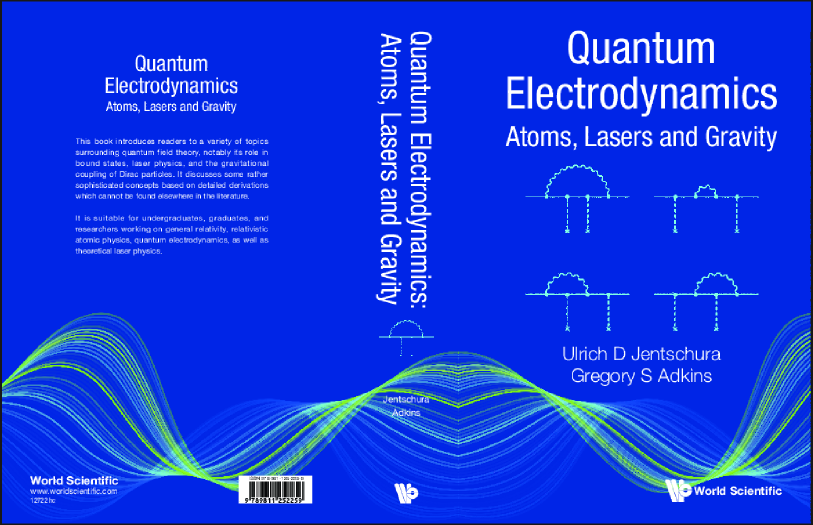
Abstract

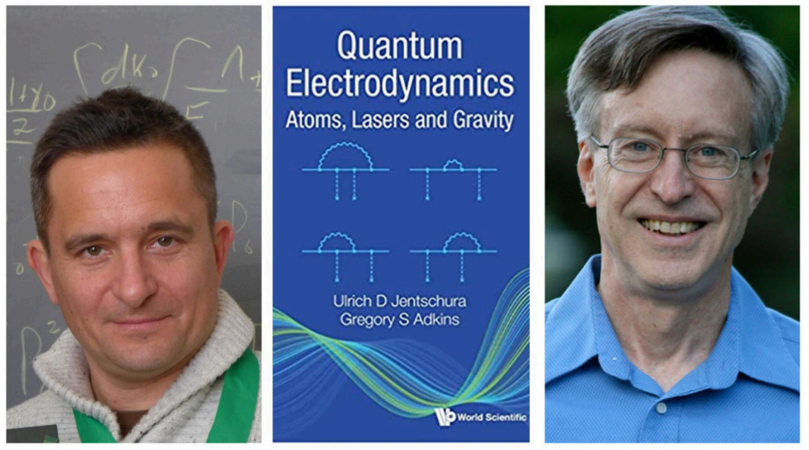
*The bound system of a deuteron and its antiparticle (deuteronium) appears to be an almost ideal candidate for the study of higher-order corrections to the electromagnetic interactions of spin-1 particles. As shown in [Phys. Rev. Research **7**, 043300 (2025)] and in [Phys. Rev. D **113**, 056029 (2026)], the fine and hyperfine structure of deuteronium give access to interesting interconnections of spin-dependent effects, angular mixing, and searches for New Physics. The bound system is essentially stable and could be explored at a future antideuteron beam facility. Due to the extreme field strengths commensurate with the small generalized Bohr radius, the deuteronium system is a good candidate for the detection of small residual dark-photon interactions coupling to neutrons (and their antiparticles).*

Textbook/Monograph
(2nd Edition in Preparation)

Textbook and Monograph



Textbook and Monograph



789 pages. Designed to explain the basis in a coherent fashion.
Second edition is being preparated.

To everyone:

*Request for the communication of typos for the 2*nd *edition.*

Extensions of the Standard Model

Search for Low–Energy Extensions of the Standard Model is a Recent Recurrent Theme in Various Investigations

Where to Look?

Two Motivations:
(i) Proton Radius Puzzle
(ii) Dark Photon Searches

...and probably many others...

