Particle Motion around Black Holes with Scalar Hair

Katherine Horton
University College London
Based on work done for a paper in preparation with M. Chivers, B. Hartmann and Y. Brihaye

Black Holes with Scalar Hair

No hair theorems famously make endowing black holes with scalar hair hard, but not impossible. Restrictions include the need for a charge on the black hole, so we typically use a spherically symmetric electric field analogous to Reissner–Nordström. Recently, a class of hairy black holes has been introduced with both electric and magnetic charges, which we consider here. Our coupling g between scalar fields and magnetic monopole with charge Q_m requires the Dirac quantization condition $gQ_m = \pm\frac{k}{2}$ with k an integer; we consider $k = 0$ (purely electric coupling) and $k = 1$ (dyonic coupling).

Geodesics and Motion for Charged and Uncharged Particles

We consider metrics written in the form

$$ds^2 = -N\sigma^2 dt^2 + \frac{1}{N}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

For geodesic motion, we take the usual Lagrangian, set $\theta = \frac{\pi}{2}$ (without loss of generality for uncharged particles, by choice for charged particles), and get conserved quantities energy $E = N\sigma^2\dot{t}$ and angular momentum $L = r^2\dot{\varphi}$. We find

$$\dot{r}^2 + V_{\text{eff}}(r) = 0$$

where

$$V_{\text{eff}} = -\frac{E^2}{\sigma^2} + N\left(1 + \frac{L^2}{r^2}\right)$$

For charged particles we find an equivalent

$$V_{\text{eff}}(r) = -\frac{(E + qV)^2}{\sigma^2} + N\left(1 + \frac{L^2}{r^2}\right)$$

Since $L = r^2\dot{\varphi}$, we integrate to get

$$\varphi - \varphi_0 = \int_{r_0}^{r} \frac{L dr}{r^2\sqrt{-V_{\text{eff}}(r)}}$$

and are hence able to solve this numerically given N and σ – which can themselves be given numerically. Stable and unstable circular orbits are hence found at stationary points of V_{eff}.

Solutions to the Field Equations

We have considered two classes of solution, $k = 0$ (purely electric black holes) and $k = 1$ (dyonic black holes). For each of these cases, we consider two sets of solutions, characterised by the coupling constant g, each of which can be varied according to the value of the scalar field on the horizon, ϕ_h. In this poster, we present one set from each class.

