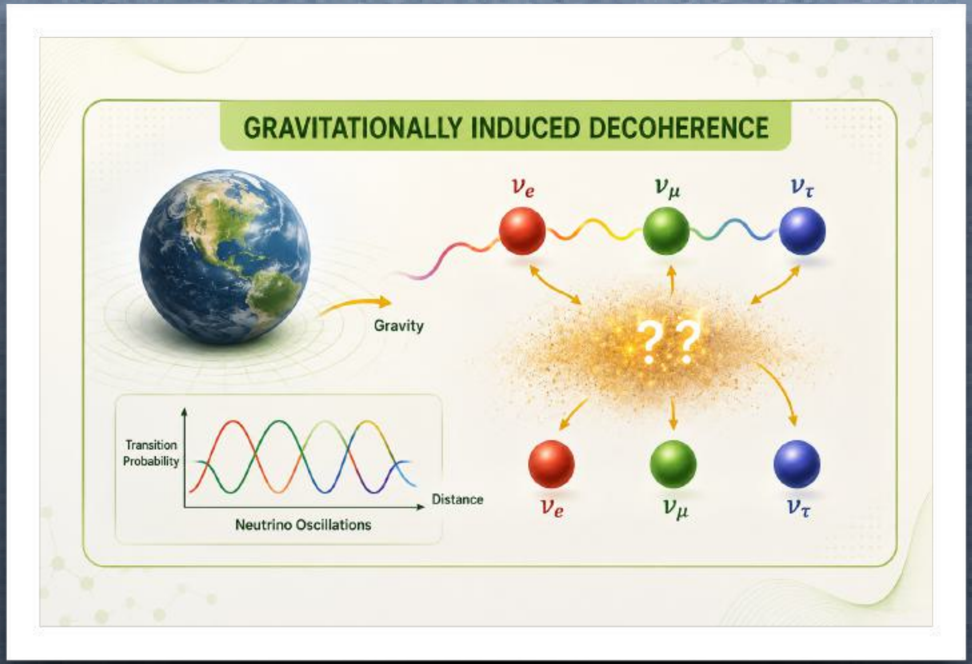
Gravitationally induced decoherence: From theoretical models to applications in neutrino oscillations



joint work with
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KM3NeT Opens a new window on our universe

I. Motivation

Current neutrino experiments: Searches for quantum gravity via decoherence

Article | Published: 26 March 2024

Search for decoherence from quantum gravity with atmospheric neutrinos

The IceCube Collaboration

Nature Physics **20**, 913–920 (2024) | Cite this article

Abstract

Neutrino oscillations at the highest energies and longest baselines can be used to study the structure of spacetime and test the fundamental principles of quantum mechanics. If the metric of spacetime has a quantum mechanical description, its fluctuations at the Planck scale are expected to introduce non-unitary effects that are inconsistent

Search for quantum decoherence in neutrino oscillations with six detection units of KM3NeT/ORCA

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Main idea:

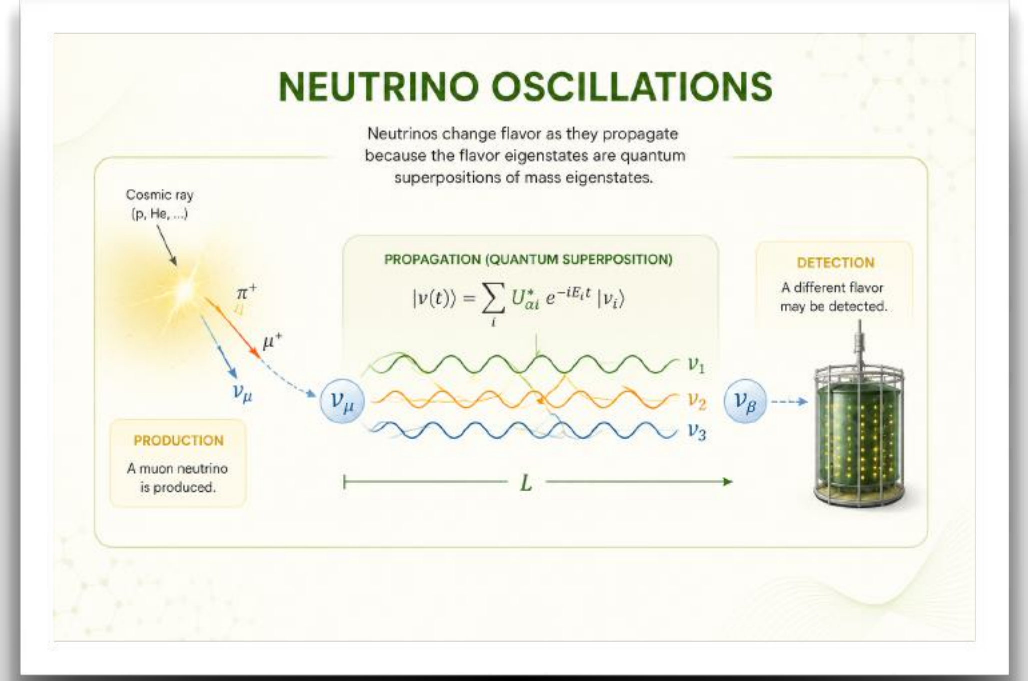
Neutrinos interact with a quantum gravitational environment

Interaction modifies neutrino oscillations due to decoherence

No direct detection yet, constrain decoherence parameters

Question: If I am asked by experimentalists what would be my perspective on this?

I. Brief introduction to neutrino oscillations



II. Neutrino oscillations without decoherence

Neutrino oscillation in a nutshell

Relation flavor and mass eigenstates (PMNS) matrix (most simple model)

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha i} |\nu_i(t)\rangle \quad \text{with } \alpha = e, \mu, \tau \quad i = 1, 2, 3.$$

Oscillation probabilities (vacuum)

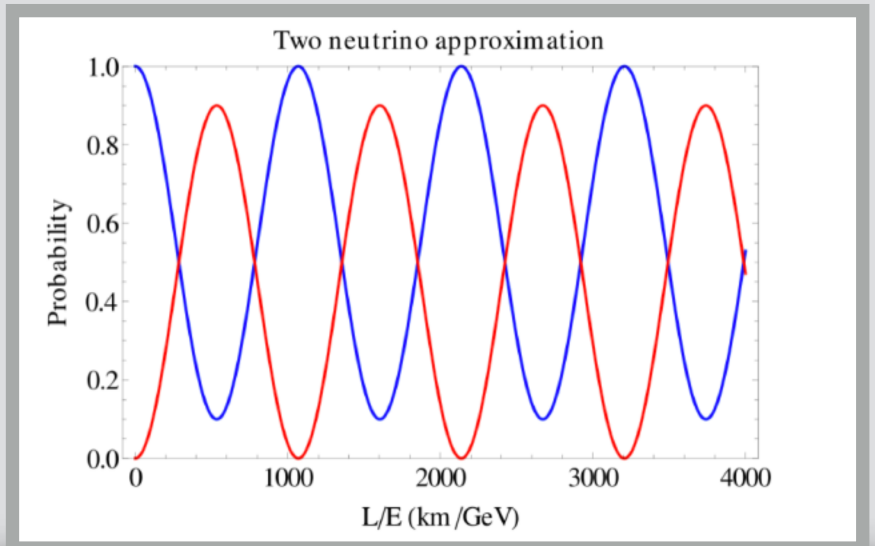
$$|\nu_i(t)\rangle = e^{-iE_i t} |\nu_i\rangle \quad H_{\text{vac}} = \text{diag}\,(E_1, E_2, E_3) \qquad E_i = E + m_i^2/2E, \quad E = \frac{1}{3}(E_1 + E_2 + E_3)$$

then $\langle \nu_\beta \mid \nu_\alpha(t)\rangle = \sum_j U^*_{\beta_j} U_{\alpha_j} e^{-iE_j t}$

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta \mid \nu_\alpha(t)\rangle|^2 = \sum_{j,k} U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} e^{-i\frac{\Delta m^2_{jk}}{2E}t}$$

$$= \delta_{\alpha\beta} - 4 \sum_{j>k} \mathcal{R}_e \left\{U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right\} \sin^2\left(\frac{\Delta_{jk} m^2 t}{4E}\right)$$

$$+ 2 \sum_{j>k} \mathcal{I}_m \left\{U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right\} \sin\left(\frac{\Delta_{jk} m^2 t}{2E}\right)$$



