# Gravity beyond a single metric
## Ghost-free interactions of spin-2 fields


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### Background

Both the Standard Model of particle physics and General Relativity are formulated in terms of **Lorentzian field theory**, where the fields can be classified by their **mass** and **spin**. By imposing theoretical consistency conditions, this structure uniquely determines the free field equations. One of the most powerful theoretical tools is to impose the **absence of ghost fields**.

Ghost-fields:

Wrong sign!

Ghost: $\mathcal{L} = -\frac{1}{2}\dot{\phi}^2 + \dots$ (Unphysical)

Healthy: $\mathcal{L} = +\frac{1}{2}\dot{\phi}^2 + \dots$ (Physical)

Ghosts make the Hamiltonian unbounded from below and give rise to pathological ghost-instabilities.

**Example: Power of imposing ghost freedom** $(m = 0, s = 1)$
Spin-1 fields are represented by vector fields $A_\mu$ and to make the field equations at most second order in time to avoid the **Ostrogradsky ghost**, the only terms allowed in the Lagrangian are,

Field redefinition

$$\mathcal{L} = a\,\partial_\mu A_\nu \partial^\mu A^\nu + b\,\partial_\mu A_\nu \partial^\nu A^\mu + \text{boundary terms} + \text{tadpoles}$$

$$= \Big[A_\mu = A^T_\mu + \partial_\mu\theta\Big] = (a+b)\ddot{\theta}^2 + \dots \qquad \Longrightarrow \qquad a + b = 0.$$

Avoid Ostrogradsky ghost!

Fixing an overall sign to make the Hamiltonian positive ($a = -\frac{1}{2}$), yields,

$$\mathcal{L} = -\tfrac{1}{2}\delta^{\mu\nu}_{\rho\sigma}\partial_\mu A_\nu \partial^\rho A^\sigma = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad \big(U(1)\text{-sym.} \quad A_\mu \mapsto A_\mu + \partial_\mu\theta\big).$$

A similar exercise for **massless spin-2**, represented by a symmetric rank-2 tensor $h_{\mu\nu}$, uniquely yields,

$$\mathcal{L} = \tfrac{1}{4}\delta^{\mu\rho\alpha}_{\nu\sigma\beta}\,\partial^\sigma h^\nu{}_\mu\,\partial_\rho h^\beta{}_\alpha \qquad \big(\text{linear diff.} \quad h_{\mu\nu} \mapsto h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}\big),$$

which is the **linearised Einstein–Hilbert** Lagrangian in Minkowski space.

The spin-1 theory in the example above can more or less uniquely be generalised to Yang–Mills. **But what is the analogue for spin-2?** According to Lovelock, General Relativity is the unique theory for a single massless spin-2 field, but **beyond one spin-2 field**, the general theory is unknown.

> Can gravity be part of a larger spin-2 structure,
> similar to how electromagnetism is part of electroweak theory?

### Multi-Gravity

At a non-linear level, spin-2 fields can be represented by **spacetime metrics** $g^I_{\mu\nu}$ $(I = 1, \dots, \mathcal{N})$. A large class of multi-gravity theories can be written as a sum of **Einstein–Hilbert** terms deformed with an interaction potential,

Matter coupling

$$\mathcal{S} = \int \mathrm{d}^4x \left[\sum_{I=1}^{\mathcal{N}} m_I^2\sqrt{-g_I}R_I - V(g_1, \dots, g_\mathcal{N})\right] + \sum_{I=1}^{\mathcal{N}} \mathcal{S}_\mathrm{M}[g^I, \psi^a_I], \tag{1}$$

where the Ricci scalar $R_I$ provides kinetic terms for 6 of the 10 components of each metric $g^I_{\mu\nu}$. For non-derivative interactions, these degrees of freedom propagate as one **massless spin-2 field** with 2 polarisations, $\mathcal{N}-1$ **massive spin-2** fields with 5 polarisations each, and $\mathcal{N}-1$ **scalar ghosts**.

The **field equations** take the form of modified Einstein equations for each metric,

$$G^I_{\mu\nu} + V^I_{\mu\nu} = \frac{1}{2m_I^2}T^I_{\mu\nu}, \qquad I = 1, \dots, \mathcal{N} \tag{2}$$

where $V^I_{\mu\nu}$ encodes the spin-2 interaction, and (1) thus naturally becomes a theory of modified gravity.