(i) Proton Radius Puzzle

Proton Radius Puzzle

Proton Radius Puzzle is No More

PHYSICAL REVIEW LETTERS **136,** 123001 (2026)

Editors' Suggestion

Precision Spectroscopy of 2S-nS Transitions in Atomic Hydrogen: A Determination of the Proton Charge Radius

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(Received 5 December 2025; accepted 20 February 2026; published 23 March 2026)

We present absolute frequency measurements of $2S_{1/2}$-$nS_{1/2}$ two-photon transitions with $n = 8$, 9, and 10 in a cryogenic beam of atomic hydrogen. Each transition has been measured with a fractional uncertainty of $\approx 2.6 \times 10^{-12}$. Combining the results from this Letter and the $1S_{1/2}$-$2S_{1/2}$ transition frequency, we extract a root-mean-square proton radius of $r_p = 0.8433(31)$ fm and a Rydberg frequency of $cR_\infty = 3\,289\,841\,960\,252.9(9.7)$ kHz. These are in good agreement with the CODATA 2022 recommended values.

DOI: 10.1103/lgl2-6cb8

Perhaps, to note:

Required line-splitting factor for the $2S$-nS measurements is only 200 to solve the proton radius puzzle!

[other recent paper: L. Maisenbacher *et al.*, Nature **650**, 845 (2026)]

Proton Radius Puzzle is No More

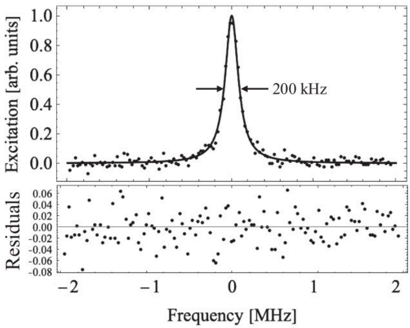

FIG. 2. Upper graph: experimentally obtained line shape of the $2S_{1/2}$-$8S_{1/2}$ transition with active ac Stark shift mitigation fit with a Lorentzian function. Bottom graph: the residuals of the fit.

Very roughly: Proton radius puzzle is r_p^2 between $(0.88\text{fm})^2$ and $(0.84\text{fm})^2$, corresponding to about 13.5 kHz for the $2S$ state of hydrogen.
A one kHz resolution of a 200 kHz line implies a splitting factor of 200; conversely, one kHz is more than ten times smaller than proton radius puzzle.

(ii) Dark (Massive) Photons

Dark (Massive) Photons

Claimed Observations by Group of Attila Krasznahorkay (ATOMKI)

PRL **116,** 042501 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016

Observation of Anomalous Internal Pair Creation in ^{8}Be: A Possible Indication of a Light, Neutral Boson

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(Received 7 April 2015; published 26 January 2016)

Electron-positron angular correlations were measured for the isovector magnetic dipole 17.6 MeV ($J^{\pi} = 1^{+}$, $T = 1$) state → ground state ($J^{\pi} = 0^{+}$, $T = 0$) and the isoscalar magnetic dipole 18.15 MeV ($J^{\pi} = 1^{+}$, $T = 0$) state → ground state transitions in ^{8}Be. Significant enhancement relative to the internal pair creation was observed at large angles in the angular correlation for the isoscalar transition with a confidence level of $> 5\sigma$. This observation could possibly be due to nuclear reaction interference effects or might indicate that, in an intermediate step, a neutral isoscalar particle with a mass of $16.70 \pm 0.35(\mathrm{stat}) \pm 0.5(\mathrm{syst})\ \mathrm{MeV}/c^{2}$ and $J^{\pi} = 1^{+}$ was created.

DOI: 10.1103/PhysRevLett.116.042501

Claimed Observations by Group of Attila Krasznahorkay (ATOMKI)

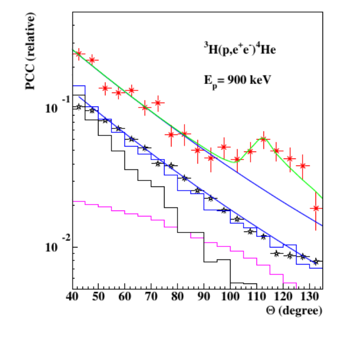
Beryllium (2016):

$$^7\mathrm{Li} + p \to {}^8\mathrm{Be}^* \to {}^8\mathrm{Be} + \gamma \to {}^8\mathrm{Be} + e^+e^- \qquad (1^+ \to 0^+)$$