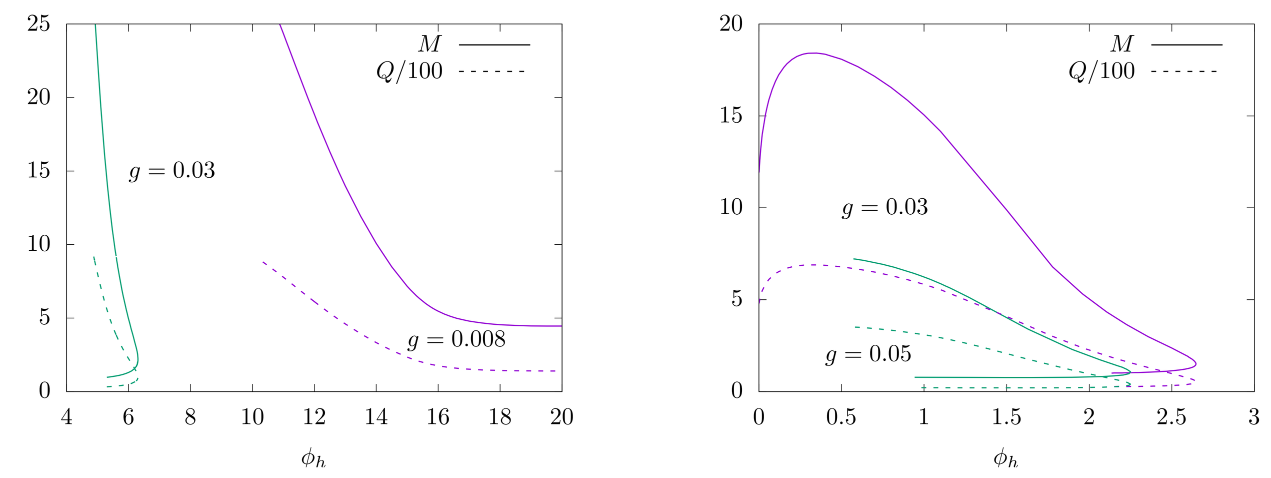


Figure: *Left*: The dependence of the ADM mass M and the electric charge Q for electrically charged black holes with scalar hair on $\phi(r_h) = \phi_h$ for $k = 0$ and two different values of g. *Right*: Same as left, but for $k = 1$. For all solutions we have chosen $\alpha = 0.001$ and $r_h = 1$.

Innermost Stable Circular Orbit

For each solution, we calculate the location of the innermost stable circular orbit (ISCO). For all sets of solution, the (scaled) r_{ISCO} and black hole charge ($K^2 = \alpha(Q^2 + Q_m^2)$) tend to those values for extremal Reissner–Nordström as ϕ_h becomes small.

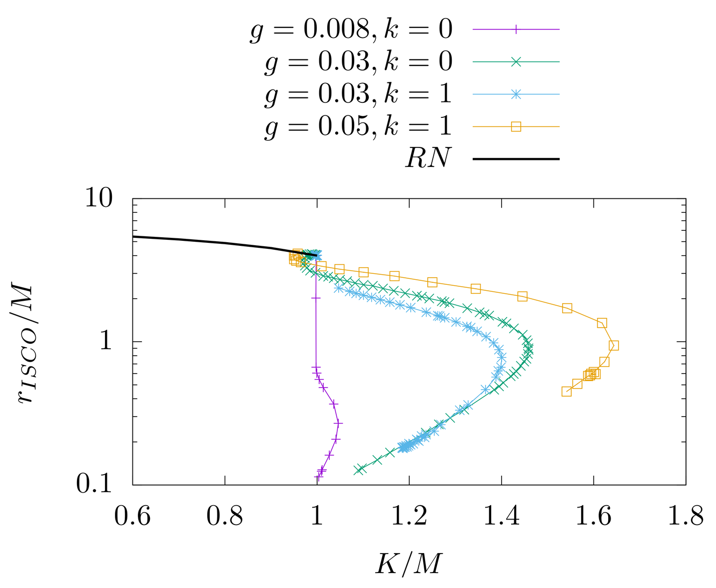


Figure: The scaled location of the ISCO, r_{ISCO}/M, against K/M for each set of solutions, as well as for Reissner–Nordström spacetimes with varying charge

Electrically charged black holes: $k = 0$, $g = 0.008$

We first consider a purely electric spacetime. For larger values of the scalar field at the horizon, we see Reissner–Nordström-like behaviour, with a defined ISCO at the minimum L, above which stationary points occur in unstable-stable pairs. For $\phi_h = 11.0$, however, we see stationary point pairs for all L. This gives a static sphere in which particles can remain at rest.