> What is the most general ghost-free multi-gravity theory?

### Where are the ghosts?

The **non-linearity** of the action makes the analysis of multi-gravity theories **complicated**. With a simplifying Ansatz, which preserves the relevant structure, one can derive **necessary conditions** for the theory to be ghost-free.

**Ghost-Isolating Ansatz**

Eliminating all spatial dependence and all the spin-2 degrees of freedom, one can make the Ansatz,

$$g^I_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu = -N_I^2(t)\mathrm{d}t^2 + f^2(\phi_I(t))\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \qquad f(\phi) = \left(\tfrac{3}{16}\right)^{1/3}\phi^{2/3},$$

where $N_I > 0$ are lapse functions and $\phi_I$ are scalar modes. Inserting the Ansatz into (1), it is clear that $\phi_I$ are ghostly,

Ghosts!

$$\mathcal{L} = -\sum_{I=1}^{\mathcal{N}}\frac{\dot{\phi}_I^2}{2N_I} - V(N_I, \phi_I) \qquad \Longrightarrow \qquad H = -\sum_{I=1}^{\mathcal{N}}\frac{N_I}{2}\pi_I^2 + V(N_I, \phi_I)$$

Linear in lapses!

where $\pi_I$ are the momenta conjugate to the ghost fields $\phi_I$.

Since the lapses $N_I$ are nondynamical, their equations of motion yield **secondary constraints**,

$$\frac{\partial H}{\partial N_I} = -\frac{\pi_I^2}{2} + \frac{\partial V}{\partial N_I} = 0,$$

which for generic potentials $V$ determine the lapses $N_I$ leaving the **ghosts propagating**. Only if all these equations are **independent of lapses** can the ghosts be eliminated and the theory be physical.

**Necessary Condition 1:**
For all ghost modes $\phi_I$ to be eliminated, the Hessian of the potential must have rank zero [3]:

$$\mathrm{Rank}\left(\frac{\partial^2 V}{\partial N_I \partial N_J}\right) = 0 \qquad \Longleftrightarrow \qquad V \text{ is linear in all } N_I. \tag{3}$$

### Vielbein potential

Imposing Necessary Condition 1 in a generally covariant way is hard in terms of metrics. However, if we instead formulate the potential in terms of **vielbeins** $e^A_{I\,\mu}$ associated with the metrics,

$$g^I_{\mu\nu} = e^A_{I\,\mu}\eta_{AB}e^B_{I\,\nu}, \qquad e^A_{I\,0} = N_I X^A_I + \dots$$

Linear in lapse!

the **most general vielbein potential**, linear in the lapses, can be shown to take the form [3],

$$V = \frac{2m^4}{4!}\sum_{IJKL=1}^{\mathcal{N}}\beta^{IJKL}\,\delta^{\alpha\beta\gamma\delta}_{ABCD}\,e^A_{I\,\alpha}e^B_{J\,\beta}e^C_{K\,\gamma}e^D_{L\,\delta}, \tag{4}$$

where $\beta^{IJKL}$ are totally symmetric coupling parameters. This potential was previously conjectured to be the most general ghost-free non-derivative interaction by Hinterbichler and Rosen [6], but is **not ghost-free for general** $\beta^{IJKL}$.

The issue lies in the solutions of the so-called **Lorentz constraints**, corresponding to the antisymmetric part of the field equations (2),

Solutions ruin (3)!

$$V^I_{[\mu\nu]} = 0 \qquad \Longleftrightarrow \qquad \sum_{JKL=1}^{\mathcal{N}}\epsilon^{\alpha\beta\gamma\delta}\beta^{IJKL}\big[e_I^{\top}\eta e_J\big]_{\alpha\beta}e^{[A}_{K\,\gamma}e^{B]}_{L\,\delta} = 0, \tag{5}$$

which generically **breaks the linearity in lapses** of $V$, thereby breaking Necessary Condition 1.