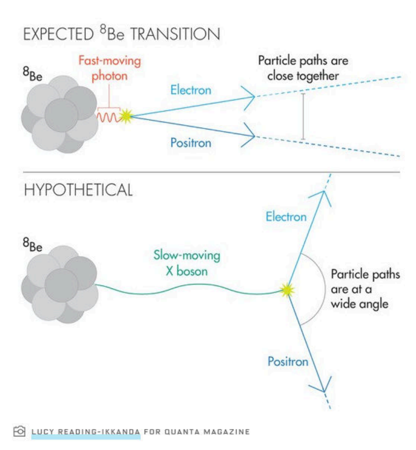
Helium (2019):

$$^3\mathrm{H} + p \to {}^4\mathrm{He}^* \to {}^4\mathrm{He} + \gamma \to {}^4\mathrm{He} + e^+e^- \qquad (0^- \to 0^+)$$

From arXiv:1910.10459:



Possible Explanation (Cartoon)



A "dark" (but not "completely black") massive vector boson (called "X17") could be "hiding" somewhere in the low-energy sector.

Here: assumption of a protophobic interaction with "heavy photon mass" of about 17 MeV [couples mainly to neutrons].

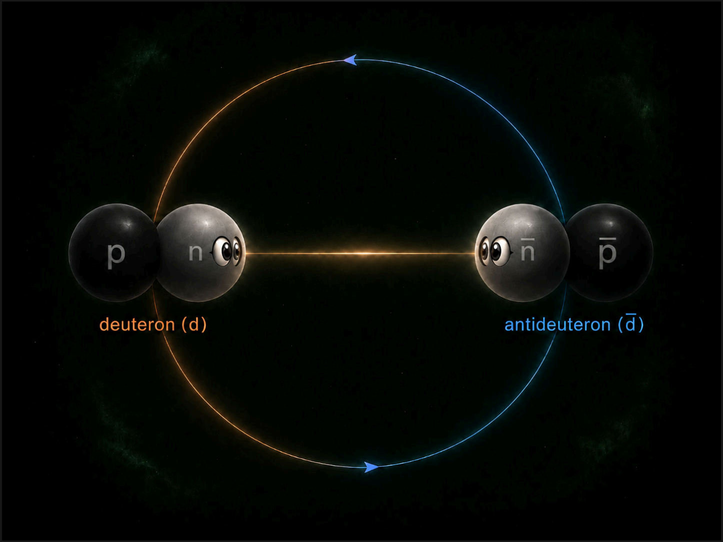
Search for “Dark” Vector Bosons (More General)

Is there a weakly coupling vector boson
(a “dark massive photon”) hiding in the
low-energy sector of the Standard Model?

Search for “Dark” Vector Bosons (Couplings)

Does this “dark vector boson” appear a bit less dark to neutrons as compared to protons?

Deuteronium as a “Double Detector”



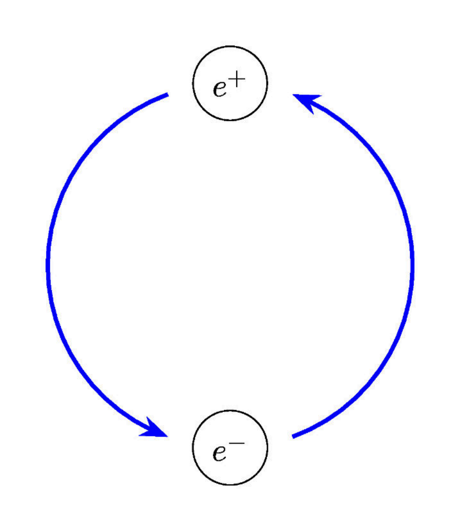
Deuteronium: Investigate Two Aspects Simultaneously