II. Phenomenological models: Decoherence in neutrino oscillations

Phenomenological models

Lindblad equation

$$H_S = H_{\mathrm{vac}} + U^\dagger H_{\mathrm{matt}} U$$ in mass basis

$$\frac{\partial \rho_\nu(t)}{\partial t} = -\frac{i}{\hbar}\left[H_S, \rho_\nu(t)\right] + \mathcal{D}\left[\rho_\nu(t)\right] \qquad H_{\mathrm{matt}} = \pm\sqrt{2} G_{\mathrm{f}} N_{\mathrm{e}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ in flavor basis

effective mass basis $\tilde{H} = \tilde{U}^\dagger H \tilde{U} = \mathrm{diag}\left(\tilde{E}_1, \tilde{E}_2, \tilde{E}_2\right)$

Vectorise Lindblad equation and then parametrise dissipator by $45 = 9 \cdot 8/2$ decoherence parameters, additional assumption reduce to 2-3 parameters

$$\mathbf{D} = -\,\mathrm{diag}\left(\Gamma_{21}, \Gamma_{21}, 0, \Gamma_{31}, \Gamma_{31}, \Gamma_{32}, \Gamma_{32}, 0\right) \qquad \Gamma_{ij}(E) = \gamma_{ij} E^n$$

Additional damping term

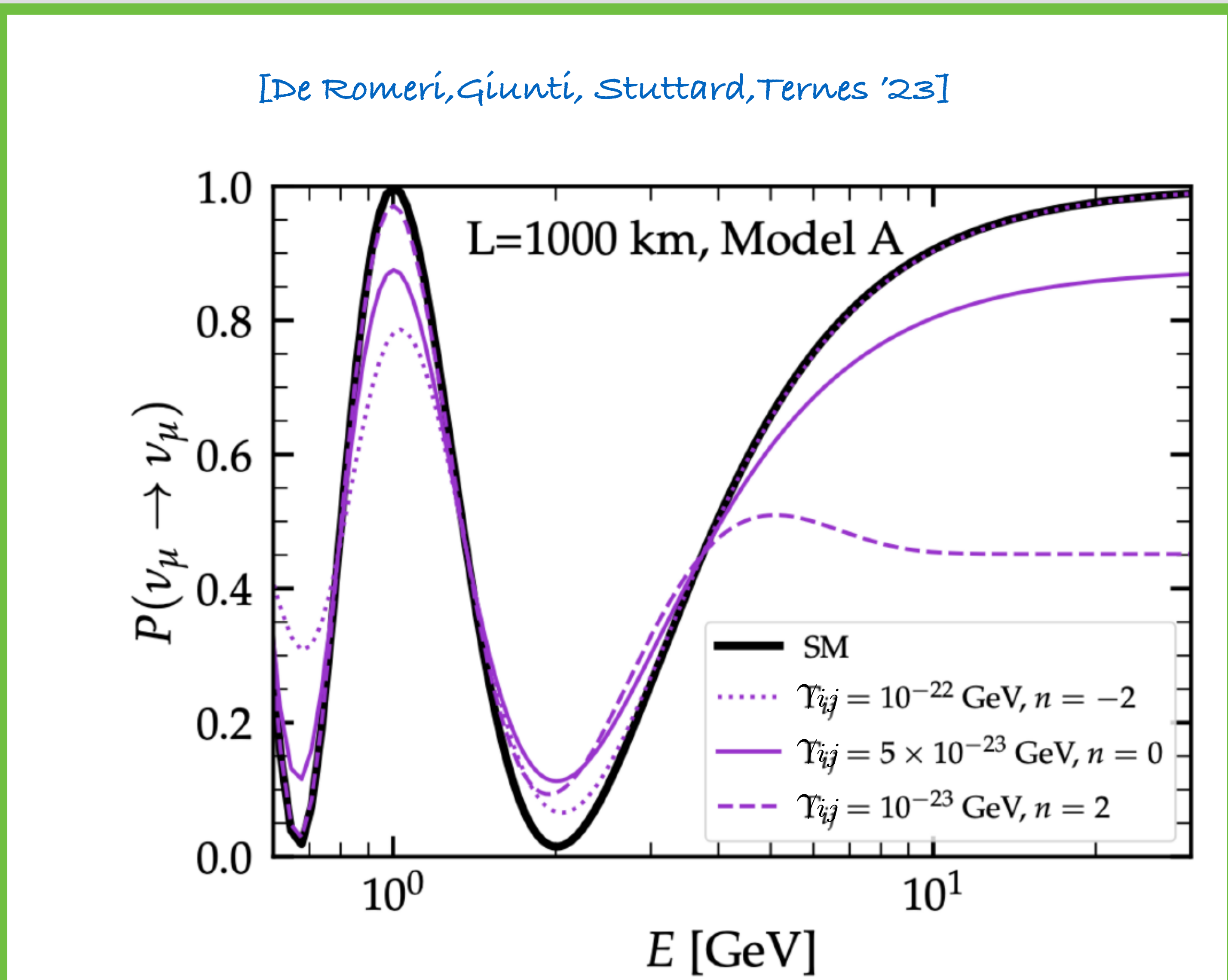
$$\tilde{\rho}_{ij} = \tilde{\rho}_{ij}(0) e^{-\frac{i}{\hbar}(\tilde{E}_i - \tilde{E}_j)t - \Gamma_{ij} t}$$ effective mass basis

Oscillation probability with decoherence

$$P\left(\nu_\alpha \to \nu_\beta\right) = \mathrm{Tr}\left[\hat{\rho}^{(\alpha)}(t)\hat{\rho}^{(\beta)}(0)\right] = \sum_{i,j} \tilde{U}_{\alpha i}\tilde{U}^*_{\beta i}\tilde{U}^*_{\alpha j}\tilde{U}_{\beta j} e^{-\frac{i}{\hbar}(\tilde{E}_i - \tilde{E}_j)t - \Gamma_{ij} t}$$

[Ellis, Lopez, Mavromatos, Nanopoulos 1996]; [Benatti, Floreanini 1999]; [Lisi, Marrone, Montanino 2000]; [Guzzo, de Holanda, Oliveira 2016]; [Gomes, Forero, Guzzo, de Holanda, Oliveira 2019]]

Characteristic of long-baseline accelerator neutrino experiments



Model A: $\gamma_{21} = \gamma_{31} = \gamma_{32}$

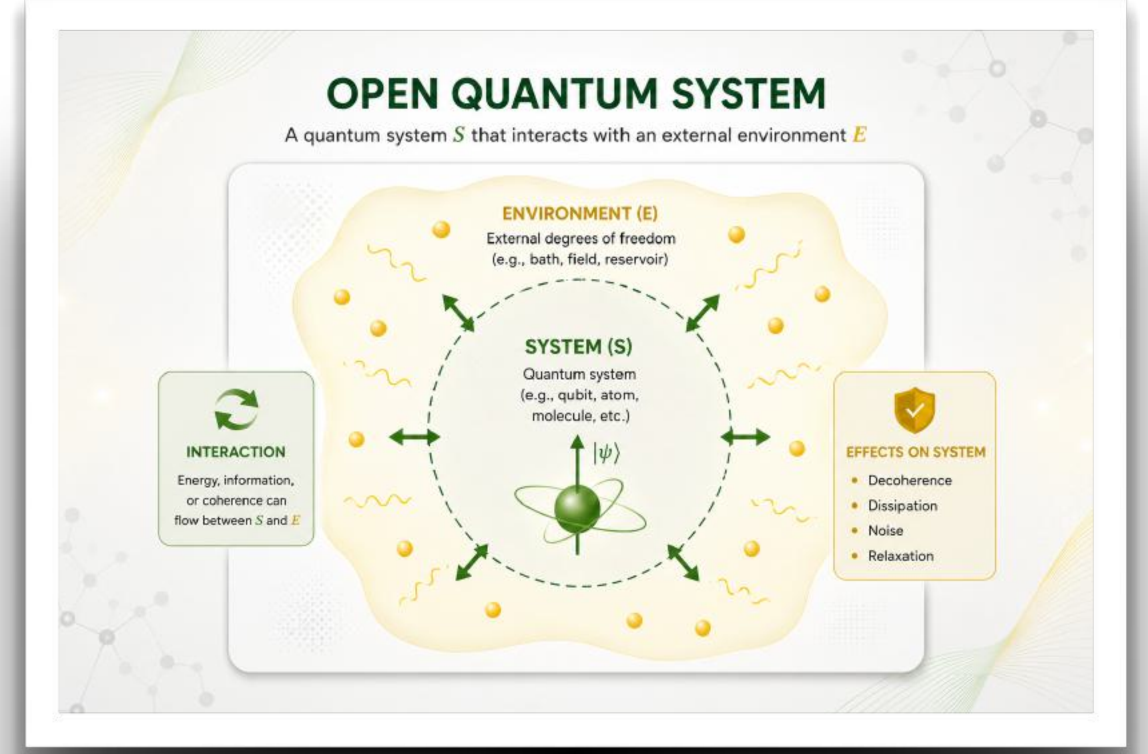
Probabilities for oscillations change depending on the values for Γ_{ij} and choice of power n

$$\Gamma_{ij}(E) = \gamma_{ij}(E_0)\left(\frac{E}{E_0}\right)^n$$

More precise questions: What would be Γ_{ij} from an QG perspective?