### Uniqueness of the ghost-free vielbein potential

For the Lorentz constraints not to reintroduce non-linearities of the lapses, the Lorentz constraints (5) must reduce to a very special form, which greatly **restricts the interaction parameters** $\beta^{IJKL}$. For **irreducible interactions**, there are only two forms that preserve the linearity [4],

**Necessary Condition 2:**

- $\mathcal{N} = 2$: Hassan–Rosen bimetric theory
  For only two vielbeins, there are no restrictions on $\beta^{IJKL}$, and the interaction potential is equivalent to the known ghost-free Hassan–Rosen bimetric theory.
- $\mathcal{N} > 2$: Hassan–Schmidt-May interaction
  For more than two vielbeins, with genuine multi-field interactions, the interaction parameter $\beta^{IJKL}$ must have **symmetric rank** 1,

$$\beta^{IJKL} = \beta^I\beta^J\beta^K\beta^L, \tag{6}$$

which is precisely the Hassan–Schmidt-May interaction [5].

With the decomposition (6), the interaction potential (4) takes the simple form,

$$V = 2m^4\det\left(\sum_{I=1}^{\mathcal{N}}\beta^I e_I\right). \tag{7}$$

The two necessary conditions (3) and (6) therefore single out the Hassan–Schmidt-May interaction as the **only genuine multi-gravity theory** ($\mathcal{N} > 2$) that can potentially be ghost-free. Its ghost-freedom has been established by a full Hamiltonian constraint analysis in [1], thereby making it the **most general ghost-free interaction** (irreducible and non-derivative).

### Building larger interactions

Theories with spin-2 fields that do not interact directly (reducible) can be constructed from the two ghost-free irreducible building blocks. However, not all such interactions are allowed:

**Necessary Condition 3:**
The interaction graph, where • represents vielbeins and ∘ an interaction vertex, must be a tree [4].

**Example:** Below is an example of a ghost-free tree interaction combining the Hassan–Rosen bimetric potential, and the genuine multi-gravity Hassan–Schmidt-May interaction.

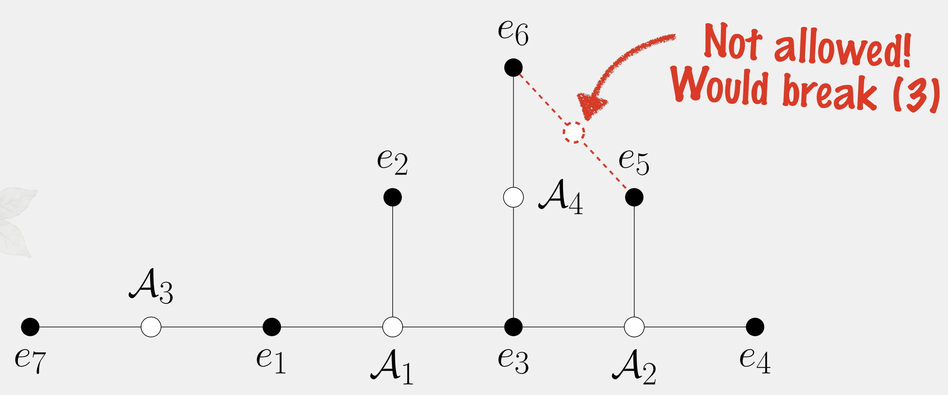


Figure 1. A mixed interaction tree on seven vielbeins. The determinant sectors $\mathcal{A}_1 = \{e_1, e_2, e_3\}$ and $\mathcal{A}_2 = \{e_3, e_4, e_5\}$ share the vielbein $e_3$, while the interactions $\mathcal{A}_3 = \{e_1, e_7\}$ and $\mathcal{A}_4 = \{e_3, e_6\}$ can be of the more general bimetric type.

### Physical applications and phenomenology

Why YOU might care!

- Multi-gravity provides natural **modifications to cosmology**, with phenomenology such as dynamical dark energy and a possible reduction of the Hubble tension ($\sim 2\sigma$).
- The massive spin-2 fields provide stable **cold dark matter** candidates which interact only gravitationally, while remaining compatible with viable cosmologies.
- For low spin-2 masses, compact-object mergers can create gravitational waves with massive modes, resulting in **gravitational wave echoes** or even waveform inversion of the "chirp".
- Due to the lack of Birkhoff's theorem, multi-gravity provides a rich yet constrained set of solutions for **black holes** that go beyond those of General Relativity.

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