(a) Dark Photon Searches
(b) Spin–1 Quantum Electrodynamics

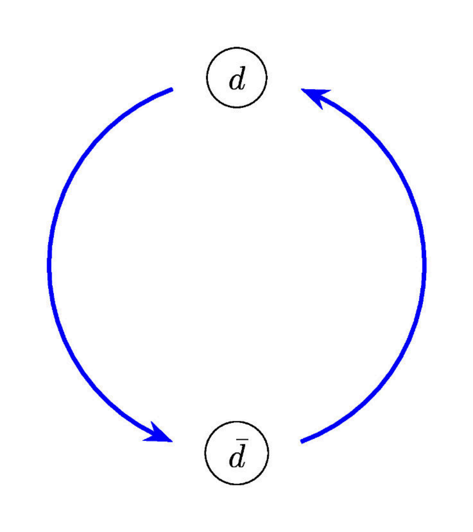
Motivation

- Deuteronium (bound system of a deuteron and its antiparticle) is paradigmatic example for a bound Coulombic system involving particles of spin other than spin-1/2.
- Motivation I: Testing higher-order relativistic corrections involving the quantum electrodynamics of spin-1 particles.
- Motivation II: Search for New Physics, notably, testing for "dark photons" coupling primarily to (perhaps "not-so-neutral") neutrons (constituents of deuteron and antideuteron).
- Electric field strength measured at one generalized Bohr radius of the deuteronium atom: $|\vec{E}|_C = 1.7320 \times 10^{18} \, \frac{\mathrm{V}}{\mathrm{m}} = 1.31 \, E_{\mathrm{cr}}$ (here, E_{cr} is Schwinger's critical field strength) $\Rightarrow$ Extreme environment!

Positronium (Cartoon)

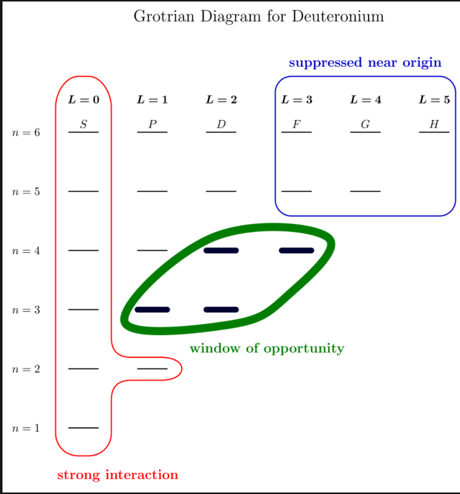


Deuteronium (Cartoon)



As compared to true muonium, the two constituent particles are stable.

"Window of Opportunity"



Transition frequencies are favorable in the "window of opportunity". For vector boson searches: levels $n = 1$ and $n = 2$ have nuclear and strong-interaction (!) effects. Very highly excited circular Rydberg states: not enough overlap with the nucleus for testing dark massive photons.

Corrections of Order $\alpha^4 m_r$ (22–Page Paper)

PHYSICAL REVIEW RESEARCH **7**, 043300 (2025)

Bound deuteron-antideuteron system (deuteronium):
Leading radiative and internal-structure corrections to bound-state energies

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(Received 6 June 2025; accepted 4 November 2025; published 16 December 2025)

We evaluate the energy levels of the deuteronium bound system, which consists of a deuteron and an antideuteron, with a special emphasis on states with nonvanishing orbital angular momenta. The excited atomic bound states of deuteronium constitute probes for the understanding of higher-order quantum electrodynamic corrections for spin-1 particles in a bound system where the typical field strength of the binding Coulomb field (at a distance of the generalized Bohr radius) exceeds Schwinger's critical field strength. For states with nonvanishing angular momenta, effects due to the internal structure of the deuteron and virtual annihilation contributions are highly suppressed. Relevant transitions are found to be in a frequency range accessible by standard laser spectroscopic techniques. We evaluate the leading and next-to-leading energy corrections of orders $\alpha^3 m_d$ and $\alpha^4 m_d$, where α is the fine-structure constant and m_d is the deuteron mass, and also investigate internal-structure corrections: hadronic vacuum polarization, finite-size effects, and strong-interaction corrections.

DOI: 10.1103/zrp8-jx3w

…tells you more about deuteronium than you ever wanted to know.

Corrections of Order $\alpha^5 m_r$ (Partial)

PHYSICAL REVIEW D **113,** 056029 (2026)

Relativistic and recoil corrections to light-fermion vacuum polarization for bound systems of spin-0, spin-1/2, and spin-1 particles

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In bound systems whose constituent particles are heavier than the electron, the dominant radiative correction to energy levels is given by light-fermion (electronic) vacuum polarization. In consequence, relativistic and recoil corrections to the one-loop vacuum-polarization correction are phenomenologically relevant. Here, we generalize the treatment, previously accomplished for systems with orbiting muons, to bound systems of constituents with more general spins: spin-0, spin-1/2, and spin-1. We discuss the application of our more general expressions to various systems of interest, including spinless systems (pionium), muonic hydrogen and deuterium, and devote special attention to the excited non-S states of deuteronium, the bound system of a deuteron and its antiparticle. The obtained energy corrections are of order $\alpha^5 m_r$, where α is the fine-structure constant and m_r is the reduced mass.