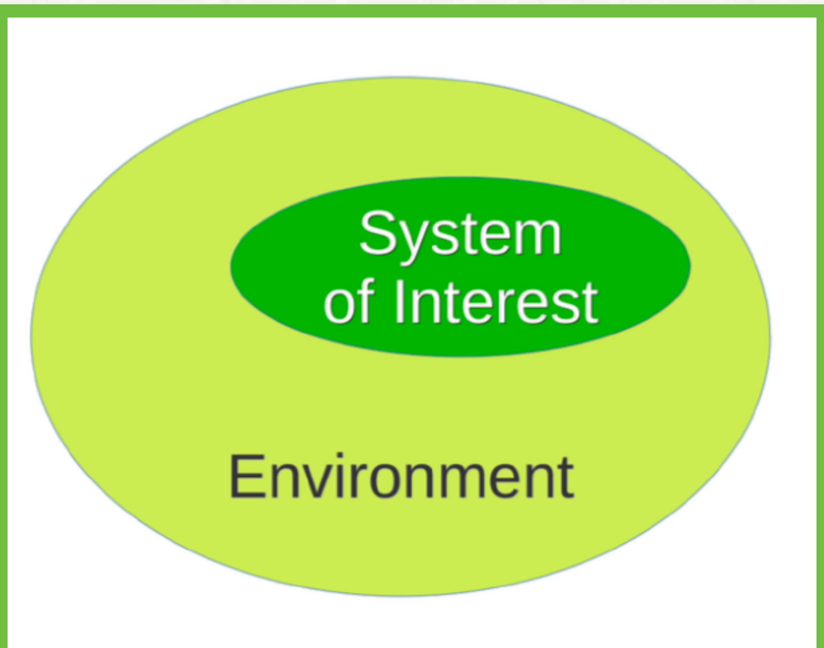
How to address this question from the theoretical microscopic perspective?

III. Brief Review on Open Quantum Models



III. Open Quantum Systems

Aim: Consider gravitationally induced decoherence



System S + environment ε : $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_\varepsilon$

Dynamics: $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_S + \mathrm{H}_\varepsilon + \mathrm{H}_{\mathrm{int}}$

$$\mathrm{H}_{\mathrm{int}}(\mathrm{t}) = \sum_\alpha \mathrm{S}_\alpha(\mathrm{t}) \otimes \mathrm{E}_\alpha(\mathrm{t})$$

Total Dynamics:

$$\partial_t \rho = \frac{1}{\mathrm{i}\hbar}[\mathrm{H}_{\mathrm{tot}}, \rho]$$

Gravitational environment interesting because all matter couples to gravity

III. Open Quantum Systems

Total dynamics is usually to complicated to solve

Microscopic model: $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_S + \mathrm{H}_\varepsilon + \mathrm{H}_{\mathrm{int}}$ $\quad \mathrm{H}_{\mathrm{int}}(\mathrm{t}) = \sum_\alpha g_\alpha \mathrm{S}_\alpha(\mathrm{t}) \otimes \mathrm{E}_\alpha(\mathrm{t})$

Aim: Effective dynamics for S: master equation

$$\partial_t \rho_S = \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{tr}_\varepsilon \left(\mathrm{U}_{\mathrm{tot}}(\mathrm{t}) \rho(0) \mathrm{U}^\dagger_{\mathrm{tot}}(\mathrm{t}) \right) = \frac{1}{\mathrm{i}\hbar} \mathrm{tr}_\varepsilon \left([\mathrm{H}_{\mathrm{tot}}, \rho] \right)$$

Master equation: encodes interaction with the environment

$$\frac{\partial \rho_S(t)}{\partial t} = -\frac{i}{\hbar} \left[H_S(t) + H_{add}(t), \rho_S(t) \right] + \mathcal{D}\left[\rho_S(t) \right]$$

contribution due to interaction with environment

Lindblad equation:

$$\frac{\partial}{\partial t} \rho_S(t) = \frac{1}{i\hbar} \left[H_S + H_{\mathrm{LS}}, \rho_S(t) \right] + \sum_k \gamma_k \left(L_k \rho_S(t) L_k^\dagger - \frac{1}{2} \left\{ L_k^\dagger L_k, \rho_S(t) \right\} \right)$$

III. What characterises a microscopic model?

Lindblad equation: Lindblad operators L_k+ environmental correlation fctns γ_k

$$\frac{\partial}{\partial t}\rho_S(t) = \frac{1}{i\hbar}\left[H_S + H_{\mathrm{LS}}, \rho_S(t)\right] + \sum_k \gamma_k \left(L_k \rho_S(t) L_k^\dagger - \frac{1}{2}\left\{L_k^\dagger L_k, \rho_S(t)\right\}\right)$$

For given H_{S} model characterised by choice of L_k, γ_k ← QG

Lindblad operators: system's operators in $\mathrm{H}_{\mathrm{int}}(\mathrm{t}) = \sum_j g_j \mathrm{S}_j(\mathrm{t}) \otimes \mathrm{E}_j(\mathrm{t})$

Environmental correlation fctns: env. operators in $\mathrm{H}_{\mathrm{int}}$, env. state ρ_ε
+ spectral density, renormalisation and approximations: Markov, rotating wave

QM vs QFT models: in QM models usually more choices of the model needs to made than in QFT

Open QFT model + one-particle sector: bridge to phenom models in QM

Strategy: increase complexity of the models step by step

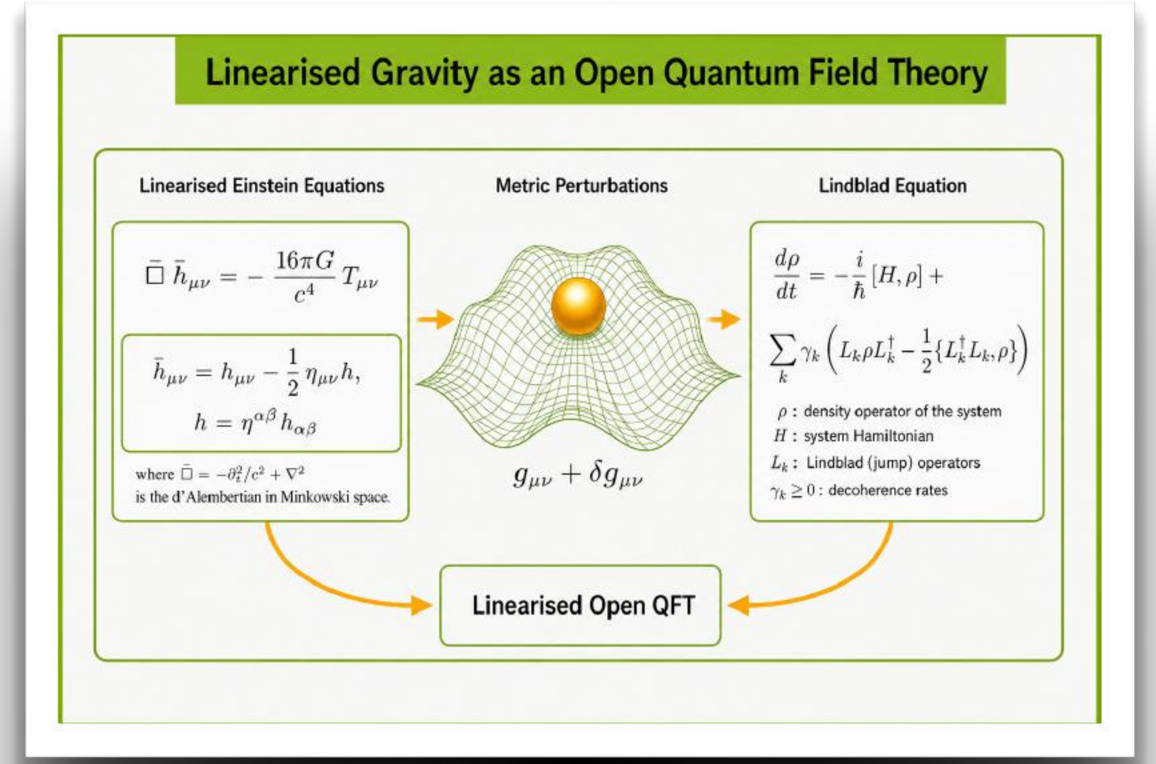
IV. Linearised Gravity as an Open QFT Model