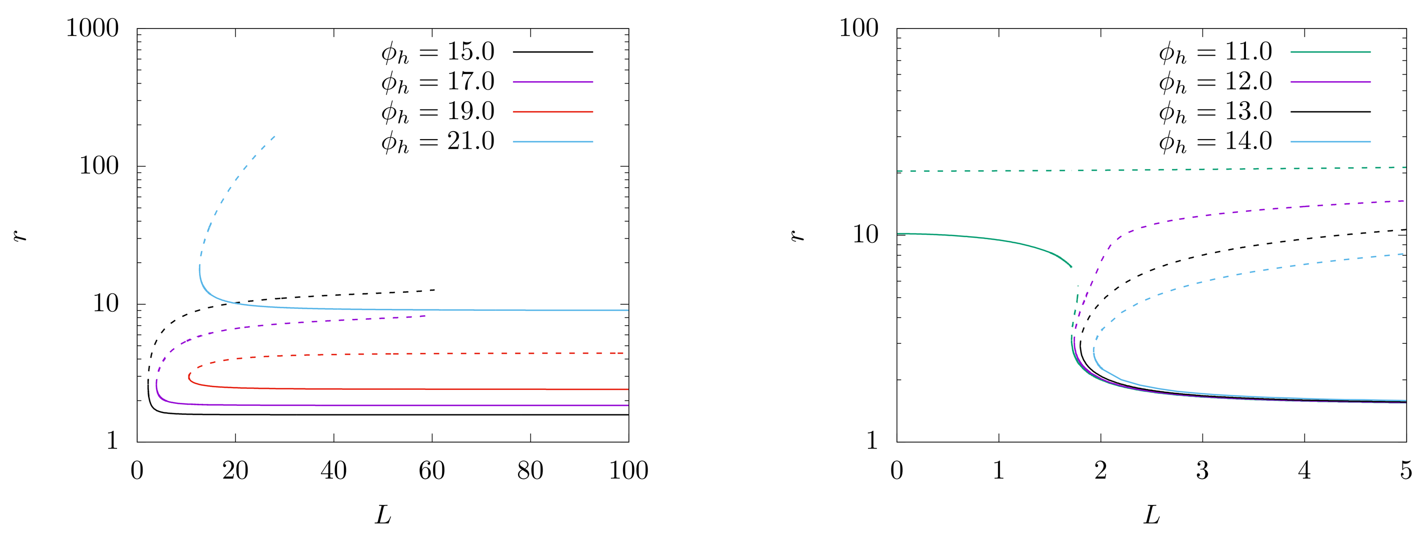
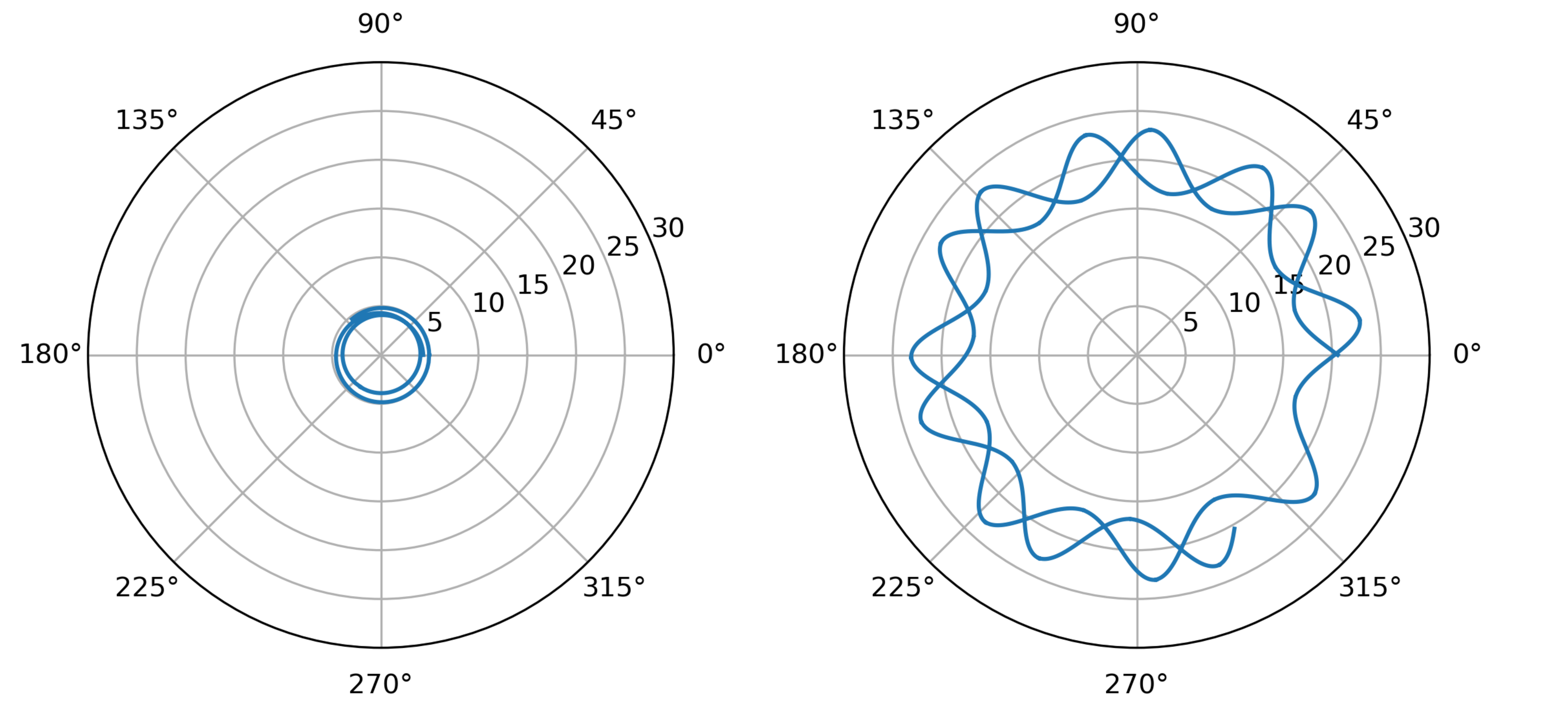