DOI: 10.1103/lll9-9wxz

Recoil corrections to electronic VP required a generalization of the “massive–photon Breit Hamiltonian” to higher spins.

Breit Hamiltonian

PHYSICAL REVIEW A **78**, 012504 (2008)

Nuclear mass correction to the magnetic interaction of atomic systems

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(Received 22 May 2008; published 11 July 2008)

The electromagnetic interaction of an atom or ion with an arbitrary spin nucleus is considered. The effective atomic interaction Hamiltonian is derived by a sequence of unitary transformations to separate out the total motion from the internal degrees of freedom. The obtained finite nuclear mass corrections modify the atomic energy levels in the magnetic field. We present resulting contributions to the electron g factor, the NMR shielding factor, and the magnetic susceptibility.

DOI: 10.1103/PhysRevA.78.012504 PACS number(s): 31.30.js, 32.10.Dk, 31.30.Gs

...generalized Breit Hamiltonian for particles with spin other than spin-1/2.

Foldy–Wouthuysen Transformation for Higher Spins

PHYSICAL REVIEW A **82**, 052520 (2010)

Electrodynamics of finite-size particles with arbitrary spin

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Institute of Theoretical Physics, University of Warsaw, Hoża 69, PL-00-681 Warsaw, Poland
(Received 20 September 2010; published 24 November 2010)

We construct the most general Hamiltonian for the electromagnetic interaction of the finite-size particle-like nucleus with arbitrary spin, magnetic dipole, and electric quadrupole moments. It includes all the terms, which are important for obtaining atomic energy levels up to the order α^6. The result is verified against spin $s = 0$, 1/2, and 1 cases, where the Foldy-Wouthuysen transformation is performed of the corresponding relativistic equation.

DOI: 10.1103/PhysRevA.82.052520 PACS number(s): 31.30.J−, 36.10.−k, 32.10.Dk

...with account for higher corrections up to order $\alpha^6 m$.

Single–Particle Hamiltonian (Unified for Spin-0, 1/2, and 1)

- Single-particle Hamiltonian:

$$H = \frac{\vec{\pi}^{\,2}}{2m} - \frac{\vec{\pi}^{\,4}}{8m^3} + qA^0 - \frac{q}{6} r_E^2 \vec{\nabla} \cdot \vec{E} - \frac{q(\tilde{g}-1)}{4m^2} \vec{S} \cdot \left(\vec{E} \times \vec{\pi} - \vec{\pi} \times \vec{E} \right)$$
$$- \frac{q}{2} Q_E (S^i S^j)^{(2)} \partial_j E^i - \frac{q\,\tilde{g}}{2m} \vec{S} \cdot \vec{B}\,, \qquad \vec{\pi} = \vec{p} - q\vec{A}\,.$$

- The quadrupole component of the spin tensor is $(S^i S^j)^{(2)} = \frac{1}{2} \left(S^i\, S^j + S^j\, S^i \right) - \frac{1}{3}\, \delta^{ij} \vec{S}^2$.
- The g factor, scaled to the mass of the individual particle, is denoted as $\tilde{g}$.
- The charge of the particle is denoted as q.
- From the photon coupling terms in the interaction Lagrangian, one can extract the Feynman rules of spin-1 NRQED (see [Phys. Rev. D **113**, 056029 (2026)]).

Breit Hamiltonian (Interaction Unified for Spin-0, 1/2, and 1)

- Interaction Hamiltonian:

$$H_{\rm BR} = H_{\rm K} + H_{\rm M} + H_{\rm SO} + H_{\rm FSS} + H_{\rm Q} + H_{\rm D}\,.$$

- Kinetic term, magnetic-photon term, spin-orbit term, Fermi spin-spin term, quadrupole term, and Darwin term.
- Individual terms (valid for spin-0, 1/2 and 1 of either particle):

$$H_K = -\frac{p^4}{8m_1^3} - \frac{p^4}{8m_2^3}\,, \qquad H_{\rm M} = -\frac{Z\alpha}{2m_1 m_2}\, p^i \left(\frac{\delta^{ij} + \hat{x}^i \hat{x}^j}{r}\right) p^j\,,$$