[Max Fahn, K.G., Roman Kemper, Michael Kobler '22, '24+'26]

Ashtekar-Barbero variables
Relational formalism

[[Anastopoulos, Hu 2013]; [Blencowe 2013]; [Oniga, Wang 2016];
[Lagouvardos, Anastopoulos 2021]; [Asprea 2021]]

ADM variables, gauge fixing



IV. Microscopic models from GR

Idea: Consider (general) relativistic action as a starting point

Lagrangian formulation: $g_{\mu\nu}, \Phi^J$ $\qquad$ $G_{\mu\nu} = \kappa T_{\mu\nu}$

Geometry & matter

Differences & challenges compared to open QM models

A. GR is a (fully) constrained system, involves gauge dof

$\longrightarrow$ quantisation of constrained systems more ambiguities

Hamiltonian formulation of weakly coupled gravity, linearised gravity

$\longrightarrow$ Post-Minkowski approximation, open QFT models, Ashtekar-Barbero variables

B. Even at classical level physical observables (gauge-inv. quantities) hard to construct

$\longrightarrow$ Dirac observables with respect to physical dynamical reference frames

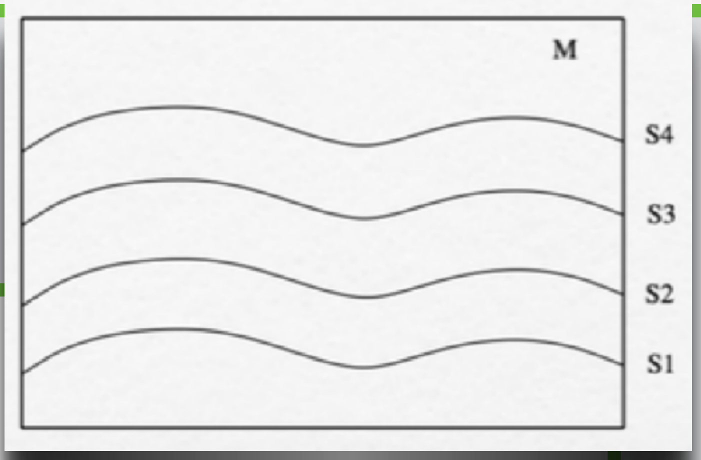
C. Choice of quantisation (representation) & Master equation

$\longrightarrow$ Here still Fock quantisation

D. Applications: one particle master equation + bridge to phenom. models

IV: Hamiltonian formulation of GR

GR in Ashtekar-Barbero variables: [Ashtekar '86, Barbero '94]
3+1 split of spacetime [ADM '59]
Causality: globally hyperbolic spacetimes



Kinematics: $(A_a^j, E_j^a) \quad \{A_a^j(x), E_k^b(y)\} = \kappa \delta_k^j \delta_a^b \delta^3(x,y)$
(Vacuum case here)

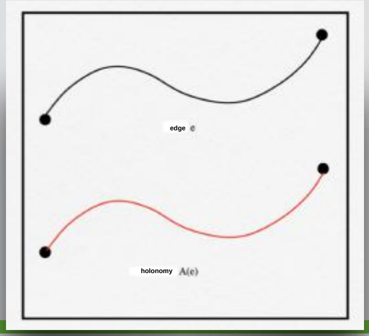
Constraints: $C(A,E) = 0, C_a(A,E) = 0, G_j(A,E) = 0$ additional SU(2) Gauß constraint

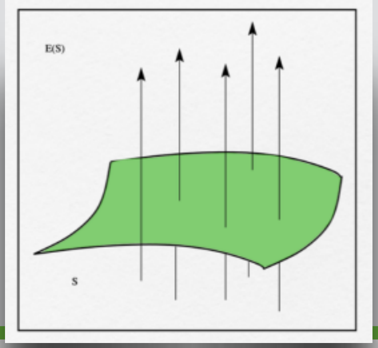
Dynamics: $\dot{A}_a^j = \{A_a^j, \mathrm{H}_{\mathrm{can}}\} \quad \dot{E}_j^a = \{E_j^a, \mathrm{H}_{\mathrm{can}}\}$

Canonical Hamiltonian $\mathrm{H}_{\mathrm{can}} = \frac{1}{\kappa}\int_\sigma d^3x \left(NC + N^a C_a + \Lambda^j G_j\right)$

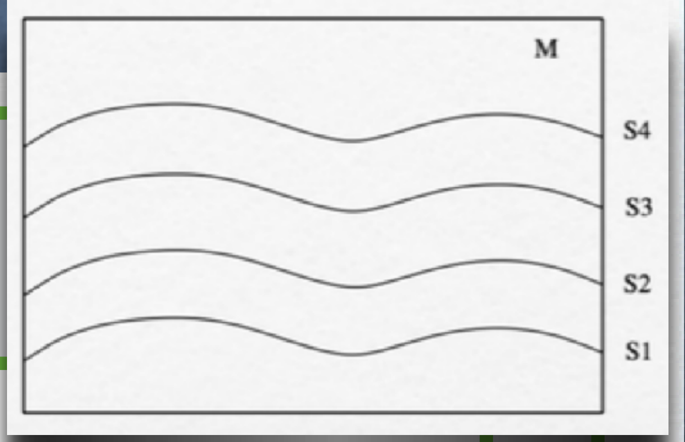
Holonomies & Fluxes: $h_e(A), E(S)$

→ Holonomy-flux algebra as classical starting point





IV.A. Hamiltonian Form: matter coupled to lin. gravity



3+1 split of spacetime [ADM '59]

GR in Ashtekar-Barbero variables+matter: [Ashtekar '86, Barbero '94]

Elementary variables $\left(\delta A_a^j, \delta E_j^a\right), (\phi, \pi) / (A_a, E^a)$

Kinematics: $\{\delta A_a^j(x), \delta E_k^b(y)\} = \frac{1}{\kappa} \delta_k^j \delta_a^b \delta^3(x,y)$

$\{\phi(x), \pi(y)\} = \delta^3(x,y)$ or $\{A_a(x), E^b(y)\} = \delta_a^b \delta^3(x,y)$

Dynamics: Linearised Constraints: $\delta C = 0, \delta C_a = 0, \delta G_j = 0$ additional SU(2) Gauß constraint

Canonical Hamiltonian

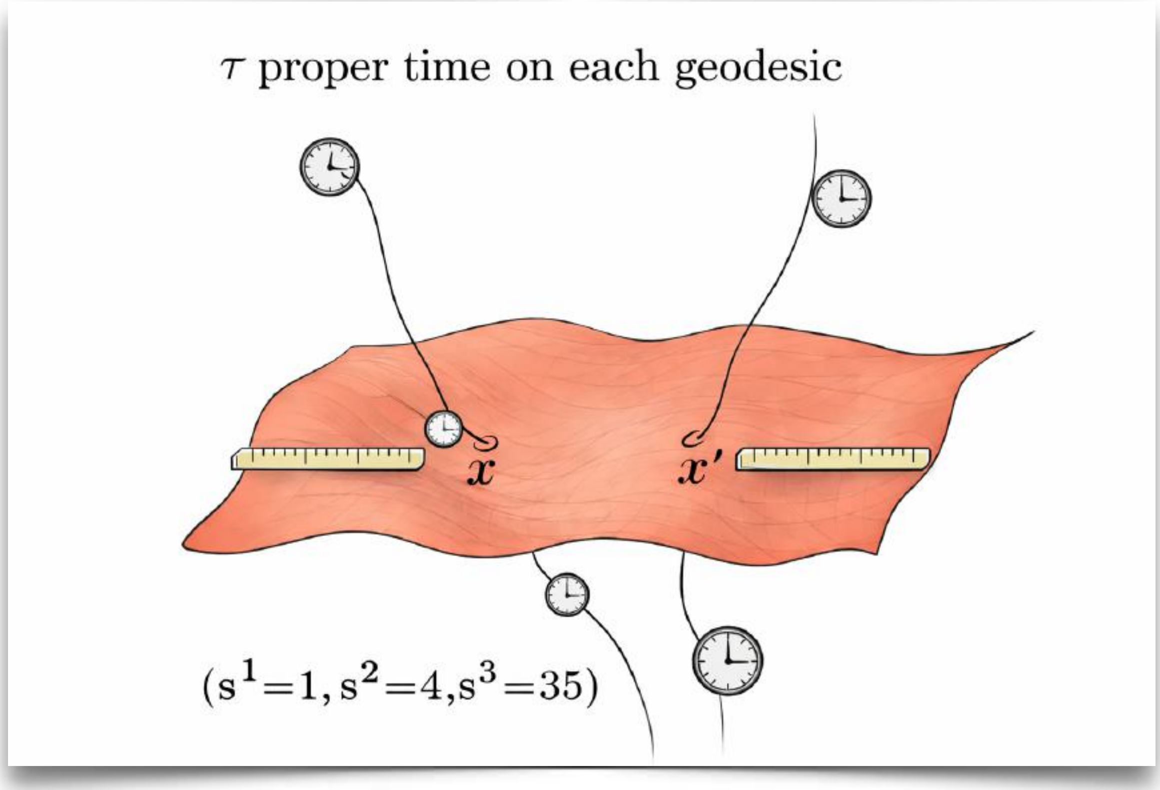
$$\delta \mathrm{H}_{\mathrm{can}} = \frac{1}{\kappa} \int_\sigma d^3x \big(\epsilon_{\mathrm{matter}}(\eta) + \delta N \delta C + \delta N^a \delta C_a + \delta \Lambda^j \delta G_j + \delta \mathcal{H}_I + \delta^2 C^{\mathrm{geo}}\big)$$