Figure: The location r of stationary points for varying angular momentum L for different values of the scalar field at the horizon for solutions with $k = 0$, $g = 0.008$. Stable orbits are shown with dashed lines, and unstable orbits with solid lines

$k = 0$, $g = 0.008$ Orbits: $\phi_h = 11.0$

For $\phi_h = 11.0$, we also see for a small range of L two pairs of stationary points. For the innermost pair, we see similar behaviour to Reissner–Nordström's circular orbits.

For the outermost pair, however, nearly-circular orbits (those with energy E slightly greater than would be required for a true circular orbit, to allow us to see the behaviour) we find the particle oscillates around the location of the circular orbit, as shown. This behaviour is related to the presence of the static sphere.



(a) $L = 1.75$, $r_0 \approx 4.34$ (b) $L = 1.75$, $r_0 \approx 20.59$

Figure: Plots of two nearly-circular orbits (r_0 at the minimum, but E slightly greater) for $L = 1.75$. *Left*: the inner circular orbit corresponding to those found Reissner–Nordström, located at $r = 4.34$. *Right*: shows a nearly-circular orbit oscillating about the minimum, located at $r = 20.59$. Note that nearly-circular orbits are used to better display the particle behaviour; setting E such that $V = 0$ would give a true circular orbit.

Dyonically charged black holes: $k = 1$, $g = 0.03$

We consider now a dyonic spacetime, with both electric and magnetic charge on the black hole. We again see Reissner–Nordström-like behaviour for larger values of ϕ_h, but for $\phi_h \leq 1$ we find stationary points for small L and hence static spheres.