$$H_{\rm SO} = \frac{Z\alpha}{2}\left(\frac{\tilde{g}_1 - 1}{m_1^2} + \frac{\tilde{g}_1}{m_1 m_2}\right)\frac{\vec{S}_1 \cdot \vec{L}}{r^3} + \frac{Z\alpha}{2}\left(\frac{\tilde{g}_2 - 1}{m_2^2} + \frac{\tilde{g}_2}{m_1 m_2}\right)\frac{\vec{S}_2 \cdot \vec{L}}{r^3}\,,$$

$$H_{\rm FSS} = \frac{2\pi\, Z\alpha\, \tilde{g}_1 \tilde{g}_2}{3m_1 m_2}\, \vec{S}_1 \cdot \vec{S}_2\, \delta^3(\vec{x}\,) + \frac{3\, Z\alpha\, \tilde{g}_1 \tilde{g}_2}{4m_1 m_2}\, \frac{S_1^i S_2^j\, (\hat{x}^i \hat{x}^j)^{(2)}}{r^3}\,,$$

$$H_{\rm Q} = -\frac{3\, Z\alpha}{2r^3}\left\{Q_{E1}(S_1^i S_1^j)^{(2)} + Q_{E2}(S_2^i S_2^j)^{(2)}\right\}(\hat{x}^i \hat{x}^j)^{(2)}\,,$$

$$H_{\rm D} = \frac{2\pi}{3}\, Z\alpha\, \left(r_{E1}^2 + r_{E2}^2\right)\delta^3(\vec{x}\,)\,.$$

Interaction Hamiltonian (Unified for Spin-0, 1/2, and 1)

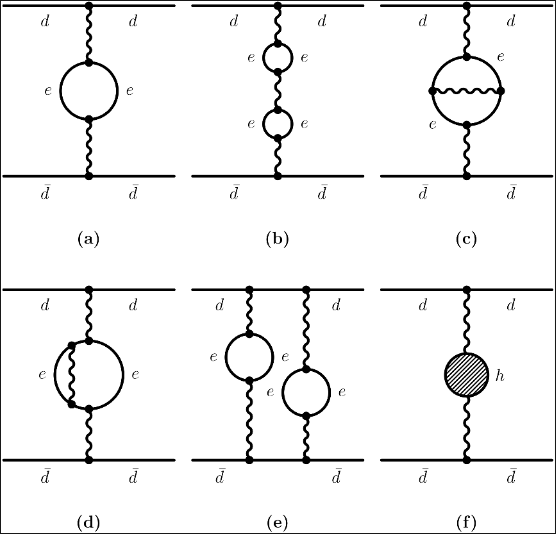
- Spectrum of deuteronium follows LSJ coupling.
- Orbital angular momentum: L.
- Spin: $\vec{S} = \vec{S}_1 + \vec{S}_2$. Spin quantum number $S = 0, 1, 2$.
- $\vec{L}$ and $\vec{S}$ couple to $\vec{J}$.
- Fine and hyperfine structures are of the same order-of-magnitude.
- There are off-diagonal matrix elements of the Breit Hamiltonian for states with $L = J$, but different $S \neq S'$.

Result for the α^4 corrections:

$$\langle H_{\rm BR} \rangle_{nLS'SJJ_z} = \alpha^4 m_d \Bigg[\left(\frac{11}{64n^4} - \frac{1}{2n^3(2L+1)} \right) \delta_{S'S} + \frac{\delta_{L\,0}}{8n^3} \left(1 + \frac{4}{3}\tilde{r}_d^2 + \frac{\tilde{g}_d^2}{3} \left[S(S+1) - 4 \right] \right) \delta_{S'S} + \frac{(2\tilde{g}_d - 1)\, B_{LSJ}}{8n^3 L(L+1)(2L+1)} \delta_{S'S} + \frac{3\left(\tilde{g}_d^2 C_{LS'SJ} - 2\tilde{Q}_d D_{LS'SJ} \right)}{16n^3 L(L+1)(2L+1)} \Bigg].$$

The angular tensors $C_{LS'SJ}$ and $D_{LS'SJ}$ couple states with $S = 0$ and $S' = 2$ for $L = J$. For details, see [Phys. Rev. Research **7**, 043300 (2025)].

Feynman Diagrams for the Leading QED Corrections



One- and two-loop vacuum-polarization corrections,
and hadronic vacuum polarization (for reference).