Open QFT model: interaction given $\frac{\kappa}{2} \int_M d^4x \delta h^{\mu\nu} T_{\mu\nu}^{\mathrm{matter}}[\eta]$ $\quad \delta h^{\mu\nu}(\delta E, \delta N, \delta N^a)$

Need true Hamiltonian system:
reduced quantisation, geometrical clocks, classical dynamical reference frame

IV.B. Relational Formalism in the Context of General Relativity

[Bergman, Komar, Ishan, Kuchar, Rovelli]



Introduce physical dynamical reference frames

IV.B: Gauge invariant formalism

Relational formalism for general relativity

[Rovelli '90, Vytheeswaran '97, Dittrich '04, Thiemann'04, Pons et al., Giddins, Marollf, Höhn,..]

Idea: Choose reference frame (Q^I, P_I) (T^I, P_{T^I})

Need: One reference field for each constraint

$$T^I \quad \text{s.t.} \quad \{T^I(x), \tilde{C}_J(y)\} \approx \delta^I_J \delta^{(3)}(x,y) \quad G_I = T^I - \tau^I \approx 0$$

Dirac observables for physical dof

(relational quantities) canon. version of dressed observables [Giddings `16]

$$\mathcal{O}_f\left(\tau^I\right) = f + \left(\tau^I - T^I\right)\left\{f, \widetilde{C}_I\right\} + \frac{1}{2!}\left(\tau^I - T^I\right)\left(\tau^J - T^J\right)\left\{\left\{f, \widetilde{C}_I\right\}, \widetilde{C}_J\right\} + \cdots$$

$$\mathcal{O}_f\left(\tau^I\right) = \left[\exp\left(\xi^I\left\{., \widetilde{C}_I\right\}\right) \cdot f\right]\Big|_{\xi^I := \tau^I - T^I}$$

dynamics in relational form

$$\frac{d}{d\tau^0}\mathcal{O}_f\left(\tau^0\right) = \left\{\mathcal{O}_f\left(\tau^0\right), \mathrm{H}_{\text{phys}}\right\}$$

phys. Hamiltonian

Formulation of GR as an unconstrained theory with true Hamiltonian

GI dynamics valid in any gauge, interpretation of observables

IV.B: Which dynamical reference system to choose?

- Main aim: Complete the quantisation programme
- Interesting class of dynamical reference system: Kuchar decomposition
- Canonical trafo s.t. new elementary variables include Dirac obs. & constraints

[Kuchar, Torre `91] [Rovelli, Smolin `93[Brown, Kuchar `95] [Kuchar, Romano`95]
[Kuchar, Romano`95] [Bicak, Kuchar `97]

$$(Q^I, P_I)(T^I, P_{T^I})$$

$$\{Q^I(t,x), P_J(t,y)\} = \delta^{(3)}(x-y)\delta^I_J$$

Poisson bracket

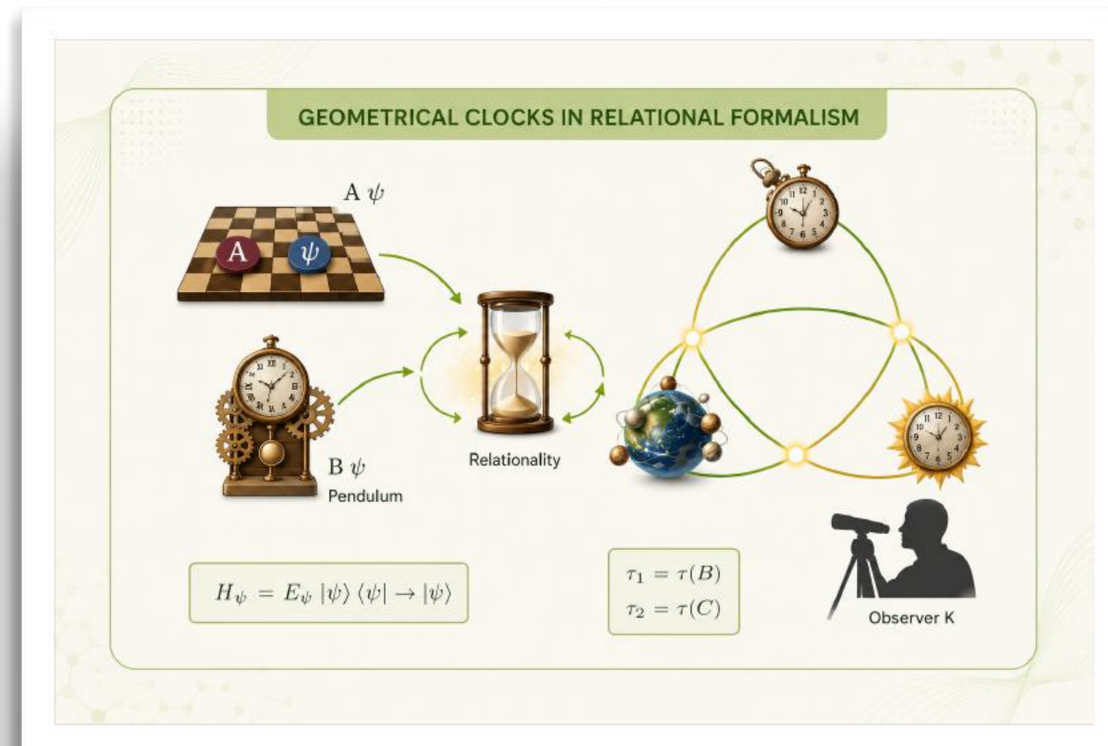
$$\left(\mathcal{O}_{Q^I}, \mathcal{O}_{P_I}\right)\left(T^I, C_I\right)$$

$$\{\mathcal{O}_{Q^I}(\tau,\sigma), \mathcal{O}_{P_J}(\tau,\sigma')\} = \mathcal{O}_{\{Q^J,P_J\}^*}(\tau,\sigma,\sigma')$$

Dirac bracket ⟵ Dynamical RF

- QT: Understand quantum dressing as similar algebra deformation [Freidel, Kirklin '26]
- Advantages: We can use same representation as for kinematical algebra [K.G., Vetter '19]
- What can go wrong in gen.: No representation, dynamics cannot be quantised
- Choice of dynamical reference frame is linked to how complicated quantisation step is
- Full LQG with matter reference frames: [K.G., Thiemann `10] [Husain, Pawlowski `11] [Domagała, K.G., Kaminski, Lewandowski '10] [K.G., Thiemann `12] [K.G., Vetter `19]

IV.B. Dynamical Reference Frames: Geometrical Clocks+ Photon Clock



VI.B: Geometrical dynamical reference frame

Here we use relational formalism in a perturbative scheme

Linearised constraints: $\delta\mathcal{C}_I := (\delta C, \delta C_a, \delta G_j) \quad \delta A_a^i, \delta E_i^a$ Photon: $G^{\mathrm{U}(1)}$

Choose geometrical clocks such that (use dual observable map)

$$\{T^I(x), \mathcal{C}'_J(y)\} = \{\delta T^I(x), \delta\mathcal{C}'_J(y) + \delta^2\mathcal{C}'_J(y)\} = \frac{1}{\kappa}\delta^I_J\delta^{(3)}(x,y) + O\left(\delta^2, \kappa^2\right)$$