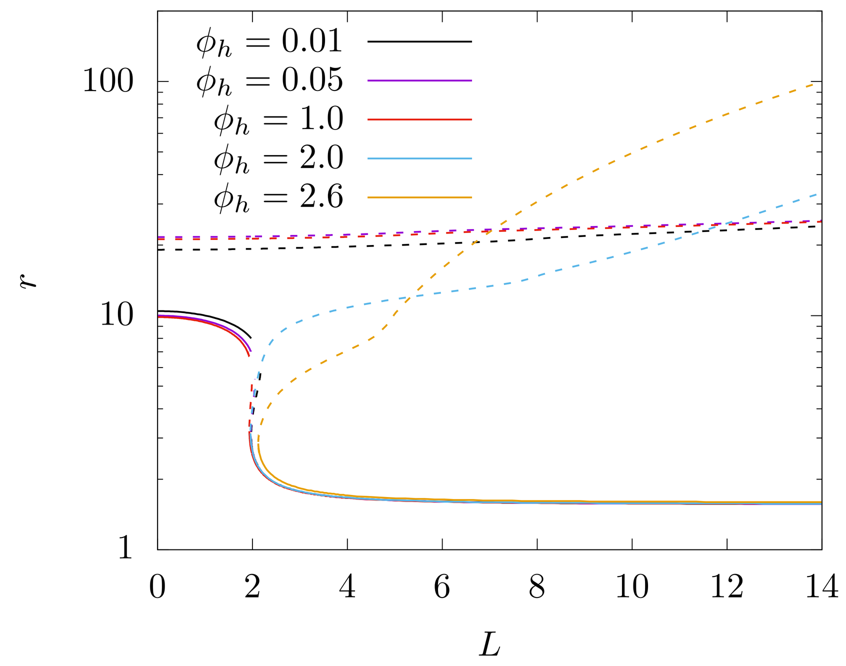


Figure: The location r of stationary points for varying angular momentum L for different values of the scalar field at the horizon for solutions with $k = 1$, $g = 0.03$

$k = 1$, $g = 0.03$ Stationary Points for Charged Particles

We are able to similarly calculate the location of stationary points and hence orbits for charged test particles, with the below showing the case for $\phi_h = 1.5$.

We see that taking negative q (the opposite sign to the charge on the black hole) acts in a similar manner to increasing ϕ_h; stationary points occur at greater L and the behaviour becomes Reissner–Nordström like.

Taking positive q, however, (for large enough q – here $q = 0.1$) leads to only a single unstable orbit for each L, and no stable orbits as the repulsive force between the charge on the black hole and the test particle counteracts the attraction of the gravitational field. This unstable orbit does exist as $L \to 0$, so an unstable static sphere remains.

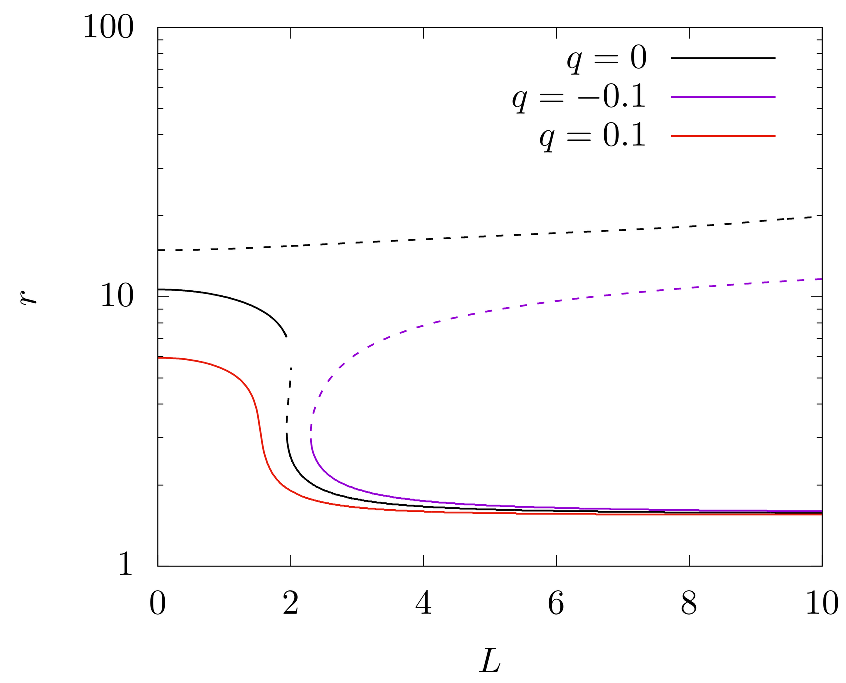


Figure: The location r of stationary points for varying angular momentum L for neutral and positively and negatively charged test particles for the solution with $k = 1$, $g = 0.03$, $\phi_h = 1.5$

Conclusions

We find a range of possible behaviours for particles in the spacetimes around hairy black holes, depending on parameters of the spacetime such as the coupling but also on the charge of the test particle. Observed phenomena not found in Reissner–Nordström include static spheres and multiple pairs of circular orbits for given L, but for large values of ϕ_h the behaviour tends towards that of Reissner–Nordström.

katherine.horton.22@ucl.ac.uk