Theoretical Predictions

Example $4D$–$4F$ Transitions:

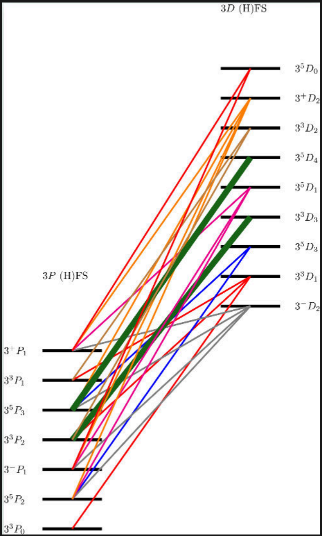
TABLE VII. Energies and wavelengths for $4F \to 4D$ transitions in deuteronium. The uncertainties for these transitions arise from the uncertainty in the quadrupole moment Q_d and the strong-interaction uncertainty for 4D states (see Tables III and V).

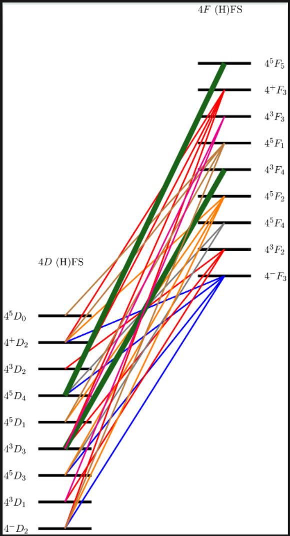
Transition	Energy ΔE (meV)	Wavelength λ (nm)
$4^-F_3 \to 4^-D_2$	314.9049(15)	3937.19(2)
$4^-F_3 \to 4^+D_2$	286.4151(14)	4328.83(2)
$4^-F_3 \to 4^5D_3$	307.2340(12)	4035.50(2)
$4^-F_3 \to 4^5D_4$	289.4061(10)	4284.091(15)
$4^+F_3 \to 4^-D_2$	324.9899(14)	3815.02(2)
$4^+F_3 \to 4^+D_2$	296.5000(14)	4181.59(2)
$4^+F_3 \to 4^5D_3$	317.3190(12)	3907.242(15)
$4^+F_3 \to 4^5D_4$	299.4910(10)	4139.830(14)
$4^3F_2 \to 4^3D_1$	312.2642(11)	3970.490(14)
$4^3F_2 \to 4^3D_2$	291.6724(11)	4250.80(2)
$4^3F_2 \to 4^3D_3$	301.3068(9)	4114.882(12)
$4^3F_3 \to 4^3D_2$	298.6482(11)	4151.51(2)
$4^3F_3 \to 4^3D_3$	308.2826(9)	4021.771(11)

The complete list of results is available from [Phys. Rev. Research **7**, 043300 (2025)]

(Hyper–)Fine Structure of $3D \to 3P$ and $4F \to 4D$

Grotrian diagrams with dipole-allowed transitions:





Favorable energy ranges:

about 1.5 eV for $3D \to 3P$, and about 0.3 eV for $4F \to 4D$.

Spin-1 “Dirac” Equations

Spin-1 Wave Equations

Dirac–Type Equations for Spin-0, 1/2, 1

- Let us remember two Lorentz-covariant relativistic wave equations.
- Dirac equation and Clifford algebra ($\hbar = c = \epsilon_0 = 1$):

$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\,\psi = 0\,, \qquad \{\gamma^\mu, \gamma^\nu\} = 2\,g^{\mu\nu}\,.$$

  The irreducible representations of the Clifford algebra of four-dimensional space-time are four-dimensional, which implies that the Dirac equation is suitable for spin-1/2 particles. The Dirac equation, simultaneously, describes spin-up and spin-down particles and their antiparticles.

- Duffin–Kemmer–Petiau equation and Duffin–Kemmer algebra:

$$(\mathrm{i}\beta^\mu \partial_\mu - m)\,\psi = 0\,, \qquad \beta_\mu\,\beta_\nu\,\beta_\rho + \beta_\rho\,\beta_\nu\,\beta_\mu = \beta_\mu\,g_{\nu\rho} + \beta_\rho\,g_{\mu\nu}\,.$$

  The irreduible representations of the Duffin–Kemmer algebra are one-dimensional (not useful), five-dimensional and ten-dimensional. The five-dimensional one describes spinless particles, in which case the wave function consists of a scalar and its four-vector gradient. The ten-dimensional representation describes spin-1 particles, in which case the wave function consists of four components forming a four-vector and six forming an antisymmetrical tensor.