ADM-like clocks extended to Ashtekar-Barbero case and matter [ADM '60, Dittrich '06]

[Fahn, K.G., Kemper, Kobler '22+ '26]

$$\delta T(\vec{x},t) := -\frac{1}{\kappa\beta}\left[\frac{1}{2}\delta^i_c\epsilon_a{}^{cb}\partial_b\left(\delta E^a{}_i * G^\Delta\right)(\vec{x},t) + \delta^a_i\left(\delta A_a{}^i * G^\Delta\right)(\vec{x},t)\right],$$

$$\delta\Xi^i(\vec{x},t) := \frac{2}{\kappa}\partial^a\left(\delta A_a{}^i * G^\Delta\right)(\vec{x},t),$$

$$\begin{aligned}\delta T^a(\vec{x},t) :=& \frac{2}{\kappa}\left(\delta^a_b\delta^i_c\partial^c - \frac{1}{2}\delta^i_b\partial^a + \delta^{ac}\delta^i_c\partial_b\right)\left(\delta E^b_i * G^\Delta\right)(\vec{x},t)\\ &+\frac{4\beta}{\kappa^2}\left[\frac{1}{2}\delta^i_b\partial^a\partial^b\left(\delta G_i * G^{\Delta\Delta}\right)(\vec{x},t) - \delta^{ab}\delta^i_b\left(\delta G_i * G^\Delta\right)(\vec{x},t)\right] + \frac{1}{\kappa^2}\partial^a\left[(\delta C - \kappa\epsilon) * G^{\Delta\Delta}\right](\vec{x},t)\end{aligned}$$

Photon: $T^{\mathrm{U}(1)}(\vec{x},t) := \partial^a\left(A_a * G^\Delta\right)(\vec{x},t)$

This allows to construct the corresponding physical Hamiltonian describing the total dynamics of system and environment

[K.G, Kabel, Wieland '24 relation to covariant phase space reference frames and harmonic gauge]

VI.B: Observable map and dual map

Usual observable map (omitted integrals here)

$$\mathcal{O}_{f,\{T\}}\left(\tau^I\right) = \left[\exp\left(\xi^I\left\{C_I,\cdot\right\}\right)\cdot f\right]\Big|_{\xi^I:=T^I-\tau^I}$$
$$= f + \left(T^I-\tau^I\right)\left\{C_I,f\right\} + \frac{1}{2!}\left(T^I-\tau^I\right)\left(T^J-\tau^J\right)\left\{C_J,\left\{C_I,f\right\}\right\} + \cdots$$

Dual map: role of constraints and clocks are interchanged [Fahn, K.G. Kobler '22]

Useful for constructing suitable reference frame, weak abelianisation of constraints
combination of both maps yields directly STT dof as physical dof in vacuum case

In non-vacuum case was useful to understand self-interaction term and simplicity of the algebra of the Dirac observables [Fahn, K.G. Kemper '26]

Either self-energy in the phys. Hamiltonian or more complicated observable algebra

Quantisation: more convenient to use first option

Relational open quantum systems: open QFT with respect to some reference frame,

observer dependent also the interaction to the environment

IV.C. Choice of Quantisation and Master Equation

IV.C. Gravitationally induced decoherence

Microscopic derivation: First steps in a given model

Assumptions of the model [Fahn, K.G.,Kobler '22] [Fahn, K.G., Kemper '26]

[[Nakajima 1958]; [Zwanzig 1960]; [Shibata, Takahashi, Hashitsume 1977]; [Chaturvedi, Shibata 1979]]

Starting point: Time-ConvolutionLess (TCL) equation truncated at second oder

Assume thermal state for the gravitational environment

As first step: Fock quantisation of physical Hamiltonian and interaction

Scalar field and photon analogue form of master equation $j_r^b(\vec{k},\vec{l})$ different

system operators

Resulting master equation

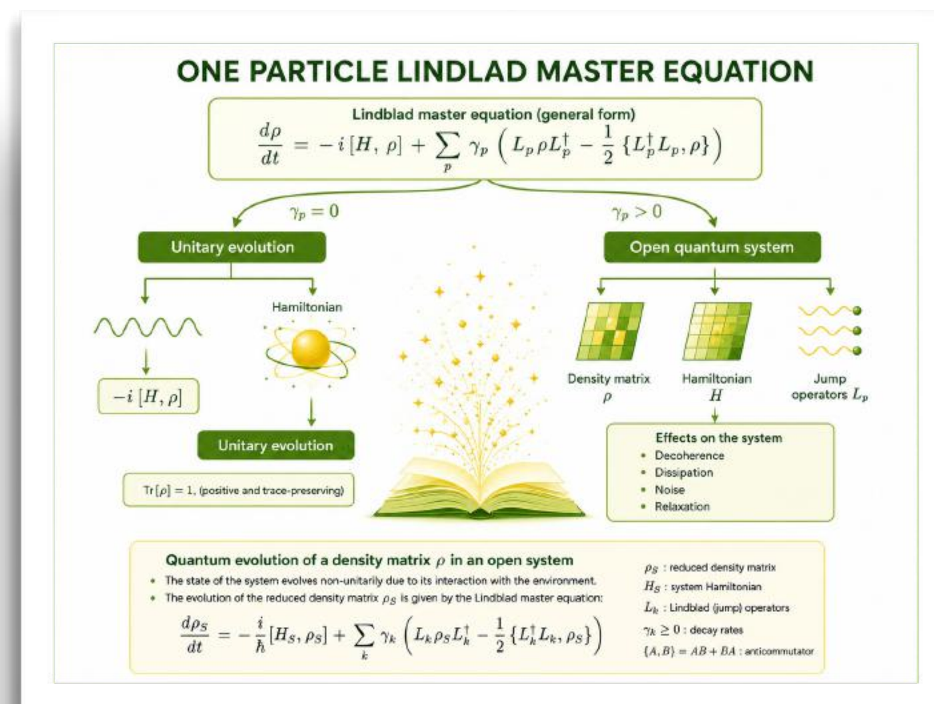
$$\frac{\partial}{\partial t}\rho_S(t) = -i\left[H_S + \kappa U + \kappa H_{LS}, \rho_S(t)\right] + \mathcal{D}_{\text{first}}\ [\rho_S]$$

Still involves divergencies

$$\mathcal{D}_{\text{first}}\ [\rho_S] := \frac{\kappa}{2}\int \frac{d^3k d^3p d^3l}{(2\pi)^{\frac{6}{2}}} \sum_{r;ab} R_{ab}(\vec{p},\vec{l};\vec{k},t)\left(j_r^b(\vec{k},\vec{l})\rho_S(t) j_r^a(\vec{k},\vec{p})^\dagger - \frac{1}{2}\left\{j_r^a(\vec{k},\vec{p})^\dagger j_r^b(\vec{k},\vec{l}), \rho_S(t)\right\}\right)^{\square}$$

not of standard Lindblad type, [confirm results of [Oniga, Wang 2016], [Anastopoulos, Hu 2013] and [Lagouvardos, Anastopoulos 2021]]

IV.D. Applications: One particle master equation



IV. One particle master equation

One particle equation and interplay with renormalisation

[Max Fahn, K.G. '24] [Burrage, Kähding,,Millington, Minar '19]

Projection: one particle density matrix

$$\rho_1(t) = \int_{\mathbb{R}^3} d^3u \int_{\mathbb{R}^3} d^3v \rho(\vec{u},\vec{v},t) a_u^\dagger |0\rangle\langle 0| a_v$$

One particle master equation

$$\begin{aligned}
\frac{\partial}{\partial t}\rho(\vec{u},\vec{v},t) = & -i\rho(\vec{u},\vec{v},t)\left(\omega_u - \omega_v\right) \\
& - \frac{\kappa}{2}\int \frac{d^3k}{(2\pi)^3}\Bigg\{\frac{P_u(\vec{k})}{\omega_{u-k}\omega_u}\left[C(\vec{u},\vec{k},t) + \delta_P C_P(\vec{u},\vec{k},t)\right] \\
& + \frac{P_v(\vec{k})}{\omega_{v-k}\omega_v}\left[C^*(\vec{v},\vec{k},t) + \delta_P C_P^*(\vec{v},\vec{k},t)\right]\Bigg\}\rho(\vec{u},\vec{v},t) \\
& + \frac{\kappa}{2}\int \frac{d^3k}{(2\pi)^3}\frac{P^{ij\,\ln}(\vec{k})u_i u_j v_l v_n}{\sqrt{\omega_{u+k}\omega_u\omega_{v+k}\omega_v}}\left\{C(\vec{u}+\vec{k},\vec{k},t) + C^*(\vec{v}+\vec{k},\vec{k},t)\right\}\rho(\vec{u}+\vec{k},\vec{v}+\vec{k},t)
\end{aligned}$$

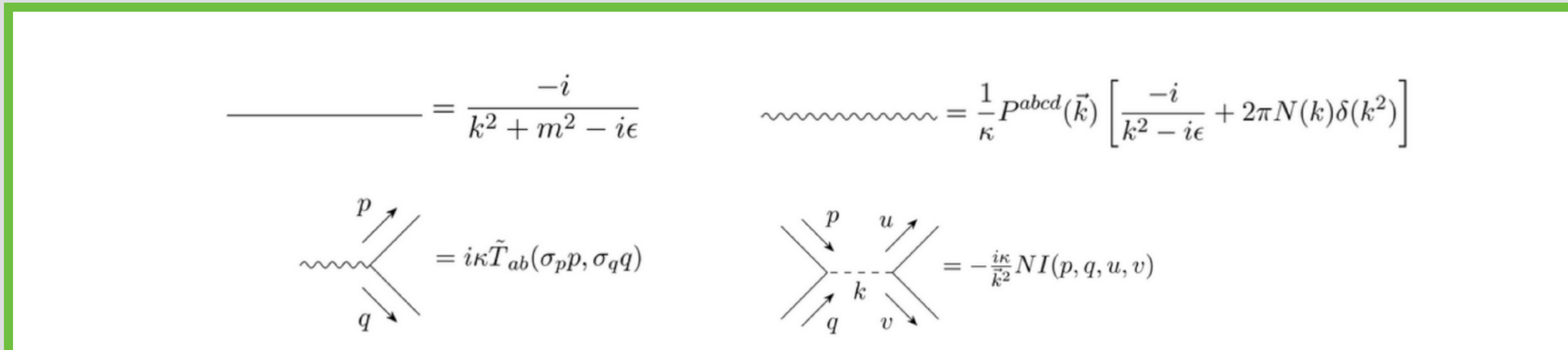
Coefficients: $C(\vec{u},\vec{k},t) = \int_0^{t-t_0} \frac{d\tau}{\Omega_k}\left\{[N(k)+1]e^{-i(\Omega_k+\omega_{u-k}-\omega_u)\tau} + N(k)e^{i(\Omega_k-\omega_{u-k}+\omega_u)\tau}\right\}$

$\delta_P = 0$ one-particle projection $\delta_P = 1$ extended one-particle projection

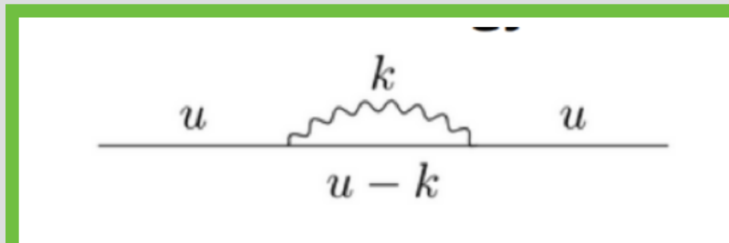
IV. One particle master equation

Strategy: perform renormalisation before any approximations are applied

Involved Feynman diagrams in 2nd order TCL equation [Fahn, K.G. '24]



UV divergent terms: vacuum self-energy



Renormalised one particle master equation

Consider Markov & rotating wave approximations + ultra-relativistic limit: toy model

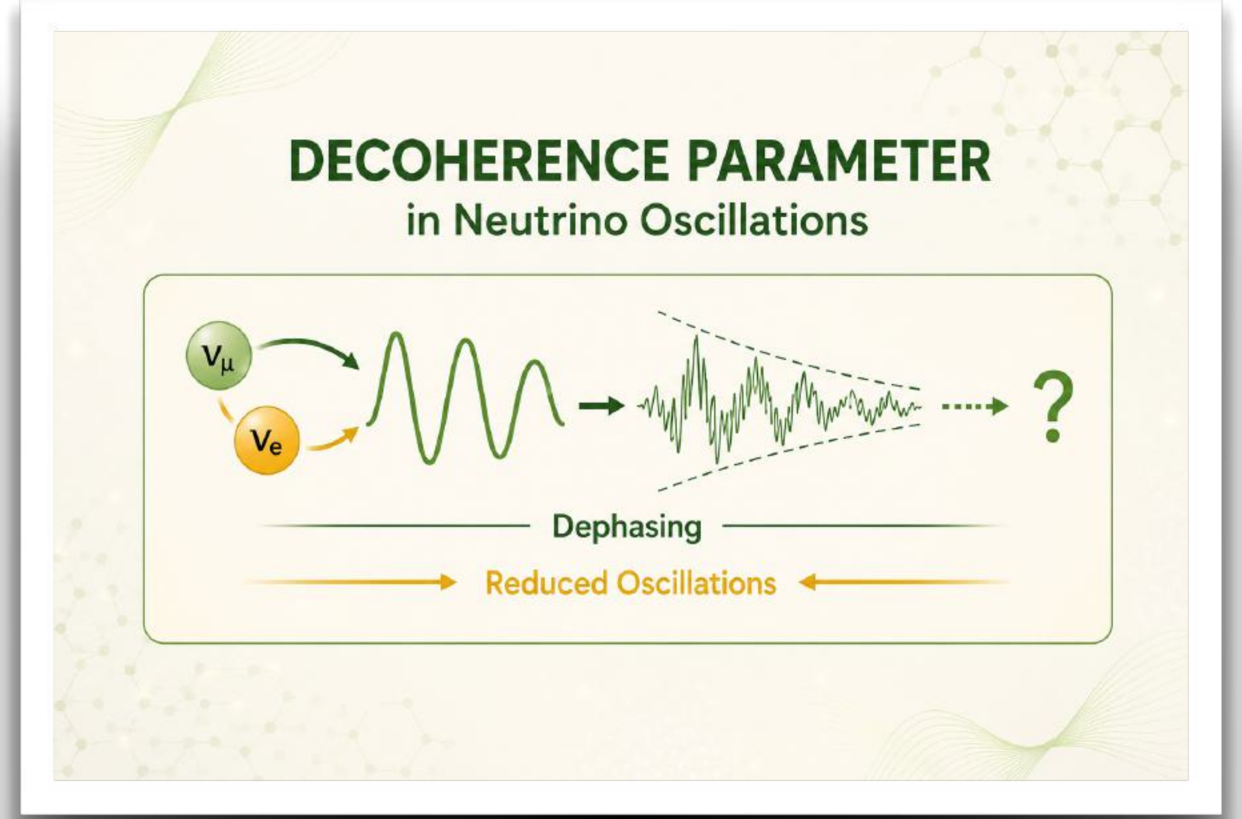
$$\frac{\partial}{\partial t}\hat{\rho}(t) = -i[\hat{H}, \hat{\rho}(t)] + \frac{\kappa k_B T}{6\pi}\left(\hat{H}\hat{\rho}(t)\hat{H} - \frac{1}{2}\left\{\hat{H}^2, \hat{\rho}(t)\right\}\right)$$

[Blencowe, Xu '22]

[Domi, Ebert, Fahn, K.G., Hennig, Katz, Kemper '24]

IV.D. Applications: Decoherence neutrino oscillations

Can we obtain some better physical intuition for decoherence parameters Γ_{ij} from a given microscopic model?