Dirac–Type Equations for Spin-0, 1/2, 1

- The five-dimensional irreducible representation of Duffin–Kemmer Petiau: spinless particles.
  Checksum: one field and its four-dimensional space-time gradient.
- The ten-dimensional irreducible representation of Duffin–Kemmer Petiau: spin-1 particles.
  Checksum: one four-vector field and the six components of the antisymmetric field tensor $F_{\mu\nu}$ derived from it.

Dirac–Type Equations for Spin-0, 1/2, 1

- Let us stress that the Clifford (γ_μ) and Duffin–Kemmer (β_μ) algebras really are different:

$$\gamma_\mu \gamma_\nu \gamma_\rho + \gamma_\rho \gamma_\nu \gamma_\mu = 2(\gamma_\mu \delta_{\nu\rho} + \gamma_\rho \delta_{\mu\nu} - \gamma_\nu \delta_{\mu\rho}) ,$$
$$\beta_\mu \beta_\nu \beta_\rho + \beta_\rho \beta_\nu \beta_\mu = \beta_\mu g_{\nu\rho} + \beta_\rho g_{\mu\nu} .$$

  Comparing the two above equations, we see that there is no term proportional to β_ρ for the Duffin–Kemmer algebra.

- After the elimination of redundant degrees of freedom, one can extract a relativistic Hamiltonian for spinless and spin-1 particles, from the Duffin–Kemmer equation. For the spin-1 equation, one has to include intrinsic quadrupole terms, and terms connected with the magnetic moment. *The procedure is complicated.* Some decisive steps:
  - R. J. Duffin, *On the Characteristic Matrices of Covariant Systems*, Phys. Rev. **54**, 1114 (1938).
  - N. Kemmer, *The particle aspect of meson theory*, Proc. Roy. Soc. London A **173**, 91 (1939).
  - S. Sakata and M. Taketani, *On the Wave Equation of Meson*, Proc. Math. Phys. Soc. Japan **22**, 757 (1940)
  - J. A. Young and S. A. Bludman, *Electromagnetic Properties of a Charged Vector Meson*, Phys. Rev. **131**, 2326 (1963).
  - J. Zatorski and K. Pachucki, *Electrodynamics of finite-size particles with arbitrary spin*, Phys. Rev. A **82**, 052520 (2010).

Dirac–Type Equations for Spin-0, 1/2, 1

- The relativistic spinless and spin-1 Hamiltonians obtained using the elimination procedure are pseudo–Hermitian (a concept popularized by Carl Bender and others). We define the metric

$$\eta = \begin{pmatrix} \mathbb{1}_{N\times N} & 0 \\ 0 & -\mathbb{1}_{N\times N} \end{pmatrix}, \qquad N = \begin{cases} 1 & \text{(spin-0)} \\ 3 & \text{(spin-1)} \end{cases} .$$

- The two-dimensional (spin-0) and six-dimensional (spin-1) Hamiltonians fulfill the pseudo–Hermiticity relation:

$$H = \eta\, H^{+}\, \eta\,.$$

- The extraction of the nonrelativistic expansion (and relativistic corrections) proceeds via a generalized Foldy–Wouthuysen transformation which is pseudo-unitary and applied to the Hermitian Hamiltonian $\mathcal{H} = \eta H$:

$$U^{+}\, \eta\, U = \eta\,, \qquad H_{\rm FW} = U^{-1}\, H\, U\,.$$

- The upper left $N \times N$ submatrix of $\mathcal{H}$ is interpreted as the effective one-particle Hamiltonian for the spin-0 and spin-1 particles.

Further Field–Theoretical Interest

Generally: Spin-1 Quantum Field Theories have some Issues
(not going into further detail, here).

Conclusions

- Analyzed generalized single-particle and interaction (Breit) Hamiltonians for spin-0, 1/2 and spin-1 particles.
- Analyzed deuteronium as a sensitive probe for higher-order (hyper-)fine structure effects involving spin-1 particles.
- $4F \to 4D$ and $3D \to 3P$ transitions are in an energetically favorable energy range. They probe QED effects of spin-1 particles in extreme fields while the exposure to details of the nuclear charge distribution is very limited.
- Deuteronium will be a good candidate for the observation of potential protophobic "dark" photons once the calculation of α^5 energy corrections is complete (currently in progress).