IV.D. Compare QFT, QM and pheno model for gravitationally induced decoherence

Microscopic models and their relations

[Blencowe, Xu1, Domi, Ebert, Fahn, K.-G., Hennig, Katz, Kemper '24]

QM microscopic inspired toy model

$$H = \underbrace{\left(H_S^{(0)} + H_S^{(C)}\right)}_{H_S} \otimes \mathbb{1}_E + \mathbb{1}_S \otimes \underbrace{\left[\frac{1}{2}\sum_{i=1}^{N}\left(\hat{p}_i^2 + \omega_i^2 q_i^2\right)\right]}_{H_\mathcal{E}} - \underbrace{H_S \otimes \sum_{i=1}^{N} g_i q_i}_{H_{\text{int}}}$$

$$\hat{\rho}_\mathcal{E} = \frac{1}{Z} e^{-\frac{1}{k_B T}\tilde{H}_\mathcal{E}}$$

$$L = H_S$$

Spectral density,
Markov,
Lamb shift
renormalisation

Lindblad equation: Solution: $\Gamma_{ij};\, \eta,\, T$

$$\frac{d\rho_{ij}}{dt} = -\frac{i}{\hbar}[\tilde{H}_i, \rho_{ij}] + \sum_k \gamma_k \left(L_k \rho_{ij} L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_{ij}\}\right)$$

ρ_{ij} : density matrix in energy eigenbasis

γ_k : decoherence parameters (k: channel/Lindblad operator)

η : coupling strength (via spectral density) T : temperature

[Fahn, K.-G., Kemper, Kobler '22+ '26]

Open QFT Model

Thermal state for grav. environment, Fock quantisation

1-particle projection+ renormalisation *[Fahn, K.-G. '25]*

ultra-rel. limit, Markov + RWA

Lindblad equation: Solution: $\Gamma_{ij};\, T$

$$\frac{d\rho_{ij}}{dt} = -\frac{i}{\hbar}[\tilde{H}_i, \rho_{ij}] + \sum_k \gamma_k \left(L_k \rho_{ij} L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_{ij}\}\right)$$

ρ_{ij} : density matrix in energy eigenbasis

γ_k : decoherence parameters (k: channel/Lindblad operator)

T : temperature G_N : Newton's constant c : speed of light

Compare now to phenom. neutrino models:

What kind of insights for decoherence parameters Γ_{ij}?

IV.D. Comparison with phenom. models

1. Pheno models and microscopic QM Models

Phenol models (PQD)

QM microscopic toy model

$$\tilde{\rho}_{ij} = \tilde{\rho}_{ij}(0) e^{-\frac{i}{\hbar}\left(\tilde{E}_i - \tilde{E}_j\right)t - \Gamma_{ij}(E)t}$$

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \frac{4\eta^2 k_B T}{\hbar^3}\left(\widetilde{H}_i - \widetilde{H}_j\right)^2 t}$$

$$\Gamma_{ij}(E) = \gamma_{ij} E^n \longleftarrow \frac{4\eta^2 k_B T}{\hbar^3}\left(\widetilde{H}_i - \widetilde{H}_j\right)^2 t \qquad \eta, T$$

For oscillations in vacuum exact match

$$\Gamma_{ij}(E) = \gamma_{ij} E^{-2} \quad \gamma_{ij} = \frac{\eta^2 c^8 k_B T}{\hbar^3}\left(\Delta m_{ij}^2\right)^2 \quad \text{and} \quad n = -2$$

For oscillations in matter: decoherence parameters include matter effects

[Carpio, Massoni, Gargo '18]

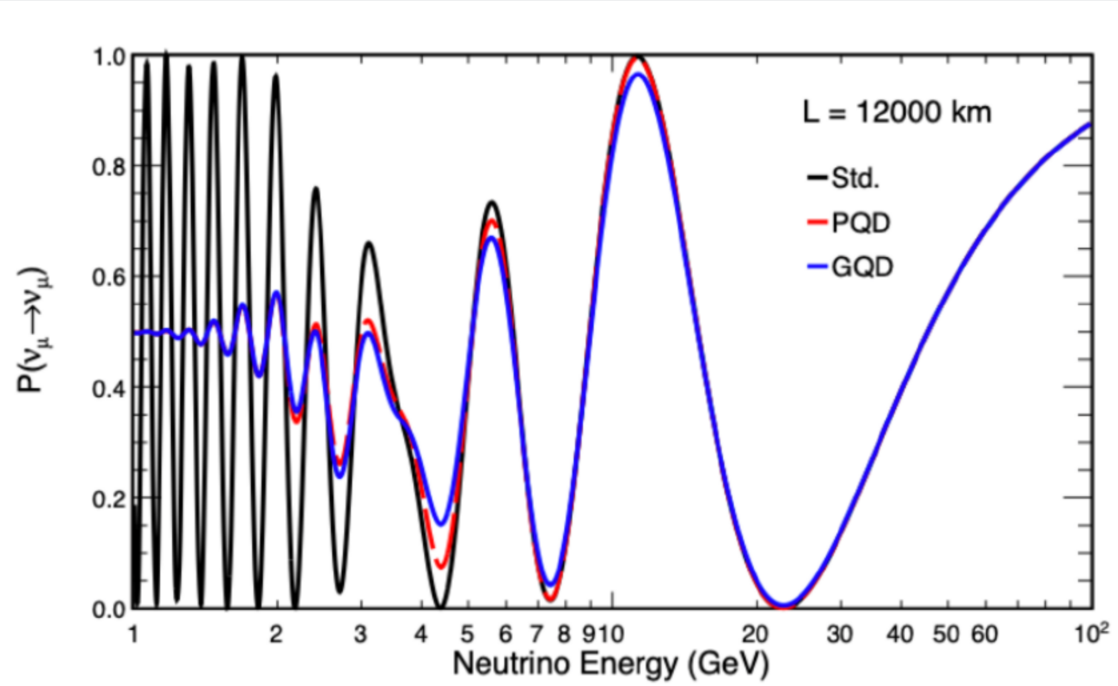
Insights: Energy dependence fixed & more general phenom. model required

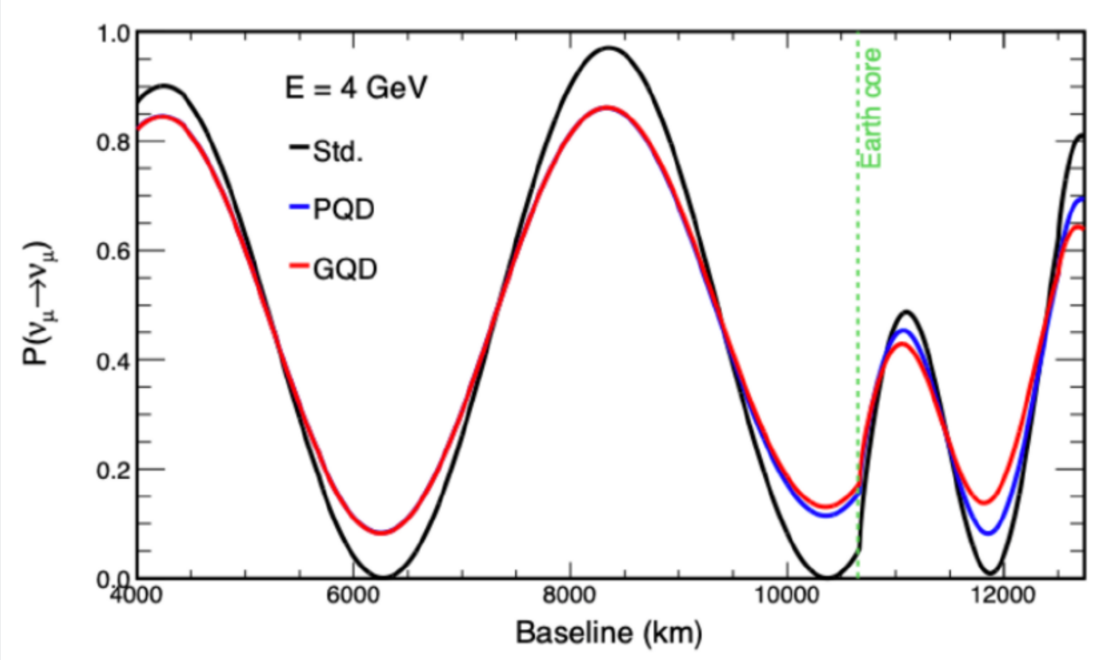
If T chosen, then experiments will constrain coupling η

IV. Comparison with phenom. models

Deviations in the probabilities for neutrino oscillations $\eta = 10^{-8}s, T = 0.9K$

thermal GW background





What can the QFT derived model say about coupling η ?

Comparing decoherence coefficients for same temperature: $\eta \sim 10^{-44} s$

Very tiny decoherence effects for chosen temperature and propagation length in these simple QFT derived decoherence models

Open question: is this the same for more general models?

Summary & Conclusions

Going back to the first question:

Whether we can constrain quantum gravity models non-trivial question

Here: considered most simple microscopic models from open QFT and QM

While coupling constant underdetermined in QM, QFT suggests very tiny one

However: new insights on assumptions+approximations for these models

Also extended methods to formulate relational open quantum systems
(reduced phase space quantisation, 1-particle projection,renormalisation)

Methods and results allow to bridge to phenomenological models

Generalising models: Open QFT and QM level: QM interaction with Weyl elements

[Ferreo, Fahn, K.G., Kemper '26]

Are there physical situations in which decoherence parameters can be larger?

Continue to collaborate with experimentalist on these questions

Theory: further investigate relativistic open QFT models beyond these applications

Nature's silence can still be informative

Thank you!

Aitäh!